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Classroom Teaching, Self-Exercise, Science Research and Engineer Computing

Earth Tide, Load Effect and Deformation Monitoring Computation

ETideLoad4.5 User Reference

- Analytically compatible geodetic and geodynamic algorithm package using the numerical standards unified and geophysical models coordinated
- Compatible with and improved the IERS conventions, some geodetic concepts clarified, all the algorithms derivated and verificated completely
- Uniform computation of solid tidal, load tidal, polar shift and mass centric variation effects on all-element geodetic variations in whole Earth space
- Analytical computation of surface load effects on all-element geodetic variations and collaborative monitoring of time-varying Earth gravity field
- Geodetic monitoring of the surface hydrological environment and ground stability variations and prediction of their spatio-temporal evolution

Chinese Academy of Surveying and Mapping

Chuanyin Zhang, October 2024, Beijing, China

Earth Tide, Load Effect and Deformation Monitoring Computation (ETideLoad4.5) is a large Windows program package for scientific computing of geophysical geodetic monitoring, Which constructs analytically compatible geodetic and geodynamic algorithm system using the numerical standards unified and geophysical models coordinated, and then uniformly computes various tidal and non-tidal effects on all-element geodetic variations outside the solid Earth. ETideLoad4.5 can be employed to approach accurately global and regional load deformation field from surface load observations such as atmosphere, sea level, soil water, lakes and glaciers, and monitor collaboratively the land water, temporal gravity field, geological environment and ground stability variations by deep fusing of multi-source heterogeneous geodetic and surface load observations.

ETideLoad4.5 includes the basic principles, main formulas and important methods of geodesy on the deforming Earth to improve higher education environment. Which could be employed to constrain and assimilate the deep fusion of multi-source heterogeneous monitoring variations strictly according to the principles of geodesy and solid geodynamics, so as to realize the collaborative monitoring of various heterogeneous geodesy, improve the science and technology of geodesy and consolidate the geodetic application.

ETideLoad4.5 is suitable for senior undergraduates, graduate students, scientific researchers, and engineering technicians in geodesy, geophysical, geoscience, geological environment, hydrodynamics, satellite dynamics, seismology and geodynamics, which can be employed in the classroom teaching, self-exercise, science research and engineer computing.

Key words: Geodesy, Geophysics, Tidal effects, Load effects, Collaborative Monitoring, Land water, Ground Stability, Geological Disasters.

http://www.zcyphygeodesy.com/en/
Chinese Academy of Surveying & Mapping
October 2024, Beijing, 100036, China
ZHANG Chuanyin, zhangchy@casm.ac.cn
WANG Wei (wangwei@casm.ac.cn)
JIANG Tao (jiangtao@casm.ac.cn)

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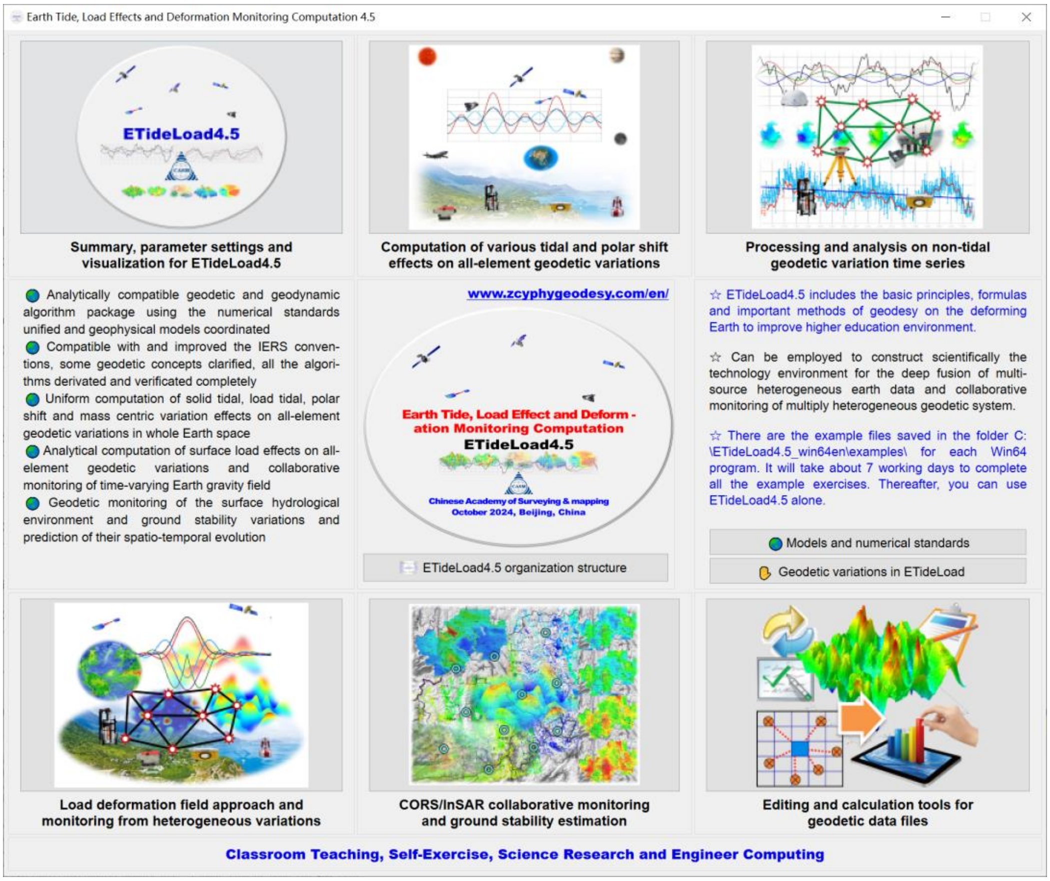
1 ETideLoad4.5's features, strengths, concepts, and usage

Earth Tide, Load Effect and Deformation Monitoring Computation (ETideLoad4.5) is a large Windows program package for scientific computing of geophysical geodetic monitoring. Which constructs analytically compatible geodetic and geodynamic algorithm system using the numerical standards unified and geophysical models coordinated, and then uniformly computes various tidal and non-tidal effects on all-element geodetic variations outside the solid Earth. ETideLoad4.5 can be employed to approach accurately global and regional load deformation field from surface load observations such as atmosphere, sea level, soil water, lakes and glaciers, and monitor collaboratively the land water, temporal gravity field, geological environment and ground stability variations by deep fusing of multi-source heterogeneous geodetic and surface load observations.

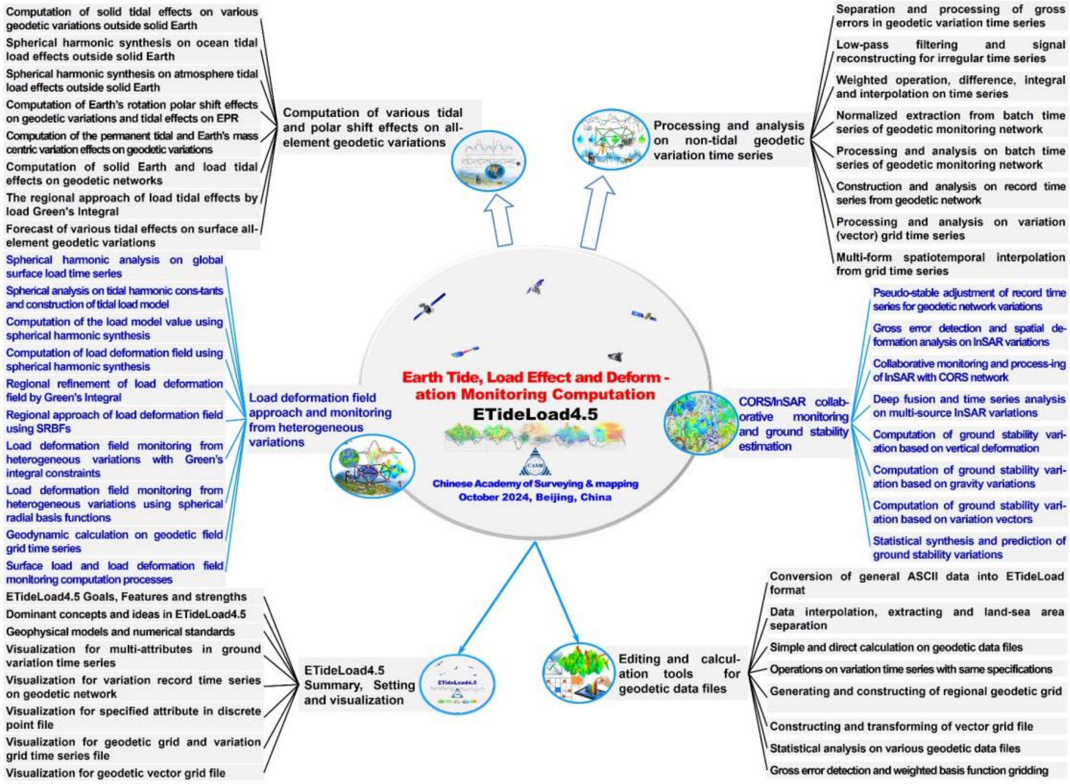


ETideLoad4.5 includes the basic principles, main formulas and important methods of geodesy on the deforming Earth to improve higher education environment. Which can be employed to constrain and assimilate the deep fusion of multi-source heterogeneous monitoring variations according to the principles of the geodesy and solid geodynamics, so as to realize the collaborative monitoring of various heterogeneous geodesy, improve the science and technology of geodesy, and consolidate the geodetic application.

1.1 ETideLoad4.5 structure of computation functions

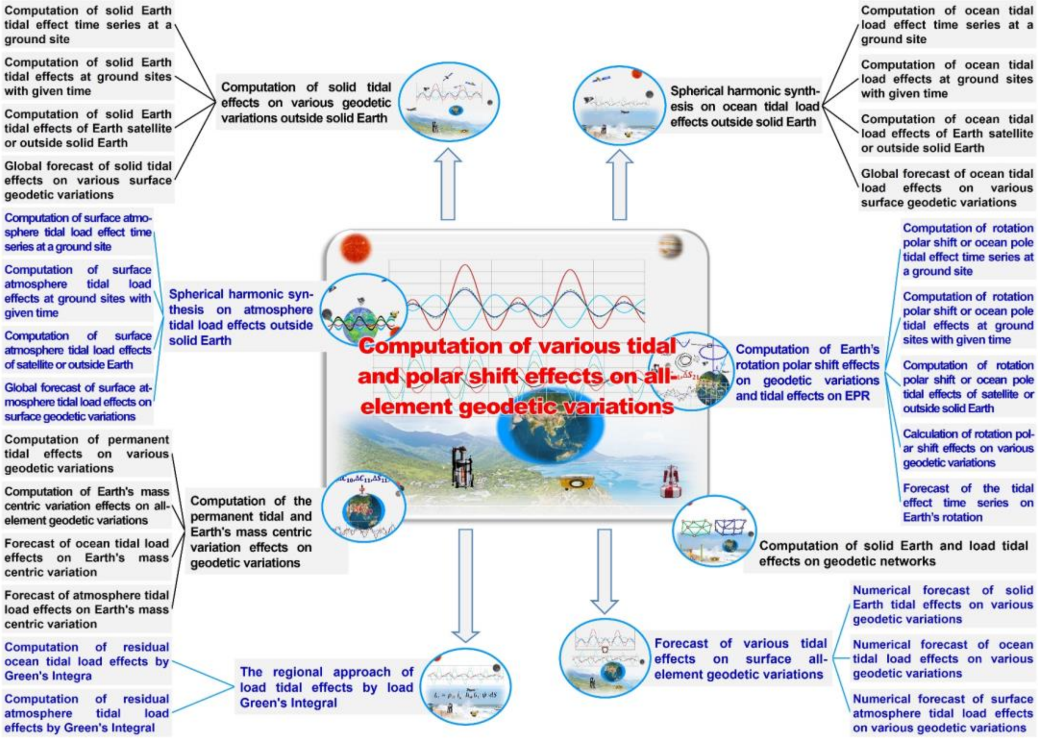
ETideLoad4.5 has five subsystems, which includes Computation of various tidal and polar shift effects on all-element geodetic variations, Processing and analysis on non-tidal geodetic variation time series, Load deformation field approach and monitoring from heterogeneous variations, CORS/InSAR collaborative monitoring and ground stability estimation as well as Editing, calculation and visualization tools for geodetic data files.

ETideLoad4.5 were developed by QT C++ (Visual C++) for the user interface, Intel Fortran (Fortran90, 132 Columns fixed format) for the core function modules and mathGL C++ for the geodetic data file visualization in the Visual Studio 2017 x64 integrated environment, which is composed of more than 50 win64 executable programs with nearly 600 function modules.



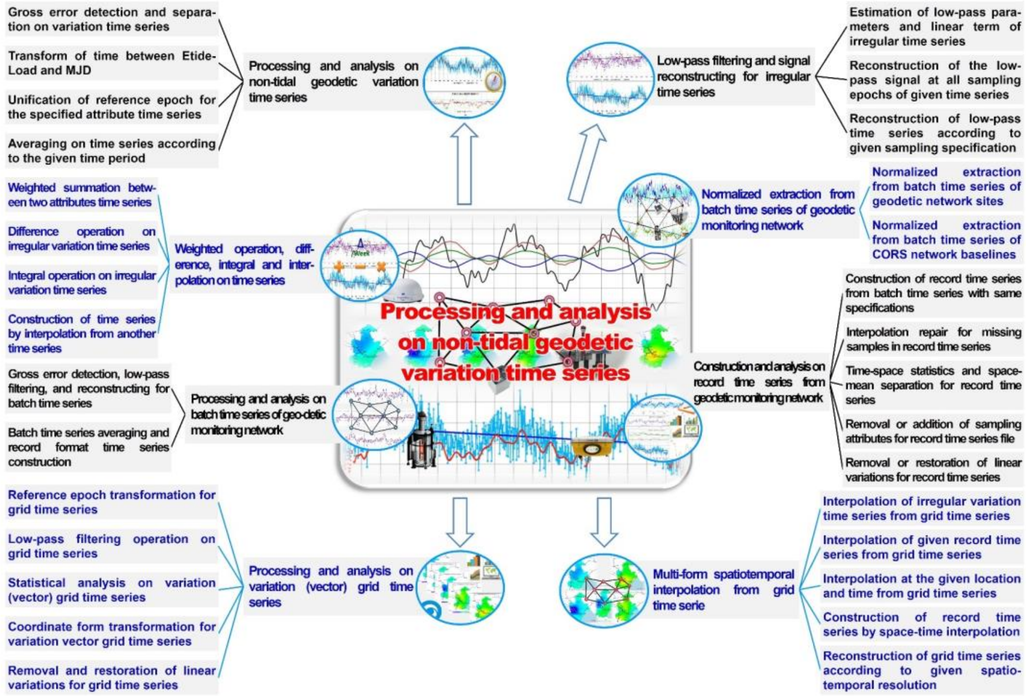
1.1.1 Computation of various tidal and polar shift effects on all-element geodetic variations

The solid Earth tidal, ocean tidal load, atmosphere tidal load, Earth's rotation polar shift, figure polar shift and mass centric variation effects on all-element geodetic variations on the ground and outside the solid Earth using the analytically compatible geodetic and geodynamic algorithm system with the numerical standards unified and geophysical models coordinated are uniformly computed. Which is the important basis and necessary condition for the collaborative monitoring of various geodetic technologies and deep fusion of multi-source heterogeneous Earth monitoring data.



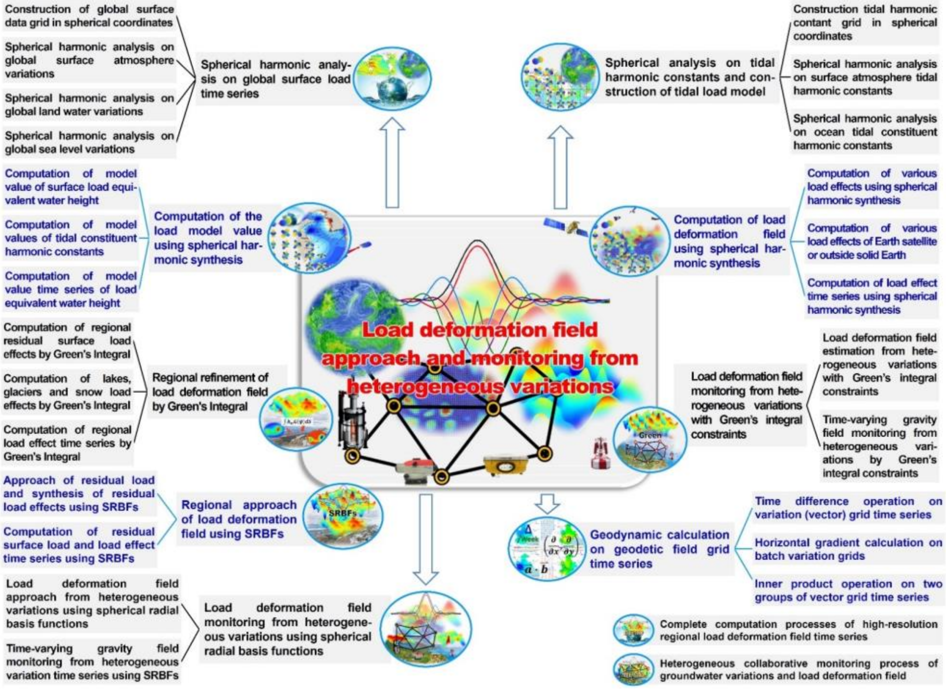
1.1.2 Processing and analysis on non-tidal geodetic variation time series

Based on the characteristics of non-tidal geodetic variations, the group of programs adopt the stable and reliable algorithms to uniformly process and analyze massive various geodetic variation time series.

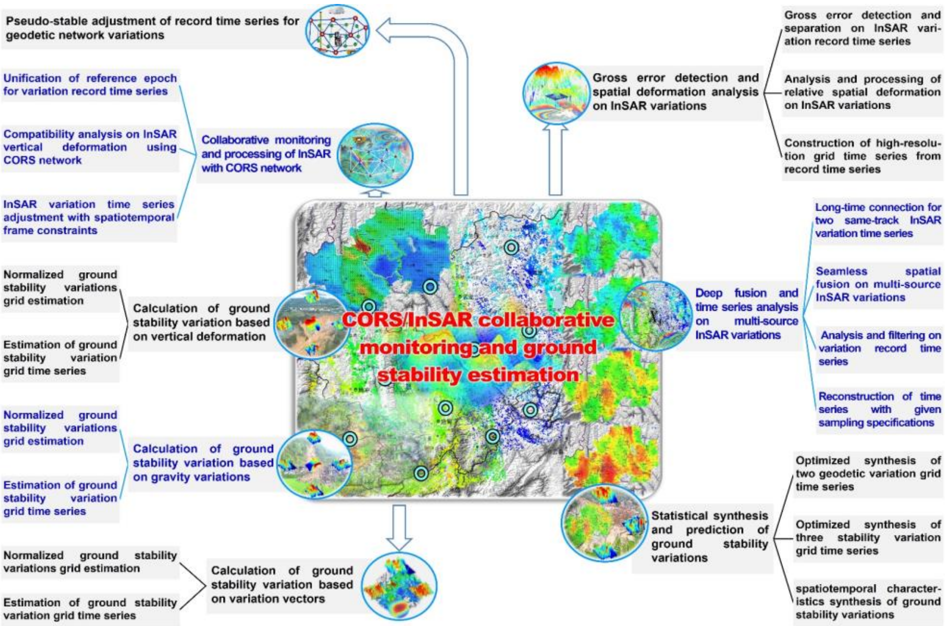


1.1.3 Load deformation field approach and monitoring from heterogeneous variations

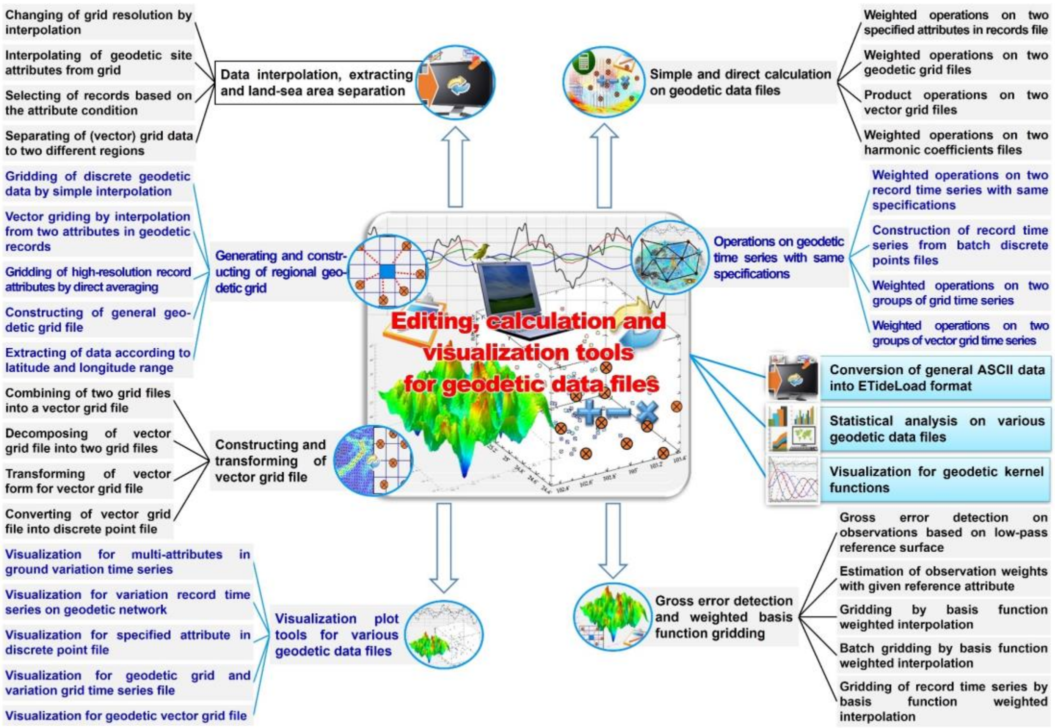
Compute and approach the global and regional non-tidal load effects on all-element geodetic variations, and then constrain and assimilate the deep fusion of multi-source heterogeneous geodetic data strictly according to the principles of the geodesy and Earth's deformation, so as to realize the collaborative monitoring of the land water variations and time-varying gravity field from various heterogeneous geodetic techniques.



1.1.4 CORS/InSAR collaborative monitoring and ground stability estimation



1.1.5 Editing, calculation and visualization tools for geodetic data files



1.2 Geodetic variations in ETideLoad4.5

1.2.1 Conventions of the geodetic variations

Geodetic variation in ETideLoad is defined as the difference between the geodetic element at the current epoch time and the mean of the elements over a period or the difference between the geodetic element at the current epoch time and that at a certain reference epoch time. The geodetic element may be a geodetic observation or parameter, and the geodetic variation refers to the difference in the geodetic element with time.

1.2.2 Type and unit of the geodetic variations

(1) Height anomaly or geoidal height variation in the unit of mm, ground gravity or gravity disturbance variation in the unit of μGal, and ground tilt or vertical deflection variation (vector) in the unit of mas namely 0.001".

(2) Ground horizontal displacement in the unit of mm, ground radial displacement namely ground ellipsoidal height variation in the unit of mm, and ground normal or orthometric height variation in the unit of mm.

(3) Gravity gradient variation in the unit of 10μE for global case, as well as in the unit of mE for regional case.

(4) External geopotential perturbation in the unit of $0.1m^2/s^2$, gravity perturbation in the unit of μGal, and gravity gradient perturbation in the unit of 10μE.

(5) Land equivalent water height variation (EWH) in the unit of cm, sea level variation in

unit of cm, ocean tidal height in unit of cm and atmosphere variation in unit of hPa (mbar).

1.2.3 The geodetic variation vectors

(1) Ground tilt or vertical deflection variation vector (SW). The first component points to the south direction,and the second component points to the west direction, which forms a right-handed rectangular coordinate system with the ground gravity direction. The coordinate system is a natural coordinate system.

(2) Ground horizontal displacement vector (EN). The first component points to the east direction and the second component points to the north direction, which forms a right-handed rectangular coordinate system with the ground radial displacement direction.

(3) Tangential gravity gradient variation vector (NW). The first component points to the north direction,and the second component points to the west direction, which forms a right-handed rectangular coordinate system with the radial gravity gradient variation direction.

(4) The harmonic constants of the tidal constituent. The first component is the in-phase amplitude for cos(argument) and the second component is the out-of-phase amplitude for sin(argument).

1.2.4 Expressions of the date and epoch

Time (date and epoch) are agreed to adopt Greenwich Time (zero time zone), which is expressed in modified Julian Date (MJD, in GPS time, and Julian Date 2000.0 = MJD 51544.5) or a long integer agreed by ETideLoad.

In most cases, the long integer time agreed by ETideLoad is employed. E.g., 20181224122642 represents 12:26:42 on December 24, 2018, 2018122412 represents 12: 0: 0 on December 24, 2018 and 20181224 represents 0: 0: 0 on December 24, 2018. But 201812, 2018 are not valid date or epoch. Here, the epoch is an instantaneous time.

1.3 Science goals and strengths of ETideLoad4.5

1.3.1 Scientific goals of ETideLoad4.5

(1) Using analytically compatible geodetic and geodynamic algorithm package with the numerical standards unified and geophysical models coordinated, computes various tidal and non-tidal effects on all-element geodetic variations and then approaches and refines accurately global or regional surface load deformation field.

(2) Strictly according to the principles of the geodesy and solid geodynamics, constrains and assimilates the deep fusion of multi-source heterogeneous geodetic and surface load observations to realize the collaborative monitoring of land water, temporal gravity field, geological environment and ground stability variations.

(3) Provides a set of scientific and practical geodetic geodynamic computation tools for the construction of heterogeneous spatiotemporal geodetic frames, deep fusion of multi-surce heterogeneous Earth observations, collaborative monitoring of various

geotechnologies, computation of solid Earth deformation, monitoring of surface hydrology environment and surveying of geological disasters.

1.3.2 Geodetic features and strengths

(1) Using the scientific uniform numerical standards and analytic compatible geophysical algorithms, computes accurately the Earth tidal, ocean tidal load, surface atmosphere tidal load, permanent tidal, rotation polar shift and Earth's mass centric variation effects on all-element geodetic variations, and forecast various tidal effects anytime and anywhere.

(2) Approaches global or regional load deformation field from surface load observations such as atmosphere, sea level, soil water, lakes, rivers, glaciers and snow, and then collaboratively monitores the land water and temporal gravity field by deep fusing of multi-source heterogeneous geodetic and surface load observations.

(3) Constructs regional uniform spatiotemporal monitoring datum frames with high robustness to fuse the CORS, InSAR and other geodetic variations. Proposes the criteria of the ground stability based on temporal geodetic field to realize quantitative monitoring of the ground stability spatiotemporal variations.

1.4 Dominant concepts and ideas integrated into ETideLoad4.5

1.4.1 Geodetic principles of the collaborative monitoring and deep fusion

(1) Using the scientific consistent geophysical models, rigorous uniform numerical standards, and analytic compatible geodetic and geodynamic algorithms, unifies the spatiotemporal monitoring datum and reference epoch to construct the theoretical basis and necessary conditions for geodetic collaborative monitoring.

(2) Based on the principle of geophysical geodesy, deeply fuses or constrainedly assimilates multi-source heterogeneous geodetic data, and then realizes collaborative monitoring with various geometric and physical geodetic technologies by reconstructing the geodetic or geodynamic spatiotemporal relationship between various variations.

(3) For the same type of multi-source geodetic monitoring variations, the basic geodetic constraints or joint adjustment methods with additional monitoring datum parameters as needed are employed to deep fusion.

(4) For different types of geodetic monitoring variations, physical geodetic, solid geophysical or environmental geodynamic constraints with additional dynamic parameters as needed are employed to deep fusion.

(5) The purpose of geodetic collaborative monitoring is not only to improve the spatiotemporal monitoring capability, but also to furtherly reveal the geodynamic structure and characteristics of the monitored objects and support the massive integration, intelligence and automation of Earth observations.

1.4.2 Tidal deformations of solid Earth and their tidal effects on geodetic variations

(1) The external celestial bodies, ocean tide and atmosphere tide excite the periodic deformation of the solid Earth and the periodic change of Earth gravity field, which are called as the tidal deformation of the solid Earth.

(2) The geodetic variations caused by the external celestial bodies, ocean tides and atmosphere tides are usually called as the tidal effects on geodetic variations.

(3) The geodetic tidal effects include the solid Earth tidal effects and tidal load effects. The solid Earth tidal effects are excited by the external celestial bodies, while the tidal load effects are excited by the ocean tide and atmosphere tide.

(4) The geodetic tidal effects can be modeled and accurately removed or restored anytime and anywhere. The geodetic tidal effect is equal to the negative value of the geodetic tidal correction.

The geodetic reference frame with only all the tidal effects removed but non-tidal effects neglected is still stationary (unchanged with time). For example, a precision leveling network or gravimetric control network, if whose observations have been corrected only using tidal effects, is still stationary.

1.4.3 Non-tidal deformation of solid Earth and their effects on geodetic variations

(1) In the Earth surface system, surface non-tidal load variations such as soil and vegetation water, river and lake water, glacier and snow, groundwater, atmosphere and sea level variations can induce the external geopotential variations, and then excite the solid Earth deformation also manifested as ground displacement, gravity and tilt variations. Which can be called as the Earth's load deformation also manifested as the variations of the Earth’s gravity field with time.

(2) Groundwater usage, underground mining, underground construction, glacier or ice sheet melting and other natural or artificial surface mass adjustments can break the mechanical balance state of the surface rock and soil layer, and then the surface rock and soil layer will tend slowly to another equilibrium state under the action of the own gravity or internal stress. The process causes plastic or viscous vertical deformation which is also called as the isostatic vertical deformation.

(3) The load deformation is excited by the surface environment load variations, and acts on the whole solid Earth. Which is an elastic deformation and can be quantitatively represented by the load Love numbers. The isostatic vertical deformation is induced by environmental geology change, whose dynamic action is in the underground rock and soil layer and transmitted by the rock and soil own as the mechanical medium. The isostatic deformation is a slow plastic or viscous deformation.

(4) Non-tidal effects are difficult to be modeled but can be measured using geodetic techniques. In most fast or real-time geodetic cases, short-time forecast estimations of non-

tidal effects are usually adopted.

The geodetic reference frame that needs to account for non-tidal effects can be only dynamic, and the reference value of the dynamic reference frame corresponds to a specific and unique reference epoch time. The reference value at the current epoch time is equal to the sum of the reference value at the reference epoch time and non-tidal effect correction. The correction here is equal to the difference between the non-tidal effects at the current epoch and that at the reference epoch. The correction process is also called as the (non-tidal effects) epoch reduction.

1.4.4 Types of ground vertical deformation and space-time quantitative natures

There are three forms of ground vertical deformation (or ground subsidence), namely, the elastic load vertical deformation, viscous or plastic isostatic vertical deformation and plastic tectonic vertical deformation near the compressive geological fracture zone. The latter two are also called as the non-load vertical deformations, both of which are plastic vertical deformations.

(1) The load vertical deformation excited by the surface mass redistribution, firstly causes the geopotential variation called as the direct effect, and then excites the solid Earth deformation simultaneously by elastic dynamic action to induce an associated geopotential variation called as the indirect effect. The load vertical deformation synchronizes with the time of load redistribution, whose time-varying characteristics are similar to that of the surface load variations with the complex nonlinearity and quasi-periodicity.

(2) The isostatic vertical deformation usually manifests as a dynamic process. In the process, the original equilibrium state of the underground rock and soil layer is firstly destroyed by the geology dynamic action, and then under the action of the own gravity or internal stress, the rock and soil layer slowly approaches another equilibrium state. For example, the compaction effects of the rock and soil layers with voids in the underground after the loss of water and the expansion effects after water infiltration, deformation of the upper rock layer (wall rock deformation) caused after underground construction and ground plastic isostatic rebound after surface mass migration.

• Spatial quantitative characteristics of the isostatic vertical deformation

The dynamic action is inside the underground rock and soil layer, and the equilibrium adjustment object is the rock and soil layer above the dynamic action point. The space influence angle of equilibrium adjustment is about 45˚, so the spatial range of ground vertical deformation is approximately equal to the buried depth of the action point.

• Temporal quantitative characteristics of the isostatic vertical deformation

The duration of the equilibrium adjustment is approximately proportional to the burial depth of dynamic action point. The isostatic vertical deformation is the opposite of its acceleration sign in a long-period of time (e,g, several years), and linear time variation in a short-period of time (e.g. several months).

(3) The tectonic vertical deformation driven by the horizontal movement of the lithospheric plate only appears near the compressive fault zone. Whose spatial influence radius is equivalent to the depth of the fault, and the deformation declines rapidly to zero with the distance of the calculation point away from the fault zone. On a centennial timescale, the tectonic vertical deformation rate would remain basically unchanged.

1.4.5 Representation and approach principles of surface load deformation field

(1) The load deformation field is a form of non-tidal geodetic load effects, which can be uniquely represented by the variations of Earth's gravity field with time. The relationship between the non-tidal load effects on the elements of Earth's gravity field is completely consistent with the relationship between the elements. Global Earth gravity field can be represented by a geopotential coefficient model (GCM). Similarly, the global load deformation field (namely temporal global gravity field) can be represented by a global surface load spherical harmonic coefficient model (LCM).

(2) From a geopotential coefficient model, you can calculate various gravity field elements on the ground or outside the solid Earth. Similarly, from a global load spherical harmonic coefficient model, you can calculate load effects on various geodetic variations outside the solid Earth. Regional gravity field (geoid) can be refined by the remove-restore scheme based on a GCM. Similarly, the regional load deformation field or temporal gravity field can also be refined by the remove-restore scheme based on an LCM.

(3) The approach theory of Earth's gravity field is linear, so that Earth's gravity field can be refined by the remove-restore scheme and cumulative residual approach method. Similarly, the approach theory of load deformation field is also linear, so load deformation field can be also refined by the remove-restore scheme and cumulative residual approach method. The total effects of various environmental loads (atmospheric pressure, land water, and sea level variation, etc.) are equal to the deformation effects of the sum of these loads.

(4) The approach methods of the Earth's gravity field can be summarized into two categories, namely, the Stokes / Hotine integral method (geodetic boundary value problem solution) in the spatial domain and the spherical basis function (e.g. surface spherical function, radial basis function and spline basis function) approach method in the spectral domain, which can integrate various global or regional gravity field data. Similarly, for load deformation field (time-varying gravity field) approach or monitoring, there are two methods namely the load Green's function integral constraint in spatial domain and spherical basis function approach in spectral domain, which can also effectively fuse global or regional multi-source heterogeneous geodetic variations.

1.4.6 CORS and InSAR collaborative monitoring principle for vertical deformation

(1) Through the gross error detection, spatial filtering and time series analysis, the InSAR vertical variation can be separated into two parts, one part is the vertical deformation of the

rock and soil layer several meters deep, and the other part is the expansion and contraction of the soil own. Only the former is compatible with most geodetic variations, while the latter is mainly affected by the temperature and rainfall and should not be regarded as a solid Earth deformation.

(2) Using the CORS network ellipsoidal height variation time series as the constraints on the multi-source InSAR vertical variation time series, separate the ground vertical deformation signal, and then realize the collaborative monitoring of the CORS network and multi-source InSAR.

(3) Only the vertical deformation of the rock and soil layer several meters deep are the useful information needed for monitoring of the ground subsidence, earthquakes, geological disasters, ground stability variations, solid Earth deformation, groundwater variations and other geodynamics.

1.4.7 Continuous quantitative monitoring scheme of ground stability variations

(1) Construct the quantitative criteria for the ground stability weakening from the regional grid time series of the geodetic vertical deformation, ground gravity and tilt variations, and then continuous quantitatively monitor the ground stability variations.

(2) Quantitative criteria of the ground stability weakening mainly include that the ellipsoidal height increases, ground gravity decreases, horizontal gradient of the height or gravity variation is larger and the inner product of the tilt variations and terrain slope vector is greater than zero.

(3) According to the objective nature of ground stability reduction at the place and time of geological disaster, optimize and synthesize the ground stability variation grid time series to adapt to the local environmental geology, and then consolidate regional stability variations monitoring capabilities.

1.4.8 Analytical compatibility between various geodetic algorithms

The consistency and analytical compatibility between various geodetic algorithms are the requirement of geodetic theory and concrete manifestation for the uniqueness of monitoring object. Which is the smallest requirement for the collaborative monitoring of various geodetic technologies and deep fusion of multi-source heterogeneous geodetic data.

Analytical compatibility between geodetic algorithms involves two issues: (1) Compatibility between various geodynamic influences for different types of geodetic variations. (2) Compatibility between different types of geodynamic influences for one kind of geodetic variations.

The first type of compatibility is the basic requirement of geodetic theory. For example, the load effect on the normal height on a site is equal to the Hotine integral of the load effect on gravity disturbances. For another example, the solid tidal effect on the normal height on a site is equal to the sum of the effects on the ellipsoidal height and geoid.

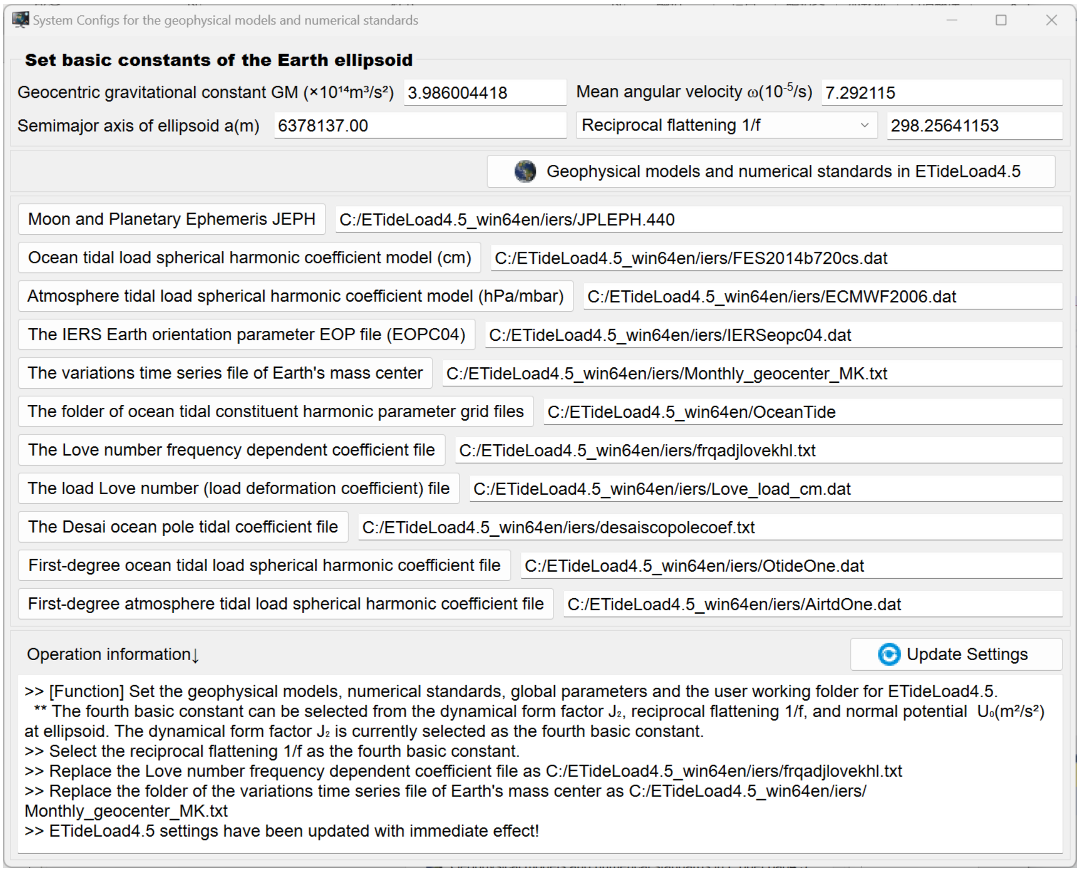
The second type of compatibility is constrained by the solid deformation geodynamic equations (including constitutive equations).

1.5 Conventions and examples in ETideLoad4.5

1.5.1 Geophysical models and numerical standards in ETideLoad4.5

ETideLoad4.5 is mainly based on the geophysical models and numerical standards recommended by IERS conventions (2010). These geophysical models and numerical standards are stored in file form in the folder of C:\ETideLoad4.5_win64en\iers, which can be updated by the program [System configs for the geophysical models and numerical standards].

Geophysical models and numerical standards in ETideLoad4.5 mainly include the surface atmosphere tidal load spherical harmonic coefficient model file, ocean tidal load spherical harmonic coefficient model file, Earth's load Love number file, IERS Earth orientation parameter time series file, Earth's mass centric variation time series file measured by SLR, ocean tidal constituent harmonic constant grid model files, JPL Moon and Planetary Ephemeris DE440 file, Love number correction file for frequency dependence and Desai ocean pole tide coefficient file.



1.5.2 Five kinds of variation time series agreed in ETideLoad4.5

The geodetic variation time series files adopt the ETideLoad own format, which include the ground variation time series file, geodetic network site record time series file, geodetic network observation record time series file, variation (vector) grid time series files and spherical harmonic coefficient (Stokes coefficient) model time series files.

(1) The ground variation time series file

A ground variation time series file can store the time series data of several kinds of variations at a certain site, a certain baseline or route, and the sampling epochs (here, the epoch is an instantaneous time) of these variations are the same. Such as CORS station coordinate solution time series, solid tide station observation or analysis result time series, GNSS baseline solution time series, etc.

(2) The geodetic site variation record time series file

A geodetic site variation record time series file can store the time series data of one kind of variations for a group of geodetic sites. Such as the station coordinate time series for the CORS network, benchmark height time series for the levelling network, observation time series for the tide station network, and InSAR monitoring time series, etc.

(3) The geodetic network observation record time series file

A geodetic network observation record time series file can store the variation record time series of the baseline component for the CORS network, the variation record time series of the height difference for the levelling network, or the variation record time series of the gravity difference for the gravity network.

(4) The variation grid time series files for geodetic field

A group of variation grid time series files are composed of a series of numerical grid model files of a certain kind of variation (vector), and the seventh attribute of the header in each grid file is agreed to be the sampling epoch time. Such as the grid time series of land water equivalent height, sea level variation, and grid time series of various regional load deformation field or temporal gravity field, etc.

(5) The spherical harmonic coefficient model time series files

A group of spherical harmonic coefficient model time series files can store the time series of the spherical harmonic coefficient (Stokes' coefficient) models of the global surface load variation, global load deformation field or temporal global gravity field.

The program [Conversion of general ASCII record data into ETideLoad format], and the function [Normalized extraction of batch time series of geodetic monitoring network] are the important interfaces for ETideLoad to accept external text data. Using the function [Global prediction of solid Earth tidal effects on various geodetic variations], or [Global prediction of surface atmosphere tidal load effects on various geodetic variations], you can construct a ground variation time series with the given location and sampling specifications. Using the program [Generating and constructing of regional geodetic grid], you can construct a numerical grid with the given grid specifications. The other programs or functions only accept the format data generated by ETideLoad own.

1.5.3 Full examples for the classroom teaching and self-study exercises

To ease the classroom teaching and self-study exercises, there are the example files saved in the folder C:\ETideLoad4.5_win64en\examples for each Win64 program. Each example includes the operation process file process.txt, some input-output data files and screenshots. The folder name of the example files is the same as the name of the window executable program.

Before using the ETideLoad4.5 programs, it is recommended to perform completely the program examples using the input-output example data files by comparing the screenshots according to the process information in process.txt. It will take about 7 working days to complete all the example exercises. Thereafter, you can use ETideLoad4.5 alone.

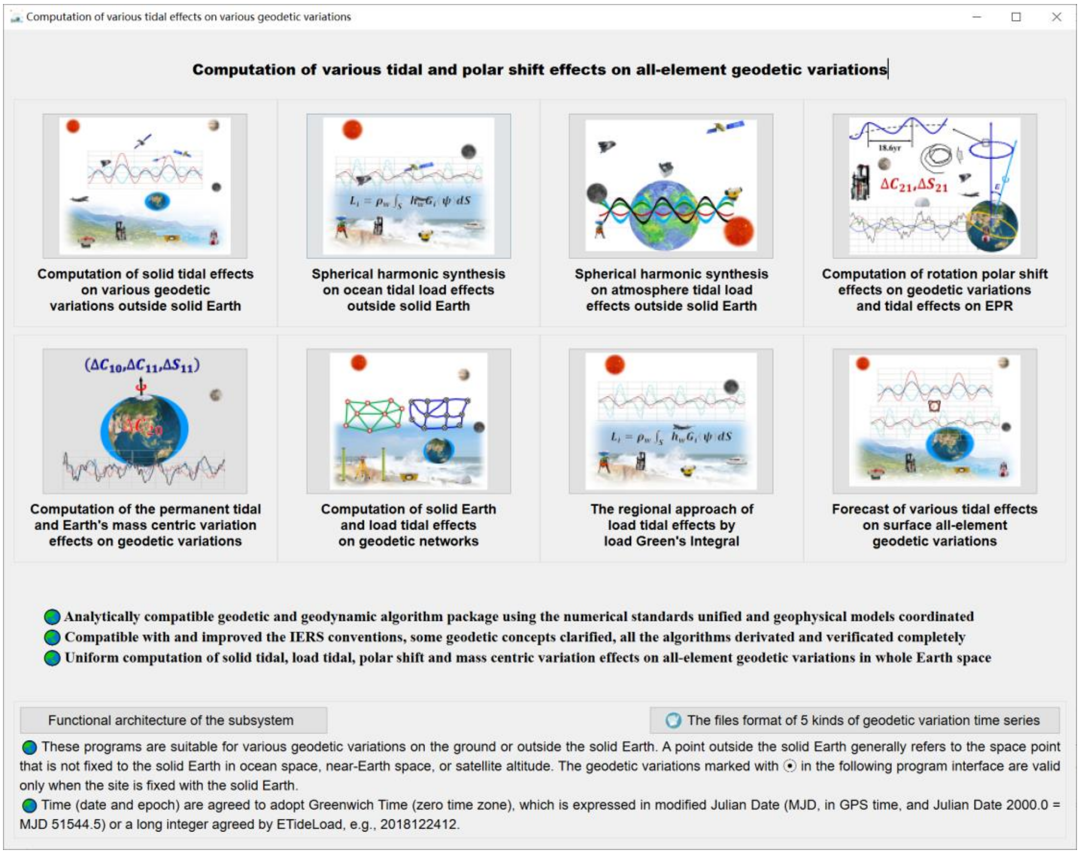
1.5.4 ETideLoad4.5's applicable professional fields and usage instructions

ETideLoad4.5 is suitable for senior undergraduates, graduate students, scientific researchers, and engineering technicians in geodesy, geophysical, geoscience, geological environment, hydrodynamics, satellite dynamics, seismology, and geodynamics, which can be employed in the classroom teaching, self-exercise, science research, and engineer computing.

You can design your own schemes and processes, then organize flexibly the related programs and functions from ETideLoad4.5, perform various scientific computations for the various tidal effects on various geodetic variations, precision approach of ground deformation field or temporal gravity field, collaborative monitoring on the land water, ground stability and surface dynamic environment, deep fusion of multi-source heterogeneous geodetic data and analysis on non-tidal geodetic variation time series.

2 Computation of various tidal and polar shift effects on all-element geodetic variations

The group of programs can be empoyed to compute uniformly the solid Earth tidal, ocean tidal load, atmosphere tidal load, Earth's rotation polar shift, figure polar shift and mass centric variation effects on all-element geodetic variations on the ground and outside the solid Earth using the analytically compatible geodetic and geodynamic algorithm system with the numerical standards unified and geophysical models coordinated. Which is the important basis and necessary condition for the collaborative monitoring of various geodetic technologies and deep fusion of multi-source heterogeneous Earth monitoring data.



These programs are suitable for various geodetic variations on the ground or outside the solid Earth. A point outside the solid Earth generally refers to the space point that is not fixed with the solid Earth in ocean space, near-Earth space, or satellite altitude. The geodetic variations marked with ⦿ in the following program interface are valid only when the site is fixed with the solid Earth.

2.1 Computation of solid tidal effects on various geodetic variations outside solid Earth

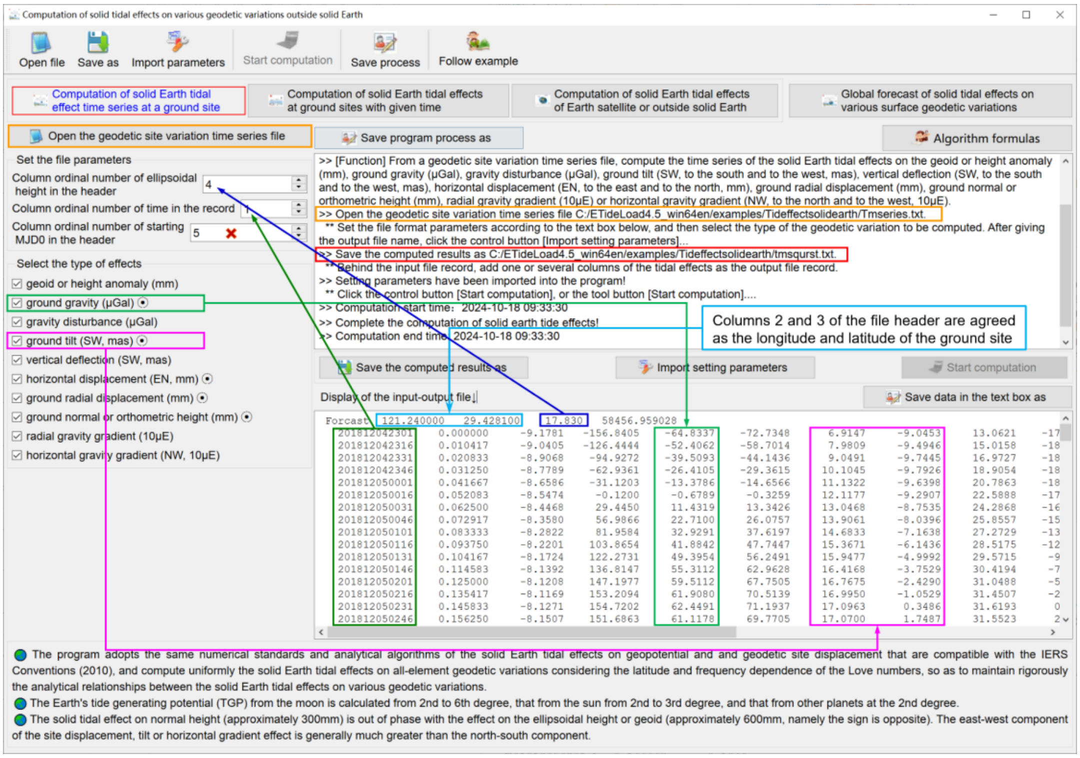
[Purpose] According to the location and time in the input time series file, compute the solid Earth tidal effect time series on various geodetic variations on the ground or outside the solid Earth. Here the point outside the solid Earth generally refers to the space point that is not fixed with the solid Earth in ocean space, near-Earth space, or satellite altitude.

The program adopts the same numerical standards and analytical algorithms of the solid Earth tidal effects on geopotential and geodetic site displacement that are compatible with the IERS Conventions (2010), and compute uniformly the solid Earth tidal effects on all-element geodetic variations considering the latitude and frequency dependence of the Love numbers, so as to maintain rigorously the analytical relationships between the solid Earth tidal effects on various geodetic variations.

The Earth's tide generating potential (TGP) from the moon is calculated from 2nd to 6th degree, that from the sun from 2nd to 3rd degree, and that from other planets at the 2nd degree.

2.1.1 Computation of solid Earth tidal effect time series at a ground site

[Function] From a geodetic site variation time series file, compute the time series of the solid Earth tidal effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).



[Input file] The geodetic site variation time series file.

The file header contains the site name, longitude (degree decimal), latitude (degree decimal), height (m) relative to the ellipsoidal surface, the starting MJD0 (optional)…

Starting from the second row of the file, each row record stores the sampling values of all the variations at one sampling epoch time. At least one column of the attribute in the record is the sampling epoch time.

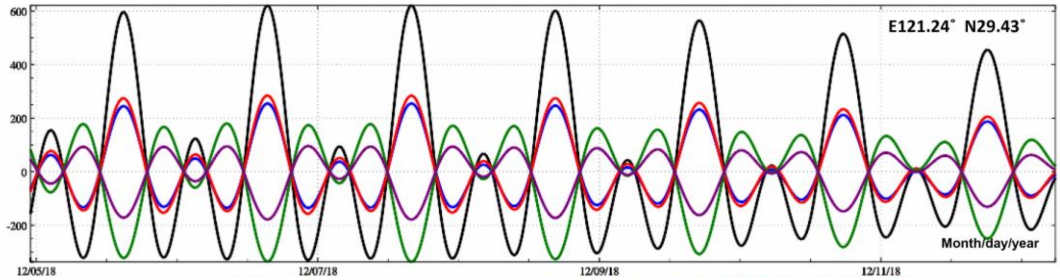
[Parameter settings] Set the input file format parameters, select the type of solid Earth tidal effects.

The geodetic variations marked with ◉ are valid only when the site is fixed with the solid Earth.

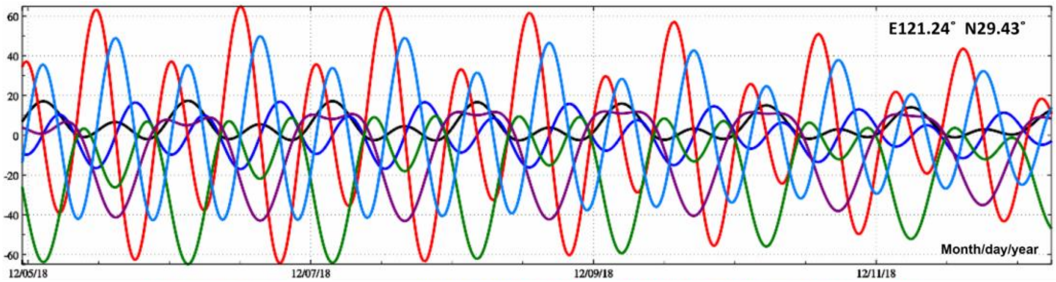
[Output file] The geodetic site solid Earth tidal effect time series file.

The file header is the same as the input time series file. Behind the input file record, adds one or several columns of the tidal effects selected as the output file record. In this example, all types are selected, and there are 14 attributes added to the record.

When the ellipsoidal height of the calculation point is equal to the ellipsoidal height of the geoid, the solid tidal effect on the height anomaly is that on the geoid.



Solid tidal effects : geoid or height anomaly (mm) ground gravity (μGal) radial displacement (mm) normal or orthometric height (mm) radial gravity gradient (10μE)



Solid tidal effects : ground tilt (S, mas) tilt (W, mas) horizontal displacement (E, mm) horizontal displacement (N, mm) horizontal gradient (N, 10μE) horizontal gradient (W, 10μE)

The solid tidal effect on normal height (approximately 300mm) is out of phase with the effect on the ellipsoidal height or geoid (approximately 600mm, namely the sign is opposite). The east-west component of the site displacement, tilt or horizontal gradient effect is generally much greater than the north-south component.

The program also outputs the solid Earth tidal effect time series file *Stdsfptm.txt on the degree-2 geopotential coefficients and figure polar shifts into the current directory. * is the input file name, which is an internal test file. The first row is the file header, and starting from the second row of the file, each row record stores the sampling epoch time, 5 columns of solid Earth tidal effects on the degree-2 normalized geopotential coefficients $\Delta\bar{C}_{20},\Delta\bar{C}_{21},\Delta\bar{S}_{21},\Delta\bar{C}_{22},\Delta\bar{S}_{22}$ (×10⁻⁸), 2 columns of tidal effects on the figure polar shifts $\Delta x_{sfp},\Delta y_{sfp}$(in ITRS space rectangular coordinate system in unit of m). The difference between the maximum and minimum values of Earth tidal effects on the figure polar shifts is generally more than 1 km.

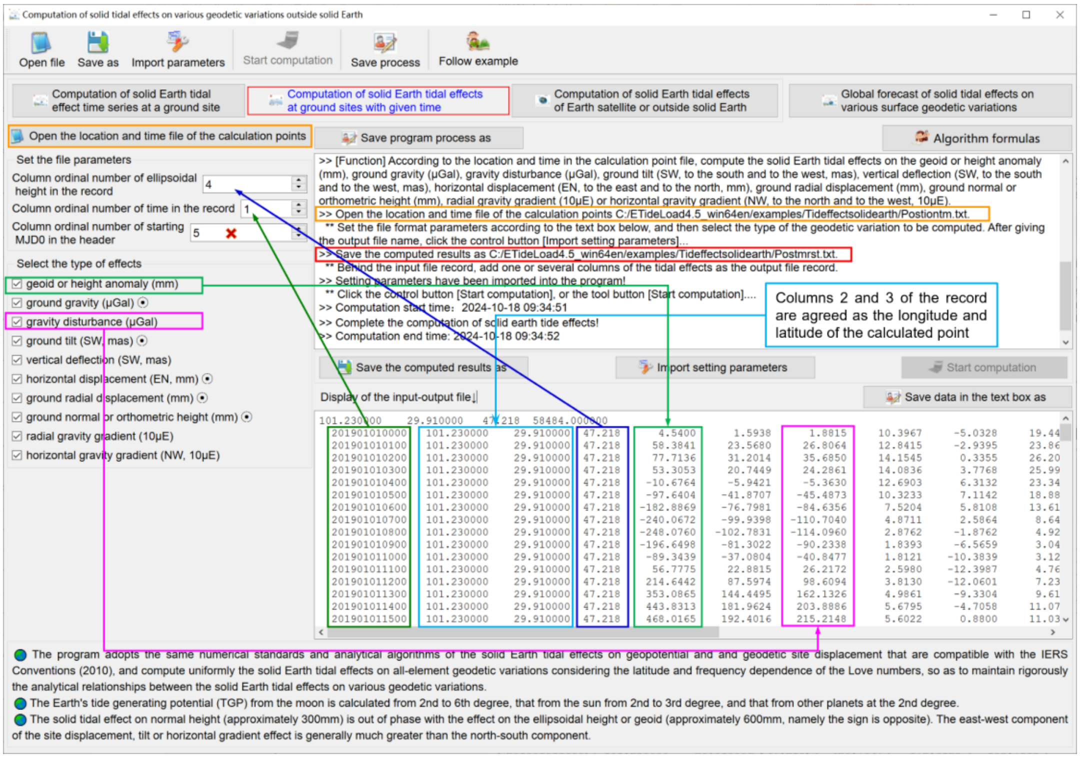
2.1.2 Computation of solid Earth tidal effects at ground sites with given time

[Function] According to the location and time in the calculation point file, compute the solid Earth tidal effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

[Input file] The location and time file of the calculation points.

The first row is the file header. From the second row onwards, the second and third attributes in the file record are conventionally longitude and latitude (degree decimals), and there are the sampling epoch time and ellipsoidal height attributes in the record.

[Parameter settings] Set the input file format parameters, select the type of solid Earth tidal effects.



[Output file] The solid Earth tidal effect file.

The file header is the same as the input file. Behind the input file record, adds one or several columns of the tidal effects selected as the output file record. In this example, all types are selected, and there are 14 attributes added to the record.

The start MJD0 attribute in the input file header is required when the date is in MJD format. In this case, the sampling epoch MJD is equal to the sum of the starting MJD0 and the number of days in the record.

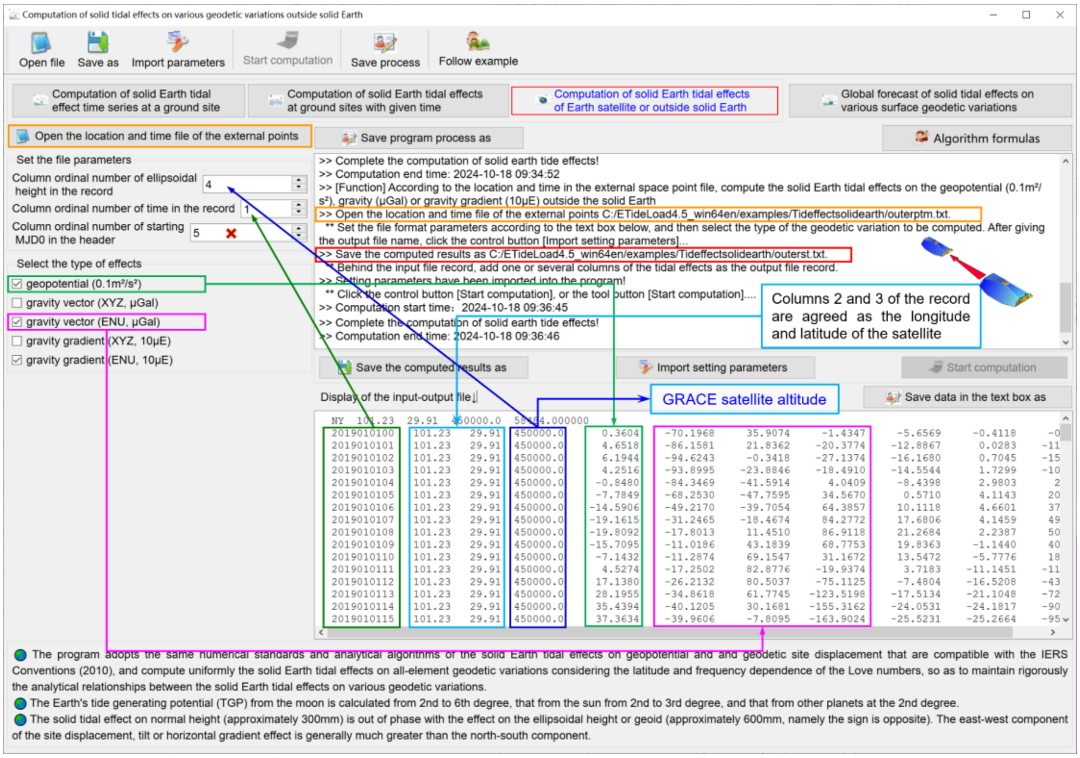
If the time (date) is in the long integer format agreed by ETideLoad, it is not necessary for the starting MJD0 attribute in the input file header, and the program automatically recognizes and ignores the selection.

$1+(2h_{nm}-(n+1)k_{nm})/n$ is the degree-n order-m body tidal factor of ground gravity. $1-(n+1)k_{nm}/n$ is that of gravity disturbance. $1+k_{nm}-h_{nm}$ is that of ground tilt. And $1+k_{nm}$ is that of the vertical deflection, geopotential or height anomaly.

In general, ΔC_{n0} mainly consists of the long-term or long period constituents of the solid tidal effects (the cycle is greater than half a lunar month, n=1, 2, …), ΔC_{n1}, ΔS_{n1} mainly consists of the diurnal tidal effects and ΔC_{n2}, ΔS_{n2} mainly consists of the semi-diurnal tidal effects. More generally, ΔC_{nm}, ΔS_{nm} is mainly composed of the 1/m diurnal tidal effects.

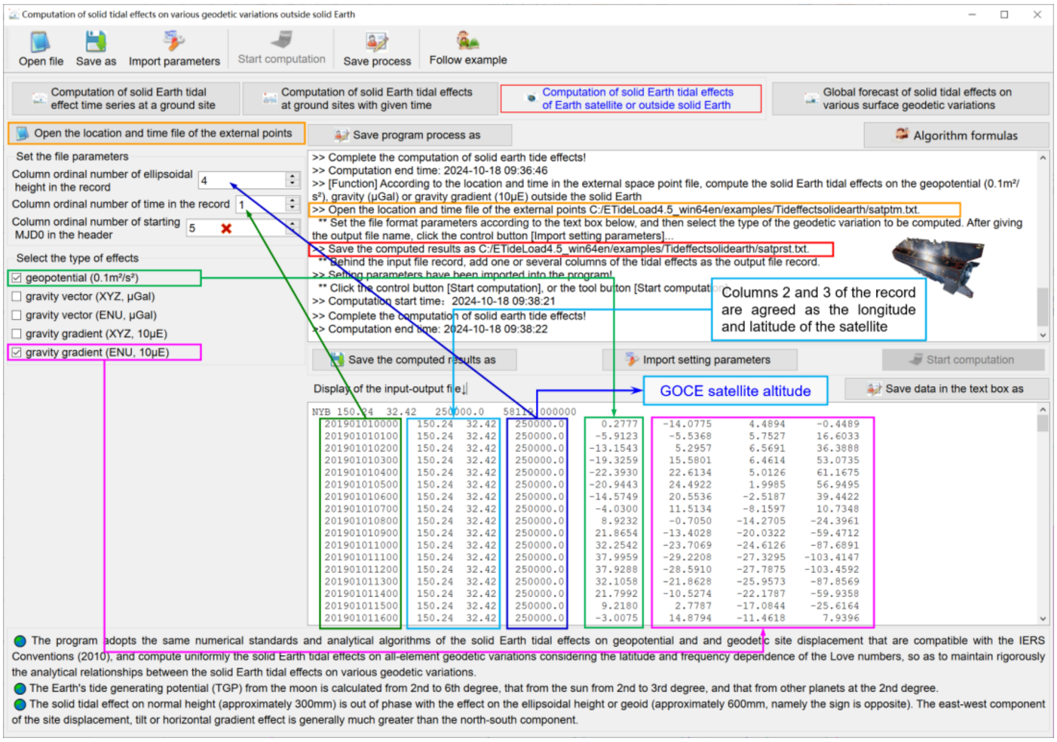
2.1.3 Computation of solid Earth tidal effects of Earth satellite or outside solid Earth

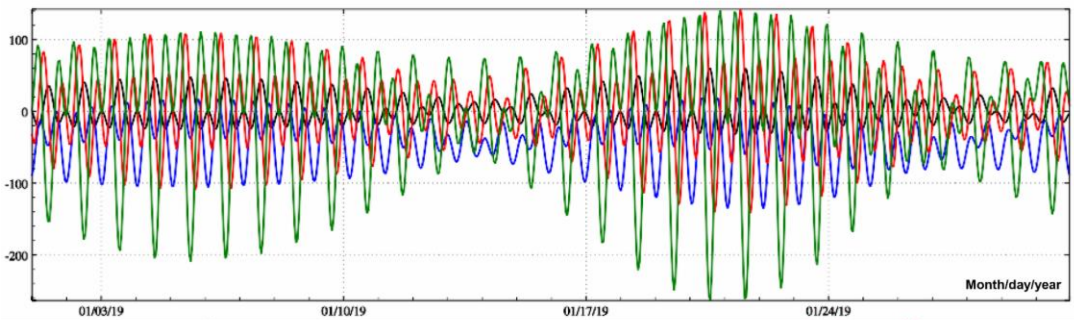
[Function] According to the location and time in the external space point file, compute the solid Earth tidal effects on the geopotential (0.1m²/s²), gravity (μGal) or gravity gradient (10μE) outside the solid Earth.



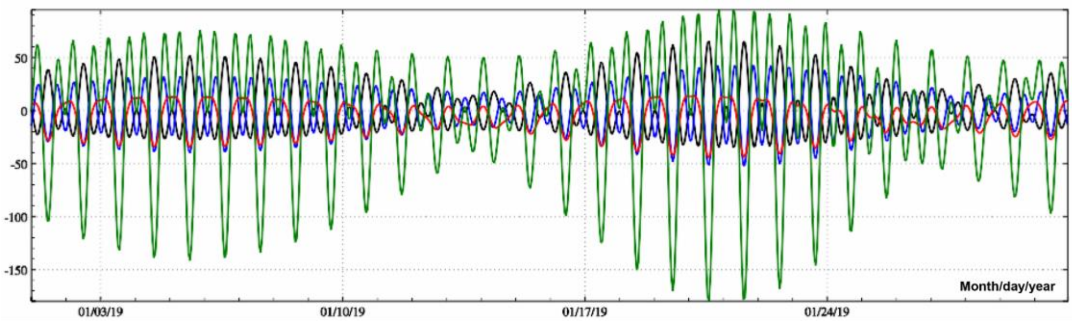
[Input file] The location and time file of the external space points.

The first row is the file header. From the second row onwards, the second and third attributes in the file record are conventionally longitude and latitude (degree decimals), and there are the sampling epoch time and ellipsoidal height attributes in the records.





The solid tidal effects at 450km altitude: geopotential (0.1m²/s²), gravity vector (E, N: along the GRACE orbit/SST-II, U, μGal)



The solid tidal effects at 250km altitude: geopotential (0.1m²/s²), gravity gradient (E, N: along the GOCE orbit, U, 10μE)

[Parameter settings] Set the input file format parameters, select the type of solid Earth tidal effects.

[Output file] The solid Earth tidal effect file.

The file header is the same as the input file. Behind the input file record, adds one or several columns of the tidal effects selected as the output file record.

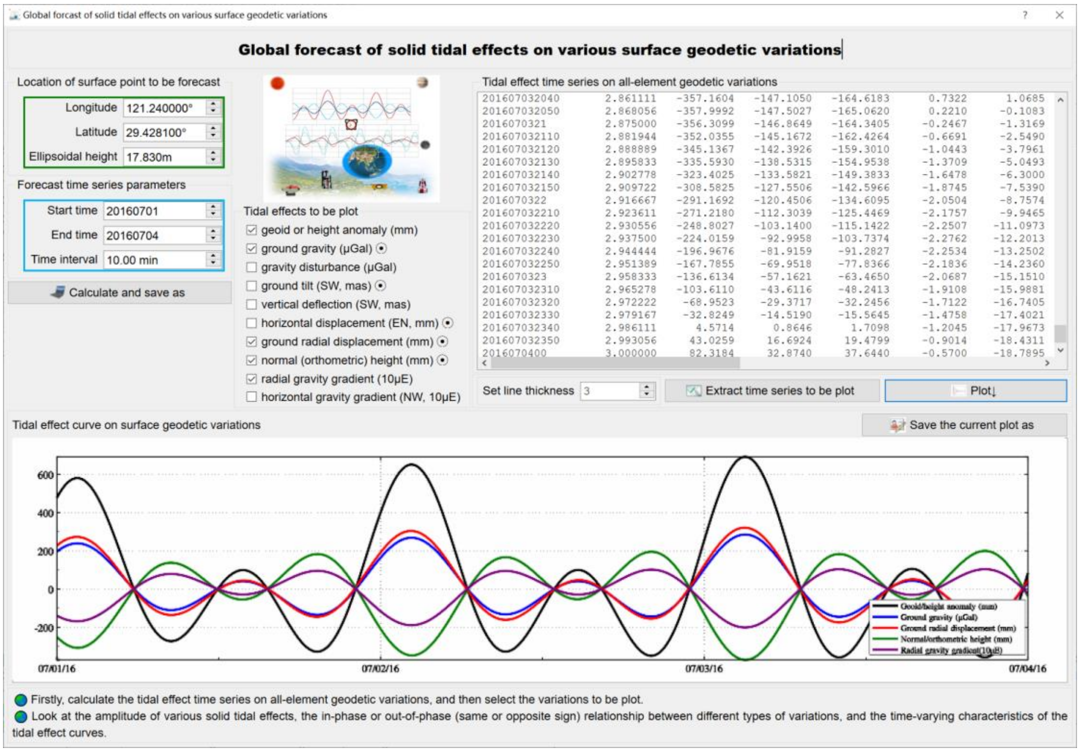
In this example, the geopotential and gravity vector are selected, and there are 4 attributes added to the record.

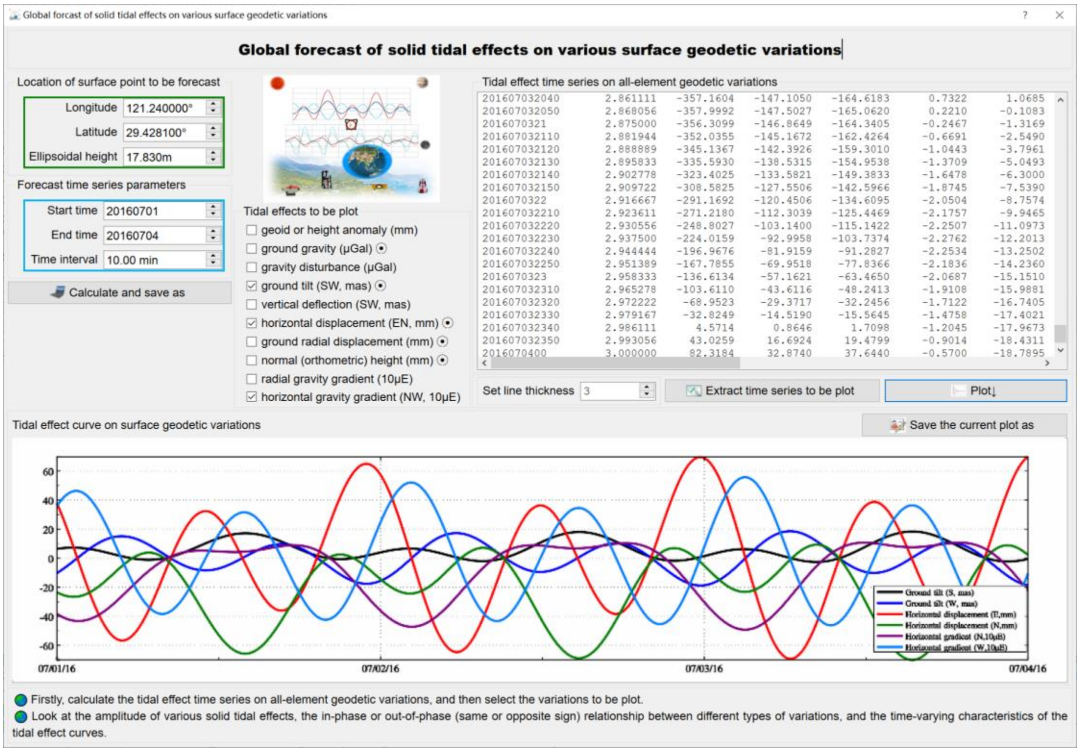
2.1.4 Global forecast of solid tidal effects on various surface geodetic variations

[Function] Input the geodetic coordinates of a global anywhere surface point and set the forecast time series parameters, calculate and display the solid Earth tidal effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

Firstly, calculate the tidal effect time series on all-element geodetic variations, and then select the variations to be plot.

Look at the amplitude of various solid tidal effects, the in-phase or out-of-phase (same or opposite sign) relationship between different types of variations, and the time-varying characteristics of the tidal effect curves.





The program outputs the solid Earth tidal effect time series file at the calculation point. The file header occupies a row and includes the name of the calculation point, longitude (degree decimal), latitude (degree decimal), ellipsoidal height (m) and starting MJD0. From the second row onwards, the first column of the record is the system format time, the second column is the difference between the MJD day and starting MJD0, and the last 14 columns are the solid Earth tidal load effects on all-element geodetic variations.

The last 14 columns of the record are the height anomaly (mm, 3rd column), ground gravity (μGal, 4th column), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas, 6th and 7th columns), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm, 10th and 11th columns), ground radial displacement (mm), ground normal or orthometric height (mm, 13th column), radial gravity gradient (14th column) or horizontal gravity gradient (NW, to the north and to the west, 10μE, 15th and 16th columns), respectively.

2.2 Spherical harmonic synthesis on ocean tidal load effects outside solid Earth

[Purpose] Using the global ocean tidal load spherical harmonic coefficient model (cm), according to the location and time in the input file, compute the ocean tidal load effects on various geodetic variations on the ground or outside the solid Earth by the spherical harmonic synthesis algorithm. Here the point outside the solid Earth generally refers to the space point that is not fixed with the soilid Earth in ocean space, near-Earth space, or satellite altitude.

2.2.1 Computation of ocean tidal load effect time series at a ground site

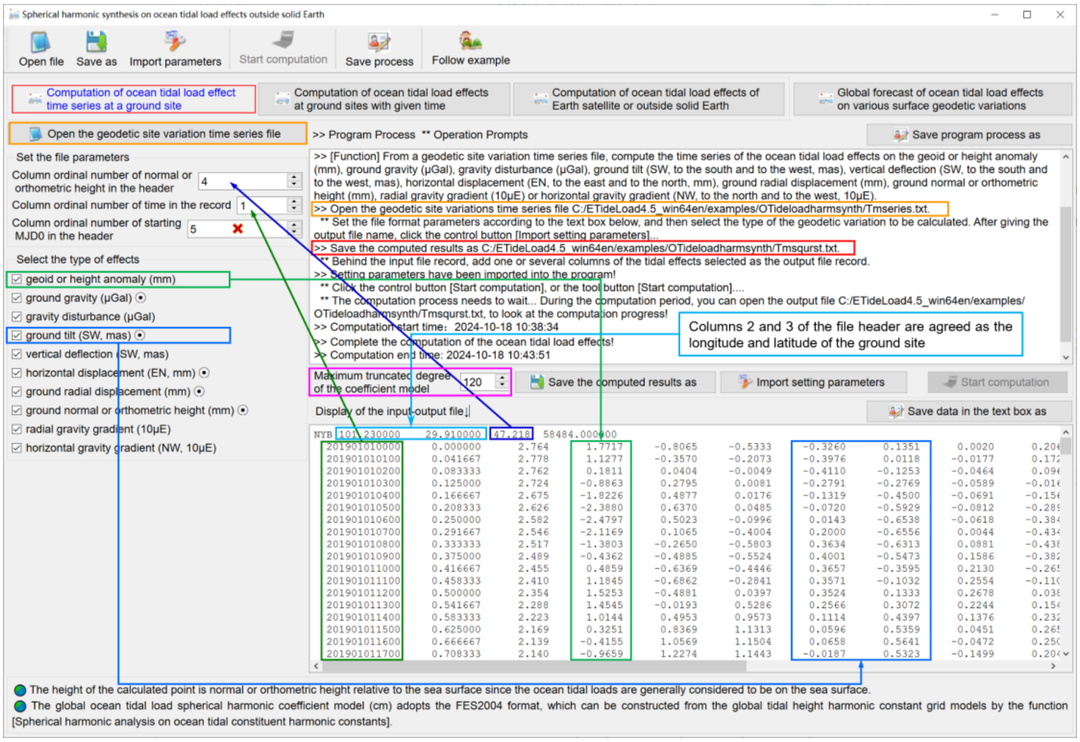
[Function] From a geodetic site variation time series file, compute the time series of the ocean tidal load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

[Input file] The geodetic site variation time series file.

The file header contains the site name, longitude (degree decimal), latitude (degree decimal), height (m) relative to the sea surface, the starting MJD0 (optional)…

Starting from the second row of the file, each row record stores the sampling values of all the variations at one sampling epoch time. At least one column of the attribute in the record is the sampling epoch time.

The height of the calculation point is normal or orthometric height relative to the sea surface since the ocean tidal loads are generally considered to be on the sea surface.

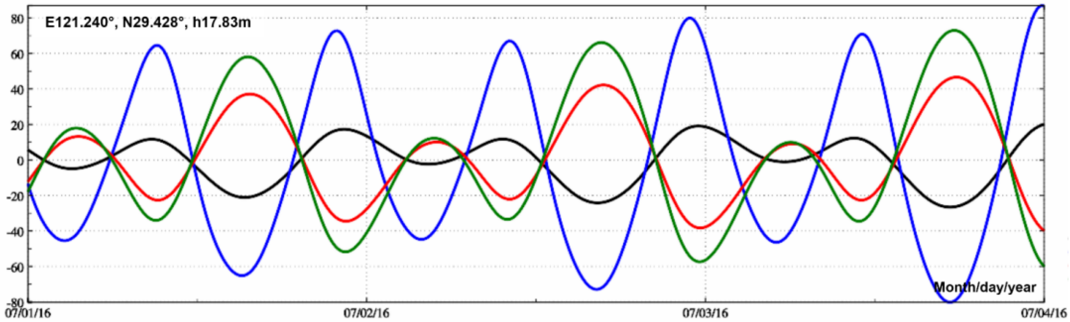


[Parameter settings] Set the input file format parameters, and select the type of ocean tidal load effects.

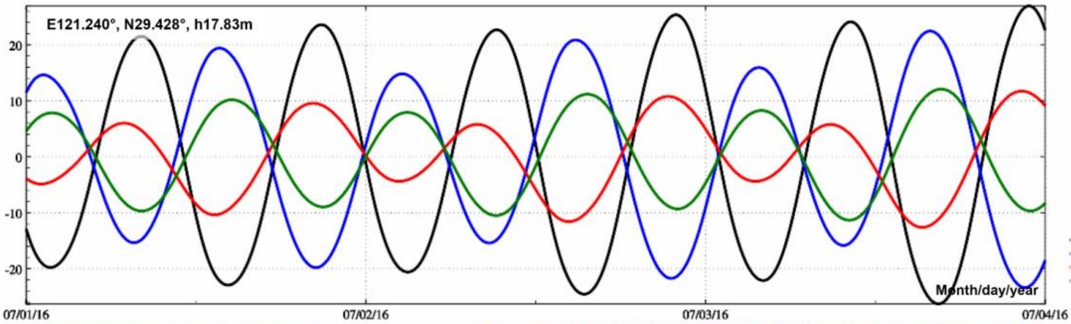
[Output file] The geodetic site ocean tidal effect time series file.

The file header is the same as the input time series file. Behind the input file record, adds one or several columns of the tidal effects selected as the output file record. In this example, all types are selected, and there are 14 attributes added to the record.

The program automatically selects the minimum value between the maximum degree of tidal load spherical harmonic coefficient model and entered maximum degree as the calculation degree.

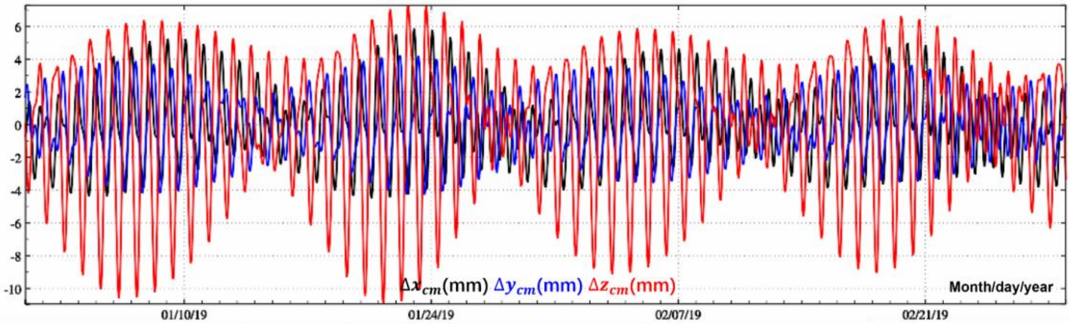


The ocean tidal load effects (360-degree) : height anomaly (mm) ground gravity (μGal) radial displacement (mm) orthometric height (mm)



The ocean tidal load effects (360-degree) : ground tilt (S, mas) tilt (W, mas) horizontal displacement (E, mm) horizontal displacement (N, mm)

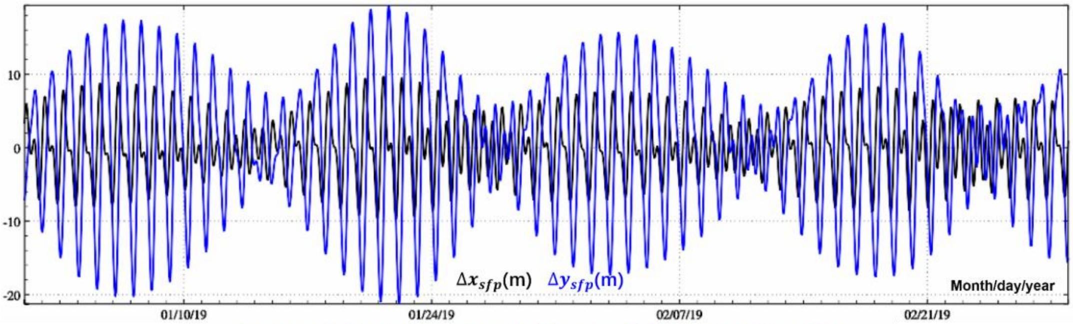
The computation process needs to wait…. During the computation period, you can open the output file to look at the computation progress!



The ocean tidal load effect time series (mm) on Earth's mass centric variations

The program also outputs the ocean tidal load effect time series file *Otdcmsfptm.txt on the degree-1 and degree-2 geopotential coefficients into the current directory. * is the input file name, which is an internal test file. The first row is the file header, and starting from the second row of the file, each row record stores the sampling epoch time, 3 columns of the ocean tidal load effects on the degree-1 normalized geopotential coefficients $\Delta\bar{C}_{10},\Delta\bar{C}_{11}$, $\Delta\bar{S}_{11}$ (×10⁻¹⁰), 5 columns of the effects on the degree-2 geopotential coefficients $\Delta\bar{C}_{20},\Delta\bar{C}_{21}$,

$\Delta\bar{S}_{21}, \Delta\bar{C}_{22}, \Delta\bar{S}_{22}$ (×10⁻¹⁰), 3 columns of the effects (mm) on the Earth's center of mass and 2 columns of the effects on the figure polar shifts $\Delta x_{sfp}, \Delta y_{sfp}$(in ITRS space rectangular coordinate system in unit of m).



The ocean tidal load effect time series (m) on Earth's figure polar shifts in ITRS

2.2.2 Computation of ocean tidal load effects at ground sites with given time

[Function] According to the location and time in the calculation point file, compute the ocean tidal load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

[Input file] The location and time file of the calculation points.

The first row is the file header. From the second row onwards, the second and third attributes in the file record are conventionally longitude and latitude (degree decimals), and there are the sampling epoch time and height attributes in the records.

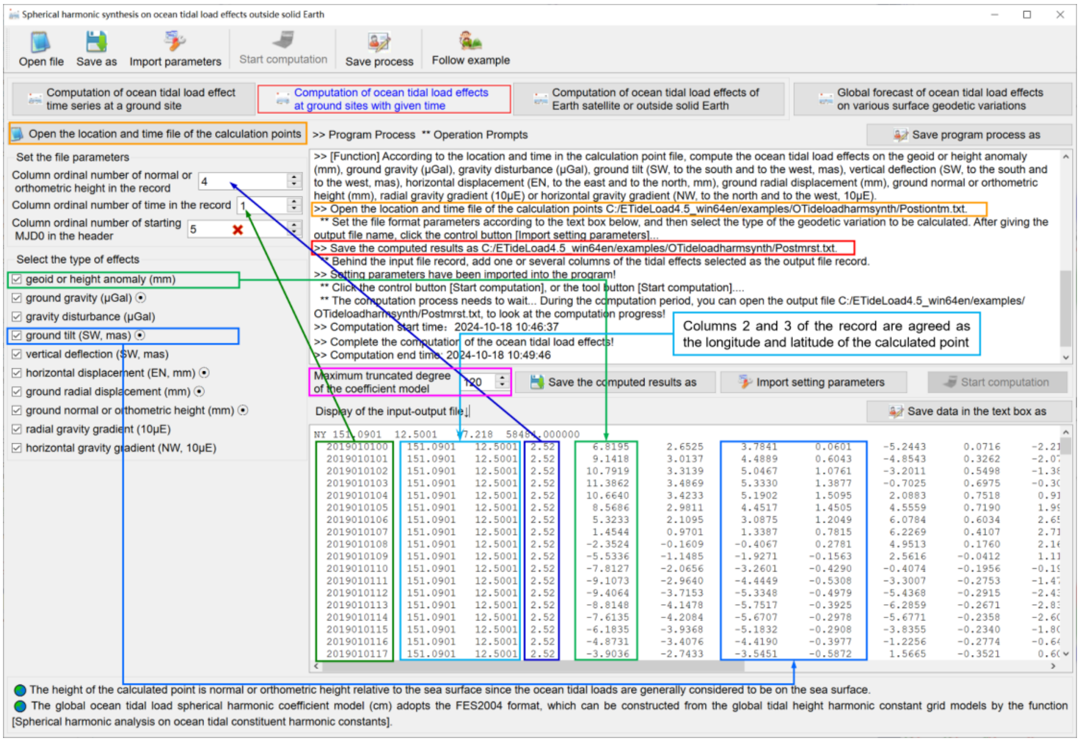
[Parameter settings] Set the input file format parameters, select the type of ocean tidal load effects.

[Output file] The ocean tidal load effect file.

The file header is the same as the input file. Behind the input file record, adds one or several columns of the tidal effects selected as the output file record. In this example, all types are selected, and there are 14 attributes added to the record.

Different from the effect of the solid Earth's tide, the load effect on the normal height is in the same phase as the effect on the ellipsoidal height, and the magnitude of the ocean tidal load effect on the normal height is about 1.75 times that on the ellipsoidal height. The east-west component of the site displacement, tilt or horizontal gradient effect is generally smaller than the north-south component.

$1+(2h'_n-(n+1)k'_n)/n$ is the degree-n load tidal factor (namely load deformation coefficient) of the ground gravity at degree-n, $1-(n+1)k'_n/n$ is that of the gravity disturbance, $1+k'_n-h'_n$ is that of the ground tilt and $1+k'_n$ is that of the vertical deflection or height anomaly.



The global ocean tidal load spherical harmonic coefficient model (cm) adopts the FES2004 format, which can be constructed from the global tidal height harmonic constant grid models by the function [Spherical harmonic analysis on ocean tidal constituent harmonic constants].

The computation speed of the program depends on the degree of the spherical harmonic coefficient model and number of the tidal constituents.

The program adopts the default global ocean tidal load spherical harmonic coefficient model. You can select other global ocean tidal load spherical harmonic coefficient models by the program [System configs for the geophysical models and numerical standards].

2.2.3 Computation of ocean tidal load effects of Earth satellite or outside solid Earth

[Function] According to the location and time in the external space point file, compute the ocean tidal load effects on the geopotential (0.1m²/s²), gravity (μGal), or gravity gradient (10μE) outside the solid Earth.

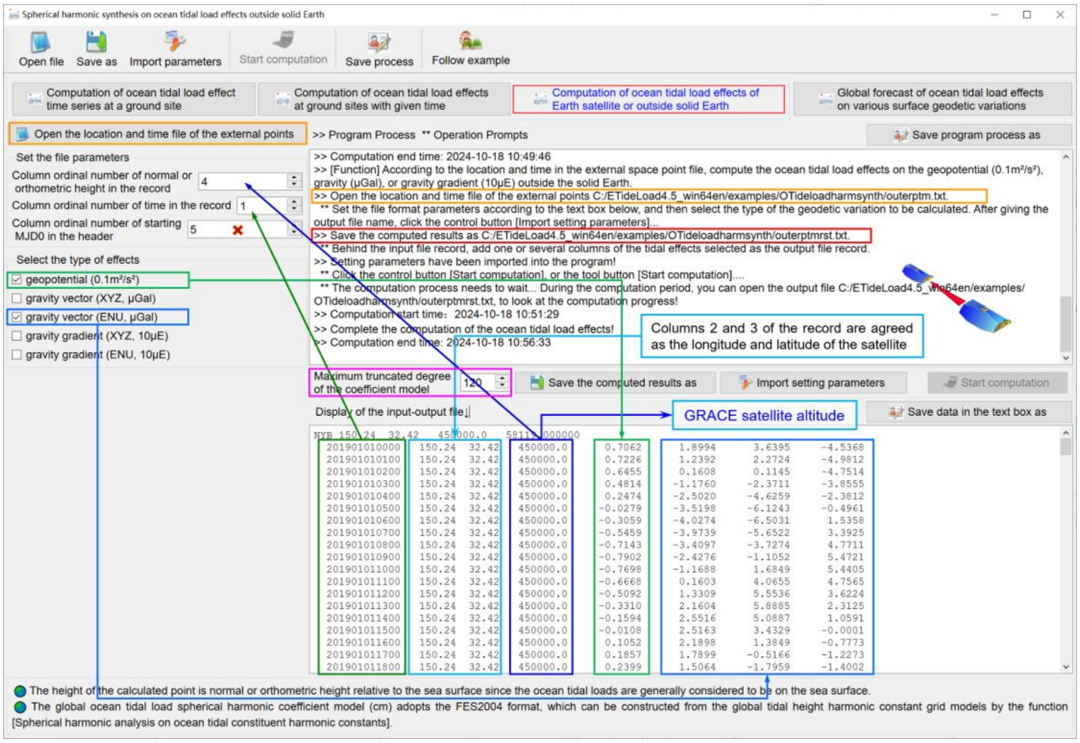
[Input file] The location and time file of the external space points.

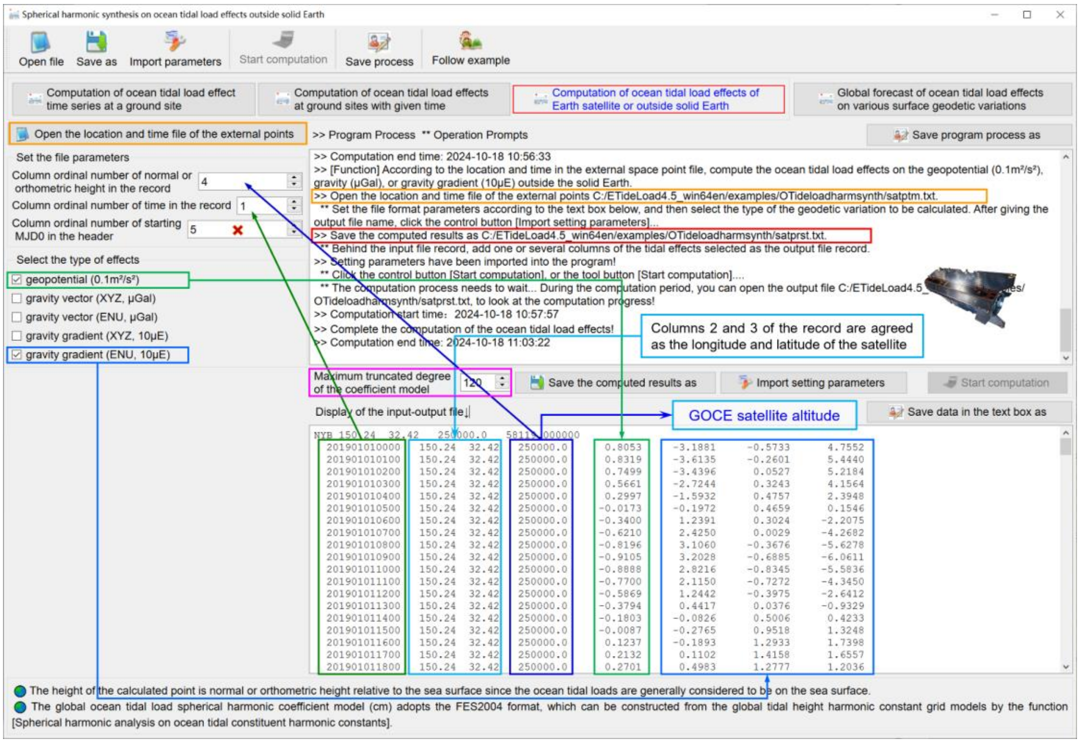
The first row is the file header. From the second row onwards, the second and third attributes in the file record are conventionally longitude and latitude (degree decimals), and there are the sampling epoch time and ellipsoidal height attributes in the record.

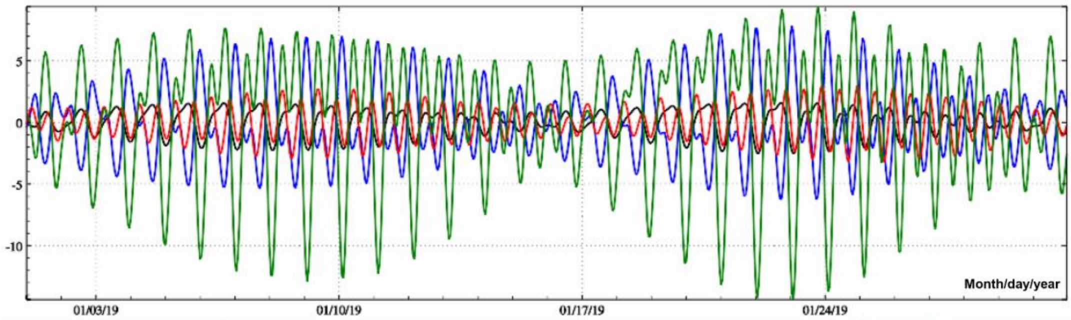
[Parameter settings] Set the input file format parameters, and select the type of ocean tidal load effects.

[Output file] The ocean tidal load effect file.

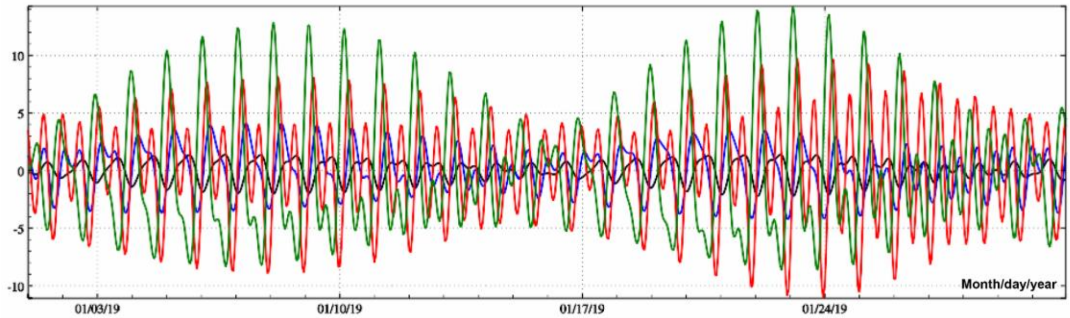
The file header is the same as the input file. Behind the input file record, adds one or several columns of the tidal effects selected as the output file record. In this example, the geopotential and gravity vector are selected, and there are 4 attributes added to the record.







The ocean tidal load effects at 450km altitude：geopotential (0.1m²/s²), gravity vector (E, N: along the GRACE orbit/SST-II, U, μGal)



The ocean tidal load effects at 250km altitude：geopotential (0.1m²/s²), gravity gradient (E, N: along the GOCE orbit, U, 10μE)

2.2.4 Global forecast of ocean tidal load effects on various surface geodetic variations

[Function] Input the geodetic coordinates of a global anywhere surface point and set the forecast time series parameters, calculate and display the ocean tidal load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

Firstly, calculate the tidal effect time series on all-element geodetic variations, and then select the variations to be plot.

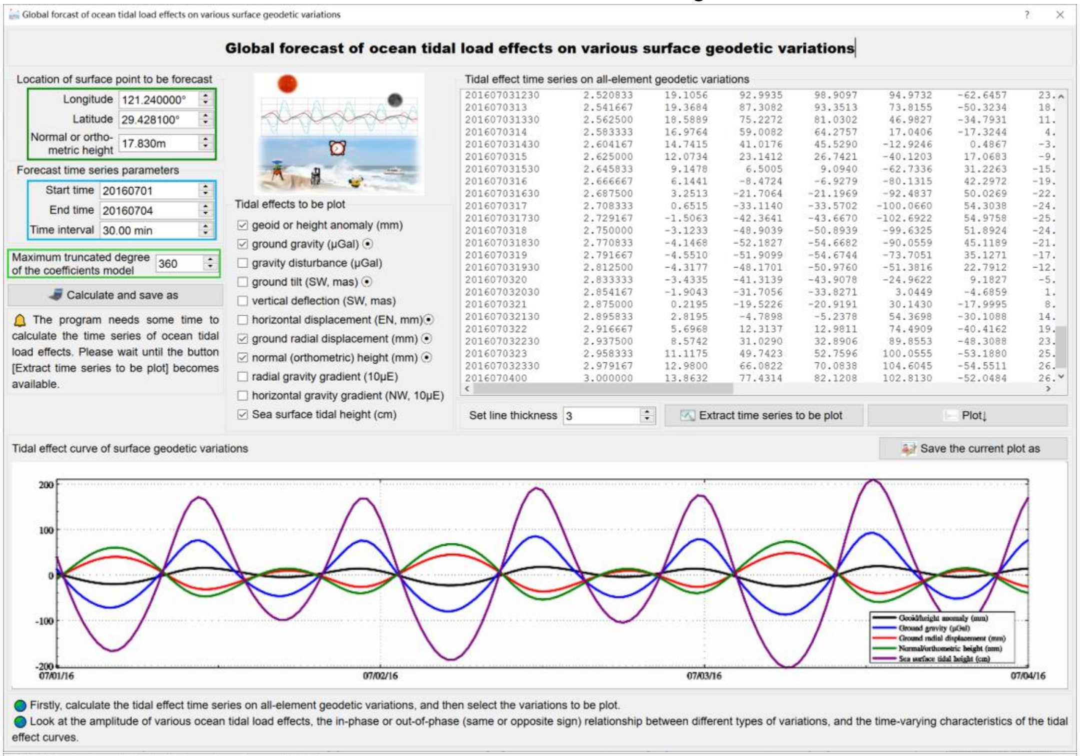
Look at the amplitude of various tidal effects, the in-phase or out-of-phase (same or opposite sign) relationship between different types of variations, and the time-varying characteristics of the tidal effect curves.

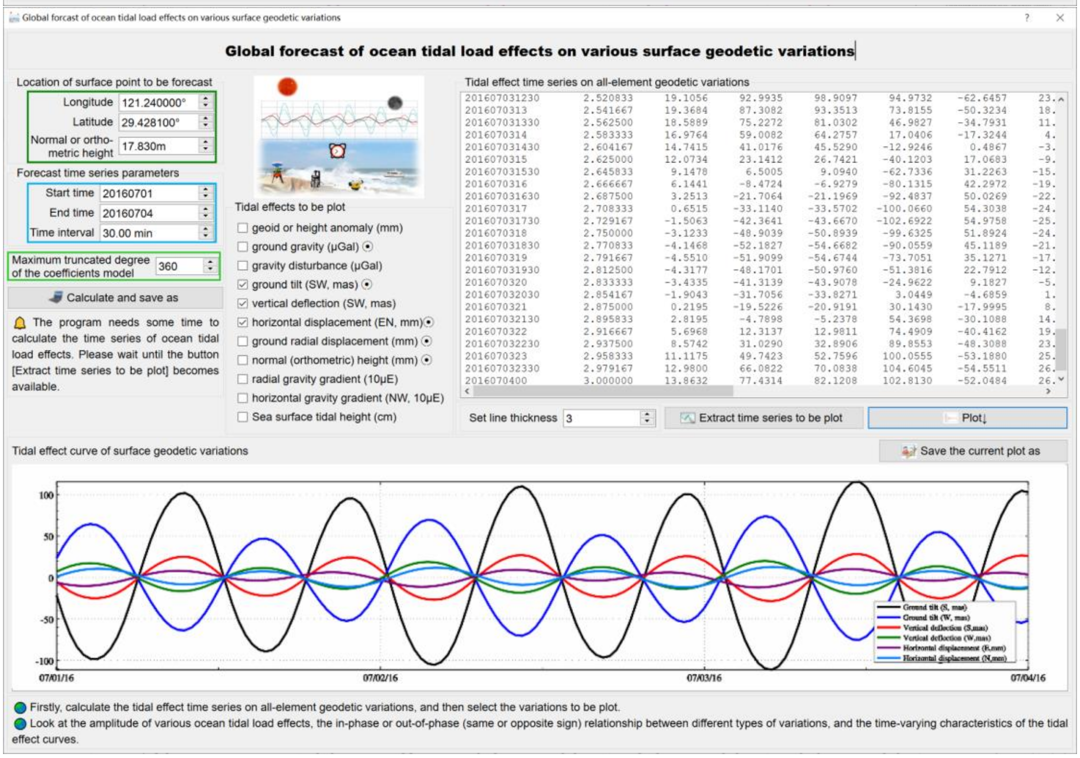
The ocean tidal load effect on gravity gradient can reach more than several of mE. The high-accuracy and high-resolution ocean tide model should be employed for high precision gravity gradient measurement in coastal areas.

The program outputs the ocean tidal load effect time series file at the calculation point. The file header occupies a row and includes the name of the calculation point, longitude (degree decimal), latitude (degree decimal), ellipsoidal height (m) and starting MJD0.

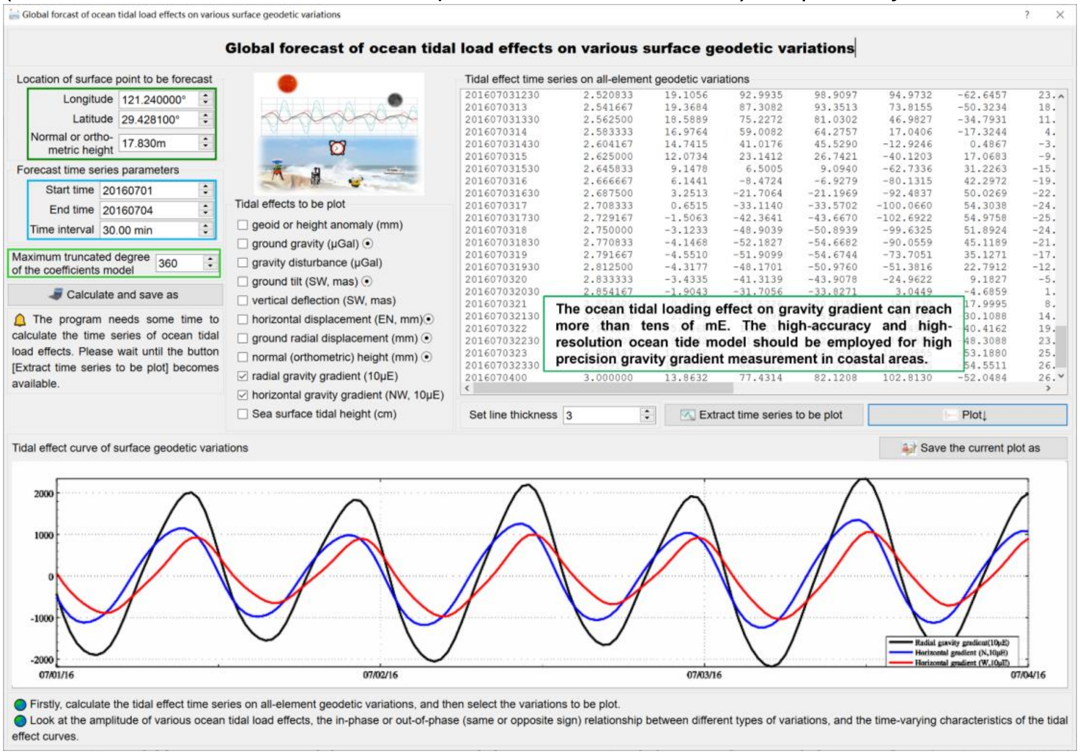
From the second row onwards, the first column of the record is the system format time, the second column is the difference between the MJD day and starting MJD0, and the last

14 columns are the ocean tidal load effects on all-element geodetic variations.





The last 14 columns of the record are the tidal load effects on the height anomaly (mm, 3rd column), ground gravity (μGal, 4th column), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas, 6th and 7th columns), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm, 10th and 11th columns), ground radial displacement (mm), ground normal or orthometric height (mm, 13th column), radial gravity gradient (10μE, 14th column) or horizontal gravity gradient (NW, to the north and to the west, 10μE, 15th and 16th columns), respectively.



2.3 Spherical harmonic synthesis on atmosphere tidal load effects outside solid Earth

[Purpose] Using the global surface atmosphere tidal load spherical harmonic coefficient model (hPa/mbar), compute the surface atmosphere tidal load effects on various geodetic variations on the ground or outside the solid Earth according to the location and time in the input file by the spherical harmonic synthesis algorithm. Here the point outside the solid Earth generally refers to the space point that is not fixed with the Earth in ocean space, near-Earth space, or satellite altitude.

The program adopts the 360-degree surface atmosphere tidal spherical harmonic coefficient model ECMWF2006.dat, which contains semi-diurnal, diurnal, semi-annual, and annual period constituents.

2.3.1 Computation of surface atmosphere tidal load effect time series at a ground site

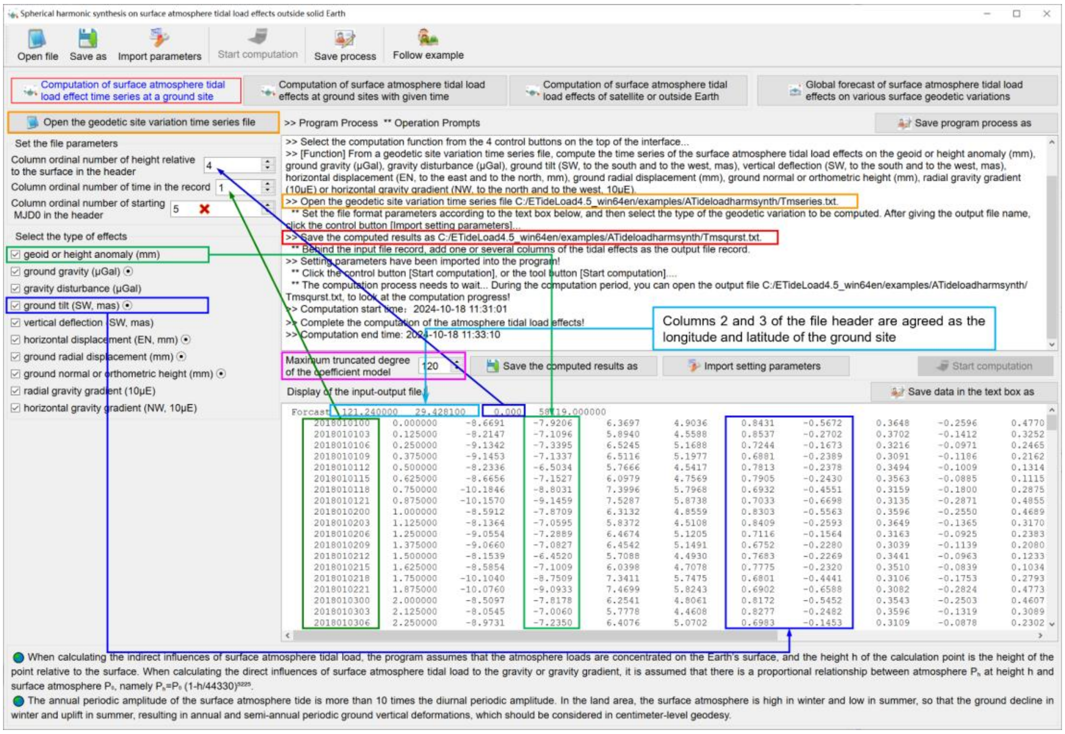
[Function] From a geodetic site variation time series file, compute the time series of the surface atmosphere tidal load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

[Input file] The geodetic site variation time series file.

The file header contains site name, longitude (degree decimal), latitude (degree decimal), height (m), starting MJD0 (optional), …

Starting from the second row of the file, each row record stores the sampling values of all the variations at one sampling epoch time. At least one column of the attributes in the record is the sampling epoch time.

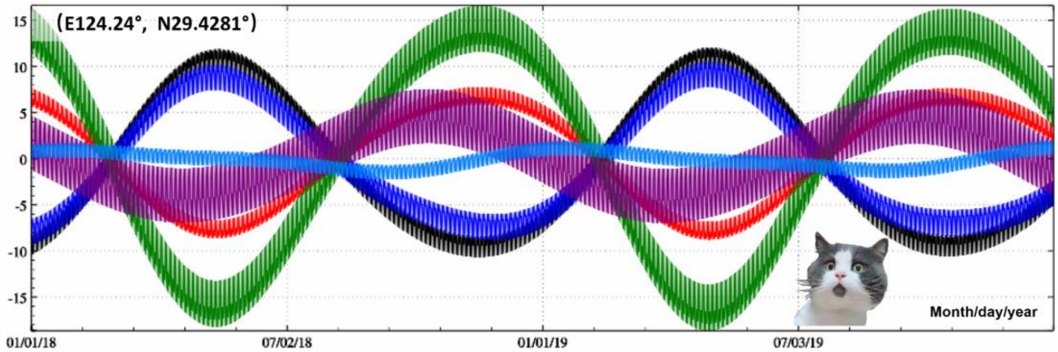
[Parameter settings] Set the input file format parameters, and select the type of the surface atmosphere tidal load effects.



[Output file] The geodetic site surface atmosphere tidal load effect time series file.

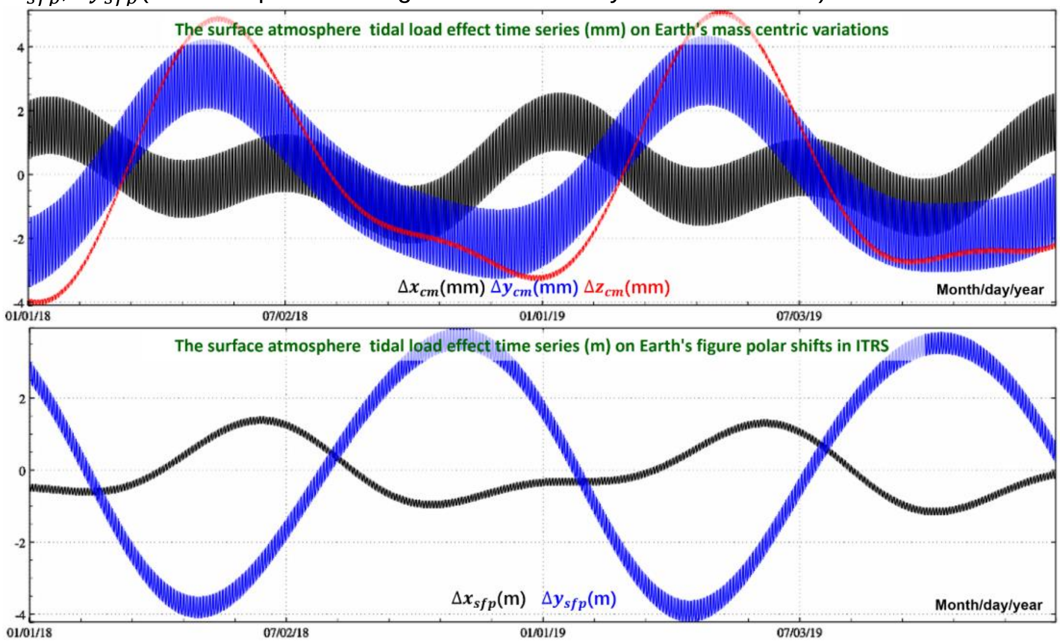
The file header is the same as the input file. Behind the input file record, adds one or several columns of the tidal effects selected as the output file record. In this example, all types are selected, and there are 14 attributes added to the record.

When calculating the indirect influences of surface atmosphere tidal load, the program assumes that the atmosphere loads are concentrated on the Earth’s surface, and the height h of the calculation point is the height of the point relative to the surface. When calculating the direct influences of surface atmosphere tidal load to the gravity or gravity gradient, it is assumed that there is a proportional relationship between atmosphere P_h at height h and surface atmosphere P_0, namely $P_h=P_0\ (1\text{-}h/44330)^{5225}$.



The Surface atmosphere tidal load effects (360-degree) : surface atmosphere(hPa/mbar) height anomaly (mm) ground gravity (μGal) orthometric height (mm) radial gravity gradient (10μE) horizontal displacement (N, 10μE)

The program also outputs the atmosphere tidal load effect time series file *Atdcmsfptm.txt on the degree-1 and degree-2 geopotential coefficients into the current directory. * is the input file name, which is an internal test file. The first row is the file header, and starting from the second row of the file, each row record stores the sampling epoch time, 3 columns of the atmosphere tidal load effects on the degree-1 normalized geopotential coefficients $\Delta\bar{C}_{10},\Delta\bar{C}_{11},\ \Delta\bar{S}_{11}$ ($\times10^{-10}$), 5 columns of the effects on the degree-2 geopotential coefficients $\Delta\bar{C}_{20},\Delta\bar{C}_{21},\ \Delta\bar{S}_{21},\Delta\bar{C}_{22},\Delta\bar{S}_{22}$ ($\times10^{-10}$), 3 columns of the effects (mm) on the Earth's center of mass and 2 columns of the effects on the figure polar shifts $\Delta x_{sfp},\Delta y_{sfp}$(in ITRS space rectangular coordinate system in unit of m).



The annual periodic amplitude of the surface atmosphere tide is more than 10 times the diurnal periodic amplitude. In the land area, the surface atmosphere is high in winter and low in summer, so that the ground decline in winter and uplift in summer, resulting in annual and semi-annual periodic ground vertical deformations, which should be considered in centimeter-level geodesy.

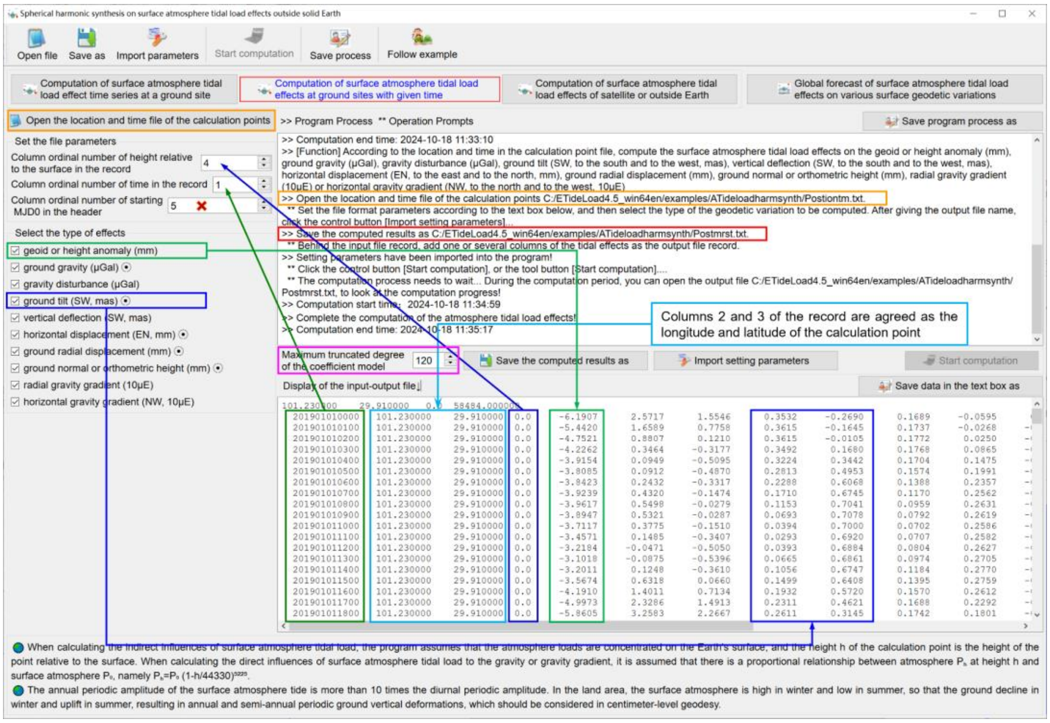
The surface atmosphere tidal load effects on the east-west component of the site displacement, tilt or horizontal gradient are generally smaller than that on the north-south component.

2.3.2 Computation of surface atmosphere tidal load effects at ground sites with given time

[Function] According to the location and time in the calculation point file, compute the surface atmosphere tidal load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

[Input file] The location and time file of the calculation points.

The first row is the file header. From the second row onwards, the second and third attributes in the file record are conventionally longitude and latitude (degree decimals), and there are the sampling epoch time and height attributes in the records.



[Parameter settings] Set the input file format parameters, and select the type of atmosphere tidal load effects.

[Output file] The surface atmosphere tidal load effect file.

The file header is the same as the input file. Behind the input file record, adds one or several columns of the tidal effects selected as the output file record. In this example, all types are selected, and there are 14 attributes added to the record.

The global surface atmosphere tidal load spherical harmonic coefficient model (hPa) adopts the FES2004 format, which can be constructed from the global surface atmosphere harmonic constant grid model by the function [Spherical harmonic analysis on surface atmosphere tidal harmonic constants]. In the program [System configs for the geophysical models and numerical standards], you can select the other global surface atmosphere tidal load spherical harmonic coefficient model.

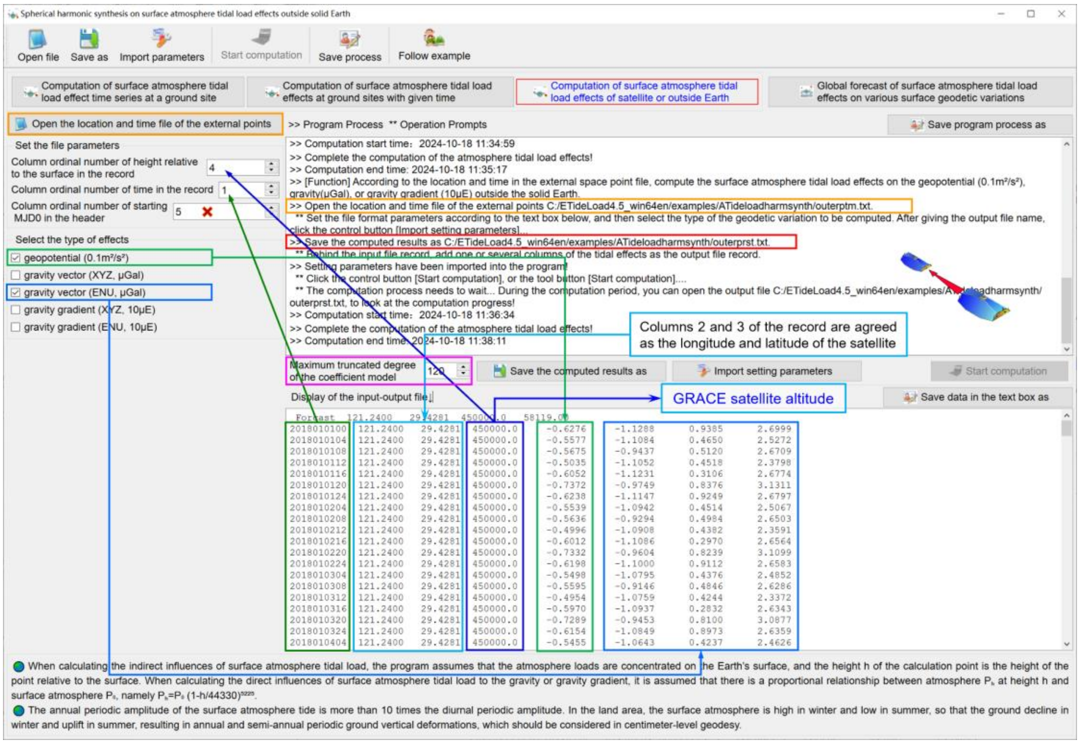
2.3.3 Computation of surface atmosphere tidal load effects of satellite or outside Earth

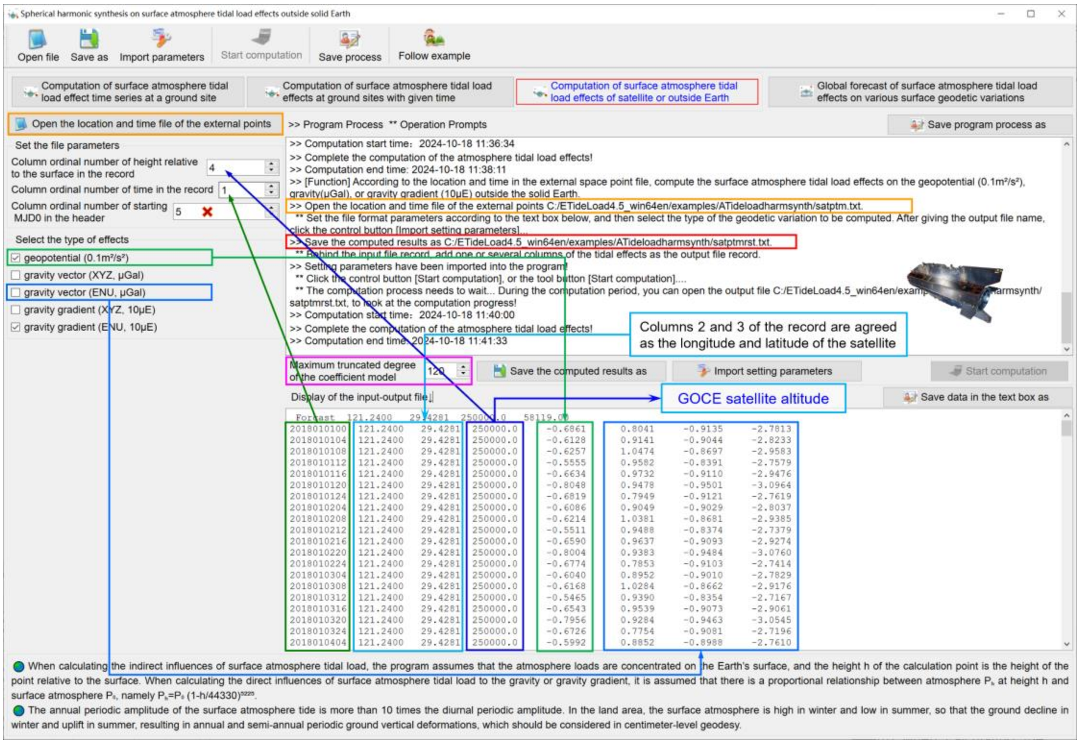
[Function] According to the location and time in the external space point file, compute the surface atmosphere tidal load effects on the geopotential ($0.1m^2/s^2$), gravity(μGal), or gravity gradient (10μE) outside the solid Earth.

[Input file] The location and time file of the external space points.

The first row is the file header. From the second row onwards, the second and third attributes in the file record are conventionally longitude and latitude (degree decimals), and there are the sampling epoch time and height attributes in the records.

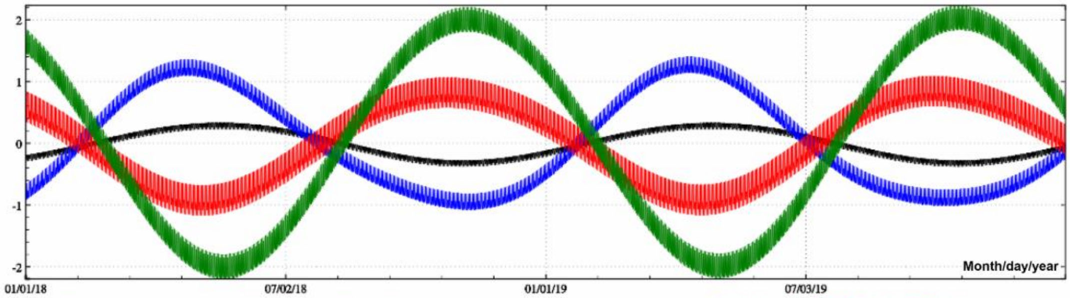
[Parameter settings] Set the input file format parameters, and select the type of surface atmosphere tidal load effects.



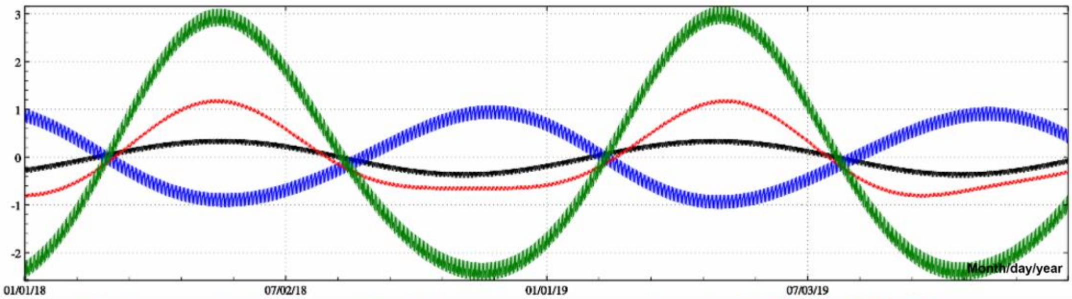


[Output file] The surface atmosphere tidal load effect file.

The file header is the same as the input file. Behind the input file record, adds one or several columns of the tidal effects selected as the output file record. In this example, the geopotential and gravity vector are selected, and there are 4 attributes added to the record.



The surface atmosphere tidal effects at 450km altitude: geopotential (0.1m²/s²), gravity vector (E, N: along the GRACE orbit/SST-II, U, μGal)



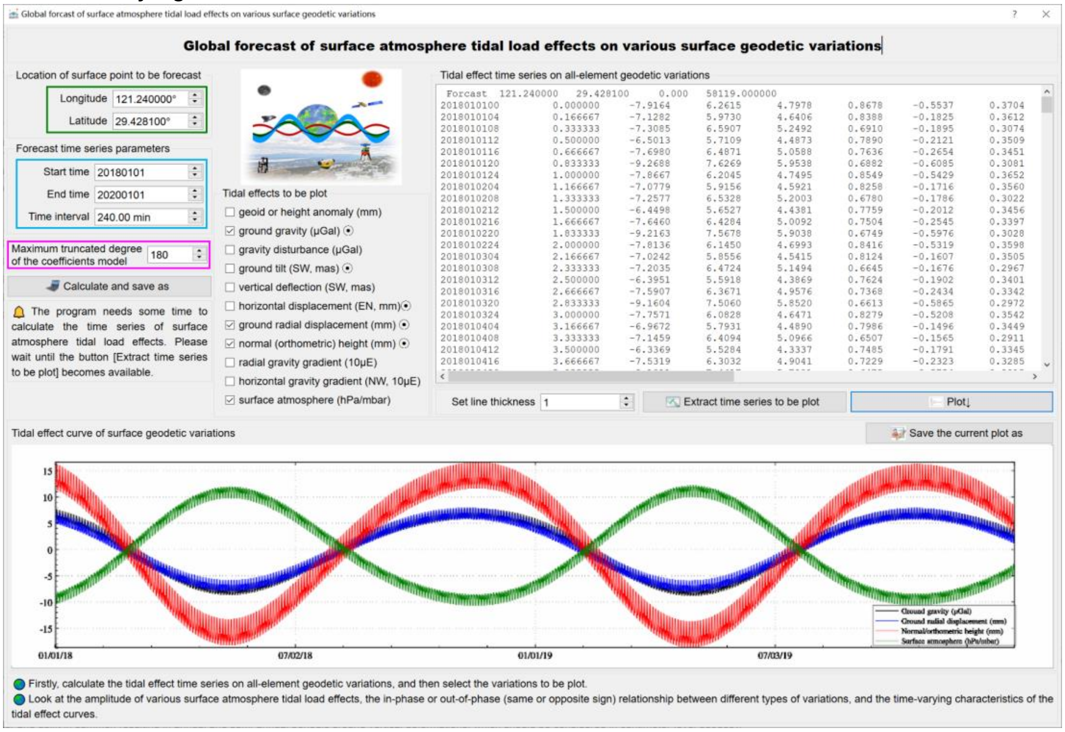
The surface atmosphere tidal effects at 250km altitude: geopotential (0.1m²/s²), gravity gradient (E, N: along the GOCE orbit, U, 10μE)

2.3.4 Global forecast of surface atmosphere tidal load effects on various surface geodetic variations

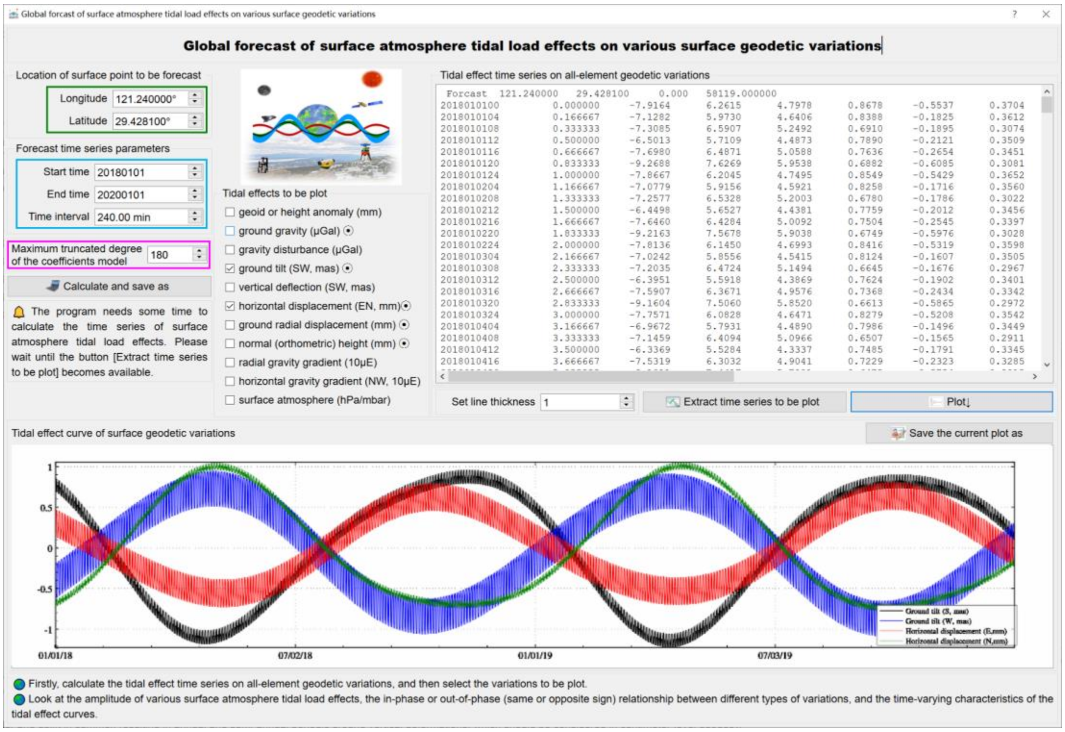
[Function] Input the geodetic coordinates of a global anywhere surface point and set the forecast time series parameters, calculate and display the surface atmosphere tidal load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

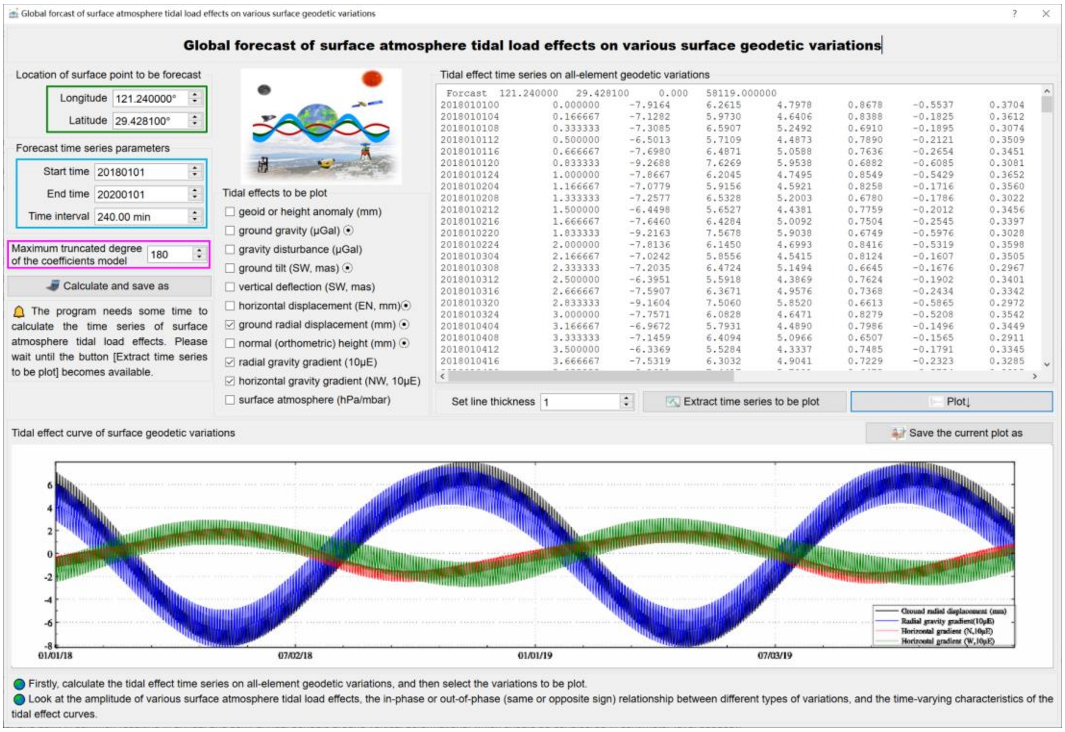
Firstly, calculate the tidal effect time series on all-element geodetic variations, and then select the variations to be plot.

Look at the amplitude of various surface atmosphere tidal load effects, the in-phase or out-of-phase (same or opposite sign) relationship between different types of variations, and the time-varying characteristics of the tidal effect curves.



The program outputs the atmosphere tidal load effect time series file at the calculation point. The file header occupies a row and includes the name of the calculation point, longitude (degree decimal), latitude (degree decimal), ellipsoidal height (m) and starting MJD0. From the second row onwards, the first column of the record is the system format time, the second column is the difference between the MJD day and starting MJD0, and the last 14 columns are the surface atmosphere tidal load effects on all-element geodetic variations.





The last 14 columns of the record are the height anomaly (mm, 3rd column), ground gravity (μGal, 4th column), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas, 6th and 7th columns), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm, 10th and 11th columns), ground radial displacement (mm), ground normal or orthometric height (mm, 13th column), radial gravity gradient (10μE, 14th column) or horizontal gravity gradient (NW, to the north and to the west, 10μE, 15th and 16th columns), respectively.

2.4 Computation of Earth's rotation polar shift effects on geodetic variations and tidal effects on EPR

[Purpose] Using IERS Earth orientation parameters (EOP) product file IERSeopc04.dat, compute the Earth's rotation polar shift and ocean pole tidal effects on various geodetic variations on the ground or outside the solid Earth, or compute the tidal effects on Earth rotation parameters (EPR).

The Earth's rotation polar shift and figure polar shift are two different types of geodetic variations. The Earth's figure polar shift is the normalized angular momentum, which is equal to the mass load excitation of the Earth's rotation motion.

2.4.1 Computation of the rotation polar shift or ocean pole tidal effect time series at a ground site

[Function] From the geodetic site variation time series file, compute the time series of the Earth's rotation polar shift or ocean pole tidal effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

[Input file] The geodetic site variation time series file.

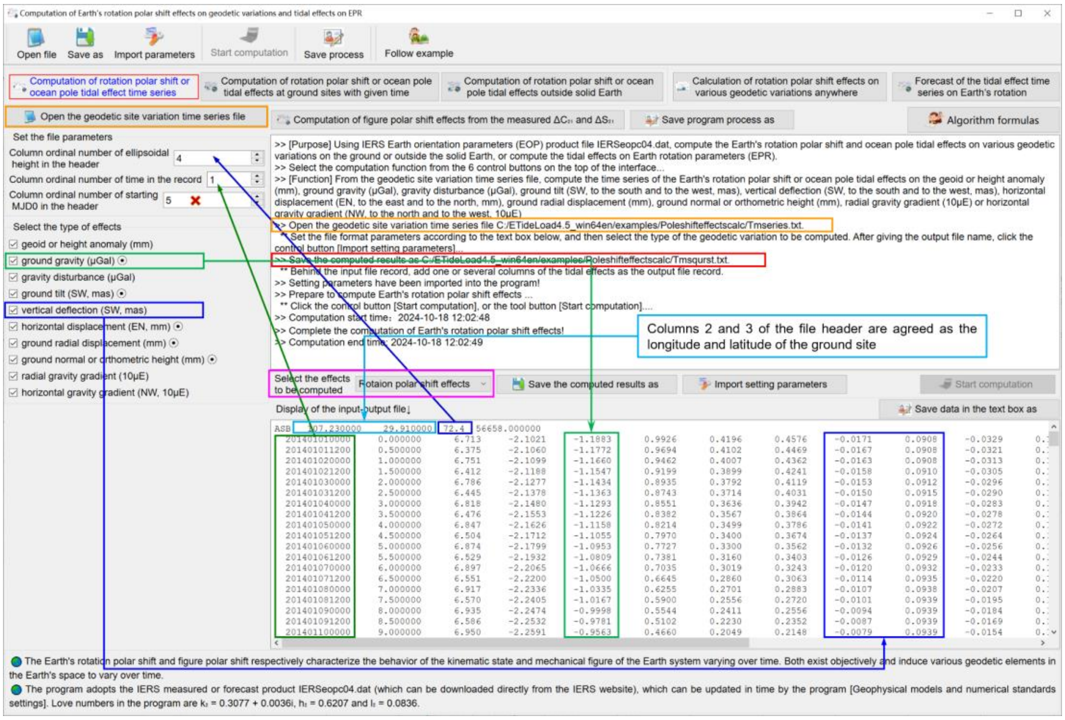
The file header contains site name, longitude (degree decimal), latitude (degree decimal), ellipsoidal height (m), starting MJD0 (optional), …

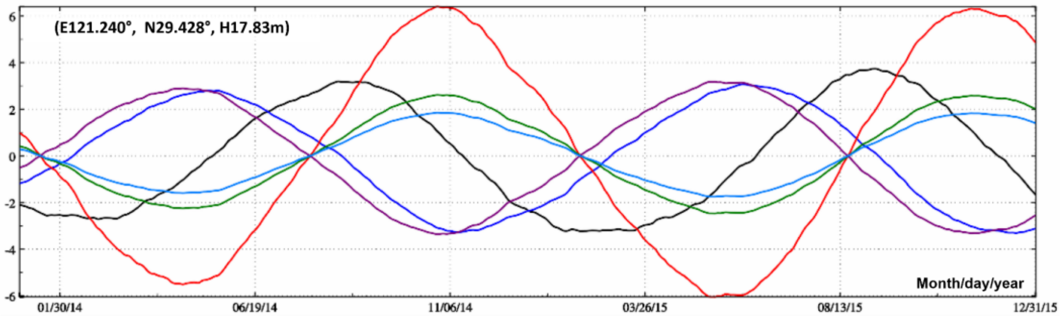
Starting from the second row of the file, each row record stores the sampling values of all the variations at one sampling epoch time. At least one column of the attributes in the record is the sampling epoch time.

[Parameter settings] Set the input file format parameters, and select the type of the rotation polar shift or ocean pole tidal effects.

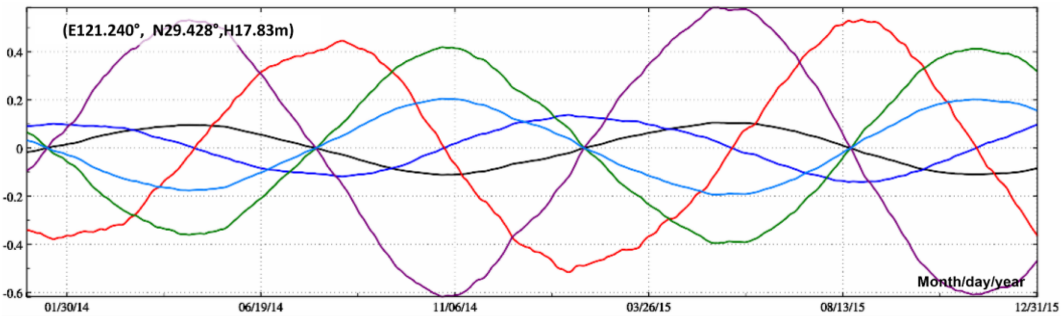
[Output file] The Earth's rotation polar shift or ocean pole tidal effect time series file at the geodetic site.

The file header is the same as the input file. When the rotation polar shift effect selected, adds 2 columns of the figure polar coordinate variations $\Delta x_{sfp}, \Delta y_{sfp}$ (in ITRS space rectangular coordinate system in unit of m) and several columns of the effects selected behind the input file record, as the output file record. When the ocean pole tidal effect selected, adds one or several columns of the effects selected as the output file record. The rotation polar shift effect selected in this example, all types are selected, and there are 2+14 attributes added to the record.





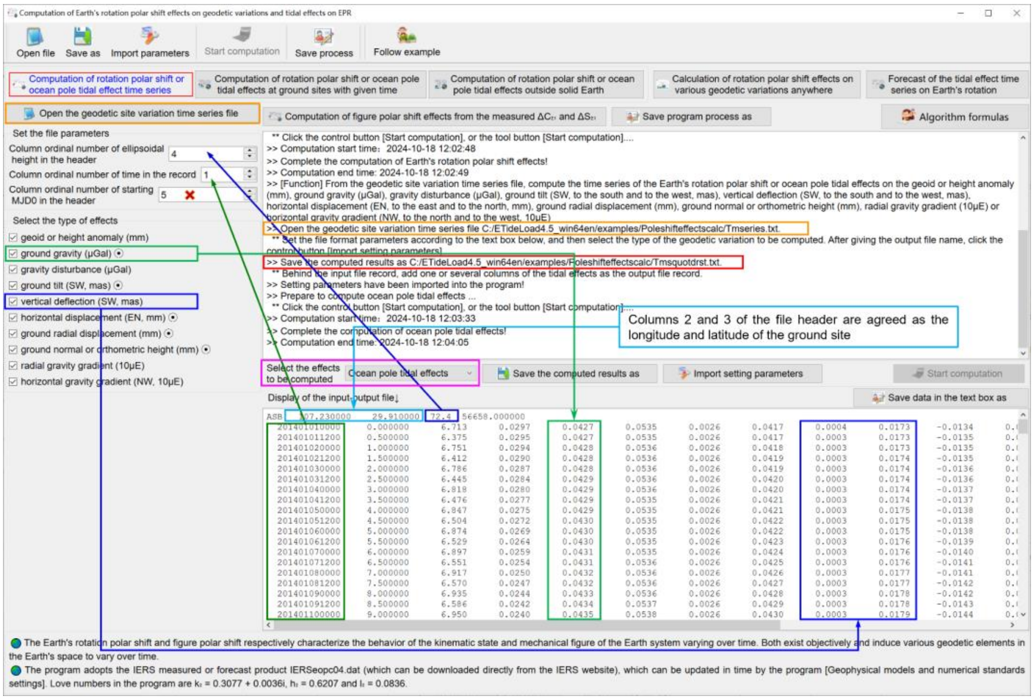
Earth's rotation polar shift (Δx_pm, Δy_pm in ITRS) effects: height anomaly (mm) ground gravity (μGal) orthometric height (mm) radial gravity gradient (10μE)

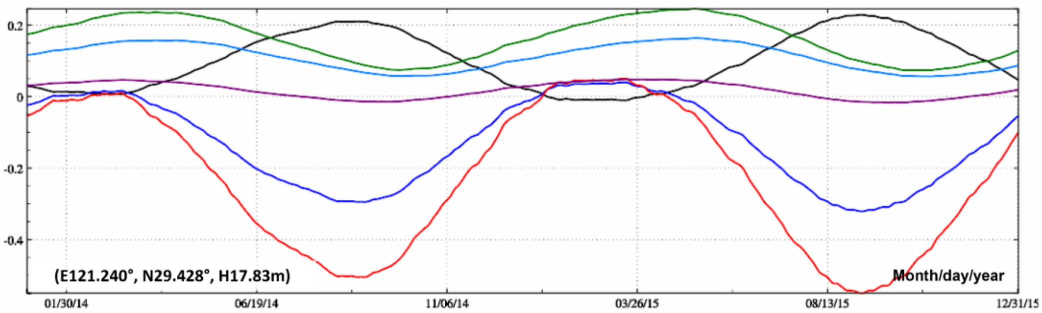


Earth's rotation polar shift effects: ground tilt (S, W, mas) horizontal displacement (E, N, mm) horizontal gravity gradient (N, W, 10μE)

Love numbers in the program are k_2 = 0.3077 + 0.0036i, h_2 = 0.6207 and l_2 = 0.0836.

If the epoch time to be calculated exceeds the time range of the Earth orientation parameter time series file, please update the Earth orientation parameter time series file.





Ocean pole tidal effects: height anomaly (mm) ellipsoidal height (mm) orthometric height (mm) radial gravity gradient (10μE)10μE horizontal gravity gradient (N, W, 10μE)

2.4.2 Computation of the rotation polar shift or ocean pole tidal effects at ground sites with given time

[Function] According to the location and time in the calculation point file, compute the Earth's rotation polar shift or ocean pole tidal effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

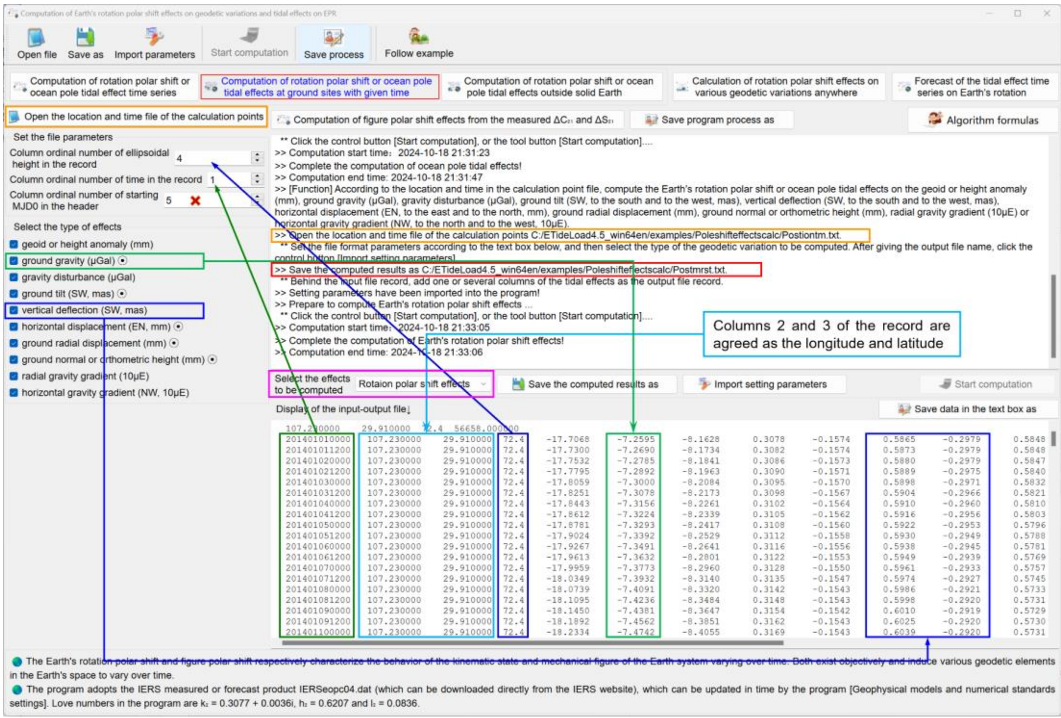
[Input file] The location and time file of the calculation points.

The first row is the file header. From the second row onwards, the second and third attributes in the file record are conventionally longitude and latitude (degree decimals), and there are the sampling epoch time and ellipsoidal height attributes in the record.

[Parameter settings] Set the input file format parameters, and select the type of the effects.

[Output file] The Earth's rotation polar shift or ocean pole tidal effect file.

The file header is the same as the input file. Behind the input file record, adds one or several columns of the Earth's rotation polar shift or ocean pole tidal effects selected as the output file record.



2.4.3 Computation of the rotation polar shift or ocean pole tidal effects of satellite or outside solid Earth

[Function] According to the location and time in the external sapce point file, compute the Earth's rotation polar shift or ocean pole tidal effects on the geopotential (0.1m²/s²), gravity(μGal), or gravity gradient(10μE) outside the solid Earth.

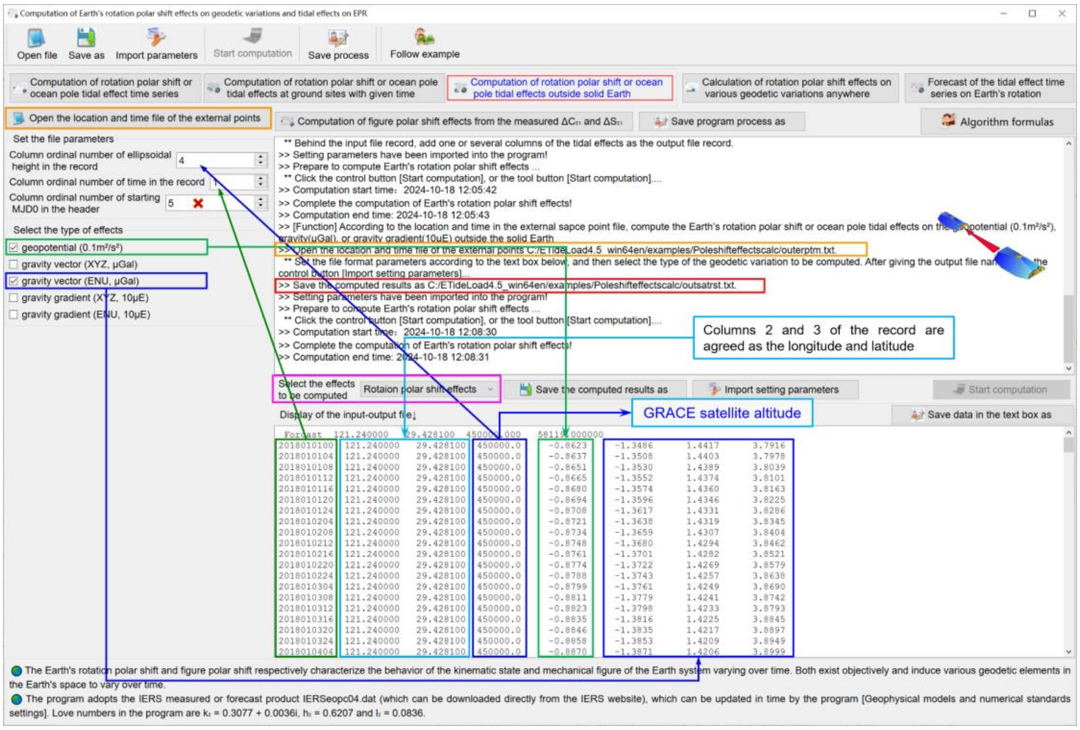
[Input file] The location and time file of the external space points.

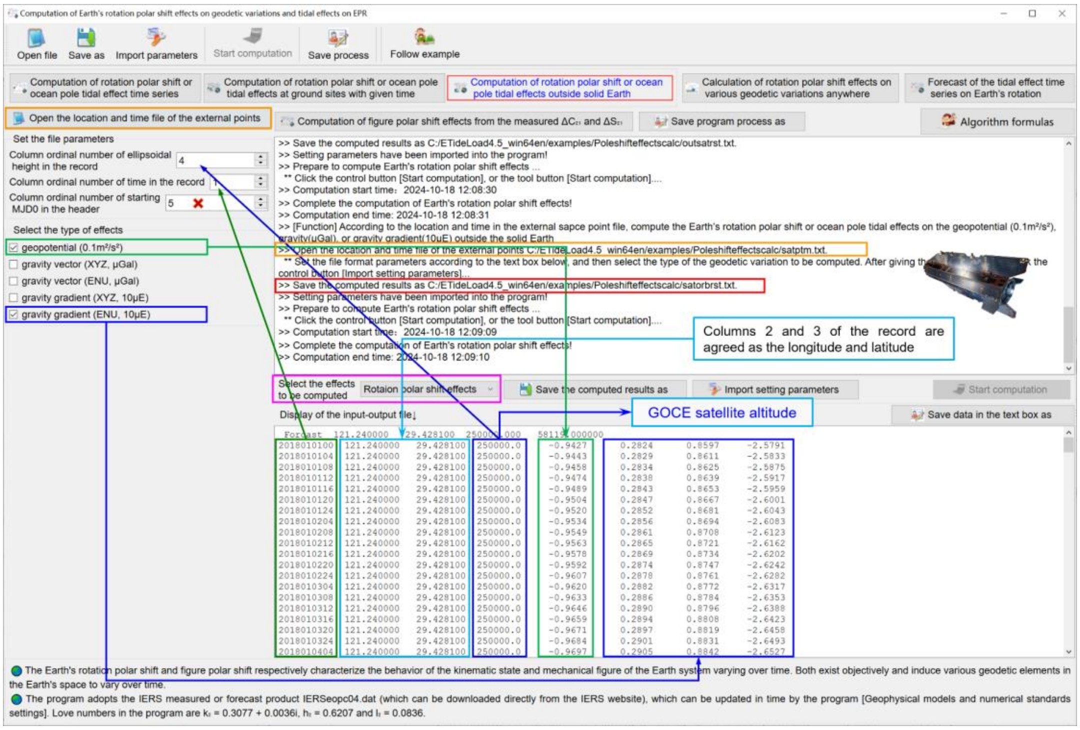
The first row is the file header. From the second row onwards, the second and third attributes in the file record are conventionally longitude and latitude (degree decimals), and there are the sampling epoch time and ellipsoidal height attributes in the records.

[Parameter settings] Set the input file format parameters and select the type of the effects.

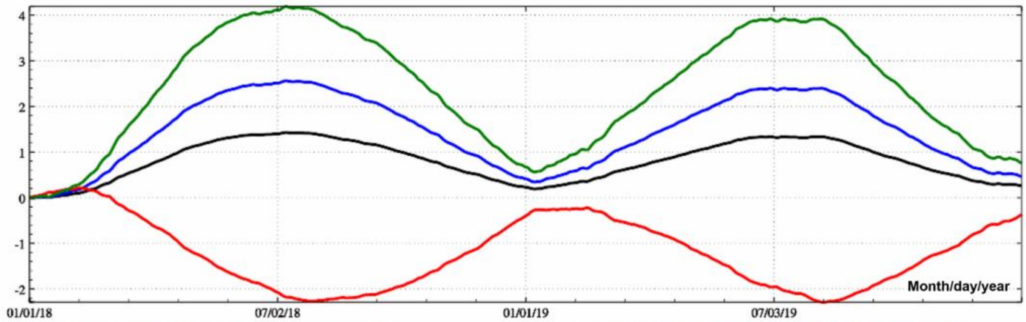
[Output file] The Earth's rotation polar shift or ocean pole tidal effect file.

For geodetic applications with an one-centimeter accuracy level, the Earth's rotation polar shift effects should be considered. The magnitude of the ocean pole tidal effects is small (less than 1cm), and it can be ignored for regional geodetic purposes.

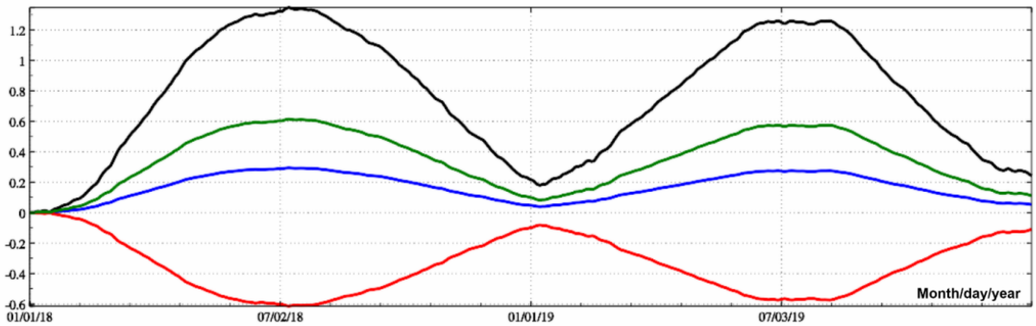




The rotation polar shift is non-tidal, so it is difficult to accurately model the non-tidal effects. The program adopts the IERS measured or forecast product IERSeopc04.dat (which can be downloaded directly from the IERS website), which can be updated in time by the program [System configs for the geophysical models and numerical standards].



Earth's rotation polar shift effects at 450km altitude：geopotential (0.1m²/s²), gravity vector (E, N: along the GRACE orbit/SST-II, U, μGal)



Earth's rotation polar shift effects at 250km altitude：geopotential (0.1m²/s²), gravity gradient (E, N: along the GOCE orbit, U, 10μE)

2.4.4 Calculation of rotation polar shift effects on various surface geodetic variations anywhere

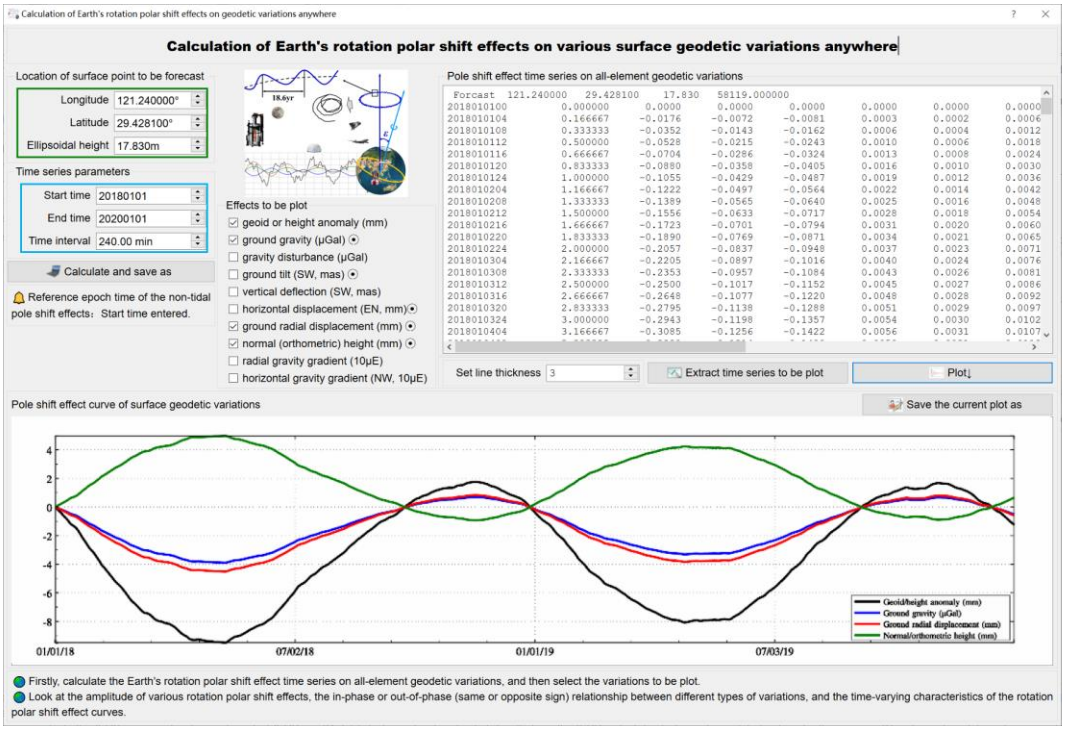
[Function] Input the geodetic coordinates of a global anywhere surface point and set the time series parameters, calculate and display the Earth's rotation polar shift effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

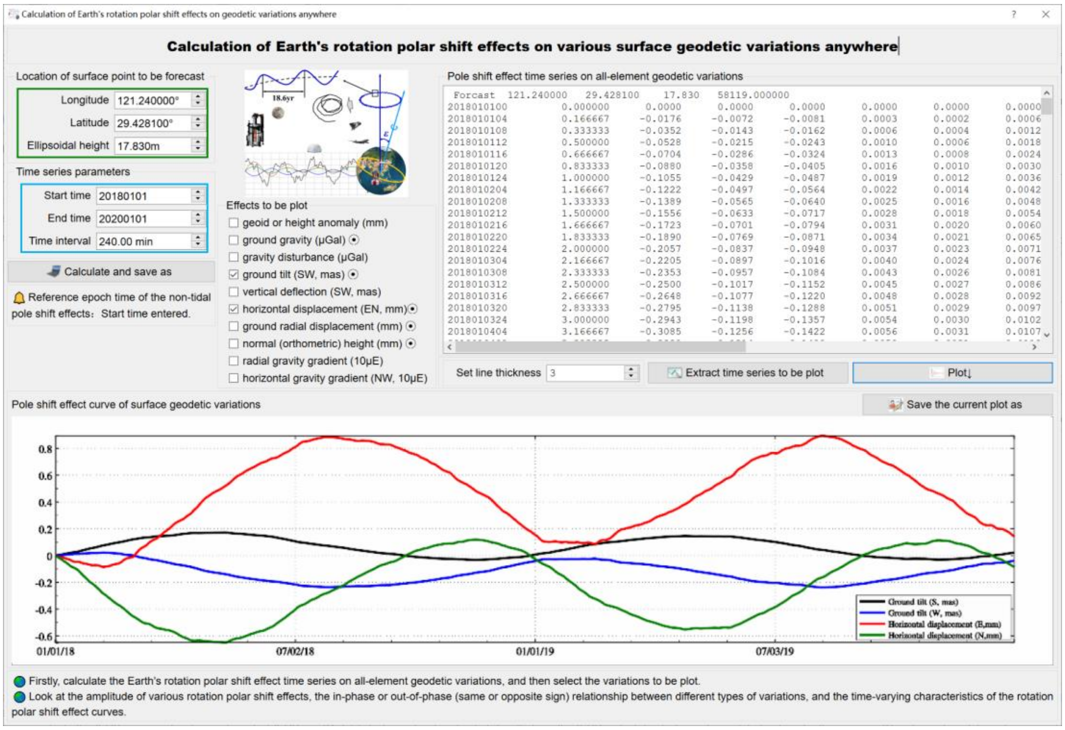
Firstly, calculate the Earth's rotation polar shift effect time series on all-element geodetic variations, and then select the variations to be plot.

Look at the amplitude of various rotation polar shift effects, the in-phase or out-of-phase (same or opposite sign) relationship between different types of variations, and the time-varying characteristics of the rotation polar shift effect curves.

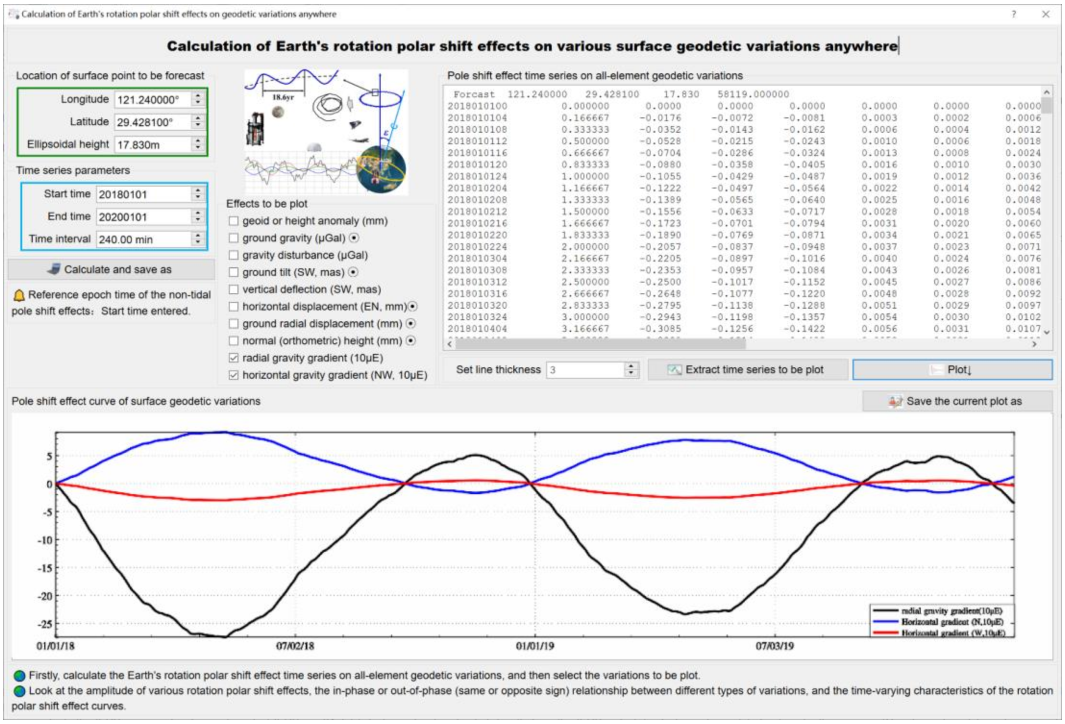
The program outputs the Earth's rotation polar shift effect time series file at the calculation point. The file header occupies a row and includes the name of the calculation point, longitude (degree decimal), latitude (degree decimal), ellipsoidal height (m) and starting MJD0.

From the second row onwards, the first column of the record is the system format time, the second column is the difference between the MJD day and starting MJD0, and the last 14 columns are the Earth's rotation polar shift effects on all-element geodetic variations.





The last 14 columns of the record are the tidal load effects on the height anomaly (mm, 3rd column), ground gravity (μGal, 4th column), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas, 6th and 7th columns), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm, 10th and 11th columns), ground radial displacement (mm), ground normal or orthometric height (mm, 13th column), radial gravity gradient (10μE, 14th column) or horizontal gravity gradient (NW, to the north and to the west, 10μE, 15th and 16th columns), respectively.



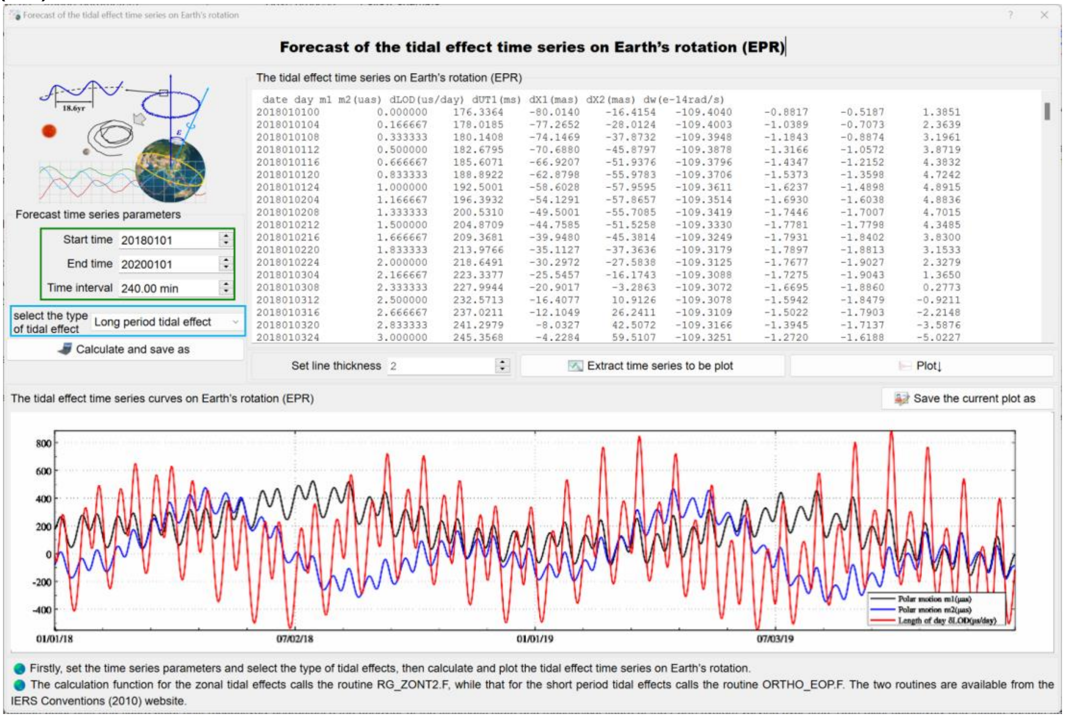
2.4.5 Forecast of the tidal effect time series on Earth's rotation

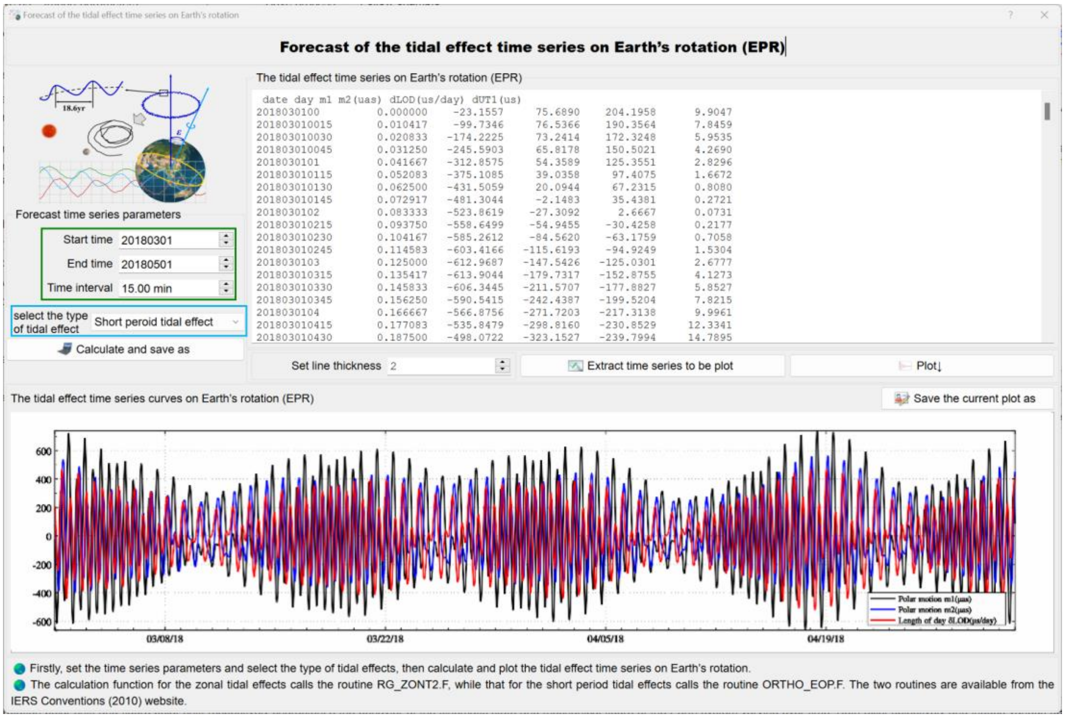
[Function] Set the time series parameters, calculate and display the long period or short peroid tidal effects on Earth's rotation. Here, the long period tidal effects include the zonal tidal effects and long period ocean tidal effects on Earth's rotation. The calculation function for the zonal tidal effects calls the routine RG_ZONT2.F, while that for the short period tidal effects calls the routine ORTHO_EOP.F. The two routines are available from the IERS conventions (2010) website.

Firstly, set the time series parameters and select the type of tidal effects, then calculate and plot the tidal effect time series on Earth's rotation.

When "Long-period tidal effect" selected, the program outputs the long-period tidal effect time series file on Earth's rotation parameters (ERP), and the file record include the epoch date (long integer agreed by ETideLoad), number of days relative to the first epoch, rotation polar shift m_1 and m_2 (μas), length variation of day (μs/day), UT1 variation (ms), effective angular momentum (EAM) variation χ_1 and χ_2 (mas), and rotation rate varition (10^{-14}rad/s).

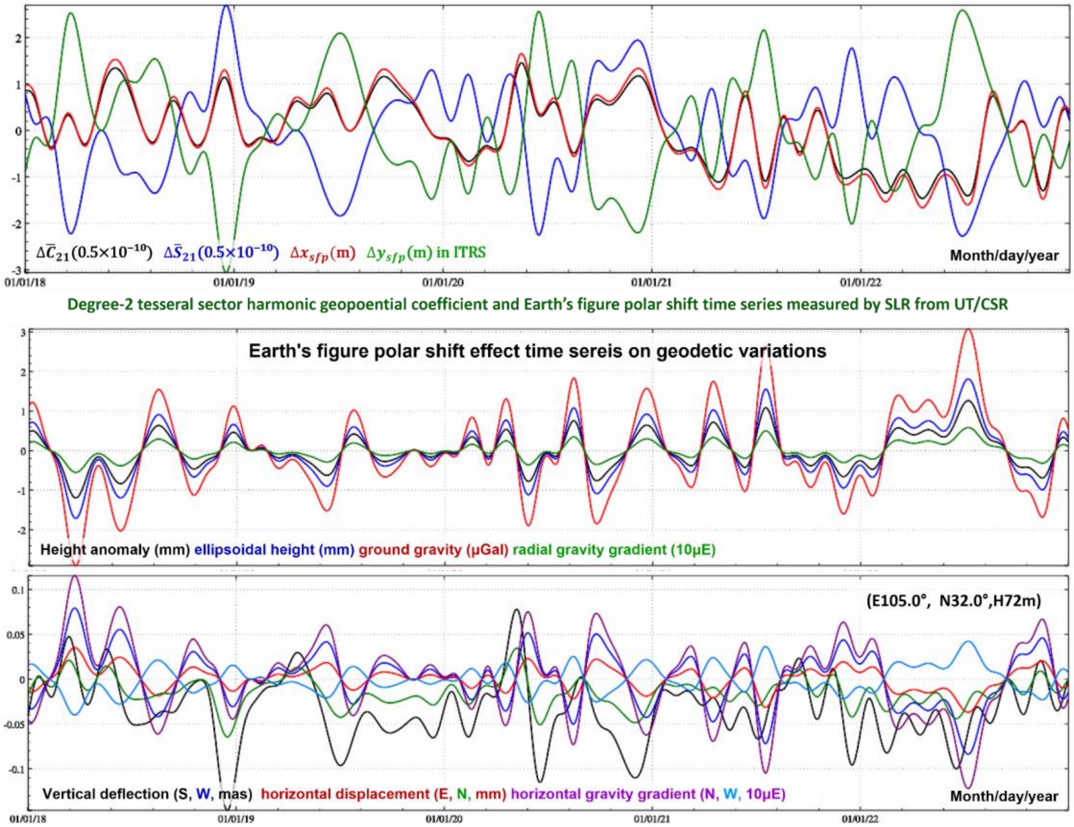
When "Short-period tidal effect" selected, the program outputs the diurnal and semi-diurnal ocean tidal effect time series file on Earth's rotation, and the file record include the epoch date (long integer agreed by ETideLoad), number of days relative to the first epoch, rotation polar shift m_1 and m_2 (μas), length variation of day (μs/day) and UT1 variation (ms).





[Computation of Earth's figure polar shift effects on geodetic variations from the measured ΔC_{21} and ΔS_{21}] (An internal test program)

Input the site time series file and the UT/CSR RL-06 ΔC_{21} and ΔS_{21} monthly time series file C21_S21_RL06.txt in the directory C:\ETideLoad4.5_win64en\iers (the first 15 rows in the file ignored by the program) to compute the Earth's figure polar shift effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).



2.5 Computation of the permanent tidal and Earth's mass centric variation effects on geodetic variations

[Purpose] Compute the permanent tidal effects and Earth's mass centric variation effects on various geodetic variations.

2.5.1 Computation of permanent tidal effects on various geodetic variations

[Function] According to the location in the point record file, compute the permanent tidal effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

[Input file] The geodetic point record file.

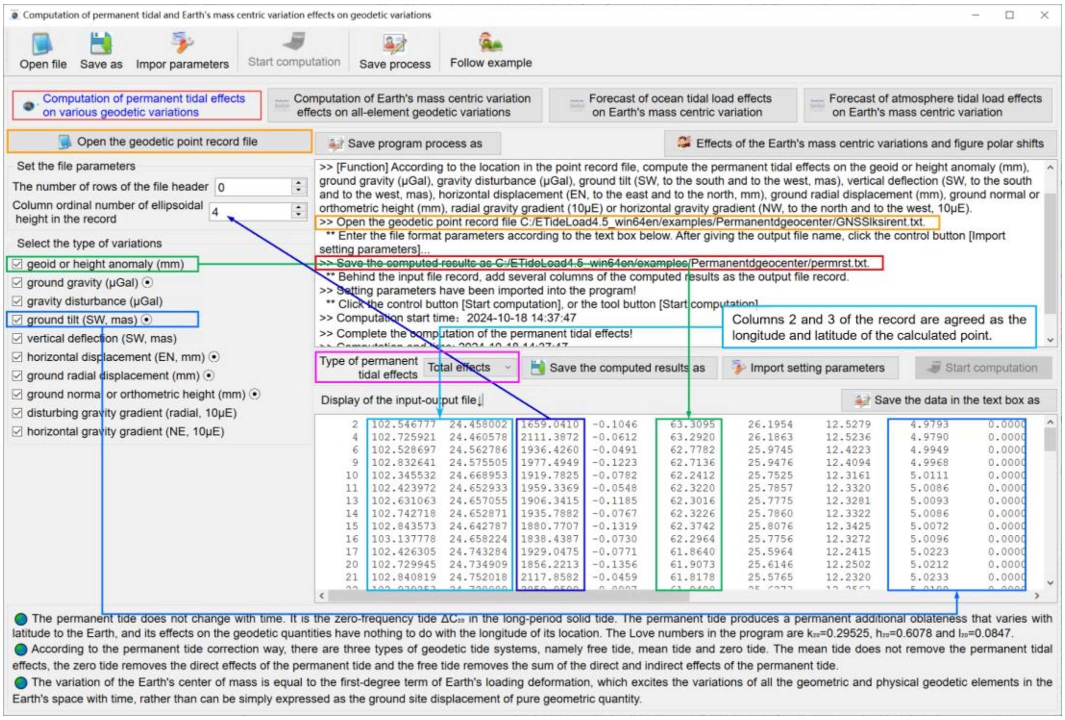
Multi-row file headers are allowed without the content and format limited.

A row of record stands for the data for a geodetic site. Each record include the site number (name), longitude (degree decimal), latitude (degree decimal), …. There is an ellipsoid height attribute in the record.

[Parameter settings] Set the input file format parameters, and select the type of permanent tidal effects.

[Output file] The permanent tidal effect file.

The file header is the same as the input file. Behind the input file record, adds one or several columns of the permanent tidal effects selected as the output file record. In this example, all types are selected, and there are 14 attributes added to the record.



The permanent tide does not change with time. It is the zero-frequency tide ΔC_{20} in the long-period solid tide. The permanent tide produces a permanent associated oblateness that varies with latitude to the Earth, and its effects on the geodetic variations have nothing to do with the longitude of its location.

The Love numbers in the program are k_{20}=0.29525, h_{20}=0.6078 and l_{20}=0.0847.

According to the permanent tide correction way, there are three types of geodetic tide systems, namely free tide, mean tide and zero tide. The mean tide does not remove the permanent tidal effects, the zero tide removes the direct effects of the permanent tide and the free tide removes the sum of the direct and indirect effects of the permanent tide.

There is no direct effect of the tidal potential on the ground geometric geodetic variations. Therefore, the zero-tide geometric geodetic variations are equal to the mean-tide geometric geodetic variations.

2.5.2 Computation of Earth's mass centric variation effects on all-element geodetic variations

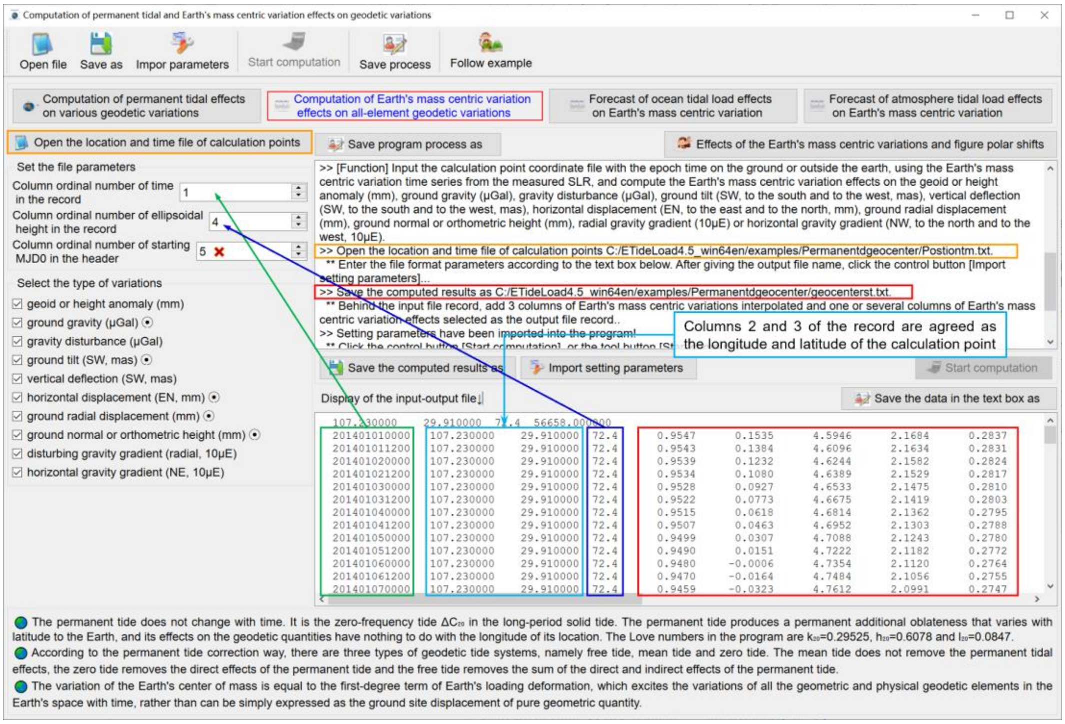
[Function] Input the calculation point coordinate file with the epoch time on the ground or outside the earth, using the Earth's mass centric variation time series from the measured SLR, and compute the Earth's mass centric variation effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE).

The variation of the Earth's center of mass is equal to the first-degree term of Earth's loading deformation, which excites the variations of all the geometric and physical geodetic elements in the Earth's space with time, rather than can be simply expressed as the ground site displacement of pure geometric quantity.

[Input file] The location and time file of the calculation points.

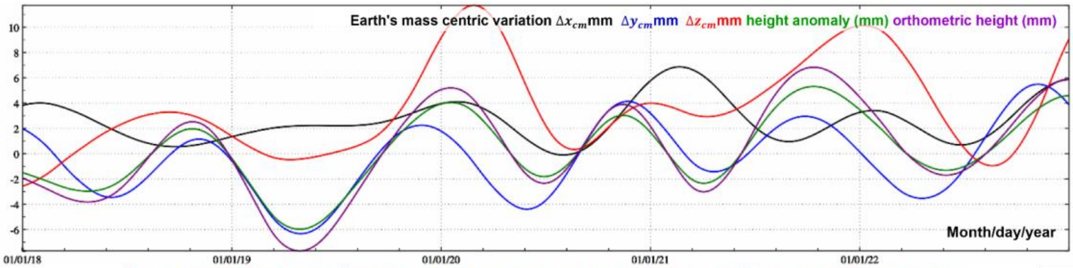
The first row is the file header. From the second row onwards, the second and third attributes in the file record are conventionally longitude and latitude (degree decimals), and there are the sampling epoch time and ellipsoidal height attributes in the records.

[Parameter settings] Set the input file format parameters, and select the type of the effects.

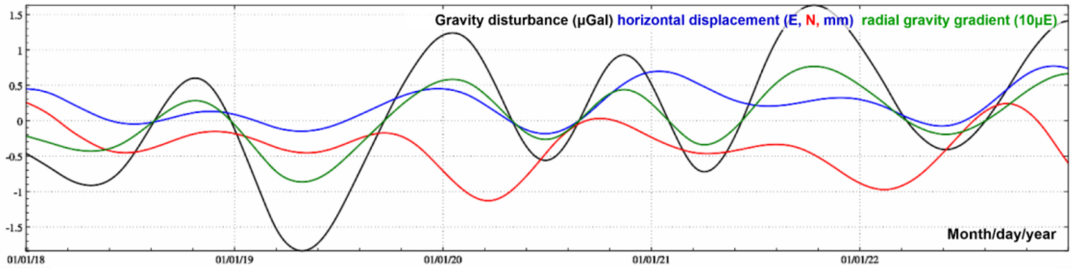


[Output file] The Earth's mass centric variation effect file.

The file header is the same as the input file. Behind the input file record, adds 3 columns of Earth's mass centric variations interpolated and one or several columns of Earth's mass centric variation effects selected as the output file record.



Earth's mass centric variation and their effects on the height anomaly geoid and orthometric height



Earth's mass centric variation effect time series on various geodetic variations

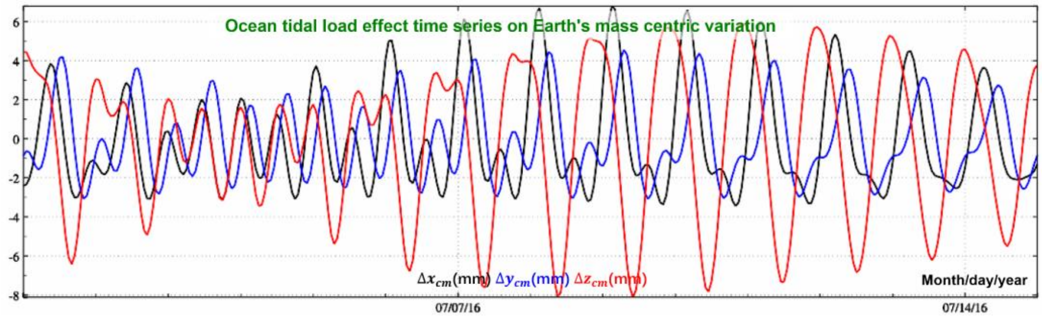
If the epoch time to be calculated exceeds the time range of Earth's mass centric variation time series from the measured SLR, please update the time series file.

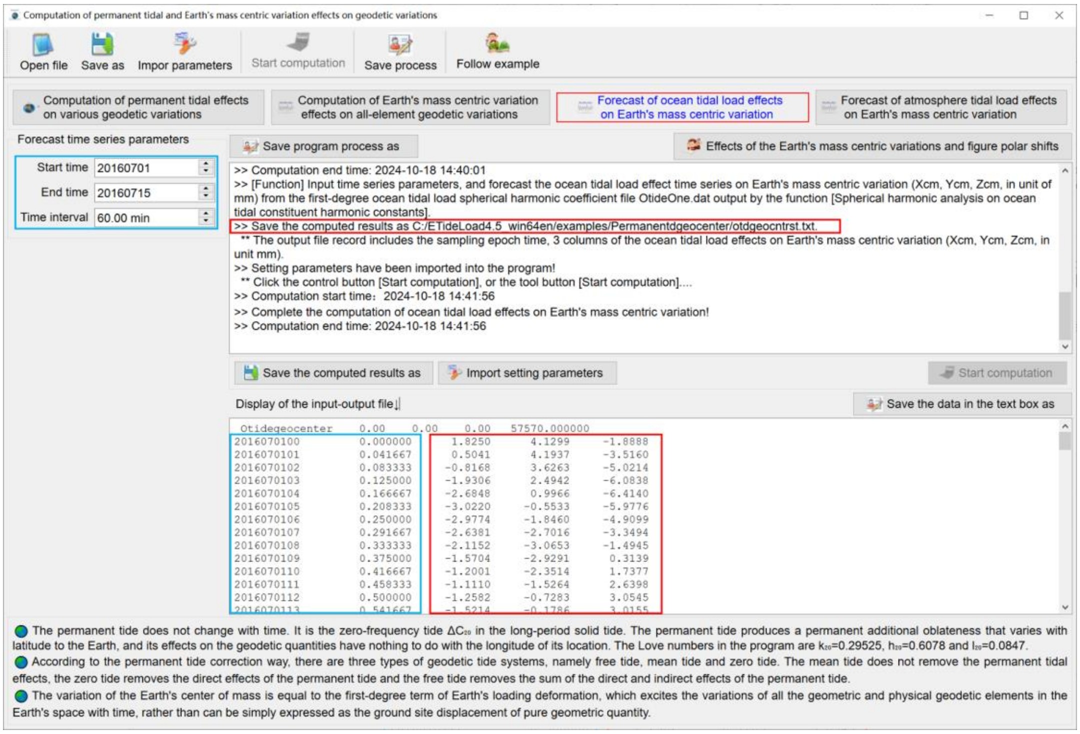
2.5.3 Forecast of ocean tidal load effects on Earth's mass centric variation

[Function] Input time series parameters, and forecast the ocean tidal load effect time series on Earth's mass centric variation (Xcm, Ycm, Zcm, in unit of mm) from the first-degree ocean tidal load spherical harmonic coefficient file OtideOne.dat output by the function [Spherical harmonic analysis on ocean tidal constituent harmonic constants].

[Output file] The ocean tidal load effect time series file on Earth's mass centric variation.

The first row is the file header, and starting from the second row of the file, each row record stores the sampling epoch time, 3 columns of the ocean tidal load effects on Earth's mass centric variation (Xcm, Ycm, Zcm, in unit of mm).



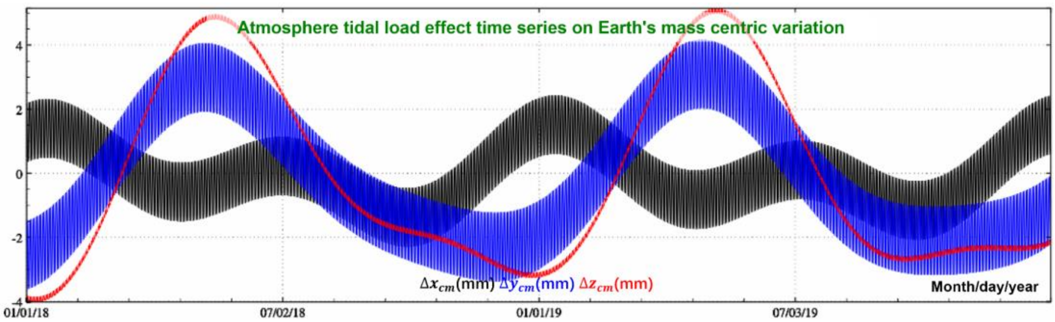


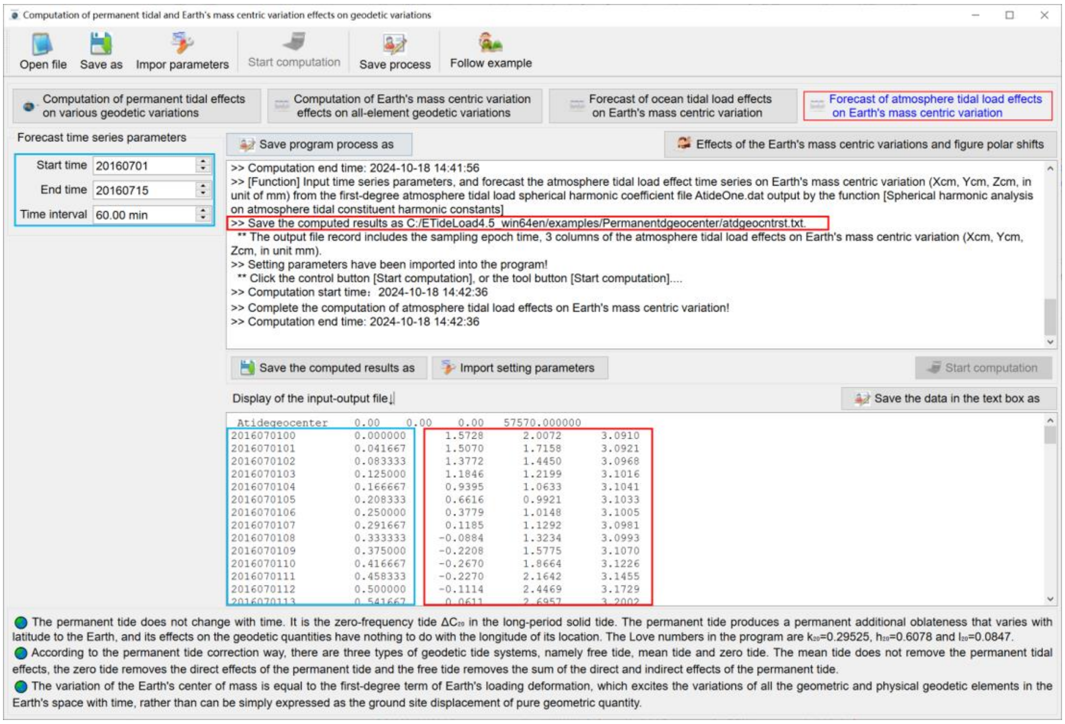
2.5.4 Forecast of atmosphere tidal load effects on Earth's mass centric variation

[Function] Input time series parameters, and forecast the atmosphere tidal load effect time series on Earth's mass centric variation (Xcm, Ycm, Zcm, in unit of mm) from the first-degree atmosphere tidal load spherical harmonic coefficient file AtideOne.dat output by the function [Spherical harmonic analysis on atmosphere tidal constituent harmonic constants].

[Output file] The atmosphere tidal load effect time series file on Earth's mass centric variation.

The first row is the file header, and starting from the second row of the file, each row record stores the sampling epoch time, 3 columns of the atmosphere tidal load effects on Earth's mass centric variation (Xcm, Ycm, Zcm, in unit of mm).





2.6 Computation of solid Earth and load tidal effects on geodetic networks

[Purpose] Compute the solid Earth, ocean tidal load or surface atmosphere tidal load effects on the GNSS baseline or level height difference according to the location and observation time in the input geodetic control network record file.

[Input file] The geodetic network observation record file.

The first row is the file header. The record format: the GNSS baseline or leveling route name, starting point longitude, latitude, height, ending point longitude, latitude, height, …, observation time, ….

The column ordinal number of the time attribute should not be less than 8.

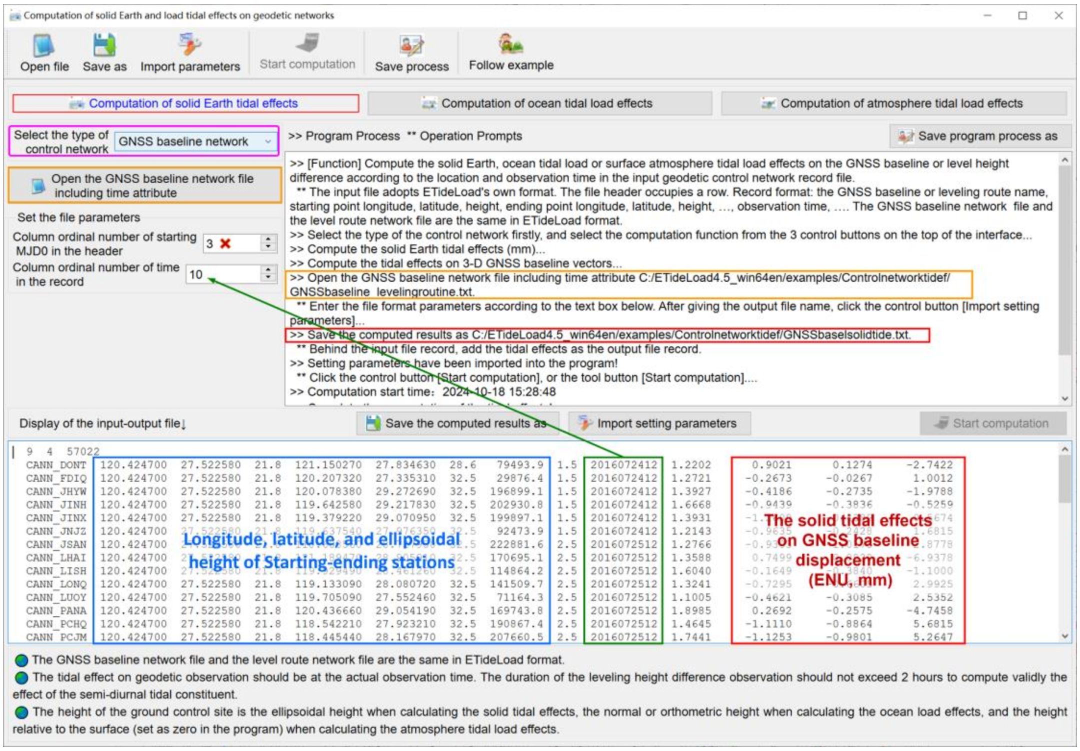
The GNSS baseline network file and the level route network file are the same in ETideLoad format.

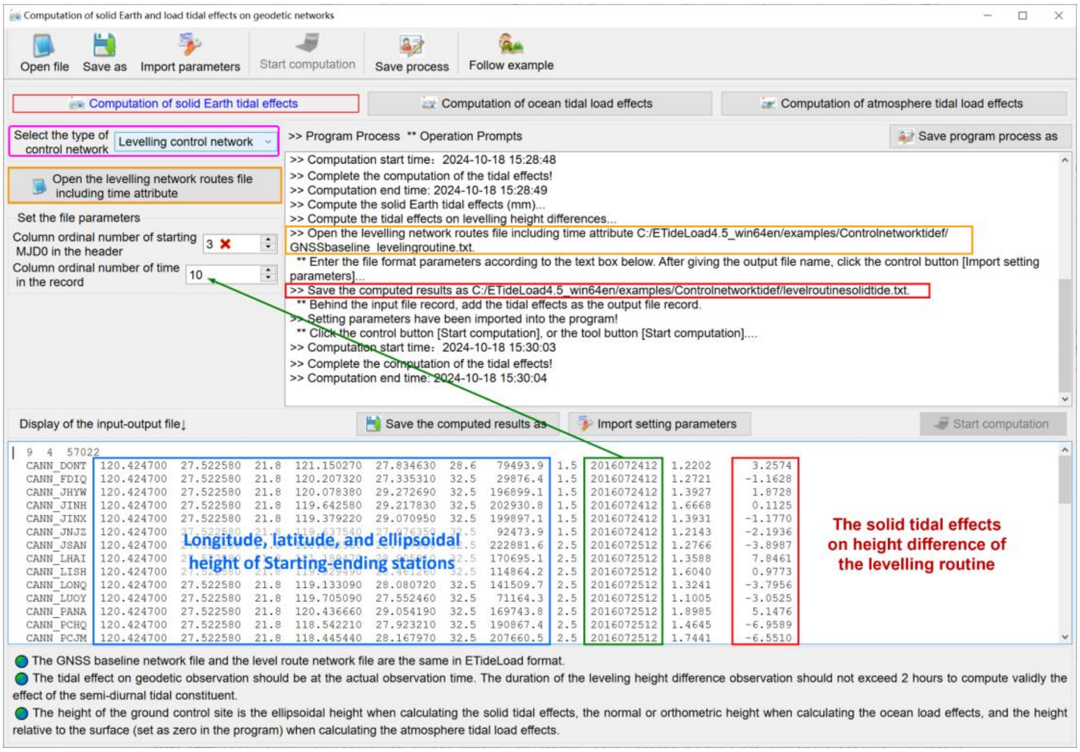
[Parameter settings] Select the type of geodetic control network, set the input file format parameters, and enter maximum truncated degree of the coefficient model when computing the tidal load effects.

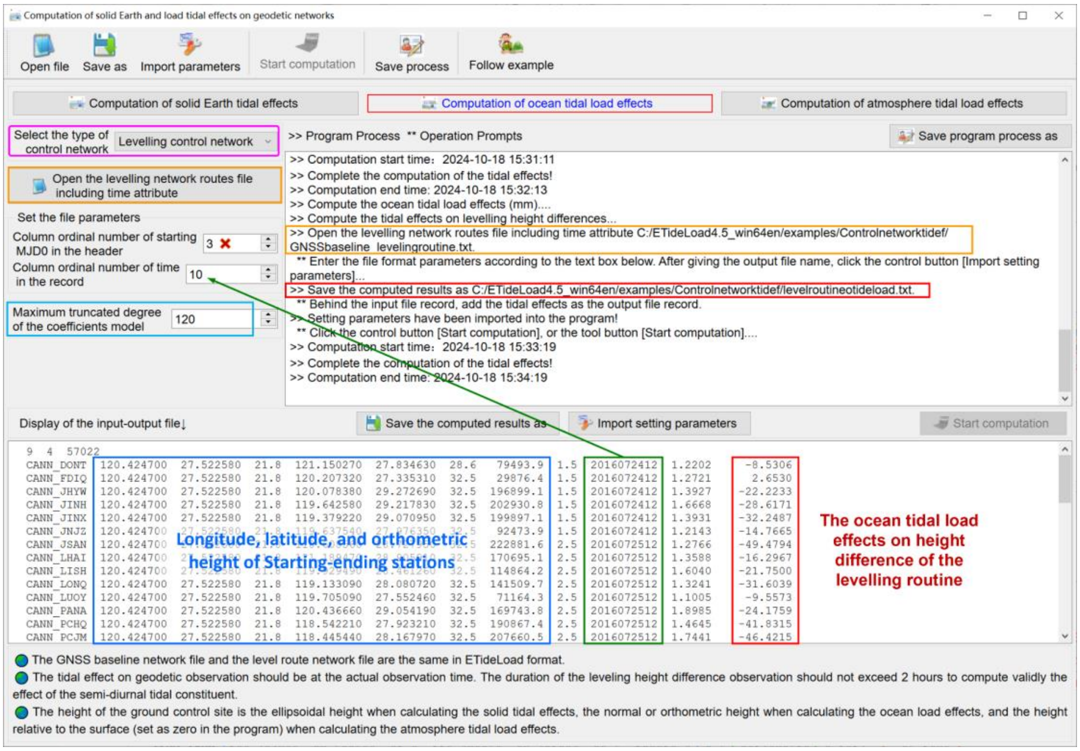
The tidal effect on geodetic observation should be at the actual observation time. The duration of the leveling height difference observation should not exceed 2 hours to compute validly the effect of the semi-diurnal tidal constituent.

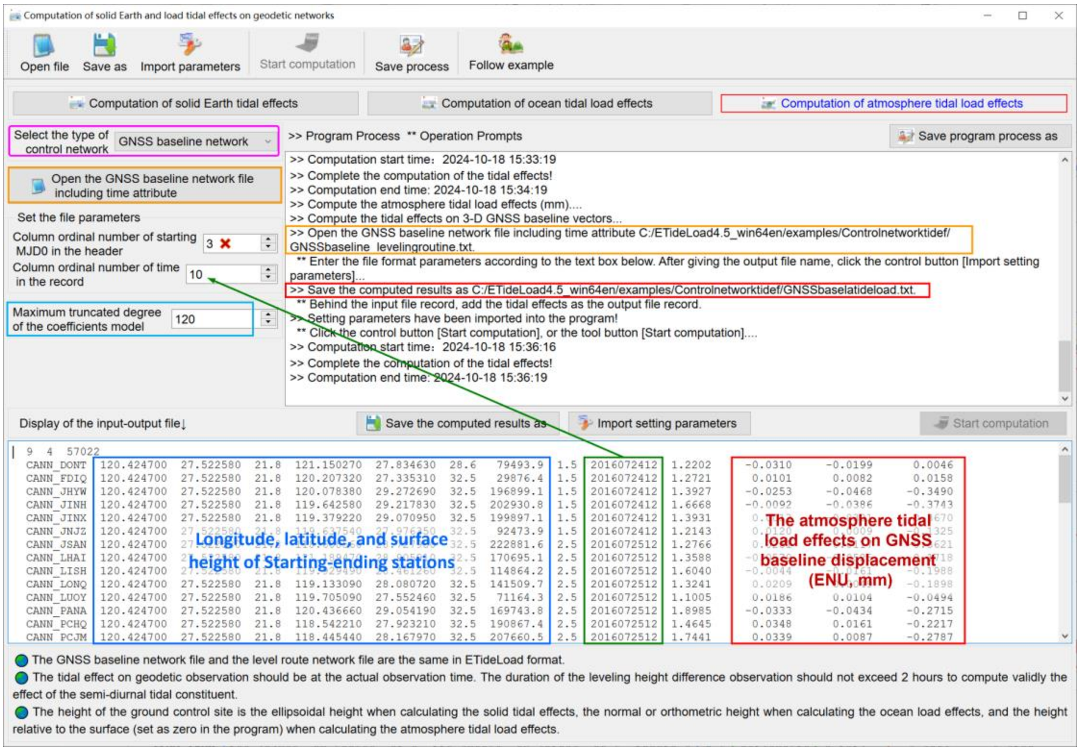
The height of the ground control site is the ellipsoidal height when calculating the solid tidal effects, the normal or orthometric height when calculating the ocean load effects, and the height relative to the surface (set as zero in the program) when calculating the atmosphere tidal load effects.

The gravity observation of the gravity control network need be only operated on the gravity sites, so the solid tidal, ocean tidal load and atmosphere tidal load effects should be calculated according to the site location and actual observation time.









2.7 The regional approach of load tidal effects by load Green's Integral

[Purpose] From the regional residual ocean or surface atmosphere tidal harmonic constant grids, compute the residual ocean or atmosphere tidal load effects near-Earth space by Green's function integral.

Here, the residual harmonic constants are equal to the regional harmonic constants minus the model value of the harmonic constants calculated using the global tidal load spherical harmonic coefficient model.

The program requires that residual harmonic constant grid files of all tidal constituents are stored in a directory. The harmonic constant grid file is saved in the form of a vector grid, and the seventh attribute of the file header is the Doodson constant.

ETideLoad4.5 takes the regional harmonic constant grid as the observations, employes global tidal load spherical harmonic coefficient model as a tidal load reference field, and refines the regional residual tidal load effects by load Green's funtion integral. Which is also called the remove-restore process. The program only calculates the regional residual value of the tidal load effects in the remove-restore process.

2.7.1 Computation of residual ocean tidal load effects by Green's Integral

[Function] From the regional residual ocean tidal harmonic constant grids, compute the residual ocean tidal load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (mE) or horizontal gravity gradient (NW, to the north and to the west, mE) by load Green's function Integral.

[Input file] The location and time file of near-Earth points, and the regional residual ocean tidal harmonic constant grid files.

The location and time file of near-Earth points. The first row is the file header. From the second row onwards, the second and third attributes in the file record are conventionally longitude and latitude (degree decimals), and there are the sampling epoch time and height attributes in the records.

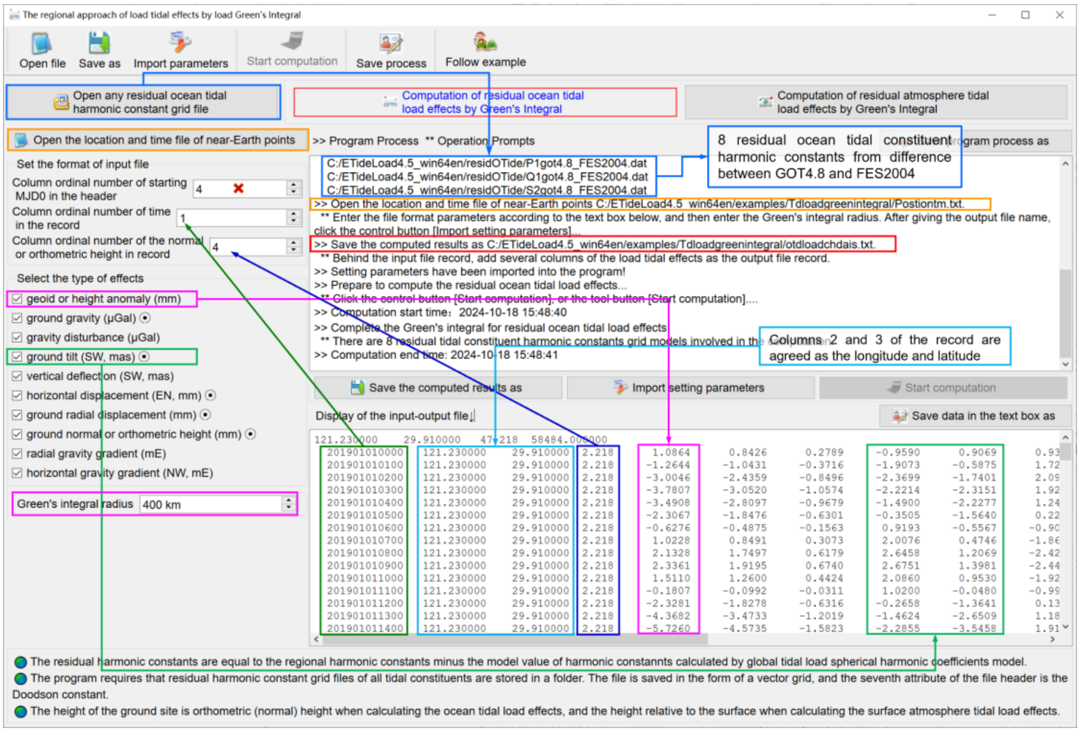
In this example, 8 residual ocean tidal constituent harmonic constant grid files are selected from the difference between the ocean tide height model GOT4.8 (0.5°×0.5° harmonic constant grid) and FES2004.

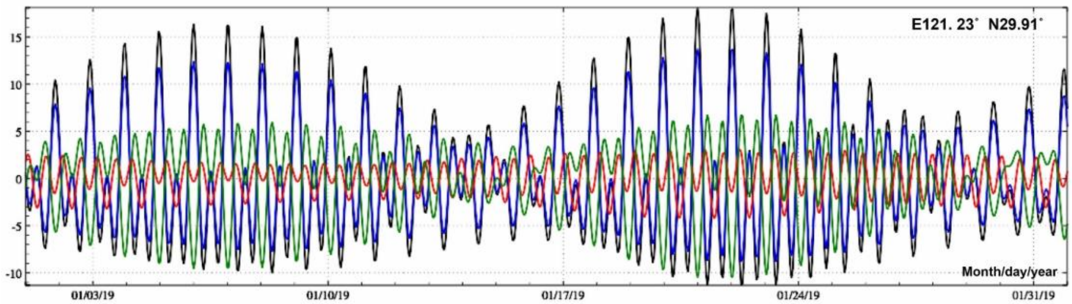
[Parameter settings] Set the input file format parameters, select the type of ocean tidal load effects, and enter Green's integral radius.

The height of the calculation point is normal or orthometric height relative to the sea surface since the ocean tidal loads are generally considered to be on the sea surface.

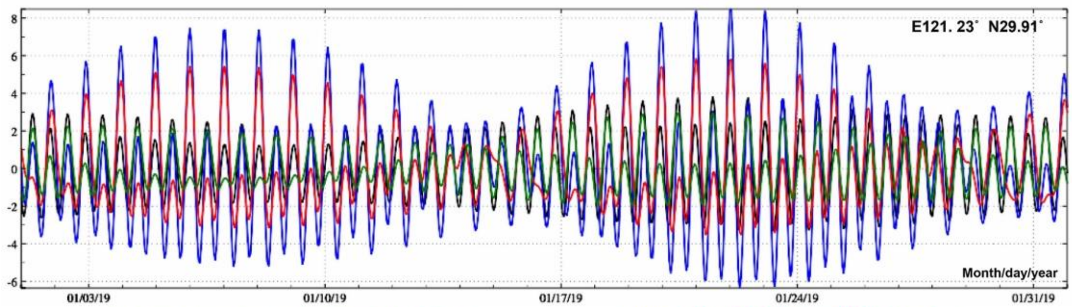
[Output file] The residual ocean tidal load effect file.

The file header is the same as the input file. Behind the input file record, adds one or several columns of the residual tidal effects selected as the output file record. In this example, all types are selected, and there are fourteen attributes added to the record.





The residual ocean tidal load effects (GOT4.8-FES2004): height anomaly (mm), ground gravity (μGal), radial displacement (mm), orthometric height (mm)



The residual ocean tidal load effects (GOT4.8-FES2004): ground tilt (S, mas), (W, mas), horizontal displacement (E, mm)), (N, mm)

2.7.2 Computation of residual atmosphere tidal load effects by Green's Integral

[Function] From the regional residual surface atmosphere tidal harmonic constant grids, compute the residual tidal load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (mE) or horizontal gravity gradient (NW, to the north and to the west, mE) by load Green's funtion integral.

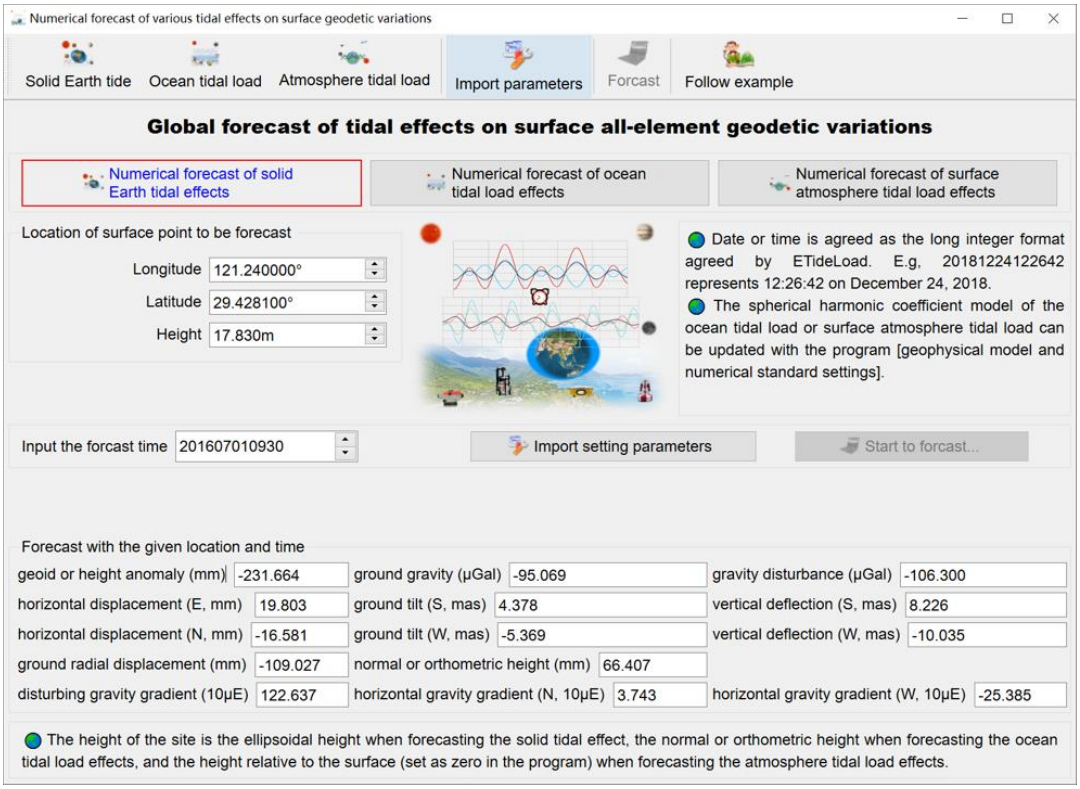
The height of the ground site is the normal or orthometric height when calculating the ocean tidal load effects, and the height relative to the surface (set as zero in the program) when calculating the surface atmosphere tidal load effects.

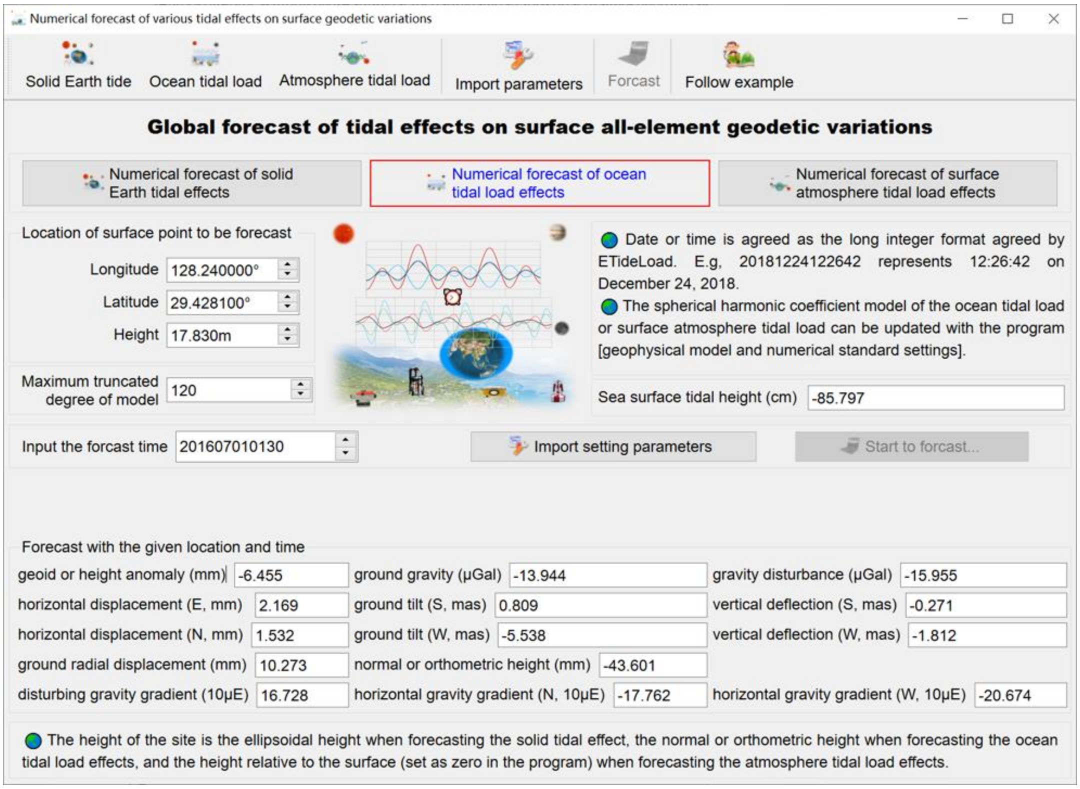
2.8 Forecast of various tidal effects on surface all-element geodetic variations

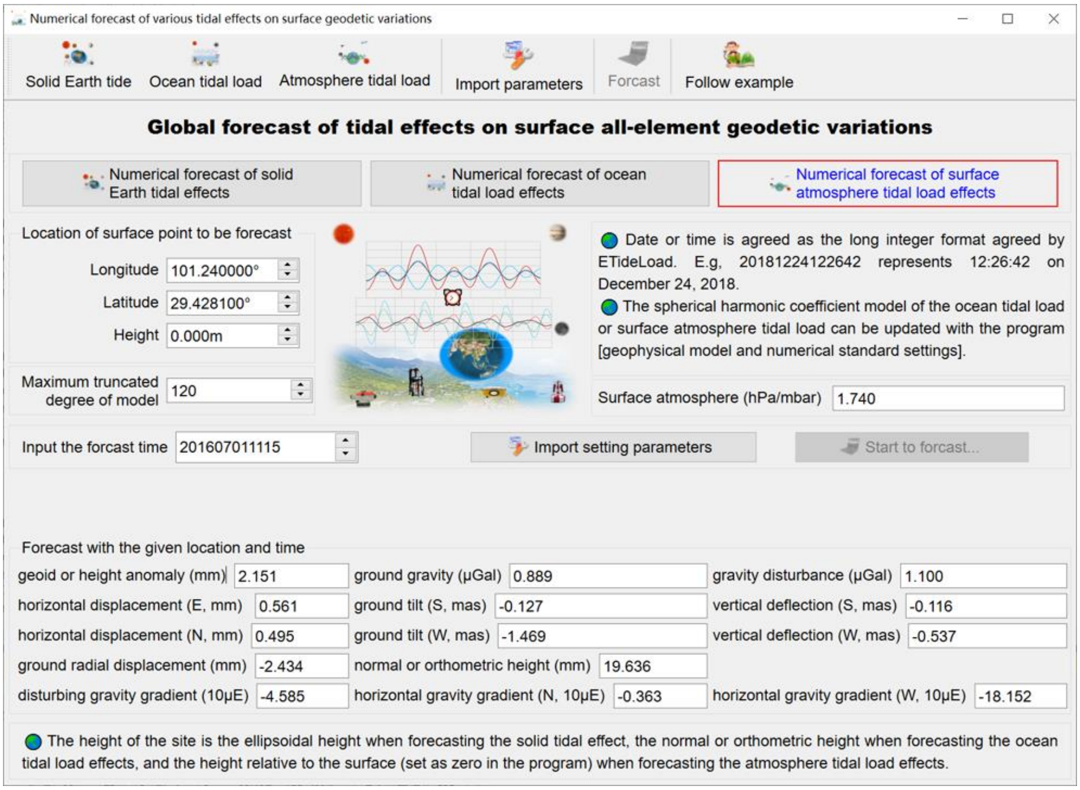
[Purpose] Forecast the solid Earth tidal, ocean tidal load or surface atmosphere tidal load effects on various surface geodetic variations anywhere and anytime.

The height of the site is the ellipsoidal height when forecasting the solid tidal effect, the normal or orthometric height when forecasting the ocean tidal load effects, and the height relative to the surface (set as zero in the program) when forecasting the atmosphere tidal load effects.

Date or time is agreed as the long integer format agreed by ETideLoad. E.g, 20181224122642 represents 12:26:42 on December 24, 2018, and 2018122412 represents 12: 0: 0 on December 24, 2018.

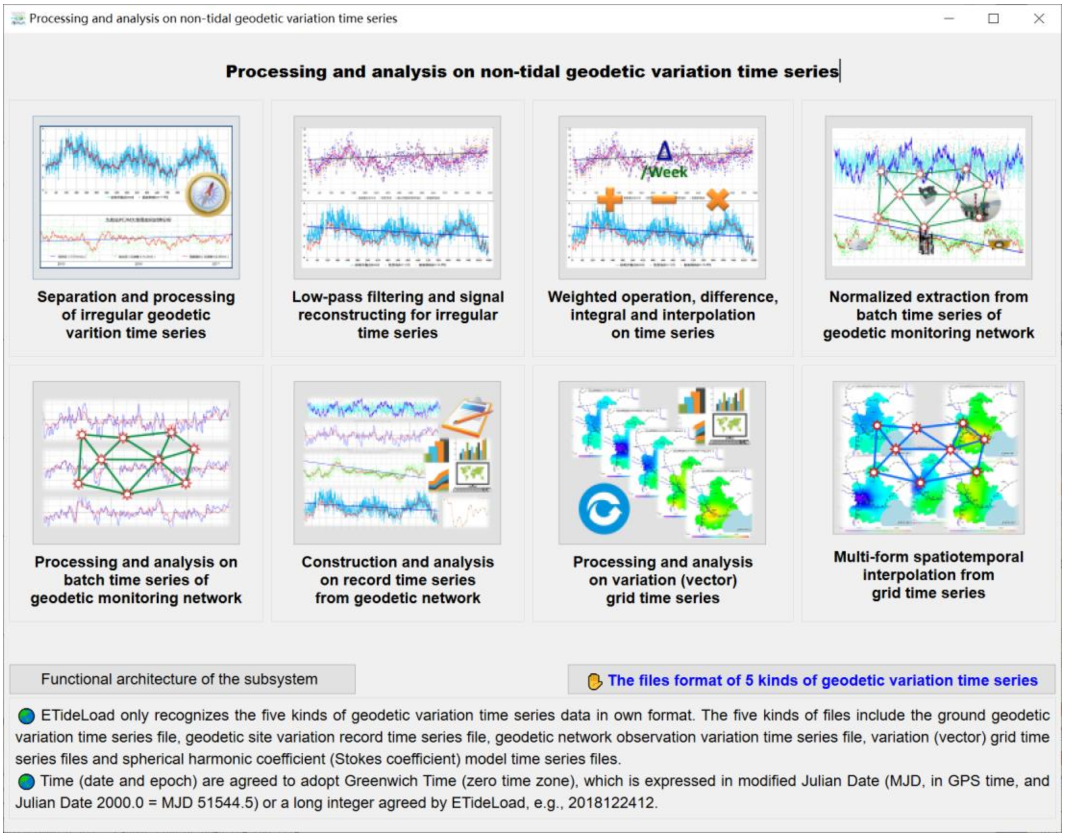



Numerical forecast of various tidal effects on surface geodetic variations
Solid Earth tide
Ocean tidal load
Atmosphere tidal load
Import parameters
Forcast
Follow example
Global forecast of tidal effects on surface all-element geodetic variations
Numerical forecast of solid Earth tidal effects
Numerical forecast of ocean tidal load effects
Numerical forecast of surface atmosphere tidal load effects
Location of surface point to be forecast
Longitude 128.240000°
Latitude 29.428100°
Height 17.830m
Maximum truncated degree of model 120
Date or time is agreed as the long integer format agreed by ETideLoad. E.g, 20181224122642 represents 12:26:42 on December 24, 2018.
The spherical harmonic coefficient model of the ocean tidal load or surface atmosphere tidal load can be updated with the program [geophysical model and numerical standard settings].
Sea surface tidal height (cm) -85.797
Input the forcast time 201607010130
Import setting parameters
Start to forcast...
Forecast with the given location and time
geoid or height anomaly (mm) -6.455
ground gravity (μGal) -13.944
gravity disturbance (μGal) -15.955
horizontal displacement (E, mm) 2.169
ground tilt (S, mas) 0.809
vertical deflection (S, mas) -0.271
horizontal displacement (N, mm) 1.532
ground tilt (W, mas) -5.538
vertical deflection (W, mas) -1.812
ground radial displacement (mm) 10.273
normal or orthometric height (mm) -43.601
disturbing gravity gradient (10μE) 16.728
horizontal gravity gradient (N, 10μE) -17.762
horizontal gravity gradient (W, 10μE) -20.674
The height of the site is the ellipsoidal height when forecasting the solid tidal effect, the normal or orthometric height when forecasting the ocean tidal load effects, and the height relative to the surface (set as zero in the program) when forecasting the atmosphere tidal load effects.


Numerical forecast of various tidal effects on surface geodetic variations
Solid Earth tide
Ocean tidal load
Atmosphere tidal load
Import parameters
Forcast
Follow example
Global forecast of tidal effects on surface all-element geodetic variations
Numerical forecast of solid Earth tidal effects
Numerical forecast of ocean tidal load effects
Numerical forecast of surface atmosphere tidal load effects
Location of surface point to be forecast
Longitude 101.240000°
Latitude 29.428100°
Height 0.000m
Maximum truncated degree of model 120
Date or time is agreed as the long integer format agreed by ETideLoad. E.g, 20181224122642 represents 12:26:42 on December 24, 2018.
The spherical harmonic coefficient model of the ocean tidal load or surface atmosphere tidal load can be updated with the program [geophysical model and numerical standard settings].
Surface atmosphere (hPa/mbar) 1.740
Input the forcast time 201607011115
Import setting parameters
Start to forcast...
Forecast with the given location and time
geoid or height anomaly (mm) 2.151
ground gravity (μGal) 0.889
gravity disturbance (μGal) 1.100
horizontal displacement (E, mm) 0.561
ground tilt (S, mas) -0.127
vertical deflection (S, mas) -0.116
horizontal displacement (N, mm) 0.495
ground tilt (W, mas) -1.469
vertical deflection (W, mas) -0.537
ground radial displacement (mm) -2.434
normal or orthometric height (mm) 19.636
disturbing gravity gradient (10μE) -4.585
horizontal gravity gradient (N, 10μE) -0.363
horizontal gravity gradient (W, 10μE) -18.152
The height of the site is the ellipsoidal height when forecasting the solid tidal effect, the normal or orthometric height when forecasting the ocean tidal load effects, and the height relative to the surface (set as zero in the program) when forecasting the atmosphere tidal load effects.

3 Processing and analysis on non-tidal geodetic variation time series

Based on the characteristics of non-tidal geodetic time series, the group of programs adopt some stable and reliable algorithms to uniformly process and analyze massive various geodetic variation time series data.



ETideLoad only recognizes the five kinds of geodetic variation time series data in own format. The five kinds of files include the ground geodetic variation time series file, geodetic site variation record time series file, geodetic network observation variation time series file, variation (vector) grid time series files and spherical harmonic coefficient (Stokes coefficient) model time series files.

Time (date and epoch) are agreed to adopt Greenwich Time (zero time zone), which is expressed in modified Julian Date (MJD, in GPS time, and Julian Date 2000.0 = MJD 51544.5) or a long integer agreed by ETideLoad. In most cases, the long integer agreed by ETideLoad is employed. E.g., 20181224122642 represents 12:26:42 on December 24, 2018. Here, the epoch is an instantaneous time.

3.1 Separation and processing of irregular geodetic varition time series

[Purpose] On the irregular geodetic varition time series in the given input file, perform preprocessing such as gross error detection and separation, time format transform, reference epoch unification of multi-column time series or averaging according to the given time period.

In the record of the geodetic variation time series file, each attribute except the sampling epoch time represents a type of variation time series, and the sampling time epoch time of all types of variation time series are the same.

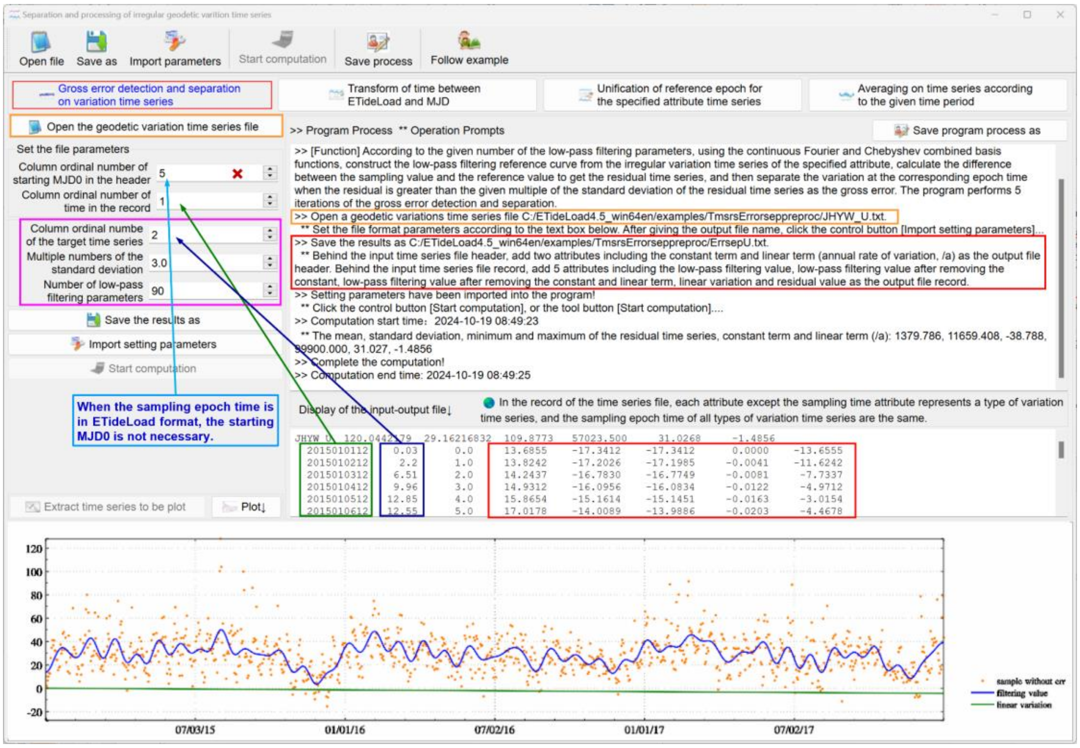
3.1.1 Gross error detection and separation on variation time series

[Function] According to the given number of the low-pass filtering parameters, using the continuous Fourier and Chebyshev combined basis functions, construct the low-pass filtering reference curve from the irregular variation time series of the specified attribute, calculate the difference between the sampling value and the reference value to get the residual time series, and then separates the variation at the corresponding epoch time when the residual is greater than the given multiple of the standard deviation of the residual time series as the gross error. The program performs 5 iterations of the gross error detection and separation.

[Input file] The geodetic variation time series file.

The first row is the file header. Starting from the second row of the file, each row record stores the sampling values of all the variations at one sampling epoch time. At least one column of the attributes in the record is the sampling epoch time.

[Parameter settings] Set the input file format parameters, enter column ordinal number of the epoch time and target attribute time series to be detected in the record, and enter the multiple of the standard deviation and the number of low-pass filtering parameters.



The entered number of the low-pass filtering parameters is not more than 1/2 of the number of time series samples, and not less than 1/30 of the number of samples. When the entered number exceeds this range, the program automatically takes the minimum or maximum value.

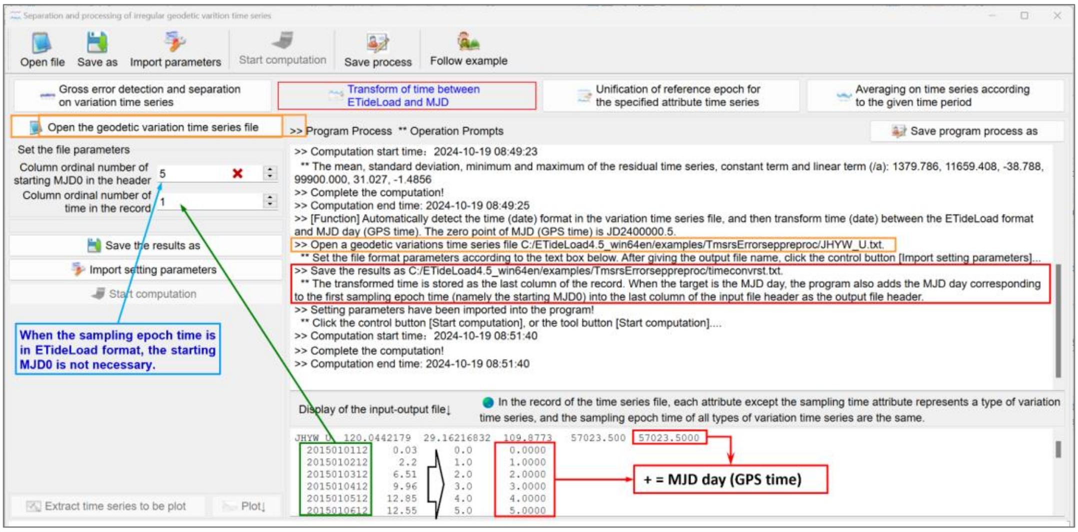
[Output file] The variation time series analysis file after removing gross errors.

Behind the input time series file header, adds two attributes including the constant term and linear term (annual rate of variation, /a) as the output file header. Behind the input time series file record, adds 5 attributes including the low-pass filtering value, low-pass filtering value after removing the constant, low-pass filtering value after removing the constant and linear term, linear variation and residual value as the output file record.

3.1.2 Transform of time between ETideLoad and MJD

[Function] Automatically detect the time (date) format in the variation time series file, and then transform time (date) between the ETideLoad format and MJD day (GPS time). The zero point of MJD (GPS time) is JD2400000.5.

The transformed time is stored as the last column of the record. When the target is the MJD day, the program also adds the MJD day corresponding to the first sampling epoch time (namely the starting MJD0) into the last column of the input file header as the output file header.



3.1.3 Unification of reference epoch for the specified attribute time series

[Function] Using the cubic spline or Gaussian function interpolation method, interpolate the sampling value of the specified attribute time series at the given reference epoch time, and then remove the corresponding sampling values from the time series, thereby unifying the reference epoch time. At the reference epoch time, the sampling value of the specified attribute time series is always zero.

[Input file] The geodetic variation time series file.

[Parameter settings] Set the input file format parameters, enter column ordinal number of the first and last attribute time series, the reference epoch time (ETideLoad long integer format), and select interpolation method.

The program requires that the reference epoch time is no earlier than the first sampling time and no later than the last sampling time, otherwise automatically set to the first or last sampling time.

The program requires that the number of the time series attributes selected are not more than 20, that is, the difference between the column ordinal number of the last attribute time series and the first attribute time series is small than 20.

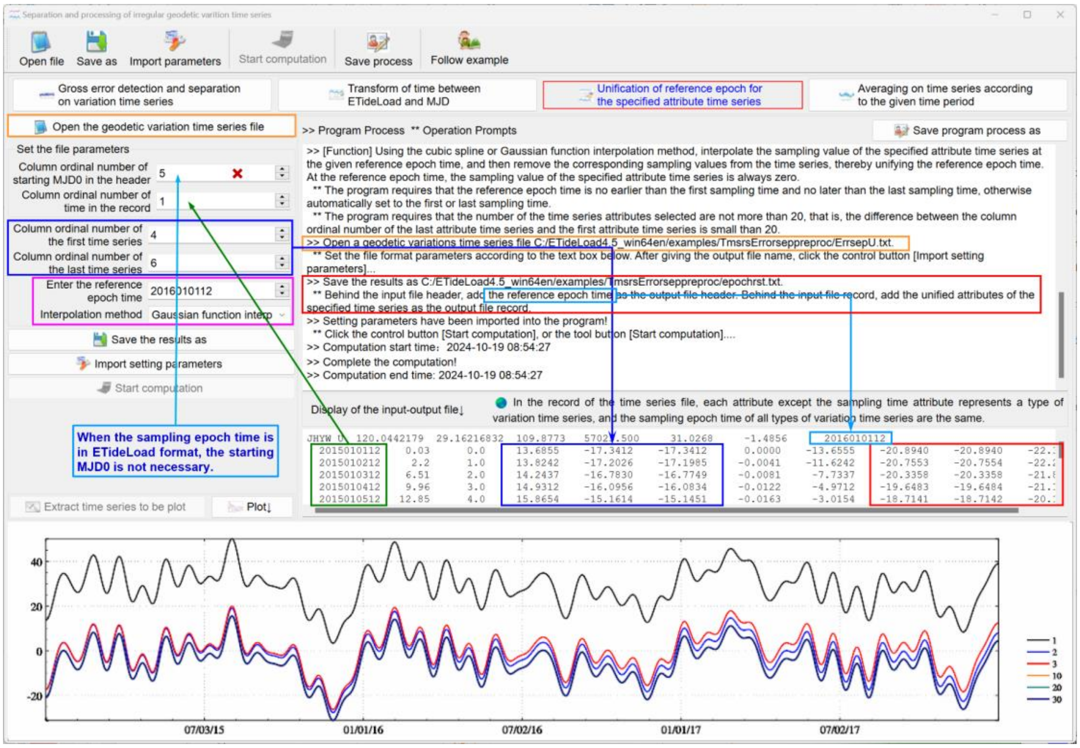
The reference epoch time need not be one of the sampling epochs of the input variation time series.

When there are more noise or missing samples in time series signals, Gaussian function interpolation is recommended.

[Output file] The variation time series analysis file.

Behind the input file header, adds the reference epoch time as the output file header. Behind the input file record, adds the unified attributes of the specified time series as the output file record.

The unification of reference epoch time for multi-source data is the most basic requirement for geodynamics monitoring.



3.1.4 Averaging on time series according to the given time period

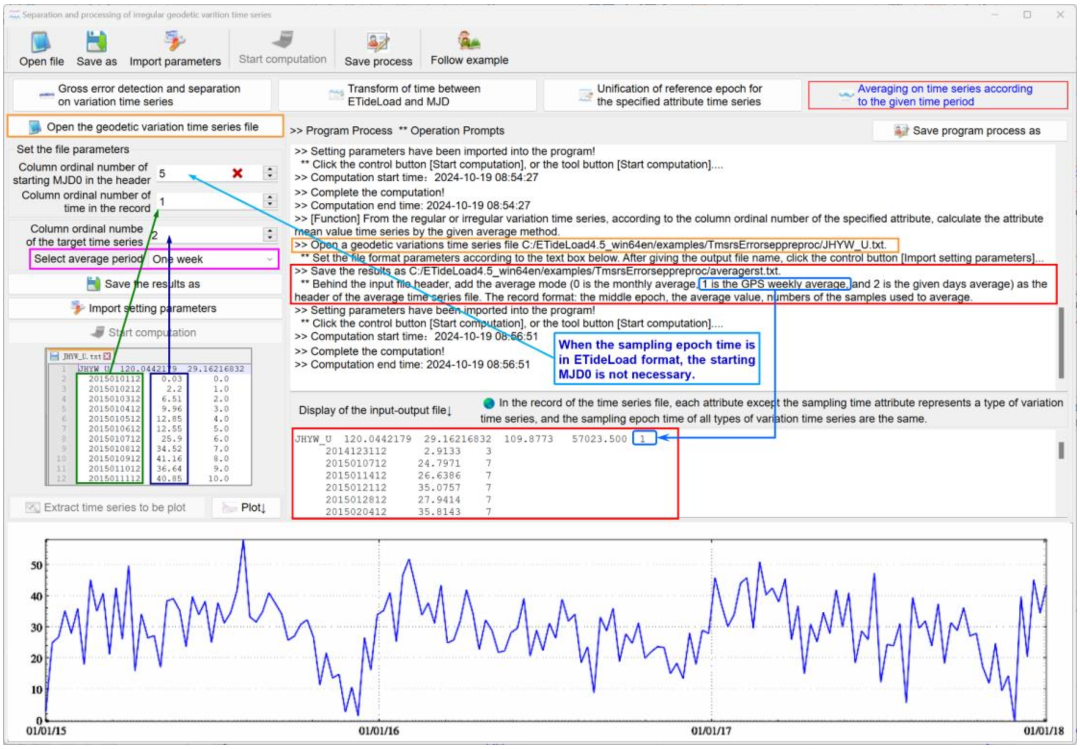
[Function] From the regular or irregular variation time series, according to the column ordinal number of the specified attribute, calculates the attribute mean value time series by the given average time period.

[Input file] The geodetic variation time series file.

[Parameter settings] Set the input file format parameters, enter column ordinal number of the specified attribute and select average period.

[Output file] The variations average time series file.

Behind the input file header, adds the average mode (0 is the monthly average, 1 is the GPS weekly average, and 2 is the given days average) as the header of the average time series file. The record format: the middle epoch, the average value, the number of the samples used to average.



3.2 Low-pass filtering and signal reconstructing for irregular time series

[Purpose] Using the continuous Chebyshev and triangular base function combination method, estimate the low-pass parameters of the irregular time series, separates the constant term and linear term, and then reconstruct the time series according to the user's requirements.

The program can separate the constant term, linear term and noise, and realize the short-time interpolation and bidirectional prediction of various irregular variation time series.

3.2.1 Estimation of the low-pass parameters and linear term of irregular time series

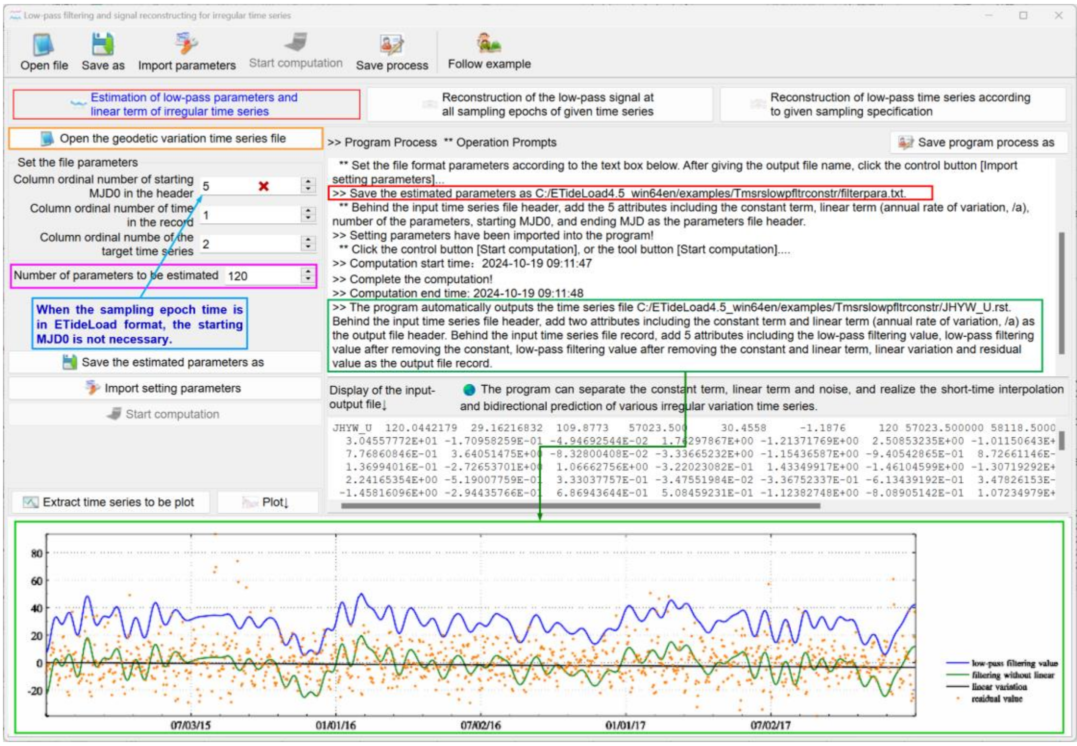
[Function] Using the continuous Chebyshev and triangular base function combination method, estimate the constant term, linear term and low-pass parameters of the irregular variation time series according to the entered number of the parameters to be estimated.

[Input file] The geodetic variation time series file.

The first row is the file header. Starting from the second row of the file, each row record stores the sampling values of all the variations at one sampling epoch time. At least one column of the attributes in the record is the sampling epoch time.

[Parameter settings] Set the input file format parameters, enter column ordinal number of the epoch time and target attribute time series in the record and enter the number of low-pass filtering parameters.

The number of the estimated low-pass parameters should not be greater than 1 / 2 of the number of samples in the input time series and should not be less than 1 / 30 of the number of samples. Otherwise, the program automatically takes the minimum or maximum values.



[Output file] The low-pass filtering parameter file. The low-pass filter variation time series analysis file.

The low-pass filtering parameter file. Behind the input time series file header, adds the 5 attributes including the constant term, linear term (annual rate of variation, /a), number of the estimated parameters, starting MJD0, and ending MJD as the parameter file header. And then all low-pass filter parameter values be saved into the file in order.

The low-pass filter variation time series analysis file *.rst. Here, * is the input geodetic variation time series file name.

Behind the input time series file header, adds two attributes including the constant term and linear term (annual rate of variation, /a) as the output file header. Behind the input time series file record, adds 5 attributes including the low-pass filtering value, low-pass filtering value after removing the constant, low-pass filtering value after removing the constant and linear term, linear variation and residual value as the output file record.

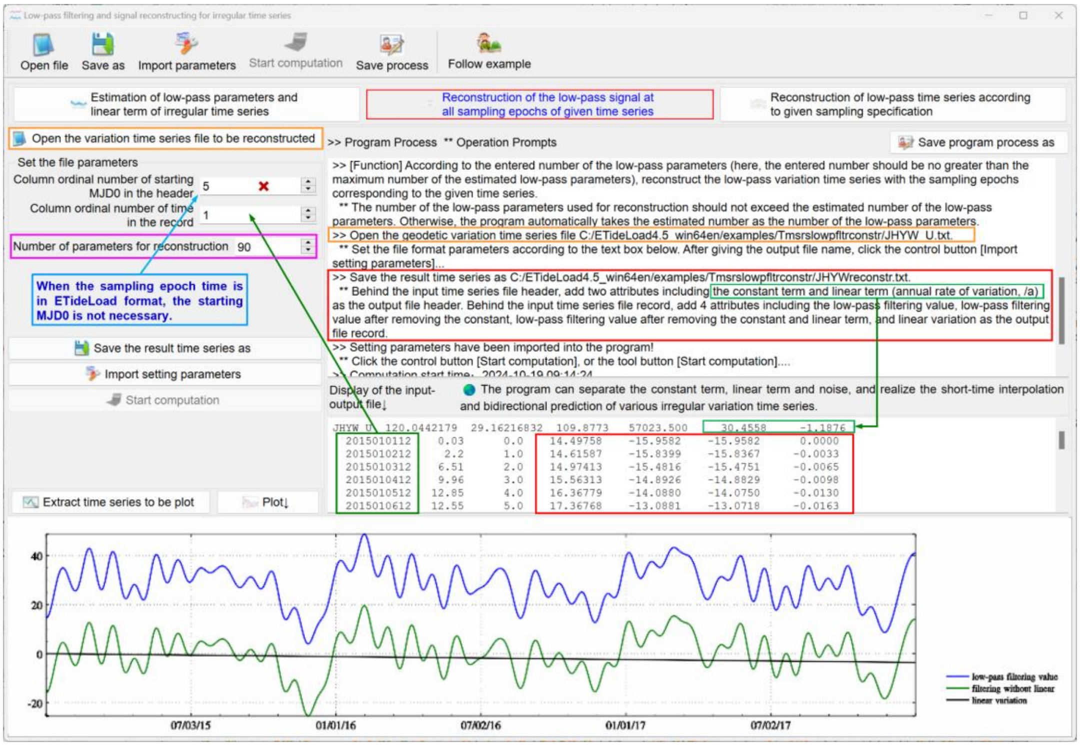
3.2.2 Reconstruction of the low-pass signal at all sampling epochs of given time series

[Function] According to the entered number of the low-pass parameters (here, the entered number should be no greater than the maximum number of the estimated low-pass parameters), reconstruct the low-pass variation time series with the sampling epochs corresponding to the given time series.

[Input files] The geodetic variation time series file to be reconstructed. The low-pass filtering parameter file, which be automatically called by the program without manual input.

[Parameter settings] Set the column ordinal number of the epoch time in the input file record and enter the number of the low-pass parameters for reconstruction.

The number of the low-pass parameters for reconstruction should not exceed the estimated number of the low-pass parameters. Otherwise, the program automatically takes the estimated number as the number of the low-pass parameters.



[Output file] The low-pass reconstruction variation time series file.

Behind the input time series file header, adds two attributes including the constant term and linear term (annual rate of variation, /a) as the output file header.

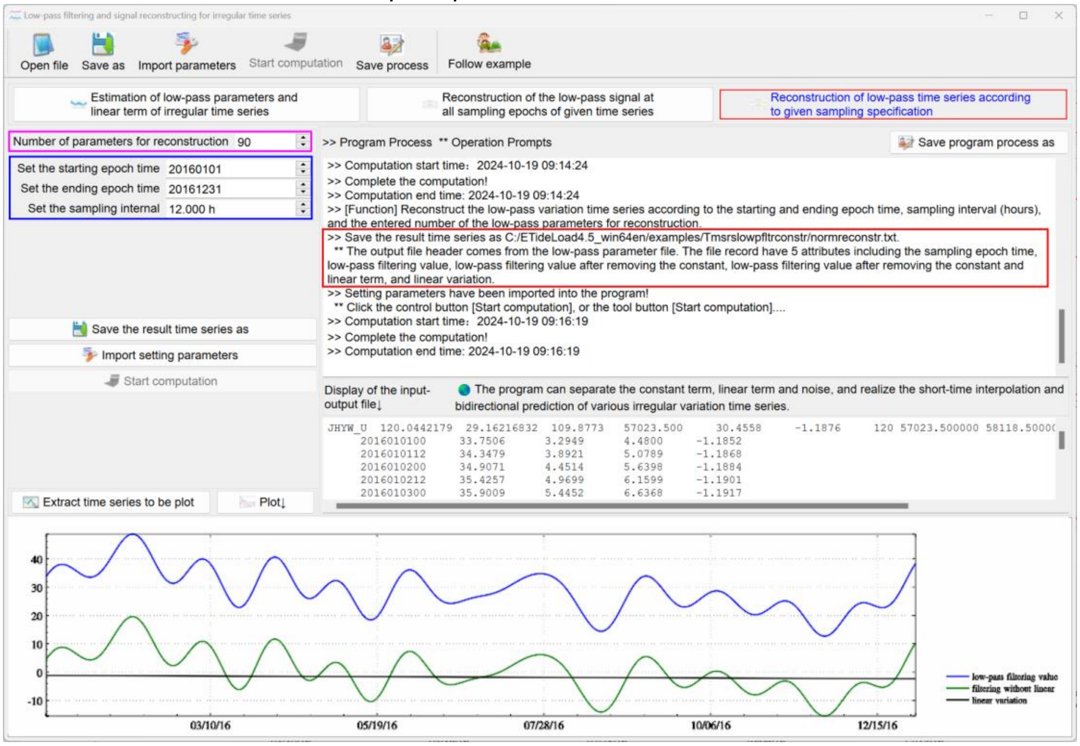
Behind the input time series file record, adds 4 attributes including the low-pass filtering value, low-pass filtering value after removing the constant, low-pass filtering value after removing the constant and linear term and linear variation as the output file record.

3.2.3 Reconstruction of low-pass time series according to given sampling specification

[Function] Reconstruct the low-pass variation time series according to the starting and ending epoch time, sampling interval (hours), and the entered number of the low-pass parameters for reconstruction.

[Input files] The low-pass filtering parameter file, which be automatically called by the program without manual input.

[Parameter settings] Set the starting and ending epoch time, sampling interval (hours) and enter the number of the low-pass parameters for reconstruction.



The starting epoch time should not be earlier (may be slightly earlier) than the first sampling epoch time of the variation time series used for parameter estimation.

The ending epoch time should not be later (may be slightly later) than the last sampling epoch time of the variation time series used for parameter estimation.

The starting-ending epoch time should be not earlier (or slightly earlier) than the starting time of the time series employed to estimate the low-pass parameters and not later (or slightly later) than the ending time of the time series.

[Output file] The low-pass reconstruction variation time series file.

The output file header comes from the low-pass parameter file. The file record have 5 attributes including the sampling epoch time, low-pass filtering value, low-pass filtering value after removing the constant, low-pass filtering value after removing the constant and linear term and linear variation.

3.3 Weighted operation, difference, integral and interpolation on time series

[Purpose] Directly perform weighted operation, difference, integral or interpolation operations on the irregular geodeic variation time series in the given manner.

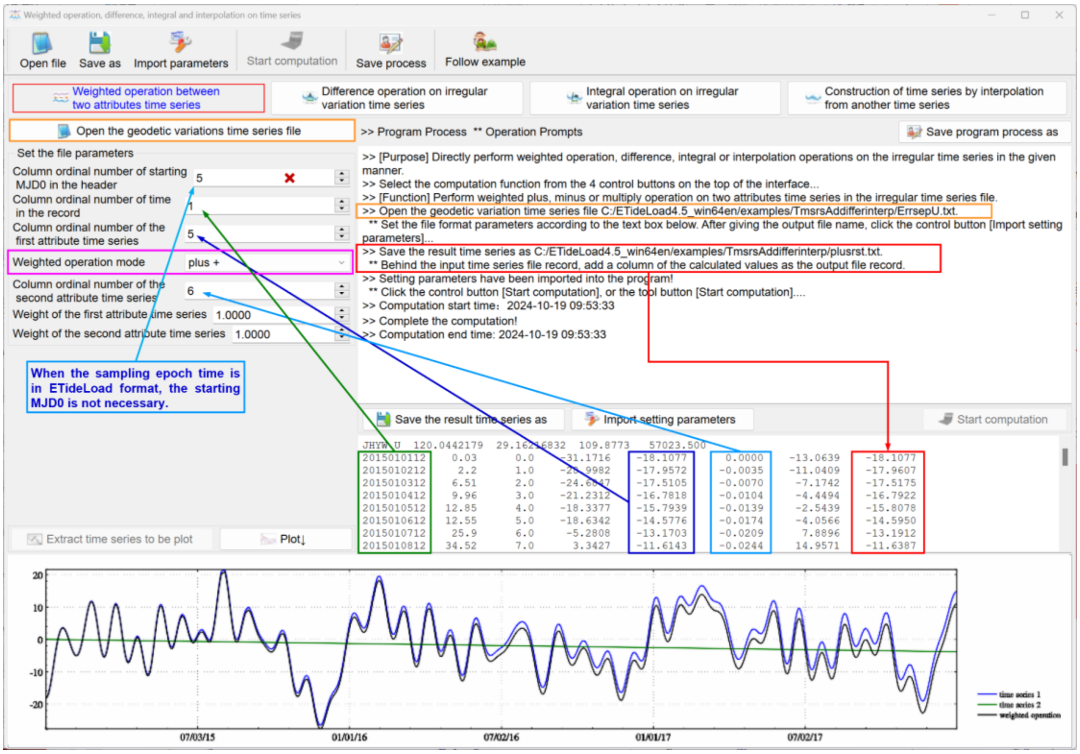
3.3.1 Weighted operation between two attributes time series

[Function] Perform weighted plus, minus or multiply operation on two attributes time series in the irregular variation time series file.

[Input file] The geodetic variation time series file.

The first row is the file header. Starting from the second row of the file, each row record stores the sampling values of all the variations at one sampling epoch time. The attributes in the record include the sampling epoch time and two attributes time series to be operated.

[Parameter settings] Set the input file format parameters, enter column ordinal number of the epoch time and two attributes time series to be operated in the record, and enter the two attribute weights.



[Output file] The weighted operation result variation time series file.

Behind the input time series file record, adds a column of the calculated values as the output file record.

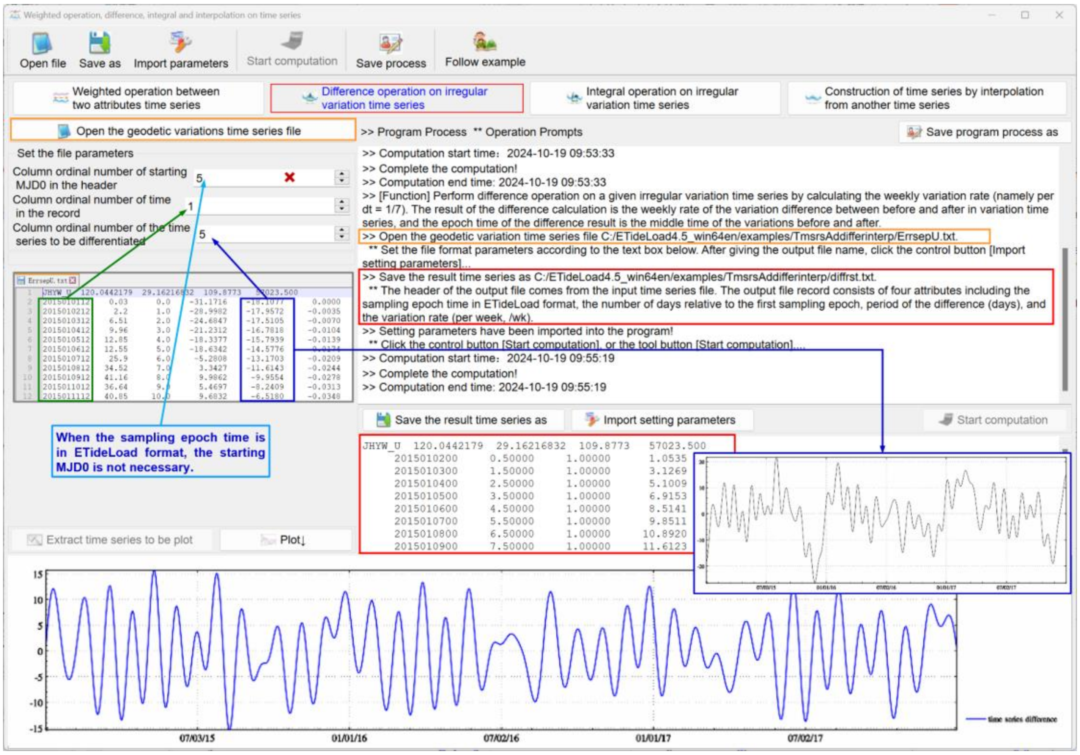
3.3.2 Difference operation on irregular variation time series

[Function] Perform difference operation on a given irregular variation time series by calculating the weekly variation rate (namely per week). The result of the difference calculation is the weekly rate of the variation difference between before and after in variation time series, and the epoch time of the difference result is the middle time of the variations before and after.

[Input file] The irregular geodetic variation time series file.

[Output file] The variation rate time series file.

The output file header comes from the input time series file. The output file record consists of four attributes including the sampling epoch time in ETideLoad format, number of days relative to the first sampling epoch, period of the difference (days) and the variation rate (per week, /wk).



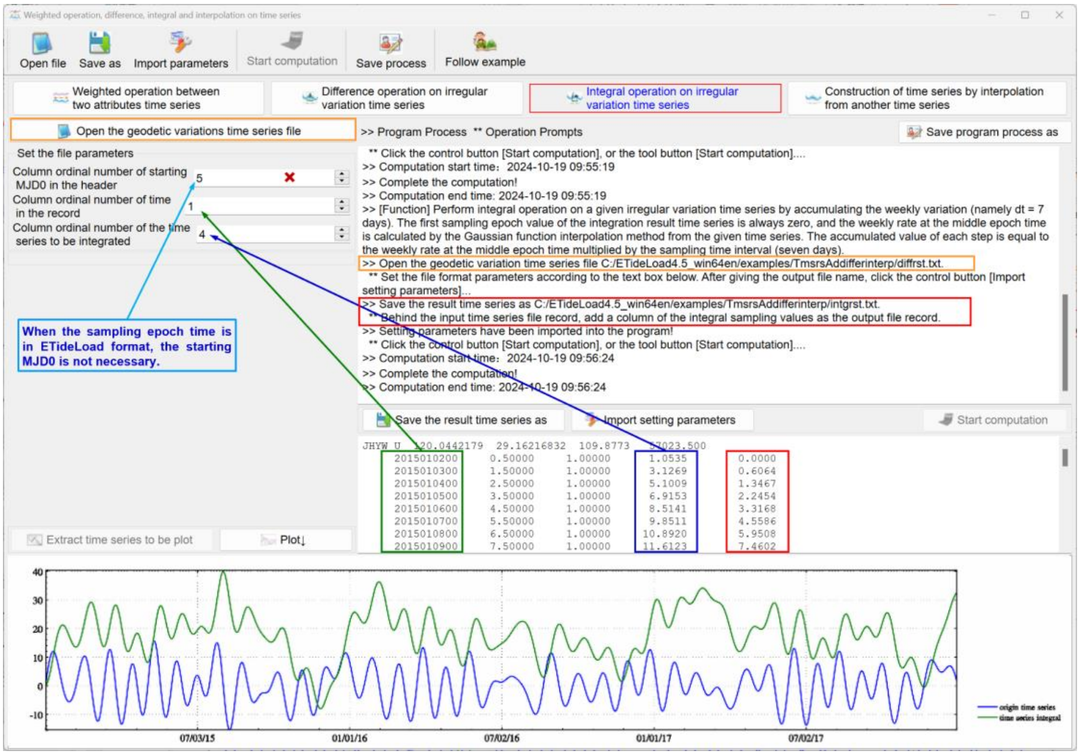
3.3.3 Integral operation on irregular variation time series

[Function] Perform integral operation on a given irregular variation time series by accumulating the weekly variation (namely dt = 7 days). The first sampling epoch value of the integration result time series is always zero, and the weekly rate at the middle epoch time is calculated by the Gaussian function interpolation method from the given time series.

The accumulated value of each step is equal to the weekly rate at the middle epoch time multiplied by the sampling time interval (seven days).

The output file header comes from the input time series file. Behind the input time series file record, adds a column of the calculated values as the output file record.

When there is some noise in the time series signal, Gaussian basis function interpolation is recommended.



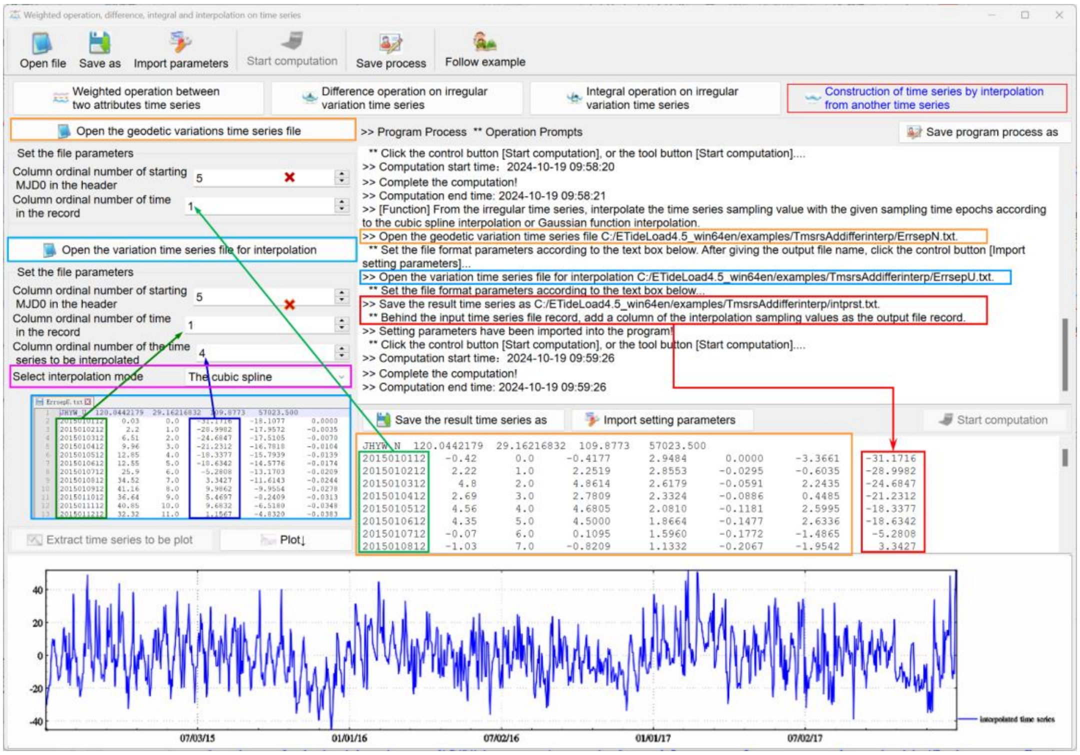
3.3.4 Construction of time series by interpolation from another time series

[Function] From the irregular time series, interpolate the time series sampling value with the given sampling time epochs according to the cubic spline interpolation or Gaussian function interpolation.

[Input file] The geodetic variation time series file to be interpolated. The geodetic variation time series file for interpolation.

[Output] The interpolation result variation time series file.

The output file header comes from the input time series file. Behind the input time series file record, adds a column of the interpolation sampling values as the output file record.



3.4 Normalized extraction from batch time series of geodetic monitoring network

[Purpose] From the text files of batch geodetic sites or batch CORS network baselines that contain the specified variation time series data with the same format, extract data and generate the corresponding variation time series files in the ETideLoad format.

The program requires all the source text files stored in a directory, and the source file name contains the site name or baseline name with the same number of the characters. The extracted time series files will be saved into another directory.

3.4.1 Normalized extraction from batch time series of geodetic network sites

[Function] From the text files of batch geodetic network sites that contain the specified time series data with the same file format, extract data and generate the corresponding time series files in the ETideLoad format, which will be saved into the specified directory.

[Parameter settings]

The program requires that wildcards can uniquely identify files in the directory, and their instance characters will be also used as the extracted time series file name.

If there is no height attribute in the source file, or the entered height column ordinal number exceeds the maximum number of the attributes, the program automatically set the height to zero.

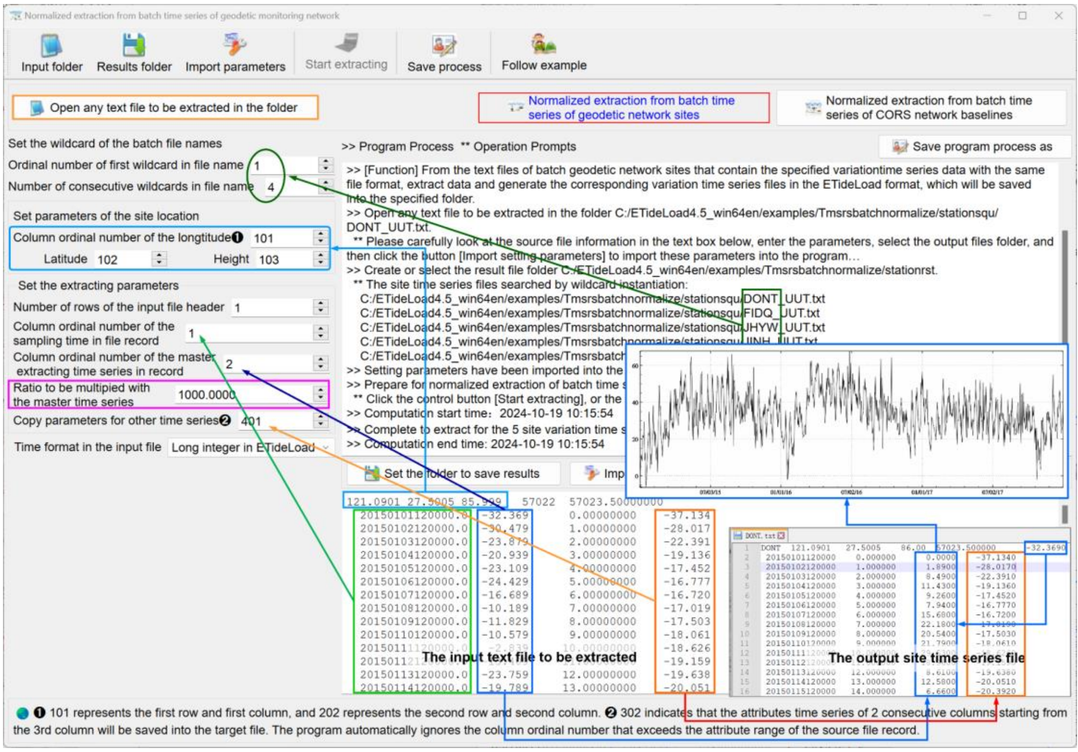
If there is not the starting MJD0 in the header of the source file, please enter the starting time agreed in ETideLoad format. After entering the epoch time in ETideLoad format, the program would automatically calculate MJD day.

[Output files] Batch geodetic variation time series files in ETideLoad format.

The file header: The site name (instance of the file name wildcard), longitude, latitude, height, starting MJD0 and constant term (the first sampling value of the target time series).

The record format: The sampling epoch time, days relative to the starting MJD0, sampling value which has removed the first sampling value, other copyed attributes.

The sum of the starting MJD0 in the header and the sampling epoch time (day) is equal to the sampling epoch time of MJD day in the record. When the sampling epoch time is in ETideLoad format, the starting MJD0 is not necessary for the file header.



3.4.2 Normalized extraction from batch time series of CORS network baselines

[Function] From batch baseline solution files of the CORS network that contain the specified time series data with the same file format, extract data and generate the corresponding baseline solution time series files in the ETideLoad format, which will be saved into the specified directory.

The program extracts the time series of one-dimension components of the ENU baseline solutions once.

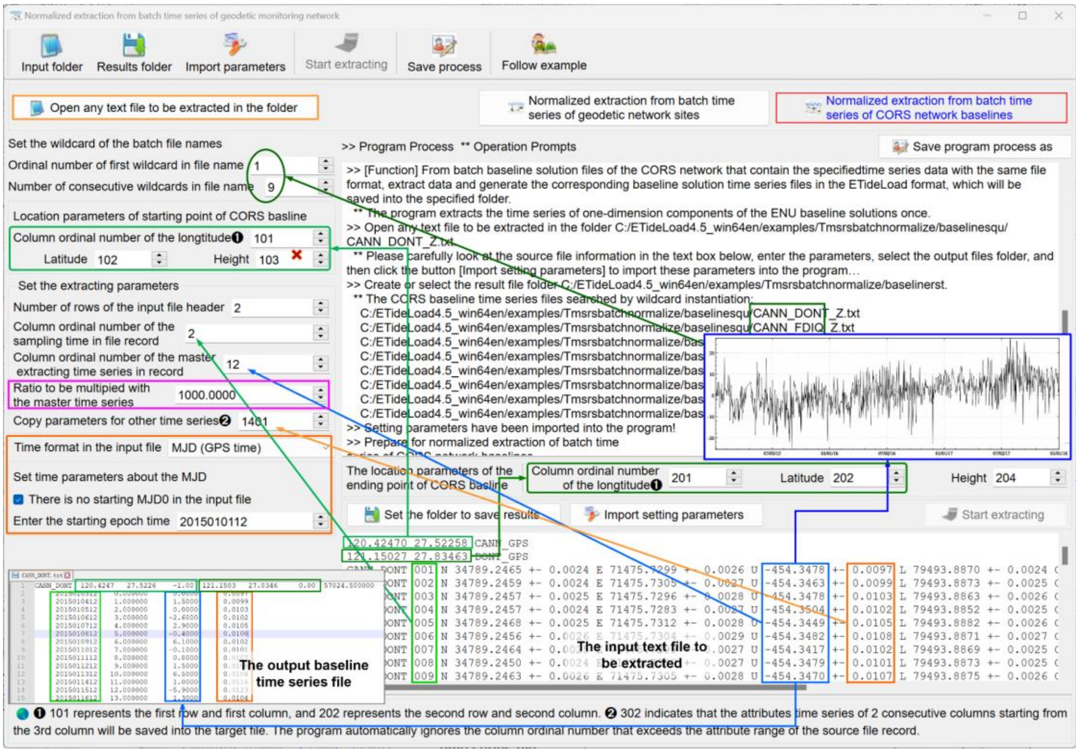
❶ 101 represents the first row and first column, and 202 represents the second row and second column.

❷ 302 indicates that the attribute time series of 2 consecutive columns starting from the 3rd column will be saved into the target file. The program automatically ignores the column ordinal number that exceeds the attribute range of the source file record.

[Output files] Batch CORS baseline solution time series files in ETideLoad format.

The file header: The baseline name (instance of the file name wildcard), starting station longitude, latitude, height, ending station longitude, latitude, height, starting MJD0 and constant term (the first sampling value of the target time series).

The record format: The sampling epoch time, days relative to the starting MJD0, sampling value which has removed the first sampling value, other copyed attributes.



3.5 Processing and analysis on batch time series of geodetic monitoring network

[Purpose] On the specified attribute time series from batch variation time series files with the same format, perform the gross error detection, linear term separation, low-pass filtering and signal reconstructing, or calculate the mean time series according to the given period.

The program requires all source time series files saved in a directory. The output time series files will be saved into the specified directory.

3.5.1 Gross error detection, low-pass filtering and reconstructing for batch time series

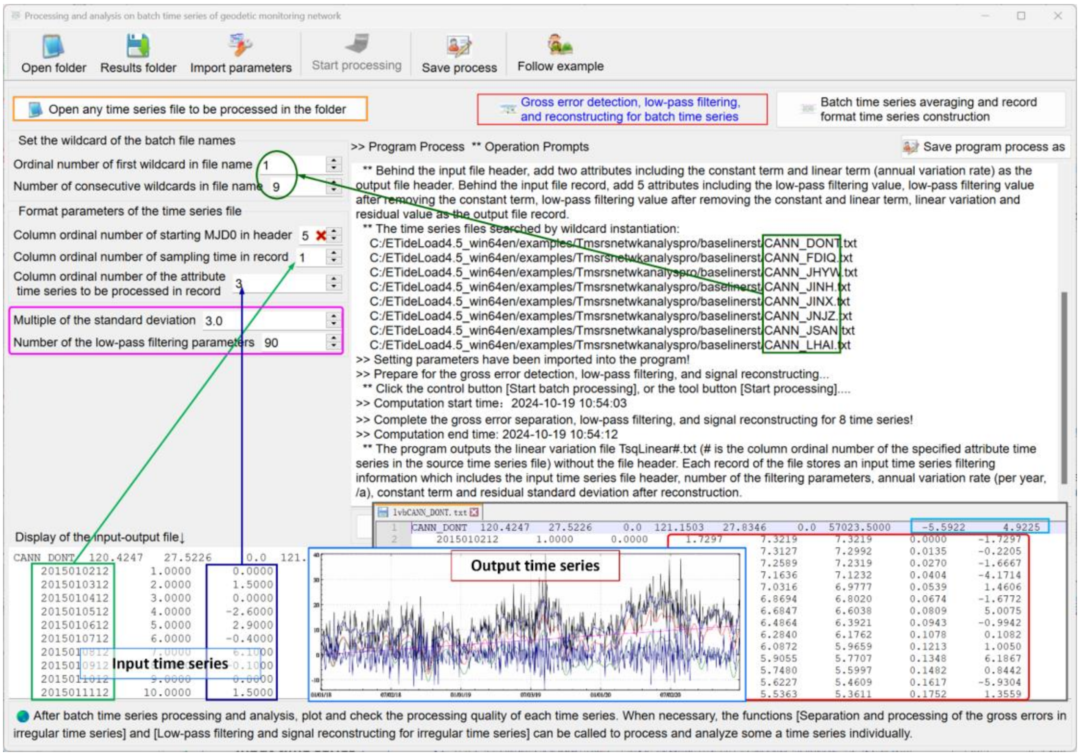
[Function] On the specified attribute time series from batch time series files with the same format, estimate the low-pass filtering parameters, separate the linear term using the low-pass filtering curve as a reference curve to detect gross errors, and then reconstruct the low-pass filtering value time series. The output time series files will be saved into the specified directory.

[Input files] Batch geodetic variation time series files with the same formats.

The first row is the file header. Starting from the second row of the file, each row record stores the sampling values of all the variations at one sampling epoch time. At least one column of the attributes in the record is the sampling epoch time.

[Parameter settings] Set the wildcard parameters for batch variation time series files, enter column ordinal number of the epoch time and target attribute time series in the record, and enter the multiple of the standard deviation and number of low-pass filtering parameters.

The entered number of the low-pass filtering parameters is not more than 1/2 of the number of time series samples, and not less than 1/30 of the number of samples. When the entered number exceeds this range, the program automatically takes the minimum or maximum value.



[Output files] Batch low-pass filtering time series files. The linear variation file.

The low-pass filtering time series file. Behind the input file header, adds two attributes including the constant term and linear term (annual variation rate) as the output file header. Behind the input file record, adds 5 attributes including the low-pass filtering value, low-pass filtering value after removing the constant term, low-pass filtering value after removing the constant and linear term, linear variation and residual value as the output file record.

The linear variation file TsqLinear#.txt (# is the column ordinal number of the specified attribute time series in the source time series file) without the file header. Each record of the file stores an input time series filtering information which includes the input time series file header, number of the filtering parameters, annual variation rate (per year, /a), constant term and residual standard deviation after reconstruction.

3.5.2 Batch time series averaging and record format time series construction

[Function] On the specified attribute time series from batch time series files with the same format, perform the average according to the given mode. The output time series is stored in two ways. The one is each time series saved as a file. The other is to arrange all the time series in rows, each record store a time series, and all the time series are stored into a record time series file.

[Input files] Batch variation time series files with the same format.

[Parameter settings] Set the wildcard parameters for batch variation time series files and input file format parameters, enter column ordinal number of the epoch time and target attribute time series in the record, and select the average period and type of the input time series files.

"The site variation time series" means that the sample of time series is the coordinate component, gravity, normal (orthometric) height or tilt component of the ground site.

"Geodetic network time series" means that sample of the time series is the GNSS baseline component, leveling height difference or gravity difference of the geodetic network.

[Output files] Batch mean variation time series files.

Behind the input file header, adds the average mode (0 is the monthly average, 1 is the GPS weekly average and 2 is the average of the given days) as the header of the average time series file. The record format: the middle epoch, average value, and number of the samples involved in the average calculation.

The program output the average value time series in the record format in the following two files.

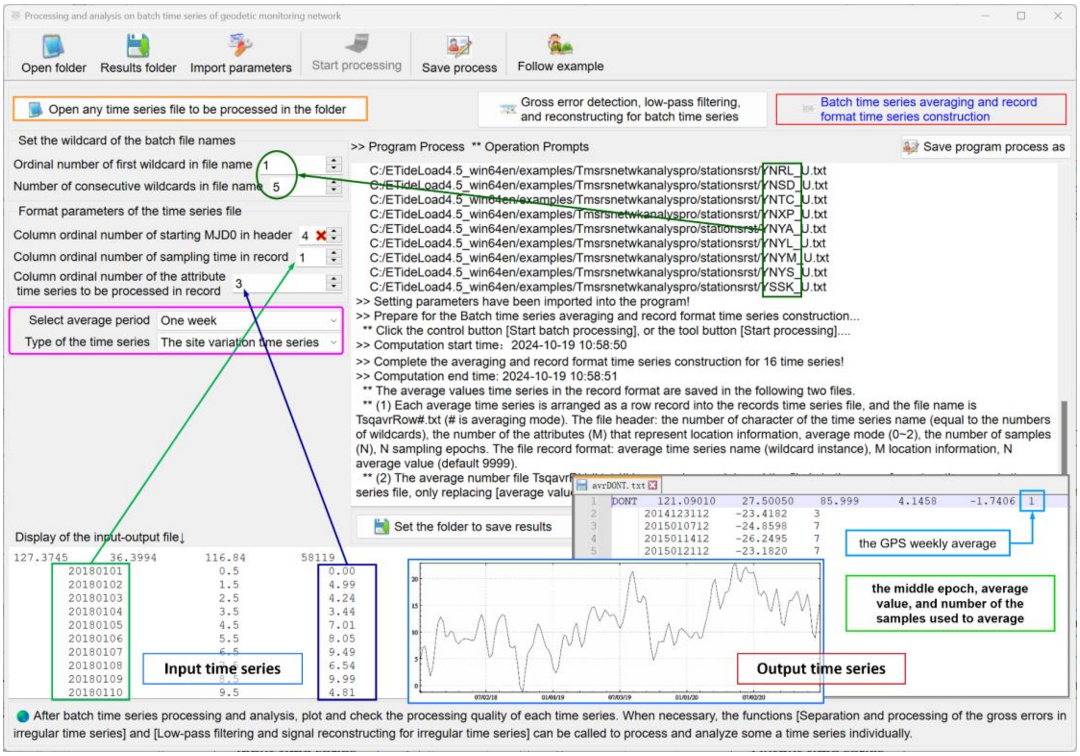
(1) The average value record time series file. Each average time series is arranged as a row record into the record time series file, and the file name is TsqavrRow#.txt (# is averaging mode).

The file header: the number of characters of the time series name (equal to the number of wildcards), the number (M) of the attributes that represent location information, average mode (0~2), the number (N) of samples, N sampling epochs.

The file record format: average time series name (wildcard instance), M location information, N average value (default 9999).

(2) The average number file TsqavrRkk#.txt (# is averaging mode). The file is in the same format as the record time series file, only replacing "average value" with the number of samples used to average.

After batch time series processing and analysis, plot and check the processing quality for each time series. When necessary, the functions [Separation and processing of the gross errors in irregular time series] and [Low-pass filtering and signal reconstructing for irregular time series] can be called to process and analyze some a time series individually.



3.6 Construction and analysis on record time series from geodetic network

[Purpose] Construct and analyze the variation record time series composed of multi-periods or continuous data from the geodetic monitoring network.

The record time series file is employed to represent the time series of a certain geodetic variation (monitoring element) in the geodetic network composed of multi-sites. One record represents a variation time series for a geodetic site, a GNSS baseline component, a gravity difference, a leveling route height difference or an InSAR monitoring point.

3.6.1 Construction of record time series from batch time series with same specifications

[Function] From batch variation time series files with the same specifications (same sampling time span and interval) stored in a directory, construct a record time series file according to the specified attribute.

The program calculates the maximum-minimum values of the sampling epochs and the minimum sampling interval in all the variation time series to build a new sampling specification. Each record stores one time series of the specified attribute, whose location information comes from the header of the corresponding input file. An attribute of 9999.000 indicates that there is no valid sampling value at the current epoch time.

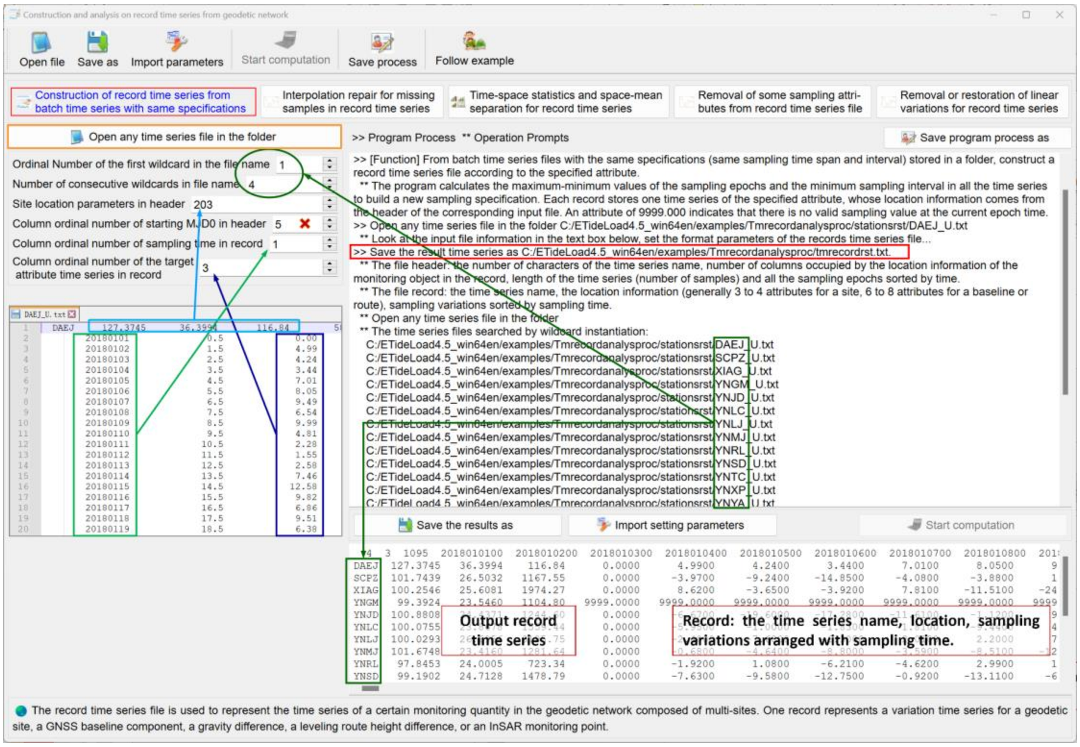
[Input files] Batch variation time series files with the same sampling specifications.

[Parameter settings] Set the wildcard parameters for batch variation time series files and site location parameters in the input file header, enter column ordinal number of the epoch time and target attribute time series in the record.

[Output files] The variation record time series file.

The file header: the number of characters of the time series name, number of columns occupied by the location information of the monitoring object in the record, length of the time series (number of samples) and all the sampling epochs arranged with time.

The file record: the time series name, location information (generally 3 to 4 attributes for a site, 6 to 8 attributes for a baseline or route), sampling variations arranged with sampling time.



3.6.2 Interpolation repair for missing samples in record time series

[Function] Interpolate and repair the missing samples in the variation record time series by the cubic spline or Gaussian function interpolation method. The function is not suitable for short-time estimation and prediction. For more missing samples repaired, please use the function [Low-pass filtering and signal reconstructing for irregular time series].

When there are more noise or missing samples in time series signals, Gaussian function interpolation is recommended.

3.6.3 Time-space statistics and space-mean separation for record time series

[Function] Firstly, calculate the time average, standard deviation, minimum and maximum of each variation record time series during the entire sampling period. Then calculate the spatial average, standard deviation, minimum and maximum of all variations at each sampling epoch time. Finally, calculate the spatiotemporal average, standard deviation, minimum and maximum of all variations during the entire sampling period.

[Input file] The variation record time series file.

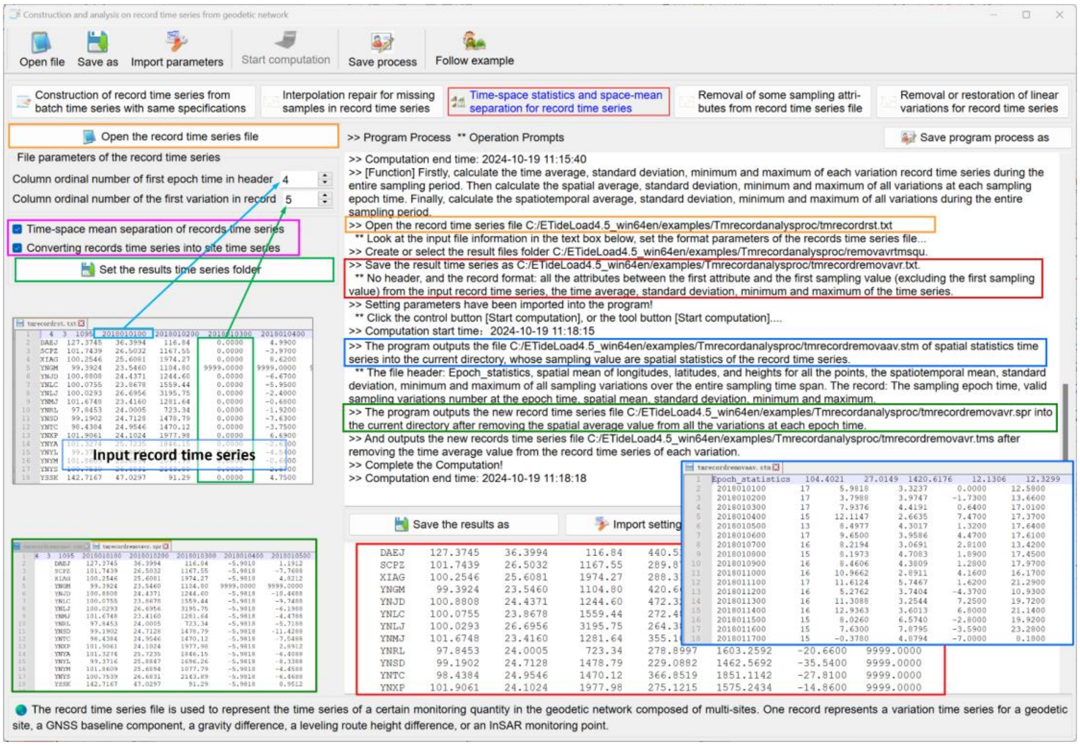
The sampling epochs in the file header should be one-by-one correspondence with the sampling variations in the file record.

[Parameter settings] Set the input file format parameters, enter column ordinal number of the epoch time in the header and target attribute time series in the record, and select the checkbox of time-space separation of record time series and converting record time series into site time series.

[Output files] The statistics result file on record time series, the variation record time series file after removing the space average, the variation record time series file after removing the time average, and the spatial statistics time series file.

The statistics result file on record time series file. No file header, and the record format: all the attributes between the first attribute and the first sampling value (excluding the first sampling value) from the input record time series, the time average, standard deviation, minimum and maximum of the time series.

The spatial statistics time series file. The file header: Epoch_statistics, spatial mean of longitudes, latitudes and heights for all the points, the spatiotemporal mean, standard deviation, minimum and maximum of all sampling variations over the entire sampling time span. The record: The sampling epoch time, valid sampling variations number at the epoch time, spatial mean, standard deviation, minimum and maximum.

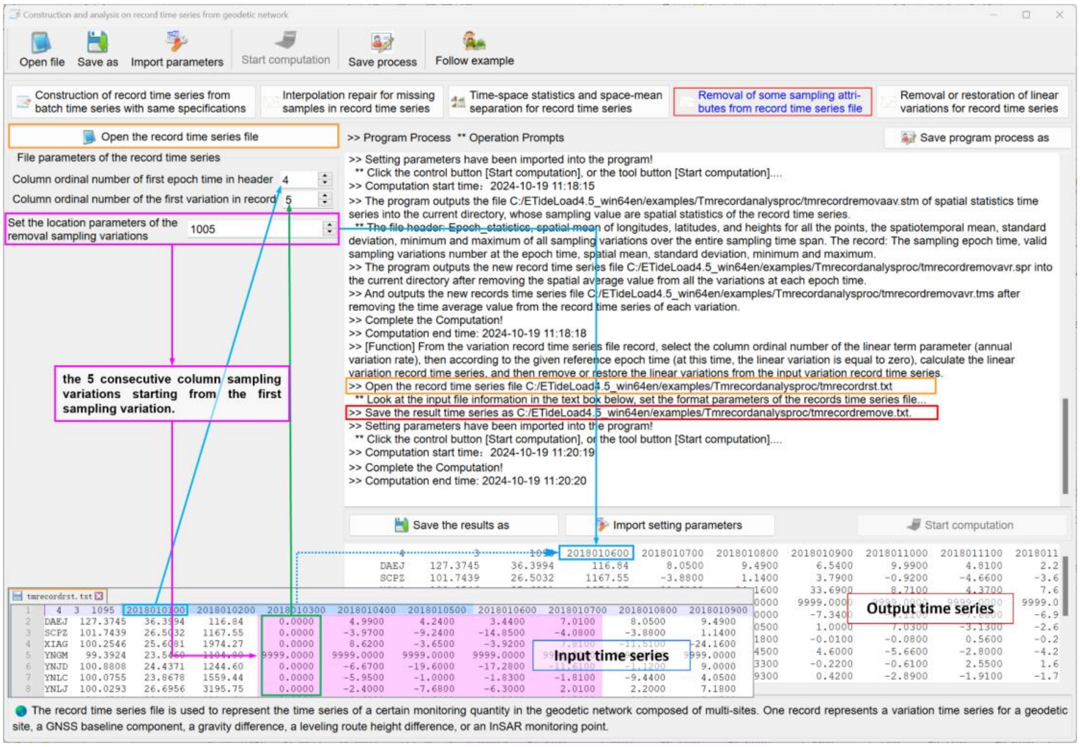


3.6.4 Removal of some sampling attributes from record time series file

[Function] Remove several consecutive columns of the sampling attributes from the record in the record time series file, and then remove the corresponding sampling epoch time in the file header.

[Input file] The variation record time series file.

[Parameter settings] Set the input file format parameters, input the location parameters of the removal sampling variations.



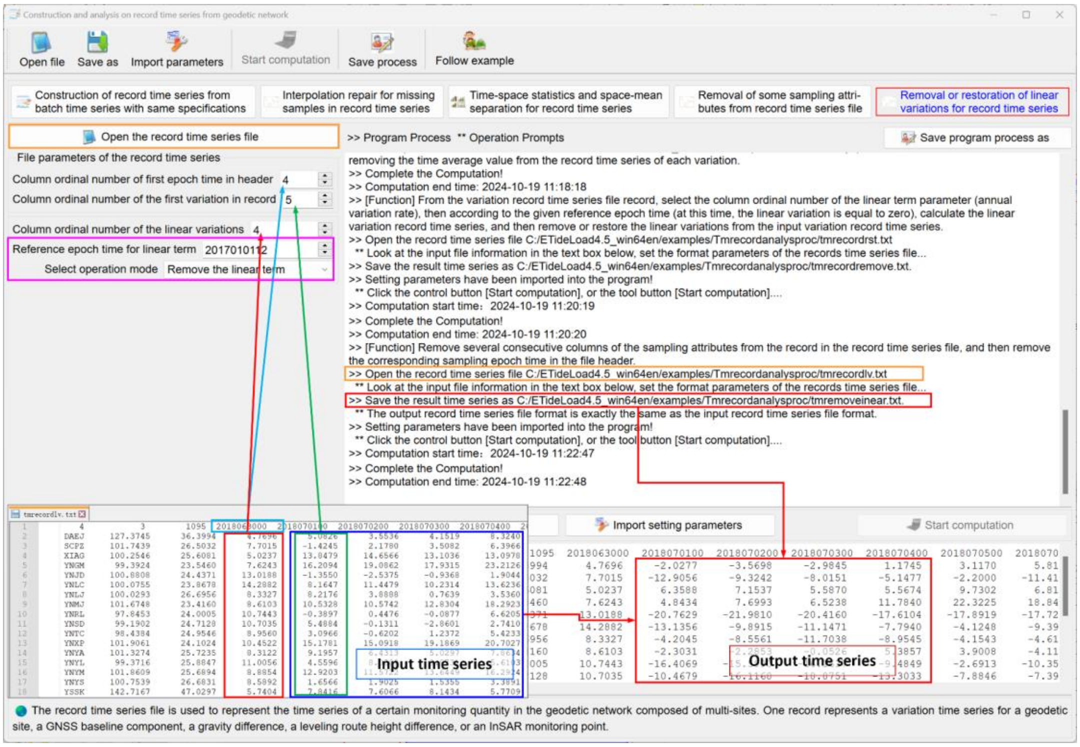
3.6.5 Removal or restoration of linear variations for record time series

[Function] From the variation record time series file record, select the column ordinal number of the linear term parameter (annual variation rate), then according to the given reference epoch time (at this time, the linear variation is equal to zero), calculate the linear variation record time series, and then remove or restore the linear variations from the input variation record time series.

[Input file] The variation record time series file.

[Parameter settings] Set the input file format parameters, enter column ordinal number of the linear term parameter in the record and the reference epoch time, and select to remove or restore the linear term.

[Output files] The result variation record time series file. The result file format is the same as that in the input record time series file.



3.7 Processing and analysis on variation (vector) grid time series

[Purpose] Perform operations such as the reference epoch transformation, difference and statistical analysis on the variation (vector) grid time series in the specified directory. The variation (vector) grid time series files are extracted according to the given wildcards.

The variation (vector) grid time series is composed of a series of numerical grid files of a certain kind of variation (vector), and the seventh attribute of the file header in each grid file is agreed to be the sampling epoch.

3.7.1 Reference epoch transformation for grid time series

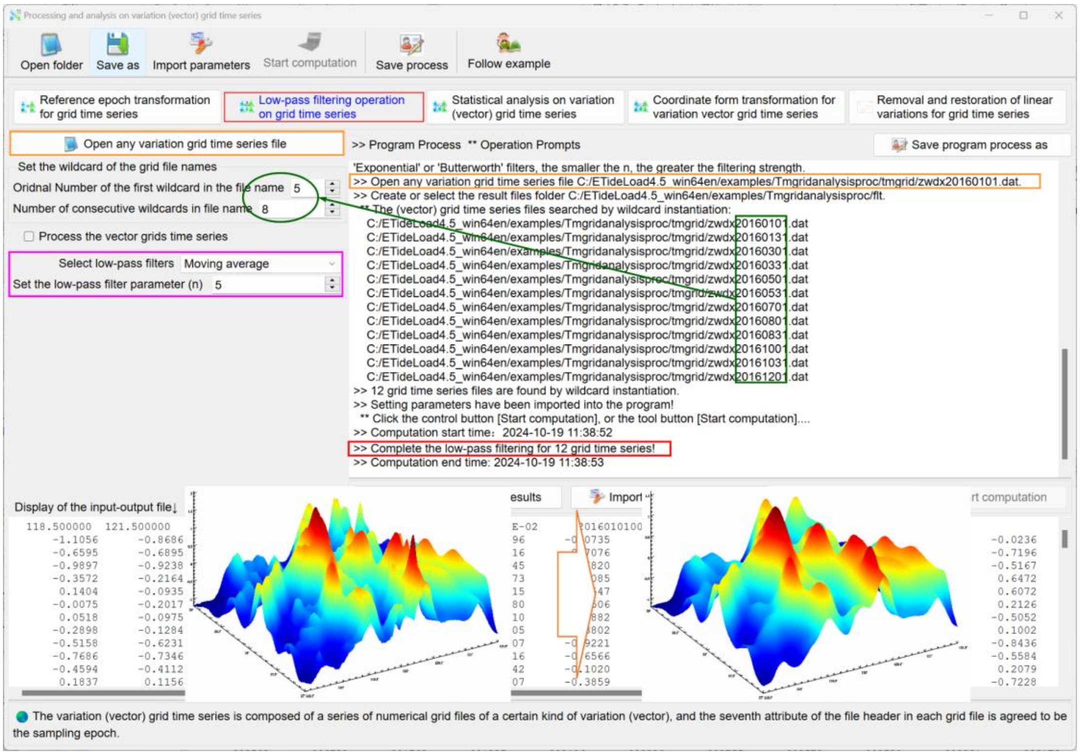
[Function] Unify the reference epoch time for all the variation (vector) grid time series by subtracting the variation (vector) grid at the given sampling time. After the epoch is unified, the variation grid values at the reference epoch time are always zero.

[Input file] The variation grid time series files. The variation grid file at the reference epoch time.

3.7.2 Low-pass filtering operation on grid time series

[Function] Using the low-pass filters such as the moving average, Gaussian, exponential or Butterworth, perform low-pass filtering on the variation grid time series. Before and after filtering, the grid specifications (Latitude and longitude range and spatial resolution) remain unchanged.

For the moving average filtering, the greater the filtering parameter n, the greater the filtering strength. For "Gaussian", "Exponential" or "Butterworth" filters, the smaller the n, the greater the filtering strength.



3.7.3 Statistical analysis on variation (vector) grid time series

[Function] Calculate the spatial mean, standard deviation, minimum and maximum of the variation (vector) grid time series at each sampling epoch time, to generate the spatial mean, standard deviation, minimum and maximum (four attributes) time series file. Then generate a new variation (vector) grid time series by removing the spatial mean grid at each epoch time. Finally, calculate the time mean, standard deviation, minimum and maximum of the time series of each (vector) cell-grid element, to generate time mean, standard deviation, minimum and maximum (vector) four grid files.

The program outputs the spatial mean, standard deviation, minimum and maximum time series file gridstatmsqu.txt of the variation (vector) grid time series.

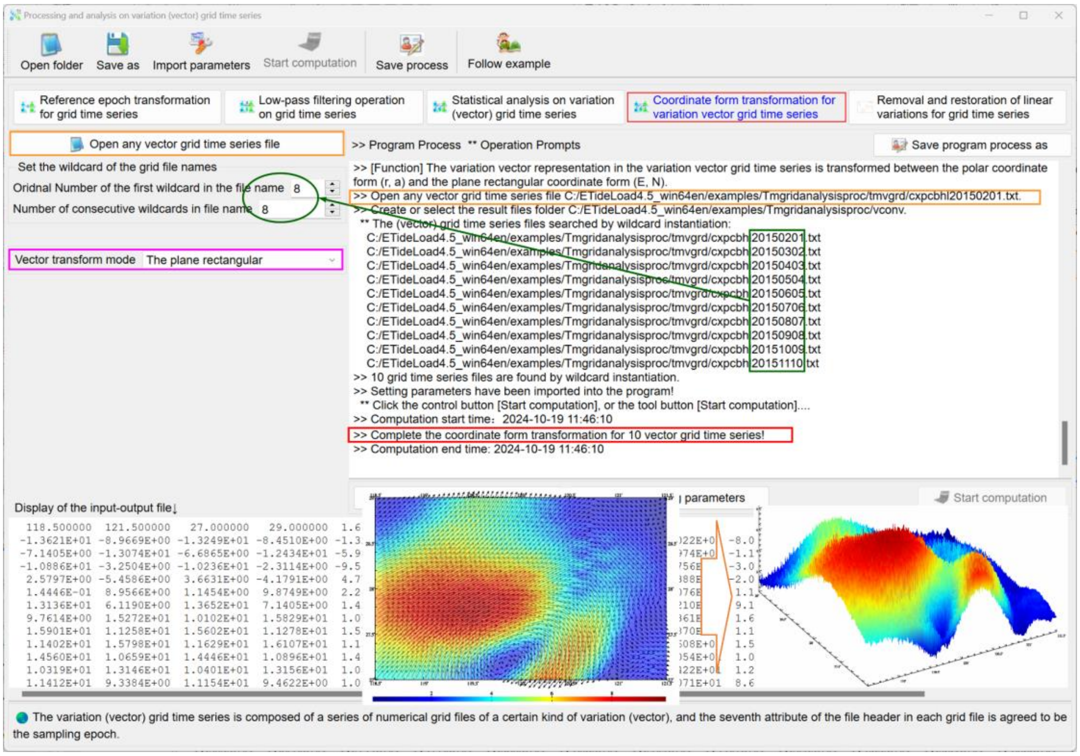
The file header: tmgridstatitics, the grid center longitude, latitude, zero value. The record: the sampling epoch time of the variation (vector) grid time series, the spatial mean, standard deviation, minimum and maximum of the grid at sampling epoch time.

The program also outputs the time mean, standard deviation, minimum and maximum (vector) grid files gridtmavr.dat, gridtmstd.dat, gridtminv.dat, and gridtmaxv.dat.

3.7.4 Coordinate form transformation for variation vector grid time series

[Function] The vector in the variation vector grid time series is transformed between the polar coordinate form (r, a) and the plane rectangular coordinate form (E, N).

[Input file] The variation vector grid time series files.



3.7.5 Removal and restoration of linear variations for grid time series

[Function] Using the annual variation (vector) rate grid, calculate the linear variation (vector) grid time series according to the given reference epoch time (the linear variation at reference epoch time is always equal to zero), and then remove or restore the linear variations of the variation (vector) grid time series.

3.8 Multi-form spatiotemporal interpolation from grid time series

[Purpose] From the variation (vector) grid time series files in the specified directory, construct the variation time series according to the location and sampling specifications by the given space and time interpolation method. The variation (vector) grid time series files are extracted according to the given wildcards.

The latitude and longitude of the site to be interpolated should not exceed the latitude and longitude range of the grid time series, and the interpolated epoch should not exceed the sampling time range of the grid time series by too much.

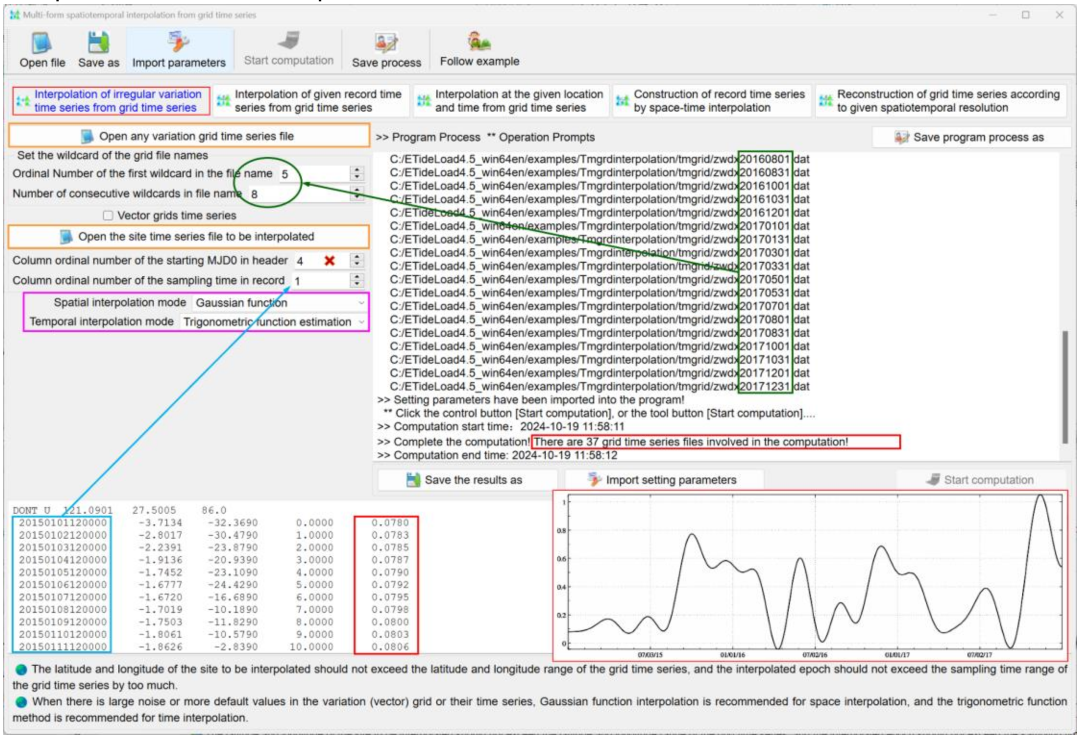
When there is large noise or more default values in the variation (vector) grids or their time series, Gaussian function interpolation is recommended for space interpolation, and the trigonometric function method is recommended for time interpolation.

3.8.1 Interpolation of irregular variation time series from grid time series

[Function] From the variation (vector) grid time series files, construct the irregular variation time series according to the location and sampling specification in the input irregular time series by the given two-dimensional space interpolation and one-dimensional time interpolation method.

[Input file] The variation (vector) grid time series files. The site variation time series file to be interpolated.

[Parameter settings] Set the wildcard parameters for the variation (vector) grid time series files and the site variation time series file format parameters. Select the space interpolation and time interpolation method.



3.8.2 Interpolation of the given record time series from grid time series

[Function] Using the given two-dimensional space interpolation and one-dimensional time interpolation method, interpolate to obtain all the sampling values of the input record time series from the variation grid time series files. The output record time series file format is the same as the input record time series file.

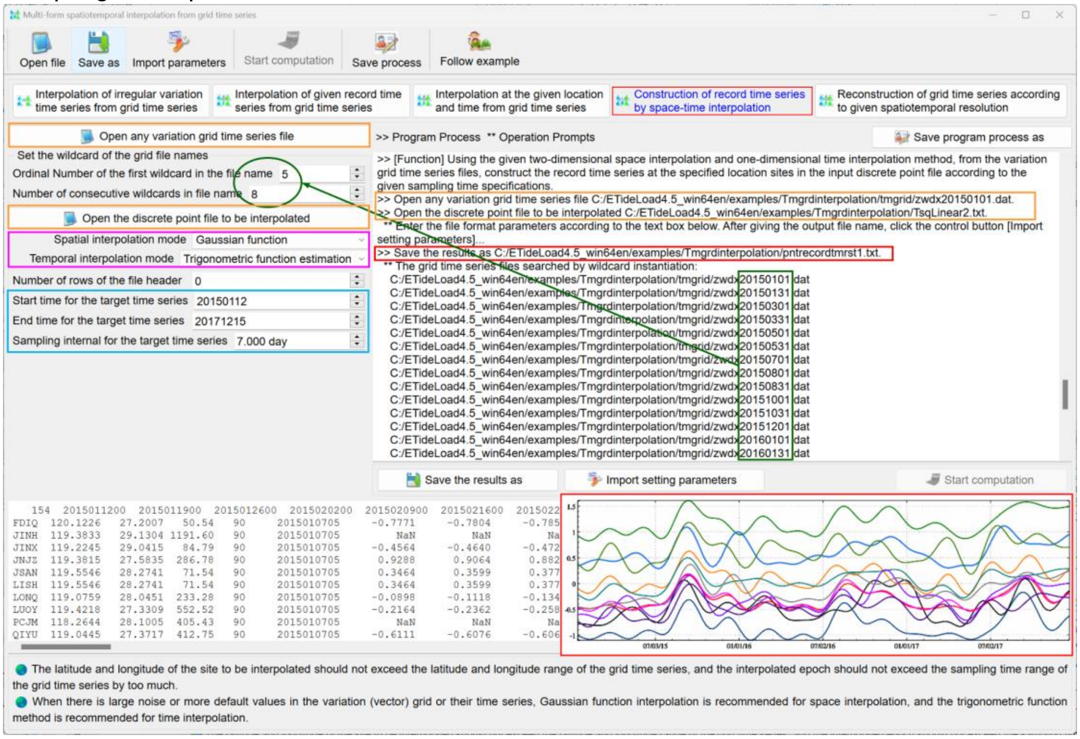
The program also outputs the remnant variation record time series file (file extension rnt) into the current directory. The format is the same as the input record time series file. Here the remnant variation is equal to the difference between the input sample value and the interpolation.

3.8.3 Interpolation at the given location and time from grid time series

[Function] Using the given two-dimensional space interpolation and one-dimensional time interpolation method, interpolate or estimate the sampling value at the given location and epoch time from the variation grid time series files.

3.8.4 Construction of record time series by space-time interpolation

[Function] Using the given two-dimensional space interpolation and one-dimensional time interpolation method, from the variation grid time series files, construct the record time series at the specified location sites in the input discrete point file according to the given sampling time specifications.



3.8.5 Reconstruction of grid time series according to given spatiotemporal resolution

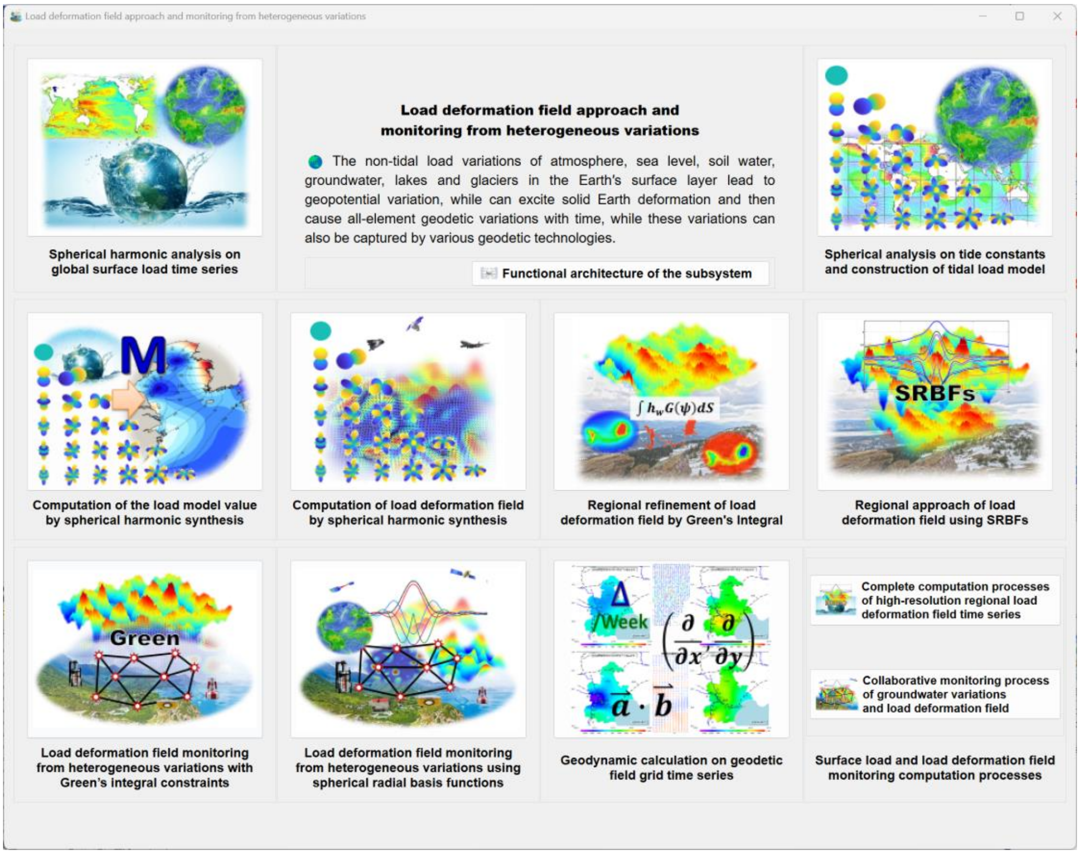
[Function] Using the given two-dimensional spatial interpolation and one-dimensional time interpolation or estimation method, increase or decrease the spatial and temporal resolution of the grid time series according to the given grid spatial resolution and time sampling specification, and then calculate time-derivative (per week, /wk) of the variation grid time series.

The program output the variation (vector) grid time series files grdtmsp*.dat and their time-derivative (vector) grid time series files grdtmdf.dat (per week, /wk).

4 Load deformation field approach and monitoring from heterogeneous variations

The non-tidal load variations of atmosphere, sea level, soil water, groundwater, lakes and glaciers in the Earth's surface layer lead to geopotential variation, while can excite solid Earth deformation and then cause all-element geodetic variations with time, while these variations can also be captured by various geodetic technologies.

The group of programs can be employed to compute and approach the global and regional non-tidal load effects on all-element geodetic variations, and then constrain and assimilate the deep fusion of multi-source heterogeneous geodetic data strictly according to the principles of the geodesy and Earth's deformation, so as to realize the collaborative monitoring of the land water variations and time-varying gravity field from various heterogeneous geodetic techniques.



Using the analytically compatible geodetic and geodynamic algorithm system with the numerical standards unified and geophysical models coordinated, to monitor and represent various geodetic non-tidal effects are the important basis for 1cm (20μGal) accuracy level of geodesy, and are also necessary conditions to realize the collaborate monitoring of various geodetic technologies, deep fusion of multi-source heterogeneous geodetic data, and construction and maintenance of high-precision geodetic datum frame.

4.1 Spherical harmonic analysis on global surface load time series

[Purpose] From the global grid model of the surface loads such as land/sea surface atmosphere, land water and sea level variation, construct a normalized surface load spherical harmonic coefficient model by spherical harmonic analysis. Using the model, the non-tidal load effects on various geodetic variations outside the solid Earth can be computed by the spherical harmonic synthesis.

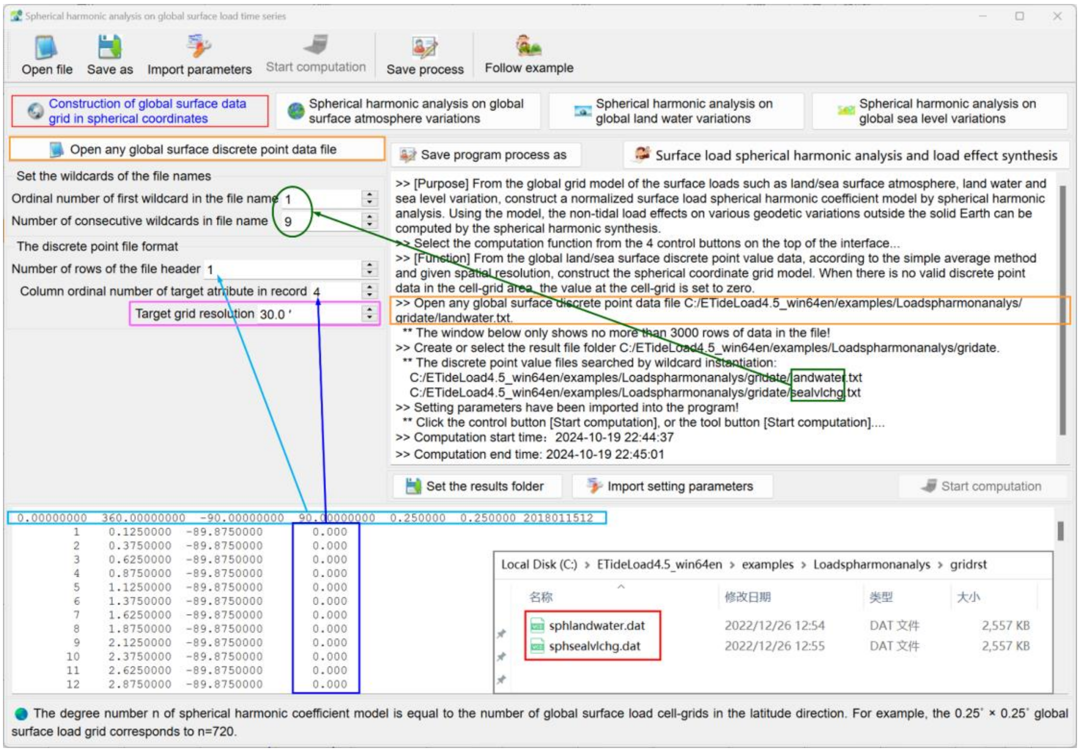
The degree number n of spherical harmonic coefficient model is equal to the number of global surface load cell-grids in the latitude direction. For example, the 0.25˚ × 0.25˚ global surface load grid corresponds to n =720.

4.1.1 Construction of global surface data grid in spherical coordinates

[Function] From the global land/sea surface discrete point value data, according to the simple average method and given spatial resolution, construct the spherical coordinate grid model. When there is no valid discrete point data in the cell-grid area, the value at the cell-grid is set to zero.

[Input files] A series of global land/sea surface discrete point value data files with the same format.

The file record format: Point number (name), longitude, latitude (decimal degrees), …, attribute to be gridded, ….



[Parameter settings] Set the wildcard parameters of the input file names, and enter the number of rows of the input file header, row ordinal number of target attribute in the file record and grid resolution.

[Output files] A series of spherical coordinate grid files that correspond one-to-one with the input discrete point value files.

4.1.2 Spherical harmonic analysis on global surface atmosphere variations

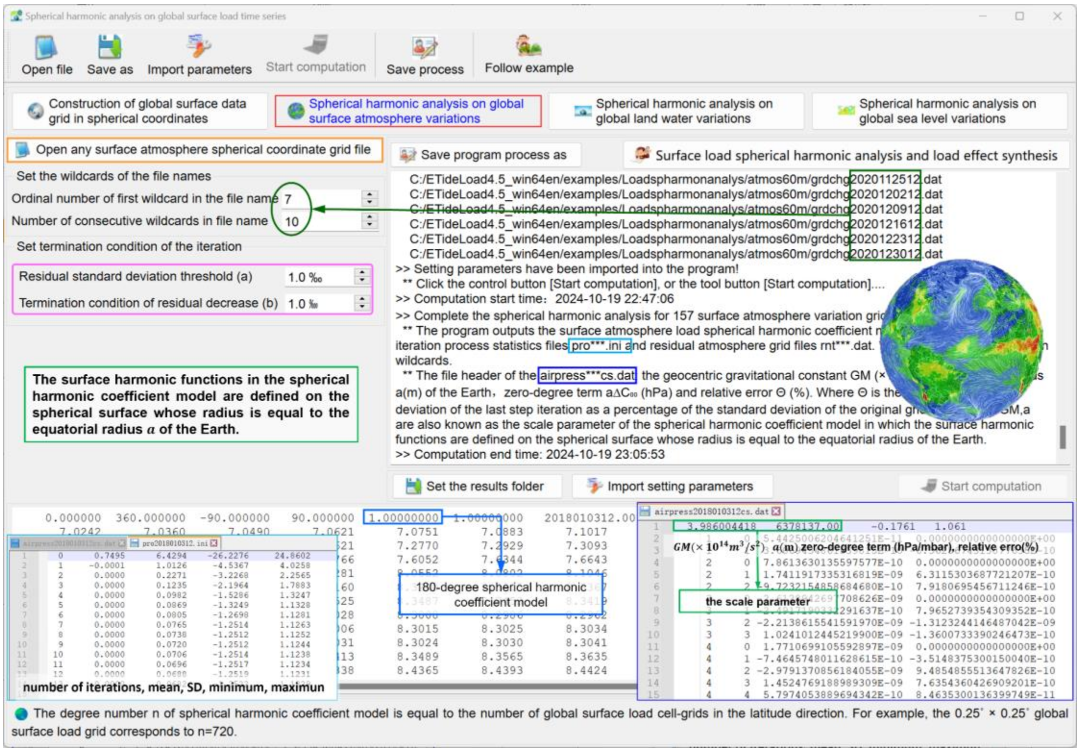
[Function] From the global land/sea surface atmosphere variation (in unit of hPa or mbar) spherical coordinate grid time series, construct the non-tidal atmosphere load spherical harmonic coefficient model (in unit of m) time series by normalized spherical harmonic analysis.

The spherical coordinate grid time series files are extracted according to the given wildcards.

[Input files] The global land/sea surface atmosphere variation spherical coordinate grid time series files.

[Parameter settings] Set the wildcard parameters for the file names of grid time series and enter the iteration condition parameters.

Iteration termination condition: The standard deviation of the residual grid value is less than a% of the standard deviation of the original grid value, or the difference of the residual standard deviation of the previous step iteration relative to the current step iteration is less than b‰ of the standard deviation of the original grid values.



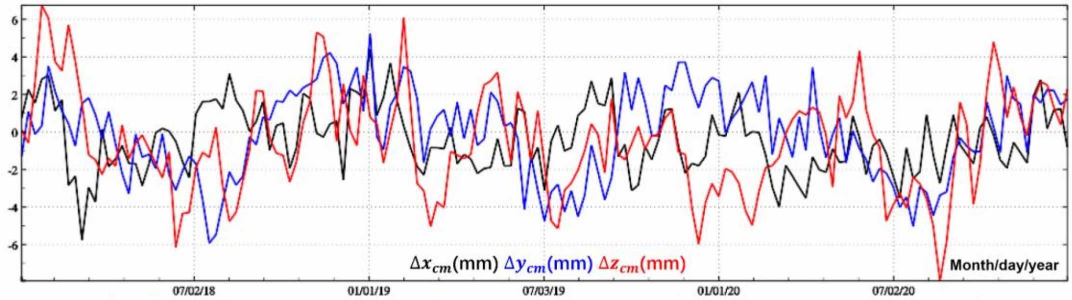
The degree number n of spherical harmonic coefficient model is equal to the number of global surface load cell-grids in the latitude direction. For example, the 0.25° × 0.25° global surface load grid corresponds to n=720.

[Output files] The surface atmosphere load spherical harmonic coefficient model files airpress***cs.dat, iteration process statistics file pro***.ini and residual atmosphere grid file rnt***.dat. Here, *** are the instance of the given wildcards.

The file header of the airpress***cs.dat: the geocentric gravitational constant GM ($\times10^{14}$m³/s²), equatorial radius a(m) of the Earth, zero-degree term aΔC_{00} (hPa) and relative error Θ (%). Where Θ is the residual standard deviation of the last step iteration as a percentage of the standard deviation of the original grid values, and GM, a are also known as the scale parameter of the spherical harmonic coefficient model in which the surface harmonic functions are defined on the spherical surface whose radius is equal to the equatorial radius of the Earth.

The zero-degree term represents the variations of the total atmospheric mass caused by the variation of global atmospheric pressure, which is meaningless under the condition of Earth's atmospheric mass conservation. The three first-degree spherical harmonic coefficients (ΔC_{10}, ΔC_{11}, ΔS_{11}) represent the variations of the Earth's center of mass due to the variations of global atmospheric pressure.

The program outputs also the global atmosphere load effect time series file geocenterairpr.txt on Earth's mass centric variations into the current directory. The record format: Epoch time (real years), Xcm (mm), Ycm (mm), Zcm (mm) and date (long integer agreed by ETideLoad).



The surface atmosphere load effect time series (mm) on Earth's mass centric variations

4.1.3 Spherical harmonic analysis on global land water variations

[Function] From the global land equivalent water height variation (in unit of cm) spherical coordinate grid time series, construct the land water non-tidal load spherical harmonic coefficient model (in unit of m) time series by normalized spherical harmonic analysis.

[Input files] The global land equivalent water height variation spherical coordinate grid time series files. The land-sea terrain spherical coordinates grid file.

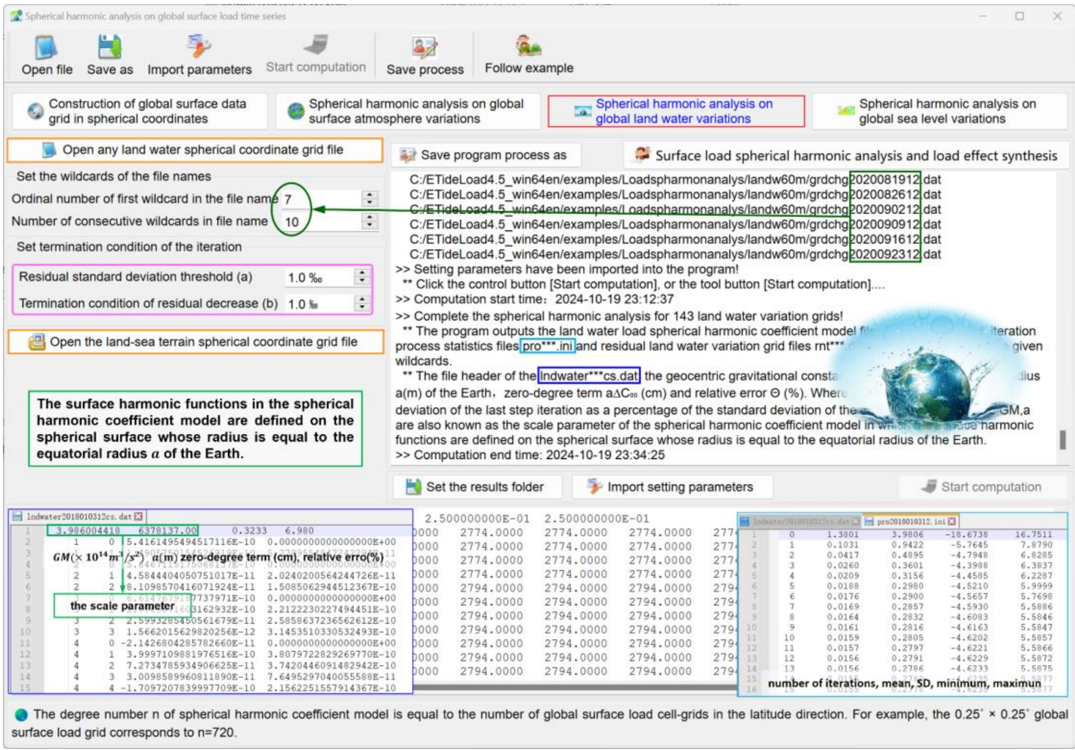
The spatial resolution of the land-sea terrain grid should not be lower than that of the surface load grid.

[Parameter settings] Set the wildcard parameters for the file names of grid time series and enter the iteration condition parameters.

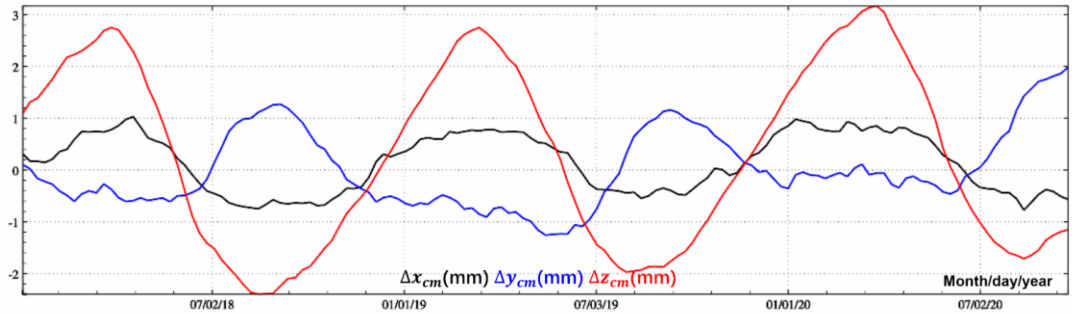
[Output files] The global land water load spherical harmonic coefficient model files lndwater***cs.dat, iteration process statistics file pro***.ini and residual equivalent water height grid file rnt***.dat. Here, *** are the instance of the given wildcards.

The file header of the Indwater***cs.dat: the geocentric gravitational constant GM ($\times10^{14}m^3/s^2$), equatorial radius a(m) of the Earth, zero-degree term $a\Delta C_{00}$ (cm) and relative error Θ (%).

The three first-degree spherical harmonic coefficients (ΔC_{10}, ΔC_{11}, ΔS_{11}) represent the variations of the Earth's center of mass due to variations of global land water. For global geodetic purposes, the first-degree spherical harmonic coefficients need to be considered. The zero-degree term can be controlled to a small value by adjusting the time datum.



The program outputs also the global land water load effect time series file geocenterlandw.txt on Earth's mass centric variations into the current directory. The record format: Epoch time (real years), Xcm (mm), Ycm (mm), Zcm (mm) and date (long integer agreed by ETideLoad).



The global land water variation load effect time series (mm) on Earth's mass centric variations

4.1.4 Spherical harmonic analysis on global sea level variations

[Function] From the global non-tidal sea level variation (in unit of cm) spherical coordinate grid time series, construct the sea level variation load spherical harmonic coefficient model (in unit of m) time series by normalized spherical harmonic analysis.

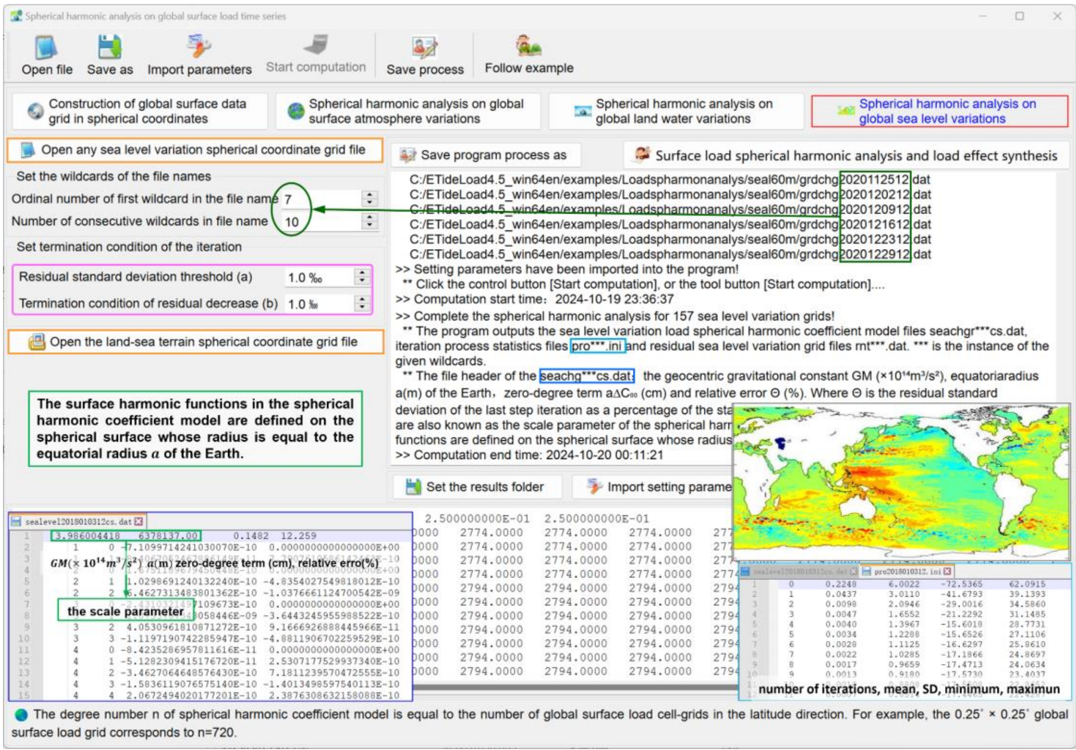
[Input files] The global sea level variation spherical coordinate grid time series files. The land-sea terrain spherical coordinates grid file.

The spatial resolution of the land-sea terrain grid should not be lower than that of the surface load grid.

[Parameter settings] Set the wildcard parameters for the file names of grid time series and enter the iteration condition parameters.

Iteration termination condition: The standard deviation of the residual grid value is less than a% of the standard deviation of the original grid value, or the difference of the residual standard deviation of the previous step iteration relative to the current step iteration is less than b‰ of the standard deviation of the original grid values.

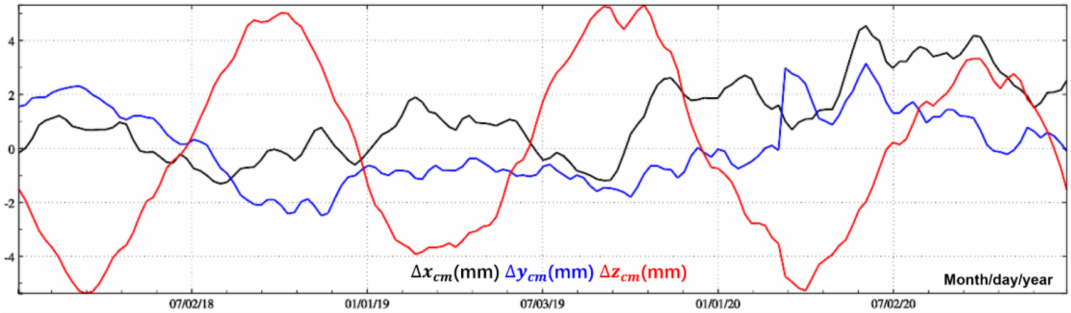
[Output files] The global sea level variation load spherical harmonic coefficient model files seachg***cs.dat, iteration process statistics files pro***.ini and residual sea level variation grid files rnt***.dat. Here, *** are the instance of the given wildcards.



The three first-degree spherical harmonic coefficients (ΔC_{10}, ΔC_{11}, ΔS_{11}) represent the variations of the Earth's center of mass due to global sea level variations. For global geodetic purposes, the first-degree spherical harmonic coefficients needs to be taken into account.

The zero-degree term can be controlled to a small value by adjusting the time datum.

The program outputs also the global sea level variation load effect time series file geocentersealv.txt on Earth's mass centric variations into the current directory. The record format: Epoch time (real years), Xcm (mm), Ycm (mm), Zcm (mm) and date (long integer agreed by ETideLoad).



The global sea level variation load effect time series (mm) on Earth's mass centric variations

For global geodetic purposes, the zero constraint should be considered that the sum of the zero-degree terms of sea, land and atmosphere at any epoch time is equal to zero, that is, the total loads of sea level, land water and atmospheric pressure variations is conserved.

Using the time series file of Earth's mass centric variations effected by surface non-tidal loads output by the program, the function [Computation of Earth's mass centric variation effects on all-element geodetic variations] in section 2.5.2 can be called to calculate the Earth's mass centric variation effects on all-element geodetic variations due to various non-tidal surface loads.

4.2 Spherical analysis on tidal harmonic constants and construction of tidal load model

[Purpose] From the tidal constituent harmonic constant grid of the global land/sea atmosphere or sea surface height, construct a normalized tidal load spherical harmonic coefficient model by spherical harmonic analysis. The model format is the same as FES2004 ocean tidal load model in the IERS conventions (2010). Using the model, the tidal load effects on various geodetic variations outside the solid Earth can be computed by the spherical harmonic synthesis.

The unit of the tidal constituent harmonic constants is the same as the unit of the tidal load spherical harmonic coefficients. The unit of the surface atmosphere tidal harmonic constants and the atmosphere tidal load spherical harmonic coefficients are hPa or mbar, and the unit of the ocean tidal harmonic constants and the load spherical harmonic coefficients are cm.

The degree number maxn of tidal load spherical harmonic coefficient model is equal to the number of harmonic constant spherical coordinate cell-grids in the latitude direction. For example, the 0.25˚ × 0.25˚ tidal harmonic constant spherical coordinate grid corresponds to maxn=720.

4.2.1 Construction tidal harmonic constant grid in spherical coordinates

[Function] From the tidal constituent harmonic constants of the surface atmosphere or sea surface height on the discrete points, according to the simple average method and given spatial resolution, construct spherical coordinate harmonic constant vector (in-phase amplitude, out-of-phase amplitude) grid model. When there is no valid discrete harmonic constant data in the cell-grid area, the vector on the cell-grid is set to zero.

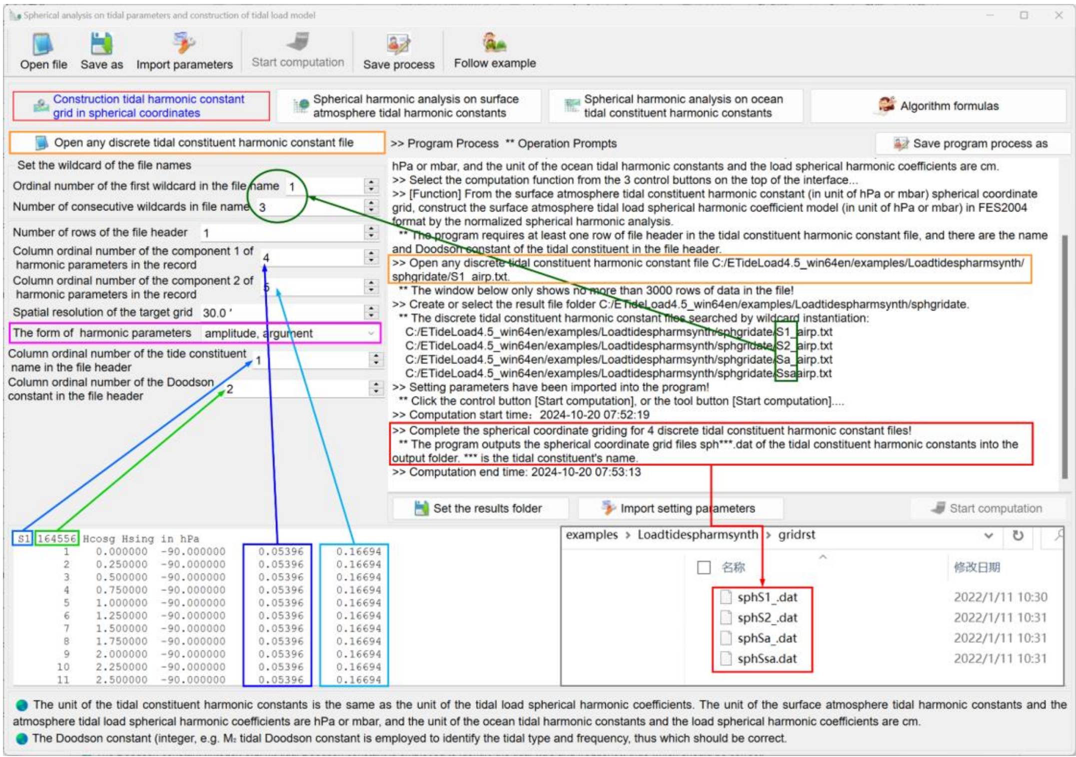
[Input files] A series of global discrete tidal constituent harmonic constant files with the same format.

The program requires at least one row of file header in the tidal constituent harmonic constant file, and there are the name and Doodson constant of the tidal constituent in the file header.

The Doodson constant (integer, e.g. M_2 tidal Doodson constant is employed to identify the tidal type and frequency, thus which should be correct.

[Parameter settings] Set the wildcard parameters of the input file names. Enter the number of rows of the input file header, column ordinal numbers of the tidal constituent name and Doodson constant in the input file header, column ordinal number of the component 1 and 2 of harmonic constants in the record and select the form of harmonic constants.

[Output files] The spherical coordinate grid files of the tidal constituent harmonic constants sph***.dat. Here, *** are the tidal constituent's name.



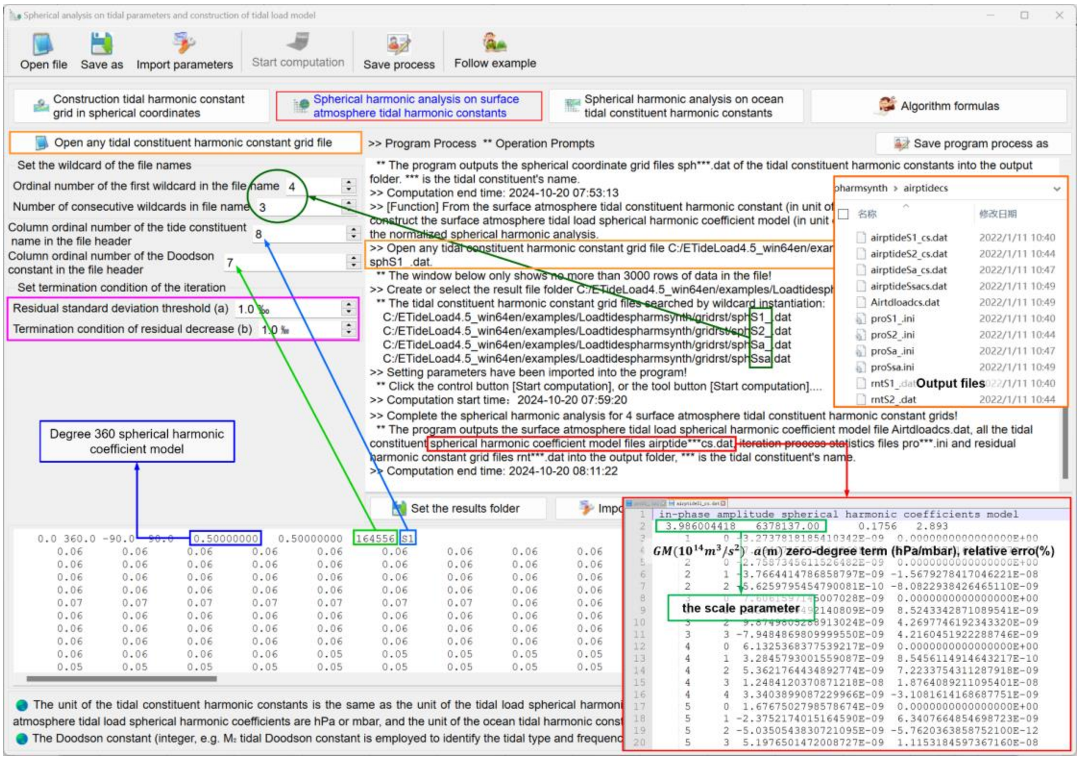
4.2.2 Spherical harmonic analysis on surface atmosphere tidal harmonic constants

[Function] From the surface atmosphere tidal constituent harmonic constant (in unit of hPa or mbar) spherical coordinate grid, construct the surface atmosphere tidal load spherical harmonic coefficient model (in unit of hPa or mbar) in FES2004 format by the normalized spherical harmonic analysis.

The tidal constituent harmonic constant vector spherical coordinate grid files are extracted according to the given wildcards.

[Input files] All the surface atmosphere tidal constituent harmonic constant vector spherical coordinate grid files.

[Parameter settings] Set the wildcard parameters for the input file names, enter the column ordinal number of the tidal constituent's name and its Doodson constant in the input file header, and set the iteration condition parameters.



[Output files] The surface atmosphere tidal load spherical harmonic coefficient model file Airtdloadcs.dat, all the tidal constituent spherical harmonic coefficient model files airptide***cs.dat, iteration process statistics files pro***.ini and residual harmonic constant grid files rnt***.dat. Here, *** are the tidal constituent's name.

The program also outputs the first-degree atmosphere tidal load spherical harmonic coefficient file AtideOne.dat into the current directory. Which should be copied into the directory c:\ETideLoad4.5_win64en\iers and can be employed to forecast the atmosphere tidal load effects on Earth's mass centric variation as section 2.5.4.

The file header inludes the geocentric gravitational constant GM ($\times 10^{14}m^3/s^2$) and equatorial radius a(m) of the Earth. The record (starting from the third row) includes the tidal constituent's name, Doodson constant, the degree-1 order-0 spherical harmonic coefficient C_{10}+ from in-phase amplitude, that C_{10}- from out-of-phase amplitude, and the degree-1 order-1 spherical harmonic coefficient C_{11}+ from in-phase amplitude, that C_{11}- from out-of-phase amplitude, and the degree-1 order-1 spherical harmonic coefficient S_{11}+ from in-phase amplitude, that S_{11}- from out-of-phase amplitude.

```
Surface atmospheric tidal load normalized spherical harmonic coefficient model in hPa or mbar.
Created by ETideLoad, ZHANG Chuanyin, Chinese Academy of Surveying and Mapping.
 Doodson name    n    m        Csin+         Ccos+         Csin-         Ccos-         C+      eps+        C-      eps-
 164.556   S1    1    0  -0.01044593    0.00562824   -0.01044593    0.00562824   0.011866  298.3157  0.011866  298.3157
 164.556   S1    1    1  -0.02016686   -0.30983778   -0.02700702    0.03082551   0.310493  183.7240  0.040983  318.7776
 164.556   S1    2    0  -0.00880807    0.02708492   -0.00880807    0.02708492   0.028481  341.9854  0.028481  341.9854
 164.556   S1    2    1  -0.00267857   -0.06099820   -0.02133360    0.03899757   0.061057  182.5144  0.044451  331.3192
 164.556   S1    2    2   0.04746516   -0.07024418   -0.05104501   -0.01871795   0.084777  145.9525  0.054369  249.8623
 164.556   S1    3    0   0.02424426    0.01222005    0.02424426    0.01222005   0.027150   63.2501  0.027150   63.2501
 164.556   S1    3    1  -0.00065416    0.08663644    0.01517276    0.03225602   0.086639  359.5674  0.035646   25.1916
 164.556   S1    3    2   0.05672425   -0.01538354    0.00625213   -0.04261689   0.058773  105.1736  0.043073  171.6539
 164.556   S1    3    3   0.01546691    0.03548381   -0.06617256    0.00859525   0.038708   23.5517  0.066728  277.4008
 164.556   S1    4    0   0.01956420   -0.01827060    0.01956420   -0.01827060   0.026769  133.0418  0.026769  133.0418
 164.556   S1    4    1  -0.01459744    0.00148107    0.03555613   -0.00398511   0.014672  275.7935  0.035779   96.3950
 164.556   S1    4    2   0.01934232    0.02790035    0.01483035   -0.01817240   0.033949   34.7322  0.023456  140.7824
 164.556   S1    4    3   0.05868605    0.05584202    0.02090025   -0.06381922   0.081009   46.4225  0.067154  161.8668
 164.556   S1    4    4   0.05071872   -0.00993816   -0.02940598    0.00988633   0.051683  101.0865  0.031023  288.5827
 164.556   S1    5    0   0.00535373   -0.01557249    0.00535373   -0.01557249   0.016467  161.0273  0.016467  161.0273
 164.556   S1    5    1  -0.01117229    0.00673870   -0.00397207   -0.03368705   0.013047  301.0968  0.033920  186.7247
 164.556   S1    5    2  -0.01642619    0.00466607   -0.01567284    0.00468926   0.017076  285.8579  0.016359  286.6570
 164.556   S1    5    3   0.01540811    0.05344217    0.01772763   -0.01768282   0.055619   16.0830  0.025039  134.9275
 164.556   S1    5    4  -0.02913706   -0.01782056    0.01890564    0.02078884   0.034155  238.5496  0.028100   42.2838
 164.556   S1    5    5   0.06196212   -0.00041678   -0.00316231    0.00014887   0.061964   90.3854  0.003166  272.6952
 164.556   S1    6    0  -0.01902007   -0.00031063   -0.01902007   -0.00031063   0.019023  269.0643  0.019023  269.0643
 164.556   S1    6    1   0.01292417    0.05007315   -0.01614491   -0.03693554   0.051714   14.4725  0.040310  203.6106
 164.556   S1    6    2  -0.02124270    0.00967981   -0.00563026    0.00828166   0.023344  294.4977  0.010014  325.7903
 164.556   S1    6    3  -0.01510560    0.02852466   -0.01900168    0.01800736   0.032277  332.0960  0.026179  313.4610
 164.556   S1    6    4  -0.01607949   -0.01730702   -0.00117357    0.01305322   0.023624  222.8943  0.013106  354.8626
```

```
  3.986004418   6378137.00
 name Doodson           C10+            C10-            C11+            C11-            S11+            S11-
   S1 164.556 -0.32755435E-08  0.17648553E-08 -0.73961840E-08 -0.43745105E-07 -0.53411096E-07 -0.10724379E-08
   S2 273.555 -0.63049967E-09  0.13744707E-08  0.80115817E-10  0.52363295E-08  0.33900139E-08 -0.10865938E-08
   Sa  56.565  0.82105514E-07 -0.16159915E-06 -0.35243498E-07 -0.82919083E-08  0.35037721E-07 -0.12165101E-06
  Ssa  57.555  0.65256321E-08  0.64837464E-07 -0.35845502E-07 -0.25039833E-07  0.12771654E-07  0.24911463E-07
```

4.2.3 Spherical harmonic analysis on ocean tidal constituent harmonic constants

[Function] From the ocean tidal constituent harmonic constant (in unit of cm) spherical coordinate grid, construct the ocean tidal load spherical harmonic coefficient model (in unit of cm) in FES2004 format using the normalized spherical harmonic analysis.

[Input files] All the ocean tidal constituent harmonic constant vector spherical coordinate grid files. The land-sea terrain spherical coordinate grid file.

The land-sea terrain spherical coordinates grid is employed for the land-sea separation for the ocean tidal harmonic constants, whose resolution should not be lower than that of the ocean tidal constituent harmonic constant grid.

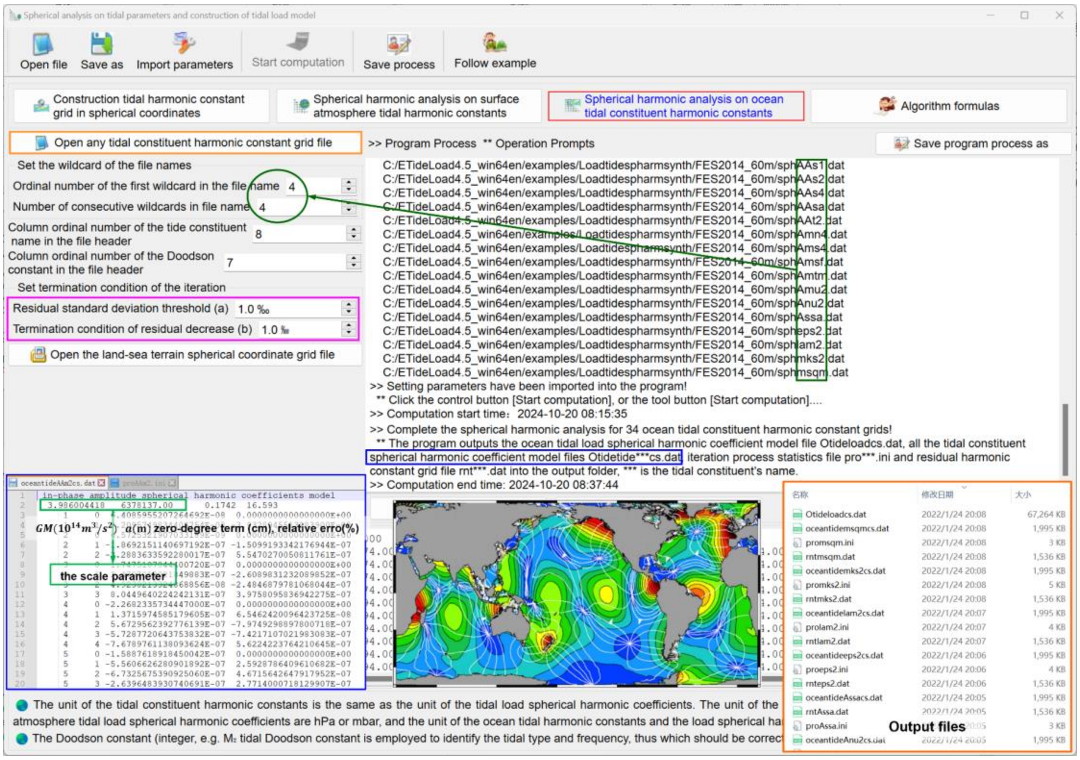
The tidal constituent harmonic constant vector spherical coordinate grid files are extracted according to the given wildcards.

[Parameter settings] Set the wildcard parameters for the input file names, enter the column ordinal number of the tidal constituent's name and its Doodson constant in the input file header, and set the iteration condition parameters.

[Output files] The ocean tidal load spherical harmonic coefficient model file Otideloadcs.dat, all the tidal constituent spherical harmonic coefficient model files

Otidetide***cs.dat, iteration process statistics file pro***.ini and residual harmonic constant grid file rnt***.dat. Here, *** are the tidal constituent's name.

The program also outputs the first-degree ocean tidal load spherical harmonic coefficient file OtideOne.dat into the current directory. Which should be copyed into the directory c:\ETideLoad4.5_win64en\iers and can be employed to forecast the ocean tidal load effects on Earth's mass centric variation as section 2.5.3.



Ocean tidal height load normalized spherical harmonic coefficient model in cm.										
Created by ETideLoad, ZHANG Chuanyin, Chinese Academy of Surveying and Mapping.										
Doodson	name	n	m	Csin+	Ccos+	Csin-	Ccos-	C+	eps+	C-	eps-
247.455	2N2	1	0	0.00458562	0.00231038	0.00458562	0.00231038	0.005135	63.2596	0.005135	63.2596
247.455	2N2	1	1	-0.00773380	0.00473565	0.01063946	-0.00152991	0.009069	301.4805	0.010749	98.1828
247.455	2N2	2	0	0.01415077	-0.00470716	0.01415077	-0.00470716	0.014913	108.3994	0.014913	108.3994
247.455	2N2	2	1	-0.01749377	0.01964053	-0.02057617	0.01244109	0.026302	318.3086	0.024045	301.1587
247.455	2N2	2	2	-0.05076973	0.15409810	0.03408330	-0.00708020	0.162246	341.7648	0.034811	101.7353
247.455	2N2	3	0	-0.00345932	-0.05402235	-0.00345932	-0.05402235	0.054133	183.6639	0.054133	183.6639
247.455	2N2	3	1	0.00459468	0.02860553	0.08674509	0.04125120	0.028972	9.1250	0.096054	64.5668
247.455	2N2	3	2	-0.01359111	-0.04803085	0.00043095	0.01917460	0.049917	195.7997	0.019179	1.2875
247.455	2N2	3	3	0.11576000	0.04745531	0.10043379	-0.03897379	0.125109	67.7090	0.107731	111.2090
247.455	2N2	4	0	-0.04607076	0.02579335	-0.04607076	0.02579335	0.052800	299.2429	0.052800	299.2429
247.455	2N2	4	1	0.03322584	0.01467790	0.01394749	0.02945707	0.036324	66.1660	0.032592	25.3369
247.455	2N2	4	2	0.06616682	-0.16308472	0.08023800	0.03608357	0.175996	157.9166	0.087978	65.7862
247.455	2N2	4	3	-0.04323293	-0.08712246	-0.08031745	0.08908738	0.097259	206.3921	0.119948	317.9635
247.455	2N2	4	4	-0.07108370	0.11911427	-0.03283587	0.04029420	0.138712	329.1726	0.051979	320.8233
247.455	2N2	5	0	0.00423674	0.05025371	0.00423674	0.05025371	0.050432	4.8190	0.050432	4.8190
247.455	2N2	5	1	-0.06599377	0.02863740	-0.06611923	-0.08775797	0.071939	293.4580	0.109878	216.9954
247.455	2N2	5	2	0.03191636	0.09160043	-0.12292118	0.09809027	0.097002	19.2099	0.157262	308.5896
247.455	2N2	5	3	-0.04622306	0.08929694	-0.03229352	-0.02331163	0.100551	332.6324	0.039828	234.1757
247.455	2N2	5	4	0.12978448	-0.00340802	-0.08015548	0.01815451	0.129829	91.5042	0.082186	282.7617
247.455	2N2	5	5	0.07170340	0.02947675	0.04405895	-0.08476786	0.077526	67.6528	0.095534	152.5364
247.455	2N2	6	0	0.03947937	-0.02794239	0.03947937	-0.02794239	0.048367	125.2898	0.048367	125.2898
247.455	2N2	6	1	-0.03340601	-0.04901155	0.00654233	-0.02479353	0.059314	214.2781	0.025642	165.2182
247.455	2N2	6	2	0.01502432	0.05093430	-0.00472606	-0.04361353	0.053104	16.4347	0.043869	186.1846
247.455	2N2	6	3	0.00272363	0.04846491	-0.00102382	0.02626808	0.048541	3.2165	0.026288	357.7680
247.455	2N2	6	4	0.05940714	-0.01371178	0.06957119	0.00812134	0.060969	102.9969	0.070044	83.3418
247.455	2N2	6	5	-0.06310363	-0.02281638	0.02184442	0.02667029	0.067102	250.1215	0.034474	39.3193
247.455	2N2	6	6	0.06505389	0.01875362	0.05082476	0.11432385	0.067703	73.9189	0.125112	23.9684

The file header inludes the geocentric gravitational constant GM ($\times 10^{14} m^3/s^2$) and equatorial radius a(m) of the Earth. The record (starting from the third row) includes the tidal constituent's name, Doodson constant, degree-1 order-0 spherical harmonic coefficient C_{10}+ from in-phase amplitude, C_{10}- from out-of-phase amplitude, C_{11}+ from in-phase amplitude, C_{11}- from out-of-phase amplitude, and S_{11}+ from in-phase amplitude, S_{11}- from out-of-phase amplitude.

oceantideA2n2cs.dat | proA2n2.ini | OtideOne.dat

```
 3.986004418    6378137.00
name Doodson             C10+            C10-            C11+            C11-            S11+            S11-
 2N2 247.455  0.14379190E-08  0.72446933E-09  0.45556662E-09  0.50261431E-09  0.98234968E-09  0.28806626E-08
  J1 175.455  0.22805765E-08 -0.14599680E-07  0.11146859E-07  0.31354016E-08  0.49073923E-08  0.50239288E-08
  K1 165.555  0.65903198E-07 -0.23618735E-06  0.15240517E-06  0.54510351E-07  0.57951321E-07  0.91115166E-07
  K2 275.555  0.58820344E-08  0.78223673E-09  0.82634785E-08  0.17098158E-07  0.28274727E-08  0.95641986E-09
  L2 265.455  0.99527541E-09  0.43369491E-10  0.27208849E-08  0.18838893E-08 -0.93316186E-09 -0.31242492E-09
  M2 255.555  0.64086749E-07  0.33741274E-07  0.82092113E-07  0.76976307E-08 -0.39331272E-07  0.74234937E-07
  M3 355.555  0.51159035E-10  0.26216133E-10  0.20622631E-10 -0.16737336E-10 -0.74054752E-10 -0.32502465E-10
  M4 455.555 -0.12877739E-09 -0.82078020E-09  0.21241775E-09  0.89312487E-09 -0.11238411E-09 -0.11882183E-08
  M6 655.555  0.18174228E-08  0.30921490E-09  0.36600543E-09  0.36841599E-09 -0.72147727E-09 -0.13743491E-09
  M8 855.555 -0.59854172E-10 -0.29503418E-11  0.41858427E-10  0.58809710E-10 -0.34465624E-10  0.81925459E-11
  Mf  75.555  0.23994538E-07  0.23160661E-08  0.14961765E-07 -0.19050356E-07  0.57231952E-08 -0.38155669E-08
  Mm  65.455  0.12211587E-07 -0.10619733E-08 -0.13680094E-08 -0.93454574E-08  0.34149364E-08 -0.61740212E-09
  N2 245.655  0.16604395E-07  0.24692742E-08  0.10060051E-07  0.75631673E-09 -0.49125733E-09  0.20845840E-07
  N4 435.755 -0.11170849E-09 -0.41029169E-10  0.37178942E-10 -0.10703469E-09 -0.53442667E-10 -0.19926918E-10
  O1 145.555  0.23239277E-07 -0.16830188E-06  0.86481239E-07  0.11802879E-07  0.58555768E-07  0.34726677E-07
  P1 163.555  0.16600812E-07 -0.74602430E-07  0.48235157E-07  0.14146460E-07  0.16888410E-07  0.27904988E-07
  Q1 135.655  0.40244812E-08 -0.29117940E-07  0.15908436E-07  0.77164577E-09  0.12770867E-07  0.14909422E-08
  R2 274.554  0.21029138E-09  0.96276767E-10  0.25728894E-09  0.46084038E-09  0.53716115E-10  0.14500876E-09
  S1 164.556 -0.40129653E-08  0.48653114E-08 -0.48716881E-08  0.11419251E-07  0.74509139E-08 -0.34899535E-09
  S2 273.555  0.22430236E-07  0.94564697E-08  0.30377828E-07  0.49157638E-07 -0.61338730E-08  0.76805145E-08
  S4 491.555  0.32089047E-09  0.14407638E-09  0.12925319E-11  0.14038268E-09  0.10308541E-09  0.11742749E-09
  Sa  56.554  0.21793187E-09  0.12972260E-09  0.71714382E-10  0.49927099E-10 -0.42733149E-10 -0.53422994E-10
  T2 272.556  0.13719484E-08  0.73425584E-09  0.20944307E-08  0.29614380E-08  0.13767437E-09  0.10318216E-08
 MN4 445.655 -0.70793273E-09 -0.76823301E-10  0.24279253E-09 -0.66374018E-09 -0.14062685E-09  0.16716883E-09
 MS4 473.555  0.32582237E-09 -0.10684852E-08  0.10873236E-08  0.38092589E-09 -0.40703836E-09 -0.28009461E-09
 Msf  73.555  0.52032006E-09  0.12958178E-08  0.20898774E-09  0.69234415E-09  0.16108594E-08  0.36734674E-09
 Mtm  85.455  0.38057222E-08  0.89028662E-09  0.47545363E-08 -0.16109463E-08  0.13034435E-08  0.46197838E-10
 mu2 237.555  0.27230195E-08 -0.54548861E-09  0.80856645E-09  0.28475772E-08  0.30945151E-08  0.39961507E-08
 nu2 245.655  0.31512988E-08  0.13274377E-08  0.16643629E-08  0.77176190E-09 -0.34369557E-09  0.49489633E-08
 Ssa  57.555  0.85592993E-08 -0.21041028E-09 -0.85777470E-08 -0.10849053E-08  0.38854237E-09 -0.73333943E-09
eps2 227.655  0.15232320E-08 -0.54284574E-09  0.18709319E-08 -0.17678032E-09  0.14037532E-08 -0.64291979E-09
lam2 263.655  0.77975910E-09 -0.46145888E-09  0.29230225E-08 -0.81098933E-09 -0.68691816E-09 -0.10714953E-08
MKS2 257.555 -0.76338045E-11 -0.81694611E-10  0.81955321E-10  0.53313693E-09  0.52931064E-09  0.23733568E-09
Msqm  93.555  0.17382639E-09 -0.21085098E-11  0.98864729E-10  0.18391545E-09 -0.15315104E-09 -0.66456652E-11
```

4.3 Computation of the load model value using spherical harmonic synthesis

[Purpose] From the tidal load spherical harmonic coefficient model or the surface non-tidal load spherical harmonic coefficient model, compute the model values of the tidal harmonic constants or the non-tidal surface loads using spherical harmonic synthesis.

In the remove-restore process, the program can be employed for regional tidal load effect refinement based on the tidal load spherical harmonic coefficient model, and for regional load deformation field refinement based on surface load spherical harmonic model.

4.3.1 Computation of model value of surface load equivalent water height

[Function] From the surface atmosphere, land water, or sea level variation load normalized spherical harmonic coefficient model (m), compute the model value of the surface atmosphere (hPa/mbar), land equivalent water height (cm) or sea level variation (cm) at the given location.

[Input files] The surface calculation point file. The surface load spherical harmonic coefficient model file.

The calculation point file can be a discrete calculation point file or a calculation surface height grid file.

The discrete calculation point file record format: Point number (name), longitude, latitude (decimal degrees), height….

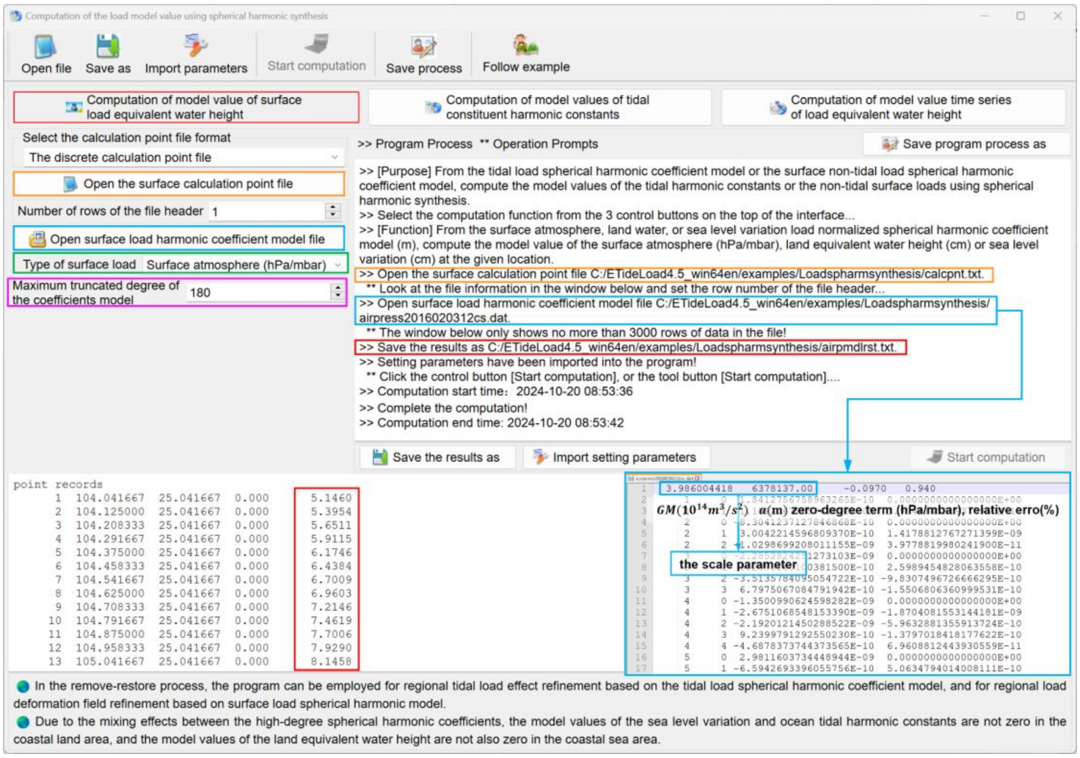
[Parameter settings] Select the format of the calculation point file, enter maximum truncated degree of the load spherical harmonic coefficient model and select the type of surface loads.

The program automatically selects the minimum value between the maximum degree of tidal load spherical harmonic coefficient model and entered maximum degree as the calculation degree.

[Output file] The surface load model value file.

When the discrete calculation point file input, the output file header is the same as the input file. Behind the input file record, adds a column of the model value of surface load as the output file record.

When the calculation surface height grid file input, the program outputs the surface load model value grid files with the same grid specifications.



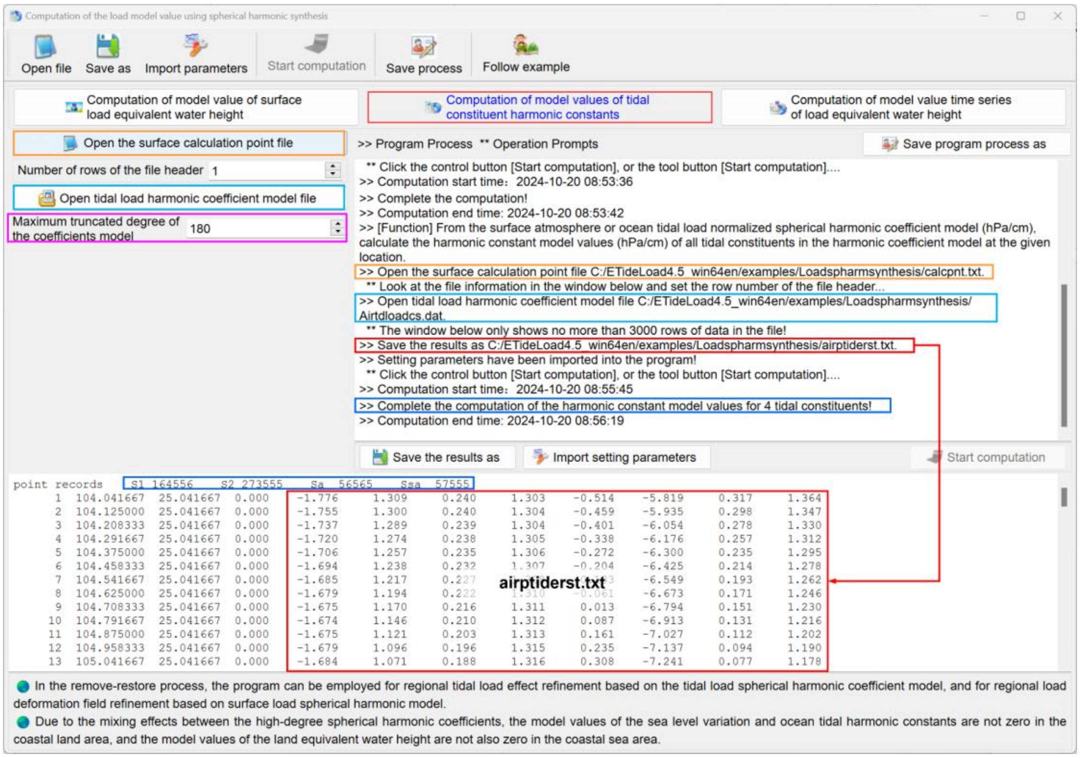
4.3.2 Computation of model values of tidal constituent harmonic constants

[Function] From the surface atmosphere or ocean tidal load normalized spherical harmonic coefficient model (hPa/cm), calculate the harmonic constant model values (hPa/cm) of all tidal constituents in the harmonic coefficient model at the given location.

[Input files] The surface calculation point file. The tidal load spherical harmonic coefficient model file.

[Output file] The tidal harmonic constant model value file.

The output file header comes from the input calculation point file. Behind the input file record, adds $2n$ columns of the tidal harmonic constant model values as the output file record. Here, n is the number of tidal constituents in the harmonic coefficient model.



4.3.3 Computation of model value time series of load equivalent water height

[Function] From the surface atmosphere, land water, or sea level variation load normalized spherical harmonic coefficient model (m) time series, compute the model value record time series of the atmosphere (hPa/mbar), land equivalent water height (cm), or sea level variation (cm) on the given points in the input file.

[Input files] The surface calculation point file. The surface load spherical harmonic coefficient model time series files.

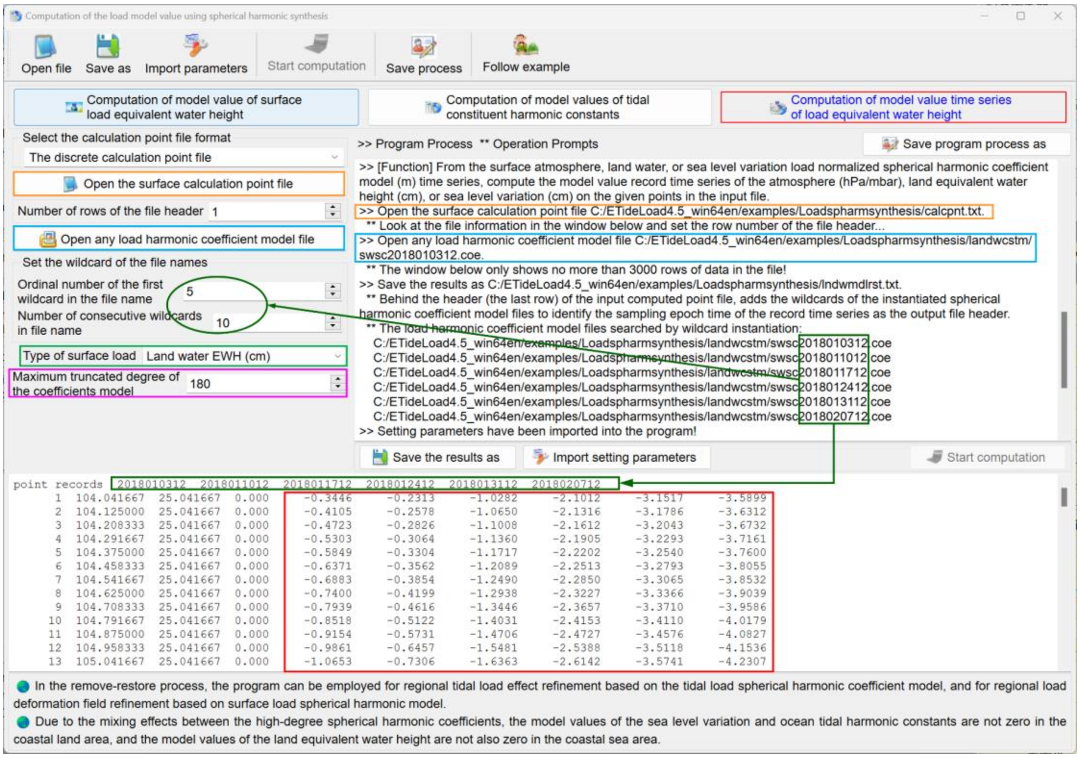
[Parameter settings] Set the wildcard parameters for the surface load spherical harmonic coefficient model time series files, and enter the number of rows of the input file header, row ordinal number of target attribute and grid resolution.

[Output file] The surface load model value record time series file.

Behind the input file header, adds n sampling epoch times of the surface load spherical harmonic coefficient model time series as the output file header. Behind the input file record, adds n load model values as the output record. Here, n is the sampling number.

The computation process needs to wait…. During the computation period, you can open the output files to look at the computation progress!

Due to the mixing effects between the high-degree spherical harmonic coefficients, the model values of the sea level variation and ocean tidal harmonic constants are not zero in the coastal land area, and the model values of the land equivalent water height are not also zero in the coastal sea area.



4.4 Computation of load deformation field using spherical harmonic synthesis

[Purpose] From the surface atmosphere, land water and sea level variation load spherical harmonic coefficient model (m), compute the non-tidal load effects on various geodetic variations on the ground or outside the solid Earth by the spherical harmonic synthesis algorithm.

The epoch time of the load effects is equal to the sampling epoch time of the load spherical harmonic coefficient model.

When computing the load effects of sea level variations, the height of the calculation point is the normal or orthometric height. When computing the load effects of surface atmosphere or land water variations, the height of the calculation point is the height relative to the Earth’s surface.

4.4.1 Computation of various load effects using spherical harmonic synthesis

[Function] From the surface atmosphere, land water or sea level variation load spherical harmonic coefficient model (m), compute the non-tidal load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (10μE) or horizontal gravity gradient (NW, to the north and to the west, 10μE) using spherical harmonic synthesis.

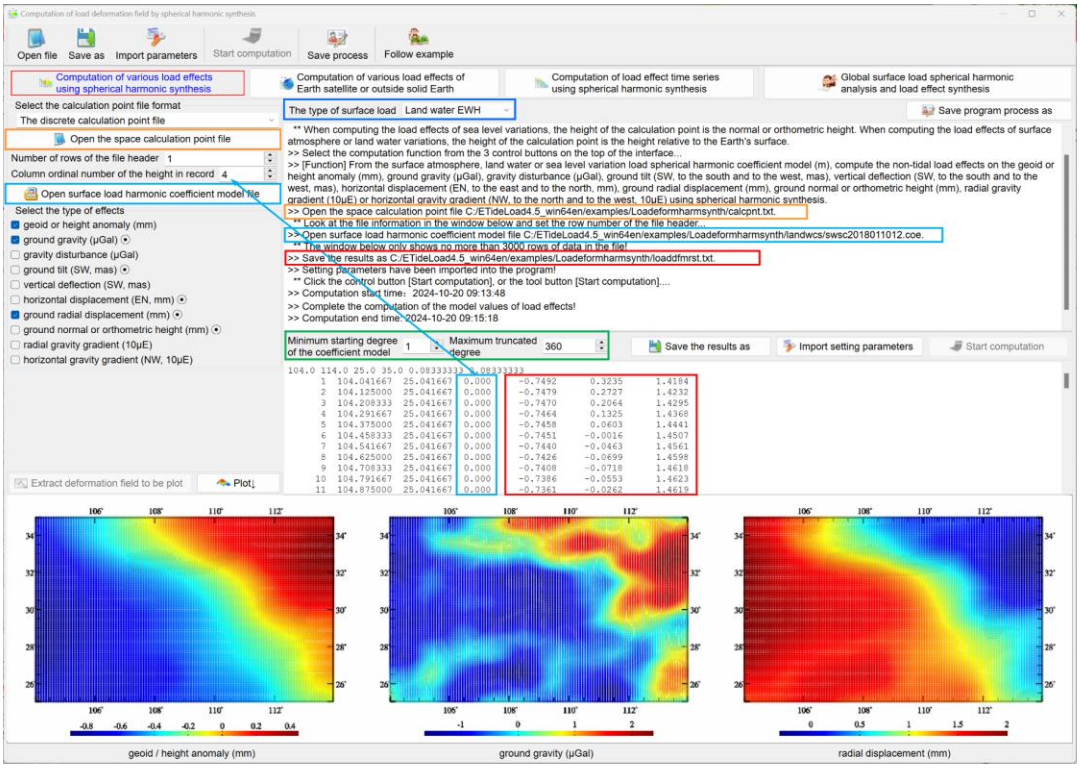
[Input files] The calculation point file. The surface load spherical harmonic coefficient model file.

The calculation point file can be a discrete calculation point file or a calculation surface height grid file.

The discrete calculation point file record format: Point number (name), longitude, latitude (decimal degrees), height….

[Parameter settings] Select the format of the calculation point file, enter column ordinal number of the height in the input file record and maximum truncated degree of the spherical harmonic coefficient model and select the type of surface loads.

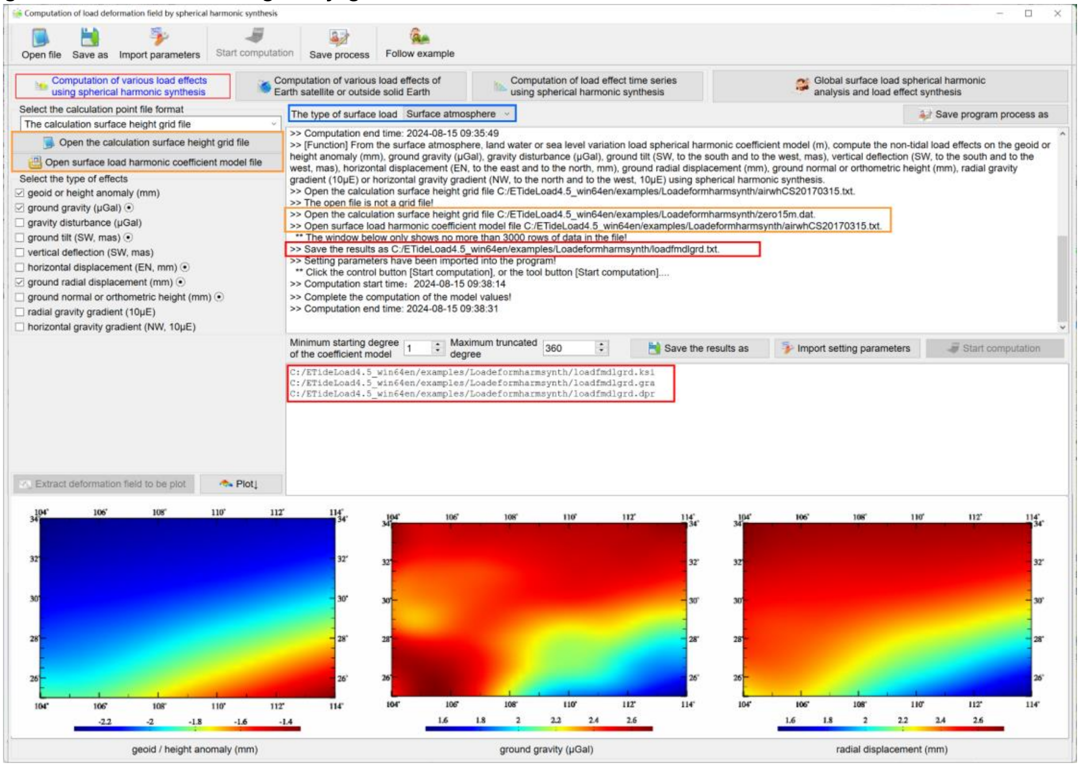
The program automatically selects the minimum value between the maximum degree of load spherical harmonic coefficient model and entered maximum degree as the calculation degree.



[Output file] The surface load effect file.

When the discrete calculation point file input, the output file header is the same as the input file. Behind the input file record, adds one or several columns of the surface load effects selected as the output file record. In this example, 3 attributes of load effects on height anomaly, ground gravity and ground radial displacement are added to the record.

When the calculation surface height grid file input, the program outputs the grid files *.??? of the surface load effects selected, where * is the input file name, and ??? = ksi, gra, rga, dft, vdf, dph, dpr, nmh, grr or hgd, respectively, representing the grid file of load effects on the height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, normal or orthometric height, radial gravity gradient or horizontal gravity gradient.



4.4.2 Computation of various load effects of Earth satellite or outside solid Earth

[Function] From the surface atmosphere, land water or sea level variation load spherical harmonic coefficient model (m), compute the non-tidal load effects on the geopotential (0.1m²/s²), gravity (μGal), or gravity gradient (10μE) outside the solid Earth using spherical harmonic synthesis.

Here the space point outside the solid Earth generally refers to the point that is not fixed with the solid Earth in ocean space, near-Earth space, or satellite altitude.

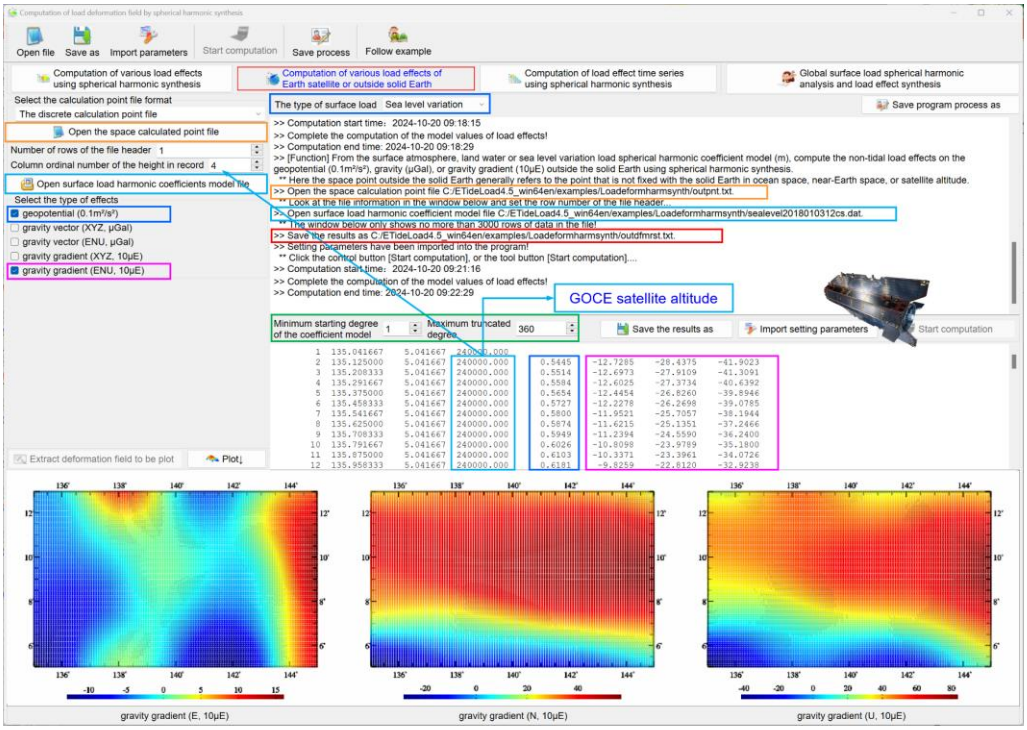
[Input files] The discrete calculation point file. The surface load spherical harmonic coefficient model file.

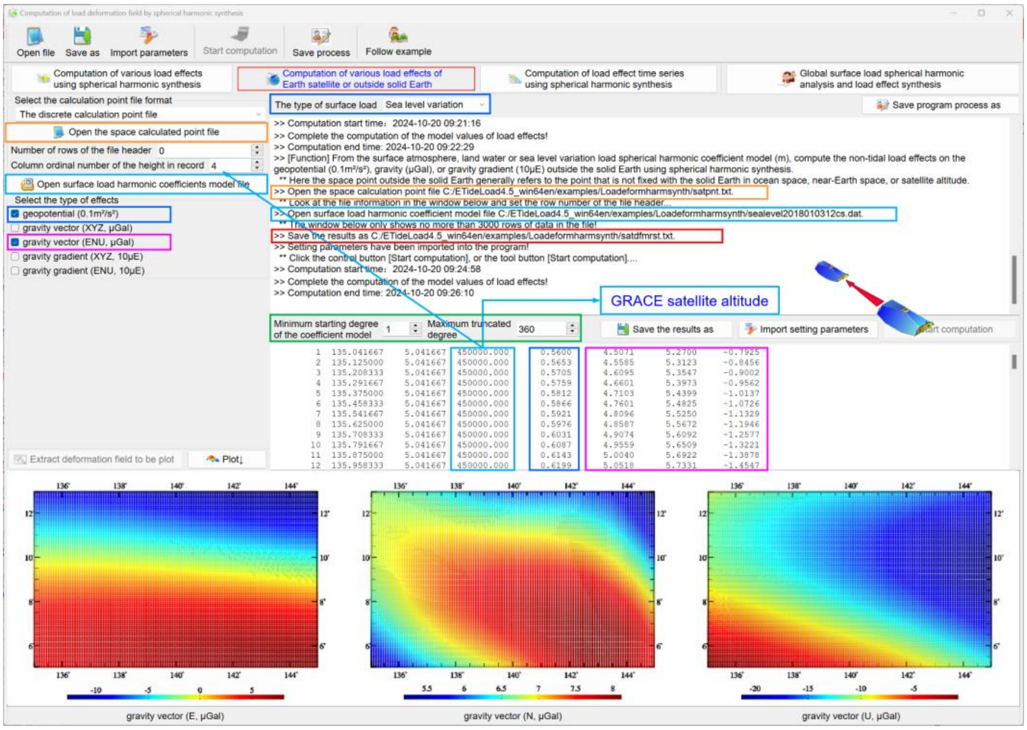
The calculation point file can be a discrete calculation point file or a calculation surface ellipsoidal height grid file.

The discrete calculation point file record format: Point number (name), longitude, latitude (decimal degrees), ellipsoidal height….

[Output file] The surface load effect file.

When the discrete calculation point file input, the output file header is the same as the input file. Behind the input file record, adds one or several columns of the surface load effects selected as the output file record. In this example, all types are selected, and there are 14 attributes added to the record.





When the calculation surface ellipsoidal height grid file input, the program outputs the grid files *.??? of the surface load effects selected, where * is the input file name, and ? ? ? = gpv, pvx, pvy, pvz, pve, pvn, pvr, vxx, vyy, vzz, or vee, vnn, vrr, respectively, representing the grid file of load effects on the geopotential, x, y, z components of gravity, E, N, U components of gravity, x, y, z components of gravity gradient or E, N, U components of gravity gradient.

4.4.3 Computation of load effect time series using spherical harmonic synthesis

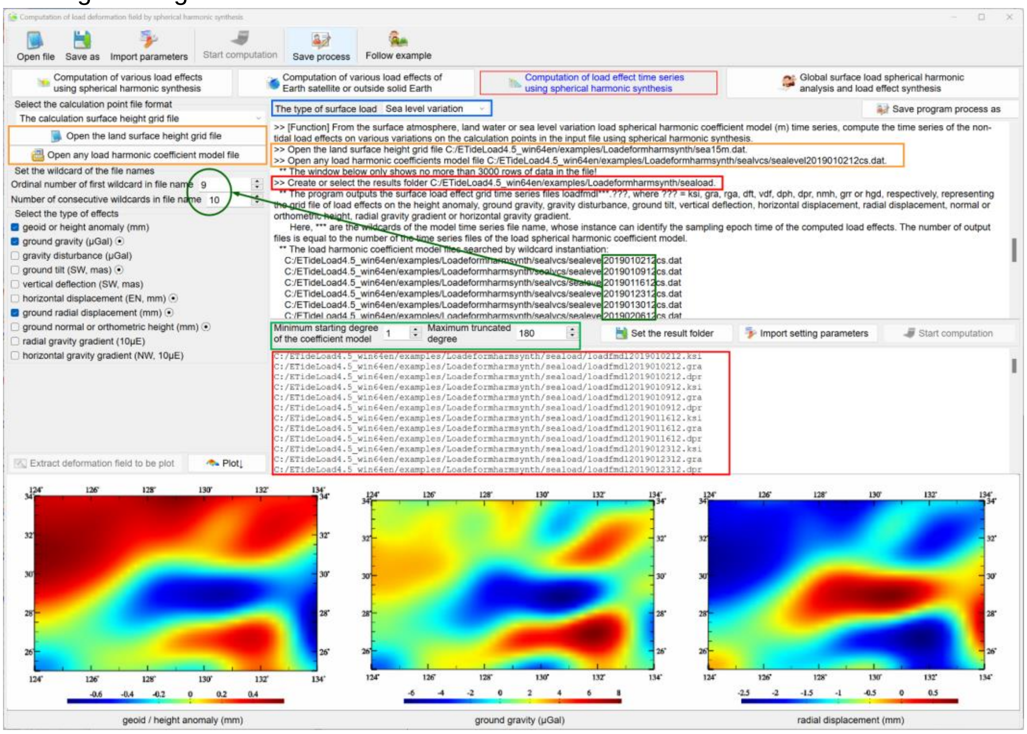
[Function] From the surface atmosphere, land water or sea level variation load spherical harmonic coefficient model (m) time series, compute the time series of the non-tidal load effects on various variations on the calculation points in the input file using spherical harmonic synthesis.

[Input files] The surface calculation point file. The surface load spherical harmonic coefficient model time series file.

The calculation point file can be a discrete calculation point file or a calculation surface height grid file.

The discrete calculation point file record format: Point number (name), longitude, latitude (decimal degrees), height….

The time series files of the load spherical harmonic coefficient model are extracted according to the given wildcards.



[Output file] The surface load effect time series files.

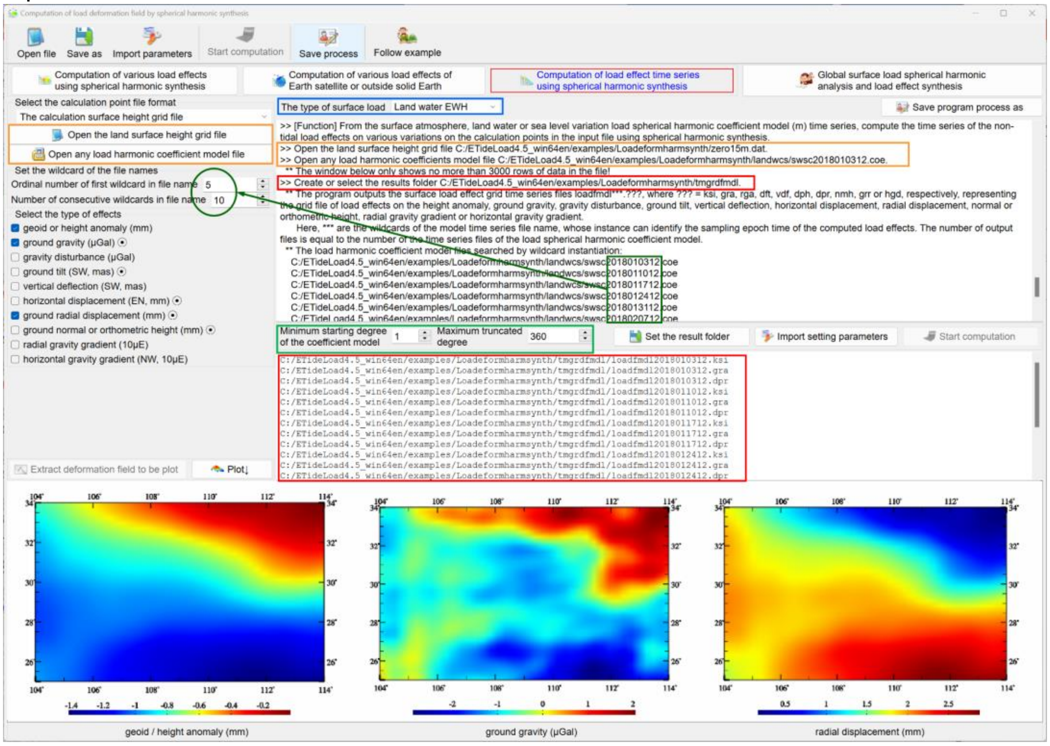
When the discrete calculation point file input, the program outputs the surface load effect record time series files loadfmdl***.txt. Each output file header is the same as the input file. Behind the input file record, adds one or several columns of the surface load effects selected as the output file record.

When the calculation surface height grid file input, the program outputs the surface load effect grid time series files loadfmdl***.???, where ??? = ksi, gra, rga, dft, vdf, dph, dpr, nmh, grr or hgd, respectively, representing the grid file of load effects on the height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, normal or orthometric height, radial gravity gradient or horizontal gravity gradient.

Here, *** are the wildcards of the load spherical harmonic coefficient model time series file name, whose instance can identify the sampling epoch time of the computed load effects. The number of output files is equal to the number of the time series files of the load spherical harmonic coefficient model.

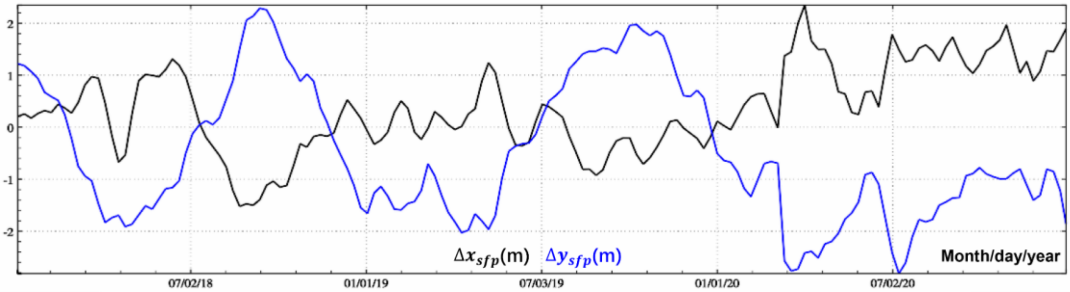
The computation process needs to wait…. During the computation period, you can open the output files directory to look at the computation progress!

The last column attribute of each output file header is the instance of the wildcards of the file name of the model time series, which represents the sampling epoch time of the output file.

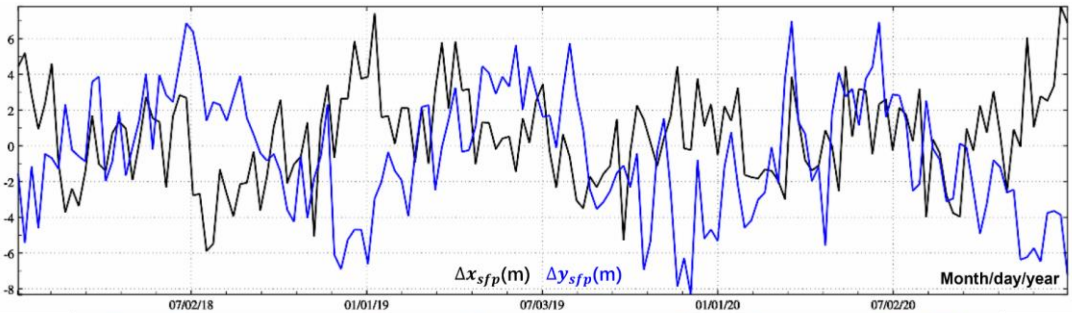


The program also outputs the time series file Loadfmcmsfptm.txt of surface load effects on the first and second degree geopotential coefficients, Earth's center of mass and Earth's figure polar shifts into the current directory, which is an internal test file. The first row is the file header, and starting from the second row of the file, each row record stores the sampling epoch time, 8 columns of surface load effects on the first and second degree normalized geopotential coefficients $\Delta\bar{C}_{10},\Delta\bar{C}_{11},\Delta\bar{S}_{11},\ \Delta\bar{C}_{20},\Delta\bar{C}_{21},\Delta\bar{S}_{21},\Delta\bar{C}_{22},\Delta\bar{S}_{22}$ (×10⁻¹⁰), 3 columns of surface load effects on Earth's mass centric variations $\Delta x_{cm},\Delta y_{cm}\Delta z_{cm}$ (in unit of mm) and 2 columns of surface load effects on Earth's figure polar shifts $\Delta x_{sfp},\Delta y_{sfp}$(in ITRS space rectangular coordinate system, in unit of m).

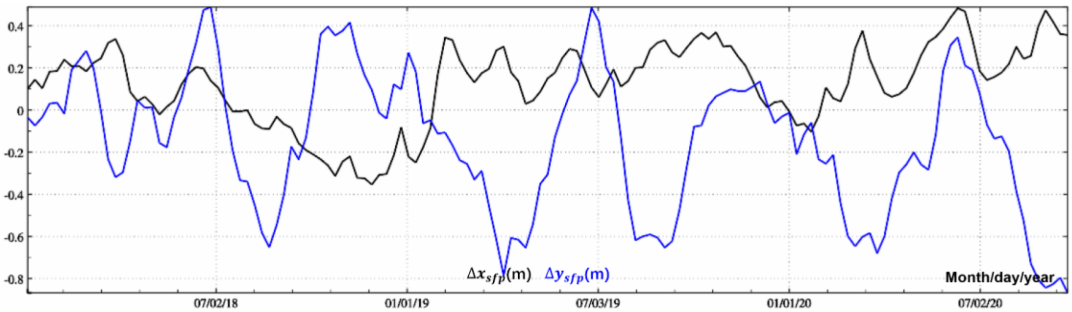
time	C10(e-10)	C11(e-10)	S11(e-10)	C20(e-10)	C21(e-10)	S21(e-10)	C22(e-10)	S22(e-10)	Xcm(mm)	Ycm(mm)	Zcm(mm)	Xf(cm)	Yf(cm)	Xp(cm)	Yp(cm)	Δlod(ms)
2018010312	-1.35502	-0.15080	1.40660	0.18793	0.09632	-0.53732	0.72810	-1.14329	-0.167	1.554	-1.497	20.681	122.410	22.532	125.687	-10.446
2018011012	-1.73900	0.01149	1.45784	0.33361	0.11743	-0.51848	0.79878	-1.10003	0.013	1.611	-1.921	25.523	118.173	27.468	121.279	-18.544
2018011712	-2.24917	0.32831	1.69397	0.49482	0.08132	-0.46954	0.72932	-1.25037	0.363	1.871	-2.485	17.423	106.962	19.022	109.832	-27.505
2018012412	-2.75292	0.77899	1.71444	0.55997	0.11999	-0.41148	0.61036	-1.29563	0.861	1.894	-3.041	26.349	93.847	28.067	96.251	-31.127
2018013112	-3.07753	0.93999	1.72085	0.54474	0.14335	-0.29383	0.50370	-1.26628	1.038	1.901	-3.400	31.930	67.144	33.532	68.730	-30.280
2018020712	-3.34000	1.01838	1.87950	0.63708	0.12638	-0.25672	0.38756	-1.10404	1.125	2.076	-3.690	28.154	58.667	29.561	60.049	-35.413
2018021412	-3.67390	1.09916	1.96699	0.62307	0.19580	-0.22456	0.36054	-0.78222	1.214	2.173	-4.059	44.014	51.512	45.800	52.527	-34.634
2018022112	-4.13098	0.97044	2.02541	0.55026	0.16328	-0.10434	0.46285	-0.47302	1.072	2.238	-4.564	36.893	24.099	38.193	24.406	-30.587
2018022812	-4.48422	0.74061	2.07043	0.52233	0.11810	0.09607	0.45245	-0.26356	0.818	2.287	-4.954	27.076	-21.579	27.626	-22.473	-29.034
2018030712	-4.78225	0.68476	2.09293	0.45141	0.20559	0.33303	0.35210	-0.03511	0.756	2.312	-5.283	47.511	-75.265	48.091	-77.901	-25.092
2018031412	-4.86443	0.62097	1.98762	0.48307	0.35331	0.41659	0.38021	0.04972	0.686	2.196	-5.374	81.295	-93.931	82.645	-97.447	-26.852
2018032112	-4.79983	0.61685	1.79772	0.46473	0.42244	0.45869	0.37763	-0.01681	0.681	1.986	-5.302	97.110	-103.347	98.814	-107.295	-25.832
2018032812	-4.52634	0.63658	1.67395	0.18362	0.40725	0.65186	0.23902	-0.03188	0.703	1.849	-5.000	94.097	-147.310	95.262	-152.480	-10.207
2018040412	-4.11887	0.63757	1.49977	-0.15645	0.19595	0.80864	0.12702	0.05165	0.704	1.657	-4.550	46.402	-183.444	45.835	-189.153	8.696
2018041112	-3.66906	0.70363	1.35652	-0.19219	-0.07753	0.76570	0.02988	0.17353	0.777	1.499	-4.053	-15.886	-174.302	-18.135	-179.108	10.683
2018041812	-3.40962	0.87752	1.05163	-0.01977	-0.30344	0.74244	-0.09346	0.35295	0.969	1.162	-3.767	-67.312	-169.526	-70.978	-173.666	1.099
2018042512	-3.24335	0.80889	0.96548	-0.11026	-0.24399	0.83963	-0.24252	0.41156	0.894	1.067	-3.583	-53.572	-191.493	-57.073	-196.401	6.129
2018050212	-2.83282	0.29777	1.08377	-0.21289	0.10242	0.82137	-0.42531	0.34882	0.329	1.197	-3.129	25.163	-186.552	23.958	-192.130	11.834
2018050912	-2.25609	-0.08185	1.10360	-0.30266	0.38522	0.74608	-0.60966	0.32724	-0.090	1.219	-2.492	89.302	-168.786	90.109	-174.519	16.824
2018051612	-1.77244	-0.13323	1.04253	-0.28504	0.43937	0.67011	-0.71584	0.38615	-0.147	1.152	-1.958	101.442	-151.386	102.775	-156.747	15.844
2018052312	-1.64067	-0.11371	1.00990	-0.36618	0.43106	0.69821	-0.83624	0.36697	-0.126	1.116	-1.812	99.616	-157.795	100.830	-163.321	20.355
2018053012	-1.21237	-0.27831	0.78354	-0.33464	0.41988	0.61118	-0.93746	0.26489	-0.307	0.866	-1.339	96.876	-138.030	98.216	-142.964	18.601
2018060612	-0.65937	-0.46525	0.47939	-0.47925	0.48181	0.52564	-0.99745	0.23370	-0.514	0.530	-0.728	110.765	-118.436	112.703	-122.954	26.640
2018061312	-0.40814	-0.48375	0.28531	-0.65272	0.57081	0.51774	-1.04357	0.19790	-0.534	0.315	-0.451	130.984	-116.438	133.519	-121.107	36.282
2018062012	-0.22753	-0.18738	0.14336	-0.39863	0.51903	0.45911	-1.11994	0.13530	-0.207	0.158	-0.251	119.077	-103.223	121.409	-107.393	22.159
2018062712	0.06792	-0.34572	0.14295	-0.31195	0.42360	0.22253	-1.05697	0.18079	-0.382	0.158	0.075	96.837	-49.639	99.086	-52.052	17.340
2018070412	0.50807	-0.66815	0.30121	-0.42541	0.23507	0.05793	-0.96644	0.17433	-0.738	0.333	0.561	53.588	-12.638	54.986	-13.550	23.647
2018071112	1.02563	-0.74940	0.22609	-0.36150	0.03626	-0.01764	-0.90758	0.16963	-0.828	0.250	1.133	8.204	4.094	8.481	4.126	20.095
2018071812	1.50456	-0.90757	0.11192	-0.22140	-0.08588	-0.05563	-0.93606	0.16397	-1.003	0.124	1.662	-19.657	12.455	-20.089	13.012	12.307



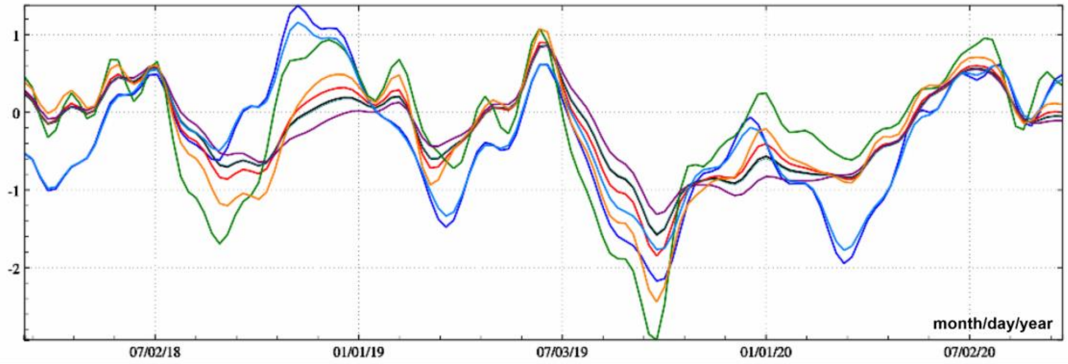
The global sea level variation load effect time series (m) on Earth's figure polar shift in ITRS



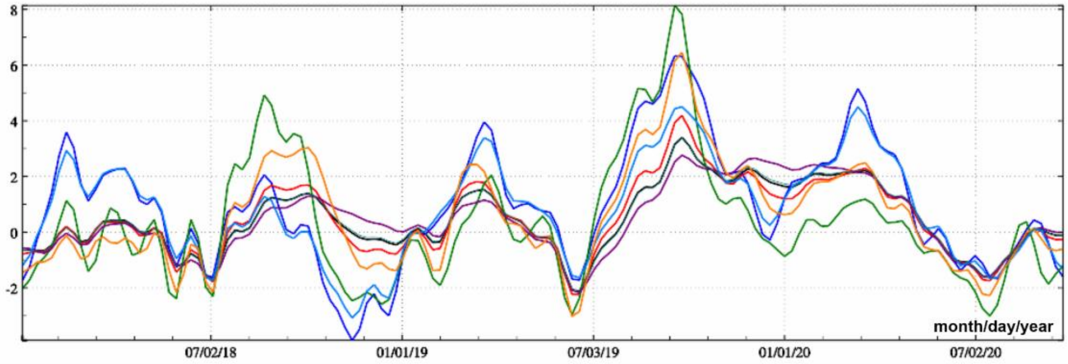
The surface atmosphere variation load effect time series (m) on Earth's figure polar shift in ITRS



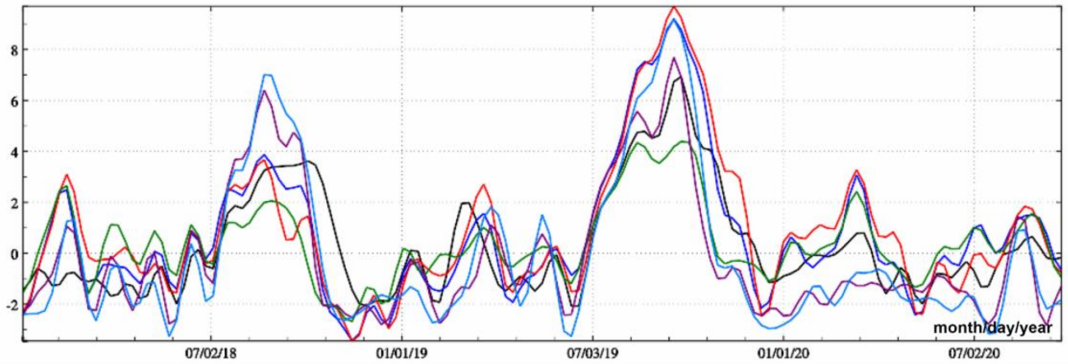
The global land water variation load effect time series (m) on Earth's figure polar shift in ITRS



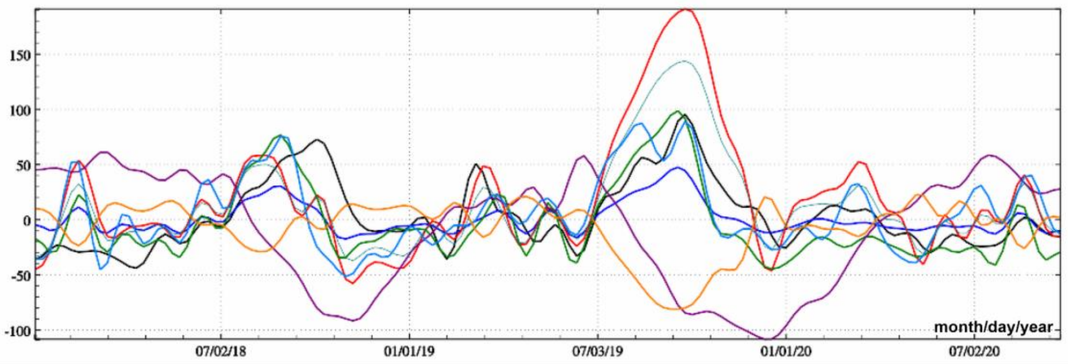
Land water load effect time series in Chinese mainland (360-degree model): geoidal variation (mm)



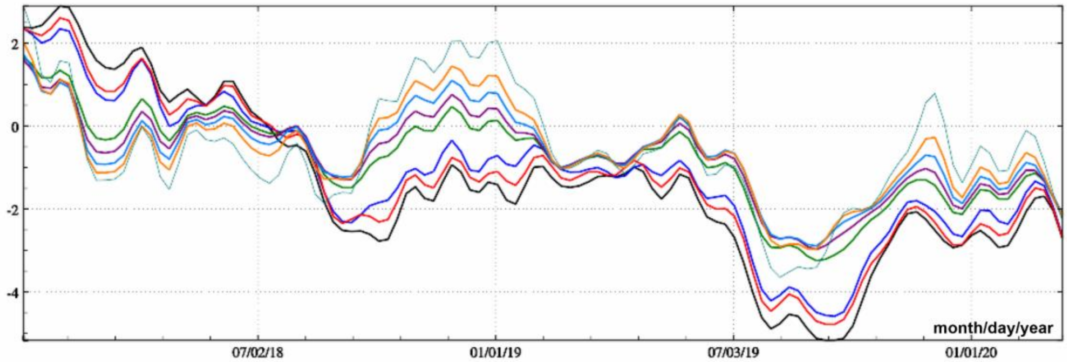
Land water load effect time series in Chinese mainland (360-degree model): ground normal height variation (mm)



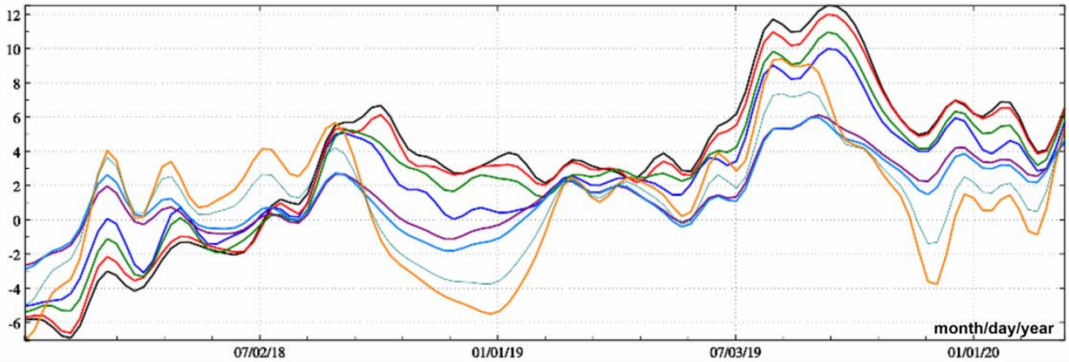
Land water load effect time series in Chinese mainland (360-degree model): gravity disturbance variation (μGal)



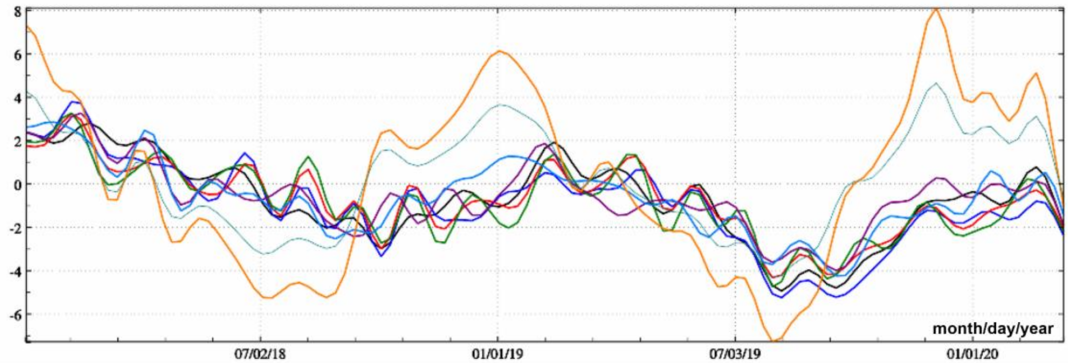
Land water load effect time series in Chinese mainland (360-degree model): radial gradient variation (10μE)



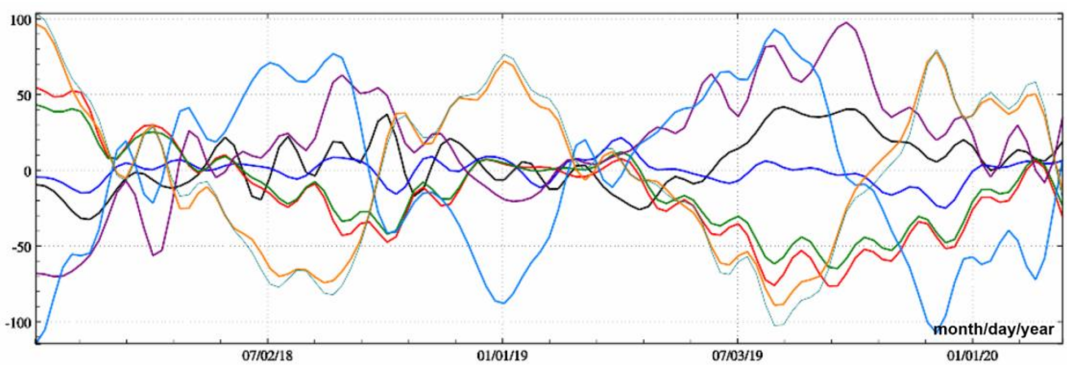
Sea level load effect time series in Chinese coastal zone (360-degree model): geoidal variation (mm)



Sea level load effect time series in Chinese coastal zone (360-degree model): normal height variation (mm)



Sea level load effect time series in Chinese coastal zone (360-degree model): gravity disturbance variation (μGal)



Sea level load effect time series in Chinese coastal zone (360-degree model): radial gradient variation (10μE)

Using the monitoring data to global surface atmosphere, land water and sea level variations, determine the non-tidal temporal Earth's gravity field, as well as the non-tidal load effects on the geopotential coefficients, and then you can calibrate various parameters of the gravity satellite's key measurement equipments, and then effectively improve and check the quality, reliability, and accuracy of the time-varying monitoring of satellite gravity field.

4.5 Regional refinement of load deformation field by Green's Integral

[Purpose] From the regional residual surface load equivalent water height grid which have been removed the reference model value with global load spherical harmonic coefficient model, calculate the residual load deformation field grid by load Green's function integral to refine the regional load deformation field and temporal gravity field.

When computing the load effects of sea level variations, the height of the calculation point is the normal or orthometric height. When computing the load effects of surface atmosphere or land water variations, the height of the calculation point is the height relative to the Earth’s surface.

4.5.1 Computation of regional residual surface load effects by Green's Integral

[Function] From the regional residual equivalent water height (EWH) variation grid (cm), compute the residual surface load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (mE) or horizontal gravity gradient (NW, to the north and to the west, mE) by load Green's function integral.

[Input files] The space calculation point file. The regional residual equivalent water height variation grid file.

The calculation point file can be a discrete calculation point file or a calculation surface height grid file.

The discrete calculation point file record format: Point number (name), longitude, latitude (decimal degrees), height….

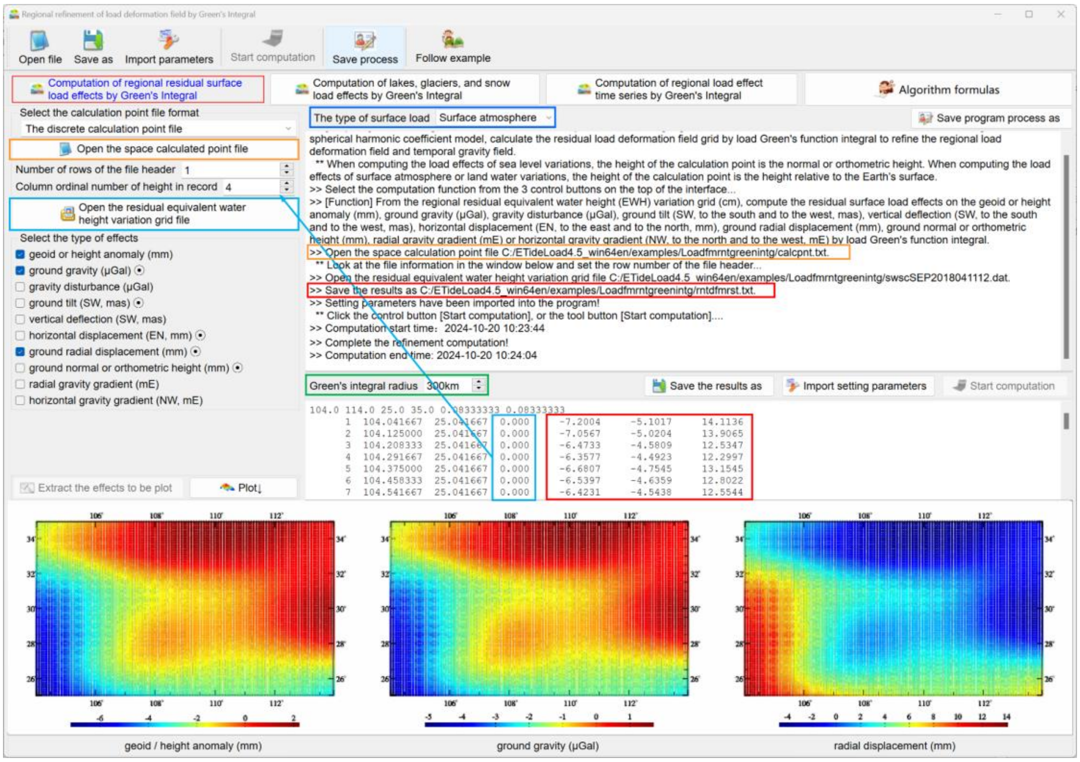
[Parameter settings] Select the format of the calculation point file, enter the load Green’s function integral radius and select the type of surface loads.

[Output file] The regional surface load effect file.

When the discrete calculation point file input, the output file header is the same as the input file. Behind the input file record, adds one or several columns of the surface load effects selected as the output file record. In this example, 4 attributes of load effects on height anomaly, ground gravity, ground radial displacement and radial gravity gradient are added to the record.

When the calculation surface height grid file input, the program outputs the grid files *.???

of the surface load effects selected, where * is the input file name, and ??? = ksi, gra, rga, dft, vdf, dph, dpr, nmh, grr or hgd, respectively, representing the grid file of load effects on the height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, normal or orthometric height, radial gravity gradient or horizontal gravity gradient.



4.5.2 Computation of lakes, glaciers and snow load effects by Green's Integral

[Function] From the load equivalent water height variation grid (cm) of the inland water-bodies such as the rivers, lakes, reservoirs, glaciers or snow-capped mountains, compute the water-body load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (mE) or horizontal gravity gradient (NW, to the north and to the west, mE) by load Green's function integral.

[Input files] The space calculation point file. The water-body equivalent water height variation grid file.

The calculation point file can be a discrete calculation point file or a calculation surface height grid file.

The height in the calculation point file refers to the height of the calculation point relative to the water surface.

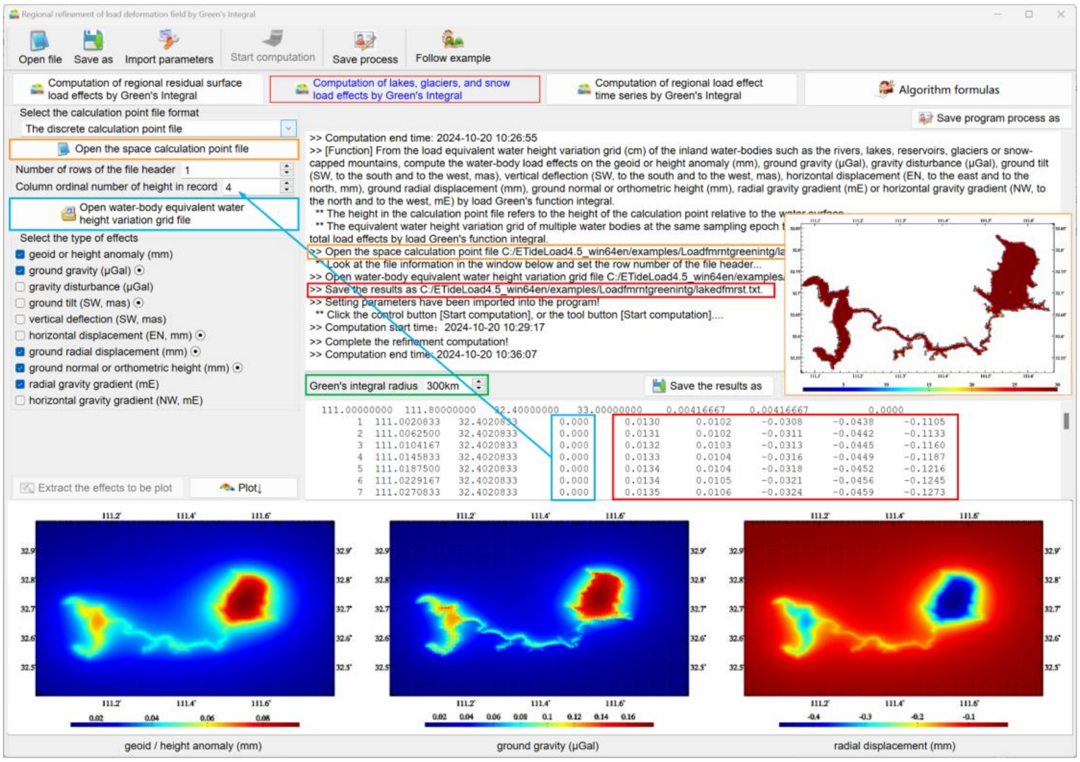
[Parameter settings] Select the format of the calculation point file and enter the load Green’s integral radius.

[Output file] The same with chapter 4.5.1.

The equivalent water height variation grid of multiple water bodies at the same sampling epoch time can be merged directly, and then you can get the total load effects by load Green's function integral.

If the changes of the inland water bodies such as the rivers, lakes, reservoirs, glaciers, and snow-capped mountains are represented by the load equivalent water height variations grid, the program can accurately compute these load effects on various geodetic variations.

Due to shortwave dominance of the residual load effects, the residual load equivalent water height grid is required to have an appropriate spatial resolution to reflect the loads shortwave characteristics. Otherwise, Green's function integral may be unstable.



4.5.3 Computation of regional load effect time series by Green's Integral

[Function] From the regional residual equivalent water height (cm) grid time series, compute the time series of the residual value of the load effects on various geodetic variations at the calculation points in the input file by load Green's function integral. The residual equivalent water height variation (cm) grid time series files are extracted according to the given wildcards.

When calculating of the lakes, glaciers, or snow load effects, please select “Land water EWH” as the type of surface load.

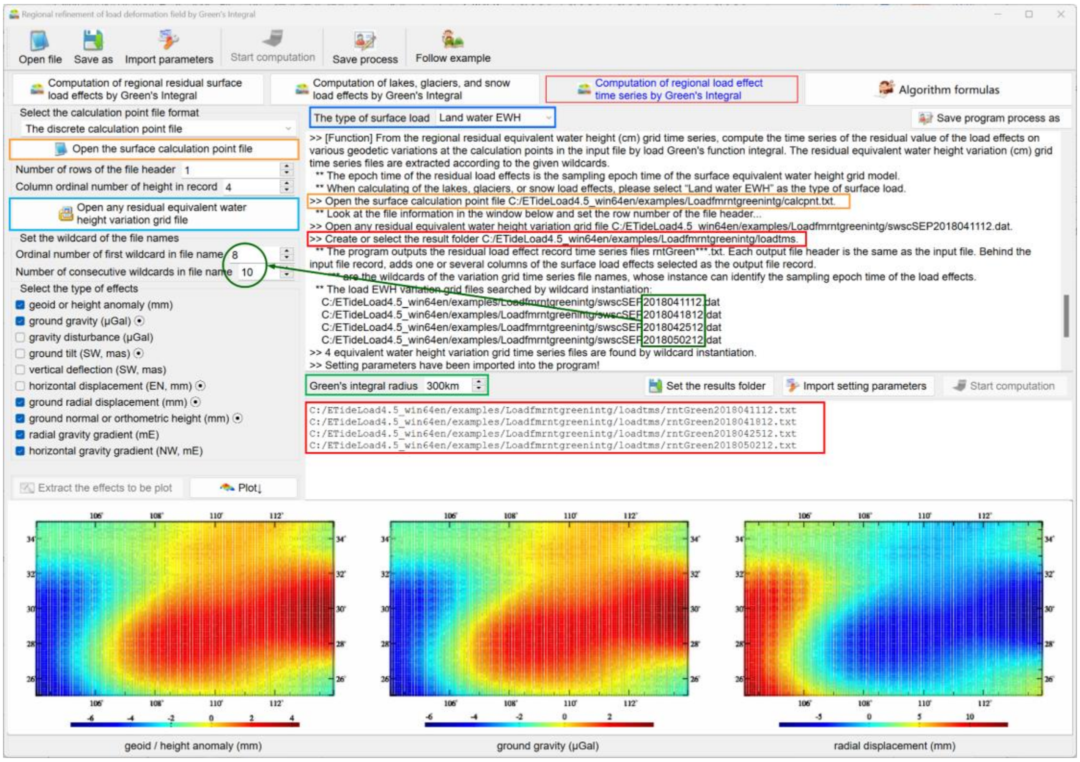
[Input files] The surface calculation point file. The regional residual equivalent water height grid time series file.

The calculation point file can be a discrete calculation point file or a calculation surface height grid file.

The discrete calculation point file record format: Point number (name), longitude, latitude (decimal degrees), height….

The time series files of the equivalent water height grids are extracted according to the given wildcards.

[Parameter settings] Select the format of the calculation point file and set the wildcard parameters for the surface load equivalent water height grid time series files, enter the load Green's integral radius, and select the type of surface loads.



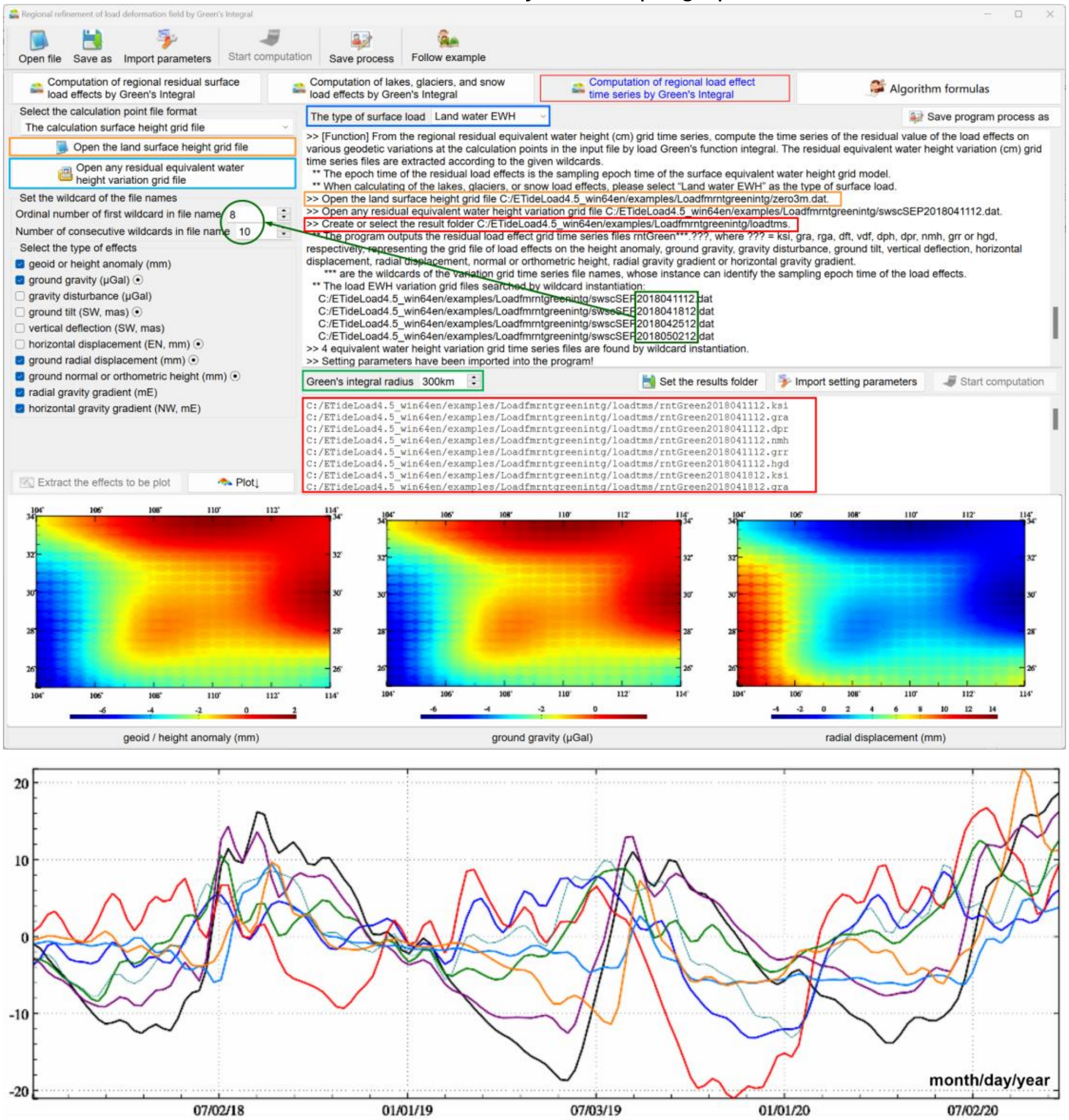
[Output file] The residual surface load effect files.

When the discrete calculation point file input, the program outputs the residual load effect record time series files rntGreen***.txt. Each output file header is the same as the input file. Behind the input file record, adds one or several columns of the surface load effects selected as the output file record.

When the calculation surface height grid file input, the program outputs the residual load effect grid time series files rntGreen***.???, where ??? = ksi, gra, rga, dft, vdf, dph, dpr, nmh, grr or hgd, respectively, representing the grid file of load effects on the height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, normal or orthometric height, radial gravity gradient or horizontal gravity gradient.

The epoch time of the residual load effects is the sampling epoch time of the surface equivalent water height grid model.

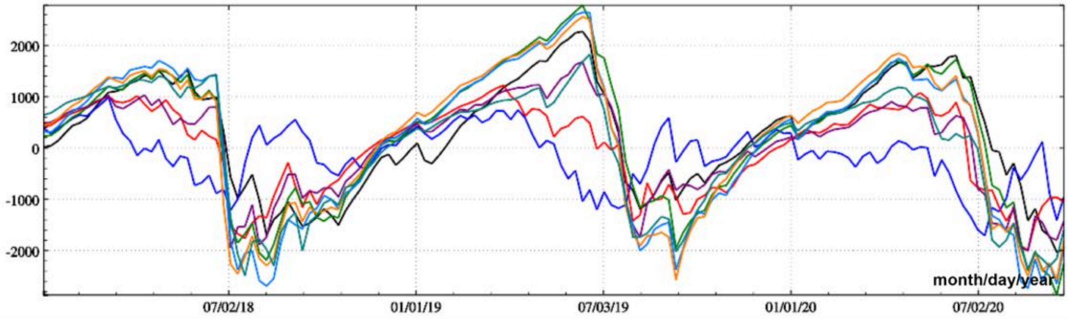
The number of the output files is equal to the number of time series files of the residual equivalent water height variation grid. Here, *** are the wildcards of the variation grid time series file names, whose instance can identify the sampling epoch time of the load effects.



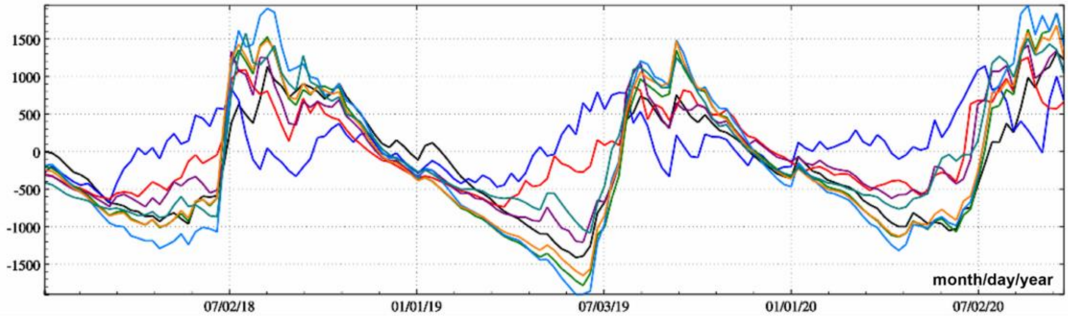
30′×30′ residual soil water load effects in Chinese mainland (Green integral): ground gravity variation (μGal)

The computation process needs to wait…. During the period, you can open the output file directory to look at the computation progress!

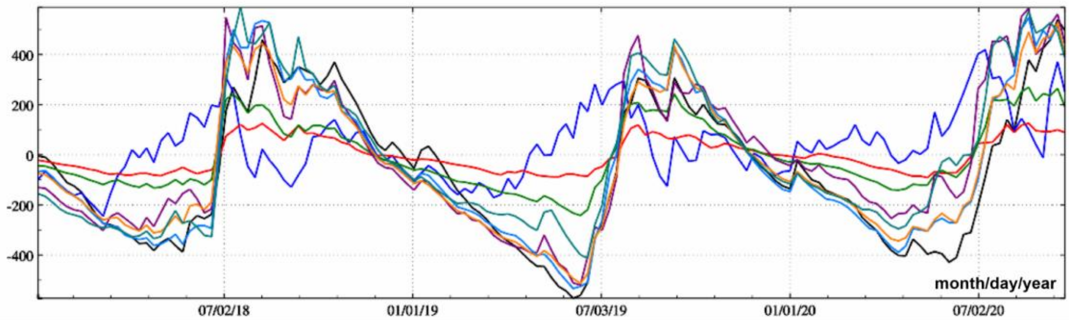
The last attribute of each output file header is the instance of the wildcards of the time series file name of the residual equivalent water height grid model, which represents the sampling epoch time of the output file.



30′×30′ residual soil water load effects in Chinese mainland (Green integral): gravity gradient variation (radial, mE)



30′×30′ residual soil water load effects in Chinese mainland (Green integral): horizontal gradient variation (N, mE)



30′×30′ residual soil water load effects in Chinese mainland (Green integral): horizontal gradient variation (W, mE)

After superimposing the load effects of ultrashort wave surface water and groundwater, the variation magnitudes of the ground gravity gradient will increase by more than several times. How to effectively deal with the surface load effects with complex space-time characteristics is a key problem that must be faced in high-precision measurement of ground gravity gradient.

4.6 Regional approach of load deformation field using SRBFs

[Purpose] From the regional residual surface load equivalent water height grid, approach the regional residual surface loads in spectral domain using spherical radial basis functions (SRBFs) and then calculate the residual load effects on all-element geodetic variations using SRBF synthesis to solve the high-degree oscillation and poor convergence troubles of load

Green′s function in the near area around the calculation point.

When computing the load effects of sea level variations, the height of the calculation point is the normal or orthometric height. When computing the load effects of surface atmosphere or land water variations, the height of the calculation point is the height relative to the Earth’s surface.

4.6.1 Approach of residual load and synthesis of residual load effects using SRBFs

[Function] From the regional residual equivalent water height (EWH) variation grid (cm), approach the regional residual surface loads using spherical radial basis functions (SRBFs) and then calculate the residual EWH estimation (cm) and residual load effects on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (mE) or horizontal gravity gradient (NW, to the north and to the west, E) using SRBF synthesis.

[Input files] The space calculation point file. The regional residual equivalent water height variation grid file.

The calculation point file can be a discrete calculation point file or a calculation surface height grid file.

The discrete calculation point file record format: Point number (name), longitude, latitude (decimal degrees), height….

[Parameter settings] Select the calculation point file format and set SRBF approach algorithm parameters.

The effectiveness principle of the parameter optimization and cumulative approach: (1) The estimated load EWH and load effects in space are continuous and differentiable, and (2) the residual standard deviation of the estimated load EWHs is obviously reduced and the residual statistical mean tends to zero.

Select the spherical radial basis functions: radial multipole kernel function, Poisson wavelet kernel function.

Enter the order number m. The order number m of the radial multipole kernel function or Poisson wavelet kernel function. The greater the m, the bigger the kurtosis of SRBF. The zero-order radial multipole kernel function is the point mass kernel function, and the zero-order Poisson wavelet kernel function is the Poisson kernel function.

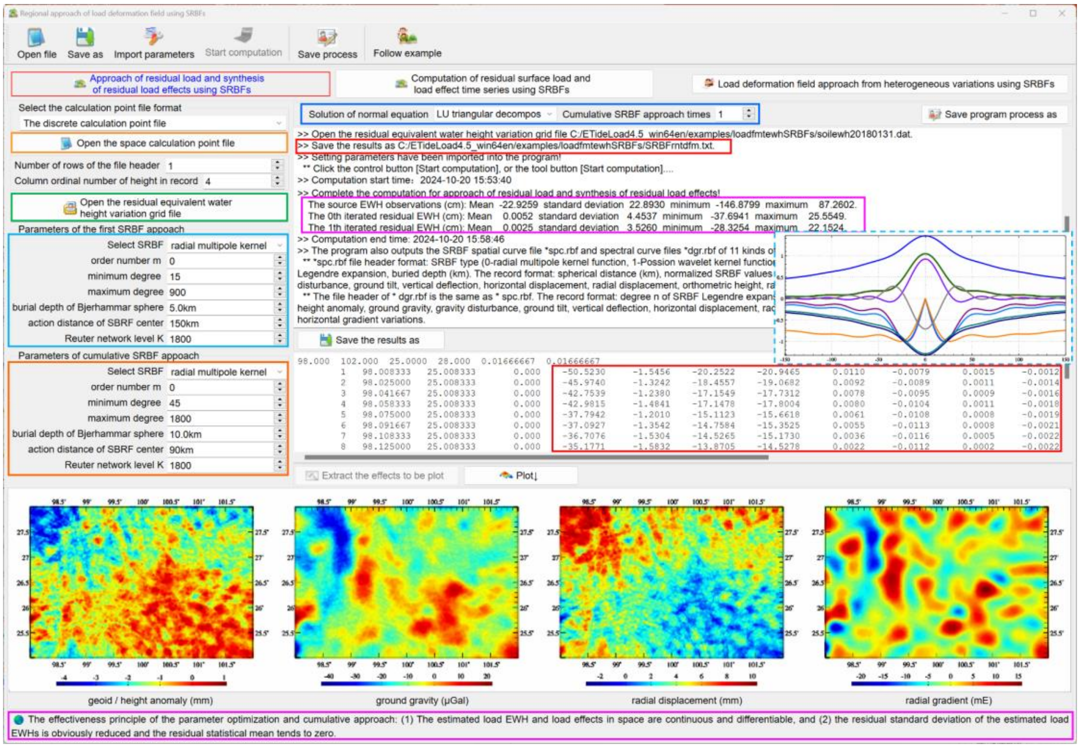
Enter minimum and maximum degree of SRBF Legendre expansion. Minimum and maximum degree can be employed to adjust SRBF bandwidth.

Input the Bjerhammar sphere burial depth: The depth of the Bjerhammar sphere relative to the mean height surface of the observed variations can be employed to adjust the spectral center and bandwidth of SRBF. The greater the burial depth, the smoother the SRBF, the smaller the kurtosis namely the wider the spectral bandwidth.

Enter the action distance of SRBF center. The action distance is also called as the radius of influence = spherical angular distance × the mean radius of the Earth. Which is equivalent to load Green's function integral radius in space domain.

A fixed action distance is adopted to ensure the coordination and consistency of the spatial and spectral figure of load deformation field.

Cumulative SRBF approach times. Every cumulative SRBF approach can be considered that the current load deformation field is refined by the remove-restore scheme with the previous load deformation field as the reference field. Generally cumulative 1 ~ 2 times can obtain a stable solution.



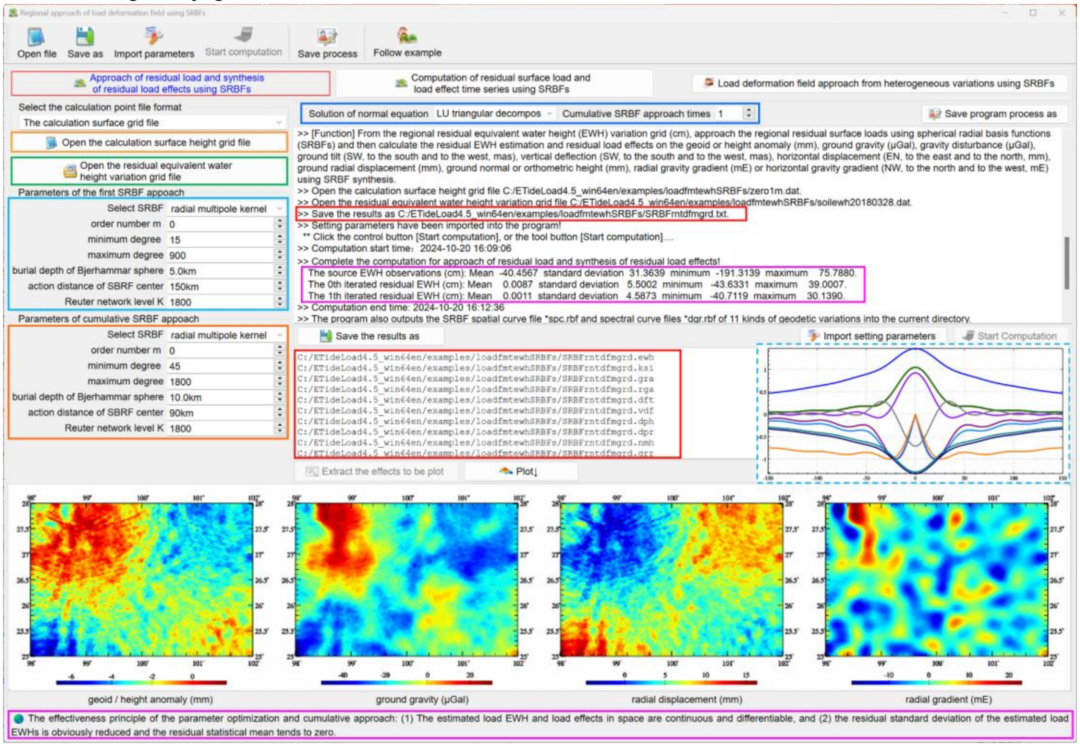
Set the Reuter network level K: The spherical surface is divided into K prime vertical circles, and the latitude interval is 180°/K. The larger the K value, the greater the spatial resolution of the spherical Reuter network. The suitable 180°/K is approximately equal to the average distance between observation points.

Select the method of the solution of normal equation. LU triangular decomposition method or Cholesky decomposition. The normal equation here does not need regularization and iterative computation.

[Output file] The residual EWH estimation and residual load effect file.

When the discrete calculation point file input, the output file header is the same as the input file. Behind the input file record, adds the residual EWH estimation and residual load effects on the height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, normal or orthometric height, radial gravity gradient or horizontal gravity gradient a total of 14 columns of attributes as the output file record.

When the calculation surface height grid file input, the program outputs the the residual EWH estimation and residual load effect all-element grid files *.???, where * is the input file name, and ??? = ewh, ksi, gra, rga, dft, vdf, dph, dpr, nmh, grr or hgd, respectively, representing the grid file of the residual EWH estimation and residual load effects on the height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, normal or orthometric height, radial gravity gradient or horizontal gravity gradient.



The program also outputs the SRBF spatial curve file *spc.rbf and spectral curve file *dgr.rbf of 11 kinds of geodetic variations into the current directory.

*spc.rbf file header format: SRBF type (0-radial multipole kernel function, 1-Poisson wavelet kernel function), order of SRBF, Minimum and maximum degree of SRBF Legendre expansion, buried depth (km). The record format: spherical distance (km), the normalized SRBF values from the load EWH, height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, orthometric height, radial gravity gradient and horizontal gradient variations.

The file header of * dgr.rbf is the same as * spc.rbf. The record format: degree n of SRBF Legendre expansion, the degree-n normalized SRBF values from the load EWH, height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, orthometric height, radial gravity gradient and horizontal gradient variations.

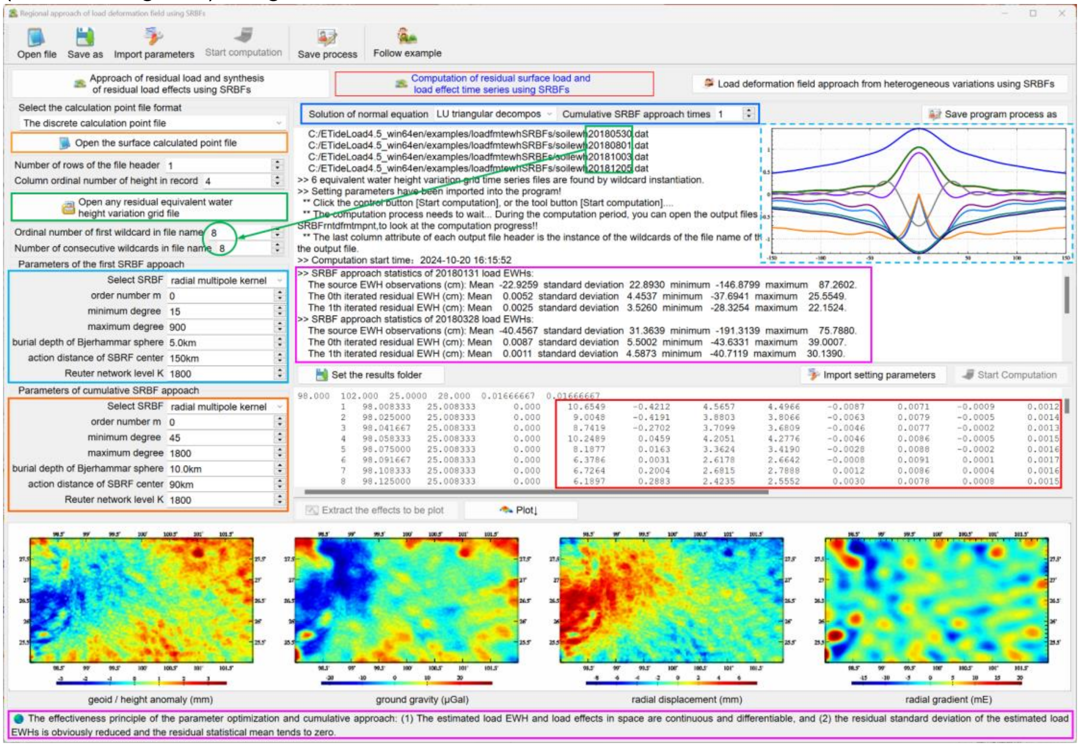
4.6.2 Computation of residual surface load and load effect time series using SRBFs

[Function] From the regional residual equivalent water height (EWH) variation grid (cm) time series, approach the regional residual surface load in spectral domain using spherical radial basis functions (SRBFs) and then calculate the residual EWH estimation and residual load effect all-element grid time series on the geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (mE) or horizontal gravity gradient (NW, to the north and to the west, mE) using SRBF synthesis.

[Input files] The space calculation point file. The regional residual equivalent water height variation grid time series files.

The calculation point file can be a discrete calculation point file or a calculation surface height grid file.

The discrete calculation point file record format: Point number (name), longitude, latitude (decimal degrees), height….

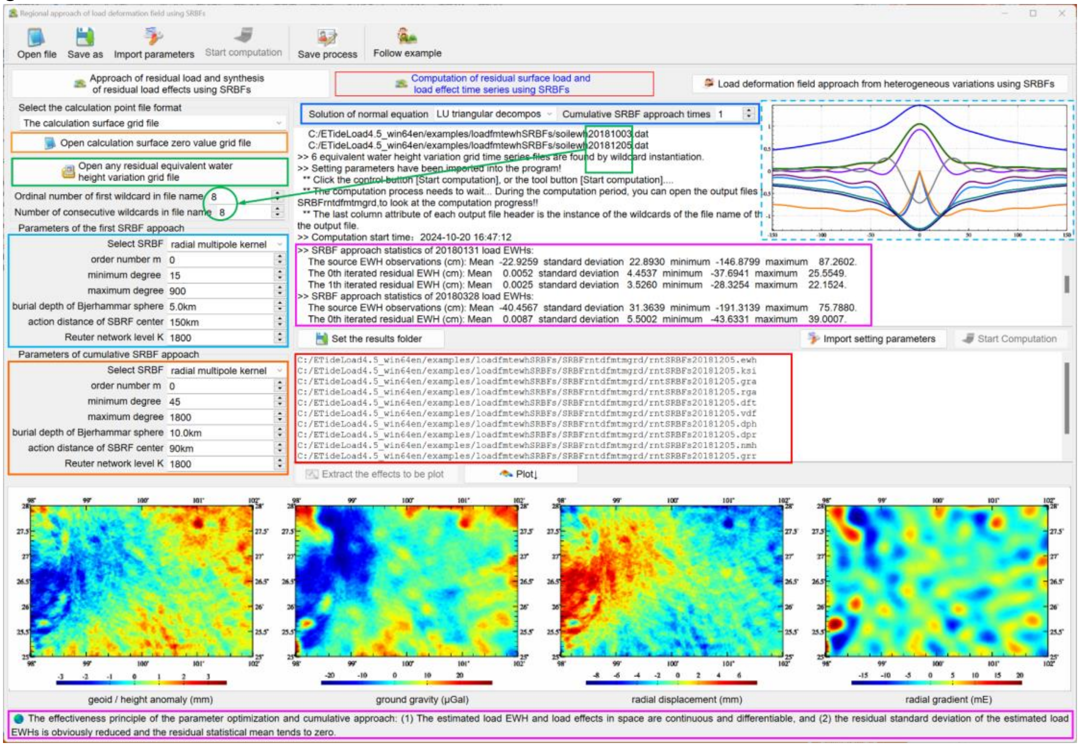


[Parameter settings] Directly employing the Parameter settings from the chapter 4.6.1.

[Output file] The residual EWH estimation and residual load effect all-element grid time series files.

When the discrete calculation point file input, the program outputs the residual load effect record time series files rntSRBFs***.txt. Each output file header is the same as the input file. Behind the input file record, adds the residual EWH estimation and residual load effects on the height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, normal or orthometric height, radial gravity gradient or horizontal gravity gradient a total of 14 columns of attributes as the output file record.

When the calculation surface height grid file input, the program outputs the the residual EWH estimation and residual load effect all-element grid time series files rntSRBFs*.???, where ??? = ewh, ksi, gra, rga, dft, vdf, dph, dpr, nmh, grr or hgd, respectively, representing the grid file of the residual EWH estimation and residual load effects on the height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, normal or orthometric height, radial gravity gradient or horizontal gravity gradient.



Here, *** are the wildcards of the variation grid time series file names, whose instance can identify the sampling epoch time of the load effects. The time of the residual load effects is the sampling epoch time of the surface equivalent water height grid model.

The last attribute of each output file header is the instance of the wildcards of the time series file name of the residual equivalent water height grid model, which represents the sampling epoch time of the output file.

The computation process needs to wait…. During the period, you can open the output file directory to look at the computation progress!

4.7 Load deformation field monitoring from heterogeneous variations with Green's integral constraints

[Purpose] Using the heterogeneous geodetic variations from the regional CORS network, gravity tide stations and various geodetic monitoring networks as the observations, and the load Green's function integral as the geodynamic constraints, estimate the regional land water variation and the load effects on all-element load deformation field (time-varying gravity field).

It is technically required that the long wave parts of the load effects on geodetic variations should be removed to satisfy the local Green's integral condition.

The geodetic variations here can be one or more of the following five types of variations. (1) Height anomaly variations (mm) from GNSS-leveling monitoring network, (2) disturbance gravity variations (μGal) from GNSS-gravity monitoring network or CORS-gravity tide stations, (3) ground gravity variations (μGal) from gravity monitoring network or gravity tide stations, (4) ellipsoidal height variations (mm) from CORS network or GNSS monitoring network, and (5) normal or orthometric height variations (mm) from leveling monitoring network.

4.7.1 Load deformation field estimation from heterogeneous variations with Green's integral constraints

[Function] Using various heterogeneous geodetic variations as the observations and the load Green's function integral as the geodynamic constraints, estimate the regional surface load equivalent water height (EWH) and all-element load effects to obtain the land water EWH, geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (mE) and horizontal gravity gradient (NW, to the north and to the west, mE) variation grids.

[Input files] The heterogeneous geodetic variation record time series file. The calculation surface height grid file.

The geodetic variation record time series file. The file header contains the time series length and the sampling epoch time arranged with time. Record format: ID (the site name / no), longitude, latitude, …, weight, variation type, …, variations arranged in time series length (default value is 9999.0000).

Variation type = 1 represents the height anomaly variation (mm), = 2 represents gravity disturbance variation (μGal), = 3 represents ground gravity variation (μGal), = 4 represents ground ellipsoidal height variation (mm), and = 5 represents normal or orthometric height variation (mm).

The calculation surface height is the height of the calculation point relative to the ground surface. When calculating the ground load deformation field, enter the zero-value grid. The calculation surface height grid specification is employed to specify the latitude and longitude range and spatial resolution of the land water EWH grid to be estimated.

The program requires that the grid range of the calculation surface height must be larger than the geodetic site distribution range to absorb the edge effect. The actual effective range of the land water EWH and its load deformation field grid to be estimated will be less than the coverage range of these geodetic sites.

[Parameter settings] Set the geodetic variation record time series file format parameters, enter the column ordinal number of the current variations, load Green's integral radius and mean distance between geodetic sites, and set the algorithm control parameters.

The weights of variations are employed only to distinguish the quality of variations in the same type.

The column ordinal number of the current variations in the file record. The program estimates an epoch time of the land water EWH and all-element load effect grids once, therefore you need to specify the column ordinal number of the variations at the epoch time.

Mean distance (km) between the geodetic sites. Input the approximate value of the mean distance between the geodetic sites. Which should not be greatly reduced intentionally, otherwise it will seriously affect the speed of parameter estimation and the stability of the solution. The mean distance is not directly related to the spatial resolution of the estimated land water EWH.

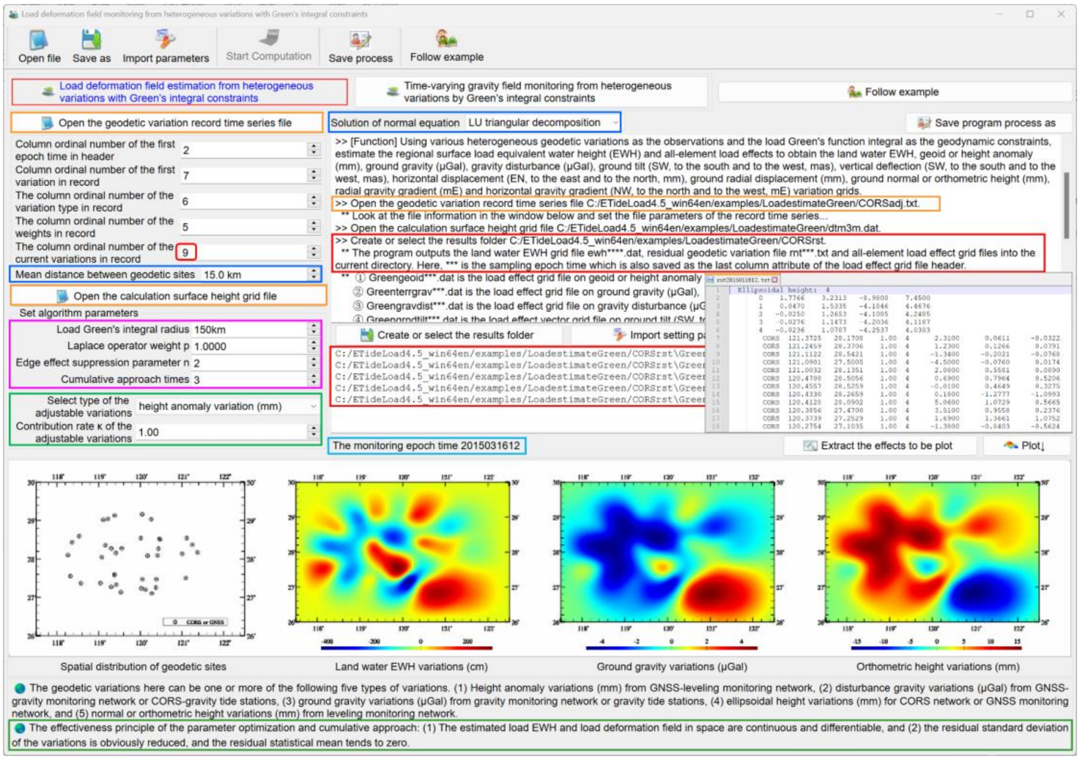
Green's integral radius (km). A fixed integral radius is adopted to ensure the coordination and consistency of the spatial and spectral figure of the load deformation field.

The Laplace operator weight p. It is employed to suppress spatial noise. The greater the weight, the greater the spatial filtering intensity. Not smooth when $p = 0$, smooth enhancing when $p > 1$ and smooth weakening when $p < 1$.

Edge effect suppression parameter n. The program lets the unknown EWHs of n cell-grids located in the region edge equal to zero as the observation equations to suppress the edge and far zone effects.

Cumulative approach times. Every cumulative approach can be considered that the current load deformation field is refined by the remove-restore scheme with the previous load deformation field as the reference field. Generally cumulative 1 ~ 3 times can obtain a stable solution.

The effectiveness principle of the parameter optimization and cumulative approach: (1) The estimated load EWH and load deformation field in space are continuous and differentiable, and (2) the residual standard deviation of the variations is obviously reduced, and the residual statistical mean tends to zero.



Select the type of adjustable variation and set the contribution rate κ of the adjustable variation.

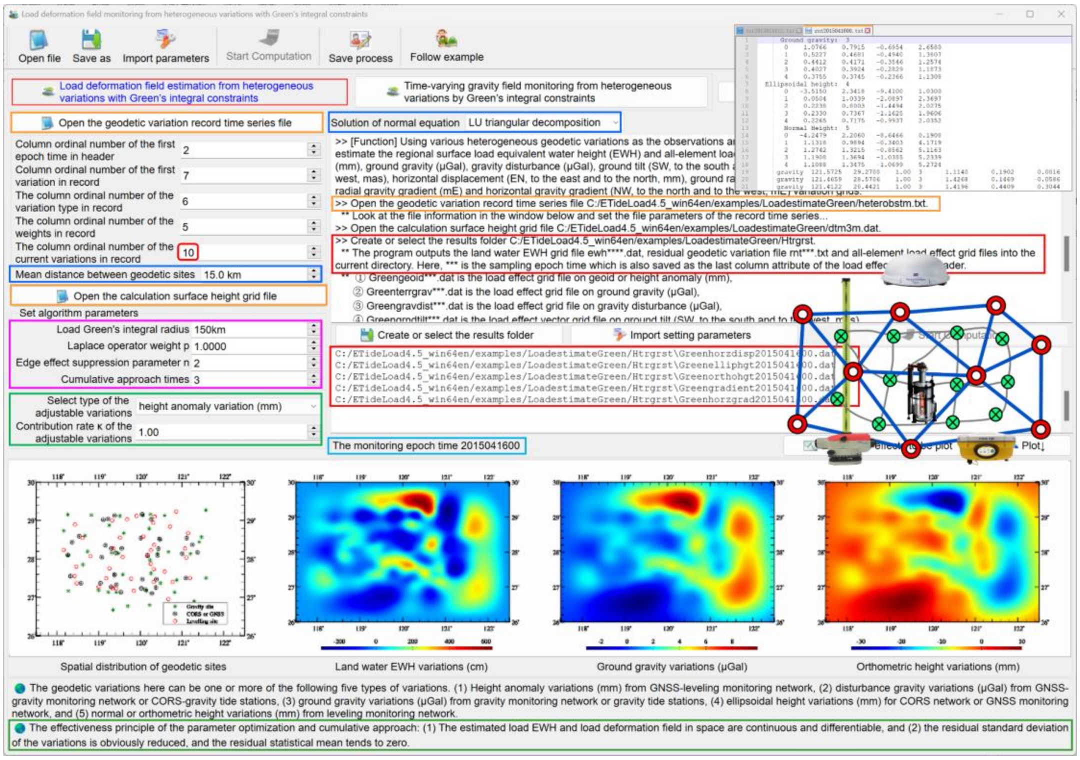
The program multiplies the normal equation coefficient matrix and constant matrix of the adjustable variations by κ, respectively, to increase $(\kappa > 1)$ or decrease $(\kappa < 1)$ the contribution of the adjustable variation. When $\kappa = 1$, it means that there is not any adjustable variation selected. When $\kappa = 0$, the adjustable variation does not participate in the EWH estimation.

Select the method of the solution of normal equation. LU triangular decomposition method or Cholesky decomposition. The normal equation here does not need regularization and iterative computation.

ETideLoad4.5 employs the normalzation of normal equations to combine multiply heterogeneous geodetic monitoring system, so that the properties of the load EWH estimation solution are only related to the space distribution of geodetic variations without influence of various monitoring error. Which can improve significantly the universality and reliability of the estimation algorithm.

[Output files] The land EWH variation and all-element load effect grid files, the residual geodetic variation file for the cumulative approach process.

The program outputs the land water EWH grid files ewh****.dat, residual geodetic variation file rnt***.txt and various load effect grid files into the current directory.



Here, *** is the sampling epoch time which is also saved as the last column attribute of the load effect grid file header.

①Greengeoid***.dat is the load effect grid file on geoid or height anomaly (mm),

②Greenterrgrav***.dat is the load effect grid file on ground gravity (μGal),

③Greengravdist***.dat is the load effect grid file on gravity disturbance (μGal),

④Greengrndtilt***.dat is the load effect vector grid file on ground tilt (SW, to the south and to the west, mas),

⑤Greenvertdefl***.dat is the load effect vector grid file on vertical deflection (SW, to the south and to the west, mas),

⑥Greenhorzdisp***.dat is the load effect vector grid file on horizontal displacement (EN, to the east and to the north, mm),

⑦Greenelliphgt***.dat is the load effect grid file on ground radial displacement (mm),

⑧Greenorthohgt***.dat is the load effect grid file on ground normal or orthometric height (mm),

⑨Greengradient***.dat is the load effect grid file on radial gravity gradient (mE) and

⑩Greenhorzgrad***.dat is the load effect vector grid file on horizontal gravity gradient (NW, to the north and to the west, mE).

After the computation is completed, the residual geodetic variation file should be opened to observe the standard deviation and mean value of the residual variations and their changes with the cumulative number of times to optimize the parameter settings and cumulative number of times, and then recompute.

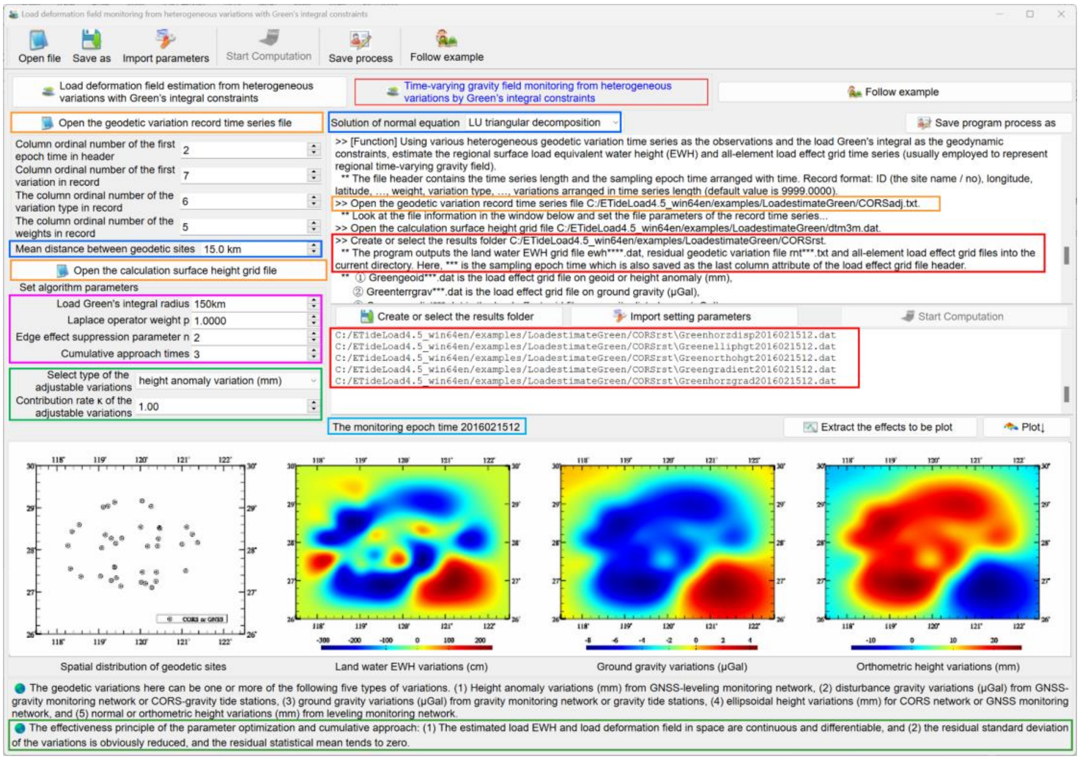
4.7.2 Time-varying gravity field monitoring from heterogeneous variations by Green's integral constraints

[Function] Using various heterogeneous geodetic variation time series as the observations and the load Green's integral as the geodynamic constraints, estimate the regional surface load equivalent water height (EWH) and all-element load effect grid time series (usually employed to represent regional time-varying gravity field).

[Input files] The heterogeneous geodetic variation record time series file. The calculation surface height grid file.

The geodetic variation record time series file. The file header contains the time series length and the sampling epoch time arranged with time. Record format: ID (the site name / no), longitude, latitude, …, weight, variation type, …, variations arranged in time series length (default value is 9999.0000).

Variation type = 1 represents the height anomaly variation (mm), = 2 represents gravity disturbance variation (μGal), = 3 represents ground gravity variation (μGal), = 4 represents ground ellipsoidal height variation (mm), and = 5 represents normal or orthometric height variation (mm).

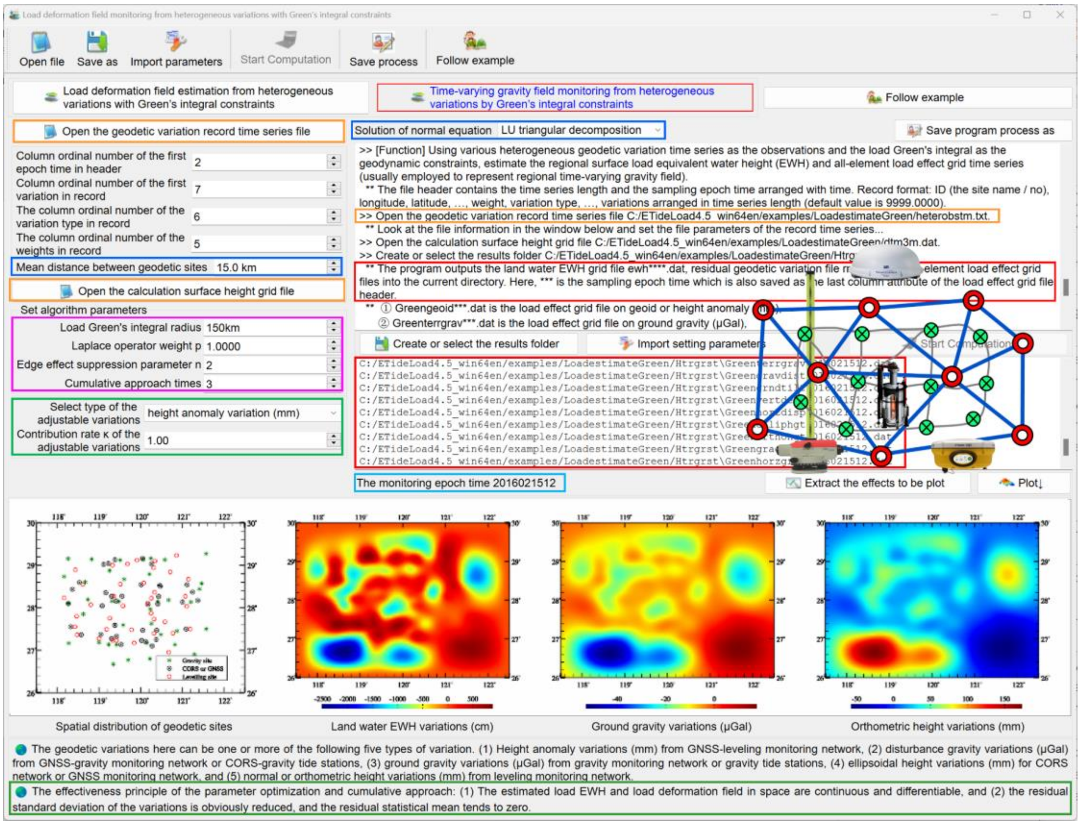


[Parameter settings] All the algorithm parameters that are same as the function [Load deformation field estimation from heterogeneous variations with Green's integral constraints].

During the monitoring period, when the spatial distribution of geodetic monitoring sites or shape of geodetic monitoring networks and the types of geodetic variations are basically unchanged, the algorithm parameters will remain unchanged, which is a typical feature of ETideLoad algorithm of load deformation field approach that is especially useful for automatic processing of multiple heterogeneous multi-period or continuous geodetic monitoring networks.

[Output files] The program outputs the land water EWH grid time series file ewh****.dat, residual geodetic variation time series files rnt***.txt and all-element load effect grid time series files.

Here, *** is the sampling epoch time which is also saved as the last column attribute of the load effect grid file header.



①Greengeoid***.dat is the load effect grid file on geoid or height anomaly (mm),

②Greenterrgrav***.dat is the load effect grid file on ground gravity (μGal),

③Greengravdist***.dat is the load effect grid file on gravity disturbance (μGal),

④Greengrndtilt***.dat is the load effect vector grid file on ground tilt (SW, to the south and to the west, mas),

⑤Greenvertdefl***.dat is the load effect vector grid file on vertical deflection (SW, to the south and to the west, mas),

⑥Greenhorzdisp***.dat is the load effect vector grid file on horizontal displacement (EN, to the east and to the north, mm), ⑦Greenelliphgt***.dat is the load effect grid file on ground radial displacement (mm),

⑧Greenorthohgt***.dat is the load effect grid file on ground normal or orthometric height (mm),

⑨Greengradient***.dat is the load effect grid file on radial gravity gradient (mE) and

⑩Greenhorzgrad***.dat is the load effect vector grid file on horizontal gravity gradient (NW, to the north and to the west, mE).

After the estimation is completed, the residual value files should be opened to check the results. The first few rows of the file indicate the change information of the mean and standard deviation of the residual value with the number of iterations. If necessary, the algorithm parameters should be adjusted and recomputed for individual epochs without ideal solutions by the function [Load deformation field estimation from heterogeneous variations with Green's integral constraints]. Particular attention should be paid at the monitoring epoch time when the distribution or types of geodetic variations occur obvious changes.

4.8 Load deformation field monitoring from heterogeneous variations using spherical radial basis functions

[Purpose] From the heterogeneous geodetic variations in the regional CORS network, gravity tide stations and various geodetic networks, approach or monitor the regional land water variations and the load effects on all-element load deformation field (time-varying gravity field) in spectral domain using spherical radial basis functions (SRBF).

It is technically required that the long wave parts of the load effects on geodetic variations should be removed to satisfy the regional SRBF approach condition.

The variations here can be one or more of the following six types of variations. (1) Height anomaly variations (mm) from GNSS-leveling monitoring network, (2) disturbance gravity variations (μGal) from GNSS-gravity monitoring network or CORS-gravity tide stations, (3) ground gravity variations (μGal) from gravity monitoring network or gravity tide stations, (4) ellipsoidal height variations (mm) for CORS network or GNSS monitoring network, (5) normal or orthometric height variations (mm) from leveling monitoring network, and (6) equivalent water height variations (cm) from hydrological monitoring stations.

4.8.1 Load deformation field approach from heterogeneous variations using spherical radial basis functions

[Function] Using spherical radial basis functions in spectral domain, approach the regional surface load equivalent water height (EWH) and all-element load effects to obtain the land water EWH, geoid or height anomaly (mm), ground gravity (μGal), gravity disturbance (μGal), ground tilt (SW, to the south and to the west, mas), vertical deflection (SW, to the south and to the west, mas), horizontal displacement (EN, to the east and to the north, mm), ground radial displacement (mm), ground normal or orthometric height (mm), radial gravity gradient (mE) and horizontal gravity gradient (NW, to the north and to the west, mE) variation grids from various heterogeneous geodetic variations.

[Input files] The heterogeneous geodetic variation record time series file. The calculation surface height grid file.

The variation record time series file. The file header contains the time series length and the sampling epoch time arranged with time. Record format: ID (the site name / no), longitude, latitude, …, weight, variation type, …, variations arranged in time series length (default value is 9999.0000).

Variation type = 1 represents the height anomaly variation (mm), = 2 represents gravity disturbance variation (μGal), = 3 represents ground gravity variation (μGal), = 4 represents ground ellipsoidal height variation (mm), = 5 represents normal or orthometric height variation (mm), and = 6 represents equivalent water height variations (cm).

The calculation surface height is the height of the calculation point relative to the ground surface. When calculating the ground load deformation field, enter the zero-value grid. The calculation surface height grid specification is employed to specify the latitude and longitude range and spatial resolution of the land water EWH grid to be estimated.

The program requires that the grid range of the calculation surface height must be larger than the monitoring site distribution range to absorb the edge effect. The actual effective range of the land water EWH and load effect grid to be estimated will be less than the coverage range of these monitoring sites.

[Parameter settings] Set the variation record time series file format parameters, enter the column ordinal number of current variations and mean distance between geodetic sites, and set the SRBF parameters and algorithm control parameters.

The weights of variations are employed only to distinguish the quality of variations in the same type.

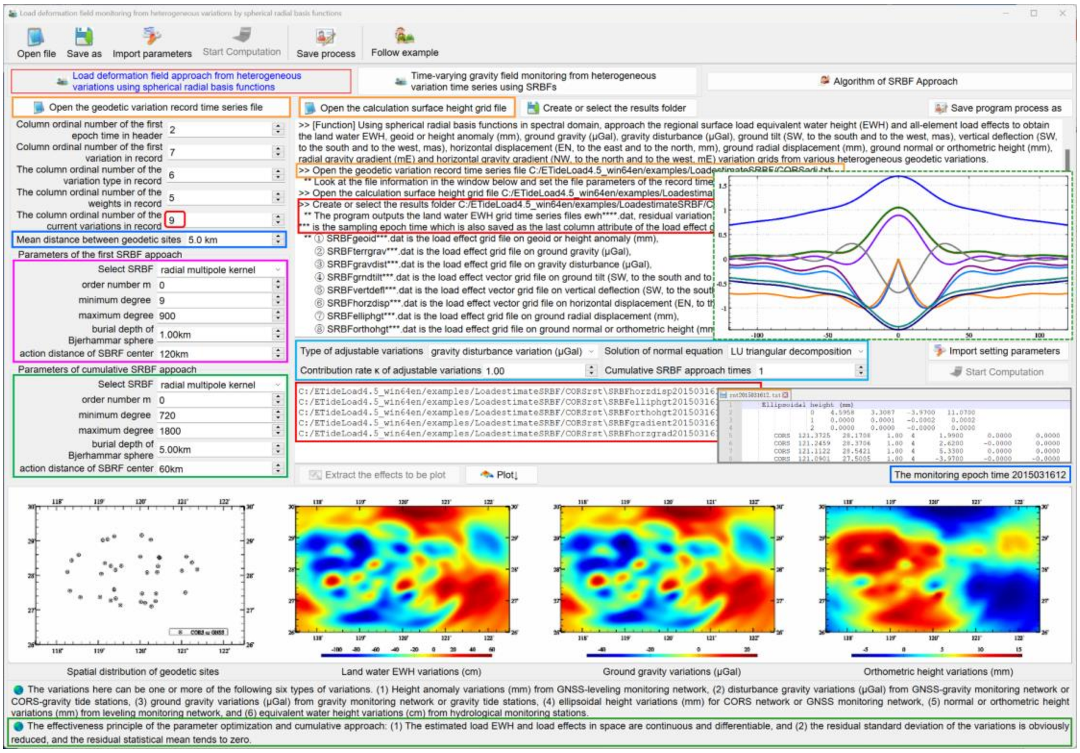
The column ordinal number of the current variations in the file record. Each time, the program estimates an epoch time of the land water EWH and load deformation field grid, you need to specify the column ordinal number of the current variations at the epoch time.

Mean distance (km) between the geodetic sites. Input the approximate value of the mean distance between the geodetic sites. Which should not be greatly reduced intentionally, otherwise it will seriously affect the speed of parameter estimation and the stability of the solution. The mean distance is not directly related to the spatial resolution of the estimated land water EWH.

Select the spherical radial basis functions: radial multipole kernel function, Poisson wavelet kernel function. The zero-order radial multipole kernel function is the point mass kernel function, and the zero-order Poisson wavelet kernel function is the Poisson kernel function.

Enter the order number m. The order number m of radial multipole kernel function and Poisson wavelet kernel function. The greater the m, the bigger the kurtosis of SRBF.

Input the Bjerhammar sphere burial depth: The depth of the Bjerhammar sphere relative to the mean height surface of the observation variations can be employed to adjust the spectral center and bandwidth of SRBF. The greater the burial depth, the smoother the SRBF, the smaller the kurtosis namely the wider the spectral bandwidth.



Enter the action distance of SRBF center. The action distance is also called as the radius of influence = spherical angular distance × the mean radius of the Earth. Which is equivalent to load Green's function integral radius in space domain.

A fixed action distance is adopted to ensure the coordination and consistency of the spatial and spectral figure of load deformation field.

Enter minimum and maximum degree of SRBF Legendre expansion. Minimum and maximum degree can be employed to adjust SRBF bandwidth.

Cumulative SRBF approach times. Every cumulative SRBF approach can be considered that the current load deformation field is refined by the remove-restore scheme with the previous load deformation field as the reference field. Generally cumulative 1 ~ 3 times can obtain a stable solution.

The effectiveness principle of the parameter optimization and cumulative approach: (1) The estimated load EWH and load effects in space are continuous and differentiable, and (2) the residual standard deviation of the variations is obviously reduced, and the residual statistical mean tends to zero.

Select the type of adjustable variation and set the contribution rate κ of the adjustable variation.

The program multiplies the normal equation coefficient matrix and constant matrix of the adjustable variations by κ, respectively, to increase ($\kappa > 1$) or decrease ($\kappa < 1$) the contribution of the adjustable variation. When $\kappa = 1$, it means that there is not any adjustable variation selected. When $\kappa = 0$, the adjustable variation does not participate in the EWH estimation.

Select the method of the solution of normal equation. LU triangular decomposition method or Cholesky decomposition. The normal equation here does not need regularization and iterative computation.

ETideLoad4.5 employs the normalzation of normal equations to combine multiply heterogeneous geodetic monitoring system, so that the properties of the load EWH estimation solution are only related to the space distribution of geodetic variations without influence of various monitoring error. Which can improve significantly the universality and reliability of the estimation algorithm.

[Output files] The land EWH variation and all-element load effect grid files, the residual variation file for the cumulative approach process.

The program outputs the land water EWH grid files ewh****.dat residual variation file rnt***.txt and various load effect grid files into the current directory.

Here, *** is the sampling epoch time which is also saved as the last column attribute of the load effect grid file header.

①SRBFgeoid***.dat is the load effect grid file on geoid or height anomaly (mm),

②SRBFterrgrav***.dat is the load effect grid file on ground gravity (μGal),

③SRBFgravdist***.dat is the load effect grid file on gravity disturbance (μGal),

④SRBFgrndtilt***.dat is the load effect vector grid file on ground tilt (SW, to the south and to the west, mas),

⑤SRBFvertdefl***.dat is the load effect vector grid file on vertical deflection (SW, to the south and to the west, mas),

⑥SRBFhorzdisp***.dat is the load effect vector grid file on horizontal displacement (EN, to the east and to the north, mm),

⑦SRBFelliphgt***.dat is the load effect grid file on ground radial displacement (mm),

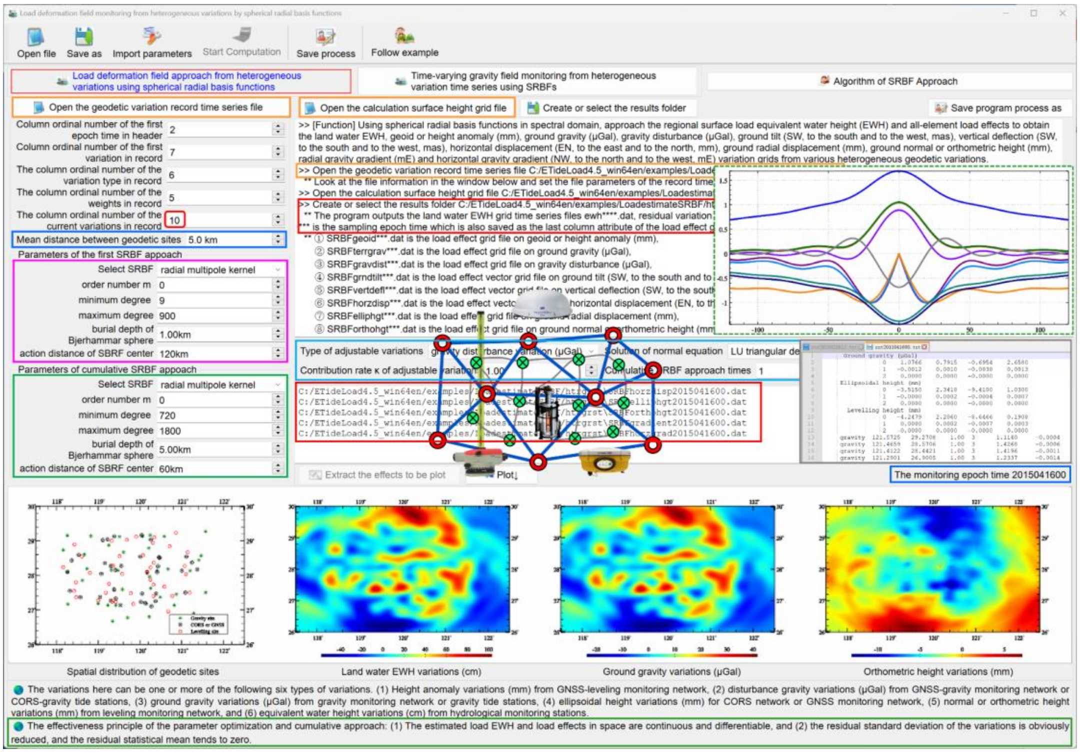
⑧SRBForthohgt***.dat is the load effect grid file on ground normal or orthometric height (mm),

⑨SRBFgradient***.dat is the load effect grid file on radial gravity gradient (mE) and

⑩SRBFhorzgrad***.dat is the load effect vector grid file on horizontal gravity gradient (NW, to the north and to the west, mE).

The program also outputs the SRBF spatial curve file *spc.rbf and spectral curve files *dgr.rbf of 11 kinds of geodetic variations into the current directory.

*spc.rbf file header format: SRBF type (0-radial multipole kernel function, 1-Poisson wavelet kernel function), order of SRBF, Minimum and maximum degree of SRBF Legendre expansion, buried depth (km) of Bjerhammar shpere. The record format: spherical distance (km), the normalized SRBF values from the load EWH, height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, orthometric height, radial gravity gradient and horizontal gradient variations.



The file header of * dgr.rbf is the same as * spc.rbf. The record format: degree n of SRBF Legendre expansion, the degree-n normalized SRBF values from the load EWH, height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, orthometric height, gravity gradient and horizontal gradient variations.

After the computation is completed, the residual geodetic variation file shoulid be opened to observe the standard deviation and mean value of the residual variations and their changes with the cumulative number of times to optimize the parameter settings and cumulative number of times, and then recompute.

4.8.2 Time-varying gravity field monitoring from heterogeneous variation time series using SRBFs

[Function] From various heterogeneous geodetic variation time series, using spherical radial basis function approach method in spectral domain, estimate the regional surface load equivalent water height (EWH) and all-element load effect grid time series (usually employed to represent regional time-varying gravity field).

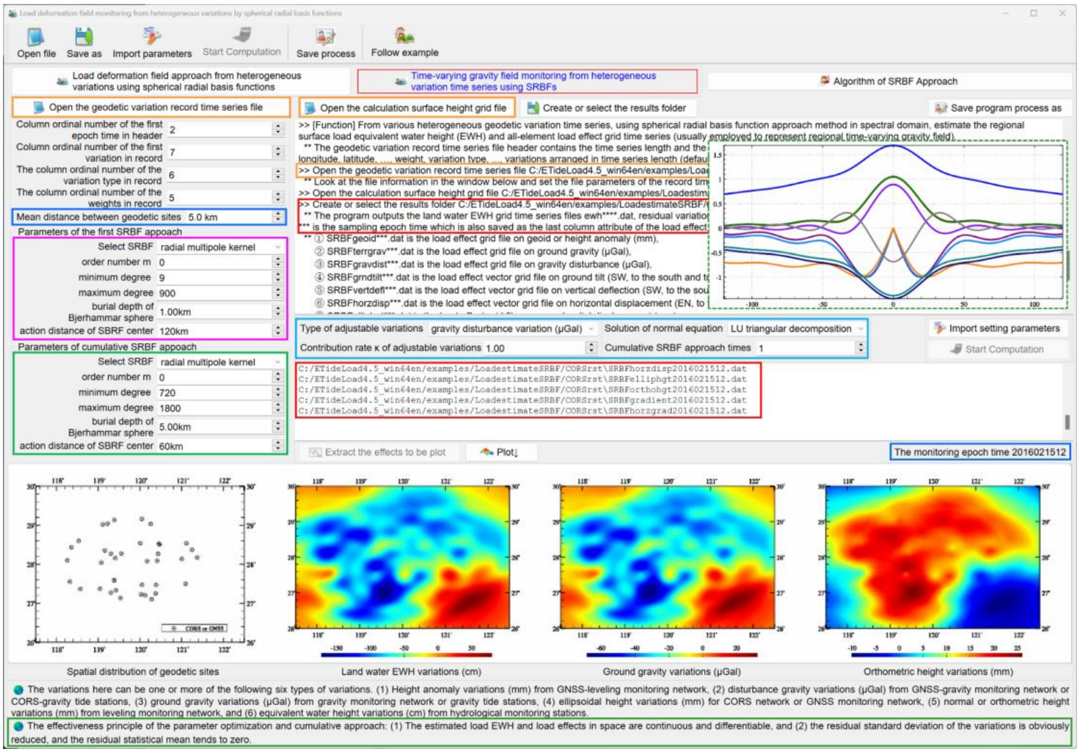
[Input files] The heterogeneous variation record time series file. The calculation surface height grid file.

The variation record time series file. The file header contains the time series length and the sampling epoch time arranged with time. Record format: ID (the site name / no), longitude, latitude, …, variation type, weight, …, variations arranged in time series length (default value is 9999.0000).

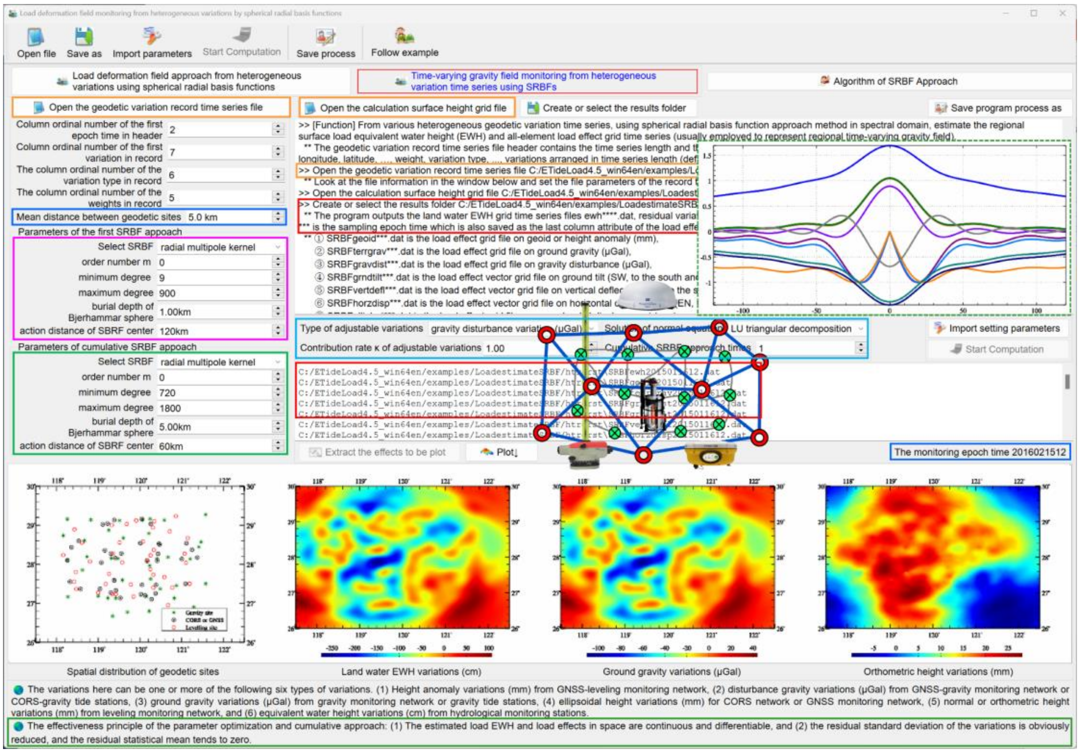
Variation type = 1 represents the height anomaly variation (mm), = 2 represents gravity disturbance variation (μGal), = 3 represents ground gravity variation (μGal), = 4 represents ground ellipsoidal height variation (mm), = 5 represents normal or orthometric height variation (mm), and = 6 represents equivalent water height variations (cm).

[Parameter settings] All the algorithm parameters that are same as the function [Load deformation field approach from heterogeneous variations by spherical radial basis functions].

During the monitoring period, when the spatial distribution of geodetic monitoring sites or shape of geodetic monitoring networks and the type of geodetic variations are basically unchanged, the algorithm parameters will remain unchanged, which is a typical feature of ETideLoad algorithm of load deformation field monitoring that is especially useful for automatic processing of multiple heterogeneous multi-period or continuous geodetic monitoring networks.



[Output files] The program outputs the land water EWH grid time series file ewh****.dat residual variation time series files rnt***.txt and all-element load effect grid time series files. Here, *** is the sampling epoch time which is also saved as the last column attribute of the load effect grid file header.



①SRBFgeoid***.dat is the load effect grid file on geoid or height anomaly (mm),

②SRBFterrgrav***.dat is the load effect grid file on ground gravity (μGal),

③SRBFgravdist***.dat is the load effect grid file on gravity disturbance (μGal),

④SRBFgrndtilt***.dat is the load effect vector grid file on ground tilt (SW, to the south and to the west, mas),

⑤SRBFvertdefl***.dat is the load effect vector grid file on vertical deflection (SW, to the south and to the west, mas),

⑥SRBFhorzdisp***.dat is the load effect vector grid file on horizontal displacement (EN, to the east and to the north, mm),

⑦SRBFelliphgt***.dat is the load effect grid file on ground radial displacement (mm),

⑧SRBForthohgt***.dat is the load effect grid file on ground normal or orthometric height (mm),

⑨SRBFgradient***.dat is the load effect grid file on radial gravity gradient (mE) and

⑩SRBFhorzgrad***.dat is the load effect vector grid file on horizontal gravity gradient (NW, to the north and to the west, mE).

After the estimation is completed, the residual value files should be opened to check the results. The first few rows of the file indicate the change information of the mean and standard deviation of the residual value with the number of iterations.

If necessary, the algorithm parameters should be adjusted and recomputed for individual epochs without ideal solutions by the function [Load deformation field approach from heterogeneous variations using spherical radial basis functions]. Particular attention should be paid at the monitoring epoch time when the distribution or types of geodetic variations occur obvious changes.

4.9 Geodynamic calculation on geodetic field grid time series

[Purpose] Calculate the time difference, space horizontal gradient, or two vector grids inner product of the ground deformation field grid time series to display their spatiotemporal geodynamic characteristics.

4.9.1 Time difference operation on variation (vector) grid time series

[Function] Sort the input variation (vector) grid time series files according to the sampling epoch time (the seventh attribute of the file header), and then calculate the variation rate at two neighboring sampling epochs to generate the variation (vector) rate grid time series. Here, the sampling epoch time of the current grid is equal to the average of the before and after sampling epochs of the variation (vector) grids, the unit of the variation rate is per k day, and k is the given differential time scale factor.

The variation (vector) grid time series files are extracted according to the given wildcards. For the variation vector grid time series, the program requires them to be in the form of horizontal coordinates.

[Input files] The variation (vector) grid time series files.

[Parameter settings] Set the wildcard parameters for the variation (vector) grid time series files, enter the differential time scale factor k.

[Output files] The variation (vector) rate grid time series files.

4.9.2 Horizontal gradient calculation on batch variation grids

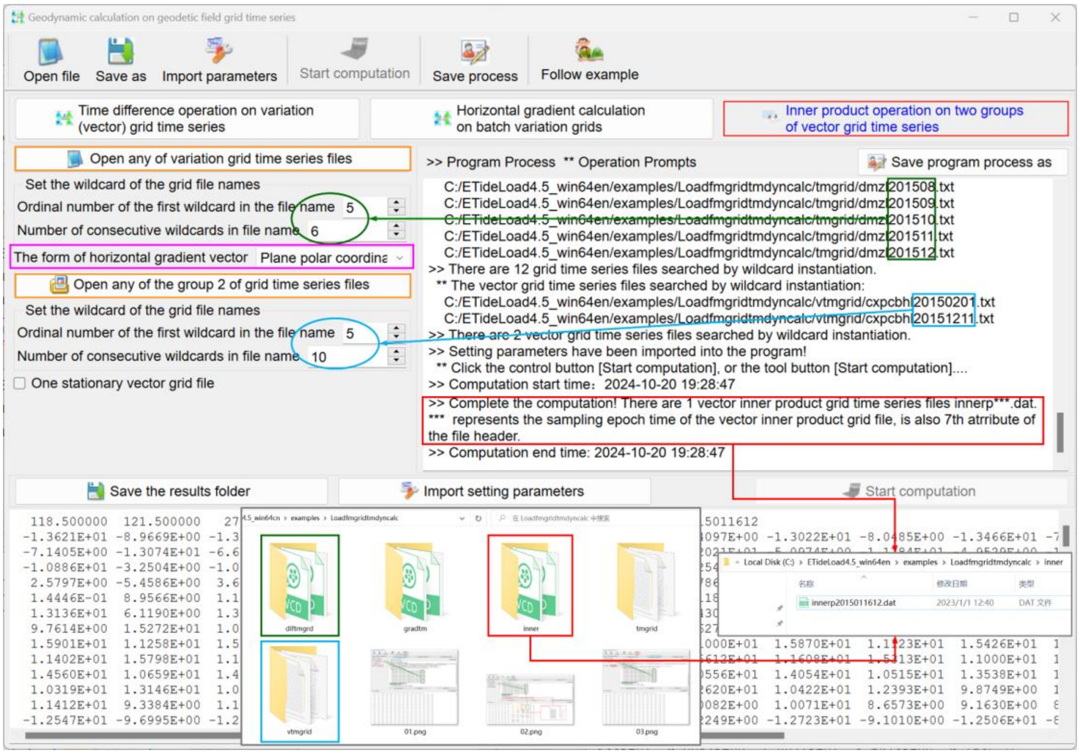
[Function] From batch variation grid files with the same grid specifications in the specified directory, calculate horizontal gradient vector grids (per km). The horizontal gradient vector can be output in the form of polar coordinates or EN horizontal coordinates. The variation grid files are extracted according to the given wildcards.

4.9.3 Inner product operation on two groups of vector grid time series

[Function] Calculate the inner product grid time series from two groups of variation vector grid time series in the form of the EN horizontal rectangular coordinates with the same grid specifications.

The variation vector grid files are extracted according to the given wildcards.

The program allows a group of vector grid files with only one sampling time. When the two groups are both vector grid time series, the program requires one-by-one correspondence between the sampling epochs.



4.10 Monitoring computation processes for the surface loads and load deformation field

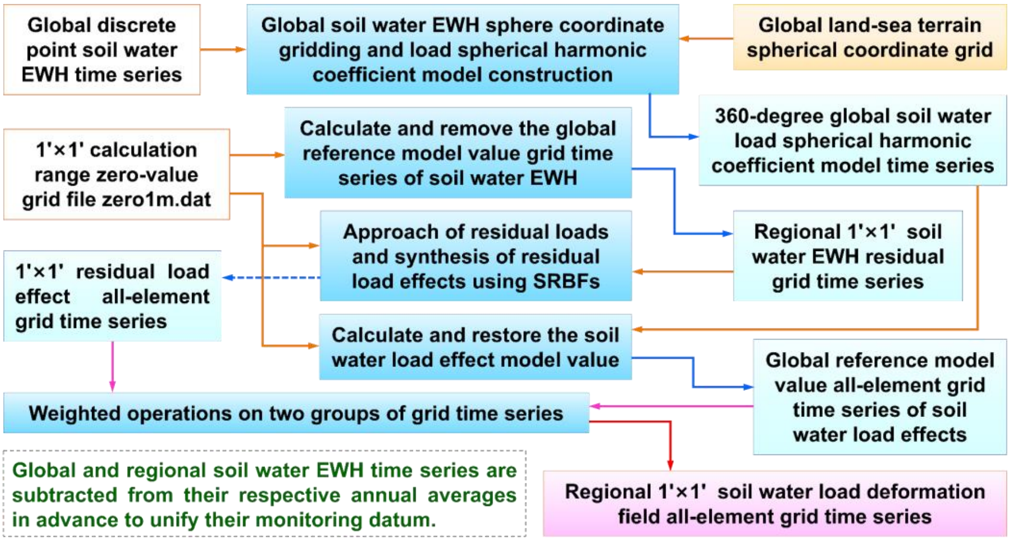
4.10.1 Complete computation processes of high-resolution regional load deformation field time series

Taking the regional soil water variations as the example, the remove-restore scheme combined the global load spherical harmonic coefficient synthesis and regional residual load SRBF (spherical radial basis function) approach is employed to compute the high-precision and high-resolution regional load deformation field all-element grid time series in the near-Earth space in the four steps.

The soil water here consists of soil water in 4 m shallow, wetland water, vegetation water, glaciers and snow mountain water, but does not include lakes, rivers and groundwater.

The global soil water equivalent water height (EWH) time series and the regional high-resolution soil water EWH time series are subtracted from their respective annual averages in advance to unify the global and regional soil water variation monitoring datum.

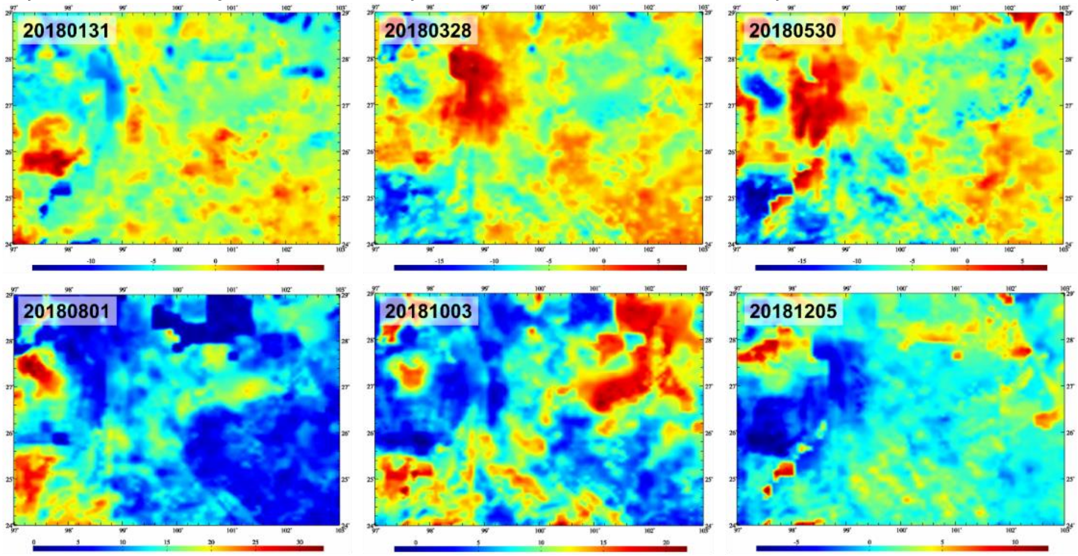
The complete computation process of high-resolution load deformation field all-element grid time series consists of four steps usually, namely global surface load spherical harmonic analysis, spherical harmonic synthesis for load deformation field, regional residual load SRBF approach and SRBF synthesis for residual load deformation field.



Complete computation processes of regional load deformation field all-element grid time series

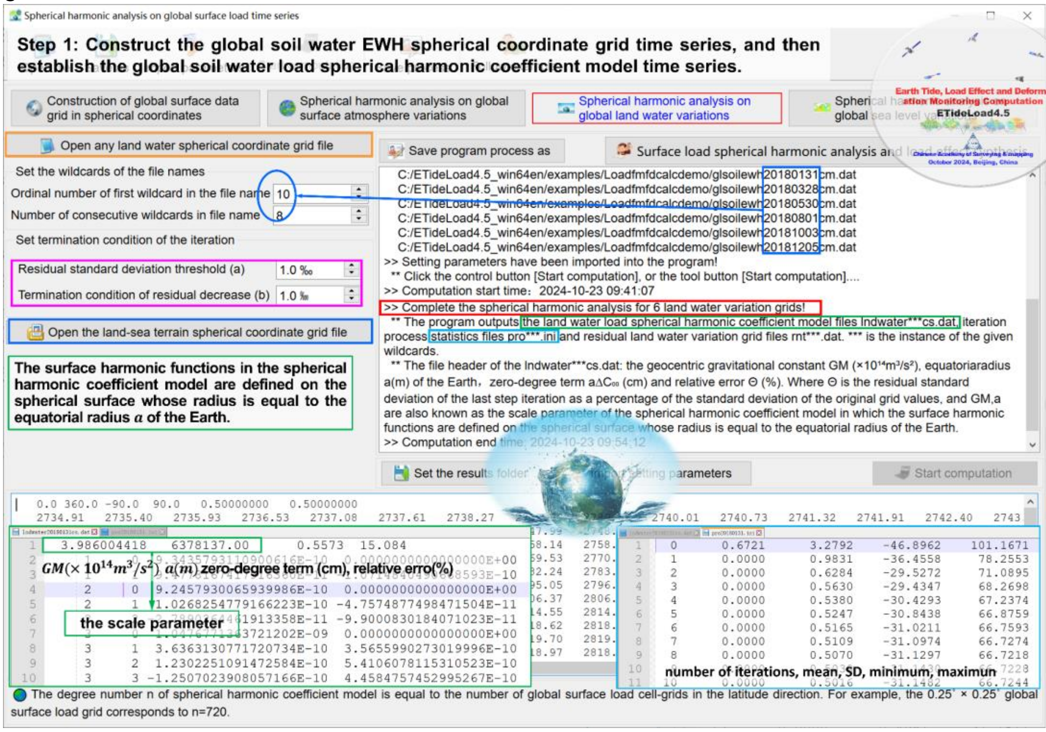
Step 1: Construct the global soil water EWH spherical coordinate grid time series, and then establish the global soil water load spherical harmonic coefficient model time series.

Call the function [Construction of global surface data grid in spherical coordinates], construct the global soil water EWH spherical coordinate grid time series glsoilewh*.dat from global soil water observations, where * is the sample epoch time and * = 20180131 represents January 31, 2018. The process is omitted in this example.



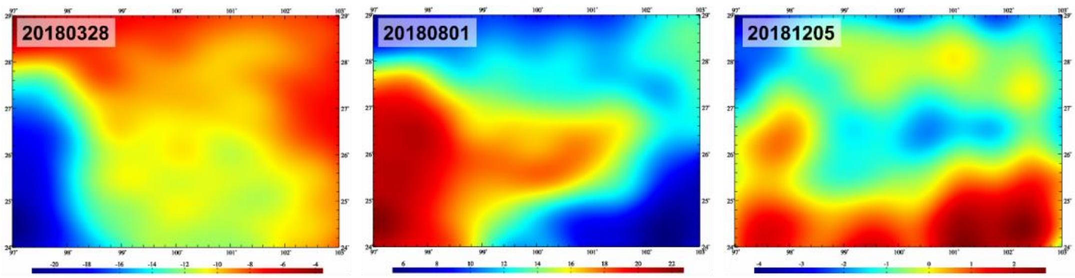
Regional 1′×1′ soil water equivalent water height (EWH, cm) grid time series

Call the function [Spherical harmonic analysis on global land water variations], input global land-sea terrain spherical coordinate grid sphETOPOnc30m.dat (EWH automatically zero in sea area), whose resolution is not less than that of the soil water EWH grid, and establish the global soil water load spherical harmonic coefficient model time series lndwater*cs.dat form the global soil water EWH spherical coordinate grid time series glsoilewh*.dat.



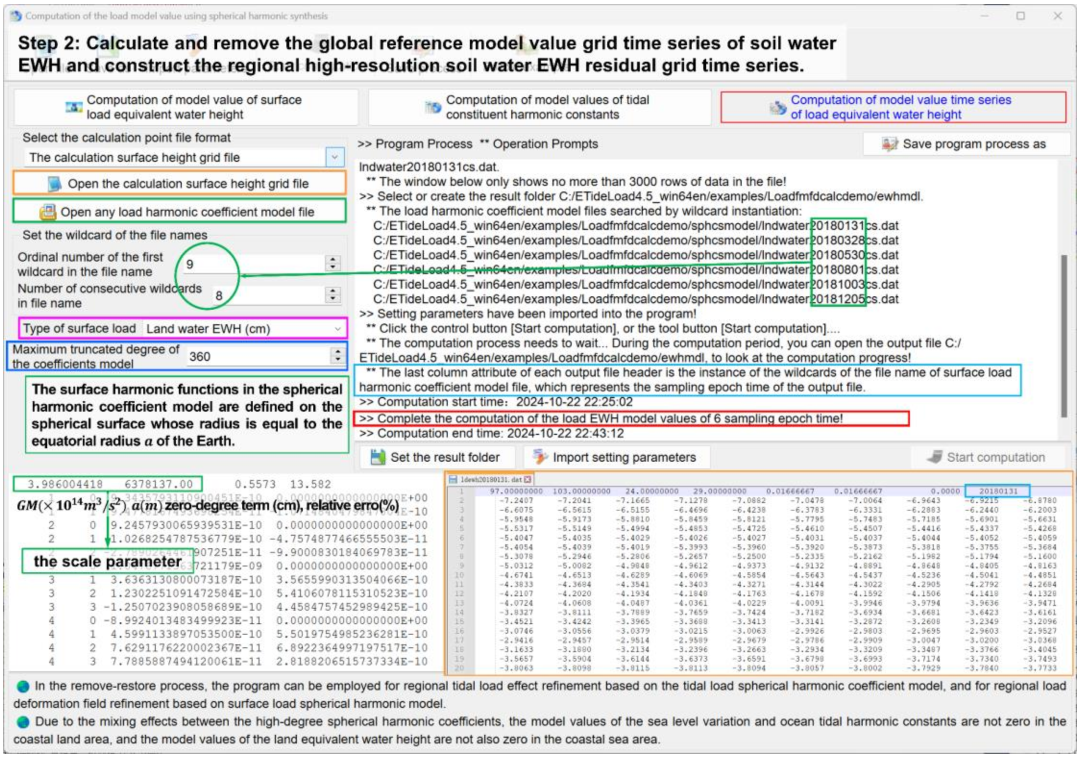
Step 2: Calculate and remove the global reference model value grid time series of soil water EWH and construct the regional high-resolution soil water EWH residual grid time series.

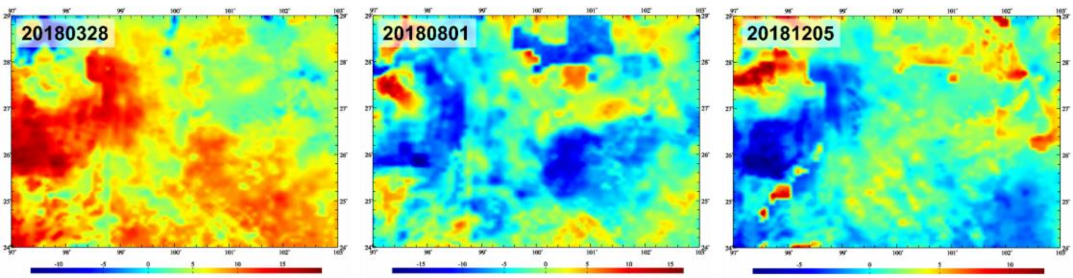
Call the function [Computation of model value time series of load equivalent water height], input the 1'×1' zero-value grid file zero1m.dat, which is employed to give the calculation range and the zero-value represents the calculation surface as the ground, let 'land water EWH (cm)' as the surface load type and the maximum calculation degree 360, calculate the global reference model value grid time series ldewh*.dat of soil water EWH from the global soil water load spherical harmonic coefficient model time series lndwater*cs.dat.



Regional 1′×1′ soil water EWH model reference value (cm) grid time series

Call the function [Weighted operations on two groups of grid time series], subtract the soil water EWH model value grid time series ldewh***.dat from the 1'×1' soil water EWH grid time series soilewh***.dat to generate the regional 1'×1' soil water EWH residual grid time series rntewh***.dat.

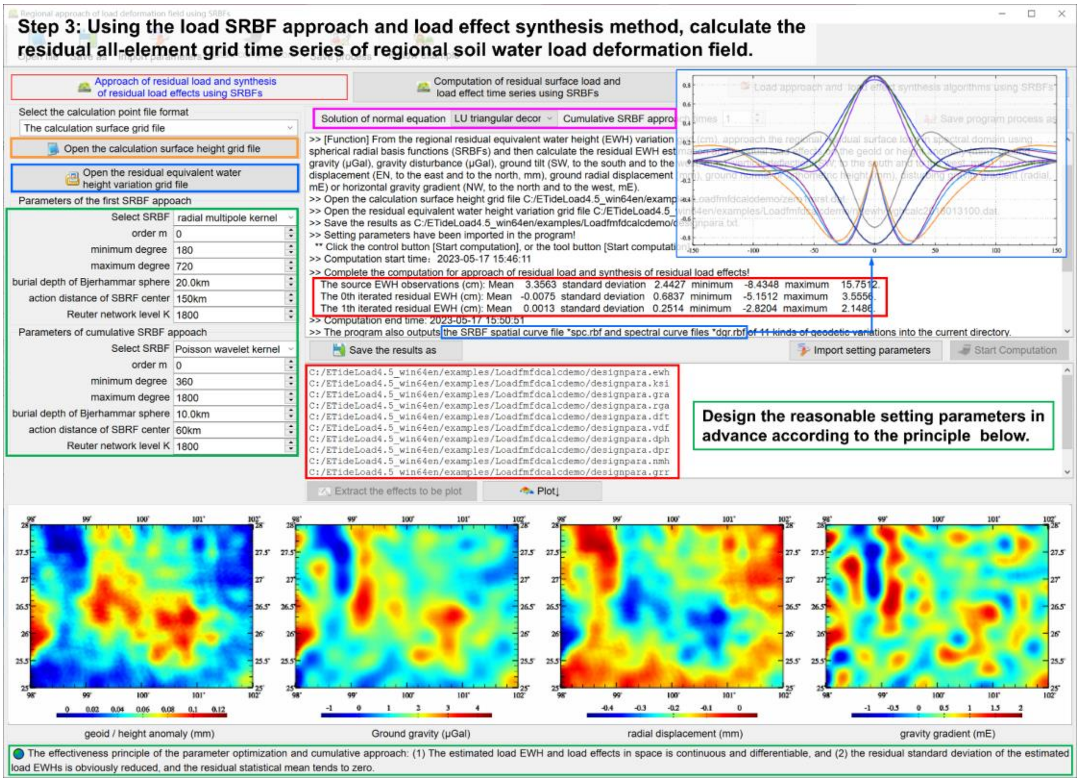


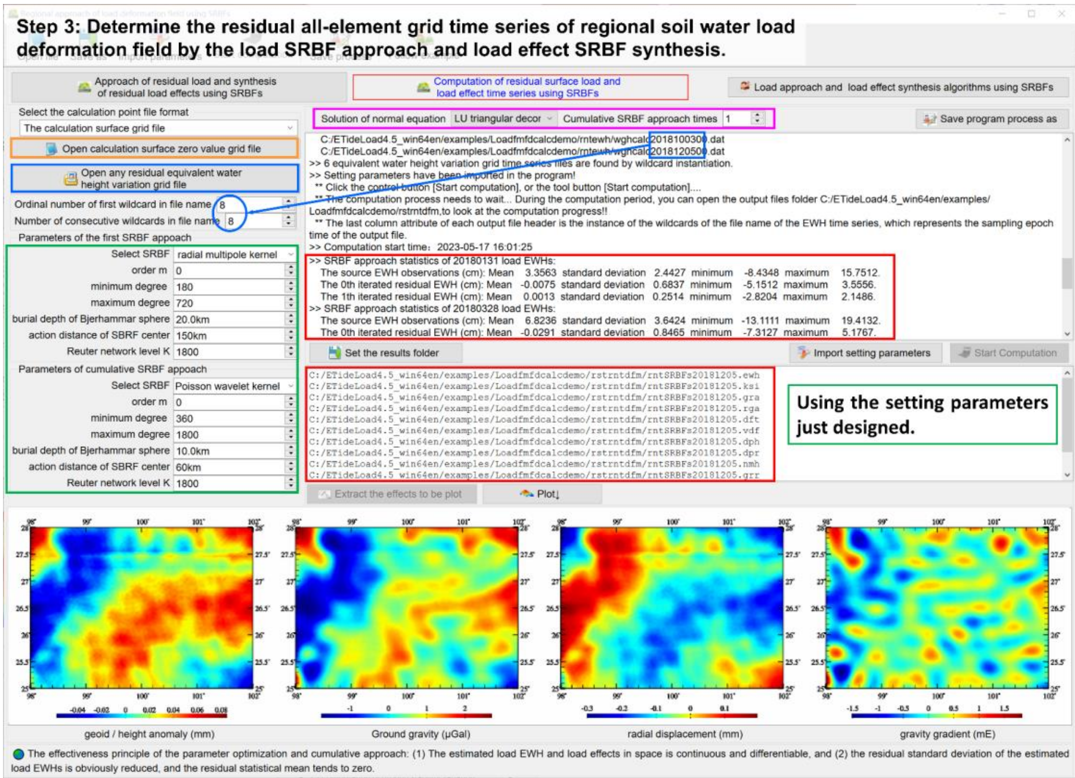


Regional 1′×1′ soil water EWH residual (cm) grid time series

Step 3: Determine the residual all-element grid time series of regional soil water load deformation field by the load SRBF approach and load effect SRBF synthesis.

Call the function [Approach of residual load and synthesis of residual load effects using SRBFs], input the calculation result area 1'×1' zero-value grid file zero1mrst.dat removed the 1° edge area around the grid zero1m.dat, and generate the residual load effect all-element grid ttt.??? from regional 1'×1' soil water EWH residual grid rntewh***.dat at any epoch time to design the reasonable setting parameters according to the principle of parameter setting optimization and cumulative approach effectiveness given below the program interface.



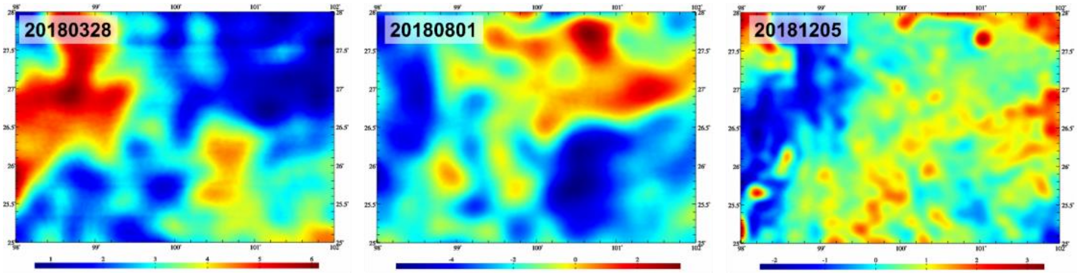


Call the function [Computation of residual surface load and load effect time series using

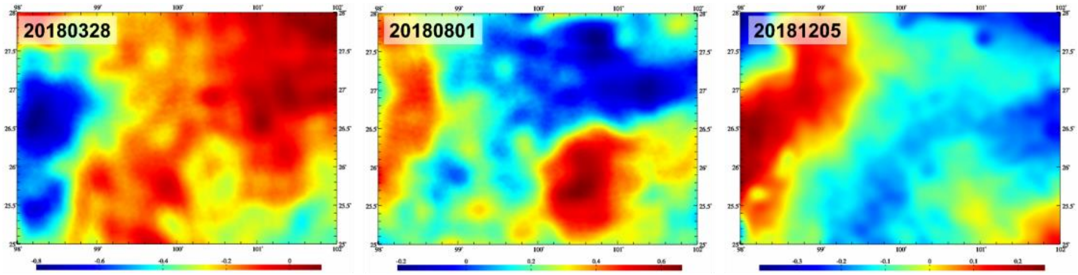
SRBFs], input the 1'×1' zero-value grid file zero1mrst.dat, and generate the residual load effect all-element grid time series rntSRBFs***.??? from regional 1'×1' soil water EWH residual grid time series wghcalc*.dat with the setting parameters above.

Where ??? = ewh, ksi, gra, rga, dft, vdf, dph, dpr, nmh, grr or hgd, respectively, representing the grid file of the residual EWH estimation and residual load effects on the height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, normal or orthometric height, radial gravity gradient or horizontal gravity gradient.

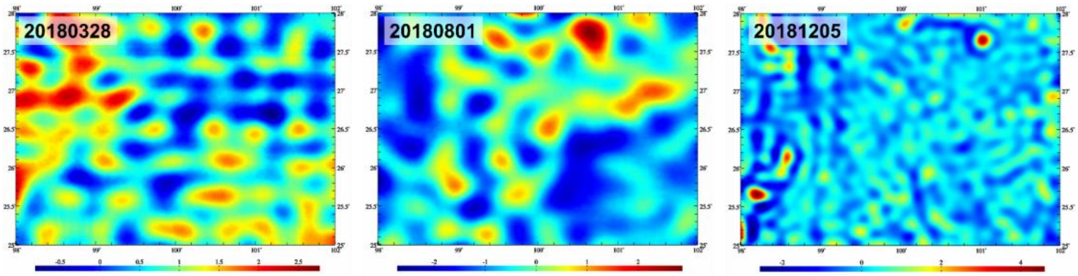
*** are the wildcards of the soil water EWH residual grid time series file names, whose instance can identify the sampling epoch time of the residual load effects.



Regional 1′×1′ soil water residual load effect (μGal) grid time series on ground gravity



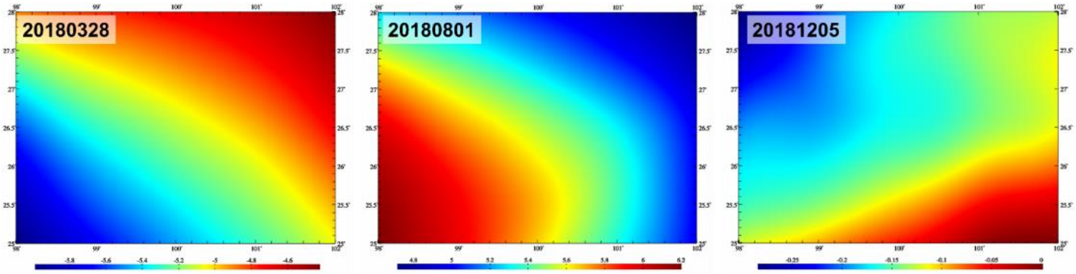
Regional 1′×1′ soil water residual load effect (mm) grid time series on ground ellipsoidal height



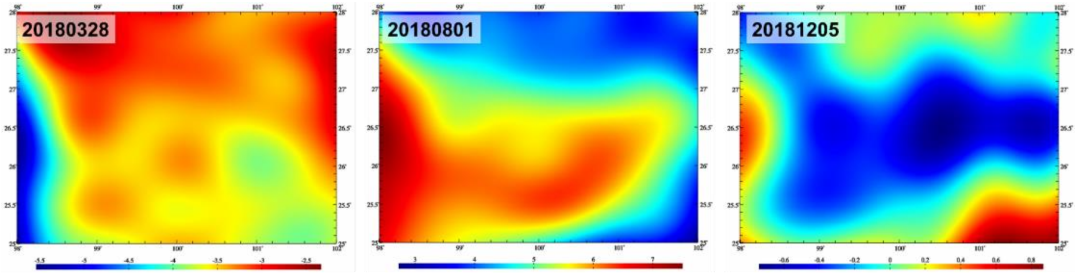
Regional 1′×1′ soil water residual load effect (mE) grid time series on gravity gradient

Step 4: Calculate and restore the soil water load effect model value grid time series and generate the regional high-resolution soil water load effect all-element grid time series.

Call the function [Computation of load effect time series by spherical harmonic synthesis], input the calculation result area 1'×1' zero-value grid file zero1mrst, let 'land water EWH (cm)' as the surface load type and the maximum calculation degree is 360, calculate the global reference model value grid time series loadfmdl***.??? of soil water load effects from the global soil water load spherical harmonic coefficient model time series lndwater*cs.dat.

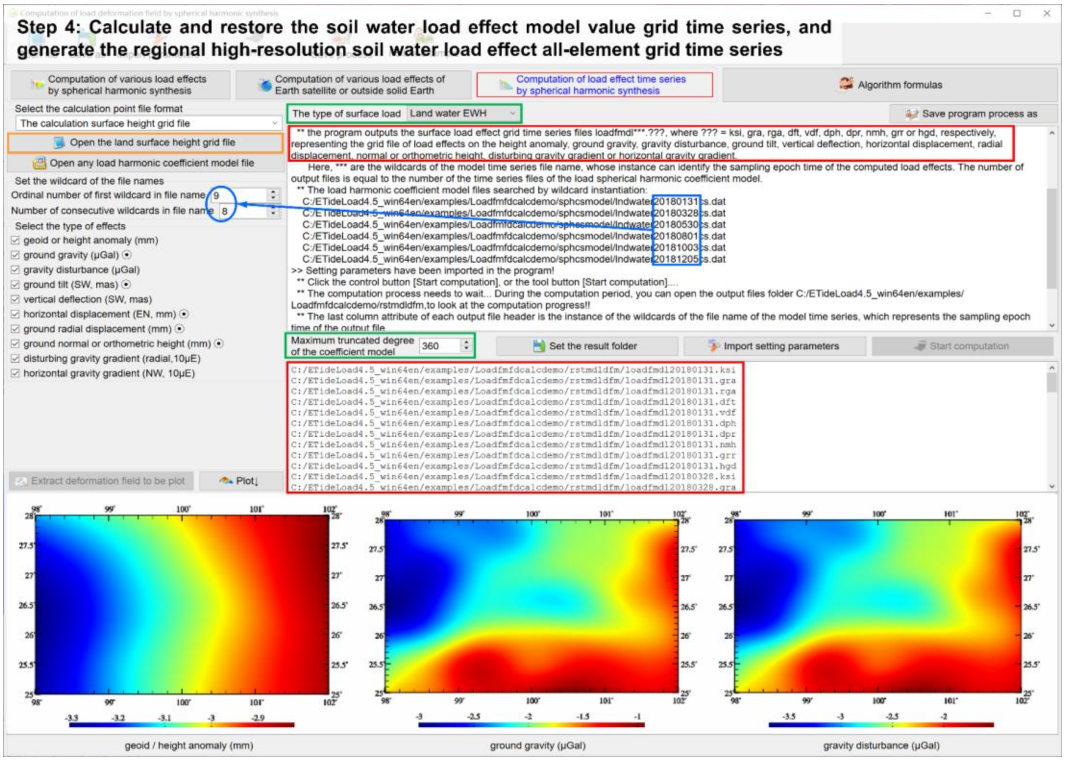


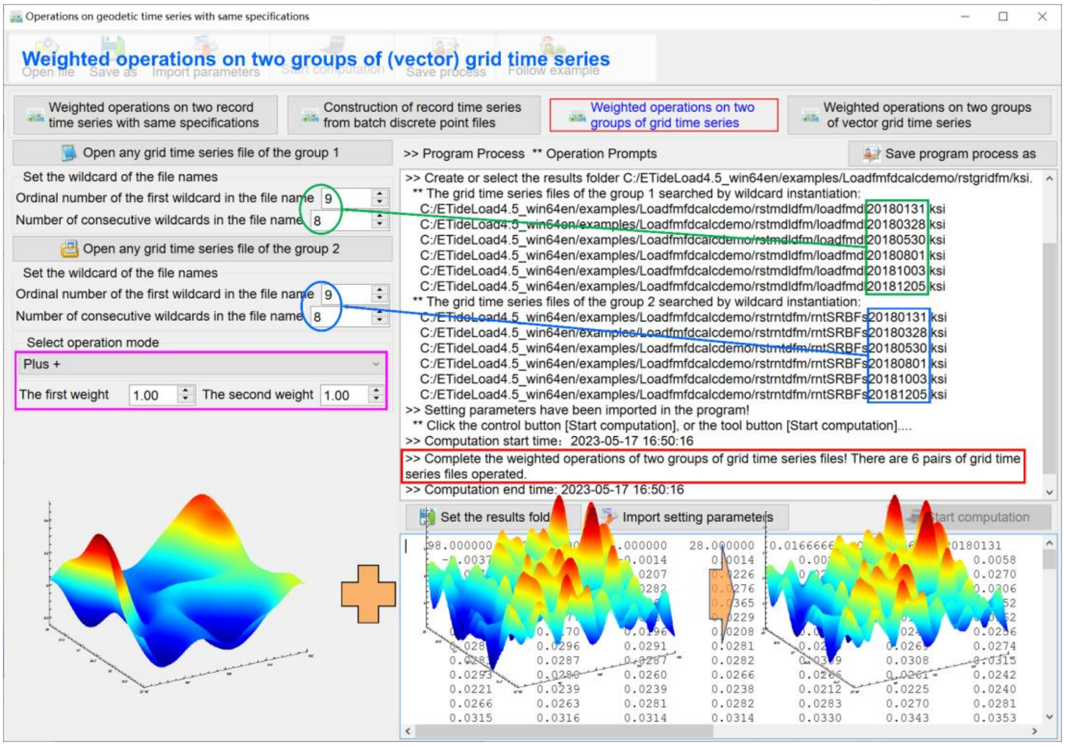
Regional 1′×1′ soil water load effect model value (mm) grid time series on geoid



Regional 1′×1′ soil water load effect model value (μGal) grid time series on ground gravity

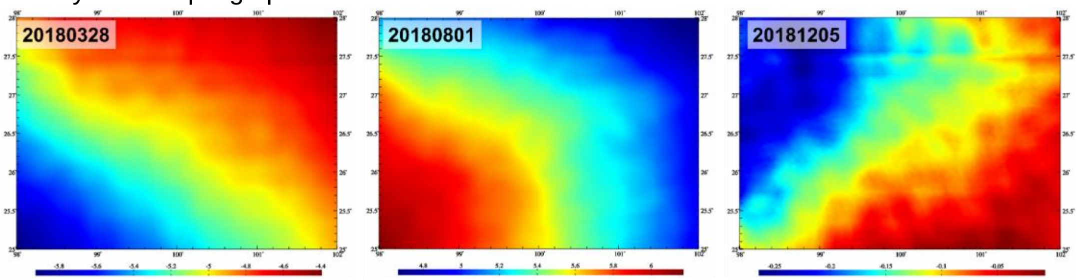
Call the function [Weighted operations on two groups of grid time series], directly add the reference model value grid time series loadfm***.??? to the residual grid time series rntSRBFs***.??? of soil water load effects to generate the regional 1'×1' all-element grid time series soilloadfm***.??? of soil water load effects.



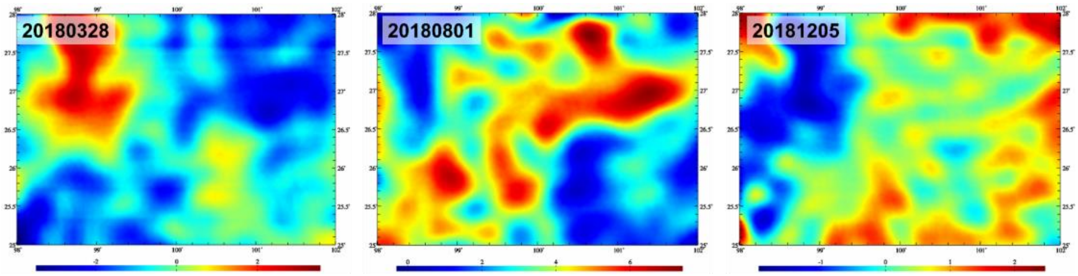


Where ??? = ksi, gra, rga, dft, vdf, dph, dpr, nmh, grr or hgd, respectively, representing the grid file of the soil water load effects on the height anomaly, ground gravity, gravity disturbance, ground tilt, vertical deflection, horizontal displacement, radial displacement, normal or orthometric height, radial gravity gradient or horizontal gravity gradient.

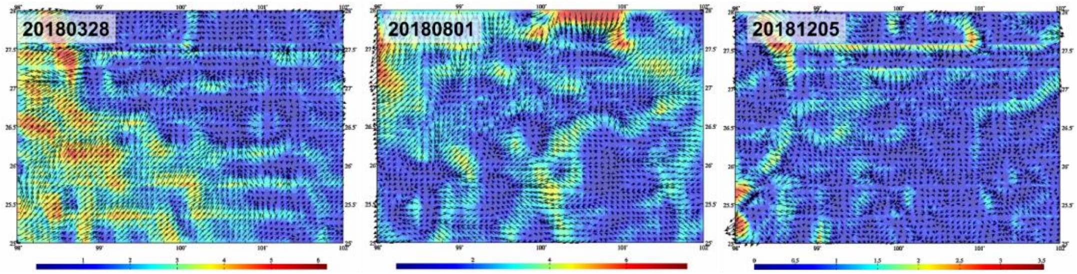
*** are the wildcards of the variation grid time series file names, whose instance can identify the sampling epoch time of the load effects.



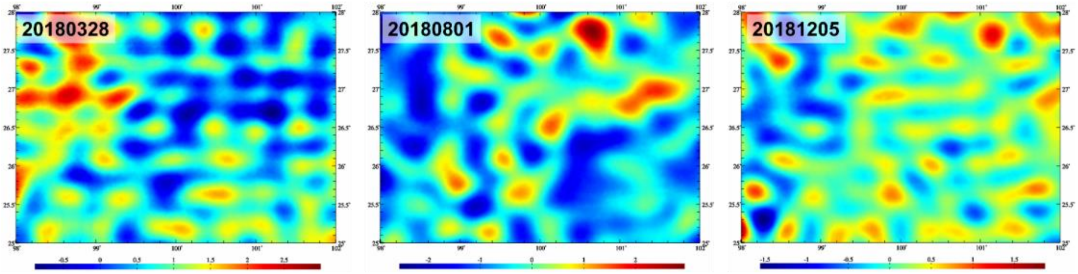
Regional 1′×1′ soil water load effect (mm) grid time series results on geoid



Regional 1′×1′ soil water load effect (μGal) grid time series results on ground gravity



Regional 1′×1′ soil water load effect (mas) vector grid time series results on ground tilt



Regional 1′×1′ soil water load effect (mE) grid time series results on gravity gradient

According to the same processes above, you can compute regional atmosphere or sea level variation load deformation field all-element grid time series.

ETideLoad4.5′s algorithm of the load approach and load effect synthesis using SRBFs can effectively solve the troubles of high-degree oscillation and poor convergence of load Green′s function and the spectrum leakage and singularity of load Green′s function integral in the near area around the calculation point.

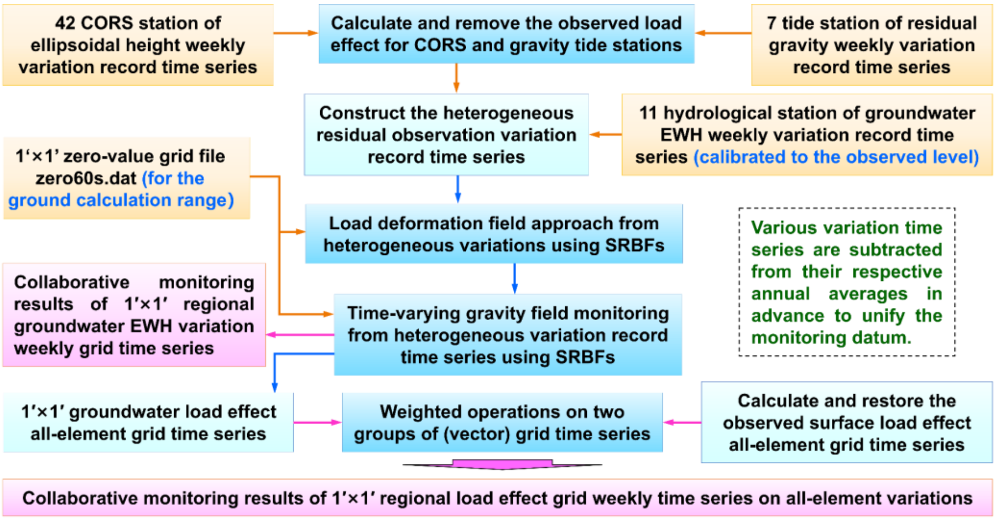
The all-element load deformation field approached can be employed to accurately calibrate the key payloads of geodetic satellite, verify the satellite geodetic monitoring ability and improve effectively the monitoring performance, reliability and accuracy level.

The regional load deformation field approached can be employed for the epoch reduction of various high-precision observations such as GNSS, leveling and gravity, which can support the realization and coordinated maintenance of heterogeneous geodetic datum.

It is the basic and lowest requirement for the deep fusion of multi-source heterogeneous data and collaborative monitoring of various heterogeneous geodetic technologies for the reduction of monitoring epoch and unification of monitoring datum based on surface load deformation field.

4.10.2 Heterogeneous collaborative monitoring process of groundwater variations and load deformation field

From the heterogeneous observation variation record time series from the regional CORS station, gravity tide station and groundwater monitoring station, compute the groundwater variations and regional load deformation field all-element grid time series in the five steps to realize the heterogeneous collaborative monitoring to regional groundwater and time-vary gravity field by the remove-restore scheme based on the observed surface loads.



Heterogeneous collaborative monitoring process of groundwater variations and load deformation field

The target monitoring area: 98°~101°E, 24°~26.5°N. Observation point distribution : 97.5°~101.5°E, 23.5°~27.0°N. Monitoring time interval: one week. Starting and ending time: March 2019 to August 2019. Spatial resolution: 1′×1′.

Step 1: Data preparation and preprocessing of various geodetic and surface load observations.

(1) CORS network data processing and calculation of the ellipsoidal height variation weekly time series at 42 CORS stations, and calculation of gravity variation weekly time series at 7 gravity tide stations.

(2) Various geodetic and surface load observation variation time series are subtracted from their respective annual averages in advance to unify variation monitoring datum.

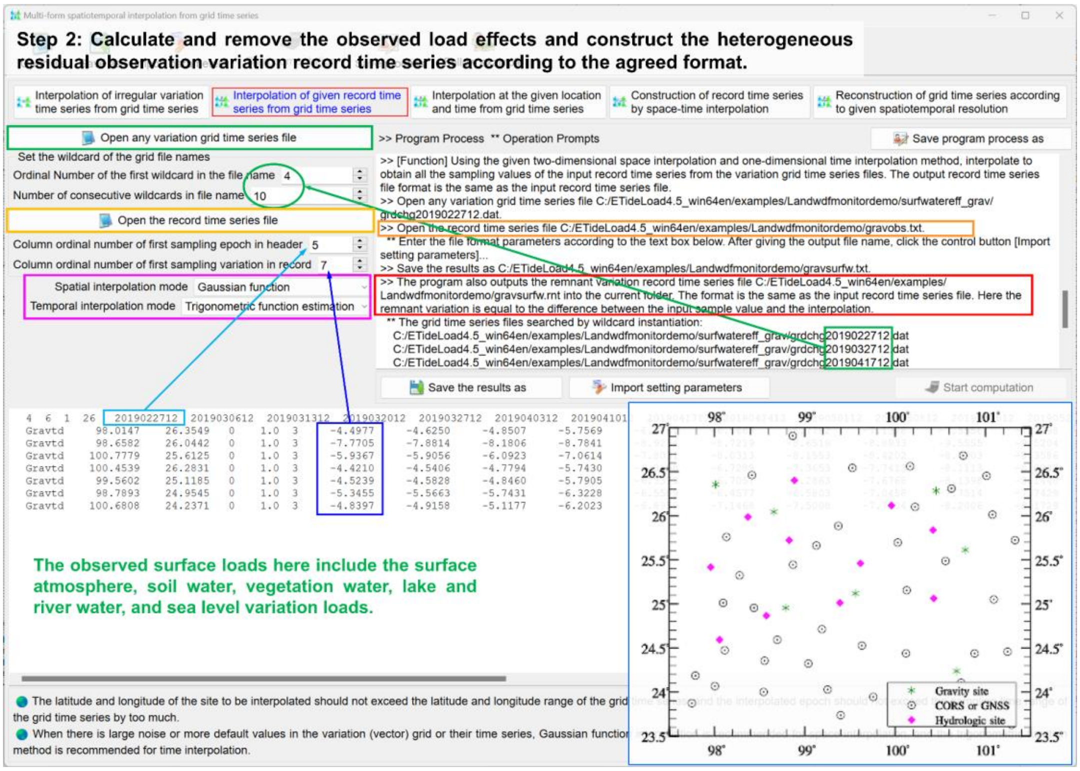
(3) Calculation of groundwater equivalent water height (EWH) variation weekly time series at 11 hydrological monitoring stations.

According to the process of this section, calculate the regional groundwater EWH variation grid weekly time series in advance only from CORS and gravity tide monitoring data, whose monitoring time span was not less than two years. Then, interpolate the groundwater EWH at the hydrological monitoring station from thecalculated groundwater EWH grid to calibrate the hydrogeological parameters and then transform the variation of observed groundwater level (head) into the variation of groundwater EWH at each hydrological monitoring station.

The process in step 1 is omitted in this example.

Step 2: Calculate and remove the observed load effect and construct the heterogeneous residual observation variation record time series according to ETideLoad4.5 agreed format.

According to the computation process of section 4.10.1, calculate the observed load effect variation grid weekly time series on ground ellipsoidal height and gravity.



Call the function [Interpolation of given record time series from grid time series], remove the observed load effects from the gravity variation weekly time series at gravity tide stations to generate the residual gravity variation weekly time series, and remove the observed load effects from the ellipsoidal height variation weekly time series at CORS stations to generate the residual ellipsoidal height variation weekly time series.

The observed surface loads in this example include surface atmosphere, soil water, vegetation water, lake and river water and sea level variation loads.

According to the agreed format in ETideLoad4.5, merge the 11 hydrological stations of groundwater EWH variation, 7 tide stations of residual gravity variation and 42 CORS of residual ellipsoidal height variation record time series to generate the heterogeneous residual observation variation record time series file.

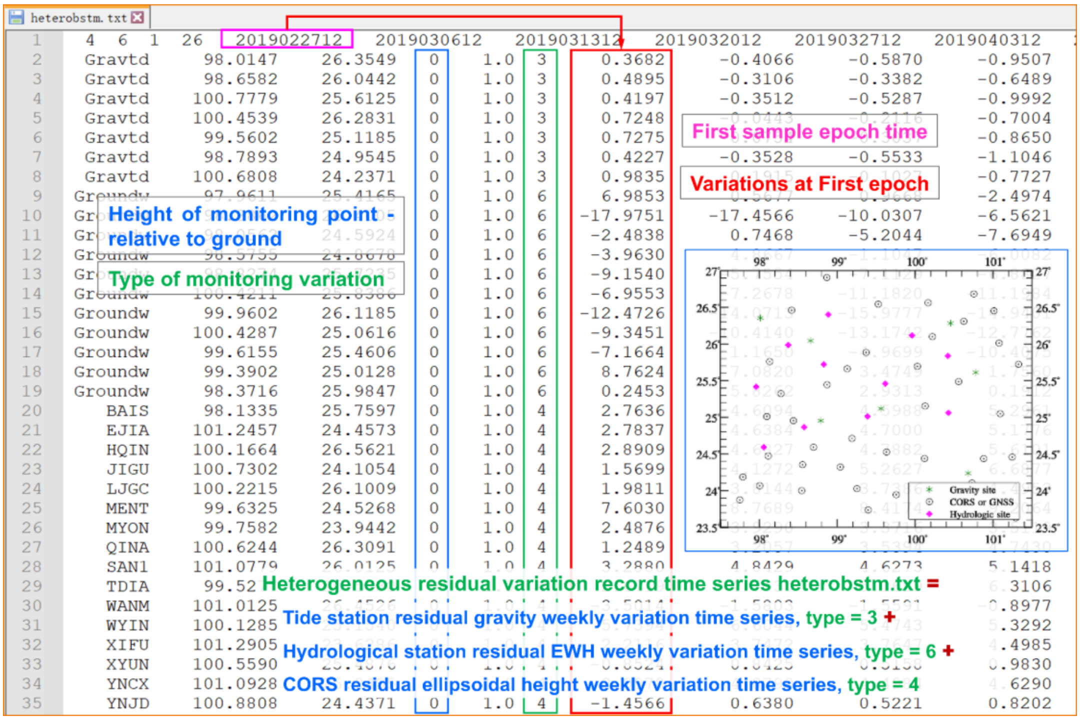
The file header contains the time series length and sampling epoch time arranged with time. Record format: ID (the site name / no), longitude, latitude, …, weight, variation type, …, variations arranged in time series length (default value is 9999.0000).

Variation type = 3 represents residual gravity variation (μGal), = 4 represents residual ellipsoidal height variation (mm), = 6 represents groundwater EWH variation (cm).

Step 3: Call the function [Load deformation field approach from heterogeneous variations using spherical radial basis functions] to design the reasonable setting parameters for time series SRBF approach.

Call the function [Load deformation field approach from heterogeneous variations using spherical radial basis functions], input the 1'×1' zero-value grid file zero60s.dat, which is employed to give the calculation range and the zero value represents the calculation surface as the ground, and estimate the residual EWH variation and 10 kinds of load effect grids from the heterogeneous residual observation variation record time series file at any epoch time to design the reasonable setting parameters according to the principle of parameter setting optimization and cumulative approach effectiveness given below the program interface.

Construct the heterogeneous residual observation variation record time series.



Step 4: Estimate the residual EWH and 10 kinds of residual load effect variation grid weekly time series.

Call the function [Time-varying gravity field monitoring from heterogeneous variation time series using SRBFs], input the 1'×1' zero-value grid file zero60s.dat, and estimate the residual EWH variation grid weekly time series ewh***.dat and the following 10 kinds of residual load effect variation grid weekly time series files from the heterogeneous residual observation variation record time series files with the setting parameters above, while output residual variation time series files rnt***.txt.

①SRBFgeoid***.dat is the residual load effect grid file on geoid or height anomaly (mm),

②SRBFterrgrav***.dat is the residual load effect grid file on ground gravity (μGal),

③SRBFgravdist***.dat is the residual load effect grid file on gravity disturbance (μGal),

④SRBFgrndtilt***.dat is the residual load effect vector grid file on ground tilt (SW, to the south and to the west, mas), ⑤SRBFvertdefl***.dat is the residual load effect vector grid file on vertical deflection (SW, to the south and to the west, mas),

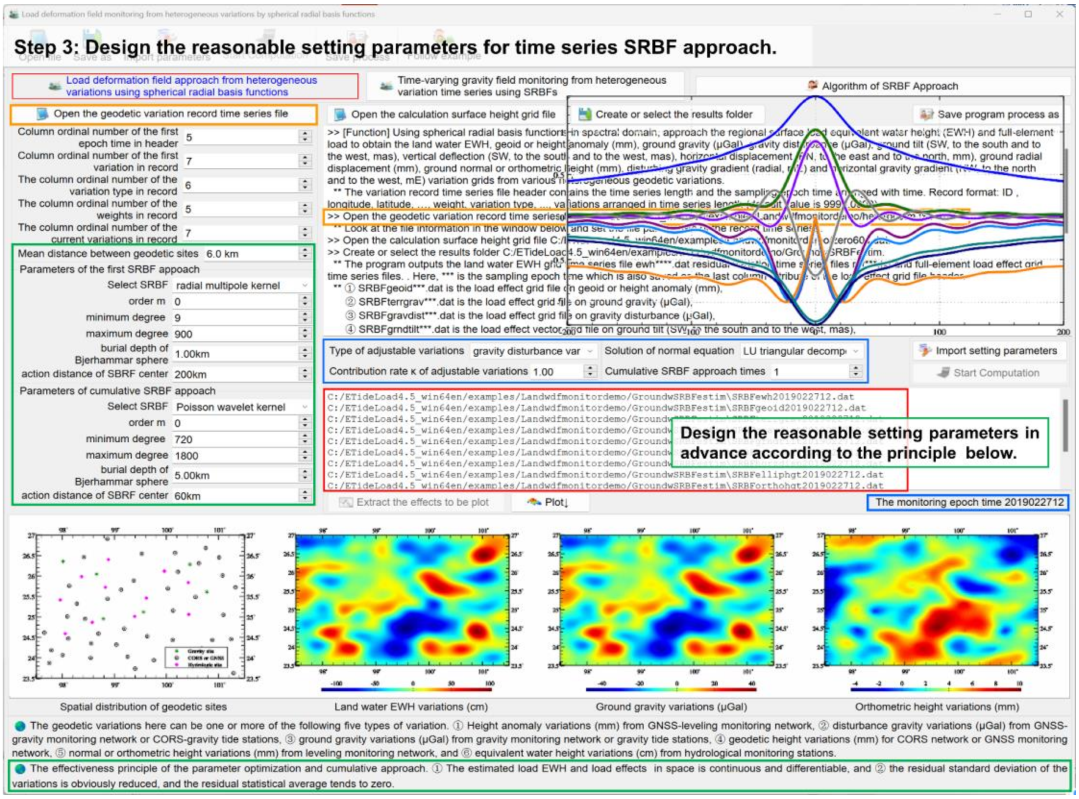
⑥SRBFhorzdisp***.dat is the residual load effect vector grid file on horizontal displacement (EN, to the east and to the north, mm),

⑦SRBFelliphgt***.dat is the residual load effect grid file on ground radial displacement (mm),

⑧SRBForthohgt***.dat is the residual load effect grid file on ground normal or orthometric height (mm),

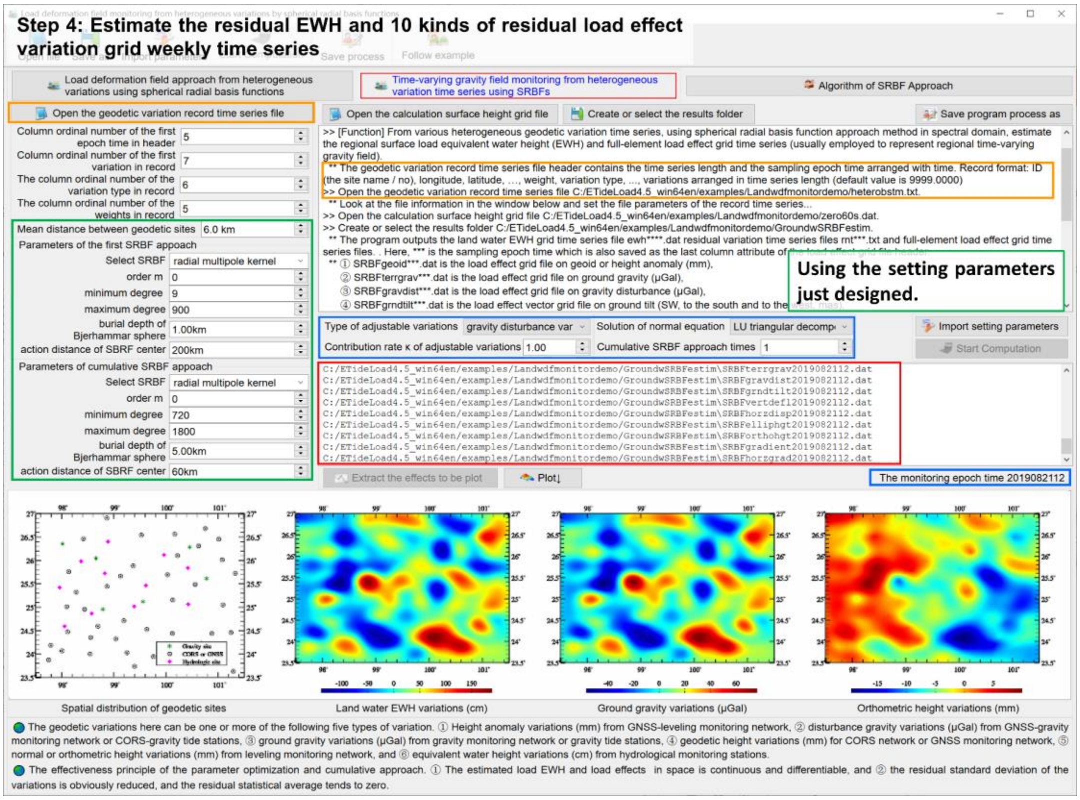
⑨SRBFgradient***.dat is the residual load effect grid file on radial gravity gradient (rmE) and ⑩SRBFhorzgrad***.dat is the residual load effect vector grid file on horizontal gravity gradient (NW, to the north and to the west, mE).

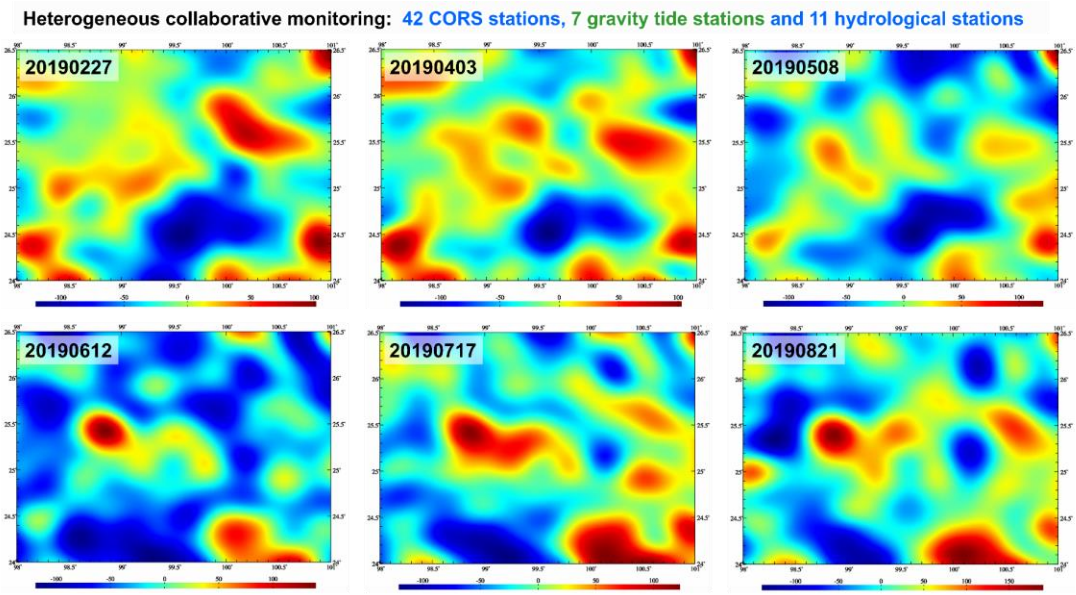
Where *** is the sampling epoch time from the heterogeneous variation record time series file header, which is also saved as the last column attribute of the load effect grid file header.



The residual EWH variation here does not contain surface observed load variation, which can be considered as groundwater EWH variation. Therefore ewh***.dat are the heterogeneous collaborative monitoring results of the groundwater EWH variations, while the residual load effect variation can be considered as the groundwater load effect variation.

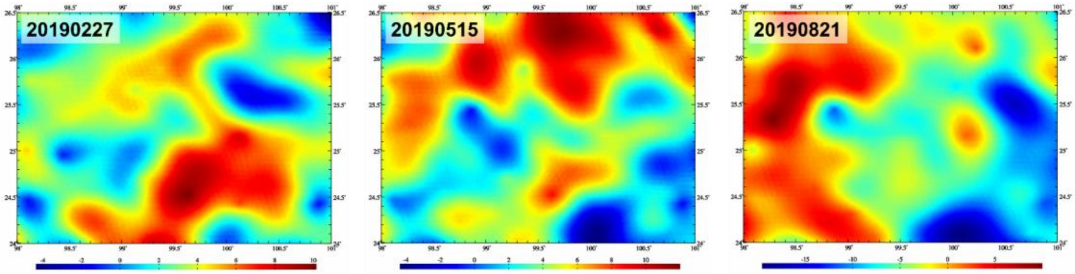
Step 5: Calculate and restore the observed surface load effect all-element grid time series to generate the heterogeneous collaborative monitoring results of land water load deformation field all-element grid time series.



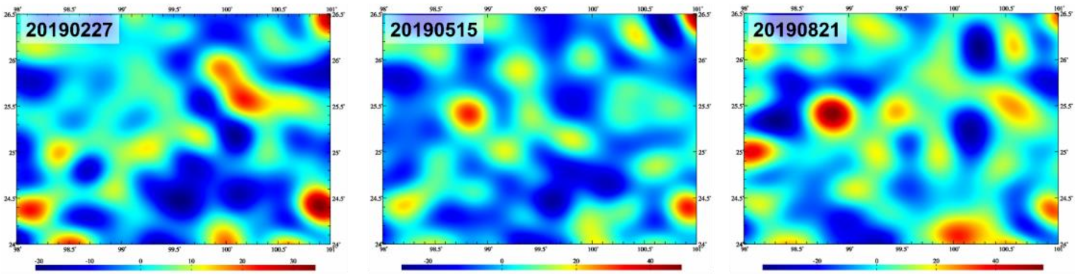


Collaborative monitoring results of 1′×1′ regional groundwater EWH variation grid weekly time series

According to the computation process of section 4.10.1, calculate the all-element variation grid weekly time series of the observed surface load effects. The process is omitted in this example.

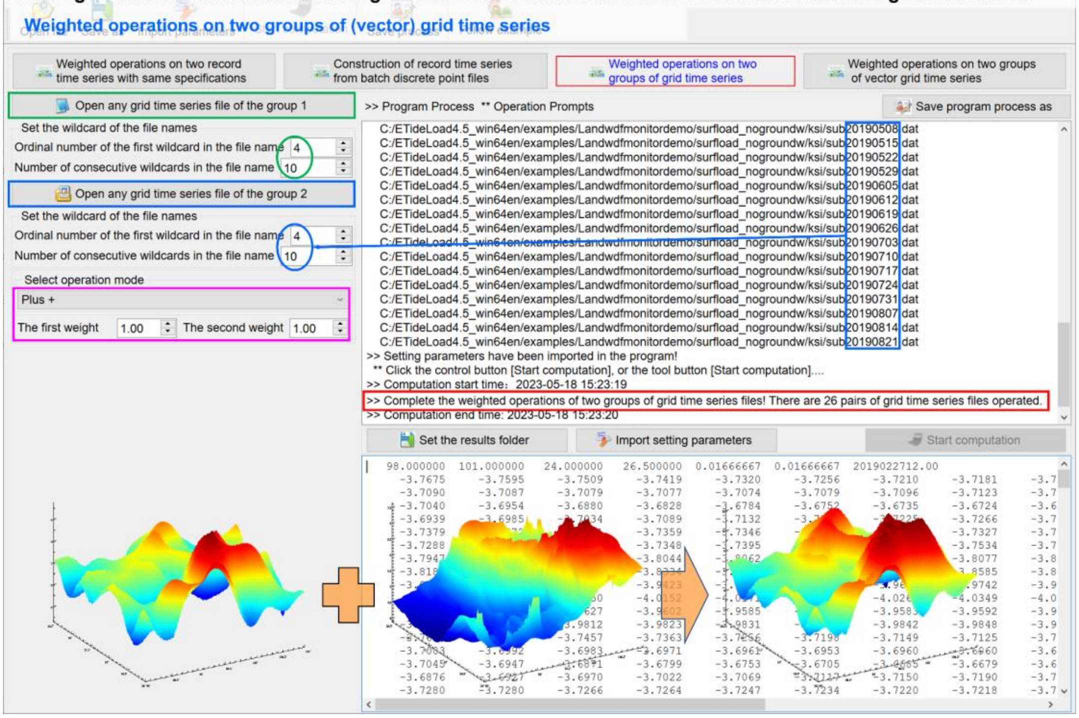


1′×1′ regional groundwater load effect variation (mm) grid weekly time series on ground orthometric height

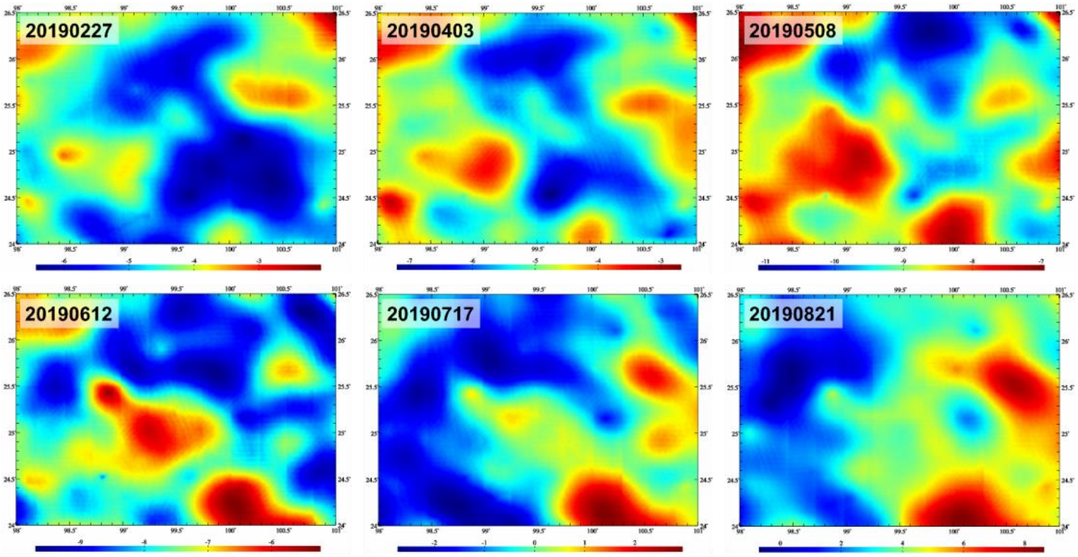


1′×1′ regional groundwater load effect variation (mE) grid weekly time series on gravity gradient

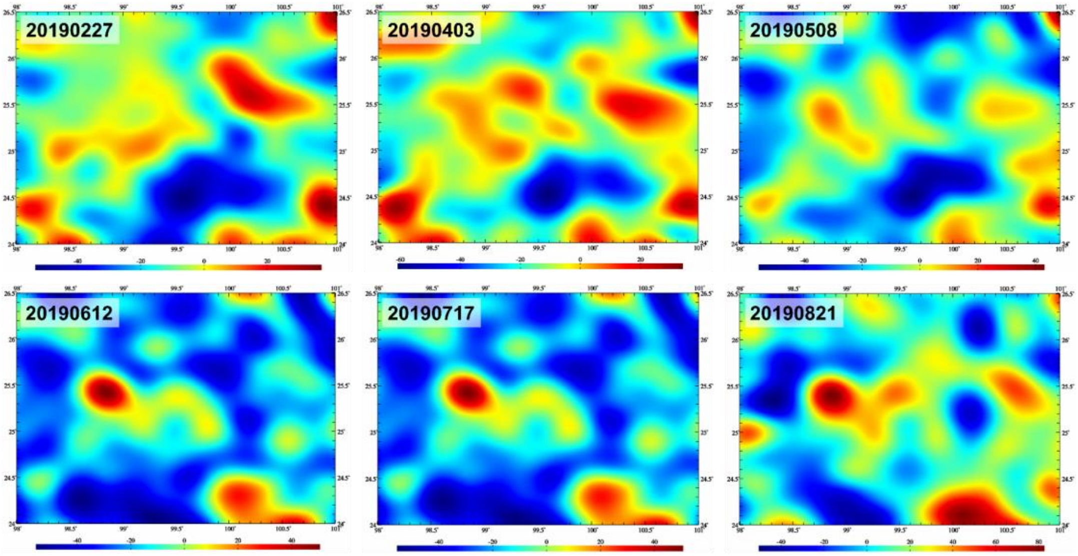
Step 5: Calculate and restore the observed surface load effect all-element grid time series to generate the heterogeneous collaborative monitoring results of land water load deformation field all-element grid time series



Call the function [Weighted operations on two groups of grid time series], directly add the all-element variation grid weekly time series of the observed surface load effects to the all-element variation grid weekly time series of the residual (groundwater) load effects, respectively, to generate the regional 1'×1' all-element variation grid weekly time series of land water load effects, which are the heterogeneous collaborative monitoring results of land water all-element load deformation field.



Collaborative monitoring results of 1′×1′ surface load effect variation (mm) grid weekly time series on geoid



Collaborative monitoring results of 1′×1′ load effect variation (μGal) grid weekly time series on ground gravity

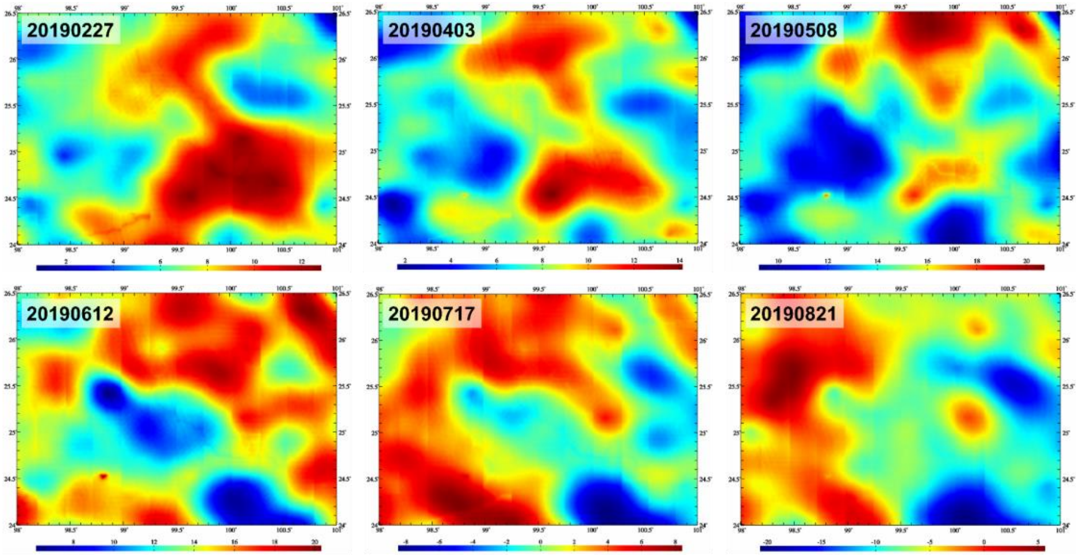
The main technical features of ETideLoad4.5's algorithm for the heterogeneous collaborative monitoring of the surface load and time-vary gravity field are in following.

(1) The algorithm can effectively solve the troubles of high-degree oscillation and poor convergence of load Green′s function and spectrum leakage and singularity of load Green′s integral in the near area around the calculation point, and then realize the collaborative monitoring of GNSS, gravity, leveling, ground tilt and groundwater strictly according to solid geophysical analytical constraints.

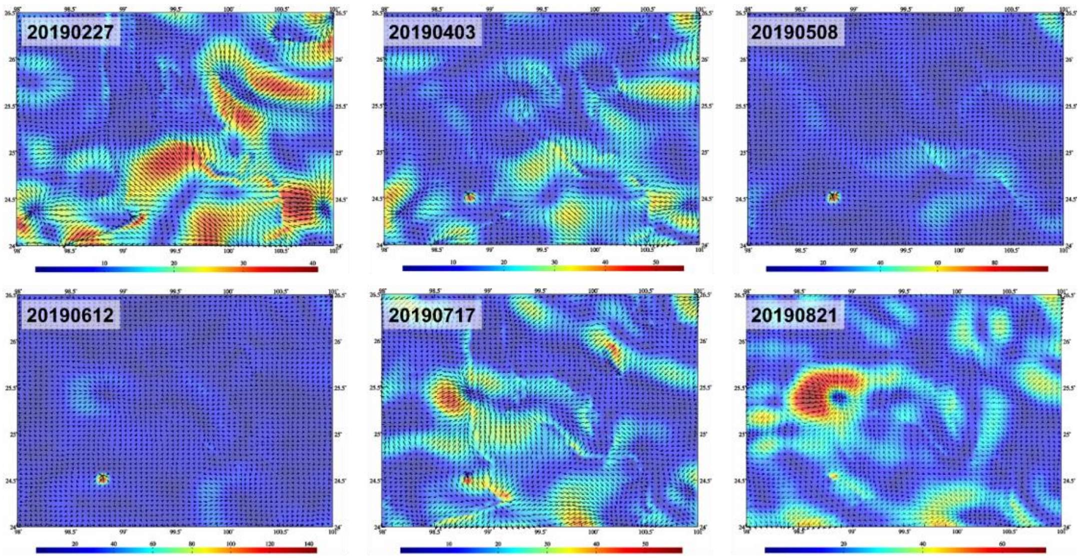
(2) There are rigorous analytical relationships between observation equations, and

various heterogeneous geodetic monitoring systems can be deeply collaborated by normalization of their normal equations to avoid the collaborative monitoring algorithms affected by the observation errors. The algorithm has high stability and universality, which is suitable for massive computation of multiple geodetic collaborative monitoring.

(3) The algorithm has the functions of the spatial and spectrum domain separation of geophysical signals and measurement equipment parameter calibration, which can improve the medium and long-term monitoring ability for the gravity tide station and make the EWH monitoring ability for groundwater monitoring station, so as to enhance the level of collaborative monitoring from space, terrestrial and marine geodesy.



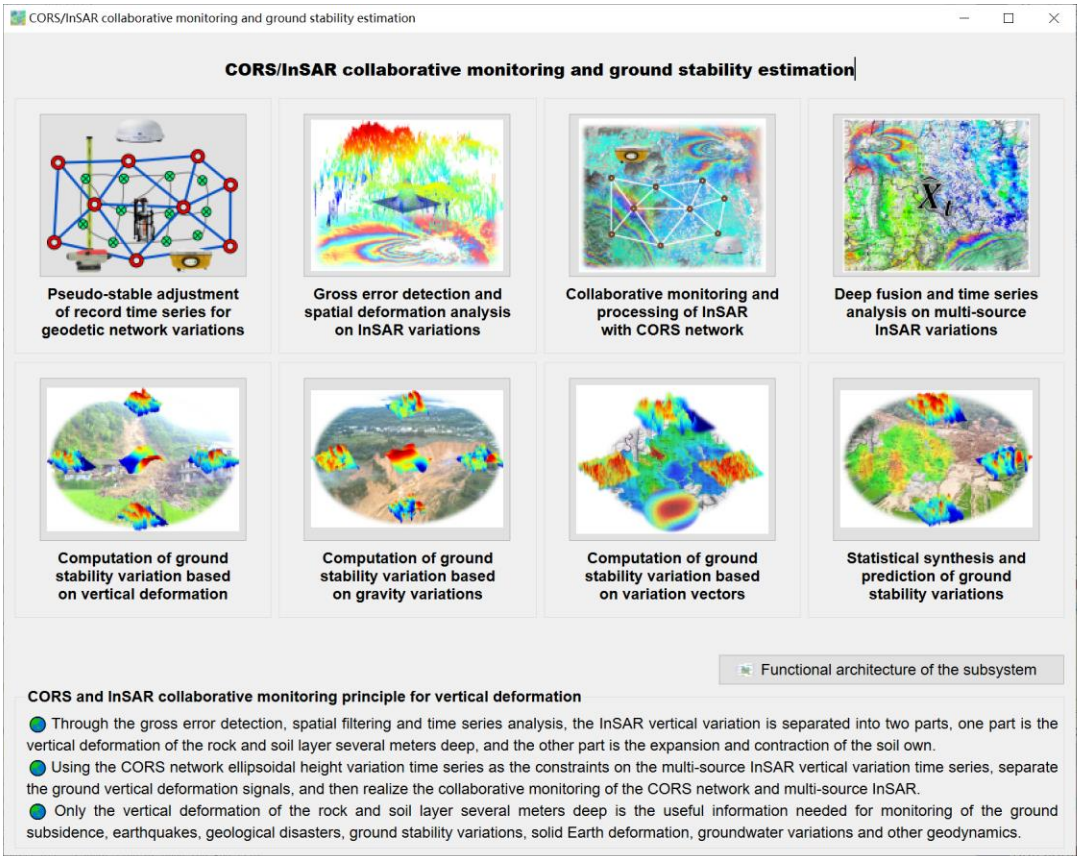
Collaborative monitoring results of 1′×1′ load effect variation (mm) grid weekly time series on ground ellipsoidal height



Collaborative monitoring results of 1′×1′ load effect variation (mas) vector grid weekly time series on ground tilt

5 CORS/InSAR collaborative monitoring and ground stability estimation

The group of programs can be employed to construct an accurate geometric and physical spatiotemporal monitoring frame with regional unification, long-term stability and high robustness, and then fuse the monitoring variations of the CORS network and multi-source InSAR. From the variation grid time series of various geodetic deformation field, quantitatively and continuously monitors the regional ground stability variations using ETideLoad own defined quantitative criteria for the ground stability weakening.



CORS and InSAR collaborative monitoring principle for vertical deformation:

(1) Through the gross error detection, spatial filtering and time series analysis, the InSAR vertical variation is separated into two parts, one part is the vertical deformation of the rock and soil layer several meters deep, and the other part is the expansion and contraction of the soil own. Only the former is compatible with most geodetic variations, while the latter is mainly affected by the temperature and rainfall and should not be regarded as a solid Earth deformation.

(2) Using the CORS network ellipsoidal height variation time series as the constraints on the multi-source InSAR vertical variation time series, separate the ground vertical deformation signals, and then realize the collaborative monitoring of the CORS network and multi-source InSAR.

(3) Only the vertical deformation of the rock and soil layer several meters deep is the useful information needed for monitoring of the ground subsidence, earthquakes, geological disasters, ground stability variations, solid Earth deformation, groundwater variations and other geodynamics.

Continuous quantitative monitoring scheme of ground stability variations:

(1) From the grid time series of the geodetic vertical deformation, ground gravity and tilt variations, quantitatively and continuously monitor ground stability variations by constructing the quantitative criteria for the ground stability weakening.

(2) Quantitative criteria of the ground stability weakening can include that the ground ellipsoidal height increases, the gravity decreases, the horizontal gradient of the height or gravity variation is large and that the inner product of the tilt variations and terrain slope vector is greater than zero.

(3) According to the objective nature of ground stability weakening at the place and time of geological disaster, optimize and synthesize the ground stability variation grid time series to adapt to the local environmental geology, and then consolidate the monitoring capabilities of regional stability variations.

5.1 Pseudo-stable adjustment of record time series for geodetic network variations

[Function] Using the variation time series of the GNSS baseline components, height differences of the leveling route or gravity differences of the gravity control network as the observations, and a given group of sites as the pseudo-stable reference datum, estimate the variation time series of the coordinate component of the CORS network sites, height of the leveling network sites or gravity of the gravity network sites by the indirect least squares adjustment method.

The program can be employed to construct an accurate geometric and physical spatiotemporal monitoring frame with regional unification, long-term stability and high robustness.

The program requires that all the variations are strictly synchronized at each sampling epoch time, and the reference epochs of all the variation record time series should be same and unique.

[Input files] The observed variation record time series file of the geodetic network. The reference variation record time series file of the reference sites.

(1) The observed variation record time series file for the geodetic network (consists of the baselines or routes). The file header includes the number of characters of the baseline or route name, number of characters of the site name, sampling length, …, and all the sampling epochs arranged with time.

The record includes the baseline or route name, starting site (longitude, latitude, height), ending site (longitude, latitude, height), …, and all the observed variations arranged with sampling time (default value is 9999).

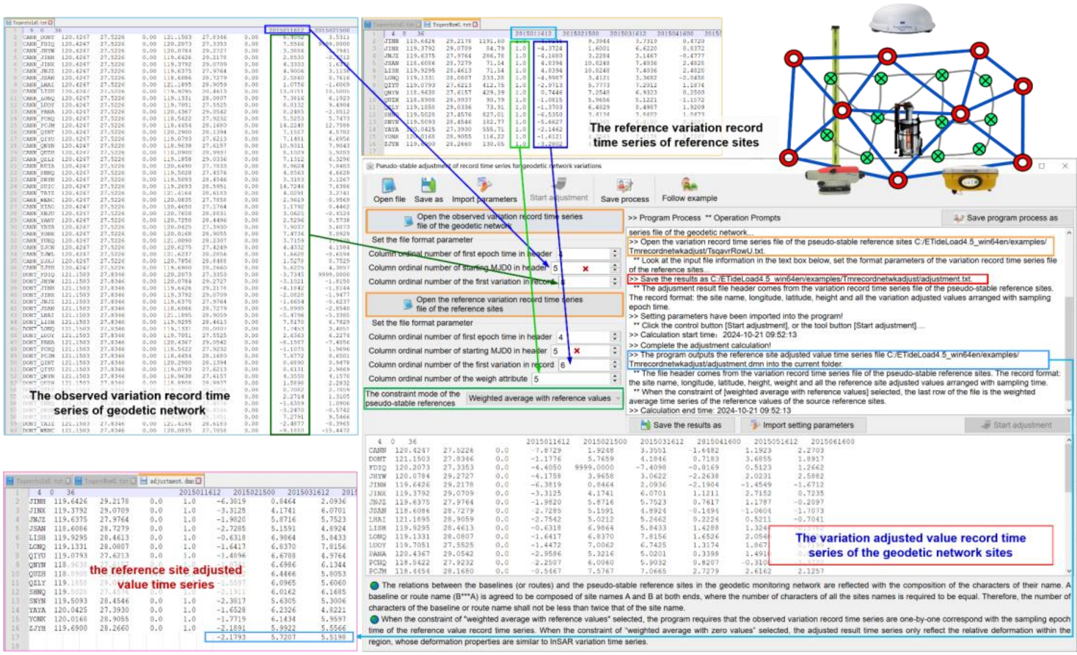
(2) The variation record time series file of the pseudo-stable reference sites. The file header contains all the sampling epochs arranged with time. The record format: the site name, longitude, latitude, height, …, and all the variations arranged with sampling time (default value is 9999).

The relations between the baselines (or routes) and pseudo-stable reference sites in the geodetic monitoring network are reflected with the composition of the characters of their name. A baseline or route name (B***A) is agreed to be composed of site names A and B at both ends, where the number of characters of all the sites names is required to be equal. Therefore, the number of characters of the baseline or route name should not be less than twice that of the site name.

[Parameter settings] Set the record time series file format parameters for the observed variations of the geodetic network and reference variations of the pseudo-stable reference sites, and set the constraint mode of the pseudo-stable references.

When the constraint of "weighted average with reference values" selected, the program requires that the observed variation record time series are one-by-one correspond with the sampling epoch time of the reference value record time series.

When the constraint of “weighted average with zero values” selected, the adjusted result time series only reflect the relative deformation within the region, whose deformation properties are similar to InSAR variation time series.



[Output files] The variation adjusted value record time series file of geodetic sites.

The file header comes from the reference variation record time series file of the reference sites. The record format: the site name, longitude, latitude, height and all the variation adjusted values arranged with sampling epoch time.

When the constraint of "weighted average with reference values" selected, the program outputs the reference site adjusted value time series file ***.dmn into the current directory.

The file header comes from the variation record time series file of the pseudo-stable reference sites. The record format: the site name, longitude, latitude, height, weight and all the reference site adjusted values arranged with sampling time. The last row of the file is the weighted average time series of the reference values of the source reference sites. Here, *** are the output file name of the variation adjusted value record time series.

5.2 Gross error detection and spatial deformation analysis on InSAR variations

[Purpose] Construct InSAR variation spatial analysis algorithms according to the spatial distribution natures of the ground deformation under the action of the environmental geology and load geodynamics, separate the outliers and gross errors from InSAR variations, suppress and weaken the impact of the soil own variations, and then extract the InSAR ground vertical deformation which is compatible with the other geodetic variations.

5.2.1 Gross error detection and separation on InSAR variation record time series

[Function] According to the spatial high-correlation characteristics of the ground deformation, construct a reference surface respectively at each sampling epoch time with the given low-pass filter to separate the outliers, gross error and abrupt signals from the input InSAR variation record time series.

Before and after gross error separation, the format of InSAR variation record time series, spatial and temporal distribution of monitoring points, number of monitoring points and the value of InSAR variation remain unchanged, and only the gross error variation in the result InSAR variation record time series are replaced by 9999.000.

[Input file] The InSAR variation record time series file.

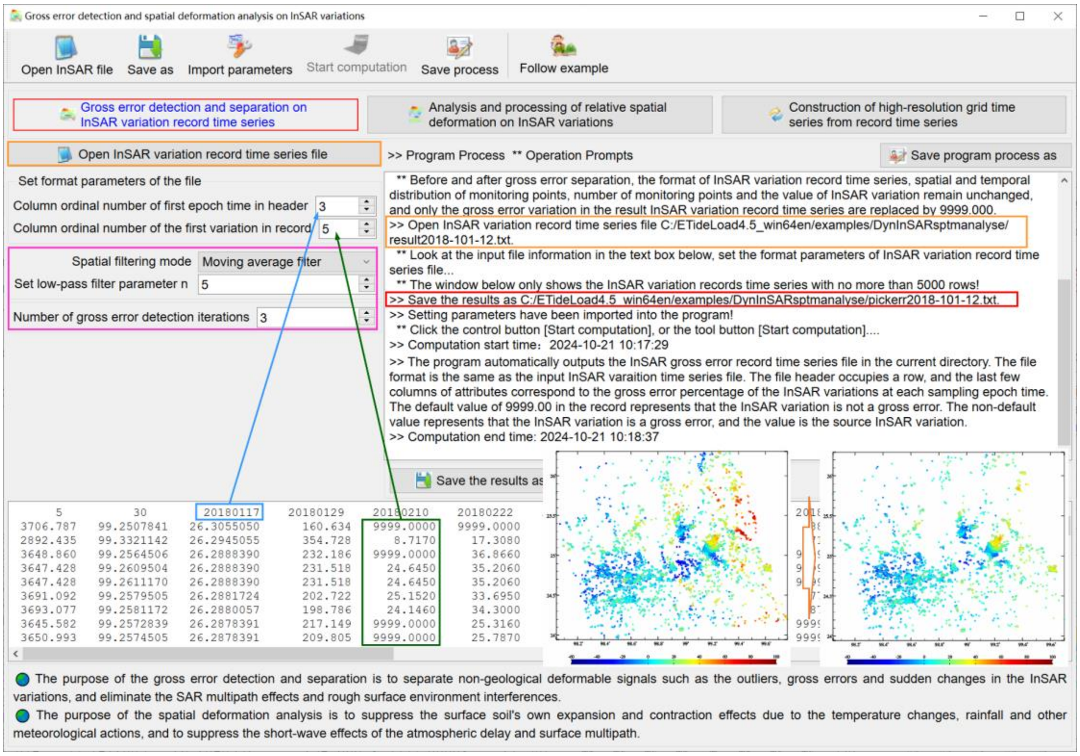
InSAR variation time series is agreed in the record time series format, and the sampling epoch time is agreed in ETideLoad format.

[Parameter settings] Set the InSAR variation record time series file format parameters, select the spatial filtering mode and enter the number of gross error detection iterations.

[Output file] The InSAR variation record time series file.

The InSAR variation record time series in the output file is the same as that in the input file with only the gross variations replaced by 9999.000.

The program automatically outputs the InSAR gross error record time series file in the current directory. The file format is the same as the input InSAR varaition time series file. The file header occupies a row, and the last few columns of attributes correspond to the gross error percentage of the InSAR variations at each sampling epoch time. The default value of 9999.00 in the record represents that the InSAR variation is not a gross error. The non-default value represents that the InSAR variation is a gross error, and the value is the source InSAR variation.



The purpose of the gross error detection and separation is to separate non-geological deformable signals such as the outliers, gross errors and sudden changes in the InSAR variations, and eliminate the SAR multipath effects and rough surface environment interferences.

5.2.2 Analysis and processing of relative spatial deformation on InSAR variations

[Function] According to the spatial distribution natures that the ground vertical deformation is inversely proportional to the distance away from the dynamic source, suppress or weaken the local changes due to non-geological dynamics on the shallow surface from the input InSAR variation record time series using the specified spatial filtering algorithm.

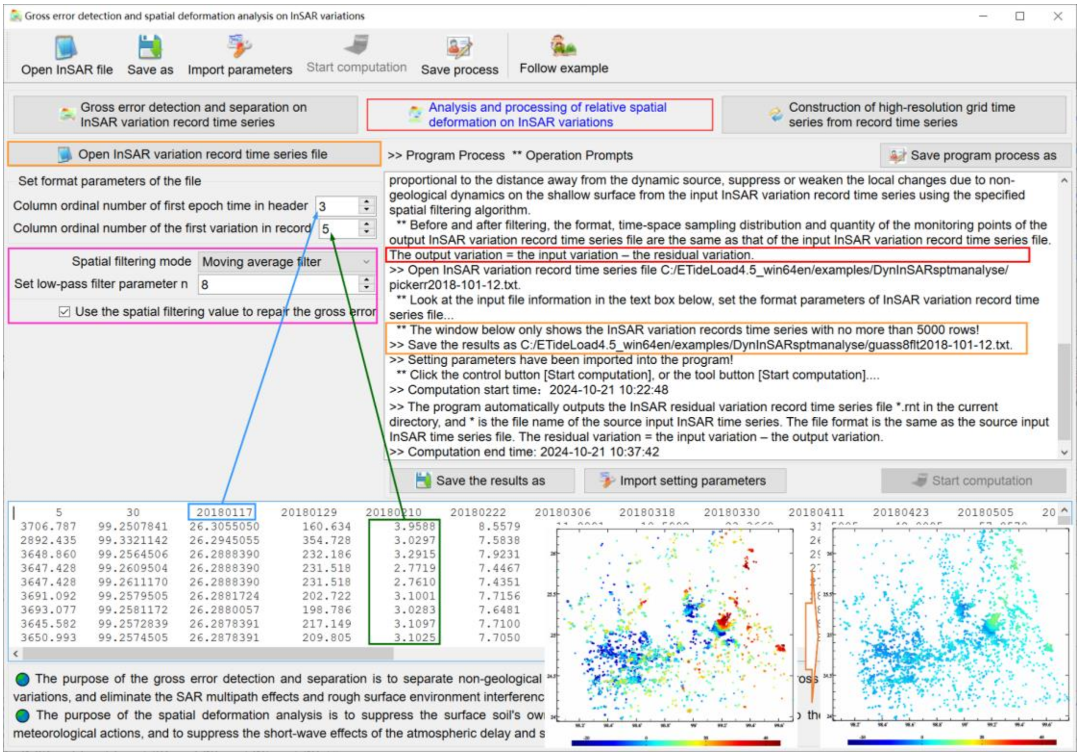
[Input file] The InSAR variation record time series file.

[Parameter settings] Set the InSAR variation record time series file format parameters, select the spatial filtering mode, enter spatial low-pass filter times and set the checkbox of [use the spatial filtering value to repair the gross error].

For the moving average filter, the greater the filtering parameter n, the greater the filtering strength, and for the spatial Gaussian filter, the smaller the n, the greater the filtering strength.

[Output file] The InSAR variation record time series file.

Before and after filtering, the format, time-space sampling distribution and quantity of the monitoring points of the output InSAR variation record time series file are the same as that of the input InSAR variation record time series file. The output variation = the input variation – the residual variation.



The program automatically outputs the InSAR residual variation record time series file *.rnt in the current directory, and * is the file name of the source input InSAR time series. The file format is the same as the source input InSAR time series file. The residual variation = the input variation – the output variation.

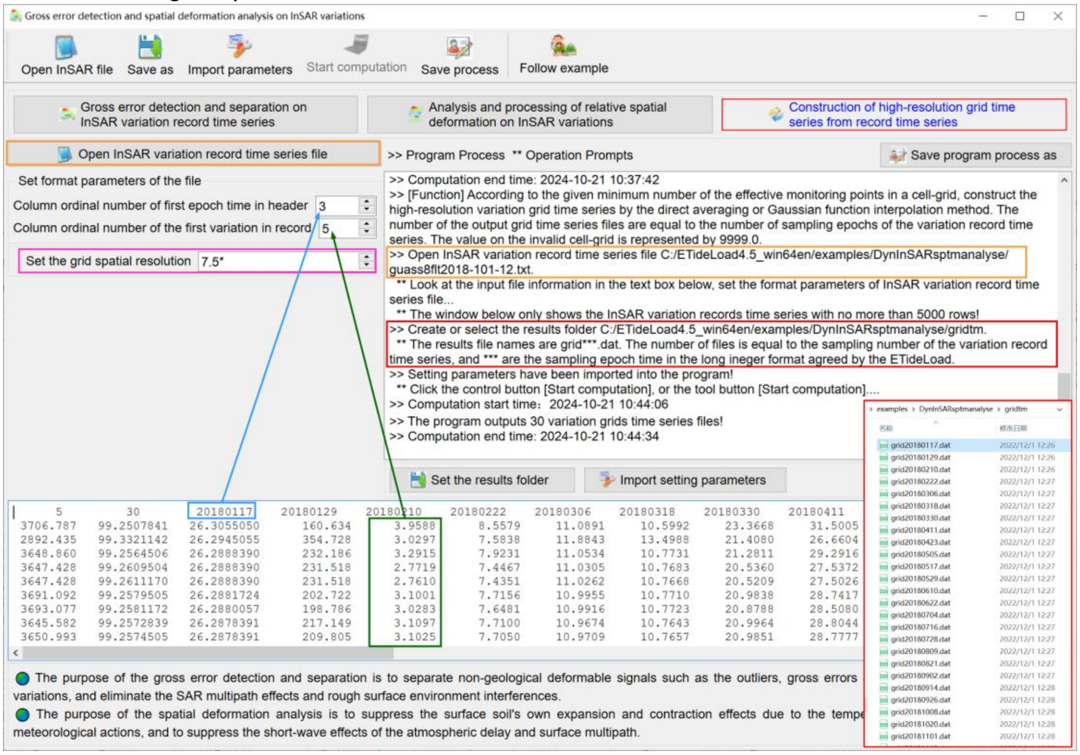
The purpose of the spatial deformation analysis is to suppress the surface soil's own expansion and contraction effects due to the temperature changes, rainfall and other meteorological actions, and to suppress the short-wave effects of the atmospheric delay and surface multipath.

5.2.3 Construction of high-resolution grid time series from record time series

[Function] According to the given minimum number of the effective monitoring points in a cell-grid, construct the high-resolution variation grid time series by the direct averaging or Gaussian function interpolation method. The number of the output grid time series files are equal to the number of sampling epochs of the variation record time series. The value on the invalid cell-grid is represented by 9999.0.

[Input file] The InSAR variation record time series file.

[Parameter settings] Set the InSAR variation record time series file format parameters and enter the grid spatial resolution.



[Output file] The results file names are grid***.dat. The number of files is equal to the sampling number of the variation record time series, and *** are the sampling epoch time in the long integer format agreed by the ETideLoad.

5.3 Collaborative monitoring and processing of InSAR with CORS network

[Purpose] Unify the reference epoch time of multi-source InSAR variation time series and CORS network height variation record time series, and then through the compatibility analysis of vertical deformation from the CORS network and InSAR, and InSAR variation adjustment with the constraint of the CORS network, unify the spatiotemporal monitoring frame of the InSAR variation time series to control the accumulation of the InSAR monitoring errors over time.

The purpose of cooperative monitoring and processing of the CORS network and InSAR: (1) Repair the tidal and non-tidal load effects on the InSAR variations, compensate the spatial long-wave troposphere model errors. (2) Compensate the temporal information which spatial wavelength larger than the InSAR monitoring area to control the cumulative errors of the InSAR variations over time. (3) When there are no less than 3 CORS stations, can precisely repair the InSAR differential interference scale error and compensate the other medium-long wave errors.

5.3.1 Unification of reference epoch for variation record time series

[Function] Using the cubic spline interpolation, Gaussian function interpolation or low-pass filtering, estimate the reference value of the variation record time series at the given reference epoch time, and then remove the reference value from the variation record time series, thereby unify the reference epoch time of the variation record time series. At the reference epoch time, the sampling values of all the variations are always zero.

The program requires that the reference epoch time be no earlier than the first sampling time and no later than the last sampling time, otherwise automatically set to the first or last sampling time.

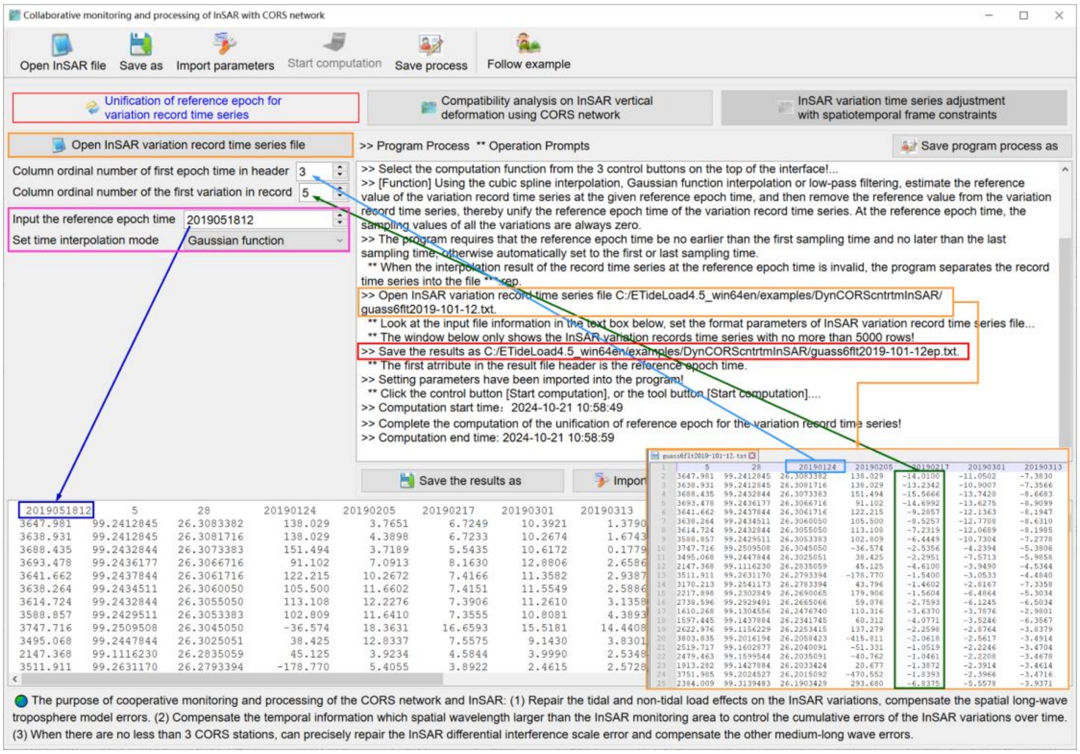
[Input file] The InSAR variation record time series file.

[Parameter settings] Set the InSAR variation record time series file format parameters, select the time interpolation mode, and enter the reference epoch time.

[Output file] The InSAR variation record time series file.

When the interpolation result of the record time series at the reference epoch time is invalid, the program separates the record time series into the file ***.rep.

When there are more noise or missing samples in the variation record time series, Gaussian function interpolation is recommended.



When the reference epoch time exceeds the effective time range of a record time series, if the cubic spline interpolation selected, the program automatically extrapolates the sampling value of the record time series at the reference epoch time by the Gaussian basis function method.

The geodetic variation with time is reflected by the difference between the monitoring elements at any two epoch moments and has nothing to do with the reference epoch time. Therefore, unifying or transforming the reference epoch time will not change the time-varying monitoring signals of geodetic variation time series.

5.3.2 Compatibility analysis on InSAR vertical deformation using CORS network

[Function] Calculate the ellipsoidal height variation time series on the CORS site from InSAR variation time series near the CORS site by the direct average method. Interpolates the ellipsoidal height variations at sampling epochs of the InSAR time series from the ellipsoidal height variation time series at the CORS sites. And then construct the CORS baselines by the complete combinations between the CORS sites, calculate all the double-difference time series respectively from the two kinds of CORS site ellipsoidal height variation time series above. Evaluate the compatibility of the vertical deformation between the CORS network and InSAR, and analyze the effectiveness of the InSAR variations gross error detection and spatial analysis algorithm.

[Input file] The InSAR variation record time series file. The CORS site ellipsoidal height variation record time series file.

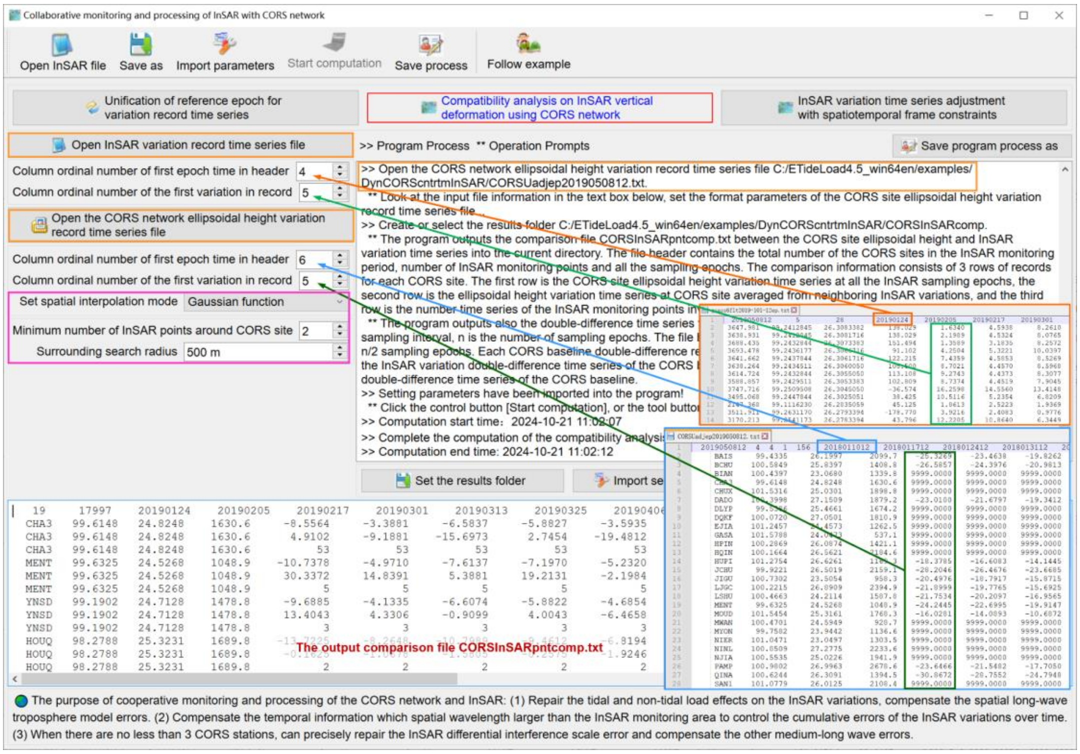
[Parameter settings] Set the InSAR and CORS site variation record time series file format parameters, select the time interpolation mode and enter the minimum number of InSAR points around CORS site and surrounding search radius.

When there are more noise or missing samples in the CORS height variation record time series, Gaussian function interpolation is recommended.

The double-difference algorithm of the InSAR variation time series on the CORS baseline: Firstly, calculate the InSAR variation at the current epoch time at the CORS site by the direct average method using the InSAR variations around the CORS site, and then calculate the InSAR variation difference between the two ends of the CORS baseline at the current epoch time, and finally calculate the time difference between the InSAR variation differences after and before the epoch time to obtain the InSAR variation double-difference time series of the CORS baseline.

[Output file] The comparison file CORSInSARpntcomp.txt between the CORS site ellipsoidal height and InSAR variation time series.

The file header contains the total number of the CORS sites in the InSAR monitoring period, number of InSAR monitoring points and all the sampling epochs. The comparison information consists of 3 rows of records for each CORS site. The first row is the CORS site ellipsoidal height variation time series at all the InSAR sampling epochs, the second row is the ellipsoidal height variation time series at CORS site averaged from neighboring InSAR variations, and the third row is the number time series of the InSAR monitoring points involved in the calculation for the second row.



The program outputs also the double-difference time series file dblediff*.txt, *=1~n/2 represents the multiple number of the sampling interval, n is the number of sampling epochs. The file header includes the number of the difference sampling epochs n/2, n/2 sampling epochs. Each CORS baseline double-difference record time series consists of two rows of records. The first row is the InSAR variation double-difference time series of the CORS baseline, and the second row is the ellipsoidal height variation double-difference time series of the CORS baseline.

5.3.3 InSAR variation time series adjustment with spatiotemporal frame constraints

[Function] From the comparison file CORSInSARpntcomp.txt output by the function [Compatibility analysis of vertical deformation from CORS network and InSAR], estimate spatiotemporal monitoring datum transfer parameters, construct spatiotemporal frame constraint equations, and performs the adjustment for the InSAR variation record time series, so as to unify the spatiotemporal monitoring frame of the InSAR variation record time series into the CORS network spatiotemporal monitoring frame.

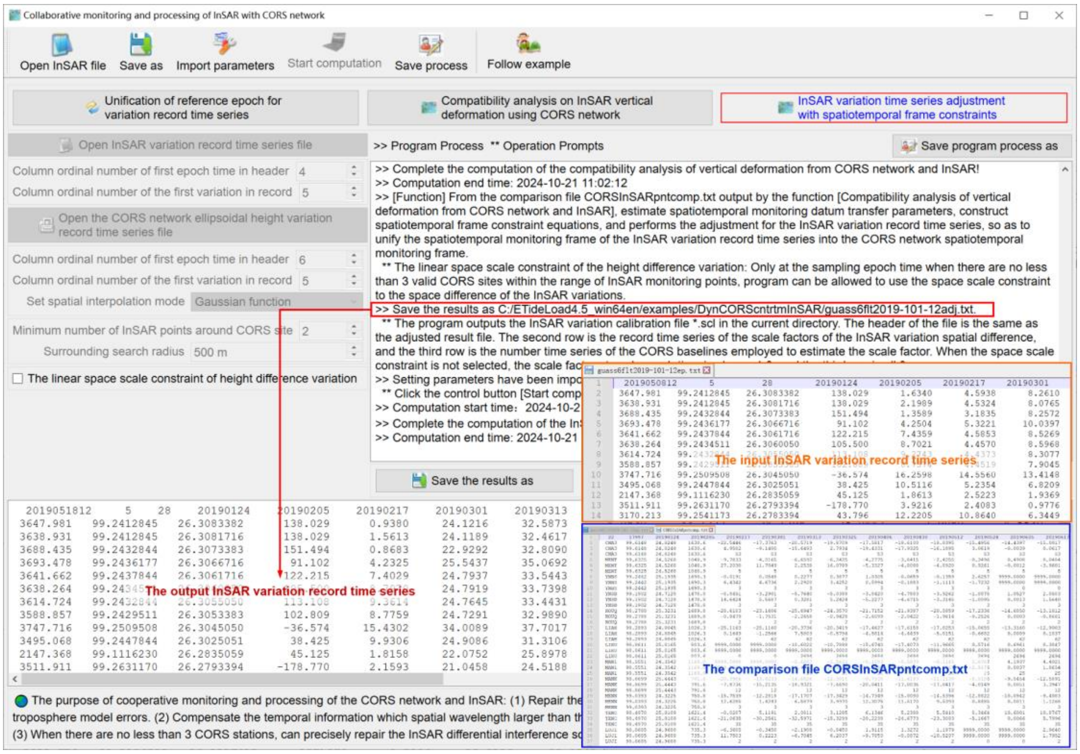
[Input files] The InSAR variation record time series file. The geodetic variation time series file to be reconstructed. The comparison file CORSInSARpntcomp.txt between the CORS site ellipsoidal height variations and InSAR variation time series. The two files can be automatically called by the program without manual input.

[Parameter settings] Set the checkbox [The linear space scale constraint of the height difference variation].

[Output file] The InSAR variation adjusted value record time series file. Whose format is same as the input InSAR variation record time series file.

The program outputs the InSAR variation calibration file ***.scl in the current directory. Here *** are the output adjusted result file name.

The header of the file is the same as the adjusted result file. The second row is the record time series of the scale factors of the InSAR variation spatial difference, and the third row is the number time series of the CORS baselines employed to estimate the scale factor. When the space scale constraint is not selected, the scale factor at each epoch time is always 1.0, and the third row is all 0.



The scale factor is an important quantitative indicator to evaluate the performance of the InSAR deformation monitoring. At sampling epoch time whose scale factor exceeds (0.5, 2.0), the vertical deformation separation of the InSAR variations is insufficient, or the quality of the InSAR variations is poor.

The linear space scale constraint of the height difference variation: Only at the sampling epoch time when there are no less than 3 valid CORS sites within the range of InSAR monitoring points, program can be allowed to use the space scale constraint to the space difference of the InSAR variations.

5.4 Deep fusion and time series analysis on multi-source InSAR variations

[Purpose] Firstly, deeply fuse multi-source InSAR variation record time series into the uniform spatiotemporal monitoring frame and reference epoch represented by the CORS variation time series respectively in time and space, and then perform time series analysis for all InSAR variation monitoring points, to realize multi-source InSAR collaborative monitoring.

5.4.1 Long-time connection for two same-track InSAR variation time series

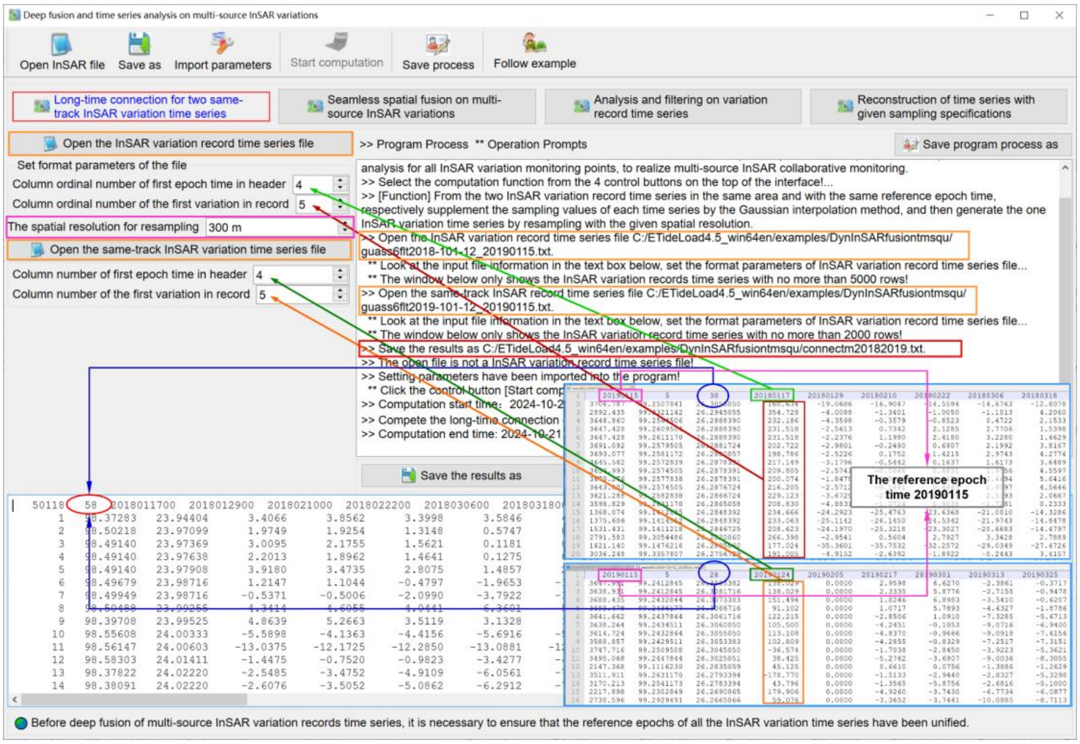
[Function] From the two InSAR variation record time series in the same area and with the same reference epoch time, respectively supplement the sampling values of each time series by the Gaussian interpolation method, and then generate the one InSAR variation time series by resampling with the given spatial resolution.

[Input files] The two InSAR variation record time series files in the same area and with the same reference epoch time.

[Parameter settings] Set the two InSAR variation record time series file format parameters and enter the resampling spatial resolution.

[Output file] The connected InSAR variation record time series file.

The output file format and reference epoch are same as that of the input InSAR variation record time series file.



5.4.2 Seamless spatial fusion on multi-source InSAR variations

[Function] According to the given spatial resolution, resample the input multi-source InSAR variation record time series to generate a new InSAR variation record time series. The input InSAR variation record time series files are extracted according to the given wildcards, and all the input files are in the same format.

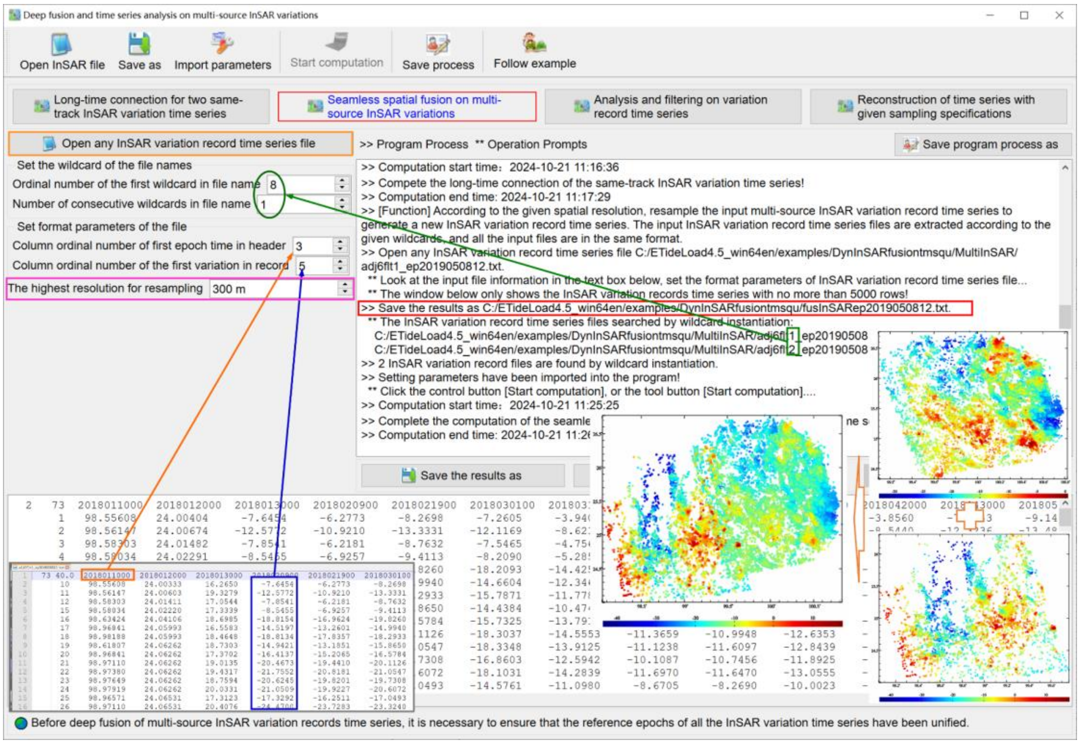
Before deep fusion of multi-source InSAR variation record time series, it is necessary to ensure that the reference epochs of all the InSAR variation time series have been unified.

[Input files] Multi-source InSAR variation record time series files.

In this example, two InSAR variation record time series are in adjacent areas, and a small number in the two groups of InSAR monitoring points are cross-distributed.

[Parameter settings] Set the file name wildcards and file format parameters of multi-source InSAR variation record time series and enter the resampling spatial resolution.

[Output file] The fused InSAR variation record time series file. The format is the same as that of the input InSAR variation record time series file.



5.4.3 Analysis and filtering on variation record time series

[Function] Using the continuous Chebyshev and triangular basis function combination method, estimate the low-pass filtering parameters for variation record time series at each monitoring point, and then calculate the filtering value and the linear variation (per year, /a) at source sampling epochs.

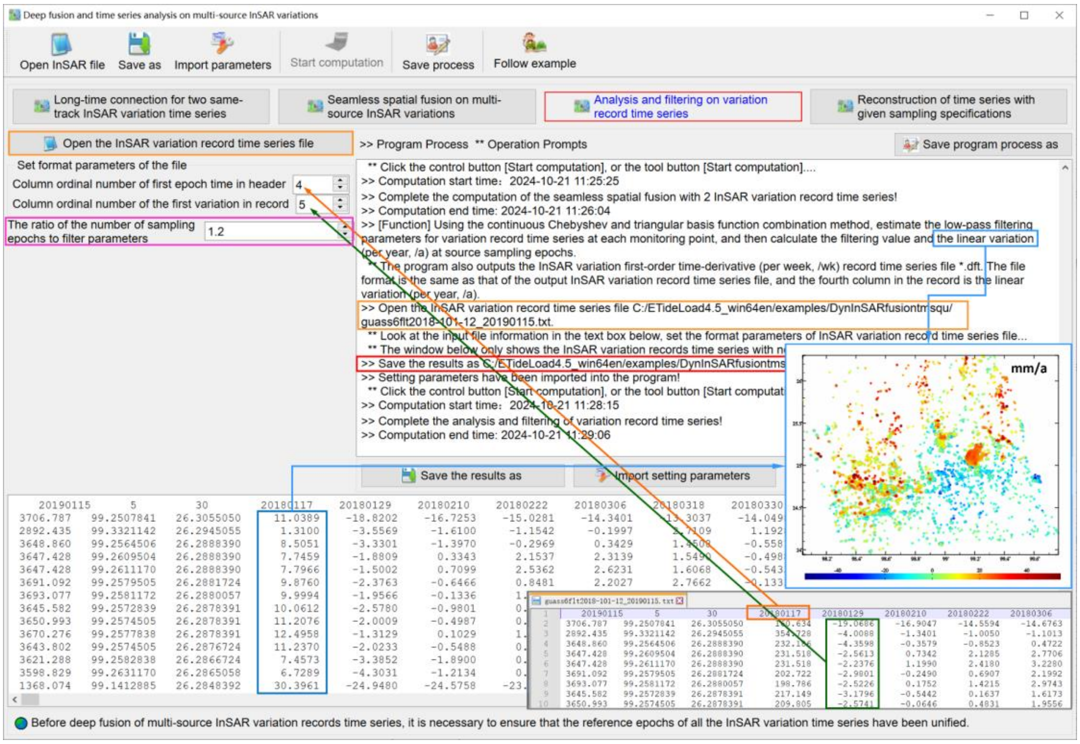
[Input files] The InSAR variation record time series files.

[Parameter settings] Set the file format parameters of InSAR variation record time series and enter the ratio of the number of sampling epochs to filter parameters.

[Output file] The filtered InSAR variation record time series file ***.txt. The InSAR variation first-order time-derivative (per week, /wk) record time series file ***.dft. Here, *** are the output file name.

The filtered variation record time series file. The file format is the same as that of the input InSAR variation record time series file and the fourth column in the file record is the linear variation (per year, /a).

The InSAR variation first-order time-derivative (per week, /wk) record time series file *.dft. The file format is the same as that of the output InSAR variation record time series file, and the fourth column in the record is the linear variation (per year, /a).



5.4.4 Reconstruction of time series with given sampling specifications

[Function] Using the continuous Chebyshev and triangular basis function combination method, estimate the filtering parameters for variation record time series at each monitoring point, and then reconstruct the variation record time series according to the given time series sampling specifications.

The program has the time-domain interpolation and short-time forecasting capabilities.

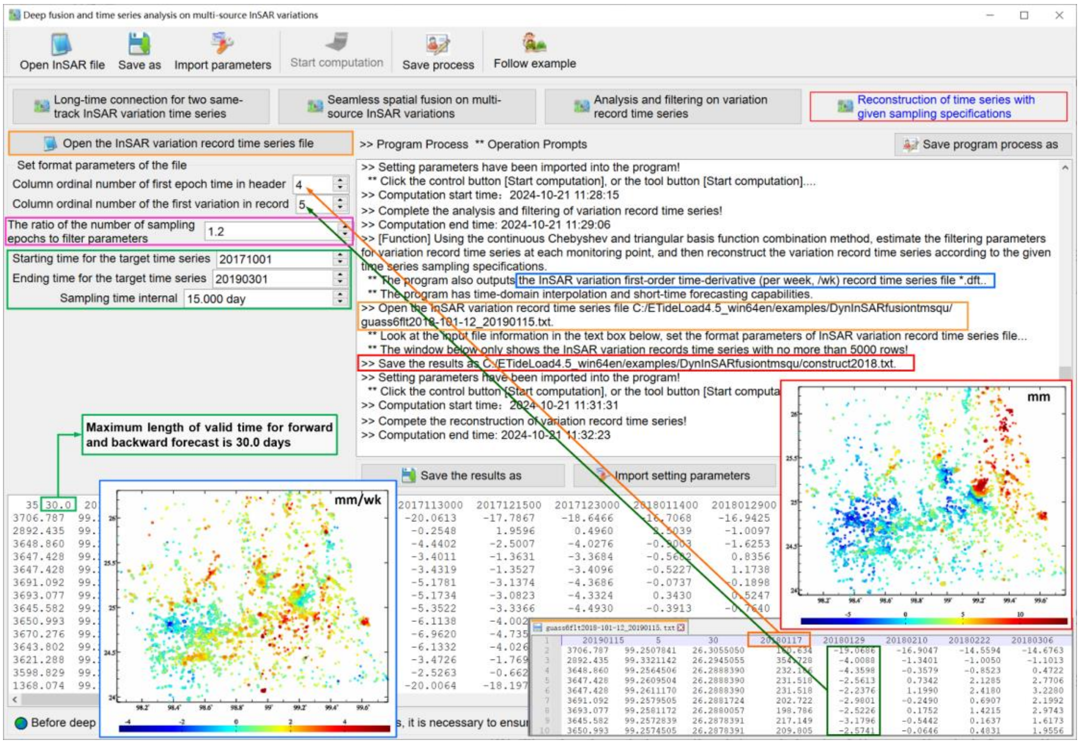
[Input files] The InSAR variation record time series files.

[Parameter settings] Set the file format parameters of InSAR variation record time series and enter the ratio of the number of sampling epochs to filter parameters and time series sampling specifications.

When the starting time is earlier than the first sampling epoch time of the source variation record time series, the program lets the starting time = the first sampling epoch time - sampling interval * total number of the samples * 5%.

When the ending time is later than the last sampling epoch time of the source variation record time series, the program lets the ending time = the last sampling epoch time + sampling interval * total number of the samples * 5%.

[Output file] The reconstructed InSAR variation record time series file ***.txt. The InSAR variation first-order time-derivative (per week, /wk) record time series file ***.dft. Here, *** are the output file name.



The filtered variation record time series file. The file format is the same as that of the input InSAR variation record time series file and the fourth column in the file record is the linear variation (per year, /a).

The InSAR variation first-order time-derivative (per week, /wk) record time series file *.dft. The file format is the same as that of the output InSAR variation record time series file, and the fourth column in the record is the linear variation (per year, /a).

5.5 Computation of ground stability variation based on vertical deformation

[Purpose] From the ground vertical deformation rate and its horizontal gradient grid model, using the normalized statistical synthesis algorithm, quantitatively estimate the ground stability variation grid according to the quantitative criteria of the ground stability weakening defined by ETideLoad.

Ground stability variation grid time series here is employed to quantitatively express the time and location, continuous influence time period and spatial distribution of ground stability weakening.

Quantitative criteria defined by ETideLoad for the ground stability weakening based on the vertical deformation grid time series are in the following.

(1) The ground vertical deformation rate is relatively large (greater than zero). At this time, the ground here is rising upward.

(2) The horizontal gradient (modulus) of the vertical deformation rate is relatively large. At this time, the ground is twisting locally.

(3) The terrain slope value is relatively large.

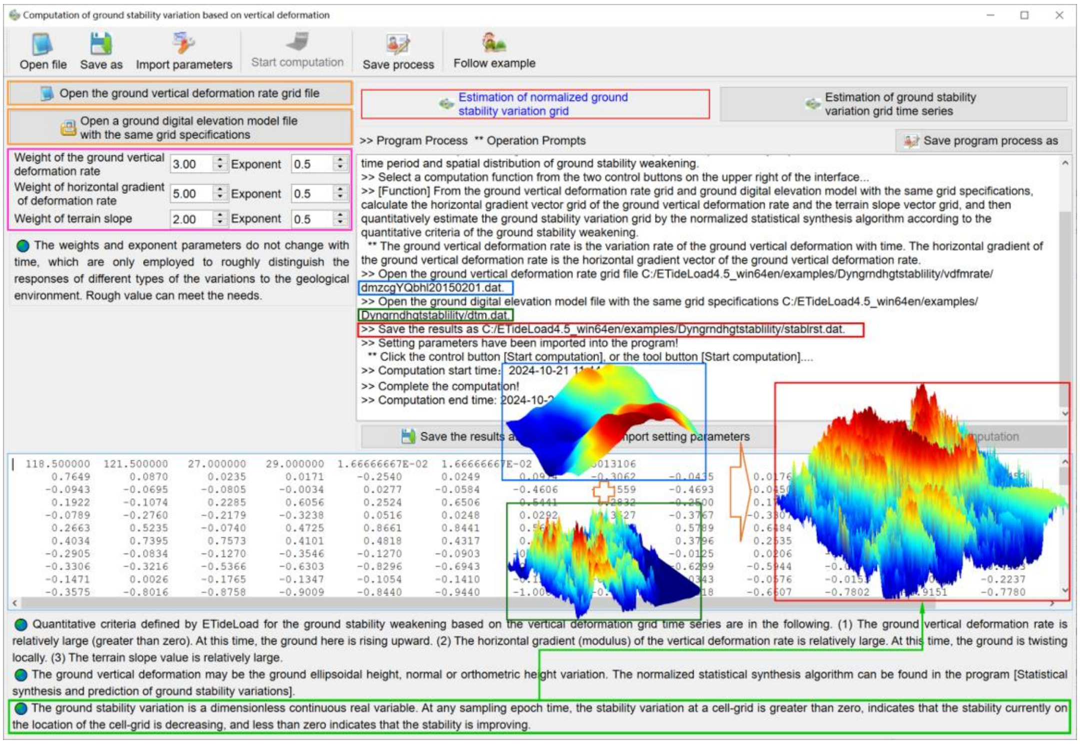
The ground stability variation is a dimensionless continuous real variable. At any sampling epoch time, the stability variation at a cell-grid is greater than zero, indicates that the stability currently on the location of the cell-grid is decreasing, and less than zero indicates that the stability is improving.

5.5.1 Estimation of normalized ground stability variation grid

[Function] From the ground vertical deformation rate grid and ground digital elevation model with the same grid specifications, calculate the horizontal gradient vector grid of the ground vertical deformation rate and the terrain slope vector grid, and then quantitatively estimate the ground stability variation grid by the normalized statistical synthesis algorithm according to the quantitative criteria of the ground stability weakening.

[Input files] The ground vertical deformation rate grid file and ground digital elevation model file with the same grid specifications.

[Parameter settings] Set the weights and exponents for ground vertical deformation rate, horizontal gradient of the ground vertical deformation rate and terrain slope.



The ground vertical deformation rate is the variation rate of the ground vertical deformation with time. The horizontal gradient of the ground vertical deformation rate is the horizontal gradient vector of the ground vertical deformation rate.

The weights and exponent parameters do not change with time, which are only employed to roughly distinguish the responses of different types of the variations to the geological environment. Rough value can meet the needs.

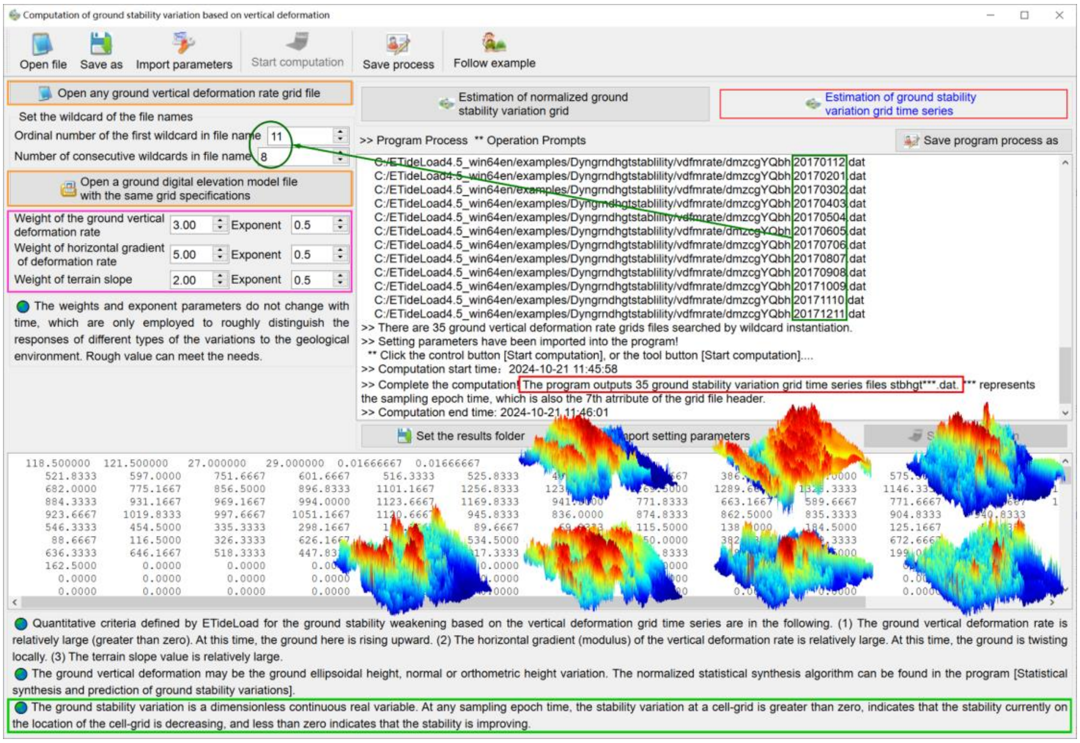
[Output file] The normalized ground stability variation grid file.

5.5.2 Estimation of ground stability variation grid time series

[Function] From the ground vertical deformation rate grid time series and ground digital elevation model with the same grid specifications, calculate the terrain slope vector grid and the horizontal gradient vector grid time series of the ground vertical deformation rate, and then quantitatively estimate the ground stability variation grid time series by the normalized statistical synthesis algorithm according to the quantitative criteria of the ground stability weakening.

[Input files] The ground vertical deformation rate grid time series files and ground digital elevation model file with the same grid specifications.

[Parameter settings] Set the wildcard parameters for the deformation rate grid time series files, enter the weights and exponents for the ground vertical deformation rate, horizontal gradient of the ground vertical deformation rate and terrain slope.



[Output file] The normalized ground stability variation grid time series files.

The ground vertical deformation may be the ground ellipsoidal height, or normal or orthometric height variation. The normalized statistical synthesis algorithm can be found in the program [Statistical synthesis and prediction of ground stability variations].

5.6 Computation of ground stability variation based on gravity variations

[Purpose] From the ground gravity (or gravity disturbance) variation rate and its horizontal gradient vector grid model, using the normalized statistical synthesis algorithm, quantitatively estimate the ground stability variation grid according to the quantitative criteria of the ground stability weakening defined by ETideLoad.

Quantitative criteria defined by ETideLoad for the ground stability weakening based on the gravity variation grid time series are in the following.

(1) The ground gravity variation rate is relatively large (less than zero). At this time, the ground here is rising upward.

(2) The horizontal gradient (modulus) of the gravity variation rate is relatively large. At this time, the ground is twisting locally.

(3) The local terrain effect (absolute value) on gravity is relatively large (the effect is always less than zero).

The ground gravity variation may be the ground gravity or gravity disturbance variation.

5.6.1 Normalized ground stability variation grid estimation

[Function] From the ground gravity variation rate grid and ground digital elevation model, calculate the horizontal gradient vector grid of the ground gravity variation rate and the local terrain effect grid on gravity, and then quantitatively estimate the ground stability variation grid by the normalized statistical synthesis algorithm according to the quantitative criteria of the ground stability weakening.

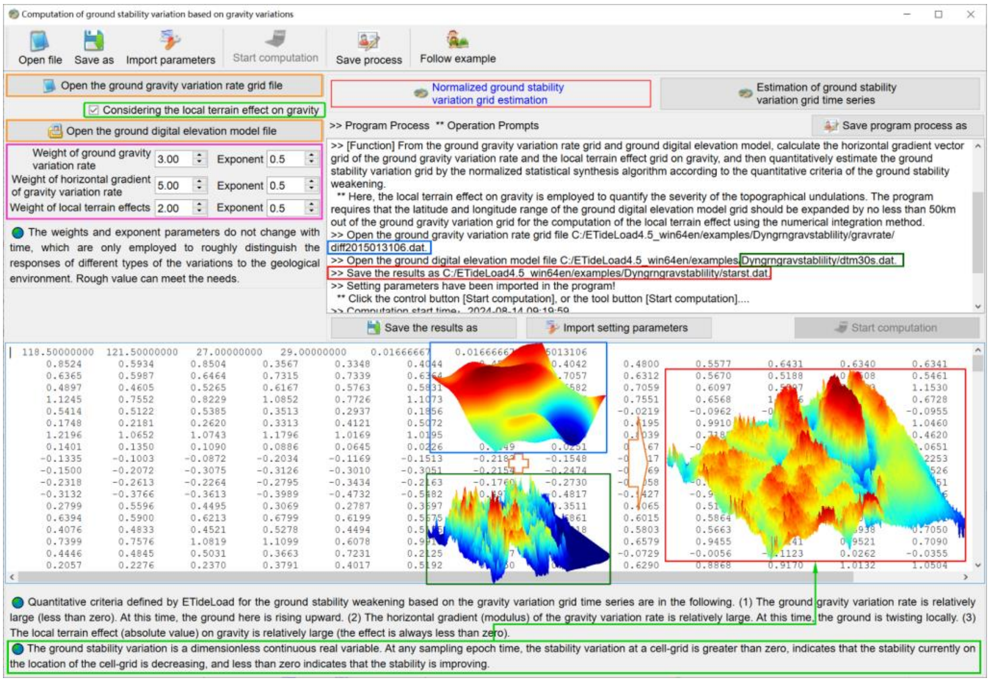
[Input files] The ground gravity variation rate grid file. The ground digital elevation model grid file.

[Parameter settings] Set the weights and exponents for ground gravity variation rate, horizontal gradient of the ground gravity variation rate and local terrain effect, and set the checkbox [Considering the local terrain effect on gravity].

The ground gravity variation is the variation rate of the ground gravity variation with time. The horizontal gradient of the ground gravity variation rate is the horizontal gradient vector of the ground gravity variation rate.

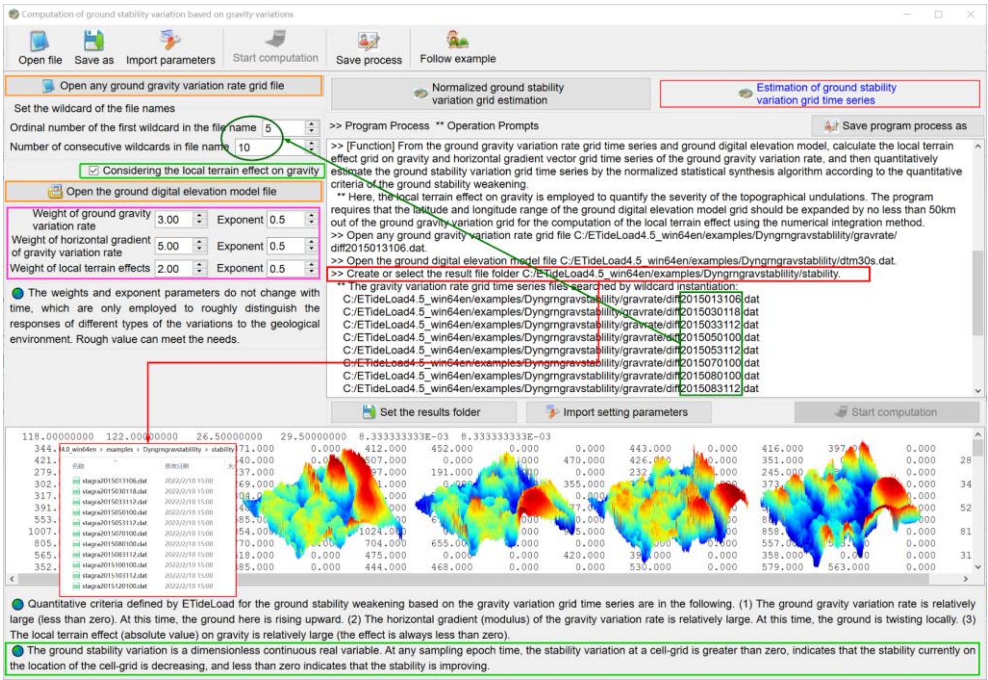
Here, the local terrain effect on gravity is employed to quantify the severity of the topographical undulations. The program requires that the latitude and longitude range of the ground digital elevation model grid should be expanded by no less than 50km out of the ground gravity variation grid for the computation of the local terrain effect using the numerical integration method.

[Output file] The normalized ground stability variation grid file.



5.6.2 Estimation of ground stability variation grid time series

[Function] From the ground gravity variation rate grid time series and ground digital elevation model, calculate the local terrain effect grid on gravity and horizontal gradient vector grid time series of the ground gravity variation rate, and then quantitatively estimate the ground stability variation grid time series by the normalized statistical synthesis algorithm according to the quantitative criteria of the ground stability weakening.



[Input files] The ground gravity variation rate grid time series files and ground digital elevation model file.

[Parameter settings] Set the wildcard parameters for the gravity variation rate grid time series files, enter the weights and exponents for the ground gravity variation rate, horizontal gradient of the ground gravity variation rate and local terrain effect.

The weights and exponent parameters do not change with time, which are only employed to roughly distinguish the responses of different types of the variations to the geological environment. Rough value can meet the needs.

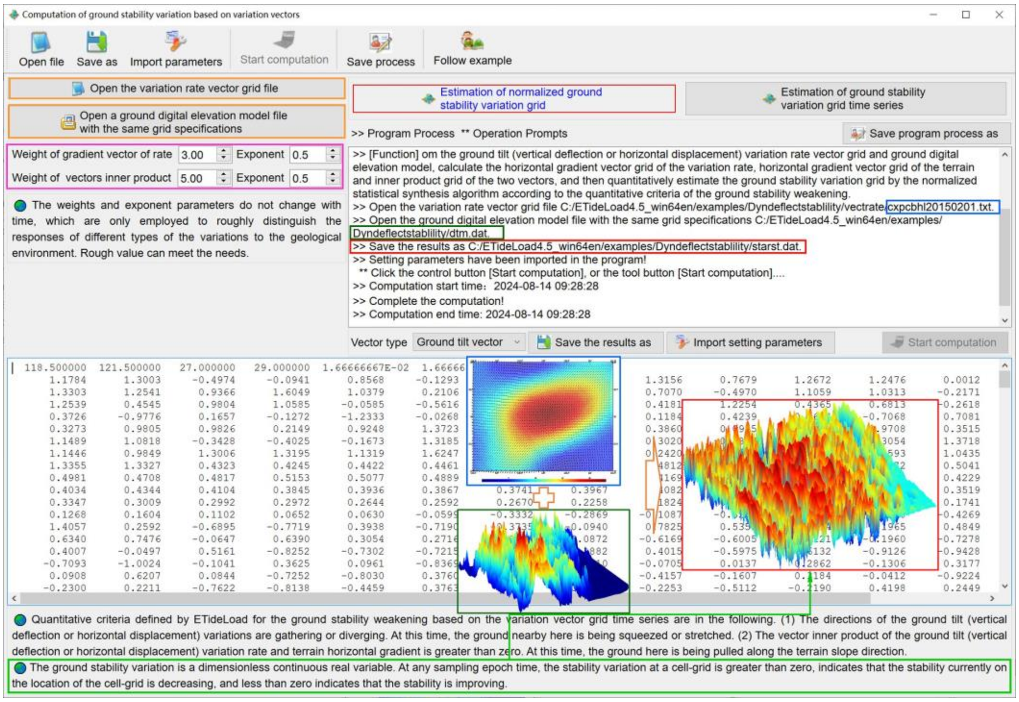
[Output file] The normalized ground stability variation grid time series files.

5.7 Computation of ground stability variation based on variation vectors

[Purpose] From the ground tilt (vertical deflection or horizontal displacement) variation rate vector grid and ground digital elevation model, using the normalized statistical synthesis algorithm, quantitatively estimate the ground stability variation grid according to the quantitative criteria of the ground stability weakening defined by ETideLoad.

5.7.1 Estimation of normalized ground stability variation grid

[Function] From the ground tilt (vertical deflection or horizontal displacement) variation rate vector grid and ground digital elevation model, calculate the horizontal gradient vector grid of the variation rate, horizontal gradient vector grid of the terrain and inner product grid of the two vectors, and then quantitatively estimate the ground stability variation grid by the normalized statistical synthesis algorithm according to the quantitative criteria of the ground stability weakening.



[Input files] The ground variation rate vector grid file and ground digital elevation model file with the same grid specifications.

[Parameter settings] Set the weights and exponents for the rate gradient and two vectors inner product, select the vector type.

[Output file] The normalized ground stability variation grid file.

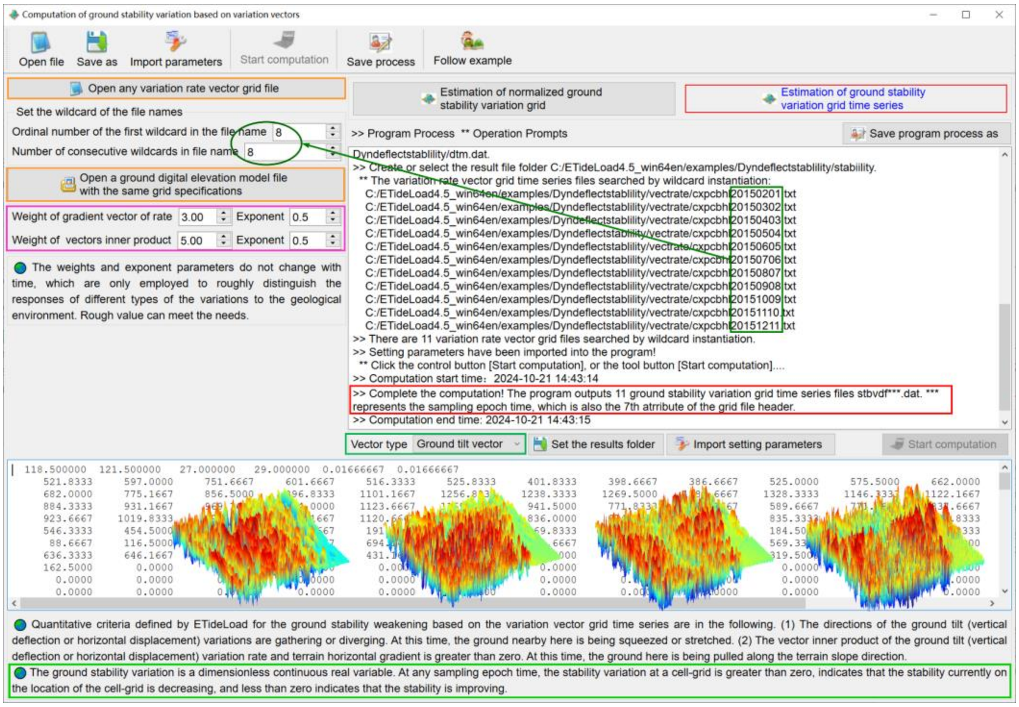
5.7.2 Estimation of ground stability variation grid time series

[Function] From the ground tilt (vertical deflection or horizontal displacement) variation rate vector grid time series and ground digital elevation model, calculate the horizontal gradient vector grid time series of the variation rate, horizontal gradient vector grid of the terrain and inner product grid time series of the two vectors, and then quantitatively estimate the ground stability variation grid time series by the normalized statistical synthesis algorithm according to the quantitative criteria of the ground stability weakening.

[Input files] The ground variation rate vector grid time series files and ground digital elevation model file with the same grid specifications.

[Parameter settings] Set the wildcard parameters for the rate vector grid time series files, enter the weights and exponents for the rate gradient and two vectors inner product, and select the vector type.

[Output file] The normalized ground stability variation grid time series files.



Quantitative criteria defined by ETideLoad for the ground stability weakening based on the variation vector grid time series are in the following.

(1) The directions of the ground tilt (vertical deflection or horizontal displacement)

variations are gathering or diverging. At this time, the ground nearby here is being squeezed or stretched.

(2) The vector inner product of the ground tilt (vertical deflection or horizontal displacement) variation rate and terrain horizontal gradient is greater than zero. At this time, the ground here is being pulled along the terrain slope direction.

The variation vector may be the ground tilt variation, vertical deflection variation or ground horizontal displacement.

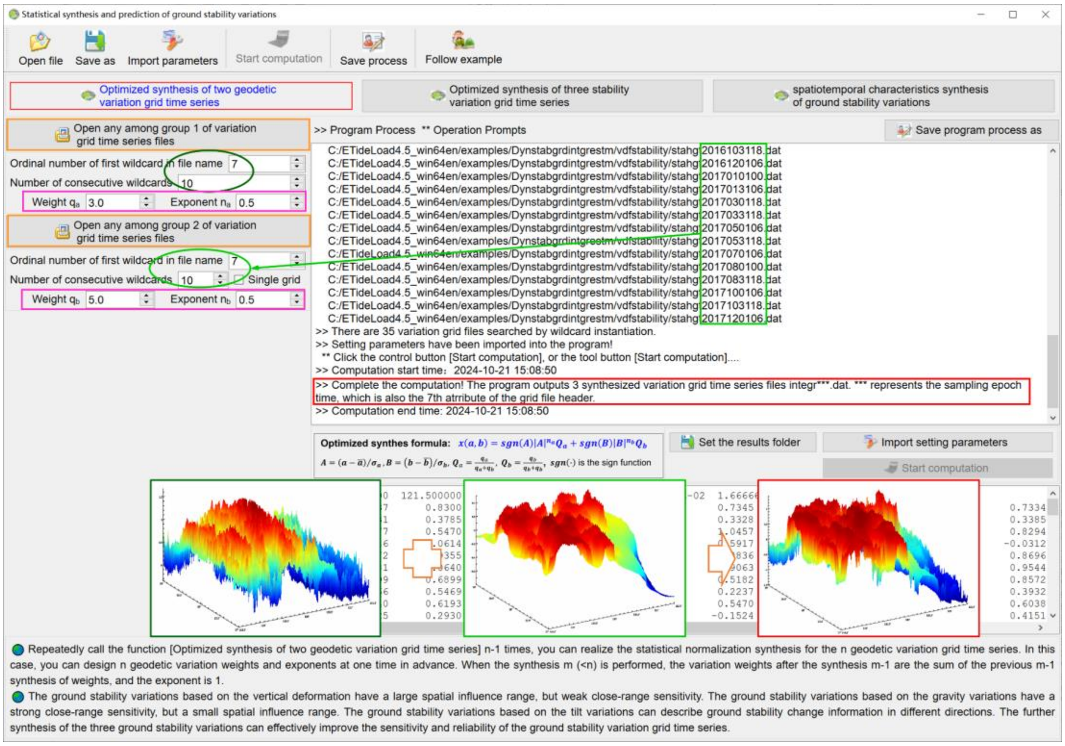
5.8 Statistical synthesis and prediction of ground stability variations

[Purpose] According to historical disaster events during the monitoring period, by adjusting the weights and exponents of multiple stability variations based on various geodetic variations, optimize the ground stability variation grid time series by the statistical normalized synthesis, to reveal the spatial distribution and temporal natures of the regional ground stability variations.

5.8.1 Optimized synthesis of two geodetic variation grid time series

[Function] From two groups of geodetic variation grid time series with the same space-grid and time-sampling specifications, generate the coupled geodetic variation grid time series by the statistical normalized synthesis, to reveal the spatiotemporal dynamic effects of the two kinds of geodetic joint monitoring.

[Input files] The two groups of geodetic variation grid time series files with the same space-grid and time-sampling specifications.



[Parameter settings] Set the wildcard parameters for grid time series files, and enter the weights and exponents.

[Output file] The synthesized variation grid time series files.

If all the characters of the file name are set as wildcards, the variation grid time series only is an epoch sampling grid. In this case, the program can realize the normalized synthesis between a group of the grid time series and a single epoch of grid.

With the two geodetic variation grid time series $\boldsymbol{a}$, $\boldsymbol{b}$, the synthesized variation grid time series $\boldsymbol{x}$ can be calculated by the following formula.

$\boldsymbol{x} = \boldsymbol{sgn}(\boldsymbol{A})|\boldsymbol{A}|^{n_a}\boldsymbol{Q_a} + \boldsymbol{sgn}(\boldsymbol{B})|\boldsymbol{B}|^{n_b}\boldsymbol{Q_b}$ where $\boldsymbol{sgn}(*)$ is the sign function,

$\boldsymbol{A} = (\boldsymbol{a} - \overline{\boldsymbol{a}})/\boldsymbol{\sigma_a}, \boldsymbol{B} = (\boldsymbol{b} - \overline{\boldsymbol{b}})/\boldsymbol{\sigma_b}, \boldsymbol{Q}_a = \frac{q_b}{q_a+q_b}, \boldsymbol{Q}_a = \frac{q_b}{q_a+q_b}$.

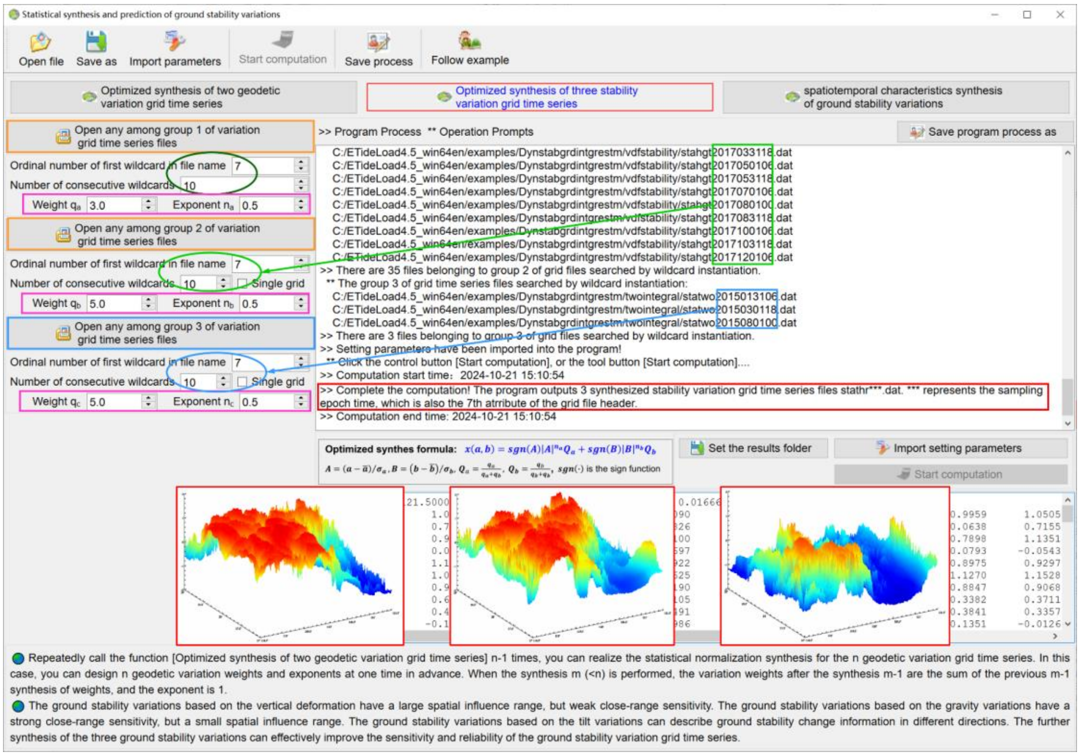
5.8.2 Optimized synthesis of three stability variation grid time series

[Function] From three groups of ground stability variation grid time series with the same space-grid and time-sampling specifications, generate the ground stability variation grid time series with spatiotemporal dynamic feature information, higher sensitivity and reliability by the statistical normalized synthesis.

[Input files] The three groups of stability variation grid time series files with the same space-grid and time-sampling specifications.

[Parameter settings] Set the wildcard parameters for grid time series files, enter the weights and exponents.

[Output file] The synthesized stability variation grid time series files.



5.8.3 Spatiotemporal characteristics synthesis of ground stability variations

[Function] From the ground stability variation grid time series, calculate its spatial horizontal gradient and time-derivative grid time series. And then using the low-pass filtering and statistical normalization synthesis algorithms, generates the grid time series files stachr*.dat of the ground stability variations that fuse spatiotemporal characteristics according to the given sampling specifications and statistical parameters.

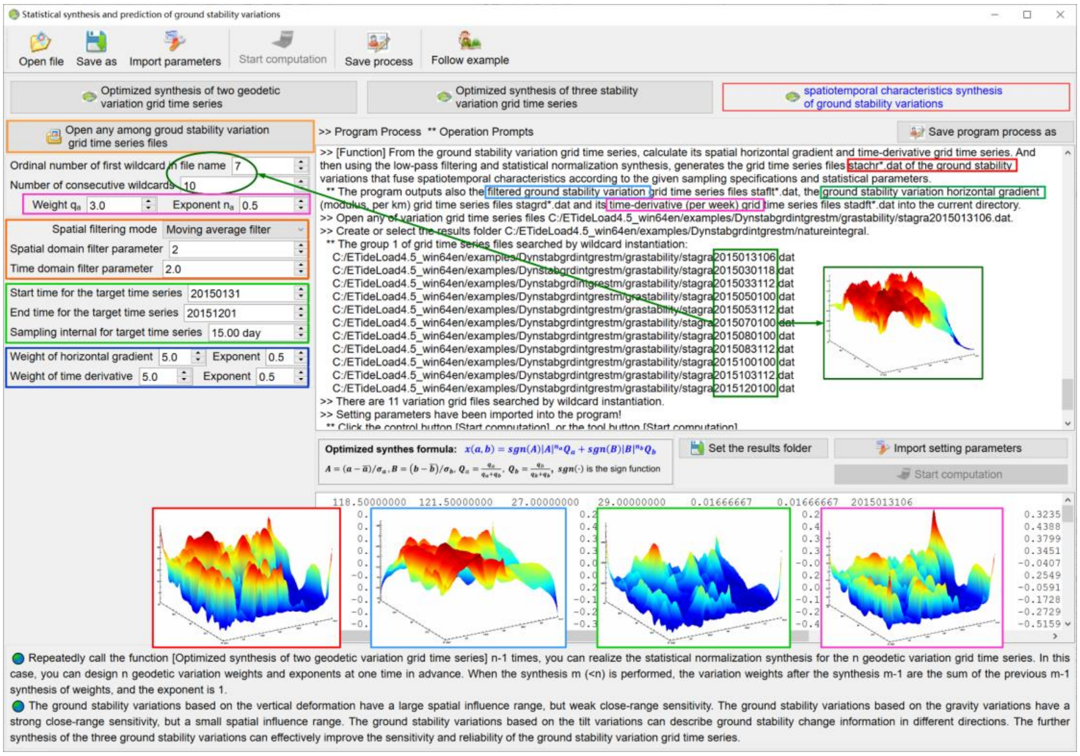
[Input files] The ground stability variation grid time series files.

[Parameter settings] Enter the weights and exponents for the ground stability variation, its horizontal gradient and time-derivative, set spatial and time domain filter parameters, and set the sampling specifications parameters.

When the starting time is earlier than the first sampling epoch time of the source variation grid time series, the program lets the starting time = the first sampling epoch time - sampling interval * total number of the samples * 5%.

When the ending time is later than the last sampling epoch time of the source variation grid time series, the program lets the ending time = the last sampling epoch time + sampling interval * total number of the samples * 5%.

[Output file] The synthesized ground stability variation grid time series files stachr*.dat, filtered ground stability variation grid time series files staflt*.dat, ground stability variation horizontal gradient (modulus, per km) grid time series files stagrd*.dat and their time-derivative (per week) grid time series files stadft*.dat.

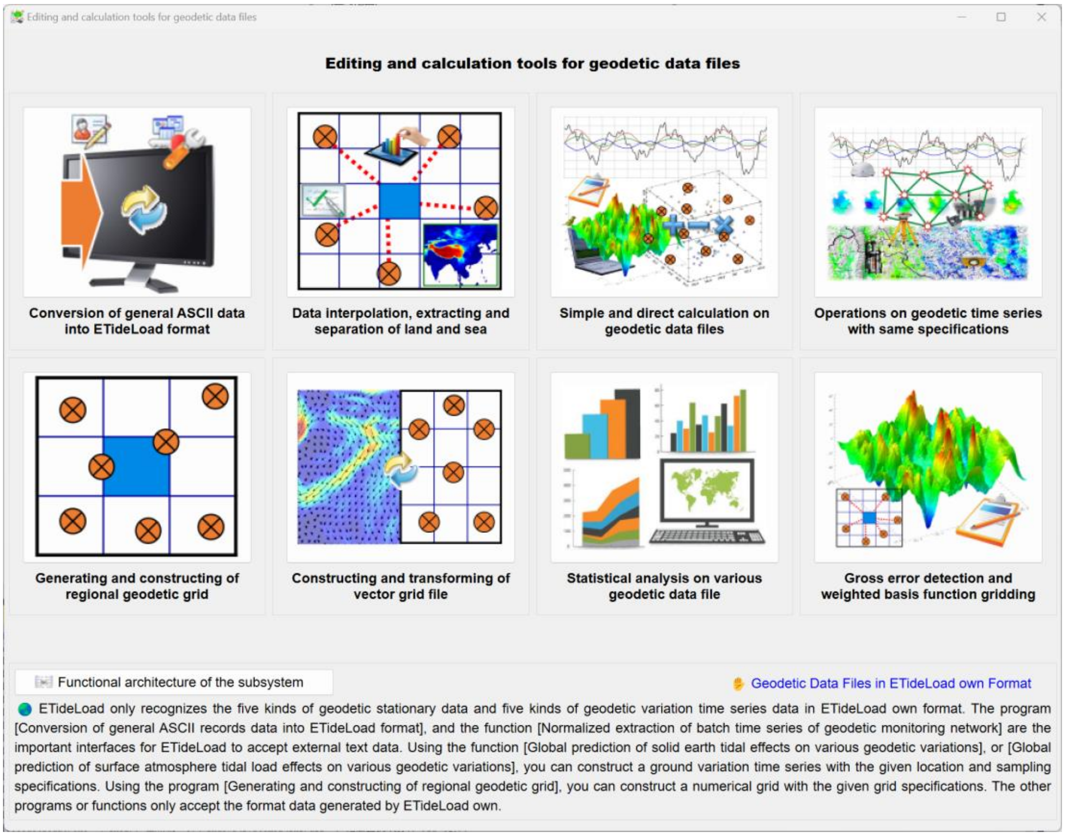


Repeatedly call the function [Optimized synthesis of two geodetic variation grid time series] n-1 times, you can realize the statistical normalization synthesis for the n geodetic variation grid time series. In this case, you can design n geodetic variation weights and exponents at one time in advance. When the synthesis m (<n) is performed, the variation weights after the synthesis m-1 are the sum of the previous m-1 synthesis of weights, and the exponent is 1.

The ground stability variations based on the vertical deformation have a large spatial influence range, but weak close-range sensitivity. The ground stability variations based on the gravity variations have a strong close-range sensitivity, but a small spatial influence range. The ground stability variations based on the tilt variations can describe ground stability change information in different directions. The further synthesis of the three ground stability variations can effectively improve the sensitivity and reliability of the ground stability variation grid time series.

6 Editing, calculation and visualization tools for geodetic data files

The group of programs can be employed to construct of geodetic data files in ETideLoad format, edit, interpolate, grid, extract, separate and merge geodetic data, and simply calculate and visualize on geodetic data files.



ETideLoad only recognizes the five kinds of geodetic stationary data and five kinds of geodetic variation time series data in ETideLoad own format. The program [Conversion of general ASCII record data into ETideLoad format], and the function [Normalized extraction of batch time series of geodetic monitoring network] are the important interfaces for ETideLoad to accept external text data. Using the function [Global prediction of solid Earth tidal effects on various geodetic variations], or [Global prediction of surface atmosphere tidal load effects on various geodetic variations], you can construct a ground variation time series with the given location and sampling specifications. Using the program [Generating and constructing of regional geodetic grid], you can construct a numerical grid with the given grid specifications. The other programs or functions only accept the format data generated by ETideLoad own.

6.1 Conversion of general ASCII data into ETideLoad format

[Function] Convert the general ASCII data record file from different sources and non-standard formats into the discrete geodetic record file in ETideLoad format.

[Input file] The general ASCII data record file.

After entering the number of rows of the input file header, click the control button [Exact and edit data] to open the dialog [Exact and edit data from the source text file].

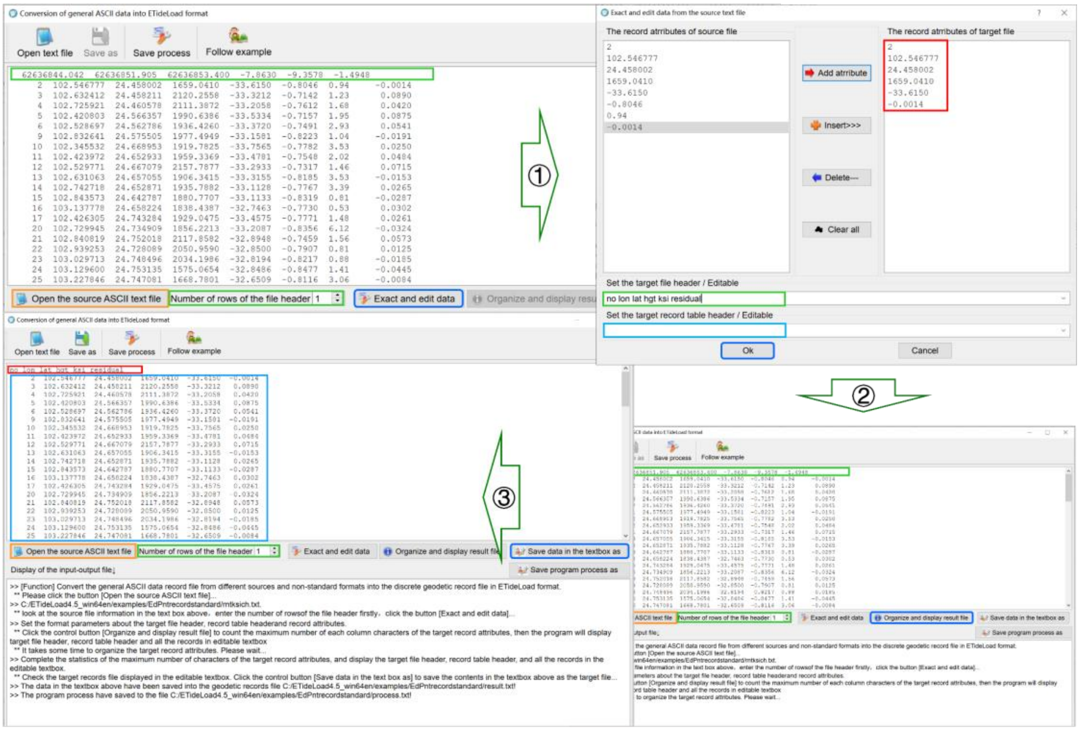
Set the format parameters about the target file header, record table header, and record attributes.

When the target file does not need the record table header, please clear the text corresponding to the input text box.

Click the button [Ok] to close the dialog. Click the control button [Organize and display result file] to count the maximum number of each column characters of the target record attributes, then the program will display target file header, record table header and all the records in the editable textbox. The program need take some time to organize the target record attributes, please wait…

Complete the statistics of the maximum number of characters of the target record attributes, and display the target file header, record table header, and all the records in the editable textbox.

[Output file] The discrete geodetic record file in ETideLoad format.



Check the target record file displayed in the editable textbox. Click the control button [Save data in the textbox as] to save the contents in the textbox above as the target file.

The program is the important interface for ETideLoad to accept the external text data.

6.2 Data interpolation, extracting and land-sea area separation

6.2.1 Changing of grid resolution by interpolation

[Function] Increase or decrease the grid spatial resolution according to the given grid resolution and specified interpolation method.

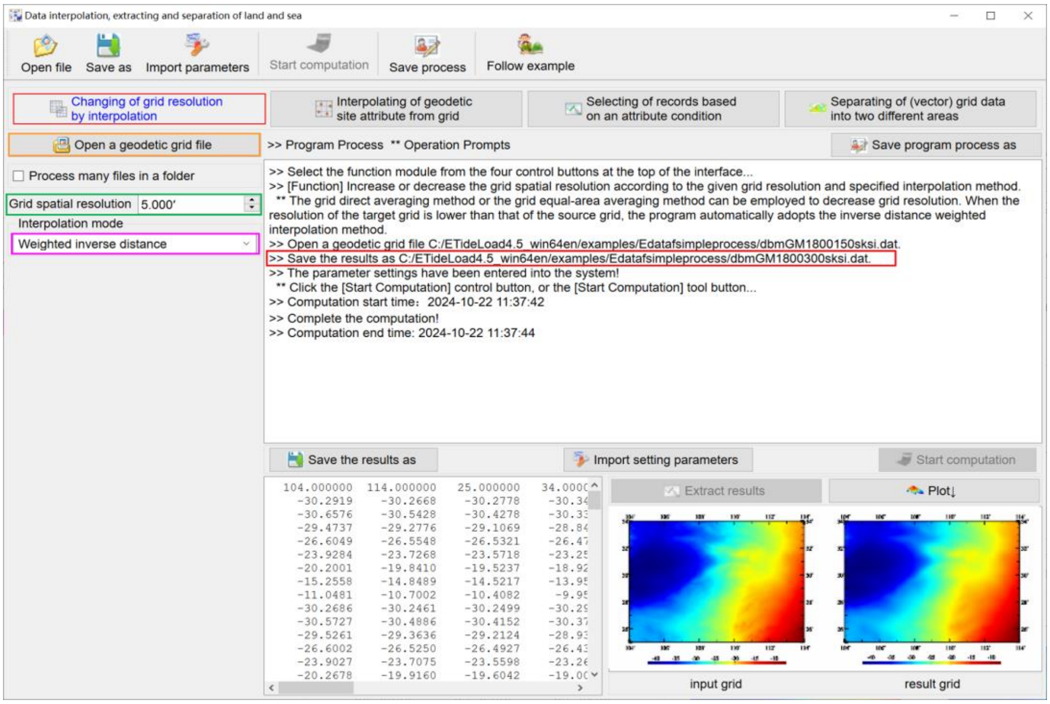
[Input file] The geodetic numerical grid file.

[Parameter settings] Enter the spatial resolution for target grid and select the interpolation mode.

[Output file] The target geodetic numerical grid file.

The grid direct averaging method is that sums up all the effective source cell-grid element values within the target cell-grid, and then divided them by the number of the effective source elements. The grid equal-area averaging method is that sums up all the effective source cell-grid element values within the target cell-grid, and then divided by the total number of source cell-grids within the target cell-grid.

It is recommended to adopt the grid equal-area averaging method when decreasing the spatial resolution of the surface loads.



The grid direct averaging method or the grid equal-area averaging method can be employed to decrease grid resolution. When the resolution of the target grid is lower than that of the source grid, the program automatically adopts the inverse distance weighted interpolation method.

6.2.2 Attribute interpolating of geodetic site from grid

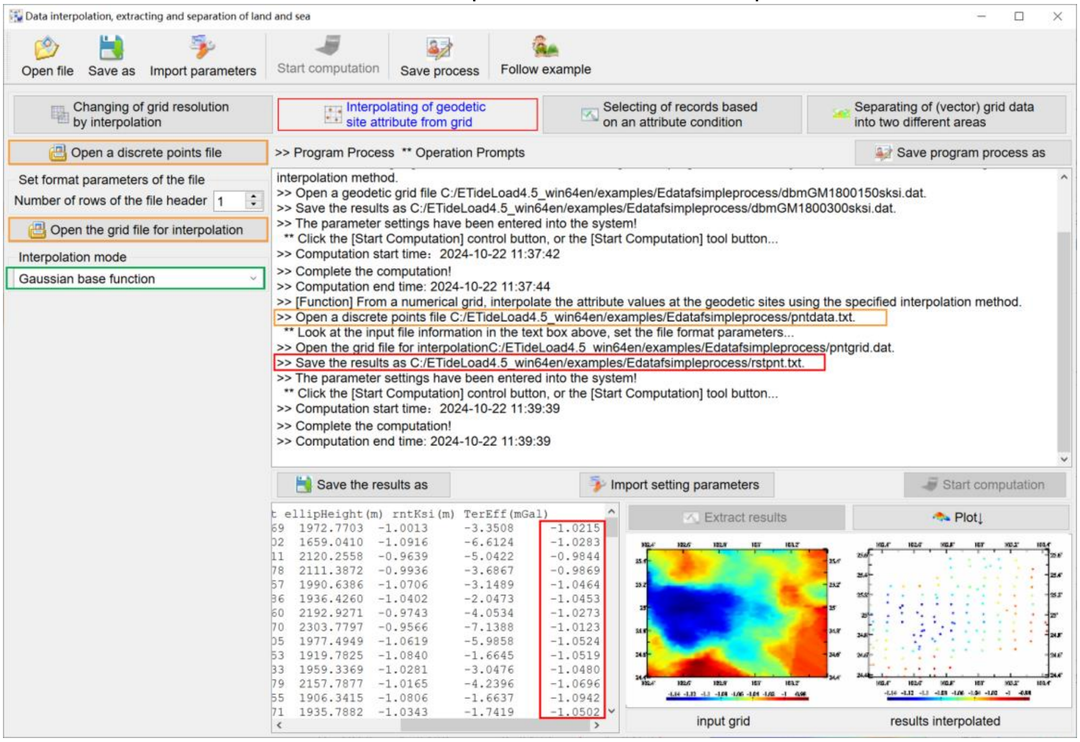
[Function] From a numerical grid, interpolate the attribute values at the geodetic sites according to the specified interpolation method.

[Input files] The discrete geodetic point file to be interpolated. The geodetic numerical grid file for interpolation.

[Parameter settings] Enter number of rows of the discrete geodetic point file header and select the interpolation mode.

[Output file] The interpolated discrete geodetic point file.

The file format is the same as the input discrete geodetic point file file. Behind the input file record, adds one column of the interpolated value as the output file record.



6.2.3 Selecting of records based on the attribute condition

[Function] Select the geodetic records from a geodetic record file according to the maximum and minimum range of the specified attribute.

[Input files] The discrete geodetic point file.

[Parameter settings] Enter number of rows of the input file header, column ordinal number of the condition attribute in the file record, and minimum and maximum of the attribute.

[Output file] The selected discrete geodetic point file.

The file format is the same as the input discrete geodetic point file file.

6.2.4 Separating of (vector) grid data into two different areas

[Function] According to the maximum and minimum range of the specified reference grid value, replace the source (vector) grid values with the given constant when the reference grid values are out of the range, to separate the source (vector) grid.

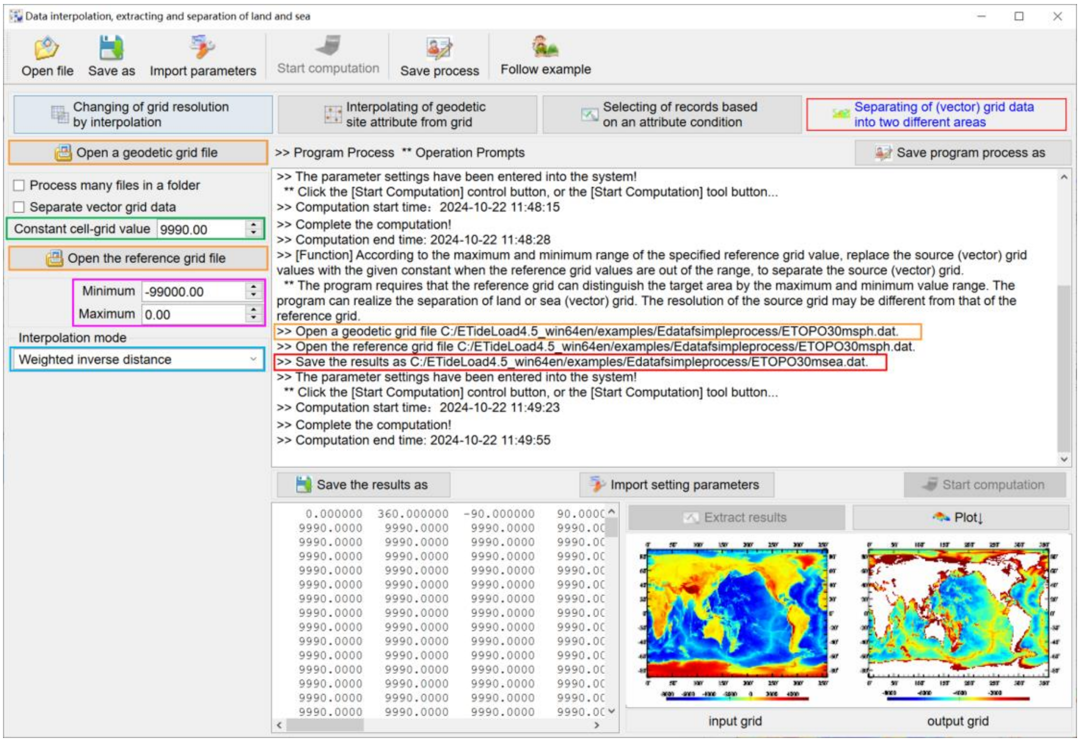
The program requires that the reference grid can distinguish the considered area by the maximum and minimum value range.

The program can realize the separation of land or sea (vector) grid.

[Input files] The source geodetic (vector) grid file. The reference grid file whose grid range and resolution are not smaller than that of the source grid file.

[Parameter settings] Enter the maximum and minimum value range and the constant grid value to replace the grid value out of the range. Select the interpolation mode.

[Output file] The separated geodetic (vector) grid file.



6.3 Simple and direct calculation on geodetic data files

6.3.1 Weighted operations on two specified attributes in record file

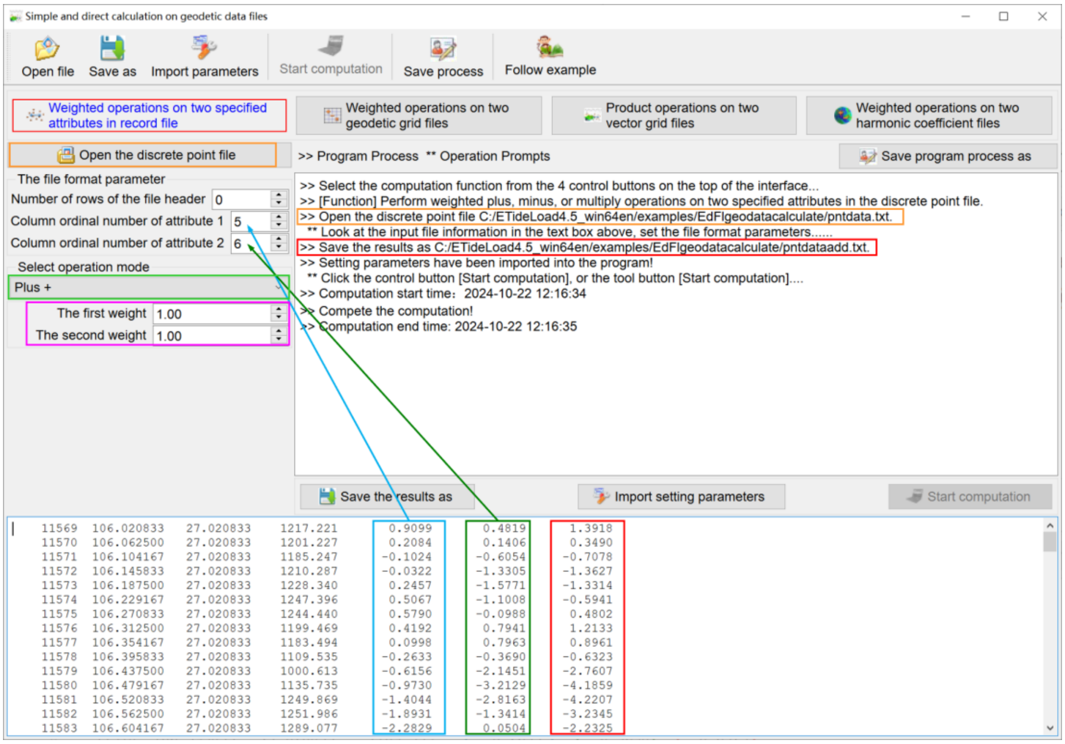
[Function] Perform weighted plus, minus, or multiply operations on two specified attributes in the discrete point file.

[Input file] The discrete geodetic point file.

[Parameter settings] Enter number of rows of the discrete geodetic point file header, column ordinal number and weight of the attribute 1, and that of attribute 2, and select the operation mode.

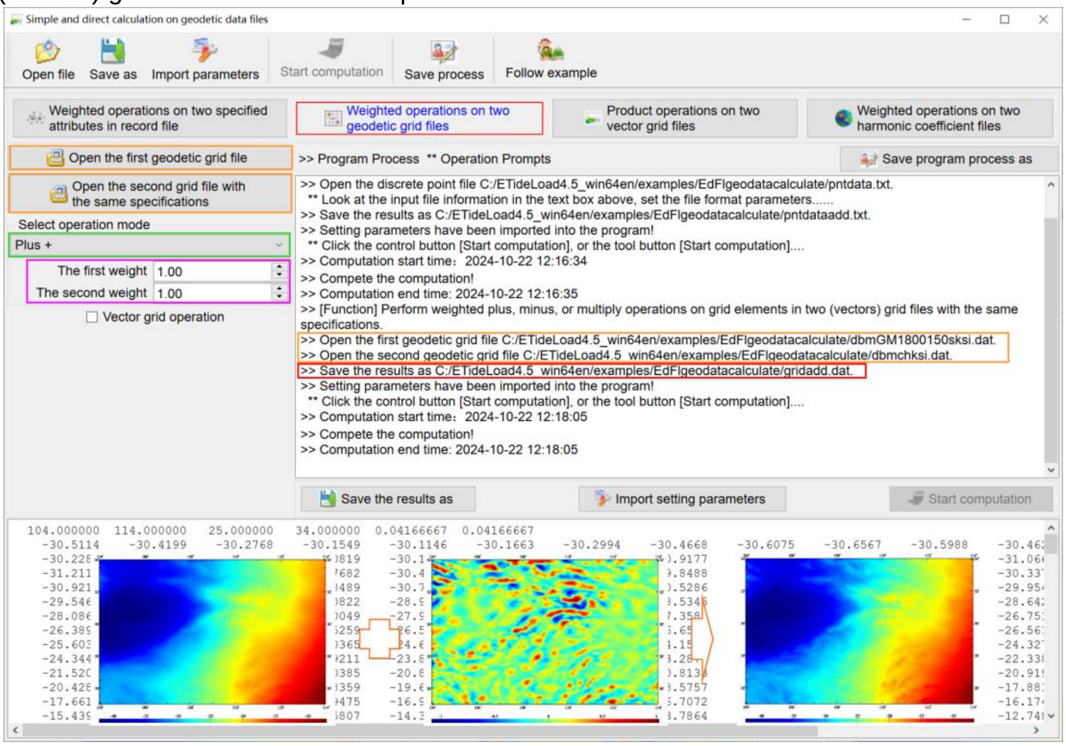
[Output file] The operated discrete geodetic point file.

The file format is the same as the input discrete geodetic point file. Behind the input file record, adds one column of the computed result as the output file record.



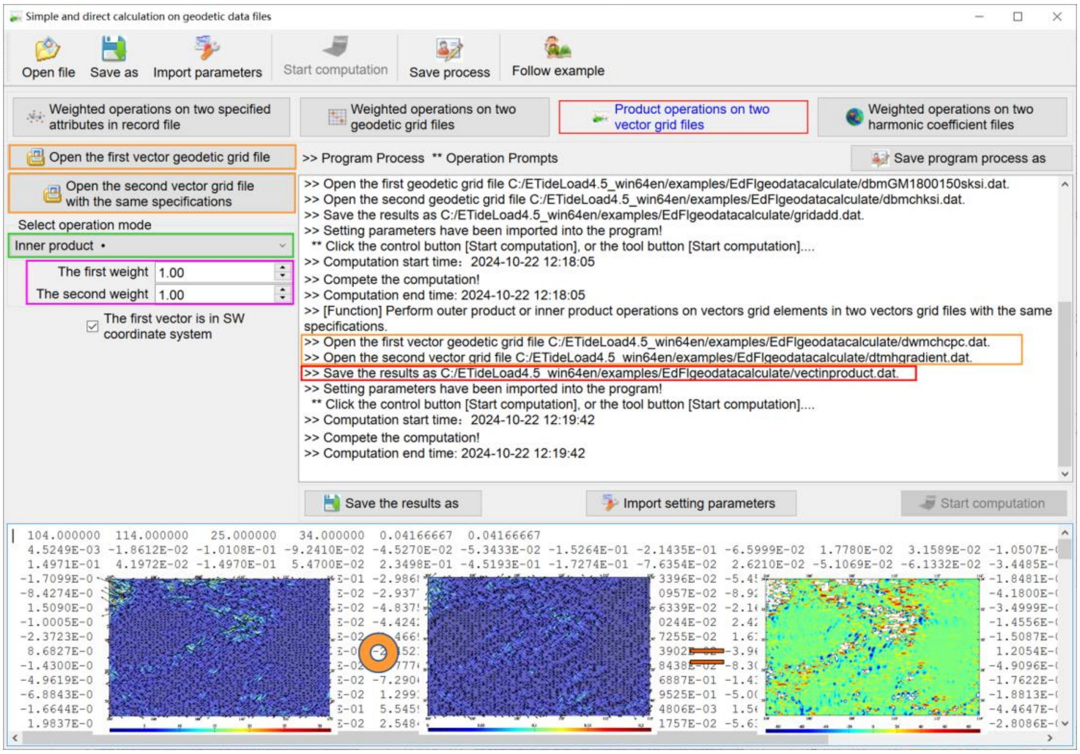
6.3.2 Weighted operations on two geodetic grid files

[Function] Perform weighted plus, minus, or multiply operations on grid elements in two (vectors) grid files with the same specifications.



6.3.3 Product operations on two vector grid files

[Function] Perform outer product or inner product operations on vectors grid elements in two vectors grid files with the same specifications.



6.3.4 Weighted operations on two harmonic coefficient files

[Function] Perform weighted operations on two normalized spherical harmonic coefficient model files.

The file header occupies a row and consists of two attributes as the scale parameters of the spherical harmonic coefficient model, namely the geocentric gravitational constant GM ($\times 10^{14}$m^2/s^2) and equatorial radius a(m) of the Earth.

6.4 Operations on variation time series with same specifications

6.4.1 Weighted operations on two record time series with same specifications

[Function] Perform weighted plus, minus or multiply operations on two variations at the same sampling epochs from two records time series.

The program requires that the records of two groups of time series are one-by-one correspondence in location and sampling epoch.

[Input files] The two groups of variation record time series files.

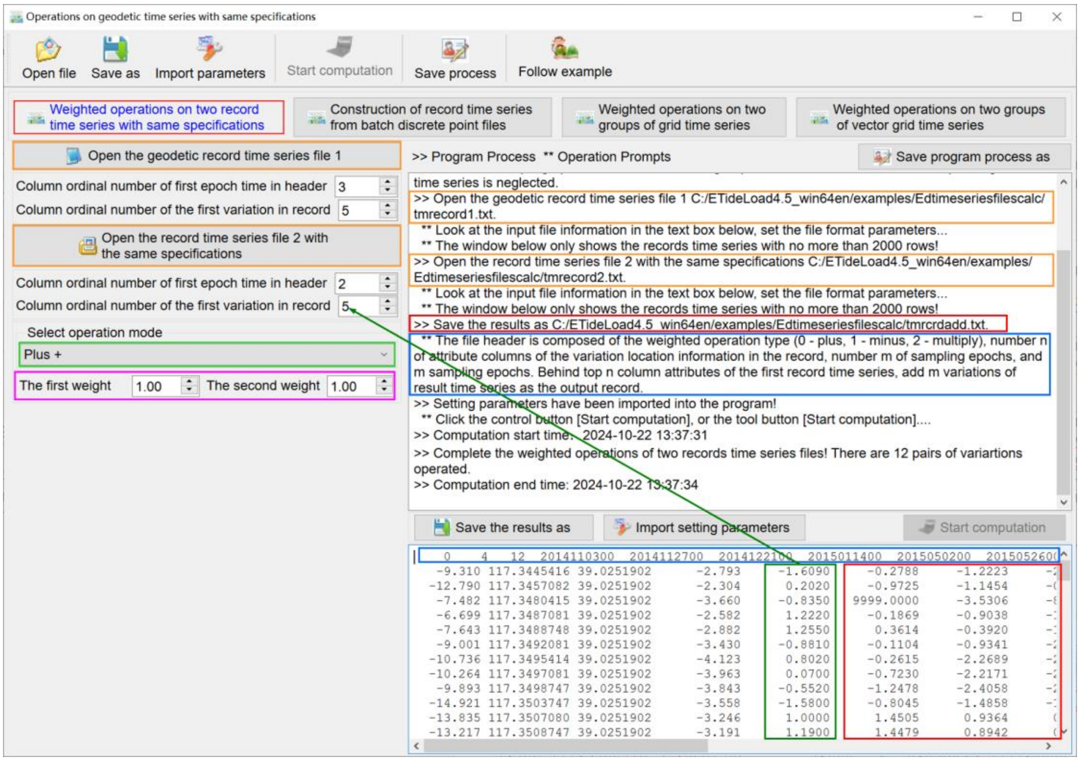
[Parameter settings] Set the record time series file format parameters, enter the weighs, and select operation mode.

If some a sampling epoch is not in both of two group of records time series, the corresponding variation time series is neglected.

[Output file] The operated variation record time series file.

The file header is composed of the weighted operation type (0 - plus, 1 - minus, 2 - multiply), number n of attribute columns of the variation location information in the record, number m of sampling epochs, and m sampling epochs.

Behind top n columns attributes of the first record time series file record, adds m variations of result time series as the output record.



6.4.2 Construction of record time series from batch discrete point files

[Function] From a series of discrete point files with the same specifications including the sampling epoch time, extract the specified attribute variation, and compose a time series by sorting with time, and then generate a record time series file with several kinds of variations.

[Input files] A series of discrete point files with the same specifications.

The program requires that the file header occupies a row that contains a sampling time epoch in ETideLoad format.

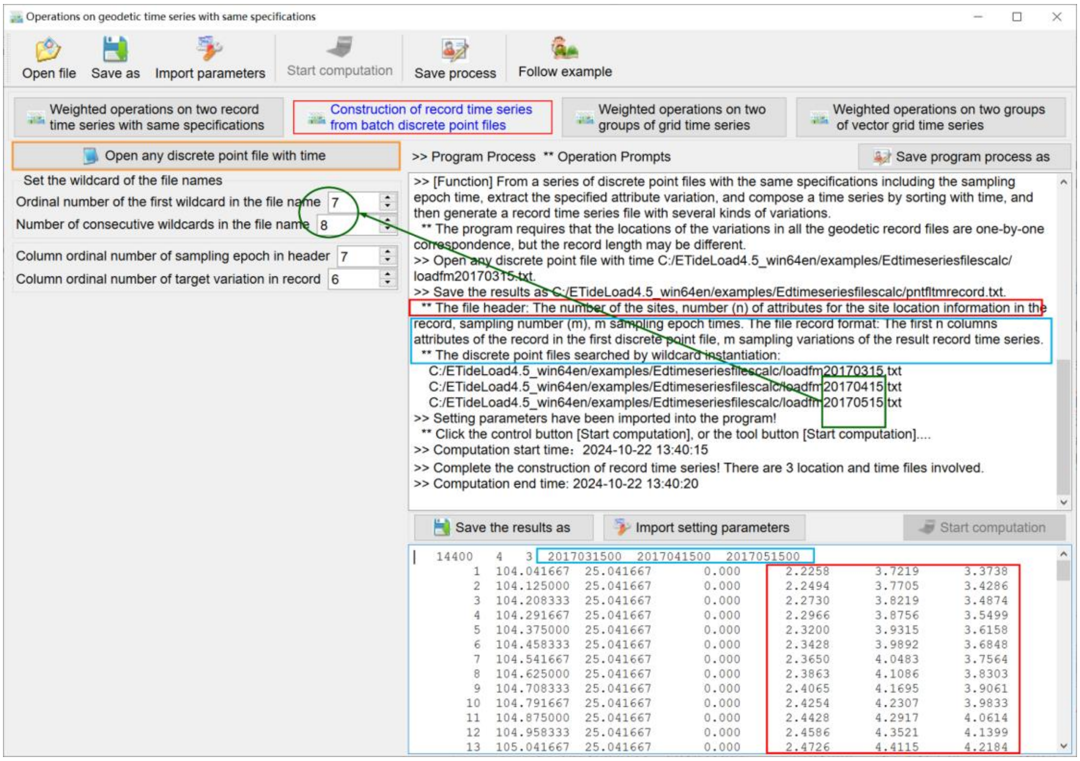
The program also requires that the locations of the variations in all the geodetic record files are one-by-one correspondence, but the record length may be different.

[Parameter settings] Set the wildcard parameters for a series of discrete point files and the file format parameters, enter column ordinal number of the epoch time in the input file header and target attribute time series in the input file record.

[Output file] The variation record time series file.

The output file header: The number of the sites, number (n) of attributes for the site location information in the record, sampling number (m), m sampling time epochs.

The output file record format: The first n columns of attributes of the record in the first discrete point file, m sampling variations of the result record time series.



6.4.3 Weighted operations on two groups of grid time series

[Function] From two groups of variation grid time series with the same grid specifications, sort the two groups of grids with time and then perform weighted plus, minus, or multiply operations.

The program automatically ignores the grid file whose sampling epoch is not one-by-one correspondence.

6.4.4 Weighted operations on two groups of vector grid time series

[Function] From two groups of variation vector grid time series with the same grid specifications, sort the two groups of vector grids with time and then perform weighted plus, minus, or outer production operations.

The program automatically ignores the vector grid file whose sampling epoch is not one-by-one correspondence.

[Input files] The two groups of variation vector grid time series files with the same grid specifications.

[Parameter settings] Set the wildcard parameters for variation vector grid time series files, enter the weighs, and select operation mode.

The weighted (w_1, w_2) outer product of two vectors is defined as $w_1(a_1, a_2) \times w_2(b_1, b_2) = w_1 a_1 b_2 - w_2 a_2 b_1$, which is a scalar.

[Output file] The variation (vector) grid time series files. Only when outer production operations, the program outputs the variation grid time series files.

6.5 Generating and constructing of regional geodetic grid

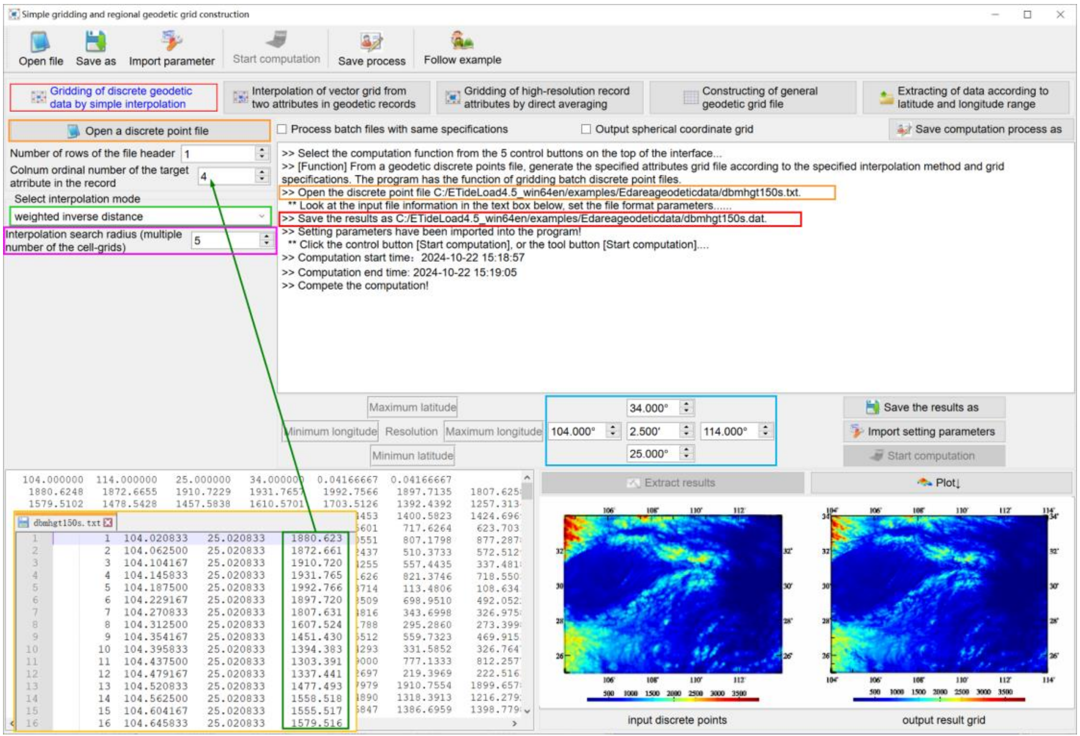
6.5.1 Gridding of discrete geodetic data by simple interpolation

[Function] From a geodetic discrete point record file, generate the specified attribute grid file according to the specified interpolation method and grid specifications.

[Input files] The discrete geodetic point file to be interpolated. The geodetic numerical grid file for interpolation.

[Parameter settings] Enter number of rows of the discrete point file header, column ordinal number of the target attribute in the file record, interpolation search radius (multiple of the cell-grid) and grid specifications parameters. Select the interpolation mode.

[Output file] The interpolated geodetic grid file.



6.5.2 Vector griding by interpolation from two attributes in geodetic records

[Function] From a geodetic discrete point file, generate the vector grid file according to the two specified component attributes, specified interpolation method and given grid specifications.

6.5.3 Gridding of high-resolution record attributes by direct averaging

[Function] Using the direct averaging method, grid the high-resolution discrete observations.

6.5.4 Constructing of general geodetic grid file

[Function] According to the given latitude and longitude range and spatial resolution, generate the constant, random number, 2D array index value or Gaussian surface grid file.

6.5.5 Extracting of data according to latitude and longitude range

[Function] According to the given latitude and longitude range, extract data from the discrete point file, grid file or vector grid file. The program can extract data from batch files.

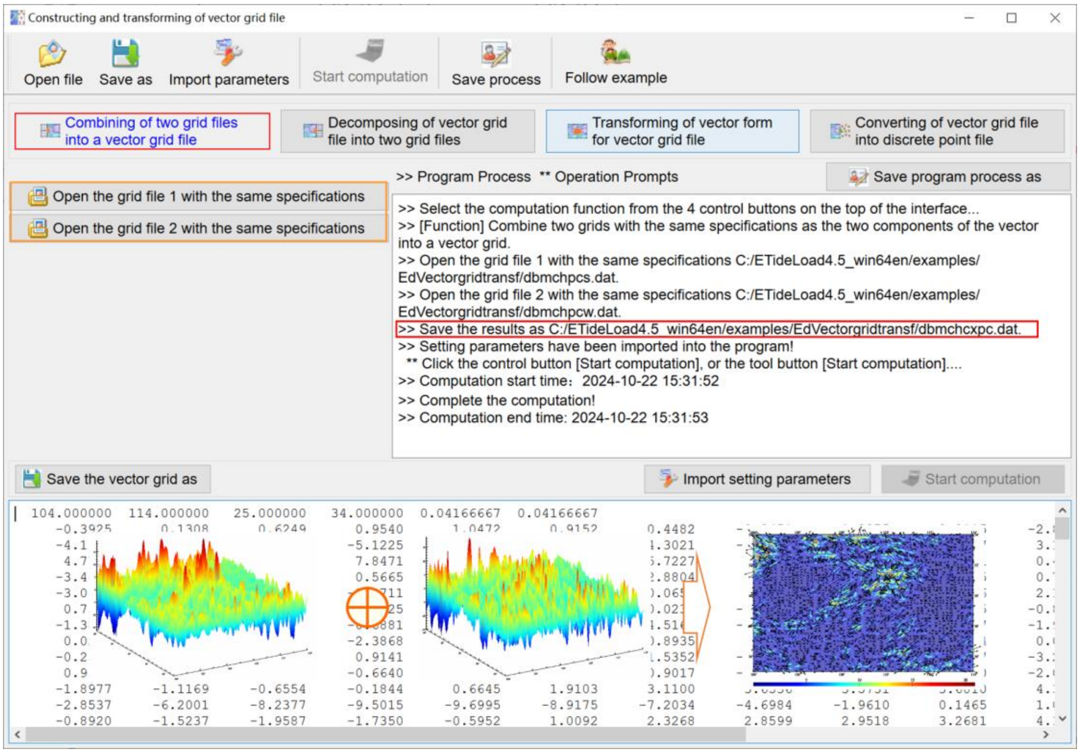
6.6 Constructing and transforming of vector grid file

6.6.1 Combining of two grid files into a vector grid file

[Function] Combine two grids with the same specifications as the two components of the vector into a vector grid.

[Input files] The two geodetic grid files.

[Output file] The geodetic vector grid file.



6.6.2 Decomposing of vector grid file into two grid files

[Function] Decompose a vector grid file into two component grid files.

[Input file] The geodetic vector grid file.

[Output files] The two geodetic grid files.

6.6.3 Transforming of vector form for vector grid file

[Function] Transform the vectors in a vector grid file between plane coordinates (in-phase/out-of-phase amplitude) and polar coordinates (amplitude/phase).

6.6.4 Converting of vector grid file into discrete point file

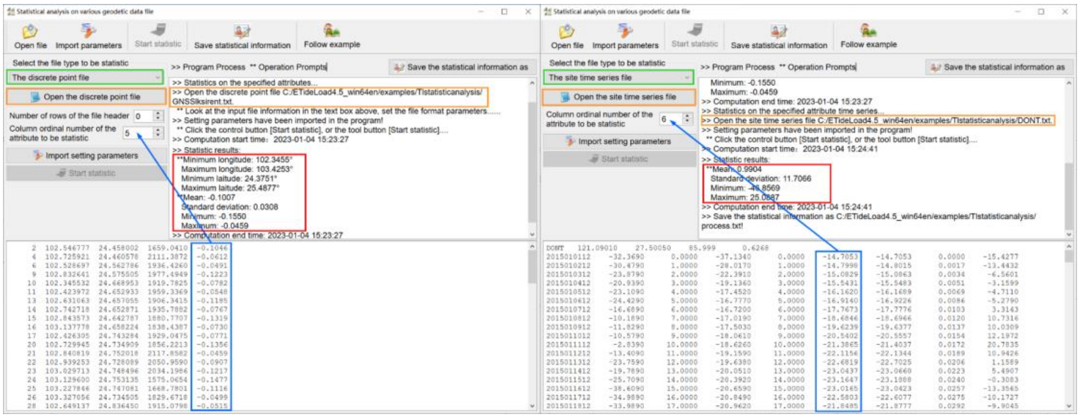
[Function] Convert the (vector) grid file into the discrete point file.

[Input file] The geodetic (vector) grid file.

[Output file] The discrete point record file.

6.7 Statistical analysis on various geodetic data files

[Purpose] Extract the latitude and longitude range, mean, standard deviation, minimum, maximum and other statistical information from the specified attributes of the discrete point file, geodetic grid file or vector grid file.



6.8 Gross error detection and weighted basis function gridding

6.8.1 Gross error detection on observations based on low-pass reference surface

[Function] Select the low-pass grid as the reference surface, interpolate the reference value of the specified attribute at the discrete point, and then detect and separate the gross error records according to the statistical properties of the differences between the specified attributes and interpolated reference values.

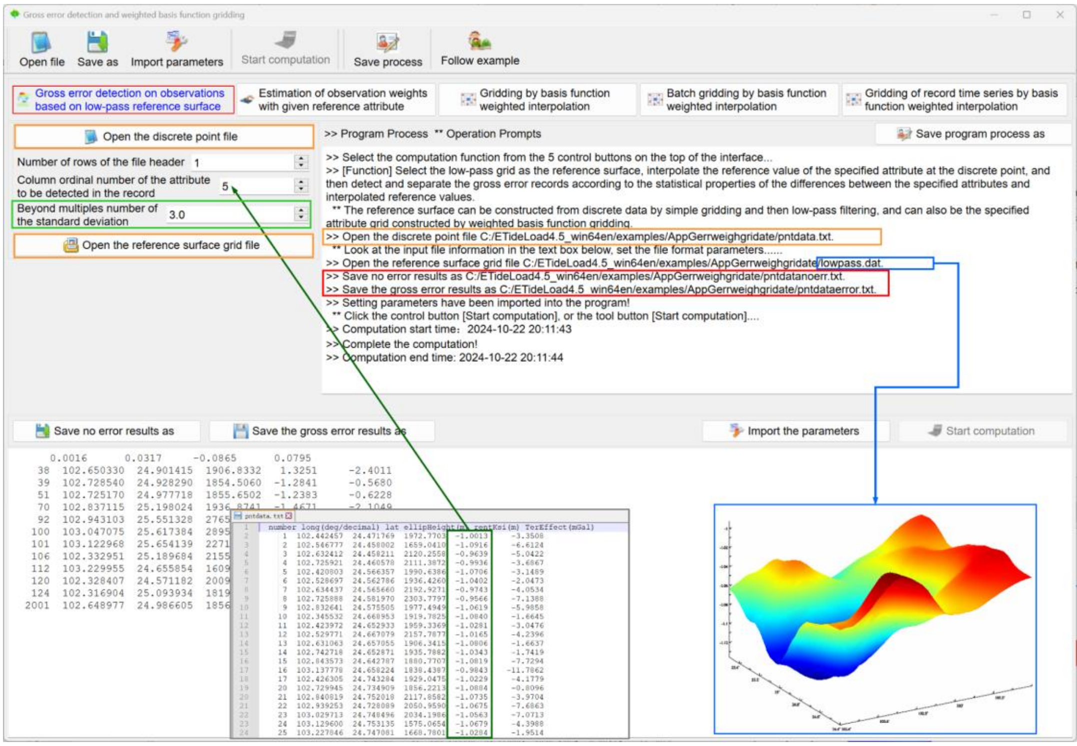
[Input files] The discrete geodetic point file to be detected. The low-pass reference surface grid file.

The reference surface can be constructed from discrete data by simple gridding and then low-pass filtering, and can also be the specified attribute grid constructed by weighted basis function gridding.

[Parameter settings] Enter number of rows of the discrete geodetic point file header, column ordinal number of the attribute to be detected in the record, and beyond multiples of the standard deviation.

When the absolute value of the difference between the attribute and its mean is greater than n times standard deviation, the record in which attribute is a gross error record.

[Output file] The discrete geodetic point file without gross error, whose format is the same with the input discrete point file. The gross error point file, whose file header include the mean, standard deviation, minimum and maximum of the differences.



6.8.2 Estimation of observation weights with given reference attribute

[Function] Using the weight function defined by ETideLoad, estimate the observation weights according to the statistical property of the specified reference attribute in the input geodetic record file.

Weight function defined by ETideLoad4.5 $w(x,a)=10\sigma\sqrt{\sigma^2+(ax)^2}$, here x is the reference attribute, a is the given smoothing factor of the weight function, σ is the standard deviation of x calculated automatically by the program.

The larger the weight function smoothing factor a, the slower the weight function w decays with distance.

6.8.3 Gridding by basis function weighted interpolation

[Function] According to the given grid specifications (grid range and spatial resolution), and specified basis function, grid the specified attribute in the input discrete geodetic record file by the weighted basis function interpolation method.

[Input files] The discrete geodetic point file.

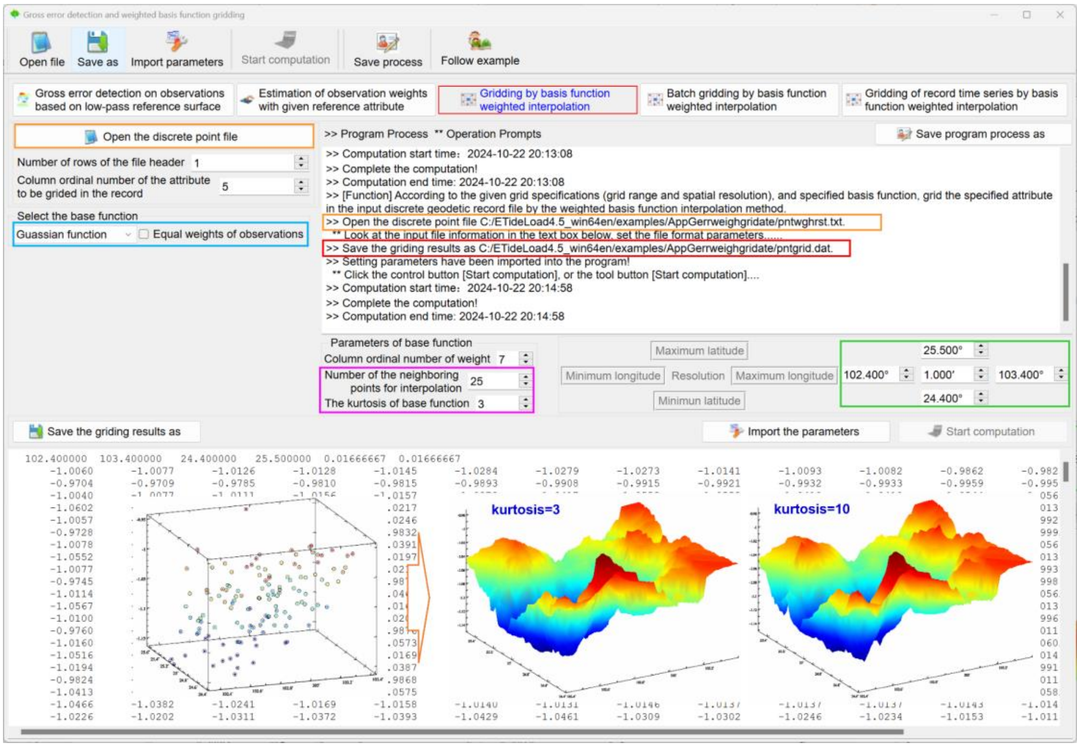
[Parameter settings] Enter number of rows of the discrete point file header, column ordinal number of the target attribute in the file record and grid specifications parameters, and set the base function and its parameters.

The smaller the kurtosis is (the slower the basis function decays with distance), the larger the number of neighboring points, the smoother the interpolation, the weaker the edge effect, and the stronger the interpolation ability for sparse data.

The interpolation weight is equal to the product of the attribute weight and base function.

[Output file] The geodetic grid file.

The program of the gridding by basis function weighted interpolation is specially designed by ETideload based on the properties of general geophysical field, which is suitable for griding of single type of multi-source heterogeneous geophysical data.



6.8.4 Batch gridding by basis function weighted interpolation

[Function] According to the given grid specifications, base function, and other parameters, respectively grid the specified attribute in each of the input discrete point files saved in a directory by the weighted basis function interpolation method.

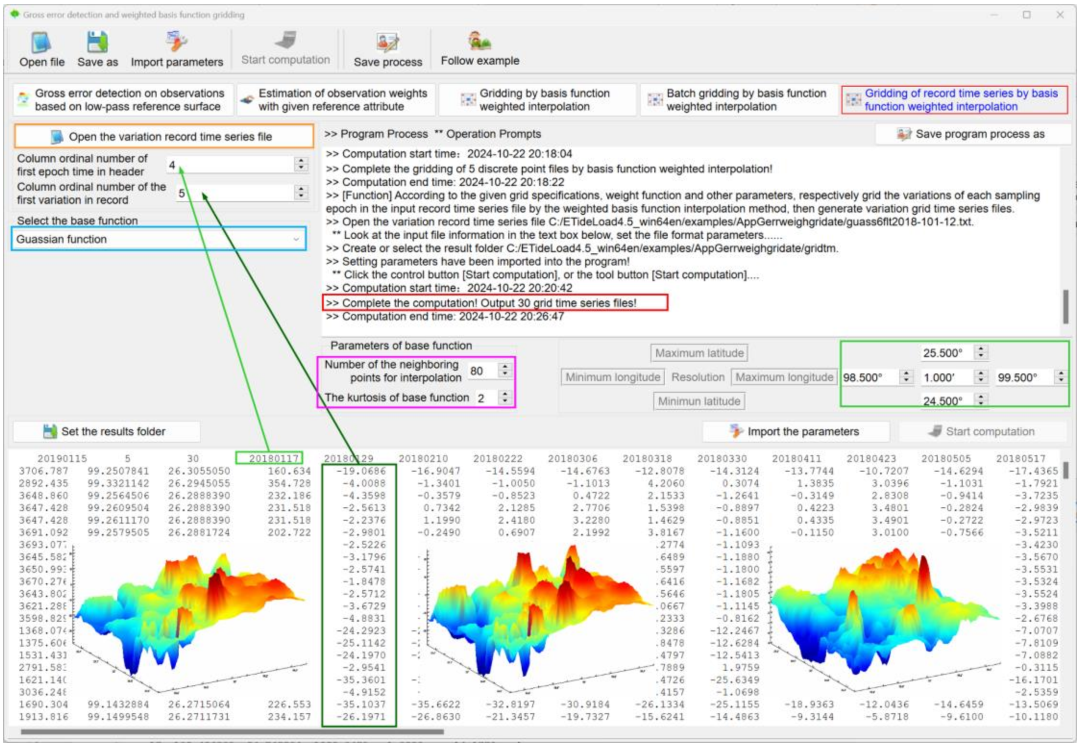
[Input files] Batch discrete geodetic point files with same format.

[Parameter settings] Set the wildcard parameters for batch discrete geodetic point files, enter number of rows of the discrete point file header, column ordinal number of the target attribute in the file record and grid specifications parameters. Select the base function, and set the number of the neighboring points and kurtosis of base function.

[Output files] A series of numerical grid files bsfgrd***.dat that correspond one-to-one with the input discrete point value files. Here, *** are the instance of the input discrete point file name wildcards.

6.8.5 Gridding of record time series by basis function weighted interpolation

[Function] According to the given grid specifications, weight function and other parameters, respectively grid the variations of each sampling epoch in the input record time series file by the weighted basis function interpolation method, then generate variation grid time series files.



6.9 Visualization plot tools for various geodetic data files

6.9.1 Visualization for multi-attributes in ground variation time series

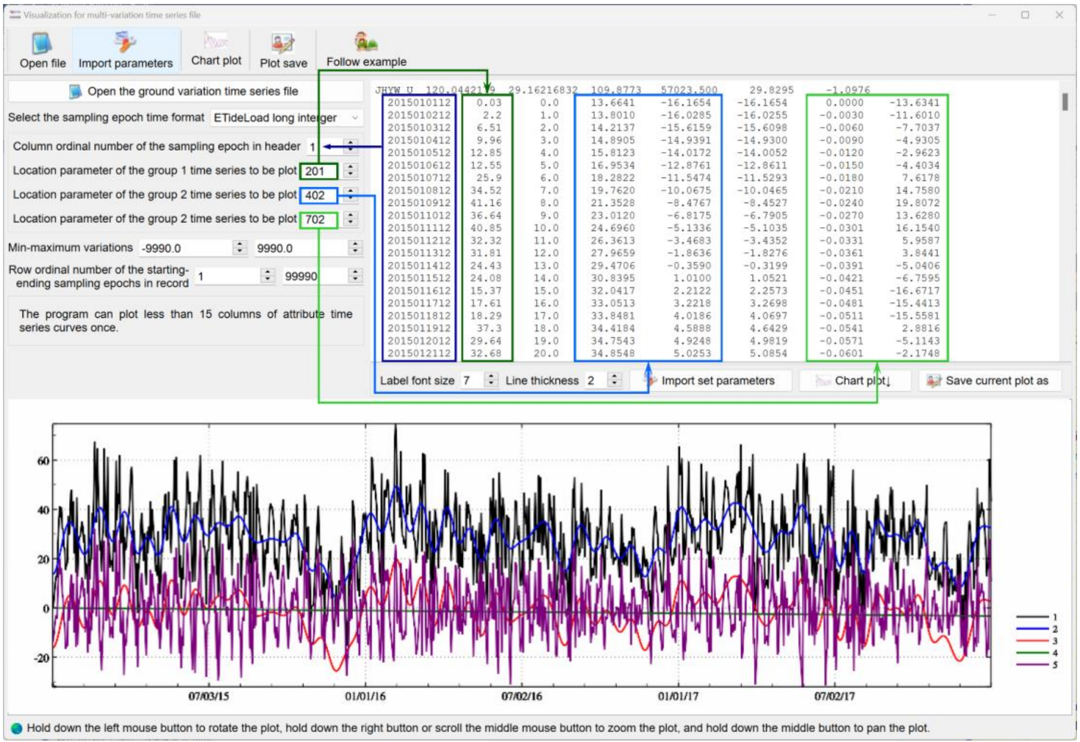
[Function] Plot multi-variation time series curves stored in a ground geodetic variation time series file.

The program can plot less than 15 columns of attribute time series curves once.

[Parameter settings] Select the sampling epoch time format, enter the column number of the sampling epoch time in the file record, set the location parameters of the attribute time series plotted in the file record, and enter minimum-maximum of the variations to be plotted and row ordinal number of the starting-ending sampling epochs.

When the location parameter corresponding to the column ordinal number in the record is greater than the record maximum column number, the program automatically set the location parameter as the serial number of the record maximum column.

When the row ordinal number of the ending sampling variation is greater than the number of samples of the time series, the program automatically set the row ordinal number of samples to be plotted of the time series as the row ordinal number of the ending sampling variation.



Hold down the left mouse button to rotate the plot, hold down the right button or scroll the middle mouse button to zoom the plot and hold down the middle button to pan the plot.

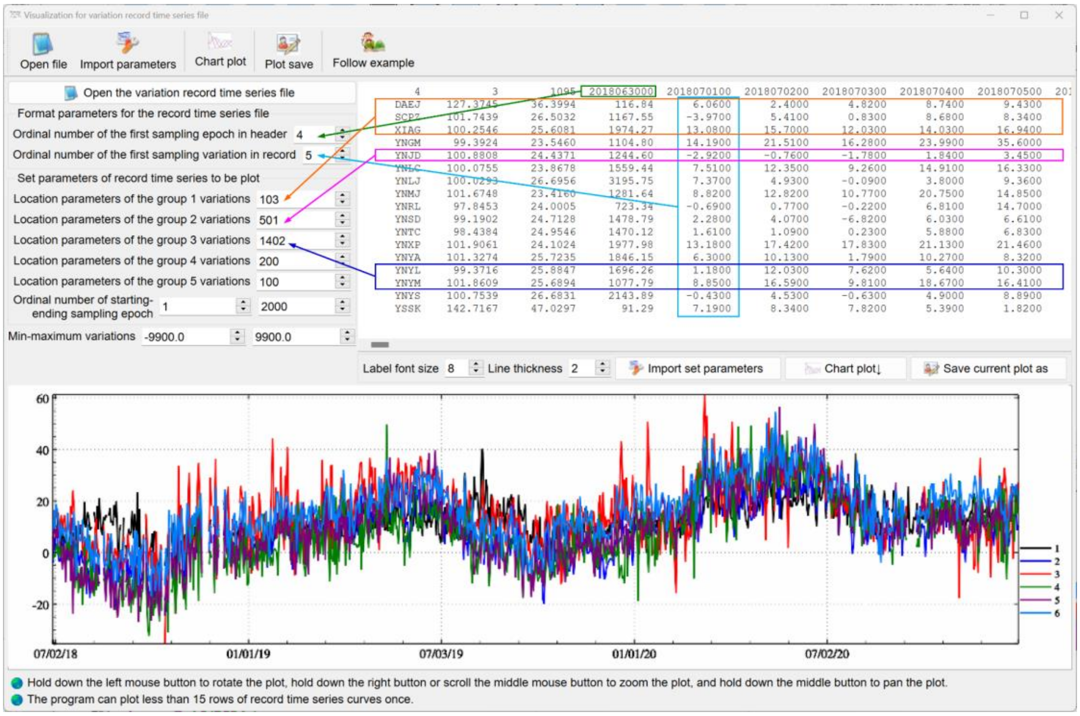
6.9.2 Visualization for variation record time series on geodetic network

[Function] Plot multi-variation time series curves stored in a geodetic variation record time series file.

[Parameter settings] Enter the ordinal number of the first sampling epoch in file header and the first sampling variation in record, set the location parameters of the time series to be plotted, and enter the ordinal number of starting-ending sampling epochs and minimum-maximum of the variations to be plotted.

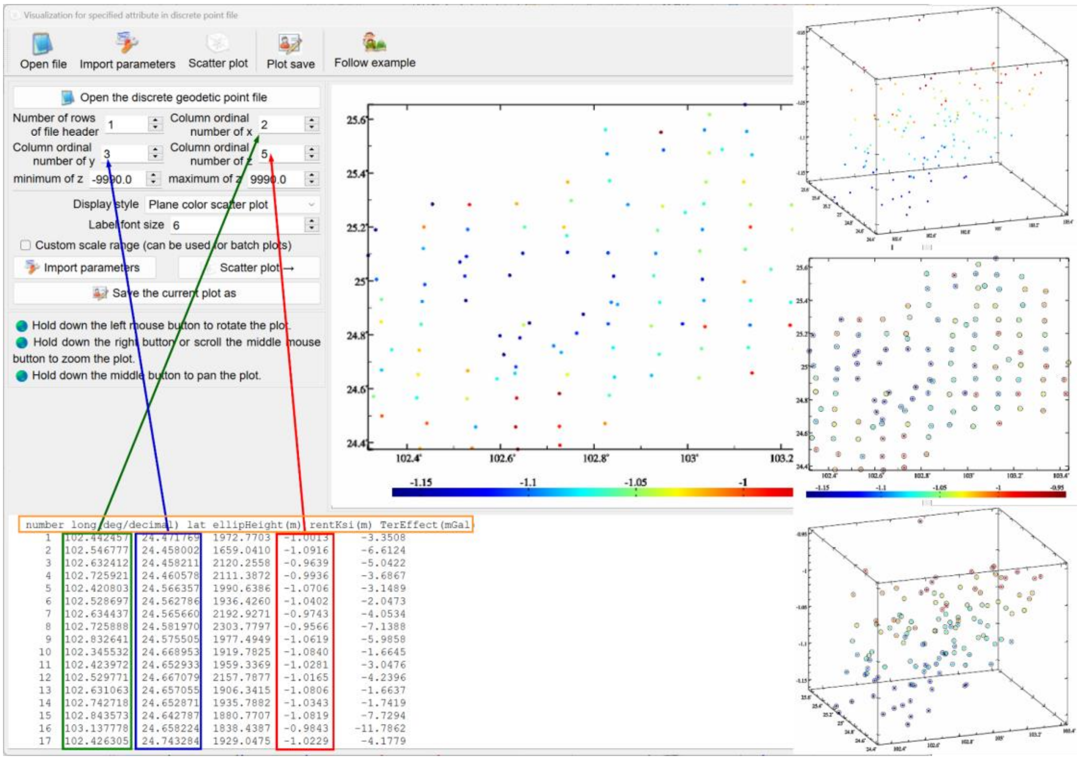
When the location parameter corresponds to the row ordinal number of the record is greater than the number of rows of the file records, the program automatically set number of rows of the file records as the row ordinal number of the last record.

The program can plot less than 15 rows of record time series curves once. When different groups of location parameters correspond to the same row, the program can automatically merge, count and plot according to one row of record time series.



6.9.3 Visualization for specified attribute in discrete point file

[Function] Display the point locations and their specified attributes in a geodetic discrete point file.



After changing the input data file, z attribute or other parameters, you need to click the control button [Import setting parameters] again to update the plot.

If needing a larger scale plot, enlarge the graphics window on the right firstly, and then click the control button [Scatter plot].

Hold down the left mouse button to rotate the plot, hold down the right button or scroll the middle mouse button to zoom the plot and hold down the middle button to pan the plot.

You can unify the scales by fixing the scale range for batch plots. Adjust the size of the graphics window on the right and the plot elements to an appropriate state before drawing batch plots. During plot period, the parameters and the size of the graphics window are kept unchanged, and no mouse operation is performed on the plot.

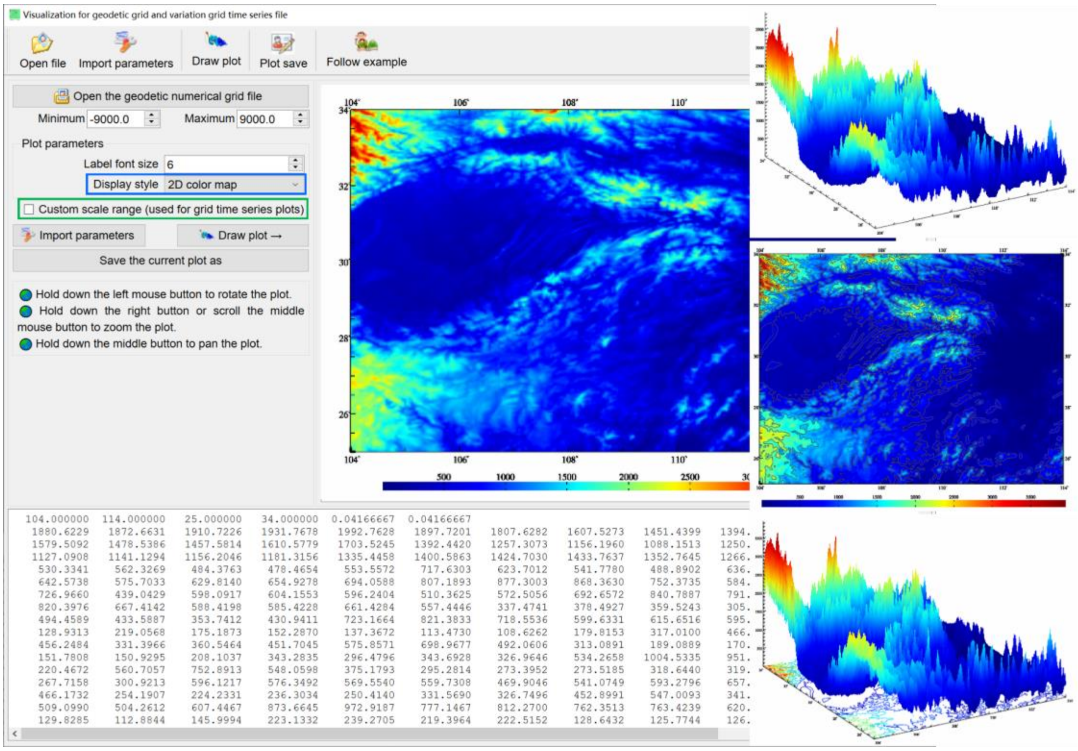
6.9.4 Visualization for geodetic grid and variation grid time series file

[Function] Plot for geodetic grid or grid time series files.

[Parameter settings] Select the display style and set the checkbox [Custom scale range (used for grid time series plots)].

The program allows the first component of a vector grid to be displayed as grid data.

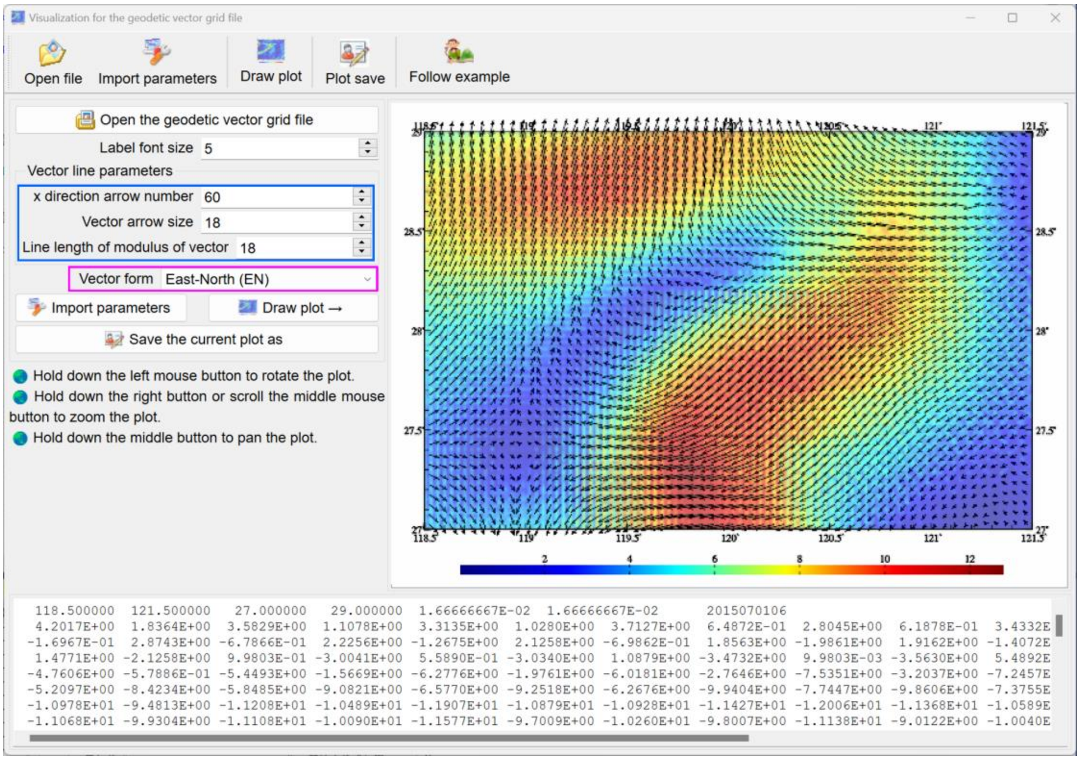
After setting the custom scale range, you can plot a series of grid time series plots.



6.9.5 Visualization for the geodetic vector grid file

The X-axis and Y-axis of the plotting coordinate system respectively point to east and to north (EN), which is the same with horizontal displacement vector.

Vector form: East-North (EN, e.g., horizontal displacement vector), South-West (SW, e.g., vertical deflection vector), North-East (NE, e.g., Tangential gravity gradient vector).



7 Data file format, geophysical models and numerical standards

7.1 Geodetic Data Files in ETideLoad own Format

ETideLoad only recognizes the five kinds of geodetic stationary data and five kinds of geodetic variation time series data in ETideLoad own format. The geodetic stationary data files include the discrete geodetic point record file, geodetic network observation record file, geodetic numerical grid file, geodetic vector grid file and spherical harmonic coefficient (Stokes coefficient) model file. The variation time series files include the ground variation time series file, geodetic network site record time series file, geodetic network observation time series file, variation (vector) grid time series files and spherical harmonic coefficient (Stokes coefficient) model time series files.

The program [Conversion of general ASCII record data into ETideLoad format], and the function [Normalized extraction of batch time series of geodetic monitoring network] are the important interfaces for ETideLoad to accept external text data. Using the function [Global prediction of solid Earth tidal effects on various geodetic variations] or [Global prediction of surface atmosphere tidal load effects on various geodetic variations], you can construct a ground variation time series with the given location and sampling specifications. Using the program [Generating and constructing of regional geodetic grid], you can construct a numerical grid with the given grid specifications. The other programs or functions only accept the format data generated by ETideLoad own.

7.2 The file format of 5 kinds of stationary geodetic data

7.2.1 The discrete geodetic point record file

A discrete geodetic point record file can store multiple element attributes of massive discrete space points. The attributes of each point are represented by a row of record.

pntdata.txt

number	long(deg/decimal)	lat	ellipHeight(m)	rentKsi(m)	TerEffect(mGal)
1	102.442457	24.471769	1972.7703	-1.0013	-3.3508
2	102.546777	24.458002	1659.0410	-1.0916	-6.6124
3	102.632412	24.458211	2120.2558	-0.9639	-5.0422
4	102.725921	24.460578	2111.3872	-0.9936	-3.6867
5	102.420803	24.566357	1990.6386	-1.0706	-3.1489
6	102.528697	24.562786	1936.4260	-1.0402	-2.0473
7	102.634437	24.565660	2192.9271	-0.9743	-4.0534
8	102.725888	24.581970	2303.7797	-0.9566	-7.1388
9	102.832641	24.575505	1977.4949	-1.0619	-5.9858
10	102.345532	24.668953	1919.7825	-1.0840	-1.6645
11	102.423972	24.652933	1959.3369	-1.0281	-3.0476
12	102.529771	24.667079	2157.7877	-1.0165	-4.2396
13	102.631063	24.657055	1906.3415	-1.0806	-1.6637
14	102.742718	24.652871	1935.7882	-1.0343	-1.7419
15	102.843573	24.642787	1880.7707	-1.0819	-7.7294
16	103.137778	24.658224	1838.4387	-0.9843	-11.7862

(1) Multiple rows of the file headers are allowed, whose content and format are not restricted.

(2) One record represents the geodetic data at one site. The attributes of each record include site number (name), longitude (degree decimal), latitude (degree decimal), attribute 4, …, attribute n.

(3) The attribute convention is a numeric format, the number (n) of the attributes is not more than 80, and the attributes are separated by spaces.

(4) A record reading code in Fortran is:

```
read(fileno,*)(record(i),i=1,n)    ! real*8 record(n)
```

7.2.2 The geodetic network observation file

A geodetic network observation file can store the baseline component data for the CORS network, height differences for the levelling network or gravity differences for the gravity network.

(1) The file header occupies a row and includes the number of characters of the baseline or route name, number of characters of the site name, …

(2) The file record includes the baseline or route name, starting site (longitude, latitude, height), ending site (longitude, latitude, height), …, observations (default value is 9999).

(3) The relations between the baselines (or routes) and the sites in the geodetic monitoring network are reflected with the composition of the characters of their name. A baseline or route name (B***A) is agreed to be composed of site names A and B at both ends, where the number of characters of all the sites names is required to be equal.

Therefore, the number of characters of the baseline or route name should not be less than twice that of the site name.

7.2.3 The geodetic numerical grid file

(1) There is a row of file header at the beginning of the file. The file header includes minimum longitude, maximum longitude, minimum latitude, maximum latitude, longitude interval of a cell-grid and latitude interval of a cell-grid. The units of all the attributes are decimal degrees.

(2) The cell-grid elements are sequentially stored in an increasing manner of row latitude and column longitude until all data is stored. The elements are separated by spaces.

(3) The Fortran reading program for the entire grid data in a geodetic grid file:

```
open(unit=fileno,file=filename,status="old")
read(fileno,*)(hd(i),i=1,6)            ! hd(6) - the file header
nlon=nint((hd(2)-hd(1))/hd(5))   ! nlon - the number of cell-grid columns along longitude direction
nlat=nint((hd(4)-hd(3))/hd(6))     ! nlat - the number of cell-grid rows along latitude direction
do i=1,nlat
   read(fileno,*)(gr(i,j),j=1,nlon)   ! gr(nlat,nlon)- two dimension array employed to store grid values
enddo
```

The grid value of cell-grid represents the mean value of the cell-grid. In the numerical integral operation, the location of the center point of the cell-grid is employed to calculate the integral distance from the cell-grid (the moving point) to the calculation point.

dbmhgt150s.dat

104.000000	114.000000	25.000000	34.000000	0.04166667	0.04166667
1880.6233	1872.6612	1910.7203	1931.7653	1992.7665	1897.7199
1579.5158	1478.5360	1457.5736	1610.5877	1703.5435	1392.4407
1127.0862	1141.1257	1156.1979	1181.3065	1335.4466	1400.5901
530.3264	562.3283	484.3702	478.4546	553.5518	717.6379
642.5849	575.7052	629.8202	654.9330	694.0609	807.1985
726.9670	439.0212	598.0862	604.1542	596.2404	510.3528
820.4032	667.4105	588.4110	585.4184	661.4350	557.4490
494.4559	433.5850	353.7288	430.9312	723.1754	821.3956
128.9223	219.0560	175.1799	152.2779	137.3618	113.4669
456.2471	331.3871	360.5383	451.7036	575.8641	698.9905
151.7805	150.9271	208.1027	343.2925	296.4793	343.6893
220.4542	560.7228	752.9326	548.0788	375.1834	295.2821
267.7073	300.9139	596.1386	576.3569	569.5556	559.7308
466.1608	254.1723	224.2118	236.2868	250.4018	331.5582
509.1123	504.2678	607.4595	873.6999	972.9491	777.1609
129.8216	112.8806	145.9967	223.1369	239.2738	219.4003
1868.6248	1859.6737	1903.7419	2051.7911	2088.7992	1910.7605
1475.5124	1382.5200	1476.5441	1626.5437	1580.4903	1318.3843
1155.2158	1193.2735	1209.3569	1255.4640	1365.5851	1386.7009
603.2370	489.2430	499.3152	525.4287	630.5491	820.6481
436.4368	493.5515	611.6602	584.7656	649.9001	678.0571

7.2.4 The geodetic vector grid file

A geodetic vector grid file is composed of the first component grid and the second component grid of the vector. The header file and the first component grid in the vector grid file are same as that in the geodetic grid file, and the second component grid follow the first component grid closely with the same way.

Vector grid such as vertical deflection and horizontal gradient vector grid in ETideLoad are stored in the form of vector grid file.

7.2.5 The spherical harmonic coefficient model file

(1) The file header occupies a row and consists of two attributes as the scale parameters of the spherical harmonic coefficient model, namely the geocentric gravitational constant GM ($\times10^{14}$m^2/s^2) and equatorial radius a(m) of the Earth.

(2) The Earth's geopotential coefficient model and surface load spherical harmonic coefficient model in ETideLoad are stored in the form of spherical harmonic coefficient file.

(3) GM, a are the scale parameters of the model. Here, the surface harmonic functions in the spherical harmonic coefficient model are defined on the spherical surface whose radius is equal to the equatorial radius a of the Earth.

(4) The degree-n order-m spherical harmonic coefficient is expressed by a record with the format "degree n, order m, C_{nm}, S_{nm} (, C_{nm} error, S_{nm} error)".

ETideLoad does not require the degrees and orders of harmonic coefficients to be arranged and allows to exist insufficient orders. For the harmonic coefficient of insufficient order, ETideLoad automatically sets to zero.

7.3 The file format of 5 kinds of geodetic variation time series

The geodetic variation time series files adopt the ETideLoad own format, which include the ground variation time series file, geodetic network site record time series file, geodetic network observation record time series file, variation (vector) grid time series files and spherical harmonic coefficient (Stokes coefficient) model time series files.

7.3.1 The ground variation time series file

A ground variation time series file can store the time series data of several kinds of variations at a certain site, a certain baseline or route, and the sampling epochs (here, the epoch is an instantaneous time) of these variations are the same. Such as CORS station coordinate solution time series, solid tide station observation or analysis result time series, GNSS baseline solution time series, etc.

(1) The file header occupies a row and includes the site name, longitude (degree decimal), latitude (degree decimal), height (m) relative to the ellipsoidal surface (sea level, or the ground), the starting MJD (optional), …

(2) Starting from the second row of the file, each row of record stores the sampling values of all the variations at one sampling epoch time. At least one column of attribute in the record is the sampling epoch time.

(3) Each attribute in the record (except the sampling epoch time) represents a type of variation time series, and the sampling epoch time of different types of variations is the same.

(4) The sum of the starting MJD0 in the header and the sampling epoch time (day) is equal to the sampling epoch time of MJD day in the record. When the sampling epoch time is the long integer agreed by ETideLoad, the starting MJD0 is not necessary in the file header.

```
tmsqurst.txt
NYB 101.230000    29.910000  47.218  58484.000000
2019010100    0.000000    2.764     5.0173     1.5712     0.3849    10.3234    -5.2424    19.3396
2019010101    0.041667    2.778    57.5174    23.2452    10.8146    12.7102    -3.1452    23.8048
2019010102    0.083333    2.762    75.5361    30.5675    14.2375    13.9736     0.1439    26.1911
2019010103    0.125000    2.724    49.9989    19.8264     8.9137    13.8655     3.6134    26.0285
2019010104    0.166667    2.675   -14.8626    -7.1040    -4.1257    12.4479     6.1922    23.4162
2019010105    0.208333    2.626  -102.4140   -43.2138   -21.3751    10.0693     7.0460    18.9862
2019010106    0.250000    2.582  -187.9254   -78.2261   -37.8487     7.2674     5.8002    13.7343
2019010107    0.291667    2.546  -245.0339  -101.3204   -48.4404     4.6323     2.6332     8.7685
2019010108    0.333333    2.517  -252.6506  -103.9667   -49.2598     2.6643    -1.7744     5.0436
2019010109    0.375000    2.489  -200.5663   -82.1562   -38.6091     1.6655    -6.4133     3.1473
2019010110    0.416667    2.455   -92.4143   -37.5190   -17.4012     1.6833   -10.1887     3.1848
2019010111    0.458333    2.410    54.6679    22.8880    11.0186     2.5175   -12.1759     4.7817
2019010112    0.500000    2.354   213.5656    88.0350    41.5647     3.7816   -11.8314     7.1990
2019010113    0.541667    2.288   353.0904   145.2782    68.4536     5.0033    -9.1200     9.5289
2019010114    0.583333    2.223   444.9509   183.1245    86.4011     5.7439    -4.5343    10.9299
2019010115    0.625000    2.169   470.2372   193.8160    91.7570     5.7101     1.0003    10.8461
2019010116    0.666667    2.139   423.9270   175.1684    83.3590     4.8356     6.3198     9.1633
2019010117    0.708333    2.140   316.2909   131.2080    62.8992     3.3117    10.2823     6.2600
2019010118    0.750000    2.176   170.7224    71.3813    34.6611     1.5535    12.0396     2.9335
2019010119    0.791667    2.245    18.3457     8.4289     4.6113     0.1037    11.2554     0.2101
2019010120    0.833333    2.337  -109.4070   -44.6375   -20.9983    -0.5077     8.2071    -0.9215
2019010121    0.875000    2.439  -188.4457   -77.7247   -37.1906     0.0868     3.7328     0.2122
2019010122    0.916667    2.535  -207.9073   -86.1738   -41.5571     1.9732    -0.9763     3.7572
2019010123    0.958333    2.611  -172.6479   -71.8776   -34.8890     4.9074    -4.6886     9.2487
2019010124    1.000000    2.657  -101.5732   -42.6368   -20.9177     8.3545    -6.4671    15.6899
2019010201    1.041667    2.671   -22.1804    -9.8860    -5.2050    11.6122    -5.9287    21.7785
2019010202    1.083333    2.655    37.2344    14.5931     6.5038    13.9841    -3.3501    26.2240
2019010203    1.125000    2.617    55.5611    22.0431     9.9729    14.9570     0.4130    28.0700
2019010204    1.166667    2.568    24.8430     9.2293     3.7078    14.3301     4.1694    26.9320
```

7.3.2 The geodetic site variation record time series file

A geodetic site variation record time series file can store the time series data of one kind of variations for a group of geodetic sites. Such as the station coordinate time series for the CORS network, benchmark height time series for the levelling network, observation time series for the tide station network and InSAR monitoring time series, etc.

(1) The file header occupies a row and includes all the sampling epochs arranged with

time.

(2) The file record: the site name, longitude, latitude, height, …, all the sampling variations arranged with sampling time.

(3) ETideLoad stipulates that the number of sampling epochs in the file header is equal to the number of sampling variations in the record, and the sampling epochs are one-by-one correspondence with the sampling variations.

(4) When receiving the input record time series file from the program interface, it is generally required to specify the column ordinal number of the first sampling epoch in the file header and the column ordinal number of the first sampling variation in the record.

TsqavrRowU.txt | TsqavrbslnU.txt

4	0	36			2015011612	2015021500	2015031612	2015041600	2015051612	2015061600
JINH	119.6426	29.2178	1191.60	1.0	-4.9145	9.3944	3.7319	0.4720	1.1566	2.7777
JINX	119.3792	29.0709	84.79	1.0	-4.3724	1.6001	6.6220	0.8372	2.9622	1.8461
JNJZ	119.6375	27.9764	286.78	1.0	-4.1680	3.2284	3.1467	-0.4777	2.3145	1.8212
JSAN	118.6086	28.7279	71.54	1.0	4.8394	10.8248	7.4036	2.4828	0.3532	-2.2769
LISH	119.9295	28.4613	71.54	1.0	4.8394	10.8248	7.4036	2.4828	0.3532	-2.2769
LONQ	119.1331	28.0807	233.28	1.0	-4.9987	3.4121	3.3682	-2.0458	-2.0137	-1.6199
QIYU	119.0793	27.6213	412.75	1.0	-2.9713	5.7773	7.2012	1.1874	-3.3157	-3.4728
QNYN	118.9638	27.6157	429.39	1.0	0.7446	7.2540	6.9323	0.2500	-1.3013	-1.8433
QUZH	118.8908	28.9937	90.79	1.0	-1.0815	5.9656	5.1221	-1.1572	0.5323	-1.6064
QZLY	119.1858	29.0336	73.91	1.0	-1.3703	6.4829	8.4987	1.9209	1.5578	0.7378
SHNQ	119.5028	27.4576	827.01	1.0	-6.5350	3.4134	3.8402	1.0473	3.2554	-2.4524
SNYN	119.5093	28.4546	182.77	1.0	-5.6627	3.1365	4.4180	0.4287	2.1431	2.2420
YAYA	120.0425	27.3930	555.71	1.0	-2.1462	5.1836	4.0938	3.6248	4.5640	1.2865
YONK	120.0168	28.9055	116.22	1.0	-1.6121	4.7569	7.1178	2.7207	0.1517	0.6173
ZJYH	119.6900	28.2660	130.05	1.0	-3.2802	4.5552	3.8968	-0.2975	0.4079	-0.3378

TsqavrRowU.txt | TsqavrbslnU.txt | InSARspdfmrst.txt

5	37			20141103	20141127	20141221	20150114	20150207	20150408	20150502	20150526
-9.310	117.3445416	39.0251902	-2.793	-0.3091	0.0866	0.0482	0.2194	0.2865	0.5720	0.3395	0.4760
-12.790	117.3457082	39.0251902	-2.304	-0.1796	0.2752	0.1906	0.2887	0.3340	0.5632	0.2861	0.2759
-7.482	117.3480415	39.0251902	-3.660	-0.3846	9999.0000	-0.0523	0.0135	0.1783	-0.0167	-0.5455	-0.1440
-6.699	117.3487081	39.0251902	-2.582	-0.2325	0.2301	0.0701	0.2610	0.4179	0.7154	0.4377	0.7507
-7.643	117.3488748	39.0251902	-2.882	-0.1876	0.3087	0.1757	0.3743	0.5347	0.9182	0.6967	0.9859
-9.001	117.3492081	39.0251902	-3.430	-0.2356	0.2486	0.2282	0.4480	0.5863	1.0390	0.8643	1.1080
-10.736	117.3495414	39.0251902	-4.123	-0.2211	0.1605	0.0977	0.4040	0.4816	1.0206	0.9163	1.0519
-10.264	117.3497081	39.0251902	-3.963	-0.2530	0.0627	0.0174	0.2817	0.3735	0.9470	0.9182	0.9979
-9.893	117.3498747	39.0251902	-3.843	-0.2733	0.0120	-0.0196	0.2078	0.3072	0.8895	0.8983	0.9588
-14.921	117.3503747	39.0251902	-3.558	-0.2906	0.0654	0.0359	0.2029	0.2907	0.8569	0.8558	0.8677
-13.835	117.3507080	39.0251902	-3.246	-0.1282	0.2632	0.1912	0.3657	0.4738	1.0386	1.0551	1.0863
-13.217	117.3508747	39.0251902	-3.191	-0.0641	0.3303	0.2465	0.4314	0.5473	1.1231	1.1658	1.2188
-12.657	117.3510413	39.0251902	-3.067	0.0039	0.3831	0.2929	0.4991	0.6149	1.2231	1.3072	1.3714
-12.424	117.3512080	39.0251902	-2.943	0.0099	0.3579	0.2776	0.4952	0.6110	1.2489	1.3458	1.3795
-12.475	117.3513747	39.0251902	-2.964	0.0053	0.3079	0.2478	0.4779	0.5912	1.2806	1.3846	1.3793
-12.682	117.3515413	39.0251902	-3.121	-0.0670	0.2179	0.1862	0.4278	0.5167	1.2285	1.3064	1.2891
-12.511	117.3517080	39.0251902	-3.357	-0.1241	0.1493	0.1503	0.3968	0.4565	1.1697	1.2314	1.2168
-11.102	117.3520413	39.0251902	-4.331	-0.1866	0.0288	0.1581	0.3671	0.4031	1.0231	1.0748	1.0342
-10.425	117.3522080	39.0251902	-4.557	-0.1592	0.0754	0.2553	0.4129	0.4636	1.0229	1.0912	1.0074
-7.999	117.3525413	39.0251902	-4.173	-0.0956	0.1919	0.4448	0.5395	0.5774	1.1620	1.2013	1.0798
-3.661	117.3528746	39.0251902	-3.783	-0.0689	9999.0000	0.4497	0.5472	0.5783	1.3511	1.2092	1.1608
-13.428	117.3543745	39.0251902	-3.938	0.1071	0.4859	0.6632	0.7978	0.9730	1.2854	1.6207	1.6274
-18.243	117.3545412	39.0251902	-3.980	0.1224	0.5127	0.6954	0.7888	0.9869	1.2956	1.6312	1.6339
-22.513	117.3547079	39.0251902	-3.825	0.1425	0.5367	0.7026	0.7356	0.9890	1.3448	1.6335	1.6239

7.3.3 The geodetic network observation record time series file

A geodetic network observation record time series file can store the variation record time series of the baseline component for the CORS network, the variation record time series of the height difference for the levelling network, or the variation record time series of the gravity difference for the gravity network.

(1) The file header occupies a row and includes the number of characters of the baseline or route name, number of characters of the site name, sampling length, …, all the sampling epochs arranged with time.

(2) The file record includes the baseline or route name, starting site (longitude, latitude, height), ending site (longitude, latitude, height), …, all the observed variations arranged with sampling time (default value is 9999).

(3) The relations between the baselines (or routes) and the sites in the geodetic

monitoring network are reflected with the composition of the characters of their name. A baseline or route name (B***A) is agreed to be composed of site names A and B at both ends, where the number of characters of all the sites names (such as A and B) is required to be equal.

Therefore, the number of characters of the baseline or route name should not be less than twice that of the site name.

TsqavrRowU.txt TsqavrbslnU.txt

9 0 36							2015011612	2015021500	2015031612	2015041600	2015051612	2015061600
CANN_DONT	120.4247	27.5226	0.00	121.1503	27.8346	0.00	6.4092	3.5311	-0.6494	3.4802	1.9057	-0.3761
CANN_FDIQ	120.4247	27.5226	0.00	120.2073	27.3353	0.00	7.5566	9999.0000	9.8866	4.9806	-3.7116	0.3579
CANN_JHYW	120.4247	27.5226	0.00	120.0784	29.2727	0.00	3.3886	1.7941	-0.5867	-0.4076	0.7054	-0.2796
CANN_JINH	120.4247	27.5226	0.00	119.6426	29.2178	0.00	2.8530	-0.7712	-1.3292	1.3058	-1.2947	-2.4476
CANN_JINX	120.4247	27.5226	0.00	119.3792	29.0709	0.00	4.3333	1.6376	2.9746	3.5908	1.9899	-1.6367
CANN_JNJZ	120.4247	27.5226	0.00	119.6375	27.9764	0.00	4.9006	3.1138	1.0930	0.3909	0.8296	-1.6116
CANN_JSAN	120.4247	27.5226	0.00	118.6086	28.7279	0.00	2.5860	0.7616	-0.7290	-0.5568	-4.1410	-5.3632
CANN_LHAI	120.4247	27.5226	0.00	121.1895	28.9059	0.00	1.0756	-1.6069	-1.8127	-0.4355	-0.0362	-0.7623
CANN_LISH	120.4247	27.5226	0.00	119.9295	28.4613	0.00	13.8711	10.5885	3.5993	4.5791	-0.7531	-6.0081
CANN_LONQ	120.4247	27.5226	0.00	119.1331	28.0807	0.00	7.3816	6.1923	2.8486	1.8460	-1.9823	-3.9091
CANN_LUOY	120.4247	27.5226	0.00	119.7051	27.5525	0.00	8.8132	9.4984	3.5027	4.0624	0.1673	-1.3212
CANN_PANA	120.4247	27.5226	0.00	120.4367	29.0542	0.00	0.2485	-2.0512	-4.4740	-2.9385	-0.0623	-0.7816
CANN_PCHQ	120.4247	27.5226	0.00	118.5422	27.9232	0.00	5.5253	5.7473	3.2665	1.6373	-1.4516	-6.3300
CANN_PCJM	120.4247	27.5226	0.00	118.4454	28.1680	0.00	14.2248	12.7588	7.9740	8.5291	3.1970	1.6813
CANN_QINT	120.4247	27.5226	0.00	120.2900	28.1394	0.00	7.1517	4.5782	2.4621	4.4460	-0.4966	-3.3975
CANN_QIYU	120.4247	27.5226	0.00	119.0793	27.6213	0.00	7.1481	6.6956	4.4206	1.9230	-5.9045	-7.7402
CANN_QNYN	120.4247	27.5226	0.00	118.9638	27.6157	0.00	10.9311	7.9043	4.5820	1.4154	-3.5702	-5.8564
CANN_QUZH	120.4247	27.5226	0.00	118.8908	28.9937	0.00	8.1029	5.9283	1.7235	1.2962	-0.8214	-5.2339
CANN_QZLY	120.4247	27.5226	0.00	119.1858	29.0336	0.00	7.1312	6.3296	4.8051	4.9249	0.4817	-2.6667
CANN_RUIA	120.4247	27.5226	0.00	120.6490	27.7833	0.00	8.9624	7.8483	0.2735	0.8812	-0.2184	-3.3412
CANN_SHNQ	120.4247	27.5226	0.00	119.5028	27.4576	0.00	4.8563	4.6628	0.3999	3.4718	0.5071	-6.6786
CANN_SNYN	120.4247	27.5226	0.00	119.5093	28.4546	0.00	3.3183	3.1267	1.3992	1.8621	1.0732	-1.1970
CANN_SUIC	120.4247	27.5226	0.00	119.2693	28.5951	0.00	14.7246	7.6386	10.1070	4.5706	6.3427	0.2411
CANN_TAIZ	120.4247	27.5226	0.00	121.4164	28.6183	0.00	4.0291	3.2741	0.3639	1.5962	-0.2033	-1.0270

7.3.4 The variation grid time series files for geodetic field

A group of variation grid time series files are composed of a series of numerical grid model files of a certain kind of variation (vector), and the seventh attribute of the header in each grid file is agreed to be the sampling epoch time. Such as the grid time series of land water equivalent height, sea level variation, and grid time series of various regional load deformation field or temporal gravity field, etc.

zwdx20150331.dat zwdx20150531.dat zwdx20150831.dat

118.500000	121.500000	27.000000	29.000000	1.66666667E-02	1.66666667E-02	2015033118			
-0.3746	-0.5686	-0.6666	-0.9356	-1.0686	-1.0836	-1.0606	-1.0586	-1.0586	-1.0566
-0.5445	-0.4746	-0.4986	-0.6746	-0.8176	-0.8646	-0.9356	-0.9356	-0.9575	-0.9565
-0.9326	-0.9128	-0.9132	-0.8647	-0.7606	-0.7255	-0.6170	-0.5403	-0.4534	-0.4873
-0.5622	-0.5214	-0.4842	-0.4644	-0.4832	-0.4767	-0.5045	-0.4994	-0.4972	-0.4458
-0.1776	-0.0795	0.0535	0.0904	0.1088	0.0885	-0.0087	-0.1460	-0.2341	-0.2686
-0.0685	0.1893	0.3498	0.4787	0.5169	0.4660	0.2675	0.1964	0.1144	0.0974
0.1248	-0.0175	-0.1520	-0.2620	-0.3917	-0.4810	-0.5751	-0.6501	-0.7137	-0.7640
-0.2918	-0.1304	0.0015	0.1363	0.2607	0.3785	0.4382	0.4686	0.4562	0.3953
-0.4998	-0.6231	-0.7580	-0.8453	-0.9315	-0.9857	-1.0331	-1.0529	-1.0680	-1.0697
-1.0796	-1.0936	-1.0966	-1.0966	-1.0996	-1.1006	-1.1006	-1.0996	-1.0997	-1.0978
-1.0184	-0.9872	-0.9330	-0.8807	-0.8047	-0.7382	-0.6460	-0.5719	-0.4788	-0.3955
-0.2513	-0.3094	-0.3723	-0.4617	-0.5417	-0.6397	-0.7159	-0.7967	-0.8500	-0.8985
-0.4647	-0.6176	-0.7077	-0.9746	-1.0456	-1.0596	-1.0546	-1.0526	-1.0576	-1.0416
-0.4536	-0.3635	-0.4356	-0.6346	-0.7596	-0.8306	-0.9016	-0.9016	-0.9385	-0.9385
-0.9086	-0.9138	-0.9142	-0.8636	-0.7696	-0.7325	-0.6810	-0.6283	-0.5264	-0.5533
-0.5762	-0.5344	-0.5152	-0.4914	-0.5093	-0.5217	-0.5505	-0.5446	-0.5243	-0.4589
-0.1982	-0.1410	-0.0271	0.0068	0.0242	0.0178	-0.0763	-0.1896	-0.2368	-0.2702
-0.0970	0.1349	0.3035	0.4214	0.4327	0.4208	0.2203	0.1393	0.0803	0.0864
0.0868	-0.0434	-0.1649	-0.2578	-0.4047	-0.5100	-0.5791	-0.6770	-0.7527	-0.7781

7.3.5 The spherical harmonic coefficient model time series files

A group of spherical harmonic coefficient model time series files can store the time series of the spherical harmonic coefficient (Stokes' coefficient) models of global surface load variation, global load deformation field or temporal global gravity field.

(1) The header file occupies one row and consists of three attributes, namely the geocentric gravitational constant GM ($\times10^{14}m^2/s^2$), equatorial radius a(m) of the Earth and sampling epoch time (in ETideLoad format).

(2) GM, a are the scale parameters of the model. Here, the surface harmonic functions

in the spherical harmonic coefficient model are defined on the spherical surface whose radius is equal to the equatorial radius a of the Earth.

(3) The degree-n order-m spherical harmonic coefficient is expressed by a record with the format: degree n, order m, C_{nm}, S_{nm} (, C_{nm} error, S_{nm} error). At different sampling epochs, n of the model files can be not the same.

(4) ETideLoad does not require the degrees and orders of harmonic coefficients to be arranged and allows to exist insufficient orders. For the harmonic coefficient of insufficient order, ETideLoad automatically sets to zero.

7.4 Geophysical models and numerical standards in ETideLoad4.5

ETideLoad4.5 is mainly based on the geophysical models and numerical standards recommended by IERS conventions (2010). You can update them by the program [System configs for the geophysical models and numerical standards]. These geophysical models and numerical standards are stored in file form in the folder C:\ETideLoad4.5_win64en\iers.

7.4.1 The surface atmosphere tidal load spherical harmonic coefficient model file

The 360-degree surface atmosphere tidal load spherical harmonic coefficient model file ECMWF2006.dat is stored in the folder C:\ETideLoad4.5_win64en\iers in FES2004 format, which were constructed by the spherical harmonic analysis programs of ETideLoad4.5 using 0.5°× 0.5° global harmonic constant grids of four atmosphere tidal constituents, to meet the basic needs of centimeter-level geodesy. The four tidal constituents are respectively the diurnal, semi-diurnal, semi-annual and annual periodic tidal constituents (S_1, S_2, Ss_a, Sa) whose harmonic constant grids come from ECMWF-DCDA2006 of European Centre for Medium-Range Weather Forecasts.

ECMWF2006.dat

Atmospheric tide model: ECMWF-DCDA2006 normalized model up to (360,360) in hPa
半日/周日/半年/年周期

Doodson	Darw	n	m	Csin+	Ccos+	Csin-	Ccos-	C+	eps+	C-	eps-
164.556	S1	1	0	-0.01055351	0.00555959	-0.01055351	0.00555959	0.01192835	297.7803	0.01192835	297.7803
164.556	S1	2	0	-0.00898730	0.02713172	-0.00898730	0.02713172	0.02858149	341.6727	0.02858149	341.6727
164.556	S1	3	0	0.02416514	0.01232573	0.02416514	0.01232573	0.02712707	62.9756	0.02712707	62.9756
164.556	S1	4	0	0.01971779	-0.01808456	0.01971779	-0.01808456	0.02675523	132.5261	0.02675523	132.5261
164.556	S1	5	0	0.00538826	-0.01556217	0.00538826	-0.01556217	0.01646859	160.9021	0.01646859	160.9021
164.556	S1	6	0	-0.01896560	-0.00055330	-0.01896560	-0.00055330	0.01897366	268.3289	0.01897366	268.3289
164.556	S1	7	0	0.00163224	0.00711629	0.00163224	0.00711629	0.00730108	12.9183	0.00730108	12.9183
164.556	S1	8	0	0.00341644	0.00607435	0.00341644	0.00607435	0.00696920	29.3550	0.00696920	29.3550
164.556	S1	9	0	-0.00469730	-0.00311697	-0.00469730	-0.00311697	0.00563739	236.4331	0.00563739	236.4331
164.556	S1	10	0	0.00442735	-0.01563001	0.00442735	-0.01563001	0.01624496	164.1847	0.01624496	164.1847
164.556	S1	11	0	0.00941838	-0.00082619	0.00941838	-0.00082619	0.00945455	95.0132	0.00945455	95.0132
164.556	S1	12	0	-0.00454013	0.00688423	-0.00454013	0.00688423	0.00824654	326.5953	0.00824654	326.5953
164.556	S1	13	0	-0.01227672	0.00310149	-0.01227672	0.00310149	0.01266243	284.1781	0.01266243	284.1781
164.556	S1	14	0	0.00203678	0.00166923	0.00203678	0.00166923	0.00263340	50.6638	0.00263340	50.6638
164.556	S1	15	0	0.00253994	0.00381849	0.00253994	0.00381849	0.00458608	33.6306	0.00458608	33.6306
164.556	S1	16	0	0.00613602	-0.00041704	0.00613602	-0.00041704	0.00615017	93.8882	0.00615017	93.8882
164.556	S1	17	0	-0.00113104	-0.00413462	-0.00113104	-0.00413462	0.00428652	195.2992	0.00428652	195.2992
164.556	S1	18	0	-0.00311700	0.00136741	-0.00311700	0.00136741	0.00340375	293.6868	0.00340375	293.6868
164.556	S1	19	0	-0.00217138	0.00053937	-0.00217138	0.00053937	0.00223737	283.9498	0.00223737	283.9498
164.556	S1	20	0	-0.00017645	0.00369644	-0.00017645	0.00369644	0.00370065	357.2671	0.00370065	357.2671
164.556	S1	21	0	0.00068441	-0.00165216	0.00068441	-0.00165216	0.00178831	157.4980	0.00178831	157.4980
164.556	S1	22	0	0.00100221	-0.00214635	0.00100221	-0.00214635	0.00236881	154.9703	0.00236881	154.9703
164.556	S1	23	0	0.00461395	-0.00179653	0.00461395	-0.00179653	0.00495136	111.2744	0.00495136	111.2744
164.556	S1	24	0	-0.00143873	0.00014453	-0.00143873	0.00014453	0.00144597	275.7366	0.00144597	275.7366
164.556	S1	25	0	-0.00083151	-0.00001238	-0.00083151	-0.00001238	0.00083160	269.1470	0.00083160	269.1470
164.556	S1	26	0	-0.00272792	-0.00095240	-0.00272792	-0.00095240	0.00288940	250.7543	0.00288940	250.7543
164.556	S1	27	0	-0.00183890	0.00217563	-0.00183890	0.00217563	0.00284868	319.7946	0.00284868	319.7946

The surface atmospherical pressure, tidal constituent harmonic constants and tidal load spherical harmonic coefficients are all in unit of hPa or mbar.

7.4.2 The ocean tidal load spherical harmonic coefficient model file

The 100-degree ocean tidal load spherical harmonic coefficient model file FES2004S1.dat is stored in the folder C:\ETideLoad4.5_win64en\iers in FES2004 format. The relationship expression between the ocean tidal load normalized spherical harmonic coefficients and the geopotential coefficients is as the formula (6.15) in the IERS conventions (2010).

ECMWF2006.dat FES2004S1.dat

```
Ocean tide model: FES2004 normalized model (fev. 2004) up to (100,100) in cm
(long period from FES2002 up to (50,50) + equilibrium Om1/Om2, atmospheric tide NOT included)
Doodson Darw   n   m    Csin+     Ccos+       Csin-     Ccos-      C+   eps+    C-     eps-
  55.565 Om1   2   0 -0.540594  0.000000   0.000000  0.000000  0.5406 270.000 0.0000  0.000
  55.575 Om2   2   0 -0.005218  0.000000   0.000000  0.000000  0.0052 270.000 0.0000  0.000
  56.554 Sa    1   0  0.017233  0.000013   0.000000  0.000000  0.0172  89.957 0.0000  0.000
  56.554 Sa    2   0 -0.046604 -0.000903   0.000000  0.000000  0.0466 268.890 0.0000  0.000
  56.554 Sa    3   0 -0.000889  0.000049   0.000000  0.000000  0.0009 273.155 0.0000  0.000
  56.554 Sa    4   0  0.012069 -0.000413   0.000000  0.000000  0.0121  91.960 0.0000  0.000
  56.554 Sa    5   0 -0.009780 -0.000421   0.000000  0.000000  0.0098 267.535 0.0000  0.000
  56.554 Sa    6   0  0.006895  0.000043   0.000000  0.000000  0.0069  89.643 0.0000  0.000
  56.554 Sa    7   0 -0.010515 -0.000287   0.000000  0.000000  0.0105 268.437 0.0000  0.000
  56.554 Sa    8   0  0.002067 -0.000011   0.000000  0.000000  0.0021  90.305 0.0000  0.000
  56.554 Sa    9   0 -0.004236 -0.000110   0.000000  0.000000  0.0042 268.512 0.0000  0.000
  56.554 Sa   10   0 -0.001781 -0.000085   0.000000  0.000000  0.0018 267.268 0.0000  0.000
  56.554 Sa   11   0 -0.001372 -0.000068   0.000000  0.000000  0.0014 267.163 0.0000  0.000
  56.554 Sa   12   0 -0.004081 -0.000048   0.000000  0.000000  0.0041 269.326 0.0000  0.000
  56.554 Sa   13   0 -0.000116 -0.000041   0.000000  0.000000  0.0001 250.534 0.0000  0.000
  56.554 Sa   14   0 -0.003043 -0.000007   0.000000  0.000000  0.0030 269.868 0.0000  0.000
  56.554 Sa   15   0  0.001109 -0.000028   0.000000  0.000000  0.0011  91.446 0.0000  0.000
  56.554 Sa   16   0 -0.002596 -0.000034   0.000000  0.000000  0.0026 269.250 0.0000  0.000
  56.554 Sa   17   0 -0.000674  0.000022   0.000000  0.000000  0.0007 271.870 0.0000  0.000
  56.554 Sa   18   0  0.000546  0.000006   0.000000  0.000000  0.0005  89.370 0.0000  0.000
  56.554 Sa   19   0 -0.000024  0.000023   0.000000  0.000000  0.0000 313.781 0.0000  0.000
  56.554 Sa   20   0  0.000867  0.000014   0.000000  0.000000  0.0009  89.075 0.0000  0.000
```

In order to meet the basic needs of satellite, coastal zone and ocean gravity gradient data processing, we adopted AVISO FES2014b global tidal height harmonic constant grid models to construct the 720-degree ocean tidal height spherical harmonic coefficient model file FES2014b720cs.dat in FES2004 format by the spherical harmonic analysis programs of ETideLoad4.5.

FES2014n720cs.dat includes spherical harmonic coefficients of the 36 tidal constituents (Ω1, Ω2；2N2, Eps2, J1, K1, K2, L2, La2, M2, M3, M4, M6, M8, Mf, MKS2, Mm, MN4, MS4, MSf, MSqm, Mtm, Mu2, N2, N4, Nu2, O1, P1, Q1, R2, S1, S2, S4, Sa, Ssa, T2), in which the spherical harmonic coefficients of the two equilibrium tidal constituents (Ω1, Ω2) come from FES2004S1.dat.

The ocean tidal height, harmonic constant of the tidal constituent and tidal load spherical harmonic coefficients are all in unit of cm.

7.4.3 The Earth's Load Love number file

The Earth's load Love numbers also called the load deformation coefficients (LDC) can be calculated using the spherically symmetric non-rotating elastic Earth model REF6371. The load Love numbers in ETideLoad4.5 come from a Regional ElAstic Rebound calculator (REAR1.0, 2015.11). The load Love number file Love_load_cm.dat were stored in the folder C:\ETideLoad4.5_win64en\iers, which includes the load Love numbers of the radial displacement, horizontal displacement and geopotential $(h'_n, l'_n, k'_n), n = 0, \cdots, 32768$ from 0 to 32768 degree, $k'_0 = k'_1 = 0$, as shown in the figure.。

In order to suppress the high-degree oscillations of the load Green's function, the load Green's function is calculated to 54000 degrees in ETideLoad, and the load Love numbers exceeding 32768 degrees (n>32768) are calculated with the following asymptotic formulas: h_n' =-6.209114, l_n' =1.890061/n, k_n' =-2.682697/n.

Love_load_cm.dat

```
The load Love numbers from the REAR package are attached. There are no
more of these oscillations at high degree, and they go up to degree 32768.
November 20, 2015. Jean-Paul
CM: center of mass reference frame
    n      h' (vert)          l' (horiz)         k' (potent)
    0   0.0000000000D+00  0.0000000000D+00   0.0000000000D+00
    1  -0.0287112988D+01  0.1045044062D+00   0.0000000000D+00
    2  -0.9945870591D+00  0.2411251588D-01  -0.3057703360D+00
    3  -0.1054653021D+01  0.7085493677D-01  -0.1962722363D+00
    4  -0.1057783895D+01  0.5958723183D-01  -0.1337905897D+00
    5  -0.1091185915D+01  0.4702627503D-01  -0.1047617976D+00
    6  -0.1149253656D+01  0.3940811757D-01  -0.9034958051D-01
    7  -0.1218363201D+01  0.3499400649D-01  -0.8205733906D-01
    8  -0.1290473661D+01  0.3225123202D-01  -0.7652348967D-01
    9  -0.1361847865D+01  0.3038562458D-01  -0.7239287690D-01
   10  -0.1430981761D+01  0.2902258995D-01  -0.6907768441D-01
   11  -0.1497377458D+01  0.2798156018D-01  -0.6629382122D-01
   12  -0.1560934855D+01  0.2716367080D-01  -0.6388475059D-01
   13  -0.1621715593D+01  0.2650554043D-01  -0.6175536119D-01
   14  -0.1679770379D+01  0.2596800569D-01  -0.5983856019D-01
   15  -0.1735198310D+01  0.2551661917D-01  -0.5808965155D-01
   16  -0.1788088250D+01  0.2512667367D-01  -0.5647488828D-01
   17  -0.1838448069D+01  0.2478452380D-01  -0.5496610314D-01
   18  -0.1886440474D+01  0.2447083426D-01  -0.5354901315D-01
   19  -0.1932084480D+01  0.2417919471D-01  -0.5220607051D-01
   20  -0.1975465902D+01  0.2389862142D-01  -0.5092726303D-01
   21  -0.2016677975D+01  0.2362510597D-01  -0.4970406011D-01
   22  -0.2055800328D+01  0.2335504487D-01  -0.4853059813D-01
   23  -0.2092911079D+01  0.2308664225D-01  -0.4740132374D-01
   24  -0.2128152865D+01  0.2281672671D-01  -0.4631386954D-01
```

7.4.4 The IERS Earth orientation parameter time series file

The IERS Earth orientation parameters (EOP) time series file IERSeopc04.dat (ITRF2008) were stored in the folder C:\ETideLoad4.5_win64en\iers. You can update the EOP time series from the IERS website. For future time epochs, the forecast EOP products can be employed. Considering the non-tidal nature of the polar shift, the forecast time should be controlled within half a year.

ECMWF2006.dat | FES2004S1.dat | IERSeopc04.dat

```


                   INTERNATIONAL EARTH ROTATION AND REFERENCE SYSTEMS SERVICE
                        EARTH ORIENTATION PARAMETERS
                          EOP (IERS) 14 C04


             FORMAT(3(I4),I7,2(F11.6),2(F12.7),2(F11.6),2(F11.6),2(F11.7),2(F12.6))
##################################################################################

      Date      MJD        x          y        UT1-UTC       LOD         dX        dY        x Err      y Err   UT1-UTC Err  LOD Err      dX Err       dY Err
                           "          "           s           s          "         "           "          "          s          s           "            "
     (0h UTC)

2001   1   1  51910  -0.073506   0.398095   0.0931626   0.0006630   0.000150  -0.000109   0.000061   0.000048  0.0000107  0.0000131    0.000028    0.000030
2001   1   2  51911  -0.072651   0.399806   0.0924546   0.0007596   0.000141  -0.000092   0.000061   0.000048  0.0000070  0.0000131    0.000028    0.000031
2001   1   3  51912  -0.071557   0.401864   0.0916573   0.0008515   0.000132  -0.000074   0.000061   0.000047  0.0000034  0.0000131    0.000028    0.000031
2001   1   4  51913  -0.071024   0.403840   0.0907195   0.0008969   0.000149  -0.000084   0.000061   0.000047  0.0000084  0.0000132    0.000029    0.000031
2001   1   5  51914  -0.070723   0.405333   0.0897667   0.0008872   0.000174  -0.000103   0.000060   0.000047  0.0000163  0.0000132    0.000029    0.000031
2001   1   6  51915  -0.070378   0.406725   0.0889292   0.0008068   0.000199  -0.000122   0.000060   0.000047  0.0000221  0.0000132    0.000029    0.000031
2001   1   7  51916  -0.070068   0.408041   0.0882375   0.0006463   0.000224  -0.000141   0.000060   0.000047  0.0000163  0.0000132    0.000029    0.000031
2001   1   8  51917  -0.070205   0.409479   0.0876861   0.0004933   0.000250  -0.000160   0.000060   0.000047  0.0000104  0.0000132    0.000029    0.000031
2001   1   9  51918  -0.070220   0.410814   0.0872445   0.0004441   0.000275  -0.000179   0.000060   0.000046  0.0000046  0.0000132    0.000029    0.000032
2001   1  10  51919  -0.069861   0.412336   0.0868199   0.0004186   0.000270  -0.000158   0.000060   0.000046  0.0000043  0.0000133    0.000029    0.000031
2001   1  11  51920  -0.069330   0.414004   0.0864003   0.0004447   0.000155  -0.000180   0.000059   0.000046  0.0000039  0.0000133    0.000029    0.000031
2001   1  12  51921  -0.068456   0.416120   0.0858451   0.0005855   0.000106  -0.000203   0.000059   0.000046  0.0000088  0.0000133    0.000028    0.000030
2001   1  13  51922  -0.067463   0.418251   0.0851161   0.0007422   0.000095  -0.000222   0.000059   0.000046  0.0000138  0.0000133    0.000028    0.000030
2001   1  14  51923  -0.066479   0.420226   0.0842390   0.0008823   0.000084  -0.000241   0.000059   0.000046  0.0000112  0.0000134    0.000028    0.000029
2001   1  15  51924  -0.065406   0.422044   0.0833100   0.0009404   0.000072  -0.000259   0.000059   0.000046  0.0000086  0.0000134    0.000027    0.000028
2001   1  16  51925  -0.063999   0.423541   0.0824180   0.0009155   0.000061  -0.000278   0.000059   0.000046  0.0000060  0.0000134    0.000027    0.000028
2001   1  17  51926  -0.062602   0.425076   0.0816384   0.0007815   0.000050  -0.000297   0.000059   0.000046  0.0000034  0.0000135    0.000027    0.000027
2001   1  18  51927  -0.061434   0.426438   0.0809369   0.0005717   0.000307  -0.000078   0.000060   0.000046  0.0000060  0.0000135    0.000026    0.000026
2001   1  19  51928  -0.060301   0.428009   0.0803992   0.0004021   0.000387  -0.000005   0.000060   0.000046  0.0000114  0.0000135    0.000026    0.000025
2001   1  20  51929  -0.059175   0.429380   0.0801026   0.0002618   0.000335  -0.000045   0.000060   0.000046  0.0000197  0.0000136    0.000025    0.000025
2001   1  21  51930  -0.058122   0.430418   0.0799970   0.0000786   0.000284  -0.000085   0.000060   0.000046  0.0000198  0.0000136    0.000025    0.000024
2001   1  22  51931  -0.056745   0.431190   0.0799904  -0.0000387   0.000232  -0.000124   0.000060   0.000047  0.0000199  0.0000136    0.000024    0.000023
2001   1  23  51932  -0.055378   0.432515   0.0800354  -0.0000794   0.000180  -0.000164   0.000061   0.000047  0.0000200  0.0000137    0.000024    0.000022
2001   1  24  51933  -0.054038   0.434299   0.0801054  -0.0000531   0.000189  -0.000183   0.000061   0.000047  0.0000090  0.0000137    0.000024    0.000022
2001   1  25  51934  -0.052227   0.436048   0.0801105   0.0000481   0.000130  -0.000240   0.000061   0.000047  0.0000025  0.0000137    0.000023    0.000021
2001   1  26  51935  -0.050435   0.438026   0.0799589   0.0001715   0.000101  -0.000252   0.000062   0.000048  0.0000160  0.0000137    0.000023    0.000021
2001   1  27  51936  -0.049130   0.439812   0.0796787   0.0002940   0.000094  -0.000242   0.000062   0.000048  0.0000312  0.0000137    0.000022    0.000020
2001   1  28  51937  -0.047602   0.441607   0.0792944   0.0004503   0.000086  -0.000232   0.000062   0.000048  0.0000276  0.0000137    0.000022    0.000019
2001   1  29  51938  -0.045537   0.443509   0.0788172   0.0005621   0.000079  -0.000221   0.000063   0.000048  0.0000239  0.0000138    0.000021    0.000019
2001   1  30  51939  -0.043660   0.444974   0.0782782   0.0006019   0.000072  -0.000211   0.000063   0.000048  0.0000203  0.0000138    0.000021    0.000018
2001   1  31  51940  -0.042067   0.446396   0.0777060   0.0005437   0.000254  -0.000159   0.000063   0.000049  0.0000063  0.0000138    0.000021    0.000019
2001   2   1  51941  -0.040683   0.447325   0.0772066   0.0004689   0.000298  -0.000141   0.000064   0.000049  0.0000064  0.0000138    0.000022    0.000020
2001   2   2  51942  -0.039012   0.448060   0.0767917   0.0003692   0.000290  -0.000134   0.000064   0.000049  0.0000143  0.0000138    0.000022    0.000020
```

7.4.5 The Earth's mass centric variation time series file

The Earth's mass centric variation time series file Monthly_geocenter_MK.txt (ITRF2014) from Center for Space Research in University of Texas in USA (UT/CSR) from LAGEOS-1/2, Stella, Starlette, AJISAI, BEC and LARES Satellite Laser Ranging (SLR) measured. For future time epochs, the forecast products can be employed, but the forecast time should be controlled within three months.

Monthly_geocenter_MK.txt

```
Description for Geocenter variations from SLR
Column 1: Year equivalent of first day of month
Column 2-4: Lower Pass Filtered X,Y,Z (mm)
Column 5-7: SLR estimated X,Y,Z (mm)
Column 8-10: Sigma for X,Y,Z (mm)
Usage: (C11,S11,C10) = (X,Y,Z)/sqrt(3.0)/6378136000.0
Remark: Monthly station range bias and zenith delay adjusted
FORMAT(F9.1,F12.4,6F9.3,3F7.2)
1993.0000    5.077   -2.796   -1.605    6.477    1.797   -6.338   0.11   0.12   0.31
1993.0849    4.639   -3.300   -1.244    1.922   -2.697    2.723   0.10   0.10   0.29
1993.1615    3.765   -3.384   -1.106    5.188   -4.895    0.714   0.09   0.10   0.26
1993.2464    2.562   -2.788   -1.336    1.450   -6.692   -3.054   0.11   0.12   0.26
1993.3285    1.198   -1.521   -1.911    2.230   -4.670   -4.583   0.11   0.10   0.25
1993.4134   -0.126    0.137   -2.656   -0.187    3.425    0.901   0.10   0.10   0.25
1993.4956   -1.214    1.754   -3.323   -0.585    2.945   -1.864   0.10   0.09   0.21
1993.5804   -1.920    2.913   -3.678   -0.832    3.154   -1.596   0.09   0.08   0.21
1993.6653   -2.178    3.347   -3.579   -4.795    3.621  -10.809   0.09   0.08   0.20
1993.7474   -2.004    2.991   -3.037   -4.289    2.258   -4.667   0.08   0.07   0.21
1993.8323   -1.466    1.952   -2.253    2.444    3.179    2.852   0.09   0.09   0.27
1993.9144   -0.645    0.449   -1.614    0.122    1.610    1.299   0.09   0.09   0.22
1994.0000    0.397   -1.237   -1.600   -0.654   -4.129   -7.715   0.08   0.09   0.19
1994.0849    1.612   -2.802   -2.625    1.034   -5.735    1.035   0.09   0.09   0.20
1994.1615    2.931   -3.928   -4.840    2.030   -1.856   -5.718   0.08   0.08   0.21
1994.2464    4.228   -4.321   -7.998    3.979   -2.408   -5.872   0.09   0.08   0.20
1994.3285    5.313   -3.784  -11.461    8.049   -1.572  -11.862   0.10   0.09   0.20
1994.4134    5.965   -2.309  -14.360    5.917   -3.452  -22.300   0.09   0.10   0.24
1994.4956    6.014   -0.151  -15.860    2.151   -2.927  -10.555   0.09   0.10   0.25
```

7.4.6 Ocean tidal constituent harmonic constant grid model files

(1) The ocean tidal height model is composed of multiple grid models of all tidal constituent harmonic constants. Each tidal constituent harmonic constants are stored as a vector grid file.

(2) All the tidal constituent grid files from an ocean tidal height model should be in a folder with the same grid specifications.

(3) The 10 vector grid files in the folder C:\ ETideLoad4.5_win64en\OceanTide represent the ocean tide model GOT4.8 with 10 global grid models of 10 tidal constituent harmonic constants.

(4) The type of the tidal constituent is identified by the seventh attribute (Doodson constant) in its grid file header. These files can be named at will.

M2_got4.8.dat

```
 0.000000  360.000000  -90.000000   90.000000  0.50000000  0.50000000    255555
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
```

(5) The ocean tidal height model can be global or regional. The ocean tidal height and the harmonic constants are all in unit of cm.

7.4.7 The JPL Planetary Ephemeris DE440 file

The JPL Planetary Ephemeris DE440 file JEPH.440 (from 1850 to 2201) was stored in the folder C:\ETideLoad4.5_win64en\iers.

7.4.8 The Love number correction file for frequency dependence

The correction file frqadjlovekhl.txt for frequency dependence was generated from Table 6.5a, 6.5b, 6.5c, 7.2, 7.3a and 7.3b in IERS conventions (2010) can be employed to calculate the Love number corrections for frequency dependence to obtain the high-accuracy solid tidal effects on all-element geodetic variations.

frqadjlovekhl.txt

doodson	°/hr	l	l'	F	D	Ω	(δR	δI)[2~72,e-5;73~120,e-4]	H(e-5m)
245655	28.43973	1	0	2	0	2	2	0	12099
255555	28.98410	0	0	2	0	2	2	0	63192
125755	12.85429	2	0	2	0	2	-29	3	-664
127555	12.92714	0	0	2	2	2	-30	3	-802
135645	13.39645	1	0	2	0	1	-45	5	-947
135655	13.39866	1	0	2	0	2	-46	5	-5020
137455	13.47151	-1	0	2	2	2	-49	5	-954
145545	13.94083	0	0	2	0	1	-82	7	-4946
145555	13.94303	0	0	2	0	2	-83	7	-26221
147555	14.02517	0	0	0	2	0	-91	9	343
153655	14.41456	1	0	2	-2	2	-168	14	194
155445	14.48520	-1	0	2	0	1	-193	16	137
155455	14.48741	-1	0	2	0	2	-194	16	741
155655	14.49669	1	0	0	0	0	-197	16	2062
155665	14.49890	1	0	0	0	1	-198	16	414
157455	14.56955	-1	0	0	2	0	-231	18	394
157465	14.57176	-1	0	0	2	1	-233	18	87

7.4.9 The Desai ocean pole tide coefficient file

The ocean pole tide is generated by the centrifugal effect of polar motion on the oceans. Desai (2002) present a self-consistent equilibrium model of the ocean pole tide. This model accounts for continental boundaries, mass conservation over the oceans, self-gravitation and load of the ocean floor. Using this model, the ocean pole tide produces the following perturbations to the normalized geopotential coefficients, as a function of the polar shift parameters (m_1, m_2).

ECMWF2006.dat | FES2004S1.dat | IERSeopc04.dat | GCN_L1_L2_30d_CF-CM.txt | desaiscopolecoef.txt

n	m	Anm(Real)	Bnm(Real)	Anm(Imaginary)	Bnm(Imaginary)
1	0	1.8736759805448e-02	0.0000000000000e+00	2.9688884960424e-02	0.0000000000000e+00
1	1	2.8258913146935e-02	2.1774643075236e-02	2.3898264393684e-02	5.6771602236635e-02
2	0	-3.9555099024374e-03	0.0000000000000e+00	6.8390464271953e-04	0.0000000000000e+00
2	1	-2.4325330521304e-01	5.4680741193318e-03	5.4680741193318e-03	-1.9252111185300e-01
2	2	1.9102047023374e-02	1.1158297399424e-02	-1.5123770169928e-02	-2.4857839911518e-04
3	0	-2.0869478248378e-02	0.0000000000000e+00	-1.0775272844125e-02	0.0000000000000e+00
3	1	3.0809252024501e-02	7.4552838003486e-03	5.5937937407386e-03	6.6496877724041e-02
3	2	2.3295703062692e-02	3.7984356463618e-02	-2.1678456242839e-03	1.1232359168959e-02
3	3	7.9776020803848e-03	1.2502542787182e-02	-2.2341399966187e-02	-2.2979590161975e-02
4	0	-1.0612668622736e-02	0.0000000000000e+00	-1.5569196271270e-02	0.0000000000000e+00
4	1	1.3606306893006e-04	2.2051992576636e-03	2.0130037501025e-03	1.6323514549038e-02
4	2	1.1139374002795e-02	1.7031544962514e-02	-7.9621127289889e-03	-8.4440848505132e-04
4	3	-1.6100794768731e-02	1.4681986705593e-02	9.5178410813713e-03	-2.1017136590507e-02
4	4	4.3132021252707e-03	-4.6836271624465e-03	-2.9309550249205e-03	1.3175690530653e-02
5	0	7.0731357453056e-03	0.0000000000000e+00	-1.8023029843730e-03	0.0000000000000e+00
5	1	2.5644907587134e-03	-1.0076857169607e-02	-9.6273922883022e-03	-1.1684145258283e-02
5	2	-7.9615162895536e-03	2.0820461332209e-03	-3.0274671879191e-03	-1.0475800274156e-02
5	3	-1.1818705609675e-02	1.2063416189422e-02	-1.6584597520384e-02	-2.8253596831795e-02
5	4	9.2731253376468e-03	1.8353138561674e-02	-1.0870088052722e-02	4.7120935900411e-03
5	5	1.4460712839068e-02	-8.5510747244577e-03	8.9167437380844e-04	1.6048852898081e-02
6	0	7.4439256593180e-03	0.0000000000000e+00	-1.0670986469176e-03	0.0000000000000e+00
6	1	1.8261459881891e-02	-3.7775168887123e-03	-3.6768761254667e-03	-1.4329108864964e-03
6	2	-8.4568708595335e-03	2.5640802224787e-03	8.0976103423504e-03	-6.3983905389798e-03
6	3	-1.5355186088842e-02	1.8642889355748e-03	-9.6956523287846e-03	-2.2353328754893e-02

The Desai calculating formula of the ocean pole tide adopts the formula (6.23) in the IERS conventions (2010), and the 360-degree ocean polar tide coefficient file desaiscopolecoef.txt were stored in the folder C:\ETideLoad4.5_win64en\iers.

7.4.10 First-degree ocean tidal load spherical harmonic coefficient file

The file OtideOne.dat could be employed to forecast of ocean tidal load effects on Earth's mass centric variations or all-element geodetic variation effects due to Earth's mass centric variation of ocean tide.

The file is output by the function [Spherical harmonic analysis on ocean tidal constituent harmonic constants]. The following figure is the first-degree ocean tidal load spherical harmonic coefficient file generated by the spherical harmonic analysis of 34 tidal constituent harmonic constants in the FES2014b ocean tide model.

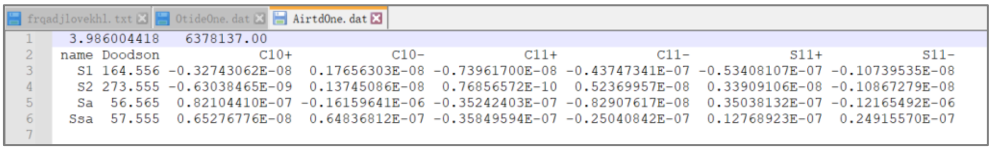
```
frqadjlovekhl.txt   OtideOne.dat
  3.986004418   6378137.00
 name Doodson         C10+            C10-            C11+            C11-            S11+            S11-
  2N2 247.455  0.14379190E-08  0.72446933E-09  0.45556662E-09  0.50261431E-09  0.98234968E-09  0.28806626E-08
   J1 175.455  0.22805765E-08 -0.14599680E-07  0.11146859E-07  0.31354016E-08  0.49073923E-08  0.50239288E-08
   K1 165.555  0.65903198E-07 -0.23618735E-06  0.15240517E-06  0.54510351E-07  0.57951321E-07  0.91115166E-07
   K2 275.555  0.58820344E-08  0.78223673E-09  0.82634785E-08  0.17098158E-07  0.28274727E-08  0.95641986E-09
   L2 265.455  0.99527541E-09  0.43369491E-10  0.27208849E-08  0.18838893E-08 -0.93316186E-09 -0.31242492E-09
   M2 255.555  0.64086749E-07  0.33741274E-07  0.82092113E-07  0.76976307E-08 -0.39331272E-07  0.74234937E-07
   M3 355.555  0.51159035E-10  0.26216133E-10  0.20622631E-10 -0.16737336E-10 -0.74054752E-10 -0.32502465E-10
   M4 455.555 -0.12877739E-09 -0.82078020E-09  0.21241775E-09  0.89312487E-09 -0.11238411E-09 -0.11882183E-08
   M6 655.555  0.18174228E-08  0.30921490E-09  0.36600543E-09  0.36841599E-09 -0.72147727E-09 -0.13743491E-09
   M8 855.555 -0.59854172E-10 -0.29503418E-11  0.41858427E-10  0.58809710E-10 -0.34465624E-10  0.81925459E-11
   Mf  75.555  0.23994538E-07  0.23160661E-08  0.14961765E-07 -0.19050356E-07  0.57231952E-08 -0.38155669E-08
   Mm  65.455  0.12211587E-07 -0.10619733E-08 -0.13680094E-08 -0.93454574E-08  0.34149364E-08 -0.61740212E-09
   N2 245.655  0.16604395E-07  0.24692742E-08  0.10060051E-07  0.75631673E-09 -0.49125733E-09  0.20845840E-07
   N4 435.755 -0.11170849E-09 -0.41029169E-10  0.37178942E-10 -0.10703469E-09 -0.53442667E-10 -0.19926918E-10
   O1 145.555  0.23239277E-07 -0.16830188E-06  0.86481239E-07  0.11802879E-07  0.58555768E-07  0.34726677E-07
   P1 163.555  0.16600812E-07 -0.74602430E-07  0.48235157E-07  0.14146460E-07  0.16888410E-07  0.27904988E-07
   Q1 135.655  0.40244812E-08 -0.29117940E-07  0.15908436E-07  0.77164577E-09  0.12770867E-07  0.14909422E-08
   R2 274.554  0.21029138E-09  0.96276767E-10  0.25728894E-09  0.46084038E-09  0.53716115E-10  0.14500876E-09
   S1 164.556 -0.40129653E-08  0.48653114E-08 -0.48716881E-08  0.11419251E-07  0.74509139E-08 -0.34899535E-09
   S2 273.555  0.22430236E-07  0.94564697E-08  0.30377828E-07  0.49157638E-07 -0.61338730E-08  0.76805145E-08
   S4 491.555  0.32089047E-09  0.14407638E-09  0.12925319E-11  0.14038268E-09  0.10308541E-09  0.11742749E-09
   Sa  56.554  0.21793187E-09  0.12972260E-09  0.71714382E-10  0.49927099E-10 -0.42733149E-10 -0.53422994E-10
   T2 272.556  0.13719484E-08  0.73425584E-09  0.20944307E-08  0.29614380E-08  0.13767437E-09  0.10318216E-08
  MN4 445.655 -0.70793273E-09 -0.76823301E-10  0.24279253E-09 -0.66374018E-09 -0.14062685E-09  0.16716883E-09
  MS4 473.555  0.32582237E-09 -0.10684852E-08  0.10873236E-08  0.38092589E-09 -0.40703836E-09 -0.28009461E-09
  Msf  73.555  0.52032006E-09  0.12958178E-08  0.20898774E-09  0.69234415E-09  0.16108594E-08  0.36734674E-09
  Mtm  85.455  0.38057222E-08  0.89028662E-09  0.47545363E-08 -0.16109463E-08  0.13034435E-08  0.46197838E-10
  mu2 237.555  0.27230195E-08 -0.54548861E-09  0.80856645E-09  0.28475772E-08  0.30945151E-08  0.39961507E-08
  nu2 245.655  0.31512988E-08  0.13274377E-08  0.16643629E-08  0.77176190E-09 -0.34369557E-09  0.49489633E-08
  Ssa  57.555  0.85592993E-08 -0.21041028E-09 -0.85777470E-08 -0.10849053E-08  0.38854237E-09 -0.73333943E-09
 eps2 227.655  0.15232320E-08 -0.54284574E-09  0.18709319E-08 -0.17678032E-09  0.14037532E-08 -0.64291979E-09
 lam2 263.655  0.77975910E-09 -0.46145888E-09  0.29230225E-08 -0.81098933E-09 -0.68691816E-09 -0.10714953E-08
 MKS2 257.555 -0.76338045E-11 -0.81694611E-10  0.81955321E-10  0.53313693E-09  0.52931064E-09  0.23733568E-09
 Msqm  93.555  0.17382639E-09 -0.21085098E-11  0.98864729E-10  0.18391545E-09 -0.15315104E-09 -0.66456652E-11
```

The file header inludes the geocentric gravitational constant GM ($\times 10^{14} m^3/s^2$) and equatorial radius a (m) of the Earth. The record (starting from the third row) includes the tidal constituent's name, Doodson constant, the degree-1 order-0 spherical harmonic coefficient C_{10}+ from in-phase amplitude, that C_{10}- from out-of-phase amplitude, and the degree-1 order-1 spherical harmonic coefficient C_{11}+ from in-phase amplitude, that C_{11}- from out-of-phase amplitude, and the degree-1 order-1 spherical harmonic coefficient S_{11}+ from in-phase amplitude, that S_{11}- from out-of-phase amplitude.

7.4.11 First-degree atmosphere tidal load spherical harmonic coefficient file

The file AirtdOne.dat could be employed to forecast of surface atmosphere tidal load effects on Earth's mass centric variations or all-element geodetic variation effects due to Earth's mass centric variation of surface atmosphere tide.

The file is output by the function [Spherical harmonic analysis on atmosphere tidal constituent harmonic constants]. The following figure is the first-degree atmosphere tidal load spherical harmonic coefficient file generated by the spherical harmonic analysis of 4 tidal constituent harmonic constants in the ECMWF-DCDA2006 atmosphere tide model.



frqadjlovekhl.txt OtideOne.dat AirtdOne.dat

3.986004418 6378137.00

name	Doodson	C10+	C10-	C11+	C11-	S11+	S11-
S1	164.556	-0.32743062E-08	0.17656303E-08	-0.73961700E-08	-0.43747341E-07	-0.53408107E-07	-0.10739535E-08
S2	273.555	-0.63038465E-09	0.13745086E-08	0.76856572E-10	0.52369957E-08	0.33909106E-08	-0.10867279E-08
Sa	56.565	0.82104410E-07	-0.16159641E-06	-0.35242403E-07	-0.82907617E-08	0.35038132E-07	-0.12165492E-06
Ssa	57.555	0.65276776E-08	0.64836812E-07	-0.35849594E-07	-0.25040842E-07	0.12768923E-07	0.24915570E-07

The file header inludes the geocentric gravitational constant GM ($\times 10^{14} m^3/s^2$) and equatorial radius a (m) of the Earth. The record (starting from the third row) includes the tidal constituent's name, Doodson constant, the degree-1 order-0 spherical harmonic coefficient C_{10}+ from in-phase amplitude, that C_{10}- from out-of-phase amplitude, and the degree-1 order-1 spherical harmonic coefficient C_{11}+ from in-phase amplitude, that C_{11}- from out-of-phase amplitude, and the degree-1 order-1 spherical harmonic coefficient S_{11}+ from in-phase amplitude, that S_{11}- from out-of-phase amplitude.

8 Main Algorithms and Formulas in ETideLoad4.5

8.1 Solid tidal effects on various geodetic variations outside solid Earth

The Earth's tidal generating potential (TGP) from celestial body outside the Earth directly cause the variation of geopotential on the ground and outside the Earth, and induce the deformation of the solid Earth, resulting in the redistribution of mass inside the Earth and generation of the associated geopotential. The former is the direct influence of the TGP, and the latter is the indirect influence of TGP. The sum of the two is the solid Earth tidal effect on the geopotential on the ground and outside the Earth, referred to as the body tidal effect.

8.1.1 The unified expression of body tidal effects on geodetic variations outside solid Earth

(1) The Earth's tidal generating potentials from celestial bodies

The tidal generating potential (TGP, or tidal potential) of celestial body outside the Earth can be expressed by the variations of geopotential coefficients $(\Delta\bar{C}_{nm}, \Delta\bar{S}_{nm})$ in the Earth-fixed coordinate system as

$$\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm} = \frac{1}{2n+1}\sum_{j=2}^{10}\frac{GM_j}{GM}\left(\frac{a}{r_j}\right)^{n+1}\bar{P}_{nm}(cos\theta_j)e^{im\lambda_j} \tag{1.1}$$

Where, $j = 2\sim10$ represents the moon, sun, Mercury, Venus, Mars, Jupiter, Saturn, Uranus and Neptune, respectively. $\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm}$ are the variations of the normalize geopotential coefficients with degree-n order-m (with $\Delta\bar{S}_{n0} = 0$), $\bar{P}_{nm}$ are the normalized associated Legendre functions, $n = 2,3,4,5,6$ for the moon, $n = 2,3$ for the sun and $n = 2$ for the other celestial bodies. $GM_j, r_j, \varphi_j, \lambda_j$ are respectively the gravitational parameter, distance from geocenter, geocentric latitude, and longitude (from Greenwich) of the celestial body j. GM, a are the geocentric gravitational constant and equatorial radius of the Earth, respectively.

At any epoch time t, the spherical coordinates $(r_j, \theta_j, \lambda_j)$ of the celestial body j in the Earth-fixed coordinate system can be calculated by the SOFA routines using the JPL Planetary Ephemeris DE440.

(2) The representation of solid Earth tidal effects on all-element geodetic variations outside solid Earth

According to the theory of spherical harmonic expansion of gravitational potential, degree-n tidal potential $\Delta V_n(r,\theta,\lambda)$ and degree-n tidal force $\Delta g_n(r,\theta,\lambda)$ at any point (r,φ,λ) on the ground or outside the solid Earth can be expressed by degree-n geopotential coefficient variations $(\Delta\bar{C}_{nm}, \Delta\bar{S}_{nm})$ as follows:

$$\begin{aligned}\Delta V_n(r,\theta,\lambda) &= \frac{GM}{r}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta)\\ \Delta g_n(r,\theta,\lambda) &= \frac{GM}{r^2}(n+1)\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta)\end{aligned} \tag{1.2}$$

For the spherically symmetric non-rotating elastic Earth such as PREM Earth, the potential Love number k_n and displacement Love number (h_n, l_n) are real numbers. When

$n \geq 4,\ k_n = h_n = l_n = 0.$

According to the solid Earth tide theory and spherical harmonic synthesis algorithm, the body tidal effect on geopotential $\Delta V(r,\varphi,\lambda)$ is equal to the sum of the tidal potential $\Delta V_n(r,\theta,\lambda)$ and associated geopotential $\Phi_n^a(r,\theta,\lambda) = k_n \Delta V_n(r,\theta,\lambda)$ of each degree.

$$\Delta V(r,\theta,\lambda) = \sum_{n=2}^{6}[\Delta V_n(r,\theta,\lambda) + \Phi_n^a(r,\theta,\lambda)] = \sum_{n=2}^{6}(1+k_n)\Delta V_n(r,\theta,\lambda)$$
$$= \frac{GM}{r}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.3)$$

Similarly, the body tidal effect expression on height anomaly on the ground or outside the solid Earth can be obtained as follows:

$$\Delta\zeta(r,\theta,\lambda) = \frac{GM}{\gamma r}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)$$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.4)$$

The body tidal effect expression on gravity (disturbance) outside the solid Earth as:

$$\Delta g^{\delta}(r,\theta,\lambda) = \frac{GM}{r^2}\sum_{n=2}^{6}(n+1)\left(\frac{a}{r}\right)^n (1+k_n)$$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.5)$$

The body tidal effect expression on vertical deflection outside the solid Earth as:

South: $\Delta\xi(r,\theta,\lambda) = \frac{GM}{\gamma r^2}\sin\theta\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{nm}(cos\theta) \quad (1.6)$$

West: $\Delta\eta(r,\theta,\lambda) = \frac{GM}{\gamma r^2 \sin\theta}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)$
$$\sum_{m=1}^{n} m(\Delta\bar{C}_{nm}sinm\lambda - \Delta\bar{S}_{nm}cosm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.7)$$

The body tidal effect expression on radial gravity gradient as:

$$\Delta T_{rr}(r,\theta,\lambda) = \frac{GM}{r^3}\sum_{n=2}^{6}(n+1)(n+2)\left(\frac{a}{r}\right)^n (1+k_n)$$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.8)$$

The body tidal effect expression on horizontal gravity gradient as:

North: $\Delta T_{NN}(r,\theta,\lambda) = -\frac{GM}{r^3}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\frac{\partial^2}{\partial\theta^2}\bar{P}_{nm}(cos\theta) \quad (1.9)$$

West: $\Delta T_{WW}(r,\theta,\lambda) = \frac{GM}{r^3 cos^2\varphi}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)$
$$\sum_{m=1}^{n} m^2(\Delta\bar{C}_{nm}sinm\lambda + \Delta\bar{S}_{nm}cosm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.10)$$

According to whether the contribution of solid Earth tidal deformation by the displacement Love number is directly included, the body tidal effects on geodetic variations are divided into two categories. One class of the body tidal effect on geodetic variations does not directly include the contribution of displacement Love number (h_n, l_n), such as the geopotential, gravity (disturbance), vertical deviation and gravity gradient outside Earth. Another class of geodetic variation whose site is fixed with the solid Earth, on which the body tidal effect includes the contribution of displacement Love number (h_n, l_n), such as the ground gravity, site displacement and ground tilt.

The body tidal effect expression on ground displacement whose site is fixed with the solid Earth◉

East: $\Delta e(r,\theta,\lambda) = -\frac{GM}{\gamma r\sin\theta}\sum_{n=2}^{3}\left(\frac{a}{r}\right)^n l_n$
$$\sum_{m=1}^{n} m(\Delta\bar{C}_{nm}sinm\lambda - \Delta\bar{S}_{nm}cosm\lambda)\bar{P}_{nm}(cos\theta) \qquad (1.11)$$

North: $\Delta n(r,\theta,\lambda) = -\frac{GM}{\gamma r}\sin\theta\sum_{n=2}^{3}\left(\frac{a}{r}\right)^n l_n$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{nm}(cos\theta) \qquad (1.12)$$

Radial: $\Delta r(r,\theta,\lambda) = \frac{GM}{\gamma r}\sum_{n=2}^{3}\left(\frac{a}{r}\right)^n h_n$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \qquad (1.13)$$

The body tidal effect expression on ground gravity whose site is fixed with the solid Earth⦿

$$\Delta g^s(r,\varphi,\lambda) = \frac{GM}{r^2}\sum_{n=2}^{6}(n+1)\left(\frac{a}{r}\right)^n\left(1+\frac{2}{n}h_n - \frac{n+1}{n}k_n\right)$$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \qquad (1.14)$$

The body tidal effect expression on ground tilt whose site is fixed with the solid Earth⦿

South: $\Delta\xi^s(r,\theta,\lambda) = \frac{GM}{\gamma r^2}\sin\theta\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n(1+k_n-h_n)$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{nm}(cos\theta) \qquad (1.15)$$

West: $\Delta\eta^s(r,\theta,\lambda) = \frac{GM}{\gamma r^2\sin\theta}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n(1+k_n-h_n)$
$$\sum_{m=1}^{n} m(\Delta\bar{C}_{nm}sinm\lambda - \Delta\bar{S}_{nm}cosm\lambda)\bar{P}_{nm}(cos\theta) \qquad (1.16)$$

In the above expressions, the body tidal effects on the geodetic variations marked ⦿ are valid only when their sites are fixed with the solid Earth, and that on the remaining geodetic variations are suitable on the ground or outside the solid Earth.

In order to uniformly represent the effects of ocean tide, atmosphere tide, external celestial bodies and non-tidal loads on various geodetic variations on the ground and outside the solid Earth, ETideLoad4.5 looks the sum of the direct and indirect influences of the tidal generating geopoential as the solid tidal effect, rather than only the indirect influence of the tidal generating geopoential as the body tidal effect in some literatures.

(3) Earth’s tidal potentials and tidal forces from celestial bodies and their time-varying analysis

Using the solar system ephemeris, degree-n Earth’s tidal potential (tidal force) from any celestial object in the solar system can be calculated by (1.1) and (1.2). Then, according to the accuracy requirements of geodesy, the celestial objects and their tidal potential expansion degree-n can be determined, and the periodicity, magnitude and time-varying characteristics of the different degree of tidal potentials can be calculated and investigated.

The Earth's tidal potential and tidal force from celestial body are related to the position of the calculation point in the Earth-fixed coordinate system. Here, the ground point P (105°N, 20°E, H100m) selected, the 2nd to 6th degree Earth’s tidal potential ΔV_n (in unit $10^{-5}m^2/s^2$) and tidal force Δg_n (in unit nGal = $10^{-11}m/s^2$) from 10 external celestial bodies in the solar system are calculated using the formula (1.2). The difference between the maximum and minimum values of the tidal potential (force) of each degree from 10 celestial bodies is calculated as Tab 1.1. 0.0000 in the table indicates that the calculation results are close to zero, and the blank indicates that the value is too small to calculate.

It can be seen from Tab 1.1 that when the tidal potential (force) are calculated, if the cutoff threshold is 10^{-8}m^2/s^2 (11^{-14}m/s^2), the moon needs to be expanded to 6 degrees, the sun needs to be expanded to 3 degrees, Venus, Jupiter, Mars, Mercury and Saturn only need to be calculated for the 2nd degree, and Uranus, Neptune and Pluto do not need to be calculated.

Tab 1.1 The difference between the maximum and minimum values of the tidal potential (force) of celestial bodes

	ΔV_2	$\dot{g}_2$	ΔV_3	$\dot{g}_3$	ΔV_4	$\dot{g}_4$
Moon	247660.1100	116532.1527	6176.8512	2906.4098	174.7919	124.7522
Sun	92514.4904	43531.0825	5.6041	2.6369	0.0004	0.0003
Venus	10.8438	5.1023	0.0014	0.0007	0.0000	0.0000
Jupiter	1.4120	0.6644	0.0000	0.0000		
Mars	0.4041	0.1901	0.0000	0.0000		
Mercury	0.0815	0.0383	0.0000	0.0000		
Saturn	0.0383	0.0164	0.0000	0.0000		
Uranus	0.000566	0.000266				
Neptune	0.000194	0.000091				
Pluto	0.00000002	0.00000001				
	ΔV_5	$\dot{g}_5$	ΔV_6	$\dot{g}_6$	ΔV_7	$\dot{g}_7$
Moon	3.0696	2.7402	0.0531	0.0567	0.0000	0.0000

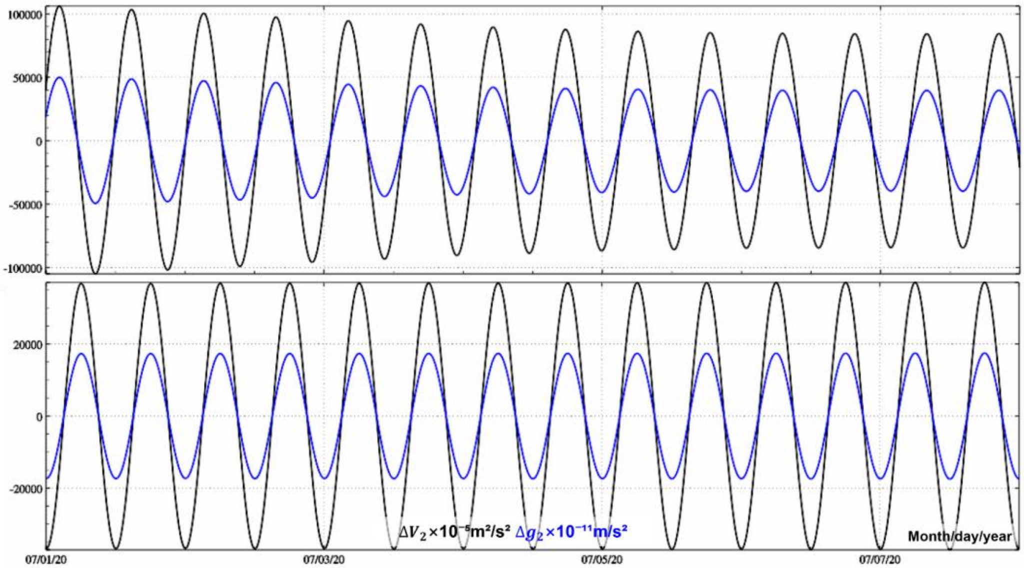


Fig 1.1 Degree-2 Earth's tidal potential (force) time series from Moon and Sun

Fig 1.1 shows degree-2 Earth's tidal potential (force) time series from Moon and Sun, and Fig 1.2 shows degree 3, 4 and 5 Earth's tidal potential (force) time series from Moon.

The time span is from 0 : 00 on July 1, 2020 to 24 : 00 on July 7, 2022 (7 days), with a time interval of 10 minutes.

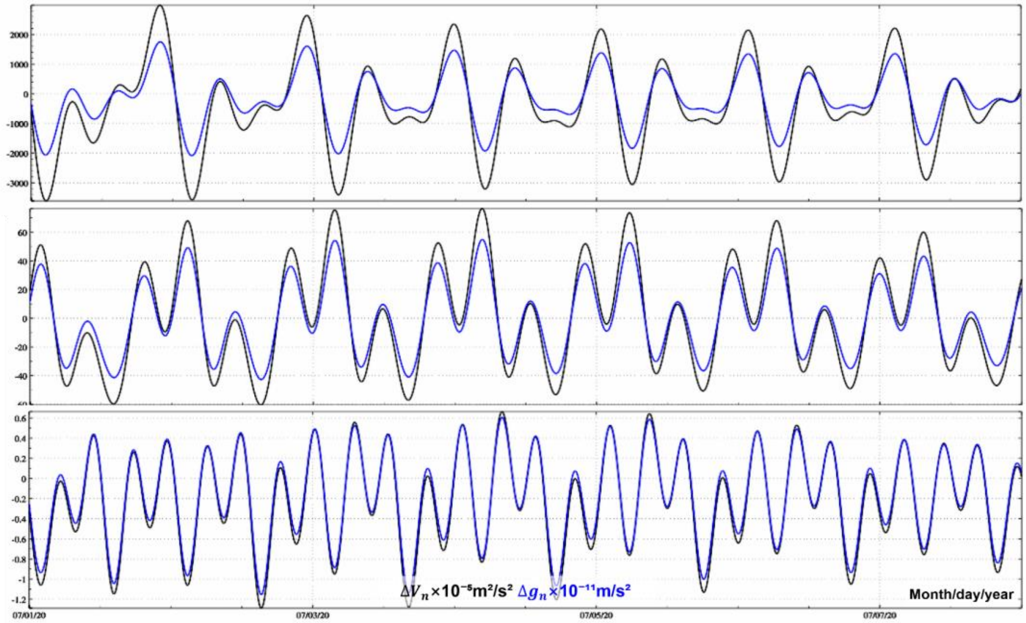


Fig 1.2 Degree 3, 4 and 5 Earth's tidal potential (force) time series from Moon (7 days)

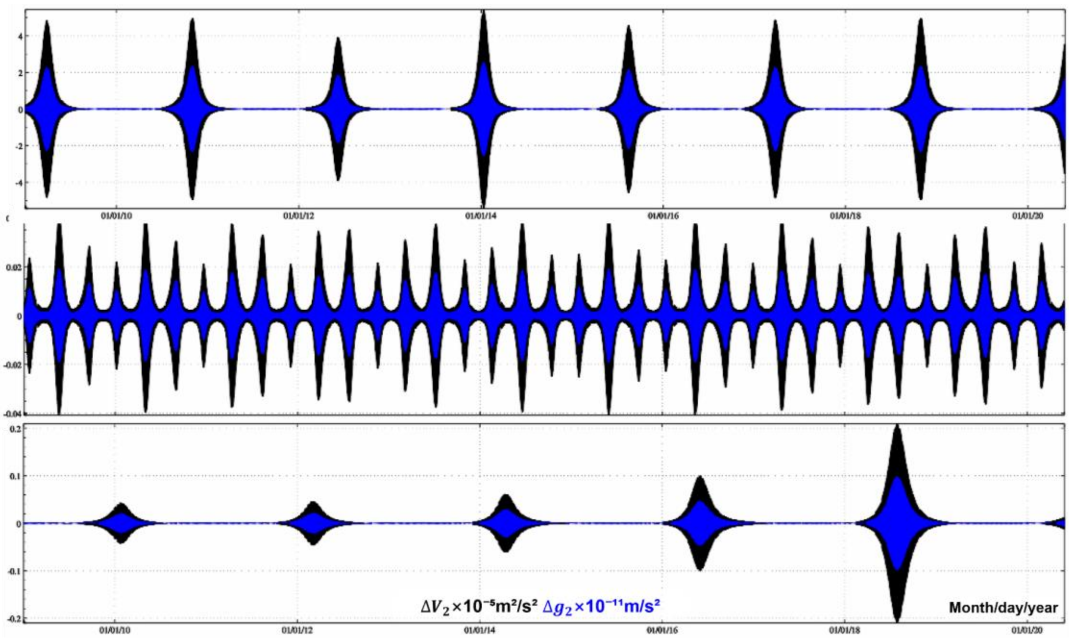


Fig 1.3 Degree-2 Earth's tidal potential (force) time series from Venus, Jupiter and Mars (12 years)

Fig 1.3 shows degree-2 Earth's tidal potential (force) time series from Venus, Jupiter and Mars from January 1, 2010 to December 31, 2022 (12 years), with a time interval of 2 hours.

Fig 1.4 shows degree-3 Earth tidal potential (force) time series from Moon and Sun from January 1, 2020 to December 31, 2022 (2 years).

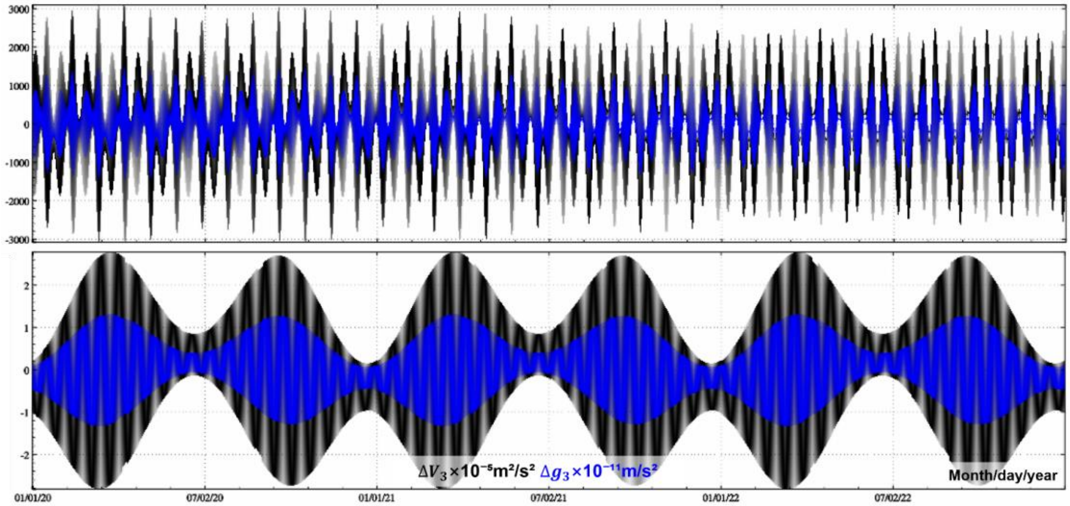


Fig 1.4 Degree 3 Earth's tidal potential (force) time series from Moon and Sun

Fig 1.5 ~ Fig 1.7 shows the geopotential coefficient time series of the direct influence of TGP calculated from the formula (1.1). From these three figures, it is not difficult to see that the TGP tesseral harmonic geopotential coefficient (m = 1) time series mainly shows the diurnal variation, the TGP sector harmonic geopotential coefficient (m = 2) time series mainly shows semi-diurnal variation, and the TGP zonal harmonic geopotential coefficient (m = 0) time series mainly shows long-period variation.

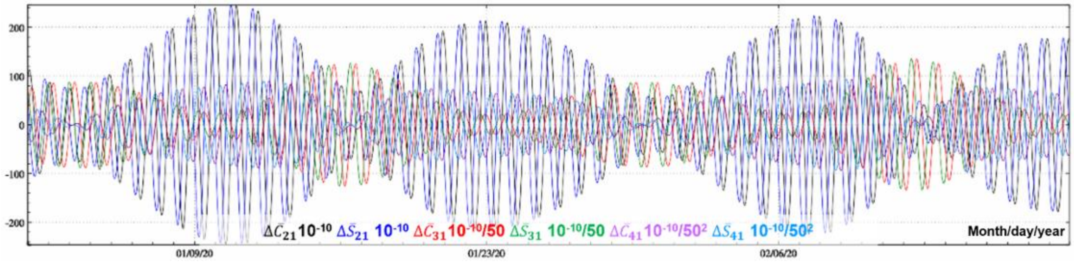


Fig 1.5 The tesseral TGP geopotential coefficient time series (diurnal variation)

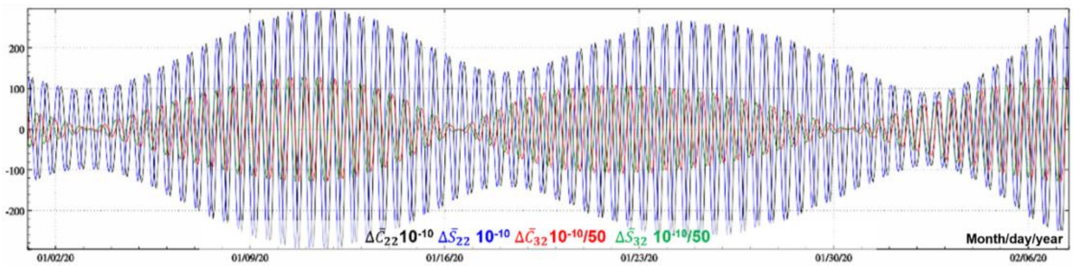


Fig 1.6 The sector TGP geopotential coefficient time series (semi-diurnal variation)

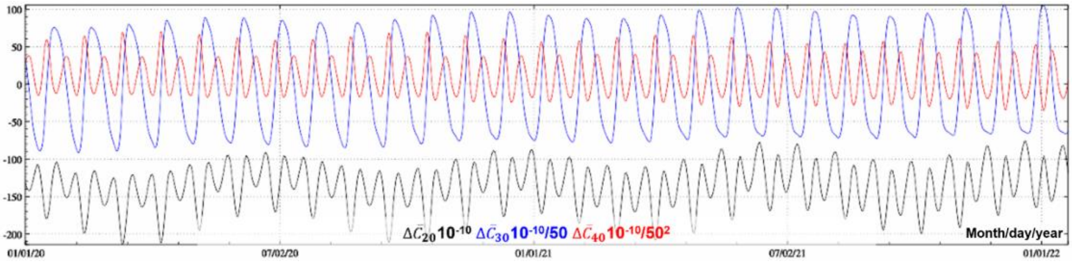


Fig 1.7 The zonal TGP geopotential coefficient time series (long-period variation)

8.1.2 The body tidal Love number for the rotating microellipsoidal anelastic Earth

The body tidal effect on geodetic variations are characterized by the linear combination of the body tidal Love numbers, that is, the body tidal factor $\delta = \mathcal{L}(k,h,l)$. There are three kinds of body tidal Love numbers, namely, the potential Love number k, radial (displacement) Love number h and horizontal (displacement) Love number l.

(1) The body tidal Love number for non-spherical rotating Earth

The non-spherical ellipticity of the elastic Earth and the rotation of the Earth make the expression of the Love number complicated. The tidal potential excites the deformation of the solid Earth, and the displacement deformation solution of the ground site excited by the degree-n order-m tidal potential is:

$$\boldsymbol{u} = \frac{W_{mn}}{g_0}\left[\boldsymbol{e}_r\left(h^0 Y_{nm} + h^+ Y_{n+2,m} + h^- Y_{n-2,m}\right) + \boldsymbol{e}_\theta \left(l^0 \frac{\partial Y_{nm}}{\partial\theta} + \omega^+ \frac{m}{sin\theta} Y_{n+1,m} + \omega^- \frac{m}{sin\theta} Y_{n-1,m} + l^+ \frac{\partial Y_{n+2,m}}{\partial\theta} + l^- \frac{\partial Y_{n-2,m}}{\partial\theta}\right) + i\boldsymbol{e}_\lambda \left(l^0 \frac{m}{sin\theta}\frac{\partial Y_{nm}}{\partial\theta} + \omega^+ \frac{m}{sin\theta}\frac{\partial Y_{n+1,m}}{\partial\theta} + \omega^- \frac{m}{sin\theta}\frac{\partial Y_{n-1,m}}{\partial\theta} + l^+ \frac{mY_{n+2,m}}{sin\theta} + l^- \frac{mY_{n-2,m}}{sin\theta}\right)\right] \quad (1.17)$$

Where, $W_{mn}Y_{nm} = W_{mn}(a)Y_{nm}(\theta,\lambda)$ is the degree-n order-m tidal potential, $g_0 = g_0(a)$ is the ground mean gravity, $W_{mn}Y_{nm}/g_0$ is the degree-n order-m equilibrium tidal height. h^0, l^0 are the spherically symmetric parts of displacement Love number, h^+, l^+, h^-, l^- are the spherical coupling parts of corresponding Love numbers, and ω^+, ω^- are the annular coupling parts of corresponding Love numbers.

Formula (1.17) indicates that for the spherically symmetric non-rotating elastic Earth, only two parameters (h and l) are needed to represent the tidal displacement field. But for the rotating microellipsoidal Earth, 8 parameters are needed. Because of the ellipticity and rotation, degree-n spherical displacement field (2 parameters) is coupled into the degree n+1, n-1 annular displacement field (2 parameters) and the degree n+2, n-2 spherical displacement field (4 parameters), forming the latitude dependent parts of the displacement Love numbers.

The tidal potential excites the deformation of the Earth's surface and internal medium, resulting in the redistribution of the Earth's internal density and the generation of associated geopotential. The associated geopotential at (r,θ,λ) can be expressed as:

$$\Phi^a(r,\theta,\lambda) = W_{mn}\left[k^0\left(\frac{a}{r}\right)^{n+1} Y_{nm} + k^+\left(\frac{a}{r}\right)^{n+3} Y_{n+2,m} + k^-\left(\frac{a}{r}\right)^{n-1} Y_{n-2,m}\right] \quad (1.18)$$

Similar to the displacement deformation solution, k^0 represents the spherically symmetric part of potential Love number, and k^+ and k^- represent the spherically coupling parts of corresponding Love number. Since the annular displacement does not involve volume expansion, it will not lead to the variation of geopotential. Therefore, there will be no annular coupling term in the potential Love number k.

The body tidal Love numbers for the microelliptical elastic Earth are still real numbers, which are independent of frequency. Whose value are shown in Tab 1.2.

Tab 1.2 The body tidal Love numbers for the microelliptical elastic Earth

n	m	periods of tidal constituents	k_{nm}	h_{nm}	l_{nm}
2	0	long period	0.29525	0.6078	0.0847
2	1	diurnal	0.29470	0.6078	0.0847
2	2	semi-diurnal	0.29801	0.6078	0.0847
3	0	long period	0.093	0.2920	0.0150
3	1	diurnal	0.093	0.2920	0.0150
3	2	semi-diurnal	0.093	0.2920	0.0150
3	3	1/3-diurnal	0.094	0.2920	0.0150
			0.041	0.175	0.010
			0.025	0.129	
			0.017	0.197	

(2) The latitude dependence of displacement Love numbers for the rotating microellipsoidal Earth

The Earth's ellipticity and rotation destroy the symmetry of the tidal response. The centrifugal force from the microellipsoidal Earth's rotation leads to the latitude dependence of displacement Love number (h_{nm}, l_{nm}). The latitude dependence formulas of degree-2 displacement Love numbers are:

$$\begin{cases} h_{2m}(\varphi) = h_{2m} + h^{\varphi} \frac{3sin^2\varphi - 1}{2} \\ l_{2m}(\varphi) = l_{2m} + l^{\varphi} \frac{3sin^2\varphi - 1}{2} \end{cases} \tag{1.19}$$

Where, $\varphi = \pi/2 - \theta$ is the geocentric latitude of the ground site, (h_{2m}, l_{2m}) are the displacement Love numbers for the microelliptic non-rotating elastic Earth with these value shown in Table 1.2, and $[h_{2m}(\varphi), l_{2m}(\varphi)]$ are the displacement Love numbers considering the latitude dependence of the site, $(h^{\varphi}, l^{\varphi})$ are the latitude dependence coefficients of displacement Love numbers. In the IERS conventions (2010), $h^{\varphi} = -0.0006, l^{\varphi} = 0.0002$.

The latitude dependence of degree-3 displacement Love numbers are very weak and do not need to be corrected.

(3) The frequency dependence of body tidal Love numbers from the mantle anelasticity

The mantle anelasticity leads to the delay of the deformation response to the tidal potential, which makes the Love numbers (including the spherical symmetry parts and the latitude dependence factors) of each tidal cluster (semi-diurnal $m = 2$, diurnal $m = 1$ and long-period $m = 0$ tidal cluster) become complex number with a relatively small imaginary part whose absolute value is less than 1 % of the real part. On the other hand, the mantle anelasticity also amplifies the tidal deformation, resulting in the frequency dependence of Love number increasing with the increase of the tidal period, so that the real and imaginary parts of Love numbers change slightly at the same time.

Long-period tidal waves ($m = 0$, also known as zonal harmonic tidal waves, with a period of 8 days to 18.6 years) have a low frequency and a large time span. The anelasticity of the mantle leads to a strong frequency dependence of Love numbers for long-period tidal waves. For semi-diurnal tidal waves $(m = 2)$, the radial Love number h increases by 1.4%. For the diurnal tidal waves $(m = 1)$, h increases by 1.4% and the potential Love number k increases by 1.7%. For long-period tidal waves ($m = 0$, such as M_f), the anelasticity of the mantle leads to a 2.5% increase in h and a 3% increase in k.

In order to represent degree-n order-m geopotential coefficient variations caused by the associated geopotential for the anelastic Earth, three forms of potential Love numbers $\left(k_{nm}^{(0)}, k_{nm}^{(\pm)}\right)$, radial Love numbers $\left(h_{nm}^{(0)}, h_{nm}^{(\pm)}\right)$ and horizontal Love numbers $\left(l_{nm}^{(0)}, l_{nm}^{(\pm)}\right)$ are needed to characterize the indirect influence of the degree-n order-m $(n \geq 2, m \leq n)$ tidal potential. Considering the mass conservation of the Earth, when $n = 2$, $k_{2m}^{(-)} = 0, h_{2m}^{(-)} = 0, l_{2m}^{(-)} = 0$, there are only two kinds of potential Love numbers $\left(k_{2m}^{(0)}, k_{2m}^{(+)}\right)$, two kinds of radial Love numbers $\left(h_{2m}^{(0)}, h_{2m}^{(+)}\right)$ and two kinds of horizontal Love numbers $\left(l_{2m}^{(0)}, l_{2m}^{(+)}\right)$.

The anelasticity of the mantle causes the Earth's response to tidal potential to be delayed, so that the Love numbers change with frequency, $\left(k_{2m}^{(0)}, k_{2m}^{(+)}\right)$, $\left(h_{2m}^{(0)}, h_{2m}^{(+)}\right)$ and $\left(l_{2m}^{(0)}, l_{2m}^{(+)}\right)$ have a small imaginary part. The frequency dependence of degree-3 Love numbers are very weak, and their delay effects to tidal potential can be ignored.

For the convenience of compulation, degree-2 long-period $(m = 0)$, diurnal $(m = 1)$ and semi-diurnal $(m = 2)$ tidal clusters for the anelastic Earth in Table 1.3 are denoted as $k_{2m} = Re(k_{2m}) + i\, Im(k_{2m})$ as the nominal potential Love numbers. The displacement Love numbers $h_{2m} = 0.6078$, $l_{2m} = 0.0847$ for the microelliptical elastic Earth in Table 1.2 are taken as the nominal displacement Love numbers.

Tab 1.3 Frequency dependence of potential Love number k for anelastic Earth

nm		periods of tidal constituents	anelastic Earth		anelastic Earth		
n	m		k_{nm}	$k_{2m}^{(+)}$	$Re(k_{nm})$	$Im(k_{nm})$	$k_{2m}^{(+)}$
2	0	long period	0.29525	-0.00087	0.30190	-0.00000	-0.00089
2	1	diurnal	0.29470	-0.00079	0.29830	-0.00144	-0.00080
2	2	semi-diurnal	0.29801	-0.00057	0.30102	-0.00130	-0.00057
3	0	long period	0.093	···			
3	1	diurnal	0.093	···			
3	2	semi-diurnal	0.093	···			

3	3	1/3-diurnal	0.094	···			

8.1.3 Frequency dependence of degree-2 body tidal Love numbers and their corrections

(1) The Earth's near-diurnal wobble and resonance parametersof Love numbers

The excitation of the Earth's near-diurnal free wobble leads to obvious resonance amplification in the observation of diurnal tides (such as P_1、K_1、ψ_1 and ϕ_1 tidal waves) and corresponding free nutation, which are close to the eigen-frequency. The change of centrifugal force from the Earth's wobble excites the deformation of solid Earth, which leads to the coupling change of the Earth’s moment of inertia. The contribution to the diurnal tidal Love numbers are proportional to the wobble response of the Earth and its nucleus.

For diurnal tides, the frequency dependent values of any load or body tide Love number L (such as $k_{21}^{(0)}$ or $k_{21}^{(+)}$) may be represented as a function of the tidal excitation frequency σ by a resonance formula

$$L(\sigma) = L_0 + \sum_{\alpha=1}^{3} \frac{L_\alpha}{\sigma - \sigma_\alpha} \tag{1.20}$$

Where, $\sigma_\alpha (\alpha = 1,2,3)$ are the respective resonance frequencies associated with the Chandler wobble (CW), the retrograde FCN and the prograde free core nutation (also known as the free inner core nutation), and the L_α are the corresponding resonance coefficients. All the parameters are complex. The σ_α and σ are expressed in cycles per sidereal day (cpsd), with the convention that positive (negative) frequencies represent retrograde (prograde) waves. (This sign convention, followed in tidal theory, is the opposite of that employed in analytical theories of nutation.) In particular, with the tidal frequency ω given in degrees per hour (°/hr), we have

$$\omega = 15\kappa\sigma, \quad \kappa = 1.002737909 \tag{1.21}$$

Here, the factor $\kappa = 1.002737909$ being the number of sidereal days per solar day. The values employed herein for the σ_α are from Mathews et al. (2002), adapted to the sign convention employed here:

$$\begin{aligned} \sigma_1 &= -0.0026010 - 0.0001361i \\ \sigma_2 &= \ \ 1.0023181 + 0.000025\ i \\ \sigma_3 &= \ \ 0.9999026 + 0.000780\ i \end{aligned} \tag{1.22}$$

Tab 1.4 shows the resonance parameters for the diurnal body tidal Love numbers $\left(k_{21}^{(0)}, k_{21}^{(+)}\right)$ calculated by the resonance formula (1.20).

Tab 1.4 Resonance parameters for diurnal Love numbers $\left(\boldsymbol{k_{21}^{(0)}}, \boldsymbol{k_{21}^{(+)}}\right)$

α	$k_{21}^{(0)}$		$k_{21}^{(+)}$	
	Re(L_α)	Im(L_α)	Re(L_α)	Im(L_α)
0	0.29954	-0.1412×10^{-2}	-0.804×10^{-3}	0.237×10^{-5}
1	-0.77896×10^{-3}	-0.3711×10^{-4}	0.209×10^{-5}	0.103×10^{-6}
2	0.90963×10^{-4}	-0.2963×10^{-5}	-0.182×10^{-6}	0.650×10^{-8}
3	-0.11416×10^{-5}	0.5325×10^{-7}	-0.713×10^{-9}	-0.330×10^{-9}

Tab 1.5 shows the resonance parameters for the diurnal load Love numbers $(k'_{21}, h'_{21}, l'_{21})$ calculated by the resonance formula (1.20). These resonance parameters are related to the geopotential variations and Earth deformation induced by ocean tidal loads. Small imaginary parts are ignored in the Tab 1.5.

Tab 1.5 Resonance parameters for diurnal load Love numbers $(\boldsymbol{k'_{21}}, \boldsymbol{h'_{21}}, \boldsymbol{l'_{21}})$

α	k'_{21}	h'_{21}	l'_{21}
0	-0.30808	-0.99500	0.02315
1	8.1874×10^{-4}	1.6583×10^{-3}	2.3232×10^{-4}
2	1.4116×10^{-4}	2.8018×10^{-4}	-8.4659×10^{-6}
3	3.4618×10^{-7}	5.5852×10^{-7}	1.0724×10^{-8}

The resonance effect of displacement Love numbers need to take into account the latitude dependence of the Love numbers and the resonance effect excited by the ocean tidal loads. Combining the formula (1.19) with (1.20), the resonance parameters of the displacement Love numbers and latitude dependent coefficients can be calculated, as shown in Tab 1.6.

Tab 1.6 Resonance parameters of the displacement Love numbers and latitude-dependent coefficients

α	$h_{2\mathrm{m}}$		h^{φ}	
	Re(L_α)	Im(L_α)	Re(L_α)	Im(L_α)
0	0.60671	-0.2420×10^{-2}	-0.615×10^{-3}	-0.122×10^{-4}
1	-0.15777×10^{-2}	-0.7630×10^{-4}	0.160×10^{-5}	0.116×10^{-6}
2	0.18053×10^{-3}	-0.6292×10^{-5}	0.201×10^{-6}	0.279×10^{-8}
3	-0.18616×10^{-5}	0.1379×10^{-6}	-0.329×10^{-7}	-0.217×10^{-8}
α	$l_{2\mathrm{m}}$		l^{φ}	
	Re(L_α)	Im(L_α)	Re(L_α)	Im(L_α)
0	0.84963×10^{-1}	-0.7395×10^{-3}	0.19334×10^{-3}	-0.3819×10^{-5}
1	-0.22107×10^{-3}	-0.9646×10^{-5}	-0.50331×10^{-6}	-0.1639×10^{-7}
2	-0.54710×10^{-5}	-0.2990×10^{-6}	-0.66460×10^{-8}	0.5076×10^{-9}
3	-0.29904×10^{-7}	-0.7717×10^{-8}	0.10372×10^{-7}	0.7511×10^{-9}

(2) The contributions to diurnal body tidal Love numbers of ocean tidal loads and their frequency dependent corrections

The diurnal resonance leads to the frequency dependence of load Love numbers, and causes the Earth's moment of inertia coupling change through the Earth's deformation (polar tide effect) caused by the centrifugal force of the tidal loads. The mantle anelasticity, the core-mantle coupling and the tidal friction dissipation mechanism lead to the frequency dependence of body tidal Love number for the diurnal tidal waves (mn = 21), which make the real and imaginary parts of body tidal Love numbers change slightly.

After considering the diurnal resonance effect, the diurnal load Love numbers need to be changed from (k_2', h_2', l_2') to $(k_{21}', h_{21}', l_{21}')$. The main contribution of the diurnal tide wave with a frequency of σ to the diurnal body tidal Love number of the same frequency is (Wahr and Sasao, 1981):

$$\delta \mathscr{k}_{21}^{ol}(\sigma) = [\mathscr{k}_{21}'(\sigma) - \mathscr{k}_2'] \frac{4\pi G \rho_w}{5 g_0} R A_{21}(\sigma) \tag{1.23}$$

Where $\mathscr{k} = k$, h or l. g_0 is the ground mean gravity, $\mathscr{k}_{21}'(\sigma)$ is the load Love number considering the diurnal resonance effect, which is a function of the tidal frequency σ, and is calculated according to Formula (1.20) using Tab 1.6. $\mathscr{k}_2' = k_2'$, h_2' or l_2' is degree-2 load Love number (real) of the spherically symmetric non-rotating elastic Earth. $A_{21}(\sigma)$ is the proportional factor (admittance) of the body or load Love number correction for the tide σ. Which can be calculated by the following formula after the harmonic analysis of the global ocean tide harmonic constant model:

$$A_{21}(\sigma) = H_{21}^{otide}(\sigma) / H_{21}^{TGP}(\sigma) \tag{1.24}$$

In the formula (1.24), $H_{21}^{TGP}(\sigma)$ is the global maximum amplitude (in unit of m) of the equilibrium tidal height of the celestial diurnal tide with the frequency of σ. $H_{21}^{otide}(\sigma)$ is the normalized harmonic amplitude (in unit of m) of the diurnal ocean tidal height with the same frequency of σ after normalized harmonic analysis.

The correction formula of frequency dependence for the diurnal body tidal Love numbers is as follows:

$$\delta \mathscr{k}_{21}(\sigma) = \mathscr{k}_{21}^{(0)}(\sigma) + \delta \mathscr{k}_{21}^{ol}(\sigma) - \mathscr{k}_{21} \tag{1.25}$$

Where $\mathscr{k} = k$, h or l. $\mathscr{k}_{21}^{(0)}(\sigma)$ is the body tidal Love numbers (complex) considering the anelasticity of mantle and near-diurnal resonance, which is a function of the tidal frequency σ. Which can be calculated according to Formula (1.20) using the resonance parameters in Tab 1.5 or Tab 1.6. $\mathscr{k}_{21}$ is the nominal diurnal Love number from Tab 1.3 or Tab 1.2.

The following is a brief summary of the three-step calculation scheme of the frequency dependent correction for diurnal tidal waves. Let degree-2 nominal load Love number $k_2' = -0.3075$, $h_2' = -1.001$, $l_2' = 0.0295$, degree-2 nominal diurnal potential Love number $k_{21} = 0.29830 - 0.00144i$ from Tab 1.3 and degree-2 nominal diurnal displacement Love number $h_{21} = 0.6078$, $l_{21} = 0.0847$ from Tab 1.2.

In the first step, from Formula (1.22), Tab 1.4, Tab 1.5 and Tab 1.6, the resonance body tidal and load Love numbers $\mathscr{k}_{21}^{0}(\sigma)$, $\mathscr{k}_{21}'(\sigma)$ considering the diurnal resonance effect are calculated according to Formula (1.20). The second step is to substitute the diurnal resonant load Love numbers $\mathscr{k}_{21}'(\sigma)$ into the formula (1.23) to calculate the ocean tidal load contributions $\delta \mathscr{k}_{21}^{ol}(\sigma)$ to the diurnal body tidal Love numbers. The third step is to substitute the diurnal resonance body tidal Love numbers $\mathscr{k}_{21}^{0}(\sigma)$ and the ocean tidal load contributions $\delta \mathscr{k}_{21}^{ol}(\sigma)$ into Formula (1.25) to calculate the frequency dependent corrections $\delta k_{21}(\sigma)$ for the diurnal potential Love numbers as Tab 1.7 and that $\delta h_{21}(\sigma), \delta l_{21}(\sigma)$ for the diurnal displacement Love numbers as Tab 1.8 and Tab 1.9.

Tab 1.7 The corrections for frequency dependence of $\boldsymbol{k_{2m}^{(0)}}$

Name	Doodson	ω°/hr	δk_{21}^{R}×10^{-5}	δk_{21}^{I}×10^{-5}	H_{21}^{TGP}×10^{-5}m
$2Q_1$	125,755	12.85429	-29	3	-664
σ_1	127,555	12.92714	-30	3	-802
	135,645	13.39645	-45	5	-947
Q_1	135,655	13.39866	-46	5	-5020
ρ_1	137,455	13.47151	-49	5	-954
	145,545	13.94083	-82	7	-4946
O_1	145,555	13.94303	-83	7	-26221
τ_1	147,555	14.02517	-91	9	343
$N\tau_1$	153,655	14.41456	-168	14	194
	155,445	14.48520	-193	16	137
Lk_1	155,455	14.48741	-194	16	741
No_1	155,655	14.49669	-197	16	2062
	155,665	14.49890	-198	16	414
χ_1	157,455	14.56955	-231	18	394
	157,465	14.57176	-233	18	87
π_1	162,556	14.91787	-834	58	-714
	163,545	14.95673	-1117	76	137
P_1	163,555	14.95893	-1138	77	-12203
	164,554	15.00000	-1764	104	103
S_1	164,556	15.00000	-1764	104	289
	165,345	15.02958	-3048	92	7
	165,535	15.03665	-3630	195	4
	165,545	15.03886	-3845	229	-730
K_1	165,555	15.04107	-4084	262	36878
	165,565	15.04328	-4355	297	5001
	165,575	15.04548	-4665	334	-108
	166,455	15.07749	85693	21013	-0.6
	166,544	15.07993	35203	2084	1.1
ψ_1	166,554	15.08214	22794	358	293
	166,556	15.08214	22780	358	-4.5
	166,564	15.08434	16842	-85	5
	167,355	15.11392	3755	-189	18
	167,365	15.11613	3552	-182	5
ϕ_1	167,555	15.12321	3025	-160	525
	167,565	15.12542	2892	-154	-20
	168,554	15.16427	1638	-93	31
θ_1	173,655	15.51259	370	-20	395
	173,665	15.51480	369	-20	78
	175,445	15.58323	325	-17	-61

J_1	175,455	15.58545	324	-17	2062
	175,465	15.58765	323	-16	409
So_1	183,555	16.05697	194	-8	342
	185,355	16.12989	185	-7	169
Oo_1	185,555	16.13911	184	-7	1129
	185,565	16.14131	184	-7	723
	185,575	16.14352	184	-7	151
υ_1	195,455	16.68348	141	-4	216
	195,465	16.68569	141	-4	138
			$\delta k_{22}^{R}\times10^{-5}$	$\delta k_{22}^{I}\times10^{-5}$	$H_{22}^{TGP}\times10^{-5}$m
N_2	245,655	28.43973	2	0	12099
M_2	255,555	28.98410	2	0	63192

Tab 1.8 The corrections for the frequency and latitude dependenc of $h_{2m}^{(0)}$

Name	Doodson	ω°/hr	$\delta h_{21}^{R}\times10^{-4}$	$\delta h_{21}^{I}\times10^{-4}$	$H_{21}^{TGP}\times10^{-5}$m
$2Q_1$	125,755	12.85429	-39	-27	-664
σ_1	127,555	12.92714	-39	-26	-802
	135,645	13.39648	-42	-26	-947
Q_1	135,655	13.39866	-42	-26	-5020
ρ_1	137,455	13.47151	-43	-26	-954
	145,545	13.94082	-50	-25	-4946
O_1	145,555	13.94303	-50	-25	-26221
τ_1	147,555	14.02517	-52	-25	343
$N\tau_1$	153,655	14.41456	-67	-24	194
No_1	155,655	14.49669	-73	-23	2062
χ_1	157,455	14.56955	-80	-23	394
π_1	162,556	14.91787	-200	-15	-714
P_1	163,555	14.95893	-261	-11	-12203
S_1	164,556	15.00000	-386	-4	289
	165,545	15.03881	-795	23	-730
K_1	165,555	15.04107	-842	30	36878
	165,565	15.04333	-896	36	5001
	165,575	15.04543	-958	43	-108
ψ_1	166,554	15.08214	4491	36	293
	166,564	15.08439	3309	-50	5
ϕ_1	167,555	15.12321	567	-59	525
θ_1	173,655	15.51259	39	-30	395
J_1	175,455	15.58545	30	-30	2062
Oo_1	185,555	16.13911	2	-28	1129
			$\delta h_{22}^{R}\times10^{-4}$	$\delta h_{22}^{I}\times10^{-4}$	$H_{22}^{TGP}\times10^{-5}$m
M_2	255,555	28.98410	0	-22	12099

Tab 1.9 The corrections for the frequency and latitude dependence of $\boldsymbol{l_{2m}^{(0)}}$

Name	Doodson	ω°/hr	δl_{21}^{R}×10⁻⁴	δl_{21}^{I}×10⁻⁴	H_{21}^{TGP}×10⁻⁵m
Q_1	135,655	13.39866	-1	-6	-5020
	145,545	13.94082	-1	-6	-4946
O_1	145,555	13.94303	-1	-6	-26221
No_1	155,655	14.49669	0	-6	2062
P_1	163,555	14.95893	6	-6	-12203
	165,545	15.03886	22	-6	-730
K_1	165,555	15.04107	23	-6	36878
	165,565	15.04328	25	-6	5001
ψ_1	166,554	15.08214	-137	-20	293
ϕ_1	167,555	15.12321	-19	-7	525
J_1	175,455	15.58545	-2	-6	395
Oo_1	185,555	16.13911	-1	-6	1129
			δl_{22}^{R}×10⁻⁴	δl×10⁻⁴	H_{22}^{TGP}×10⁻⁵m
M_2	255,555	28.98410	0	-7	12099

(3) Long-period body tidal Love number frequency dependent corrections of anelastic Earth

The anelasticity of the mantle further enhances the frequency dependence of body tidal Love numbers for the long-period (zonal) tidal waves $(nm = 20)$. The frequency of the long-period tidal constituent is assumed to be σ, and the long-period Love number considering the frequency dependence can be expressed as:

$$k_{20}(\sigma) = 0.29525 - 5.796 \times 10^4 \left\{ctg\frac{\epsilon\pi}{2}\left[1-\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right] + i\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right\} \tag{1.26}$$

$$h_{20}(\sigma) = 0.5998 - 9.96 \times 10^4 \left\{ctg\frac{\epsilon\pi}{2}\left[1-\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right] + i\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right\} \tag{1.27}$$

$$l_{20}(\sigma) = 0.0831 - 3.01 \times 10^4 \left\{ctg\frac{\epsilon\pi}{2}\left[1-\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right] + i\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right\} \tag{1.28}$$

Here, σ_m is the reference frequency with period of 200s, and $\epsilon = 0.15$.

Let $k_{20} = 0.30190, h_{20} = 0.6078, l_{20} = 0.0847$, the frequency dependent correction formula of the zonal body tidal Love numbers for the anelastic Earth are:

$$\delta \ell_{20}(\sigma) = \ell_{20}(\sigma) - \ell_{20}, \quad (\ell = k, h, l) \tag{1.29}$$

The formulas (1.26) ~ (1.28) is substituted into the formula (1.29) respectively, and the frequency dependent correction value of zonal body tidal Love number for the anelastic Earth are calculated as Tab 1.10 ~ Tab 1.12.

Tab 1.10 The corrections for frequency dependence of $\boldsymbol{k_{20}^{(0)}}$

Name	Doodson	ω°/hr	δk_{20}^{R}×10⁻⁵	δk_{20}^{I}×10⁻⁵	H_{20}^{TGP}×10⁻⁵m
Ω_1	55,565	0.00221	1347	-541	2793
Ω_2	55,575	0.00441	1124	-488	-27
S_a	56,554	0.04107	547	-349	-492

S_{sa}	57,555	0.08214	403	-315	-3100
	57,565	0.08434	398	-313	77
S_{ta}	58,554	0.12320	326	-296	-181
M_{sm}	63,655	0.47152	101	-242	-673
	65,445	0.54217	80	-237	231
M_m	65,455	0.54438	80	-237	-3518
	65,465	0.54658	79	-237	229
	65,655	0.55366	77	-236	188
M_{sf}	73,555	1.01590	-9	-216	-583
	75,355	1.08875	-18	-213	-288
M_f	75,555	1.09804	-19	-213	-6663
	75,565	1.10024	-19	-213	-2762
	75,575	1.10245	-19	-213	-258
M_{stm}	83,655	1.56956	-65	-202	-242
M_{tm}	85,455	1.64241	-71	-201	-1276
	85,465	1.64462	-71	-201	-529
M_{sqm}	93,555	2.11394	-102	-193	-204
M_{qm}	95,355	2.18679	-106	-192	-169

Tab 1.11 The corrections for the frequency and latitude dependence of $\boldsymbol{h_{20}^{(0)}}$

Name	Doodson	ω°/hr	$\delta h_{20}^R\times10^{-4}$	$\delta h_{20}^I\times10^{-4}$	$H_{20}^{TGP}\times10^{-5}$m
Ω_1	55,565	0.00221	266	-93	2793
S_{sa}	57,555	0.08214	104	-54	-3100
M_m	65,455	0.54438	48	-41	-3518
M_f	75,555	1.09804	31	-37	-6663
	75,565	1.10024	31	-37	-2762

Tab 1.12 The corrections for the frequency and latitude dependence of $\boldsymbol{l_{20}^{(0)}}$

Name	Doodson	ω°/hr	$\delta l_{20}^R\times10^{-4}$	$\delta l_{20}^I\times10^{-4}$	$H_{20}^{TGP}\times10^{-5}$m
Ω_1	55,565	0.00221	89	-28	2793
S_{sa}	57,555	0.08214	39	-16	-3100
M_m	65,455	0.54438	23	-12	-3518
M_f	75,555	1.09804	17	-11	-6663
	75,565	1.10024	17	-11	-2762

(4) The geopotential coefficient adjustments from frequency dependent corrections of potential Love number

The contribution of different frequency tidal constituent to potential Love number is different, and thus the frequency dependent correction $\delta k_{2\mathrm{m}}$ for each tidal constituent needs to be calculated one by one. The Love number correction of frequency dependence for some a constituent σ is $\delta k_{2\mathrm{m}}(\sigma)$, and the direct influence of the constituent σ on degree-2 order-m geopotential coefficient is $\Delta\bar{C}_{2m}^{(\sigma)}-i\Delta\bar{S}_{2m}^{(\sigma)}$, then the product $k_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)}-i\Delta\bar{S}_{2m}^{(\sigma)}\right)$ is degree-2 order-m geopotential coefficient variation caused by the Love number frequency dependent correction for the corresponding constituent σ. The geopotential coefficient adjustments $\Delta\bar{C}_{2m}^{\delta}-i\Delta\bar{S}_{2m}^{\delta}$ can be obtained by summing the contribution of frequency dependent corrections for all tidal constituents.

Let degree-2 order-m potential Love number frequency dependent correction be $\delta k_{2m}=\delta k_{2m}^{R}+i\delta k_{2m}^{I}$, then degree-2 tesseral and sector harmonic geopoential coefficient adjustments $(m=1,2)$ caused by the diurnal and semi-diurnal Love number frequency dependent corrections are:

$$\Delta\bar{C}_{2m}^{\delta}-i\Delta\bar{S}_{2m}^{\delta}=\sum_{\sigma}\delta k_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)}-i\Delta\bar{S}_{2m}^{(\sigma)}\right)=\eta_m\left(\sum_{\tau=1}^{\tau_{20}}A_m\delta k_{2m}^{\tau}H_{2m}^{\tau}e^{i\phi^{\tau}}\right)=\eta_m A_m \sum_{\tau=1}^{\tau_{2m}}H_{2m}^{\tau}[(\delta k_{2m}^{\tau R}cos\phi^{\tau}-\delta k_{2m}^{\tau I}sin\phi^{\tau})+i(\delta k_{2m}^{\tau R}sin\phi^{\tau}+\delta k_{2m}^{\tau I}cos\phi^{\tau})] \quad (1.30)$$

$$\Delta\bar{C}_{21}^{\delta}-i\Delta\bar{S}_{21}^{\delta}= A_1\sum_{\tau=1}^{\tau_{20}}H_{21}^{\tau}[(\delta k_{21}^{\tau R}sin\phi^{\tau}+\delta k_{22}^{\tau I}cos\phi^{\tau})-i(\delta k_{21}^{\tau R}cos\phi^{\tau}-\delta k_{21}^{\tau I}sin\phi^{\tau})] \quad (1.31)$$

$$\Delta\bar{C}_{22}^{\delta}-i\Delta\bar{S}_{22}^{\delta}= A_2\sum_{\tau=1}^{\tau_{20}}H_{22}^{\tau}[(\delta k_{22}^{\tau R}cos\phi^{\tau}-\delta k_{22}^{\tau I}sin\phi^{\tau})+i(\delta k_{22}^{\tau R}sin\phi^{\tau}+\delta k_{22}^{\tau I}cos\phi^{\tau})] \quad (1.32)$$

Similarly, degree-2 zonal harmonic geopoential coefficient adjustments caused by the long-period Love number frequency dependent corrections are:

$$\Delta\bar{C}_{20}^{\delta}=Re\left[\sum_{\sigma}\delta k_{20}(\sigma)\Delta\bar{C}_{20}^{\sigma}\right]=Re\left(\sum_{\tau=1}^{\tau_{20}}\delta k_{20}^{\tau}A_0H_{20}^{\tau}e^{i\phi^{\tau}}\right)=A_0\sum_{\tau=1}^{\tau_{20}}H_{20}^{\tau}(\delta k_{20}^{\tau R}cos\phi^{\tau}-\delta k_{20}^{\tau I}sin\phi^{\tau}) \quad (1.33)$$

In the formulas (1.30) ~ (1.33), $\eta_1=-i$, $\eta_2=1$,

$$A_0=\frac{1}{R\sqrt{4\pi}}=4.4228\times10^{-8}\ \text{(in unit of /m)}$$

$$A_m=\frac{(-1)^m}{R\sqrt{8\pi}}=(-1)^m(3.1274\times10^{-8})\ \text{(in unit of /m)},\ m=1,2$$

Where, $\tau_{2m}(m=0,1,2)$ is the number of effective tidal constituents of degree-2 order-m tidal waves and in Tab 5.6, $\tau_{20}=21$, $\tau_{21}=48$, $\tau_{22}=2$. H_{2m}^{τ} is the global maximum equilibrium tidal height amplitude (in unit of m) for the corresponding tidal constituent, and that is the last column in Tab 1.6. ϕ^{τ} is the astronomical argument (in unit of radian) of the tidal constituent τ, which can be calculated by six Doodson astronomical arguments or five basic Delaunay variables.

(5) Equivalent treatment of displacement Love number frequency dependent corrections

For the ground sites fixed with the solid Earth, when the solid tidal effects on geodetic variations contain the tidal deformation contribution characterized by the displacement Love number, the frequency dependent correction of the displacement Love number needs to be considered. The displacement Love number represents the displacement of the ground site excited by the tidal generating potential, and its effect on geodetic variations always appears in the form of proportional factor. From Section 8.1.1, it is not difficult to see that in the expression of solid Earth's tidal effect on radial and horizontal displacement, ground gravity and ground tilt at ground site, the displacement Love number and geopotential coefficient for the same degree-n order-m tidal waves always appears in the form of product.

According to the theory of solid Earth deformation mechanics, the solid tidal effect on ground site displacement is harmonic and can be expressed in the form of spherical harmonic series. The product of the frequency dependent correction of displacement Love number for the degree-n order-m tidal waves and the direct influence of the same degree-n order-m geopotential coefficient is the contribution of the frequency dependent correction of the displacement Love number to the degree-n order-m spherical harmonic coefficient. Thus, degree-2 spherical harmonic coefficient adjustments caused by the frequency dependent corrections of degree-2 dispalcement Love number are:

$$\Delta\hat{C}_{2m}^{\delta} - i\Delta\hat{S}_{2m}^{\delta} = \sum_{\sigma} \delta h_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}\right) \tag{1.34}$$

$$\Delta\tilde{C}_{2m}^{\delta} - i\Delta\tilde{S}_{2m}^{\delta} = \sum_{\sigma} \delta l_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}\right) \tag{1.35}$$

Comparing the algorithm formulas of geopotential coefficient adjustment from the potential Love number frequency dependent correction, degree-2 tesseral and sector spherical harmonic coefficient adjustments $(m = 1,2)$ caused by the diurnal and semi-diurnal radial Love number frequency dependent corrections are:

$$\Delta\hat{C}_{2m}^{\delta} - i\Delta\hat{S}_{2m}^{\delta} = \sum_{\sigma} \delta h_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}\right) = \eta_m\left(\sum_{\tau=1}^{\tau_{20}} A_m \delta h_{2m}^{\tau} H_{2m}^{\tau}\, e^{i\phi^{\tau}}\right) = \eta_m A_m \sum_{\tau=1}^{\tau_{20}} H_{2m}^{\tau}[(\delta h_{2m}^{\tau R} cos\phi^{\tau} - \delta h_{2m}^{\tau I} sin\phi^{\tau}) + i(\delta h_{2m}^{\tau R} sin\phi^{\tau} + \delta h_{2m}^{\tau I} cos\phi^{\tau})] \tag{1.36}$$

Degree-2 zonal spherical harmonic coefficient adjustments $(m = 0)$ caused by the long-period radial Love number frequency dependent corrections are:

$$\Delta\hat{C}_{20}^{\delta} = Re\left[\sum_{\sigma} \delta h_{20}(\sigma)\Delta\bar{C}_{20}^{(\sigma)}\right] = Re\left(\sum_{\tau=1}^{\tau_{20}} \delta h_{20}^{\tau} A_0 H_{20}^{\tau}\, e^{i\phi^{\tau}}\right) = A_0 \sum_{\tau=1}^{\tau_{20}} H_{20}^{\tau}(\delta h_{20}^{\tau R} cos\phi^{\tau} - \delta h_{20}^{\tau I} sin\phi^{\tau}) \tag{1.37}$$

Similarly, degree-2 tesseral and sector spherical harmonic coefficient adjustments $(m = 1,2)$ caused by the diurnal and semi-diurnal horizontal Love number frequency dependent corrections are:

$$\Delta\tilde{C}_{2m}^{\delta} - i\Delta\tilde{S}_{2m}^{\delta} = \sum_{\sigma} \delta l_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}\right) = \eta_m\left(\sum_{\tau=1}^{\tau_{20}} A_m \delta l_{2m}^{\tau} H_{2m}^{\tau}\, e^{i\phi^{\tau}}\right) = \eta_m A_m \sum_{\tau=1}^{\tau_{20}} H_{2m}^{\tau}[(\delta l_{2m}^{\tau R} cos\phi^{\tau} - \delta l_{2m}^{\tau I} sin\phi^{\tau}) + i(\delta l_{2m}^{\tau R} sin\phi^{\tau} + \delta l_{2m}^{\tau I} cos\phi^{\tau})] \tag{1.38}$$

Degree-2 zonal spherical harmonic coefficient adjustments $(m = 0)$ caused by the long-period horizontal Love number frequency dependent corrections are:

$$\Delta\tilde{C}_{20}^{\delta} = Re\left[\sum_{\sigma} \delta k_{20}(\sigma)\Delta\bar{C}_{20}^{(\sigma)}\right] = Re\left(\sum_{\tau=1}^{\tau_{20}} \delta k_{20}^{\tau} A_0 H_{20}^{\tau}\, e^{i\phi^{\tau}}\right) = A_0 \sum_{\tau=1}^{\tau_{20}} H_{20}^{\tau}(\delta k_{20}^{\tau R} cos\phi^{\tau} - \delta k_{20}^{\tau I} sin\phi^{\tau}) \tag{1.39}$$

Here, the frequency dependent correction formulas of the displacement Love number is expressed as the adjustments of displacement spherical harmonic coefficients, which is the same as the frequency dependent correction formulas of potential Love number for the adjustments of geopotential coefficients. The purpose of this treatment is to standardize the computation process of the solid tidal effect on various geodetic variations, and to realize the algorithm compatibility and unified calculation of the solid tidal effects on various geometric and physical geodetic variations.

8.1.4 Unified computation scheme of the body tidal effects on all-element geodetic variations in whole Earth space

Using the analytically compatible geodetic and geodynamic algorithms with the numerical standards unified and geophysical models collaborated to calculate uniformly the solid Earth tidal effects on various geodetic variations on the ground and outside the solid Earth is an important basis and necessary condition for the collaborative monitoring of multi-geodetic technologies and deep fusion of heterogeneous Earth monitoring data.

(1) Love number frequency dependent correction algorithm for solid tidal effect

Firstly, the appropriate nominal body tidal Love numbers are selected, so that the frequency dependent corrections of Love numbers include the contribution of all imaginary parts of Love numbers, and then let the nominal Love number be real to simplify the calculation scheme of solid tidal effect.

For the reason mentioned, here let the real part of degree-2 potential Love number for the anelastic Earth in Tab 1.3 as the nominal potential Love number and the imaginary part of degree-2 potential Love number will be considered in the frequency dependent correction algorithm of the potential Love number. That is, the imaginary part of degree-2 diurnal potential Love number in Tab 1.6 is uniformly added to − 0.00144, which becomes $k_{21}^{I}(\sigma) = \delta k_{21}^{I}(\sigma) - 0.00144$, and the imaginary part of degree-2 semi-diurnal potential Love number is uniformly added to − 0.00130, which becomes $\delta k_{22}^{I}(\sigma) = \delta k_{22}^{I}(\sigma) - 0.00130$. In this way, the nominal tidal Love number value is as shown in Tab 1.13.

Tab 1.13 The values for nominal body tidal Love numbers

n	m	periods of tidal constituents	k_{nm}	h_{nm}	l_{nm}
2	0	long period	0.30190	0.6078	0.0847
2	1	diurnal	0.29830	0.6078	0.0847
2	2	semi-diurnal	0.30102	0.6078	0.0847
3	0	long period	0.093	0.2920	0.0150
3	1	diurnal	0.093	0.2920	0.0150
3	2	semi-diurnal	0.093	0.2920	0.0150
3	3	1/3-diurnal	0.094	0.2920	0.0150

The Love number frequency dependent correction algorithms for the adjustments of degree-2 geopotential coefficients have been given by Formulas (1.30) to (1.33). Substituting degree-2 geopoential coefficient adjustments $\Delta\bar{C}_{2m}^{\delta} - i\Delta\bar{S}_{2m}^{\delta}$ into the solid Earth tidal effect expression (1.4) on height anomaly, degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on height anomaly on the ground or outside solid Earth can be obtained:

$$\delta\zeta(r,\theta,\lambda) = \frac{GM}{\gamma r}\left(\frac{a}{r}\right)^2 \sum_{m=0}^{2}\left(\Delta\bar{C}_{2m}^{\delta} cosm\lambda + \Delta\bar{S}_{2m}^{\delta} sinm\lambda\right)\bar{P}_{2m}(cos\theta) \tag{1.40}$$

Here, $\Delta\bar{S}_{20}^{\delta} = 0$.

Similarly, degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on gravity can be obtained:

$$\delta g^{\delta} = 3\frac{GM}{r^2}\left(\frac{a}{r}\right)^2 \sum_{m=0}^{2}\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda + \Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\bar{P}_{2m}(cos\theta) \tag{1.41}$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on vertical deflection can be obtained:

South: $$\delta\xi = \frac{GMsin\theta}{\gamma r^2}\left(\frac{a}{r}\right)^2 \sum_{m=0}^{2}\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda + \Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\frac{\partial}{\partial\theta}\bar{P}_{2m}(cos\theta) \tag{1.42}$$

West: $$\delta\eta = \frac{GM}{\gamma r^2 \sin\theta}\left(\frac{a}{r}\right)^2 \sum_{m=1}^{2} m\left(\Delta\bar{C}_{2m}^{\delta}sinm\lambda - \Delta\bar{S}_{2m}^{\delta}cosm\lambda\right)\bar{P}_{2m}(cos\theta) \tag{1.43}$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on gravity gradient (radial) can be obtained:

$$\delta T_{rr} = 12\frac{GM}{r^3}\left(\frac{a}{r}\right)^2 \sum_{m=0}^{2}\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda + \Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\bar{P}_{2m}(cos\theta) \tag{1.44}$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on horizontal gravity gradient can be obtained:

North: $$\delta T_{NN} = -\frac{GM}{r^3}\left(\frac{a}{r}\right)^2 \sum_{m=0}^{2}\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda + \Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\frac{\partial^2}{\partial\theta^2}\bar{P}_{2m}(cos\theta) \tag{1.45}$$

West: $$\delta T_{WW} = \frac{GM}{r^3 sin^2\theta}\left(\frac{a}{r}\right)^2 \sum_{m=1}^{2} m^2\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda + \Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\bar{P}_{2m}(cos\theta) \tag{1.46}$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on site displacement can be obtained ◉ (◉ marked indicates its site fixed with the solid Earth):

East: $$\delta e = -\frac{GM}{\gamma r \sin\theta}\left(\frac{a}{r}\right)^2 \sum_{m=1}^{2} m\left(\Delta\tilde{C}_{2m}^{\delta}sinm\lambda - \Delta\tilde{S}_{2m}^{\delta}cosm\lambda\right)\bar{P}_{2m}(cos\theta) \tag{1.47}$$

North: $$\delta n = -\frac{GM\sin\theta}{\gamma r}\left(\frac{a}{r}\right)^2 \sum_{m=0}^{2}\left(\Delta\tilde{C}_{2m}^{\delta}cosm\lambda + \Delta\tilde{S}_{2m}^{\delta}sinm\lambda\right)\frac{\partial}{\partial\theta}\bar{P}_{2m}(cos\theta) \tag{1.48}$$

Radial: $$\delta r = \frac{GM}{\gamma r}\left(\frac{a}{r}\right)^2 \sum_{m=0}^{2}\left(\Delta\hat{C}_{2m}^{\delta}cosm\lambda + \Delta\hat{S}_{2m}^{\delta}sinm\lambda\right)\bar{P}_{2m}(cos\theta) \tag{1.49}$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on ground gravity can be obtained ◉:

$$\delta g^{s}(r,\varphi,\lambda) = \frac{3GM}{r^2}\left(\frac{a}{r}\right)^2 \sum_{m=0}^{2}\left[\left(\Delta\hat{C}_{2m}^{\delta} - \frac{3}{2}\Delta\bar{C}_{2m}^{\delta}\right)cosm\lambda + \left(\Delta\hat{S}_{2m}^{\delta} - \frac{3}{2}\Delta\bar{S}_{2m}^{\delta}\right)sinm\lambda\right]\bar{P}_{2m}(cos\theta) \tag{1.50}$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on ground tilt can be obtained ◉:

South: $$\delta\xi^{s} = \frac{GM\sin\theta}{\gamma r^2}\left(\frac{a}{r}\right)^2 \sum_{m=0}^{2}\left[\left(\Delta\bar{C}_{2m}^{\delta} - \Delta\hat{C}_{2m}^{\delta}\right)cosm\lambda + \left(\Delta\bar{S}_{2m}^{\delta} - \Delta\hat{S}_{2m}^{\delta}\right)sinm\lambda\right]\frac{\partial}{\partial\theta}\bar{P}_{2m}(cos\theta) \tag{1.51}$$

West: $$\delta\eta^{s} = \frac{GM}{\gamma r^2 \sin\theta}\left(\frac{a}{r}\right)^2 \sum_{m=1}^{2} m\left[\left(\Delta\bar{C}_{2m}^{\delta} - \Delta\hat{C}_{2m}^{\delta}\right)sinm\lambda - \left(\Delta\bar{S}_{2m}^{\delta} - \Delta\hat{S}_{2m}^{\delta}\right)cosm\lambda\right]\bar{P}_{2m}(cos\theta) \tag{1.52}$$

(2) The solid Earth tidal effects on degree-4 geopotential coefficients

The solid Earth tidal effects on degree-4 geopotential coefficients are calculated by the direct influence $(\Delta\bar{C}_{2m} - i\Delta\bar{S}_{2m})$ of degree-2 geopotential coefficients using the frequency dependent Love numbers $k_{2m}^{+}, (m = 0,1,2)$.

$$\Delta\bar{C}_{4m} - i\Delta\bar{S}_{4m} = k_{2m}^{+}(\Delta\bar{C}_{2m} - i\Delta\bar{S}_{2m}), \quad m = 0,1,2 \tag{1.53}$$

Although the solid Earth tidal effects on degree-4 geopotential coefficients are calculated by the direct influence of degree-2 geopotential coefficients according to Formula (1.53), their contributions to geodetic variations should be calculated according to the variations of degree-4 geopotential coefficients. Substituting the formula (1.53) into the formulas (1.4) ~ (1.16), the contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on the height anomaly can be obtained as follows:

$$\varepsilon\zeta(r,\varphi,\lambda) = \frac{GM}{\gamma r}\left(\frac{a}{r}\right)^4 \sum_{m=0}^{2} k_{2m}^{+}(\Delta\bar{C}_{2m}cosm\lambda + \Delta\bar{S}_{2m}sinm\lambda)\bar{P}_{4m}(cos\theta) \tag{1.54}$$

The contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on gravity can be obtained as follows:

$$\varepsilon g^{\delta} = \frac{5GM}{r^2}\left(\frac{a}{r}\right)^4 \sum_{m=0}^{2} k_{2m}^{+}(\Delta\bar{C}_{2m}cosm\lambda + \Delta\bar{S}_{2m}sinm\lambda)\bar{P}_{4m}(cos\theta) \tag{1.55}$$

The contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on ground tilt or vertical deflection can be obtained as follows:

$$\text{South: } \varepsilon\xi = \frac{GM\sin\theta}{\gamma r^2}\left(\frac{a}{r}\right)^4 \sum_{m=0}^{2} k_{2m}^{+}(\Delta\bar{C}_{2m}cosm\lambda + \Delta\bar{S}_{2m}sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{4m}(cos\theta) \tag{1.56}$$

$$\text{West: } \varepsilon\eta = \frac{GM}{\gamma r^2\sin\theta}\left(\frac{a}{r}\right)^4 \sum_{m=1}^{2} k_{2m}^{+}m(\Delta\bar{C}_{2m}sinm\lambda - \Delta\bar{S}_{2m}cosm\lambda)\bar{P}_{4m}(cos\theta) \tag{1.57}$$

The contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on gravity gradient (radial) can be obtained as follows:

$$\varepsilon T_{rr} = \frac{30GM}{r^3}\left(\frac{a}{r}\right)^4 \sum_{m=0}^{2} k_{2m}^{+}(\Delta\bar{C}_{2m}cosm\lambda + \Delta\bar{S}_{2m}sinm\lambda)\bar{P}_{4m}(cos\theta) \tag{1.58}$$

The contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on horizontal gravity gradient can be obtained as follows:

$$\text{North: } \varepsilon T_{NN} = -\frac{GM}{r^3}\left(\frac{a}{r}\right)^4 \sum_{m=0}^{2} k_{2m}^{(+)}(\Delta\bar{C}_{2m}cosm\lambda + \Delta\bar{S}_{2m}sinm\lambda)\frac{\partial^2}{\partial\theta^2}\bar{P}_{4m} \tag{1.59}$$

$$\text{West: } \varepsilon T_{WW} = \frac{GM}{r^3 sin^2\theta}\left(\frac{a}{r}\right)^4 \sum_{m=1}^{2} m^2 k_{2m}^{+}(\Delta\bar{C}_{2m}sinm\lambda + \Delta\bar{S}_{2m}cosm\lambda)\bar{P}_{4m} \tag{1.60}$$

The contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on ground displacement is always zero.

(3) The unified computation scheme of the solid Earth tidal effects on all-element geodetic variations

(a) The geopotential coefficient variations $\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm}, (n = 2,3,4,5,6)$ from the Earth's tidal generating potential (TGP) of celestial bodies outside the Earth are calculated directly from Formula (1.1).

(b) The nominal Love numbers from Tab 1.13 are used, and the nominal displacement Love number and the calculation point geocentric latitude φ are employed to calculate the latitude dependent displacement Love number according to formula (1.19). Then, according to formulas (1.3) to (1.16), the nominal value as x^0 of the solid tidal effects on various geodetic variations are calculated from the geopotential coefficient variations $\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm}$.

(c) The frequency dependent Love numbers $k_{2m}^{+}, (m = 0,1,2)$ are employed to calculate the contribution as εx of degree-4 geopotential coefficient to the solid tidal effects on geodetic variations according to formulas (1.54) to (1.60) from degree-2 geopotential coefficient variations $\Delta\bar{C}_{2m} - i\Delta\bar{S}_{2m}$.

(d) According to the formulas (1.30) and (1.31), (1.36) and (1.37), the frequency dependent corrections $\left(\Delta\bar{C}_{2m}^{\delta} - \Delta\hat{C}_{2m}^{\sigma}\right)$, $\left(\Delta\hat{C}_{2m}^{\sigma} - i\Delta\hat{S}_{2m}^{\sigma}\right)$ and $\left(\Delta\tilde{C}_{2m}^{\sigma} - i\Delta\tilde{S}_{2m}^{\sigma}\right)$ of the geopotential coefficient variations and displacement spherical harmonic coefficient variations are calculated by the frequency dependent corrections of the degree-2 Love numbers and ocean tide harmonic amplitudes. Then, according to the formula (1.40) to (1.52), degree-2 Love number frequency dependent corrections as δx of the solid tidal effects on various geodetic variations are calculated.

(e) The frequency dependent corrections of the degree-2 potential Love number and ocean tide harmonic amplitudes are shown in Tab 1.6 to Tab 1.11. The frequency dependent corrections of the potential Love numbers in these Tables are uniformly added using the imaginary parts of degree-2 Love number of the anelastic Earth in Tab 1.3, that is, $k_{21}^{I}(\sigma) = \delta k_{21}^{I}(\sigma) - 0.00144$, $\delta k_{22}^{I}(\sigma) = \delta k_{22}^{I}(\sigma) - 0.00130$.

(f) The sum of the nominal value x^0 of solid tidal effects, degree-4 geopotential coefficient contributions εx and degree-2 Love number frequency dependent corrections δx is the high-precision calculation result of the solid tidal effects on various geodetic variations at the calculation point on ground or outside the solid Earth.

8.1.5 Characteristics and analysis of solid Earth tidal effects in geodesy

The Earth's tidal potential and tidal force from celestial body are related to the position of the calculation point in the Earth-fixed coordinate system. In the following, the ground point P (105°N, 20°E, H100m) selected, the time-varying properties of the solid Earth tidal effects on various geodetic variations are investigated by calculating the time series of solid Earth tidal effects and the contribution time series of each part at the point P.

(1) Solid Earth tidal effects on all-element geodetic variations

Firstly, The various contributions considered in the solid Earth tidal effects, the solid Earth tidal effect time series on all-element geodetic variations at the ground point P (105°N, 20°E, H100m) are calculated, as shown in Fig 1.8. The time span is from 0 : 00 on June 1, 2020 to 24 : 00 on June 7, 2022 (7 days), with a time interval of 10 minutes.

When analyzing the time-varying properties of the tidal effect time series curve, it is generally possible to focus on the amplitude variation with time, and to investigate the phase relationship between the tidal effects on different types of geodetic variations. Because the difference between the maximum and minimum values of the time series is independent of the reference time epoch, the difference between the maximum and minimum values of tidal effect time series can effectively reflect the relationship between the tidal effects on different types of geodetic variations.

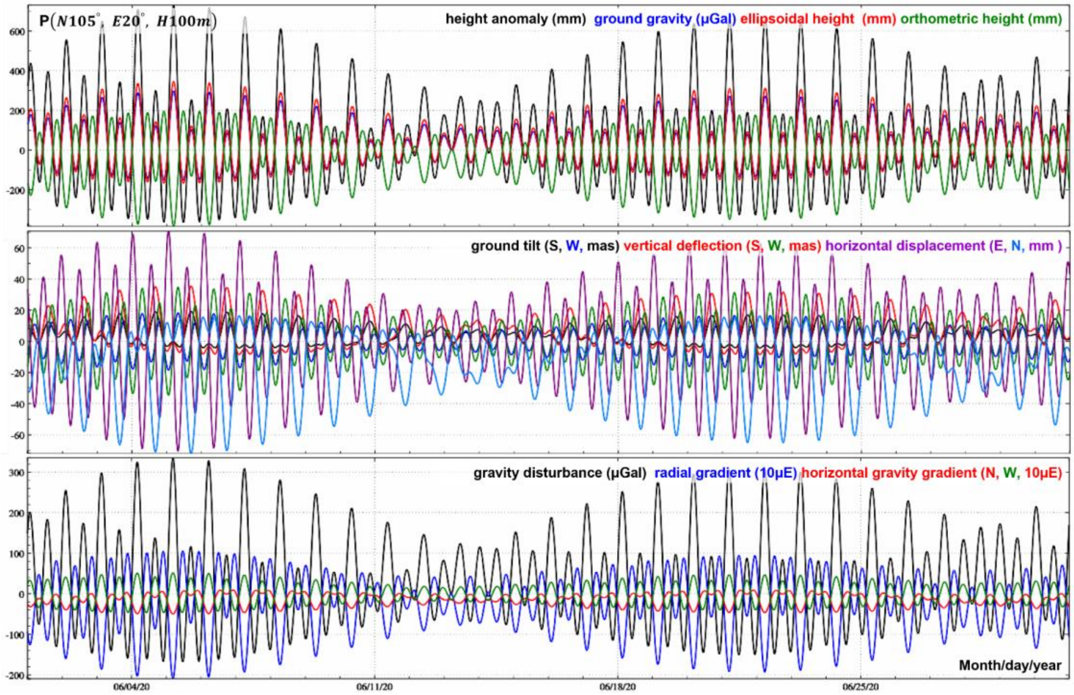


Fig 1.8 Solid Earth tidal effect time series on all-element geodetic variations

Fig 1.8 shows that the difference between the maximum and minimum values of the solid tidal effects on geoid can reach 0.99m, that on ground ellipsoidal height can reach 0.51m, that on normal height can reach 0.58m, that on the ground gravity can reach 447.5μGal, that on the ground tilt can reach 45mas, that on the horizontal displacement can reach 0.16m, that on the radial gravity gradient can reach 3.20mE, and the difference between the maximum and minimum values of the solid tidal effects on the horizontal gravity gradient can reach 1.15mE.

The solid tidal effects on the ground ellipsoidal height are different from that on the normal height (the symbol is opposite at the same time epoch), the solid tidal effects on ground tilt in the south and west directions are different, and the solid tidal effects on the horizontal gravity gradient in the north and west directions are different. The solid tidal effect on the horizontal displacement, ground tilt (vertical deviation) and horizontal gravity gradient vector are generally larger in the east-west direction than that in the north-south direction.

(2) Solid tidal effects of the planets outside Earth in the solar system

Here takes all the planets outside Earth in solar system as the tidal celestial bodies, and calculates the solid tidal effect time series on all-element geodetic variations at the ground point P(105°N, 20°E, H100m), as shown in Fig 1.9. The time span is from January 1, 2020 to December 31, 2022 (12 years), with a time interval of 2 hours. The solid tidal effects from planets do not contain the contributions of the Love number frequency dependence.

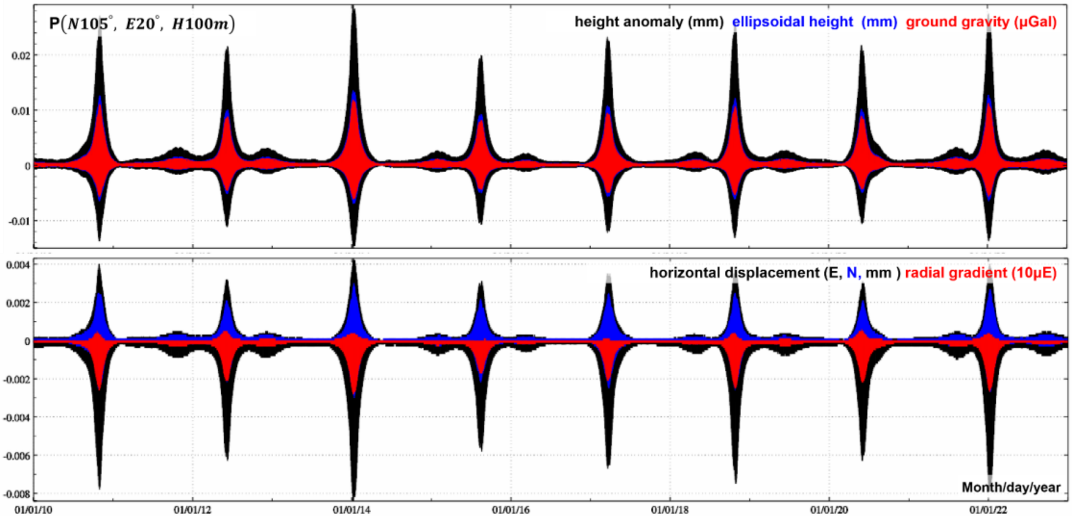


Fig 1.9 Solid tidal effect time series from the planets outside Earth

(3) The indirect influences of tidal potential to geodetic variations

The indirect influence is the contribution of tidal potential to the geodetic variations through the action of body tidal Love numbers, which is also called the solid tidal effect by some literatures. Fig 1.10 is the indirect influence time series curve of tidal potential to all-element geodetic variations at the ground point P(105°N, 20°E, H100m). The time span is from 0 : 00 on June 1, 2020 to 24 : 00 on June 7, 2022 (7 days), with a time interval of 10 minutes.

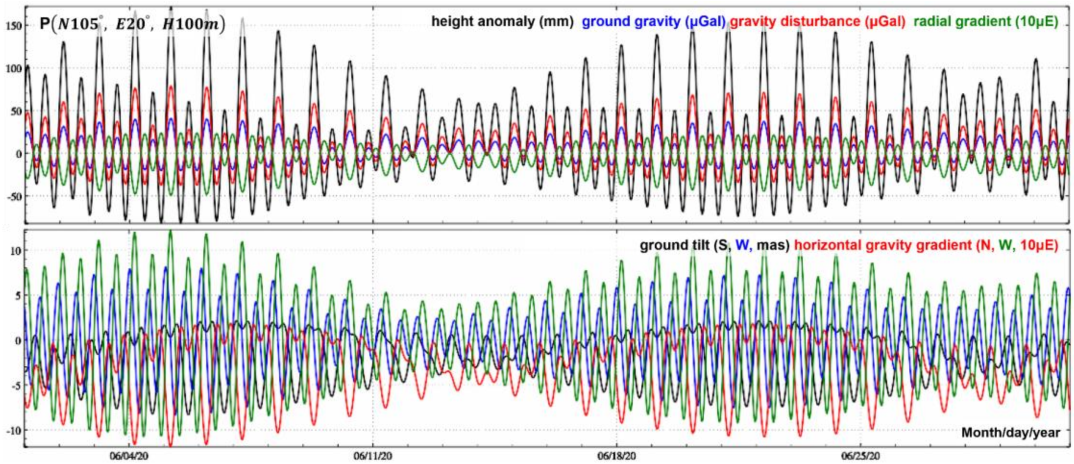


Fig 1.10 The indirect influence time series of tidal potential to geodetic variations

Figure 1.10 shows that the difference between the maximum and minimum indirect influences of tidal potential to geoid can reach 0.24 m, that to ground gravity can reach 40μGal, and the difference between the maximum and minimum indirect influences of tidal potential to radial gravity gradient can reach 0.7mE. Compared with the solid tidal effects (the sum of the direct and indirect influences of tidal potential), the phase relationship between the indirect influences to different types of geodetic variations are not completely consistent.

(4) Total contributions of Love number frequency dependent corrections to solid tidal effects

In the following, the contribution time series of potential Love number frequency dependent correction to the solid tidal effect on all-element geodetic variations at ground point P(105°N, 20°E, H100m) are calculated as Fig 1.11. The time span is from January 1, 2018 to January 31, 2018 (1 month), with a time interval of 30 minutes.

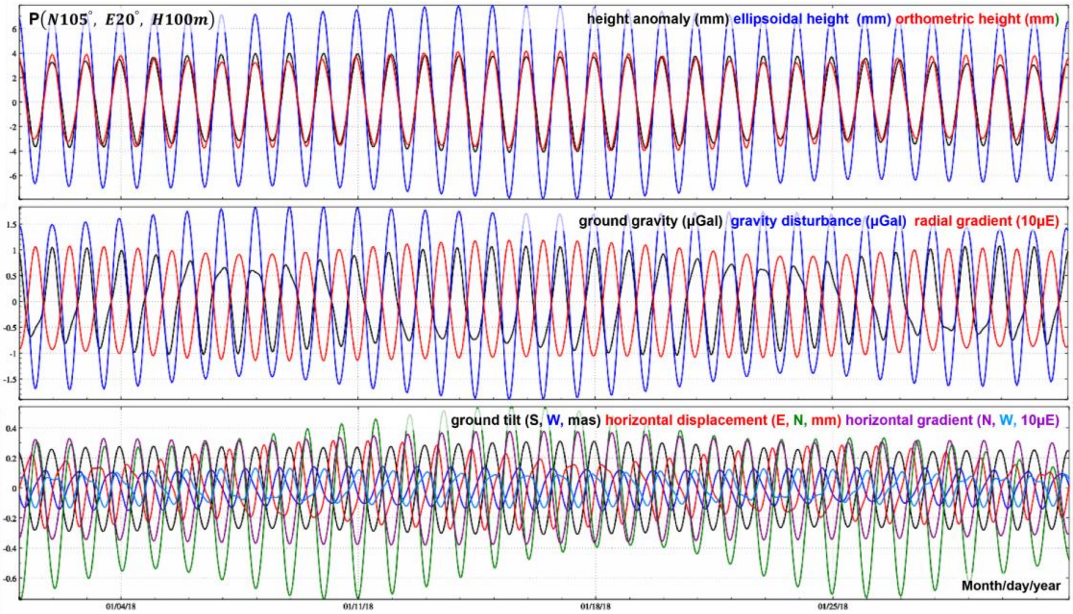


Fig 1.11 Contributions of potential Love number frequency dependent corrections

8.2 Global surface load spherical harmonic analysis and load effect synthesis

8.2.1 Spherical harmonic series representation for equivalent water heights of surface loads

The non-tidal load variations of atmosphere, sea level, soil water, groundwater, lakes and glaciers in the Earth′s surface layer system can be expressed by the variations of surface equivalent water height (EWH) h_w or unit point mass load $q_w = \rho_w h_w$ (also known as surface density, ρ_w is the density of water).

Surface non-tidal load variations h_w directly cause the variations of geopotential outside the Earth, which are the direct influences $\Delta V^*(r,\theta,\lambda)$ of surface load geopotential variations and can be expressed as:

$$\Delta V^*(r,\theta,\lambda) = \frac{GM}{r}\sum_{n=0}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{*}cosm\lambda + \Delta\bar{S}_{nm}^{*}sinm\lambda)\,\bar{P}_{nm}(cos\theta) \qquad (2.1)$$

Where, r is the geocentric distance of the calculation point, $(\Delta\bar{C}_{nm}^{*}, \Delta\bar{S}_{nm}^{*})$ are the normalized geopotential coefficient variations directly caused by surface non-tidal load varaitions, that is, the direct influence of geopotential (Stokes) coefficients, which can be calculated according to the gravitation potential definition:

$$\begin{Bmatrix}\Delta\bar{C}^*_{nm}\\ \Delta\bar{S}^*_{nm}\end{Bmatrix}=\frac{3\rho_w}{4\pi a\rho_e(2n+1)}\left(\frac{r}{a}\right)^n\int_S h_w\bar{P}_{nm}(cos\theta)\begin{Bmatrix}cosm\lambda\\ sinm\lambda\end{Bmatrix}sin\theta d\theta d\lambda dr \tag{2.2}$$

Here, $\int_S \cdot dS$ represents the global surface integral, $dS=sin\theta d\theta d\lambda dr$, and ρ_e is the mean density of the Earth.

The equivalent water height variation h_w at the ground point $(r_0\approx a,\theta,\lambda)$ can also be expressed as a normalized load spherical harmonic series:

$$h_w(r_0,\theta,\lambda)=r_0\sum_{n=1}^{\infty}\left(\frac{a}{r_0}\right)^n\sum_{m=0}^{n}(\Delta\bar{C}^w_{nm}cosm\lambda+\Delta\bar{S}^w_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \tag{2.3}$$

Here, $\Delta\bar{C}^w_{nm},\Delta\bar{S}^w_{nm}$ are the degree-n order-m normalized load spherical harmonic coefficients.

Considering that in general, the long wave is dominant in the global surface load variations, n will not be too large, and the geocentric distance of the surface load is $r_0\approx a$, so $(a/r)^n\approx 1$, then Formula (2.3) can be simplified as:

$$h_w=a\sum_{n=1}^{\infty}\sum_{m=0}^{n}(\Delta\bar{C}^w_{nm}cosm\lambda+\Delta\bar{S}^w_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \tag{2.4}$$

Comparing the formulas (2.2) and (2.4), we have:

$$\begin{Bmatrix}\Delta\bar{C}^*_{nm}\\ \Delta\bar{S}^*_{nm}\end{Bmatrix}=\frac{3\rho_w}{\rho_e}\frac{1}{2n+1}\begin{Bmatrix}\Delta\bar{C}^w_{nm}\\ \Delta\bar{S}^w_{nm}\end{Bmatrix} \tag{2.5}$$

Formula (2.5) is the relationship between the normalized spherical harmonic coefficient variations $\{\Delta\bar{C}^w_{nm},\Delta\bar{S}^w_{nm}\}$ of surface equivalent water height variations and the direct influences of surface equivalent water height variaions to the normalized geopotential coefficient $(\Delta\bar{C}^*_{nm},\Delta\bar{S}^*_{nm})$.

8.2.2 The normalized spherical harmonic series expansion for surface load deformation field

According to the theory of Earth's load deformation, the variations h_w of ground equivalent water height also lead to the deformation of solid Earth, which further make mass adjustment in Earth and produce associated geopotential, which indirectly causes the geopotential variations called the indirect influence of surface load variations, and can be characterized by load Love numbers or load tidal factor.

The total influence of the degree-n order-m normalized spherical harmonic coefficient $\{\Delta\bar{C}^w_{nm},\Delta\bar{S}^w_{nm}\}$ of the ground equivalent water height variations to geopotential coefficients are equal to the sum of the direct influence and indirect influence of ground equivalent water height variations. The sum is also called the load effects on geopotential coefficient variations.

$$\begin{Bmatrix}\Delta\bar{C}_{nm}\\ \Delta\bar{S}_{nm}\end{Bmatrix}=(1+k'_n)\begin{Bmatrix}\Delta\bar{C}^*_{nm}\\ \Delta\bar{S}^*_{nm}\end{Bmatrix}=\frac{3\rho_w}{\rho_e}\frac{1+k'_n}{2n+1}\begin{Bmatrix}\Delta\bar{C}^w_{nm}\\ \Delta\bar{S}^w_{nm}\end{Bmatrix} \tag{2.6}$$

Here, k'_n is degree-n load potential Love number.

From the load spherical harmonic coefficient variations $\{\Delta\bar{C}^w_{nm},\Delta\bar{S}^w_{nm}\}$, the spherical harmonic synthesis algorithm formula of the load effect $\Delta V(r,\theta,\lambda)$ on geopotential at the calculation point (r,θ,λ) on the ground or outside the solid Earth is calculated as follows:

$$\Delta V=\frac{GM}{r}\frac{3\rho_w}{\rho_e}\sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^n\frac{1+k_n'}{2n+1}\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{w}cosm\lambda+\Delta\bar{S}_{nm}^{w}sinm\lambda)\bar{P}_{nm}(cos\theta) \qquad (2.7)$$

From the Bruns formula, the spherical harmonic synthesis formula of the load effect $\Delta\zeta(r,\theta,\lambda)$ on the height anomaly can be obtained:

$$\Delta\zeta=\frac{GM}{r\gamma}\frac{3\rho_w}{\rho_e}\sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^n\frac{1+k_n'}{2n+1}\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{w}cosm\lambda+\Delta\bar{S}_{nm}^{w}sinm\lambda)\bar{P}_{nm}(cos\theta) \qquad (2.8)$$

Where, γ is the normal gravity. Similarly, the spherical harmonic synthesis formula of the load effect $\Delta g^s(r_0,\theta,\lambda)$ on the ground gravity can be obtained ◉:

$$\Delta g^s(r_0,\theta,\lambda)=\frac{GM}{r_0^2}\frac{3\rho_w}{\rho_e}\sum_{n=1}^{\infty}\frac{n+1}{2n+1}\left(1+\frac{2}{n}h_n'-\frac{n+1}{n}k_n'\right)\left(\frac{a}{r_0}\right)^n$$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{w}cosm\lambda+\Delta\bar{S}_{nm}^{w}sinm\lambda)\bar{P}_{nm}(cos\theta) \qquad (2.9)$$

Here, h_n' is the degree-n load radial Love number and (r_0,θ,λ) is the spherical coordinates of the ground calculation point.

The spherical harmonic synthesis formula of the load effect $\Delta g^\delta(r,\theta,\lambda)$ on gravity (disturbance) at the calculation point (r,θ,λ) on the ground or outside the solid Earth is:

$$\Delta g^\delta(r,\theta,\lambda)=\frac{GM}{r^2}\frac{3\rho_w}{\rho_e}\sum_{n=1}^{\infty}\frac{n+1}{2n+1}(1+k_n')\left(\frac{a}{r}\right)^n$$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{w}cosm\lambda+\Delta\bar{S}_{nm}^{w}sinm\lambda)\bar{P}_{nm}(cos\theta) \qquad (2.10)$$

Compared with formula (2.9), formula (2.10) does not include the influence due to ground radial displacement. Therefore, formula (2.9) is only suitable for calculating the load effect on ground gravity at calculation point fixed with the solid Earth, while formula (2.10) is suitable for calculating the load effect on gravity on the ground and outside the ground (such as aviation height, satellite height or ocean space). In order to distinguish these two cases, the formula in the case that the calculation point fixed with the Earth is marked ◉ here.

The normal gravity field is the starting datum of the anomalous Earth gravity field, and the normal gravity field elements do not change with time. Therefore, there is no difference between the tidal or non-tidal load effect on gravity, gravity disturbance and gravity anomaly.

The spherical harmonic synthesis formula of the load effect on ground tilt is ◉:

South: $$\Delta\xi^s(r_0,\theta,\lambda)=\frac{GM}{r_0^2}\frac{3\rho_w}{\gamma\rho_e}\sin\theta\sum_{n=1}^{\infty}\frac{1+k_n'-h_n'}{2n+1}\left(\frac{a}{r_0}\right)^n$$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{w}cosm\lambda+\Delta\bar{S}_{nm}^{w}sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{nm}(cos\theta) \qquad (2.11)$$

West: $$\Delta\eta^s(r_0,\theta,\lambda)=\frac{GM}{r_0^2\sin\theta}\frac{3\rho_w}{\gamma\rho_e}\sum_{n=1}^{\infty}\frac{1+k_n'-h_n'}{2n+1}\left(\frac{a}{r_0}\right)^n$$
$$\sum_{m=1}^{n}m(\Delta\bar{C}_{nm}^{w}sinm\lambda-\Delta\bar{S}_{nm}^{w}cosm\lambda)\bar{P}_{nm}(cos\theta) \qquad (2.12)$$

The spherical harmonic synthesis formula of the load effect on vertical deflection is:

South: $$\Delta\xi(r,\theta,\lambda)=\frac{GM}{r^2}\frac{3\rho_w}{\gamma\rho_e}\sin\theta\sum_{n=1}^{\infty}\frac{1+k_n'}{2n+1}\left(\frac{a}{r}\right)^n$$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{w}cosm\lambda+\Delta\bar{S}_{nm}^{w}sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{nm}(cos\theta) \qquad (2.13)$$

West: $$\Delta\eta(r,\theta,\lambda)=\frac{GM}{r^2\sin\theta}\frac{3\rho_w}{\gamma\rho_e}\sum_{n=1}^{\infty}\frac{1+k_n'}{2n+1}\left(\frac{a}{r}\right)^n$$
$$\sum_{m=1}^{n}m(\Delta\bar{C}_{nm}^{w}sinm\lambda-\Delta\bar{S}_{nm}^{w}cosm\lambda)\bar{P}_{nm}(cos\theta) \qquad (2.14)$$

The spherical harmonic synthesis formula of the load effect on the displacement of ground site is ⊙:

East: $\Delta e(r_0,\theta,\lambda) = -\frac{GM}{r_0\gamma sin\,\theta}\frac{3\rho_w}{\rho_e}\sum_{n=1}^{\infty}\frac{l_n'}{2n+1}\left(\frac{a}{r_0}\right)^n$
$$\sum_{m=1}^{n} m(\Delta\bar{C}_{nm}^{w} sinm\lambda - \Delta\bar{S}_{nm}^{w} cosm\lambda)\bar{P}_{nm}\,(cos\theta) \quad (2.15)$$

North: $\Delta n(r_0,\theta,\lambda) = -\frac{GM}{r_0\gamma}\frac{3\rho_w}{\rho_e} sin\,\theta \sum_{n=1}^{\infty}\frac{l_n'}{2n+1}\left(\frac{a}{r_0}\right)^n$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{w} cosm\lambda + \Delta\bar{S}_{nm}^{w} sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{nm}(cos\theta) \quad (2.16)$$

Radial: $\Delta r(r_0,\theta,\lambda) = \frac{GM}{r_0\gamma}\frac{3\rho_w}{\rho_e}\sum_{n=1}^{\infty}\frac{h_n'}{2n+1}\left(\frac{a}{r_0}\right)^n$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{w} cosm\lambda + \Delta\bar{S}_{nm}^{w} sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (2.17)$$

The spherical harmonic synthesis formula of the load effect on gravity gradient is:

Radial: $\Delta T_{rr}(r,\theta,\lambda) = \frac{GM}{r^3}\frac{3\rho_w}{\rho_e}\sum_{n=1}^{\infty}\frac{(n+1)(n+2)}{2n+1}(1+k_n')\left(\frac{a}{r}\right)^n$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{w} cosm\lambda + \Delta\bar{S}_{nm} sinm\lambda)\bar{P}_{nm}\,(cos\theta) \quad (2.18)$$

North: $\Delta T_{NN}(r,\theta,\lambda) = -\frac{GM}{r^3}\frac{3\rho_w}{\rho_e}\sum_{n=1}^{\infty}\frac{1+k_n'}{2n+1}\left(\frac{a}{r}\right)^n$
$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^{w} cosm\lambda + \Delta\bar{S}_{nm}^{w} sinm\lambda)\frac{\partial^2}{\partial\theta^2}\bar{P}_{nm}\,(cos\theta) \quad (2.19)$$

West: $\Delta T_{WW}(r,\theta,\lambda) = -\frac{GM}{r^3 sin^2\theta}\frac{3\rho_w}{\rho_e}\sum_{n=1}^{\infty}\frac{1+k_n'}{2n+1}\left(\frac{a}{r}\right)^n$
$$\sum_{m=1}^{n} m^2(\Delta\bar{C}_{nm}^{w} sinm\lambda + \Delta\bar{S}_{nm}^{w} cosm\lambda)\bar{P}_{nm}(cos\theta) \quad (2.20)$$

In the formulas (2.8) ~ (2.20), the first degree term $(n = 1)$ represents the contribution of the variations of Earth's center of mass caused by the surface load deformation to the corresponding geodetic elements, which can be called as the Earth's center of mass variation effects on geodetic variations. The variation of Earth's center of mass plays an important role in geodesy, and the first degree term in the mentioned formulas cannot be ignored.

The load Love number of unit point mass load (1kg/m²) can be calculated using the relevant parameters of spherically symmetric non-rotating elastic Earth model. The degree-n load radial, horizontal and potential Love numbers h_n', l_n' and k_n' are shown in Tab 2.1.

Tab 2.1 The value of load Love numbers

Degree-n	h_n'	l_n'	k'_n
1	-0.2871129880	0.1045044062	0
2	-0.9945870591	0.0241125159	-0.3057703360
3	-1.0546530210	0.0708549368	-0.1962722363
4	-1.0577838950	0.0595872318	-0.1337905897
5	-1.0911859150	0.0470262750	-0.1047617976
6	-1.1492536560	0.0394081176	-0.0903495805
7	-1.2183632010	0.0349940065	-0.0820573391
8	-1.2904736610	0.0322512320	-0.0765234897
9	-1.3618478650	0.0303856246	-0.0723928769
10	-1.4309817610	0.0290225900	-0.0690776844

12	-1.5609348550	0.0271636708	-0.0638847506
14	-1.6797703790	0.0259680057	-0.0598385602
16	-1.7880882500	0.0251266737	-0.0564748883
18	-1.8864404740	0.0244708343	-0.0535490132
20	-1.9754659020	0.0238986214	-0.0509272630
25	-2.1615247260	0.0225448633	-0.0452625739
30	-2.3044581340	0.0211578086	-0.0405033192
35	-2.4152406280	0.0197609745	-0.0364524519
40	-2.5028874800	0.0184188171	-0.0329970228
45	-2.5741299450	0.0171690959	-0.0300450548
50	-2.6337485520	0.0160264262	-0.0275153569
60	-2.7300189390	0.0140651027	-0.0234487653
70	-2.8076818590	0.0124702089	-0.0203629907
80	-2.8746338100	0.0111640070	-0.0179658948
90	-2.9350553590	0.0100800427	-0.0160636283
100	-2.9913054190	0.0091686192	-0.0145257169
120	-3.0965116190	0.0077267323	-0.0122109806
140	-3.1965444360	0.0066448758	-0.0105711243
150	-3.2455767690	0.0062018042	-0.0099238838
160	-3.2942117980	0.0058106942	-0.0093636844
180	-3.3907532400	0.0051551676	-0.0084470364
200	-3.4867370690	0.0046324760	-0.0077337989
250	-3.7248624300	0.0037212221	-0.0065109062
300	-3.9588101480	0.0031642726	-0.0057493979
350	-4.1853482260	0.0028105951	-0.0052320414
400	-4.4014325530	0.0025772705	-0.0048534799
450	-4.6045856190	0.0024162122	-0.0045579733
500	-4.7931516890	0.0022987082	-0.0043145187
600	-5.1234075730	0.0021315364	-0.0039191204
700	-5.3914177940	0.0020034613	-0.0035936423
800	-5.6025165630	0.0018887552	-0.0033104524
1000	-5.8875374130	0.0016743075	-0.0028324828
1500	-6.1543113080	0.0012327687	-0.0020071634
2000	-6.2038470670	0.0009427101	-0.0015226332

3000	-6.2137113920	0.0006307787	-0.0010176493
4000	-6.2144649520	0.0004731032	-0.0007634795
5000	-6.2148224370	0.0003784752	-0.0006108869
6000	-6.2150593160	0.0003153917	-0.0005091296
8000	-6.2153555850	0.0002365398	-0.0003819009
12000	-6.2156520860	0.0001576905	-0.0002546364
18000	-6.2158498910	0.0001051258	-0.0001697735
25000	-6.2159607070	0.0000756901	-0.0001222433
30000	-6.2160082030	0.0000630749	-0.0001018717
32768	-6.2160282710	0.0000577468	-0.0000932672
∞	-6.2091440000	0.0000000000	0.0000000000

8.2.3 The normalized associated Legendre functions and thier derivative to θ

When using (2.8) ~ (2.20) spherical harmonic synthesis formulas to calculate the load effects on geodetic variations, it is necessary to calculate the normalized associated Legendre function $\bar{P}_{nm}(cos\theta)$ and their first and second derivatives to θ. Here, let $t = cos\theta, u = sin\theta$, several fast algorithms are given directly as follows.

(1) Standard forward column recursion algorithm for $\bar{P}_{nm}(t)$ $(n < 1900)$

$$\begin{cases} \bar{P}_{nm}(t) = a_{nm} t \bar{P}_{n-1,m}(t) - b_{nm} \bar{P}_{n-2,m}(t) \quad \forall n > 1, m < n \\ \bar{P}_{nn}(t) = u\sqrt{\frac{2n+1}{2n}} \bar{P}_{n-1,n-1} \quad n > 1 \end{cases} \tag{2.21}$$

$$a_{nm} = \sqrt{\frac{(2n-1)(2n+1)}{(n+m)(n-m)}}, \quad b_{nm} = \sqrt{\frac{(2n+1)(n+m-1)(n-m-1)}{(2n-3)(n+m)(n-m)}}$$

$$\bar{P}_{00}(t) = 1, \quad \bar{P}_{10}(t) = \sqrt{3}t, \quad \bar{P}_{11}(t) = \sqrt{3}u \tag{2.22}$$

(2) Improved Belikov recursion algorithm for $\bar{P}_{nm}(t)$ $(n < 64800)$

When $n = 0,1$, uses formula (2.22) to calculate $\bar{P}_{nm}(t)$. When $n \geq 2$:

$$\bar{P}_{n0}(t) = a_n t \bar{P}_{n-1,0}(t) - b_n \frac{u}{2} \bar{P}_{n-1,1}(t), \quad m = 0 \tag{2.23}$$

$$\bar{P}_{nm}(t) = c_{nm} t \bar{P}_{n-1,m}(t) - d_{nm} u \bar{P}_{n-1,m+1}(t) + e_{nm} u \bar{P}_{n-1,m-1}(t), \ m > 0 \tag{2.24}$$

$$a_n = \sqrt{\frac{2n+1}{2n-1}}, \quad b_n = \sqrt{\frac{2(n-1)(2n+1)}{n(2n-1)}} \tag{2.25}$$

$$c_{nm} = \frac{1}{n}\sqrt{\frac{(n+m)(n-m)(2n+1)}{2n-1}}, \quad d_{nm} = \frac{1}{2n}\sqrt{\frac{(n-m)(n-m-1)(2n+1)}{2n-1}} \tag{2.26}$$

When m >0:

$$e_{nm} = \frac{1}{2n}\sqrt{\frac{2}{2-\delta_0^{m-1}}}\sqrt{\frac{(n+m)(n+m-1)(2n+1)}{2n-1}} \tag{2.27}$$

ETideLoad4.5 adopts mainly the improved Belikov recursion algorithm to calculate the normalized associated Legendre functions $\bar{P}_{nm}(t)$。

(3) Cross-degree recursive algorithm for $\bar{P}_{nm}(t)$ $(n < 20000)$

When $n = 0,1$, uses formula (2.22) to calculate $\bar{P}_{nm}(t)$. When $n \geq 2$:

$$\bar{P}_{nm}(t) = \alpha_{nm}\bar{P}_{n-2,m}(t) + \beta_{nm}\bar{P}_{n-2,m-2}(t) - \gamma_{nm}\bar{P}_{n,m-2}(t) \tag{2.28}$$

$$\alpha_{nm} = \sqrt{\frac{(2n+1)(n-m)(n-m-1)}{(2n-3)(n+m)(n+m-1)}}$$

$$\beta_{nm} = \sqrt{1+\delta_0^{m-2}}\sqrt{\frac{(2n+1)(n+m-2)(n+m-3)}{(2n-3)(n+m)(n+m-1)}} \tag{2.29}$$

$$\gamma_{nm} = \sqrt{1+\delta_0^{m-2}}\sqrt{\frac{(n-m+1)(n+m-3)}{(n+m)(n+m-1)}}$$

(4) Non-singular recursive algorithm for $\frac{\partial}{\partial\theta}\bar{P}_{nm}(\cos\theta)$

$$\frac{\partial}{\partial\theta}\bar{P}_{nm}(\cos\theta) = -\sin\theta\frac{\partial}{\partial t}\bar{P}_{nm}(t) \tag{2.30}$$

$$\begin{cases}\frac{\partial}{\partial\theta}\bar{P}_{n0}(t) = -\sqrt{\frac{n(n+1)}{2}}\bar{P}_{n1}(t), \quad \frac{\partial}{\partial\theta}\bar{P}_{n1}(t) = \sqrt{\frac{n(n+1)}{2}}\bar{P}_{n0} - \frac{\sqrt{(n-1)(n+2)}}{2}\bar{P}_{n2} \\ \frac{\partial}{\partial\theta}\bar{P}_{nm}(t) = \frac{\sqrt{(n+m)(n-m+1)}}{2}\bar{P}_{n,m-1}(t) - \frac{\sqrt{(n-m)(n+m+1)}}{2}\bar{P}_{n,m+1}(t), \quad m > 2\end{cases} \tag{2.31}$$

$$\frac{\partial}{\partial\theta}\bar{P}_{00}(t) = 0, \quad \frac{\partial}{\partial\theta}\bar{P}_{10}(t) = -\sqrt{3}u, \quad \frac{\partial}{\partial\theta}\bar{P}_{11}(t) = \sqrt{3}t \tag{2.32}$$

(5) Non-singular recursive algorithm for $\frac{\partial^2}{\partial\theta^2}\bar{P}_{nm}(\cos\theta)$

$$\begin{cases}\frac{\partial^2}{\partial\theta^2}\bar{P}_{n0}(t) = -\frac{n(n+1)}{2}\bar{P}_{n0}(t) + \sqrt{\frac{n(n-1)(n+1)(n+2)}{8}}\bar{P}_{n2}(t) \\ \frac{\partial^2}{\partial\theta^2}\bar{P}_{n1}(t) = -\frac{2n(n+1)+(n-1)(n+2)}{4}\bar{P}_{n1}(t) + \frac{\sqrt{(n-2)(n-1)(n+2)(n+3)}}{4}\bar{P}_{n3}(t)\end{cases} \tag{2.33}$$

$$\begin{aligned}\frac{\partial^2}{\partial\theta^2}\bar{P}_{nm}(t) = &\frac{\sqrt{(n-m+1)(n-m+2)(n+m-1)(n+m)}}{4}\bar{P}_{n,m-2}(t) \\ &-\frac{(n+m)(n-m+1)+(n-m)(n+m+1)}{4}\bar{P}_{nm}(t) \\ &+\frac{\sqrt{(n-m-1)(n-m)(n+m+1)(n+m+2)}}{4}\bar{P}_{n,m+2}(t), \quad m > 2\end{aligned} \tag{2.34}$$

$$\frac{\partial^2}{\partial\theta^2}\bar{P}_{00}(t) = 0, \quad \frac{\partial^2}{\partial\theta^2}\bar{P}_{10}(t) = -\sqrt{3}t, \quad \frac{\partial^2}{\partial\theta^2}\bar{P}_{11}(t) = -\sqrt{3}u \tag{2.35}$$

8.2.4 Spherical harmonic analysis of global sea level variations and synthesis of load effects

Without loss of generality, we can always decompose the global sea water mass change and transport into two effects, one is the sea surface height variation when the sea water density does not change with time, and the other is the sea water density variation when the volume and spatial distribution of sea water remain unchanged while the sea surface height remains unchanged in the case. In the first case, the sea level variation is the total sea surface height variation acted by all factors, which obviously includes the sea surface height variation caused by the sea water temperature and salt change. This part of the sea level variation contributes more than 98 % to the global sea water mass change and transportation, and can be monitored efficiently and accurately by ocean tide gauge and satellite altimetry. However, the change of seawater density no longer includes the change of sea surface height caused by temperature and salinity effect, so its contribution to global seawater quality change and transportation is generally less than 2 %, and it is difficult to accurately measure.

In most geodetic cases, sea level variations can be employed to represent global seawater mass changes and transport, while the impact of seawater density changes over time is left to other higher level of geodetic techniques (such as satellite geodetic measurements combined with on-site hydrological monitoring) to solve.

(1) Spherical harmonic analysis calculation of global sea level variations

The spherical harmonic analysis of global sea level variation can be calculated by fast Fourier algorithm using Formula (2.4). Firstly, the sea level variation grid time series in spherical coordinate system are constructed by integrating various sea surface height observation data (removing the mean sea surface height grid in a certain period of time). Then, the spherical harmonic analysis is carried out on the sea level variation grid at each sampling epoch according to the formula (2.4) to generate the load spherical harmonic coefficient model time series of sea level variation. The maximum degree number of the load spherical harmonic model depends on the spatial resolution of the sea level variation grid. The sampling epoch time of the time series of the load spherical harmonic coefficient model corresponds to the time series of the sea level variation grid one by one.

In Formula (2.4), the sea level variation is directly expressed as a linear combination of harmonic functions on the spherical surface. Therefore, the cumulative residual spherical harmonic analysis method can be effectively employed to improve the approach level of the load spherical harmonic coefficient model of sea level variation.

Fig 2.1 is the calculation results of global sea level variation spherical harmonic analysis program. The program inputs 0.5°×0.5° global sea level variation spherical coordinate grid time series, where the sea level variation grid at the first epoch time is shown in the right middle figure, and the land area is set to zero. According to Formula (2.4), the cumulative approach method is employed to construct the 360-degree sea level variation load spherical harmonic coefficient model time series, where the iterative residuals are shown in the lower right figure, and the sea level variation load spherical harmonic coefficient model at the first epoch is shown in the lower left figure.

The file header of the load spherical harmonic coefficient model (lower left figure) of sea level variation includes the geocentric gravitational constant GM ($\times 10^{14} m^3/s^2$), equatorial radius a (m) of the Earth, zero-degree term $a\Delta C_{00}$ (cm), relative error Θ (%). Here, Θ is the percentage of the standard deviation of the final iteration residual to the standard deviation of the input grid. The maximum degree n of the spherical harmonic coefficient is equal to the number of global sea level variation cell-grids in the latitude direction. In the example, a 0.5°×0.5° grid model is input, corresponding to the maximum degree n=360.

GM, a are also known as the scale parameters of the spherical harmonic coefficient model in which the surface harmonic functions are defined on the spherical surface whose radius is equal to the equatorial radius of the Earth.

The three first-degree spherical harmonic coefficients (ΔC_{10}, ΔC_{11}, ΔS_{11}) represent the variations of Earth's center of mass due to global sea level variations. The zero-degree term can be controlled to a small value by adjusting the time datum.

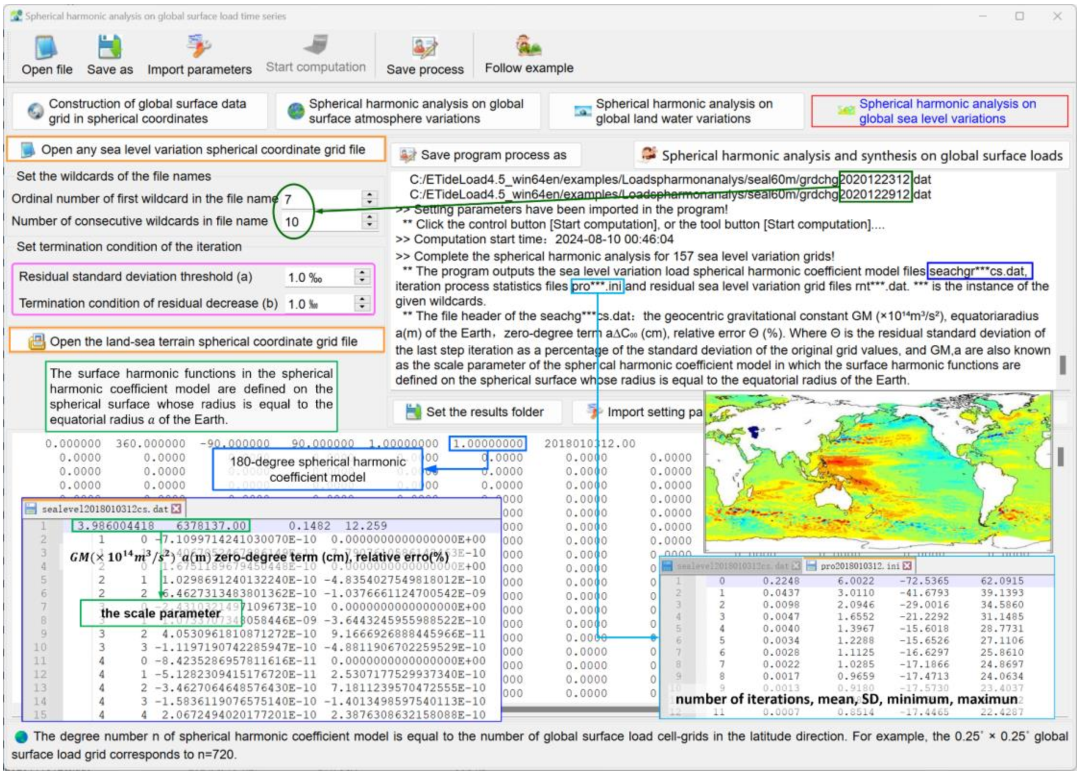


Fig 2.1 Spherical harmonic analysis on global sea level variations and construction of load spherical harmonic coefficient model

For high-precision geodesy, the contributions of the short-wave component of sea level variations cannot be ignored, and a grid model with a higher spatial resolution is needed to meet the accuracy requirements. Accordingly, a higher degree load spherical harmonic coefficient model is needed. The maximum degree number of the load spherical harmonic coefficient model can be generally determined by the global spectrum structure of the load and accuracy requirements to the load effects of sea level variations. Tab 2.2 shows the change of load spherical harmonic analysis results of global sea level variation with grid resolution (maximum degree) at a certain epoch time.

Tab 2.2 The change of spherical harmonic analysis residual of sea level variations with grid resolution

Input grid	maximum degree	Zero-degree item (cm)	First-degree item ($\times 10^{-10}$)			Relative error (%)
			ΔC_{10}^{sea}	ΔC_{11}^{sea}	ΔS_{11}^{sea}	
1°×1°	180	0.1278	-7.14017	-0.74191	6.93210	6.519
30′×30′	360	0.1419	-7.29329	-0.81169	7.57094	5.075
15′×15′	720	0.1273	-7.19655	-0.71797	6.86062	3.566

Tab 2.2 shows that the short and medium wave components of global sea level variations are obvious at this epoch time. Considering the accuracy requirements and computational efficiency, the appropriate maximum degree of the load spherical harmonic coefficient model at the epoch time can be selected as 360.

(2) Spherical harmonic synthesis calculation of sea level variation load effects

From the load spherical harmonic coefficient model of sea level variations, the spherical harmonic synthesis algorithm formulas (2.8) ~ (2.20) can be employed to calculate the sea level variation load effects on all-element geodetic variations at any point on the global ground or outside the ground, and that on geopotential, gravity (acceleration) or gravity gradient outside the solid Earth such as ocean space, aviation or satellite altitude.

Fig 2.2 is the calculation result of spherical harmonic synthesis program of sea level variation load effects. The program inputs the calculation area digital elevation model grid (employed to specify the area location and range of the calculation point), from the sea level variation load spherical harmonic coefficient model time series, selects the maximum calculation degree 360, and calculates the load effect grid time series on all-element geodetic variations according to formulas (2.8) ~ (2.20).

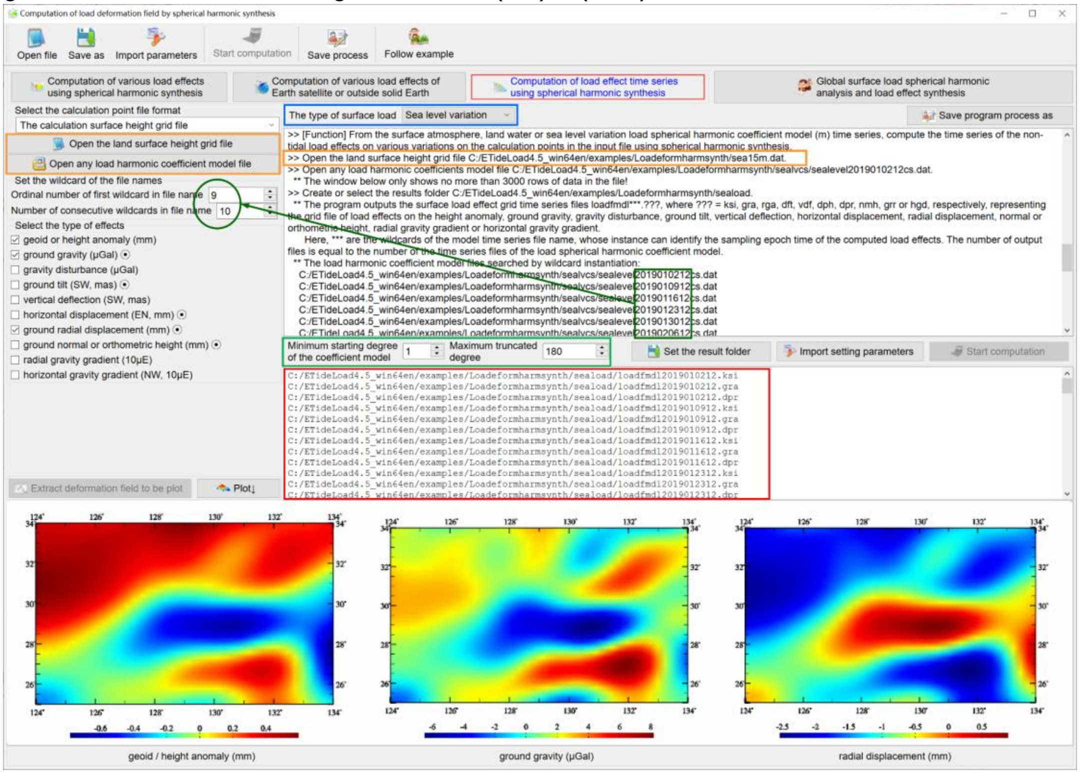


Fig 2.2 Calculation of the sea level variation load effect grid time series on all-element geodetic variations

Fig 2.3 is the calculation results of the sea level variation load effects on geopotential and gravity gradient of the Earth satellite.

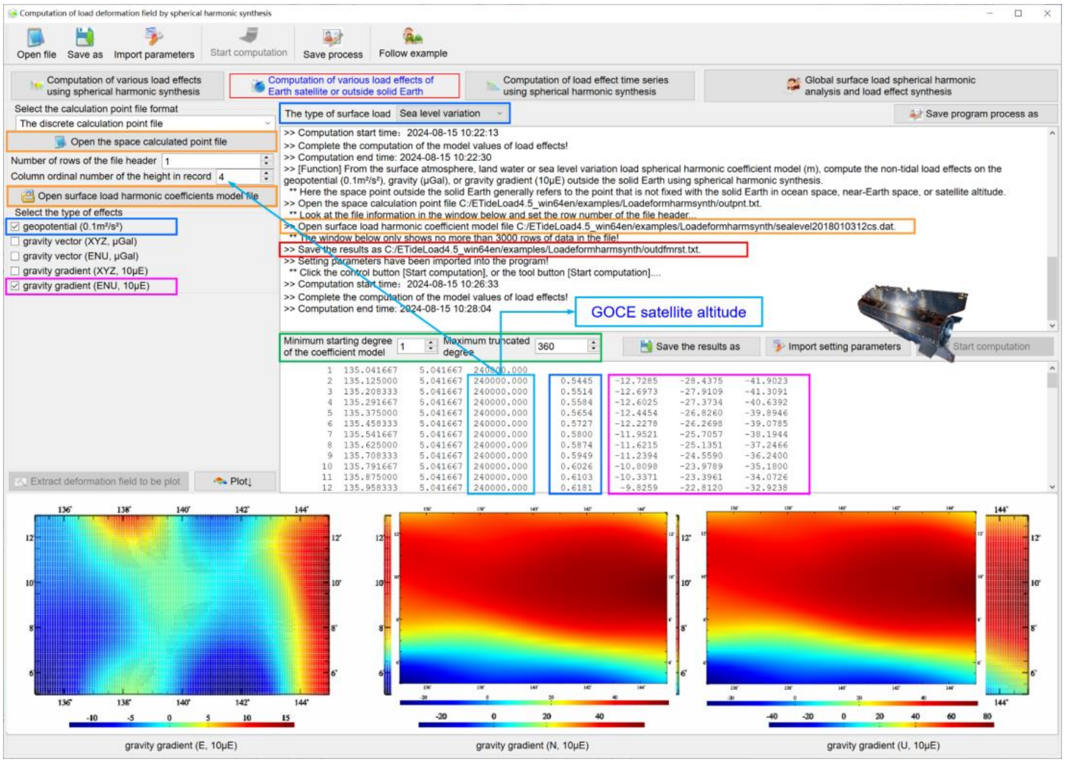


Fig 2.3 Load perturbation calculation of sea level variations of Earth satellite

In the following, the 15′×15′ global sea level weekly variations (sea level anomaly) combined by Aviso from multiple altimeter data is employed. After removing the year mean value of 2018, the 0.5°×0.5° global sea level variation (cm) spherical coordinate grid weekly time series (157 sampling epochs) from January 2018 to December 2020 are constructed. Then, the 360-degree sea level variation load spherical harmonic coefficient model (m) weekly time series are constructed by using Formula (2.4) and fast Fourier algorithm. Finally, according to the load effect spherical harmonic synthesis algorithm formulas (2.8) ~ (2.20), the sea level variation load effect weekly time series at 12 tide gauges (latitude 18°N ~ 40°N) along Chinese coast are calculated.

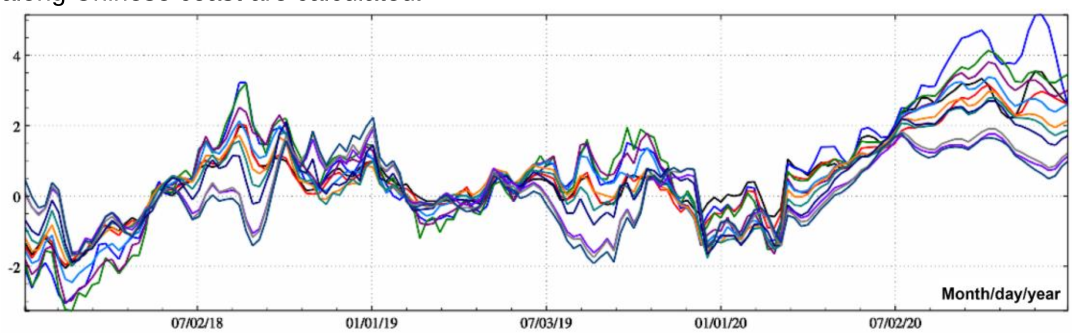


Fig 2.4 Sea level variation load effect weekly time series on the geoid (mm) at 12 tide gauges along Chinese coast

Fig 2.4 ~ Fig 2.7 are the sea level variation load effect weekly time series on the geoid (in unit of mm), ground gravity (μGal), ellipsoidal height (mm) and radial gravity gradient (10μE) respectively at 12 tide gauges from January 2018 to December 2020.

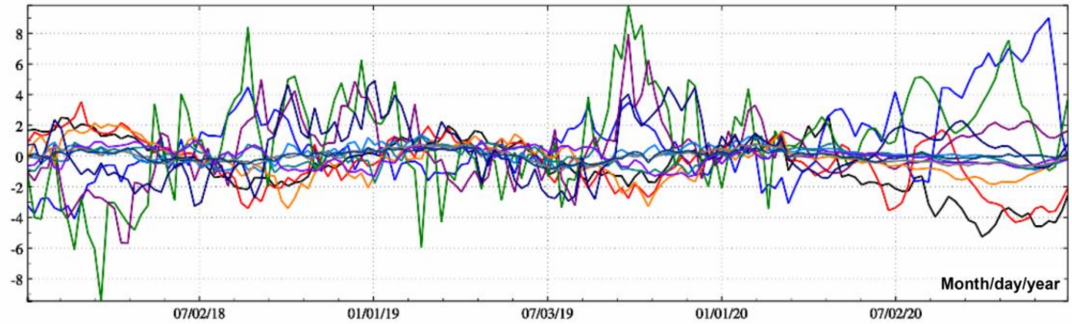


Fig 2.5 Sea level variation load effect weekly time series on the ground gravity (μGal) at 12 tide gauges along Chinese coast

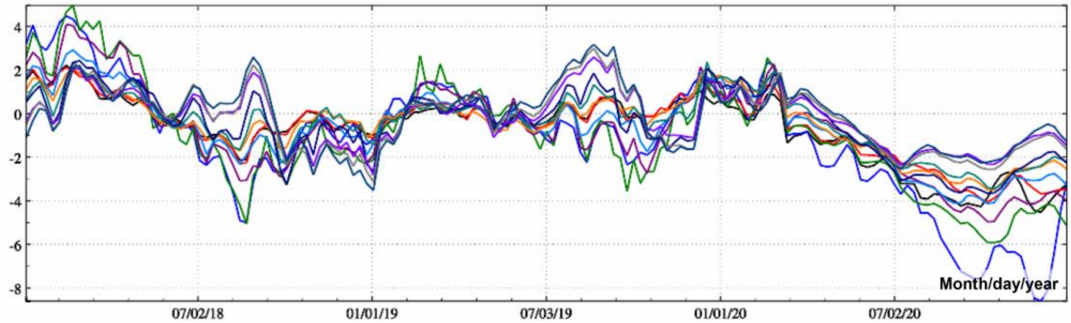


Fig 2.6 Sea level variation load effect weekly time series on the ellipsoidal height (mm) at 12 tide gauges along Chinese coast

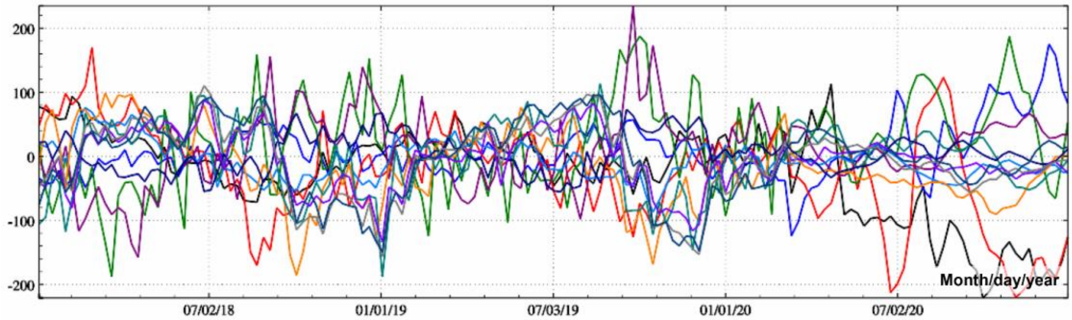


Fig 2.7 Sea level variation load effect weekly time series on the radial gravity gradient (10μE) at 12 tide gauges along Chinese coast

8.2.5 Spherical harmonic analysis of surface atmosphere variations and synthesis of load effects

(1) Load effects of atmospheric density changes and that of surface atmosphere variations

In principle, the atmospheric density load effect need three-dimensional integration of the density changes in the entire atmospheric layer space. The atmospheric load effect is usually calculated from the variation of ground atmospheric pressure by using the approximate relationship between the variation of ground atmospheric pressure and the change of atmospheric spatial density. The approximation can meet the accuracy requirements of geodesy in most cases (Guo, 2010).

A simplified calculation scheme is recommended here. When calculating the indirect influence of atmospheric load, it is assumed that the atmospheric pressure loads are concentrated on the ground, and the contribution of 1hPa (mbar) is equivalent to that of 1cm equivalent water high load, that is, 1hPa=1cm EWH, and the calculation point height h is the height of the point relative to the ground. When calculating the direct influence of atmospheric load to the gravity or radial gravity gradient, it is assumed that there is a proportional relationship between atmospheric pressure P_h at ground height h and ground atmospheric pressure P_0 as follow:

$$P_h = P_0(1 - h/44330)^{5225} \tag{2.36}$$

When calculating the load effect of atmosphere change, it is not necessary to know the atmospheric pressure P_h at the calculation point at the current epoch time. It is only necessary to determine the difference between the atmospheric pressure P_h at the current epoch and that P_h^* at the reference epoch, that is, $\Delta P_h = P_h - P_h^*$. From the difference between the ground atmospheric pressure P_0 at the current epoch and that P_0^* at the reference epoch, the atmosphere variation ΔP_h at the calculation point is obtained:

$$\Delta P_h = P_h - P_h^* = P_0\left(1 - \frac{h}{44330}\right)^{5225} - P_0^*\left(1 - \frac{h}{44330}\right)^{5225} \approx \Delta P_0\left(1 - \frac{h}{44330}\right)^{5225} \tag{2.37}$$

Using the formula (2.37), the atmospheric pressure variation ΔP_h at the height h (relative to ground) can be directly calculated from the ground atmospheric pressure change ΔP_0 without the atmospheric pressure P_0^* of the ground point at the reference epoch.

The spherical harmonic analysis process of global surface atmosphere variations is the same as that of global sea level variations. It also uses Formula (2.4) and can be calculated by fast Fourier algorithm. Firstly, global surface atmosphere variation grid time series in spherical coordinate system are constructed from the ground atmospheric pressure observations. Then, the spherical harmonic analysis is carried out for each sampling epoch of the surface atmosphere variation grid according to the formula (2.4), and the global surface atmosphere load spherical harmonic coefficient model time series are generated.

Fig 2.8 is the calculation results of surface atmosphere variation spherical harmonic analysis program. The program inputs 1°×1° surface atmosphere variation spherical coordinate grid time series. According to Formula (2.4), the cumulative approach method is employed to construct the 180-degree surface atmosphere variation load spherical harmonic coefficient model time series, where the iterative residuals are shown in the lower right figure, and the atmosphere variation load spherical harmonic coefficient model at the first epoch time is shown in the lower left figure.

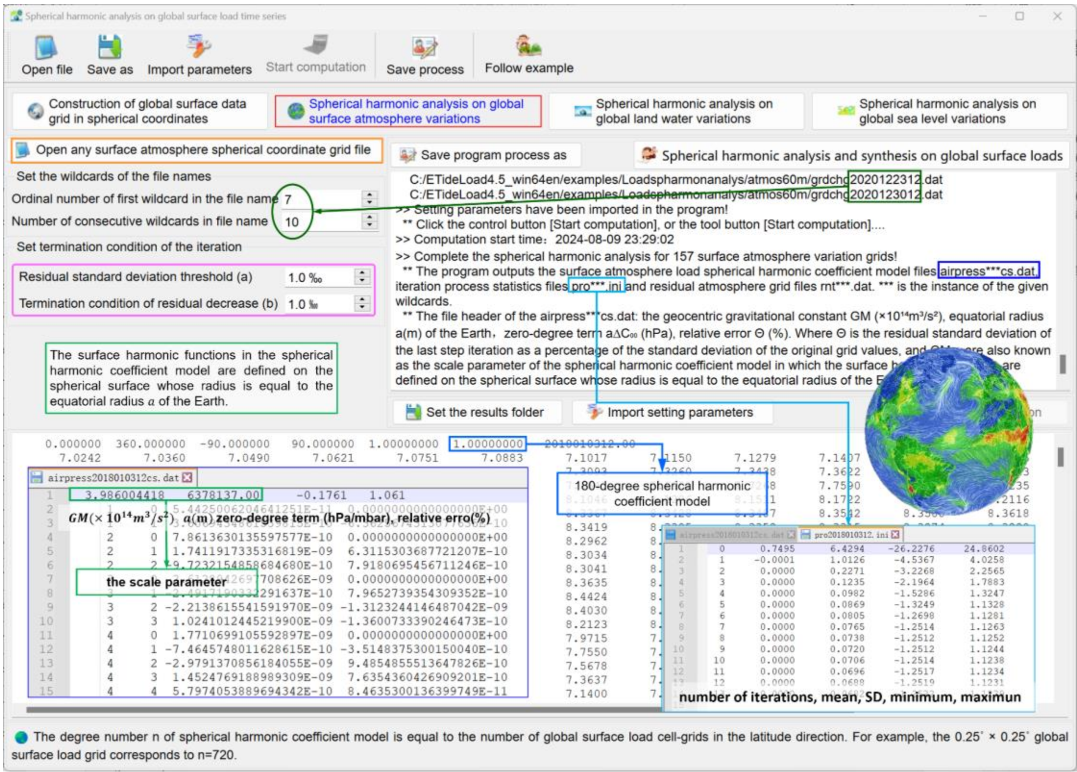


Fig 2.8 Spherical harmonic analysis on global surface atmosphere variations and construction of load spherical harmonic coefficient model

Similar to the spherical harmonic analysis of sea level variations, the maximum degree number of the load spherical harmonic coefficient model can be generally determined by the global spectrum structure of the load and accuracy requirements to the load effects of surface atmosphere variations. Tab 2.3 shows the change of load spherical harmonic analysis results of global surface atmosphere variations with grid resolution (maximum degree) at a certain epoch time.

Tab 2.3 The change of spherical harmonic analysis residual of global surface atmosphere variations with grid resolution

Input grid	maximum degree	Zero-degree item (hPa)	First-degree item ($\times 10^{-10}$)			Relative error (%)
			$\Delta\bar{C}_{10}^{air}$	$\Delta\bar{C}_{11}^{air}$	$\Delta\bar{S}_{11}^{air}$	
2°×2°	90	-1.7539	0.55043	3.60270	-6.35702	2.707
1°×1°	180	-1.7614	0.54424	3.60695	-8.36343	1.215
0.5°×0.5°	360	-1.7620	0.54251	3.60748	-8.36912	2.043

Table 3.2 shows that the long wave components are dominant in the global surface atmosphere variations at this epoch, which can be expressed by the load spherical harmonic coefficient model with the maximum degree not less than 180.

(2) Spherical harmonic synthesis calculation of surface atmosphere variation load effects

From the load spherical harmonic coefficient model of surface atmosphere variations, the spherical harmonic synthesis algorithm formulas (2.8) ~ (2.20) can be employed to calculate the surface atmosphere variation load effects on all-element geodetic variations at any point on the global ground or outside the ground, and that on geopotential, gravity (acceleration) or gravity gradient outside the solid Earth such as ocean space, aviation or satellite altitude.

Fig 2.9 is the calculation result of spherical harmonic synthesis program of surface atmosphere variation load effects. The program inputs the zero-value grid (employed to specify the area location and range of the calculation point, the zero value indicates that the calculation point is located on the ground), from the surface atmosphere variation load spherical harmonic coefficient model time series, selects the maximum calculation degree 360, and calculates the load effect grid time series on all-element geodetic variations according to formulas (2.8) ~ (2.20).

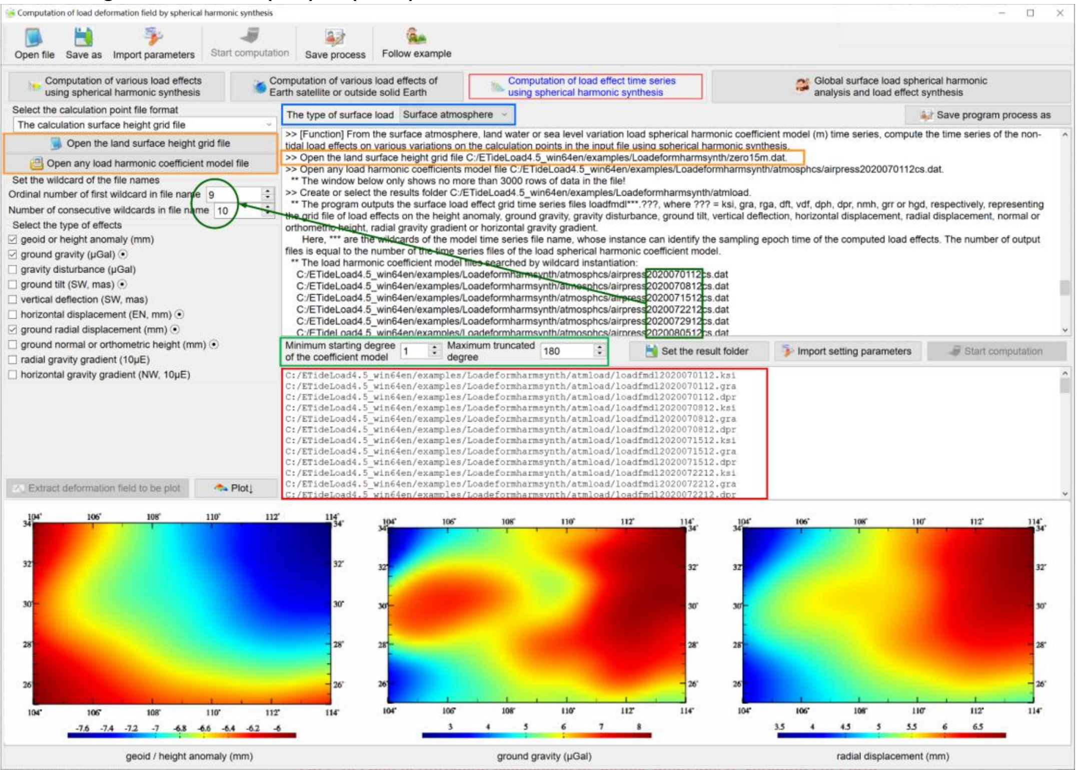


Fig 2.9 Calculation of the surface atmosphere variation load effect grid time series on all-element geodetic variations

Fig 2.10 is the calculation results of the surface atmosphere variation load effects on geopotential and gravity gradient of the Earth satellite.

In the following, the 0.5°×0.5° global surface atmospheric pressure diurnal variations in the global reanalysis data ERA-40/ERA-Interim from the European Centre for Medium-Range Weather Forecasts (ECMWF) are employed to construct the 1°×1° global surface atmosphere variation (hPa) spherical coordinate grid weekly time series (157 sampling epochs) from January 2018 to December 2020. Then, the 180-degree surface atmosphere variation load spherical harmonic coefficient model (m) weekly time series are constructed by using Formula (2.4) and fast Fourier algorithm. Finally, according to the load effect spherical harmonic synthesis algorithm formulas (2.8) ~ (2.20), the surface atmosphere load effect weekly time series at 14 CORS stations in mainland China are calculated.

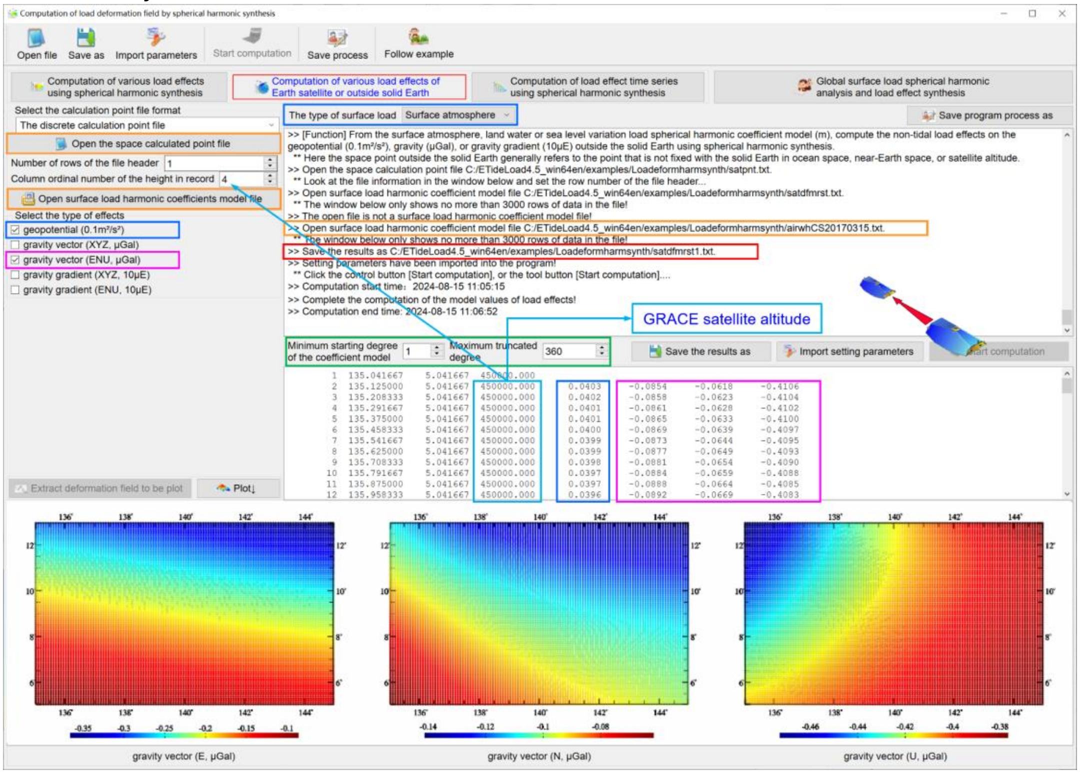


Fig 2.10 Load perturbation calculation of surface atmosphere variations of Earth satellite

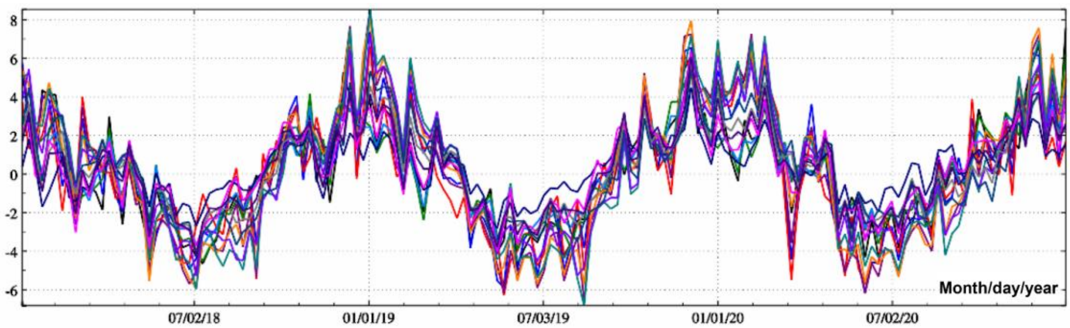


Fig 2.11 Surface atmosphere variation load effect weekly time series on the geoid (mm) at 14 CORS stations in mainland China

Fig 2.11 ~ Fig 2.14 are the surface atmosphere variation load effect weekly time series on the geoid (in unit of mm), ground gravity (μGal), ellipsoidal height (mm) and radial gravity gradient (10μE) respectively at 14 CORS stations from January 2018 to December 2020.

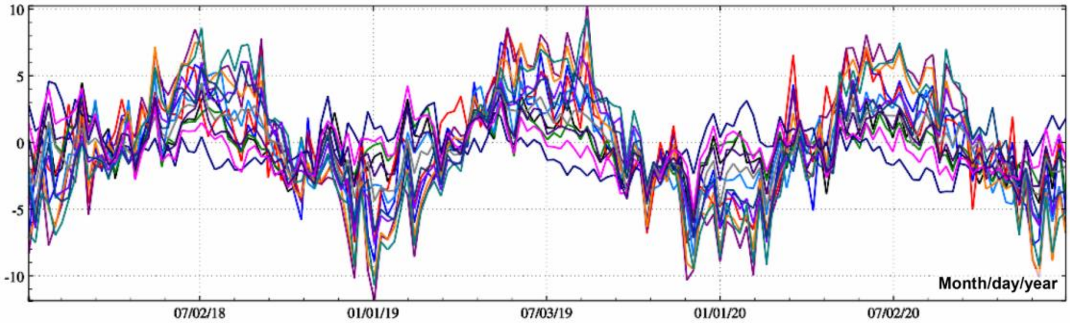


Fig 2.12 Surface atmosphere variation load effect weekly time series on the ground gravity (μGal) at 14 CORS stations in mainland China

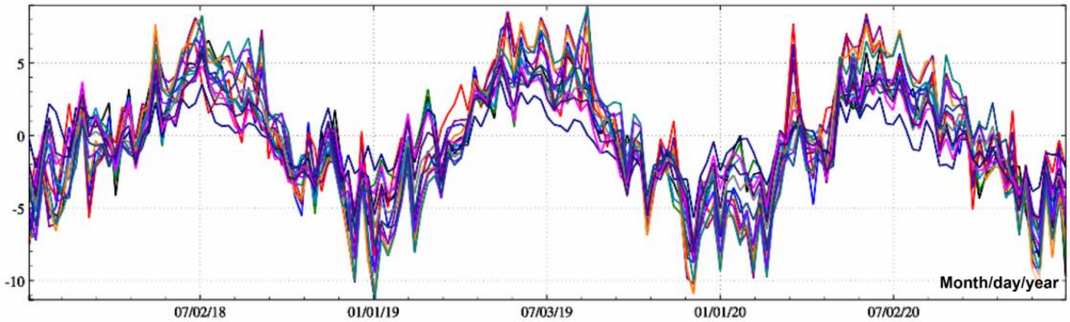


Fig 2.13 Surface atmosphere variation load effect weekly time series on the ellipsoidal height (mm) at 14 CORS stations in mainland China

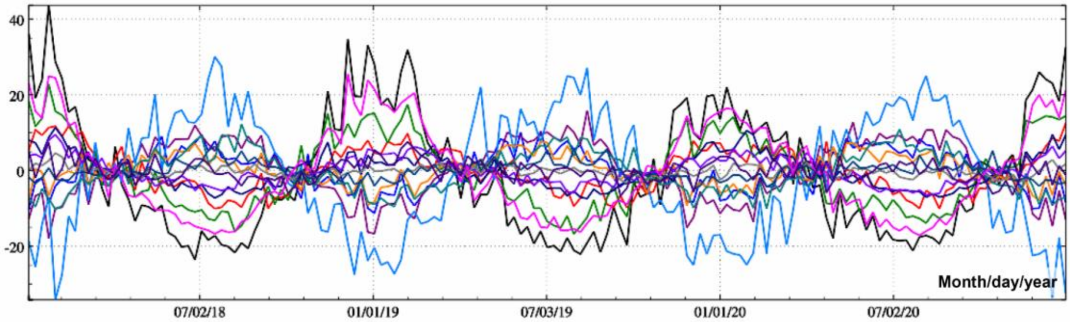


Fig 2.14 Surface atmosphere variation load effect weekly time series on the radial gravity gradient (10μE) at 14 CORS stations in mainland China

8.2.6 Spherical harmonic analysis of global land water variations and synthesis of load effects

The spherical harmonic analysis process of global land water variations expressed by the ground equivalent water height (EWH) variations is the same as that of global sea level variations. Fig 2.8 is the calculation results of land water variation spherical harmonic analysis program. The program inputs 0.25°×0.25° land water variation spherical coordinate grid (EWH equal to zero in the ocean area) time series to construct the 720-degree land water variation load spherical harmonic coefficient model time series.

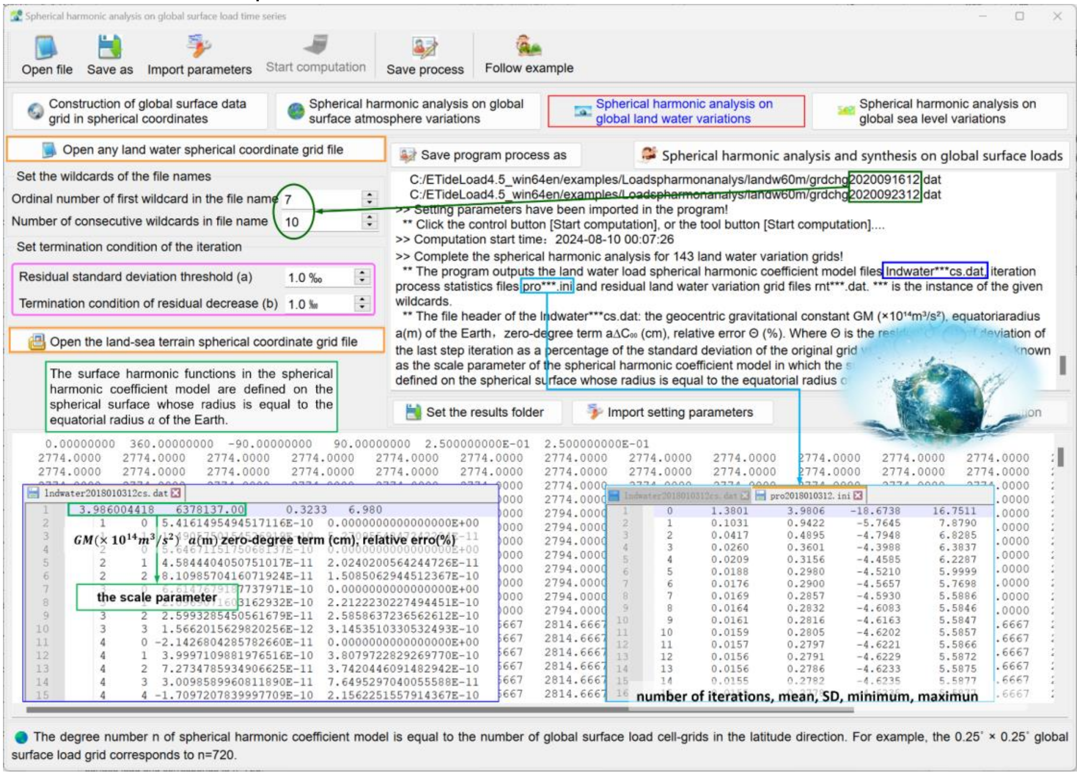


Fig 2.15 Spherical harmonic analysis on global land water variations and construction of load spherical harmonic coefficient model

Tab 2.4 shows the change of load spherical harmonic analysis results of land water variations with grid resolution (maximum degree) at a certain epoch time.

Tab 2.4 The change of spherical harmonic analysis residual of global land water variations with grid resolution

Input grid	maximum degree	Zero-degree item (cm)	First-degree item (×10⁻¹⁰)			Relative error (%)
			$\Delta\bar{C}_{10}^{lnd}$	$\Delta\bar{C}_{11}^{lnd}$	$\Delta\bar{S}_{11}^{lnd}$	
30′×30′	360	0.3242	5.46047	1.49947	0.52091	5.851
15′×15′	720	0.3207	5.32556	1.51216	0.50261	4.291
9′×9′	1200	0.3236	5.43533	1.50154	0.51493	3.094

Tab 2.4 shows that the short-wave components of global land water variations at this epoch are obvious, and the appropriate maximum degree of the load spherical harmonic coefficient model can be selected as 720.

In the following, the 0.25°×0.25° GLDAS data from NASA Global Land Data Assimilation System are employed to construct the 15′×15′ global land water variation (cm) spherical coordinate grid weekly time series (157 sampling epochs) from January 2018 to September 2020. Then, the 720-degree land water variation load spherical harmonic coefficient model

(m) weekly time series are constructed by using Formula (2.4) and fast Fourier algorithm. Finally, according to the load effect spherical harmonic synthesis algorithm formulas (2.8) ~ (2.20), the land water variation load effect weekly time series at 14 CORS stations in mainland China are calculated.

Here, the land water includes 4m shallow soil water, wetland water, vegetation water and glacier snow mountain water, but does not include lake, river water and groundwater. Fig 2.16 ~ Fig 2.19 are the land water variation load effect weekly time series on the geoid (in unit of mm), ground gravity (μGal), ellipsoidal height (mm) and radial gravity gradient (10μE) respectively at 14 CORS stations from January 2018 to December 2020.

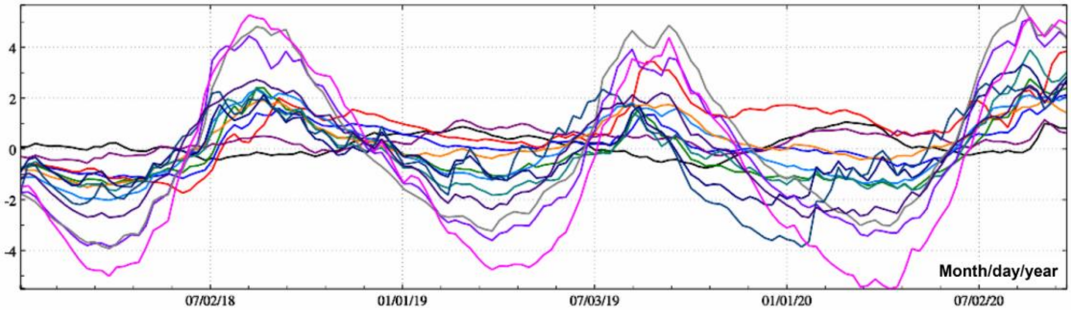


Fig 2.16 Global land water variation load effect weekly time series on the geoid (mm) at 14 CORS stations in mainland China

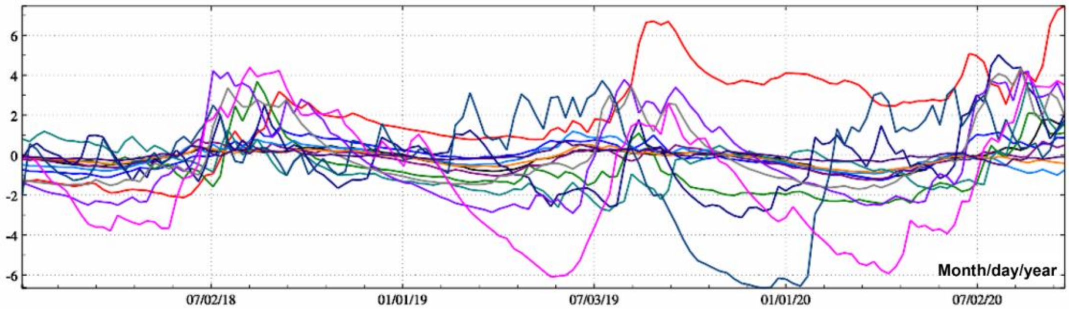


Fig 2.17 Global land water variation load effect weekly time series on the ground gravity (μGal) at 14 CORS stations in mainland China

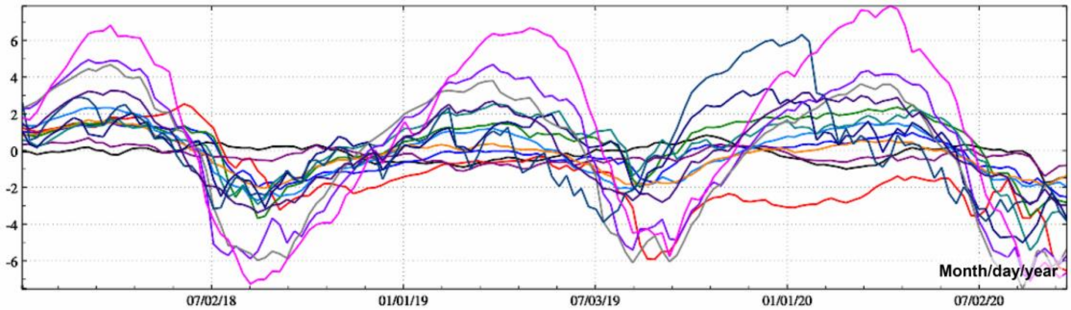


Fig 2.18 Global land water variation load effect weekly time series on the ellipsoidal height (mm) at 14 CORS stations in mainland China

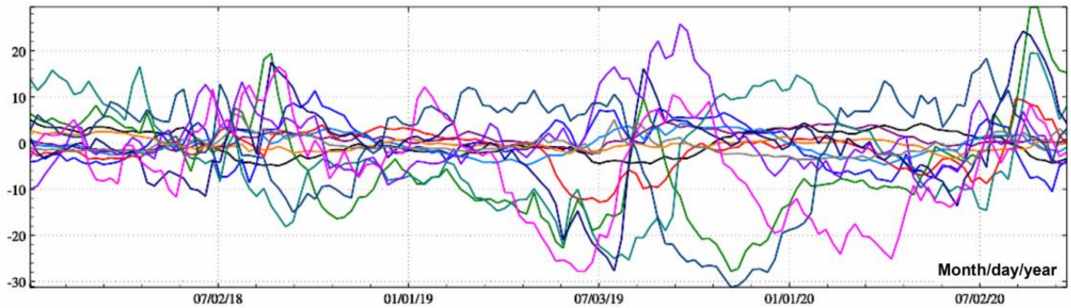


Fig 2.19 Global land water variation load effect weekly time series on the radial gravity gradient (10μE) at 14 CORS stations in mainland China

Using the monitoring data to global surface atmosphere, land water and sea level variations, determine the non-tidal temporal Earth's gravity field, as well as the non-tidal load effects on the geopotential coefficients, and then you can calibrate various parameters of the satellite's key geodetic payloads, and then effectively improve and check the quality, reliability and accuracy of the time-varying monitoring for satellite gravity field.

8.3 Surface load effects on various geodetic variations by Green's Integral

The load Green's function is defined as the response function of the unit point mass load variation (kg/m²), and the load effect on the ground geodetic variation is equal to the convolution of the load Green's function and surface density $\sigma_w(=\rho_w h_w)$ of the ground load on the global surface. In general, similar to the Stokes' integral formula in the theory of Earth's gravity field, the load effect $F(\theta,\lambda)$ on any type of geodetic variation at the ground calculation point (θ,λ) can be expressed as the load Green's function integral under the spherical approximation:

$$F(\theta,\lambda) = R^2\rho_w \iint_\sigma h_w G(\psi) d\sigma \tag{3.1}$$

Where, σ is the unit spherical surface, R is the mean radius of the Earth, ψ is the spherical angular distance from the surface load area-element $d\sigma$ to the ground calculation point (θ,λ) and $G(\psi)$ is the load Green's function with the spherical angular distance ψ as the independent variable, and its form is related to the effect type.

The load Green's function integral $F(\theta,\lambda)$ here is divided into two parts. The first part is the direct influence of load, and the second part is the indirect influence of load.

$$F(\theta,\lambda) = F^d(\theta,\lambda) + R^2\rho_w \iint_\sigma h_w G^i(\psi) d\sigma \tag{3.2}$$

In the formula (3.2), (θ,λ) is the spherical coordinates of the ground calculation point. $F^d(\theta,\lambda)$ is the direct influence of load effect at the ground calculation point, which can be calculated by the load equivalent water height variation h_w according to the rigorous integral. $G^i(\psi)$ is called as the indirect influence of load Green's function.

8.3.1 The integral of direct influence of the load effect on ground geodetic variation

(1) The integral formula of direct influence of the load effect on geopotential

Given the surface load equivalent water height (EWH) variation h_w, whose direct effect V_w on the geopotential near Earth space directly given by the universal gravitation formula

$$V^d(r,\theta,\lambda) = G\rho_w \iint_S \frac{h_w}{L} dS \ , \ L = \sqrt{r^2 + r'^2 - 2rr'cos\psi} \qquad (3.3)$$

Where, L is the spatial distance between the calculation point (r,θ,λ) near Earth space and center (r',θ',λ') of integral area-element dS on the surface S, r,θ,λ are the spherical geocentric coordinates of the calculation point, namely distance from geocenter, co-latitude and longitude, respectively. G is Newton's gravitational constant, $\rho_w = 1000kg/m^3$ is the water density. ψ is the spherical angle between the calculation point (r,θ,λ) and center (r',θ',λ') of the area-element.

$$cos\psi = cos\theta cos\theta' + sin\theta sin\theta' cos(\lambda' - \lambda) \ ,$$

$$sin\psi = sin\theta cos\theta' + cos\theta sin\theta' cos(\lambda' - \lambda) \qquad (3.4)$$

$$sin\psi cos\alpha = sin\theta cos\theta' - cos\theta sin\theta' cos(\lambda' - \lambda) \ ,$$

$$sin\psi sin\alpha = sin\theta' sin(\lambda' - \lambda) \qquad (3.5)$$

$$\frac{\partial\psi}{\partial\theta} = -\frac{\partial\psi}{\partial\varphi} = cos\alpha \ , \ \frac{\partial\psi}{\partial\lambda} = -sin\alpha sin\theta \qquad (3.6)$$

Here, α is the geodetic azimuth of ψ.

Considering $d\sigma = \psi d\psi d\alpha$, when the calculation point is located on the surface and overlaps with the center of integral area-element, we have

$$L = r\psi \ , \ r - r'cos\psi = r\psi^2/2 \qquad (3.7)$$

$$A = dS = r^2 \int_{\alpha=0}^{2\pi} \int_0^{\psi_0} \psi d\psi d\alpha = \pi r^2 \psi_0^2 \rightarrow \psi_0 = \frac{1}{r}\sqrt{\frac{A}{\pi}} \qquad (3.8)$$

Where, $A = dS$ is the area of integral area-element. In this case, the formula (3.3) on the calculation point is an integral singularity. From the formulas (3.7) and (3.8), the singular value of the integral can be obtained:

$$V_d^0(r,\theta,\lambda) = G\rho_w r^2 \int_{\alpha=0}^{2\pi} \int_0^{\psi_0} \frac{h_w}{r\psi} \psi d\psi d\alpha = 2\pi G\rho_w h_w r\psi_0 \qquad (3.9)$$

(2) The integral formula of direct influence of the load effect on gravity disturbance

According to the definition of gravity disturbance, from Formula (3.3), the direct influence of the load effect on gravity disturbance at the calculation point (r,θ,λ) is:

$$\delta g^d(r,\theta,\lambda) = -\frac{\partial V^d(r,\theta,\lambda)}{\partial r} = -G\rho_w \iint_S h_w \frac{\partial}{\partial r}\left(\frac{1}{L}\right) dS = G\rho_w \iint_S h_w \frac{r - r'cos\psi}{L^3} dS \qquad (3.10)$$

When the calculation point is located on the surface and overlaps with the center of integral area-element, the formula (3.10) on the calculation point is an integral singularity, and the singular value of the integral is

$$\delta g_0^d(r,\theta,\lambda) = 2\pi G\rho_w h_w \int_0^{\psi_0} \frac{\psi^2/2}{\psi^3} \psi d\psi = \pi G\rho_w h_w \psi_0 \qquad (3.11)$$

(3) The integral formula of direct influence of the load effect on vertical deflection

According to the definition of vertical deflection, from Formula (3.3), the direct influence of the load effect on vertical deflection at the calculation point (r,θ,λ) is:

$$\Theta^d(r,\theta,\lambda)=\frac{1}{\gamma r}\frac{\partial V^d(r,\theta,\lambda)}{\partial\psi}=\frac{G\rho_w}{\gamma r}\iint_S h_w\frac{\partial}{\partial\psi}\left(\frac{1}{L}\right)dS=-\frac{G\rho_w}{\gamma}\iint_S h_w r'\frac{sin\psi}{L^3}dS \quad (3.12)$$

$$\xi^d(r,\theta,\lambda)=\Theta^d(r,\theta,\lambda)\frac{\partial\psi}{\partial\theta}=-\frac{G\rho_w}{\gamma}\iint_S h_w r'\frac{sin\psi}{L^3}cos\alpha dS \ ,$$

$$\eta^d(r,\theta,\lambda)=-\Theta^d(r,\theta,\lambda)\frac{\partial\psi}{\partial\lambda}=-\frac{G\rho_w}{\gamma}sin\theta\iint_S h_w r'\frac{sin\psi}{L^3}sin\alpha dS \quad (3.13)$$

Where, γ is the normal gravity on the calculation point, and $\Theta^d(r,\theta,\lambda)$ is the direct influence of the load effect on total vertical deviation at the calculation point.

(4) The integral formula of direct influence of the load effect on radial gravity gradient

According to the definition of radial gravity gradient, we have

$$T_{rr}=\frac{\partial^2 V_w}{\partial r^2}=G\rho_w\int_S h_w\frac{\partial}{\partial r}\left(\frac{r-r'cos\psi}{L^3}\right)dS=G\rho_w\int_S h_w\left[\frac{1}{L^3}-\frac{3(r-r'cos\psi)^2}{L^5}\right]dS \quad (3.14)$$

$$\frac{\partial}{\partial r}\left(\frac{r-r'cos\psi}{L^3}\right)=\frac{1}{L^3}-\frac{3(r-r'cos\psi)}{L^4}\frac{\partial}{\partial r}L=\frac{1}{L^3}-\frac{3(r-r'cos\psi)^2}{L^5}\ ,\ \frac{\partial}{\partial r}L=\frac{r-r'cos\psi}{L} \quad (3.15)$$

When the calculation point is located on the surface and overlaps with the center of integral area-element, the formula (3.14) on the calculation point is an integral singularity, and the singular value of the integral is

$$T_{rr}^0=-2\pi G\rho_w h_w r^2\int_0^{\psi_0}\left(\frac{1}{r^3\psi^3}-\frac{3\psi^4}{4r^3\psi^5}\right)\psi d\psi$$
$$=-\frac{2\pi G\rho_w h_w}{r}\int_0^{\psi_0}\left(\frac{1}{\psi^2}-\frac{3}{4}\right)d\psi\approx\frac{\pi G\rho_w h_w}{r\psi_0^2} \quad (3.16)$$

(5) The integral formula of direct influence of the load effect on horizontal gravity gradient

$$\Gamma\Gamma=\frac{\partial^2 V_w}{r^2\partial\psi^2}=-\frac{G\rho_w}{r}\int_S h_w r'\frac{\partial}{\partial\psi}\left(\frac{sin\psi}{L^3}\right)dS=-\frac{G\rho_w}{r}\int_S h_w r'\left(\frac{cos\psi}{L^3}-\frac{3rr'sin^2\psi}{L^5}\right)dS \quad (3.17)$$

$$T_{NN}=\Gamma\frac{\partial^2\psi}{\partial\theta^2}=\frac{G\rho_w}{r}\int_S h_w r'\left(\frac{cos\psi}{L^3}-\frac{3rr'sin^2\psi}{L^5}\right)ctg\psi(1-cos\alpha)dS \quad (3.18)$$

$$T_{WW}=-\Gamma\frac{\partial^2\psi}{\partial\lambda^2}=\frac{G\rho_w}{r}\int_S h_w r'\left(\frac{cos\psi}{L^3}-\frac{3rr'sin^2\psi}{L^5}\right)$$
$$\left[ctg\psi-ctg\psi(sin\theta sin\alpha)^2-\frac{cos\theta cos\theta'}{sin\psi}\right]dS \quad (3.19)$$

$$\frac{\partial^2\psi}{\partial\theta^2}=\frac{\partial}{\partial\theta}cos\alpha=\frac{\partial}{\partial\theta}\frac{sin\theta cos\theta'-cos\theta sin\theta' cos(\lambda'-\lambda)}{sin\psi}=ctg\psi(1-cos^2\alpha) \quad (3.20)$$

$$\frac{\partial^2\psi}{\partial\lambda^2}=-sin\theta\frac{\partial}{\partial\lambda}sin\alpha$$
$$=-sin\theta sin\theta'\frac{\partial}{\partial\lambda}\frac{sin(\lambda'-\lambda)}{sin\psi}=sin\theta sin\theta'\left[\frac{cos(\lambda'-\lambda)}{sin\psi}-\frac{sin(\lambda'-\lambda)cos\psi}{sin^2\psi}sin\alpha sin\theta\right]$$
$$=\frac{cos\psi-cos\theta cos\theta'}{sin\psi}-\frac{cos\psi}{sin\psi}(sin\theta sin\alpha)^2=(1-sin^2\theta sin^2\alpha)ctg\psi-\frac{cos\theta cos\theta'}{sin\psi} \quad (3.21)$$

8.3.2 Green's function integral of the indirect influence of the load effect

Substituting the load spherical harmonic coefficient $\{\Delta\bar{C}_{nm}^w,\Delta\bar{S}_{nm}^w\}$ into Formula (2.7), the indirect influence $\Delta V^i(r,\theta,\lambda)$ of load effect on geopotential can be obtained:

$$\Delta V^i=\frac{GM}{r}\frac{3\rho_w}{\rho_e}\sum_{n=0}^{\infty}\frac{k_n'}{2n+1}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\Delta\bar{C}_{nm}^w cosm\lambda+\Delta\bar{S}_{nm}^w sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (3.22)$$

Let $\boldsymbol{e}=(\theta,\lambda)$ be the spherical coordinate of point on the unit spherical surface, then the formula (3.22) can be expressed as a linear combination of normalized spherical basis functions $\{\bar{Y}_{nm}(\boldsymbol{e})=\bar{Y}_{nm}(\theta,\lambda)\}$ as follows:

$$\Delta V^i(r,\theta,\lambda)=\frac{GM}{r}\frac{3\rho_w}{\rho_e}\sum_{n=0}^{\infty}\frac{k'_n}{2n+1}\left(\frac{a}{r}\right)^n\sum_{m=-n}^{n}\bar{F}_{nm}^w\bar{Y}_{nm}(\boldsymbol{e}) \tag{3.23}$$

Where, $\bar{F}_{nm}^w=\Delta\bar{C}_{nm}^w, m\geq 0$ and $\bar{F}_{nm}^w=\Delta\bar{S}_{n|m|}^w, m<0$. Let

$$Y_n^w(\boldsymbol{e})=\sum_{m=-n}^{n}\bar{F}_{nm}^w\bar{Y}_{nm}(\boldsymbol{e}) \tag{3.24}$$

then Formula (3.23) can be expressed as:

$$\Delta V^i(r,\theta,\lambda)=\frac{GM}{r}\frac{3\rho_w}{\rho_e}\sum_{n=0}^{\infty}\frac{k'_n}{2n+1}\left(\frac{a}{r}\right)^n Y_n^w(\boldsymbol{e}) \tag{3.25}$$

The load equivalent water height spherical harmonic expansion (2.3) can be also expressed by the linear combination of the normalized spherical basis functions $\{\bar{Y}_{nm}(\boldsymbol{e})=\bar{Y}_{nm}(\theta,\lambda)\}$ as:

$$h_w(r\approx R,\theta,\lambda)=h_w(\boldsymbol{e})=R\sum_{n=1}^{\infty}\left(\frac{a}{R}\right)^n\sum_{m=-n}^{n}\bar{F}_{nm}^w\bar{Y}_{nm}(\boldsymbol{e})$$
$$=R\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\bar{F}_{nm}^w\bar{Y}_{nm}(\boldsymbol{e})=a\sum_{n=1}^{\infty}Y_n^w(\boldsymbol{e}) \tag{3.26}$$

According to the theory of spherical function expansion, from the formula (3.25):

$$Y_n^w(\boldsymbol{e})=\frac{2n+1}{4\pi a}\iint_\sigma h_w(\boldsymbol{e}')P_n(\psi)d\sigma \tag{3.27}$$

Here, ψ is the spherical angular distance from the integral area-element $\boldsymbol{e}'$ on the sphere to the calculation point $\boldsymbol{e}$.

Considering $dS=R^2d\sigma$, the formula (3.27) is substituted into the formula (3.26), and the summation and integral are exchanged to obtain:

$$\Delta V^i(r,\theta,\lambda)=\frac{1}{R^2}\iint_S \rho_w h_w(\boldsymbol{e}')\frac{GM}{4\pi ra}\frac{3}{\rho_e}\sum_{n=0}^{\infty}\left(\frac{a}{r}\right)^n k'_nP_n(\psi)\,dS$$
$$=\rho_w\iint_S h_w(\boldsymbol{e}')G_V^i(\psi)dS \tag{3.28}$$

Where ,

$$G_V^i(\psi)=\frac{GM}{4\pi R^2ra}\frac{3}{\rho_e}\sum_{n=0}^{\infty}\left(\frac{a}{r}\right)^n k'_nP_n(\psi) \tag{3.29}$$

is the general form of the load Green's function of indirect influence to geopotential, which represents the indirect influence of unit mass load (kg/m²) to geopotential.

When the calculation point is also located on the ground, that is, $r\approx a\approx R$ (R is the mean radius of the Earth), considering the total mass $M=4\pi R^3\rho_e/3$ of the Earth, the formula (3.29) can be simplified as follows:

$$G_V^i(\psi)=\frac{GM}{4\pi R^4}\frac{3}{\rho_e}\sum_{n=0}^{\infty}k'_nP_n(\psi)=\frac{G}{R}\sum_{n=1}^{\infty}k'_nP_n(\psi) \tag{3.30}$$

Similarly, the load Green's function of indirect influence to height anomaly can be obtained as follows:

$$G_\zeta^i(\psi)=\frac{R}{M}\sum_{n=0}^{\infty}k'_nP_n(\psi) \tag{3.31}$$

The load Green's function of indirect influence to ground gravity is◉:

$$G_g^i(\psi)=\frac{g_0}{M}\sum_{n=0}^{\infty}(n+1)\left(\frac{2}{n}h'_n-\frac{n+1}{n}k'_n\right)P_n(\psi) \tag{3.32}$$

Where, $g_0=GM/R^2$.

The load Green's function of indirect influence to gravity (disturbance) is:

$$G_{\delta g}^i(\psi)=-\frac{g_0}{M}\sum_{n=0}^{\infty}(n+1)k'_nP_n(\psi) \tag{3.33}$$

The load Green's function of indirect influence to ground tilt is◉:

$$G_t^i(\psi) = \frac{1}{M}\sum_{n=0}^{\infty}(k_n' - h_n')\frac{\partial P_n(\psi)}{\partial\psi} \quad (3.34)$$

The load Green's function of indirect influence to vertical deflection is:

$$G_\Theta^i(\psi) = -\frac{1}{M}\sum_{n=0}^{\infty} k_n' \frac{\partial P_n(\psi)}{\partial\psi} \quad (3.35)$$

The load Green's function of indirect influence to ground horizontal displacement is⦿:

$$G_l^i(\psi) = \frac{R}{M}\sum_{n=0}^{\infty} l_n' \frac{\partial P_n(\psi)}{\partial\psi} \quad (3.36)$$

The load Green's function of indirect influence to ground radial displacement is⦿:

$$G_r^i(\psi) = \frac{R}{M}\sum_{n=0}^{\infty} h_n' \frac{\partial P_n(\psi)}{\partial\psi} \quad (3.37)$$

The load Green's function of indirect influence to horizontal gravity gradient is:

$$G_{V_{rr}}^i = \frac{g_0}{RM}\sum_{n=0}^{\infty}(n+1)(n+2)k_n' P_n(\psi) \quad (3.38)$$

The load Green's function of indirect influence to radial gravity gradient is:

$$G_{V_{ss}}^i(\psi) = \frac{g_0}{RM}\sum_{n=0}^{\infty} k_n' \frac{\partial^2 P_n(\psi)}{\partial\psi^2} \quad (3.39)$$

Guo (2001) furtherly derived the asymptotic approach formula of the ground load Green's functions to suppress the high-degree oscillation of load Green's functions. Where, The load Green's functions of indirect influence are taken as follows:

$$G_\zeta^i(\psi) = \frac{R}{M}\frac{k_\infty'}{2sin\frac{\psi}{2}} + \frac{R}{M}\sum_{n=0}^{\infty}(k_n' - k_\infty')P_n(\psi) \quad (3.40)$$

$$G_g^i(\psi) = -\frac{g_0}{M}\frac{k_\infty' - 2h_\infty'}{2sin\frac{\psi}{2}} - \frac{g_0}{M}\sum_{n=0}^{\infty}[(n+1)k_n' - k_\infty' - 2(h_n' - h_\infty')]P_n(\psi) \quad (3.41)$$

$$G_{\delta g}^i(\psi) = -\frac{g_0}{M}\frac{k_\infty'}{2sin\frac{\psi}{2}} - \frac{g_0}{M}\sum_{n=0}^{\infty}[(n+1)k_n' - k_\infty']P_n(\psi) \quad (3.42)$$

$$G_t^i(\psi) = -\frac{1}{M}\frac{h_\infty' cos\frac{\psi}{2}}{4sin^2\frac{\psi}{2}} + \frac{1}{M}\frac{k_\infty' cos\frac{\psi}{2}\left(1+2sin\frac{\psi}{2}\right)}{2sin\frac{\psi}{2}\left(1+sin\frac{\psi}{2}\right)} - \frac{1}{M}\sum_{n=1}^{\infty}\left(k_n' - \frac{k_\infty'}{n} - h_n' + h_\infty'\right)\frac{\partial P_n(\psi)}{\partial\psi} \quad (3.43)$$

$$G_\Theta^i(\psi) = \frac{1}{M}\frac{k_\infty' cos\frac{\psi}{2}\left(1+2sin\frac{\psi}{2}\right)}{2sin\frac{\psi}{2}\left(1+sin\frac{\psi}{2}\right)} - \frac{1}{M}\sum_{n=1}^{\infty}\left(k_n' - \frac{k_\infty'}{n}\right)\frac{\partial P_n(\psi)}{\partial\psi} \quad (3.44)$$

$$G_l(\psi) = -\frac{R}{M}\frac{l_\infty' cos\frac{\psi}{2}\left(1+2sin\frac{\psi}{2}\right)}{2sin\frac{\psi}{2}\left(1+sin\frac{\psi}{2}\right)} + \frac{R}{M}\sum_{n=1}^{\infty}\left(l_n' - \frac{l_\infty'}{n}\right)\frac{\partial P_n(\psi)}{\partial\psi} \quad (3.45)$$

$$G_r(\psi) = \frac{R}{M}\frac{h_\infty'}{2sin\frac{\psi}{2}} + \frac{a}{M}\sum_{n=0}^{\infty}(h_n' - h_\infty')P_n(\psi) \quad (3.46)$$

Let $\mathcal{G}^i(l) = 2Rsin\frac{\psi}{2}G^i(\psi) = lG^i(\psi)$, the load Love number is substituted into the formulas (3.31) ~ (3.39) to obtain the load Green's function of indirect influence with the integral distance under the action of unit point mass load (1 kg/m²), as shown in Tab 3.1.

Tab 3.1 Load Green's function values of the indirect influence of unit point mass

l(km)	$\mathcal{G}_\zeta^i$×10-13	$\mathcal{G}_g^i$×10-17	$\mathcal{G}_{\delta g}^i$×10-18	$\mathcal{G}_t^i$×10-14	$\mathcal{G}_\Theta^i$×10-19	$\mathcal{G}_l$×10-12	$\mathcal{G}_r$×10-11	$\mathcal{G}_{rr}^i$×10-15	$\mathcal{G}_{ss}^i$×10-15
0.1	-0.0249	-11.3315	15.8795	42.2955	-2.1192	-0.8369	-42.1264	-40.7525	20.0337
0.2	-0.0439	-9.8972	29.6981	21.1510	-8.0632	-3.1842	-41.9553	-73.6102	34.1831
0.3	-0.0625	-8.8334	39.7946	14.1058	-16.6878	-6.5901	-41.7788	-92.3770	37.9744
0.4	-0.0804	-8.2348	45.2182	10.5853	-26.3601	-10.4097	-41.5956	-93.8712	29.4189

0.5	-0.0975	-8.1095	45.8894	8.4739	-35.3064	-13.9425	-41.4057	-78.5612	9.4993
0.6	-0.1139	-8.3807	42.5773	7.0657	-41.9834	-16.5790	-41.2101	-50.3867	-18.0490
0.7	-0.1294	-8.9073	36.7009	6.0583	-45.3905	-17.9241	-41.0109	-15.8142	-47.6055
0.8	-0.1444	-9.5157	30.0034	5.3006	-45.2558	-17.8704	-40.8109	17.6468	-72.9744
1.0	-0.1727	-10.3454	20.4992	4.2343	-36.8762	-14.5596	-40.4173	55.8494	-91.9157
1.2	-0.1998	-10.1321	21.4749	3.5210	-26.2416	-10.3574	-40.0402	39.6641	-61.0517
1.4	-0.2261	-9.1669	30.0077	3.0153	-22.8895	-9.0304	-39.6752	-8.4433	-7.5471
1.6	-0.2518	-8.3519	37.0350	2.6419	-28.6871	-11.3158	-39.3091	-42.4515	24.9158
2.0	-0.3003	-8.9633	28.5858	2.1198	-40.5309	-15.9830	-38.5476	4.3817	-24.2022
2.5	-0.3570	-9.1242	24.1119	1.6843	-25.9871	-10.2232	-37.6133	17.0612	-27.2278
3.0	-0.4112	-7.9718	32.8632	1.4080	-35.2424	-13.8576	-36.7093	-28.7167	17.2271
3.5	-0.4621	-8.9437	20.3140	1.2022	-32.5321	-12.7629	-35.7866	31.1746	-40.2655
4.0	-0.5112	-7.7218	29.8481	1.0465	-28.2814	-11.0562	-34.9109	-22.8507	15.9355
5.0	-0.6036	-7.8959	22.7679	0.8291	-26.3578	-10.2305	-33.1702	5.9459	-11.1019
6.0	-0.6903	-7.8527	18.1028	0.6858	-29.9324	-11.5649	-31.5082	23.6048	-28.4842
7.0	-0.7725	-7.2943	18.8748	0.5827	-33.7803	-12.9988	-29.9389	13.5281	-18.2480
8.0	-0.8510	-6.5206	22.0921	0.5013	-33.1161	-12.6452	-28.4652	-9.3638	5.3150
10.0	-0.9991	-6.0125	18.9937	0.3784	-24.7530	-9.1540	-25.7982	-5.3162	2.8950
12.0	-1.1387	-5.9045	13.1167	0.2999	-27.9718	-10.2454	-23.5296	16.1892	-18.4692
14.0	-1.2726	-4.9048	17.3988	0.2398	-26.5722	-9.5373	-21.6664	-13.0654	11.2087
16.0	-1.4019	-4.8896	12.8941	0.1911	-21.0009	-7.2164	-20.1480	4.3047	-5.5888
20.0	-1.6520	-4.0437	14.8205	0.1306	-20.9145	-7.0582	-18.0179	-12.2601	11.2369
25.0	-1.9534	-3.6904	13.7959	0.0872	-19.8016	-6.6584	-16.5317	-10.0949	9.3198
30.0	-2.2455	-3.5544	12.9067	0.0638	-18.9897	-6.5141	-15.7982	-5.5325	4.9129
35.0	-2.5296	-3.5250	12.0811	0.0505	-18.1729	-6.4230	-15.4331	-0.0753	-0.4331
40.0	-2.8059	-3.5272	11.4345	0.0423	-17.1945	-6.2698	-15.2297	4.7358	-5.1568
50.0	-3.3365	-3.4643	11.2395	0.0322	-14.9772	-5.7725	-14.9607	8.1685	-8.4622
60.0	-3.8395	-3.2518	12.5464	0.0262	-13.6029	-5.4612	-14.6941	2.7549	-2.9775
70.0	-4.3177	-3.0073	14.0654	0.0229	-13.9783	-5.7205	-14.3923	-4.6469	4.4506
80.0	-4.7741	-2.8804	14.3310	0.0210	-15.3999	-6.3101	-14.0649	-6.2127	6.0235
100.0	-5.6311	-2.9117	11.9306	0.0171	-15.7804	-6.3810	-13.3843	4.6763	-4.8316
120.0	-6.4270	-2.6545	12.4755	0.0129	-14.0249	-5.5346	-12.7235	-0.1761	0.0607
140.0	-7.1738	-2.4359	12.7461	0.0120	-15.5946	-5.9880	-12.0989	-3.7448	3.6348
160.0	-7.8804	-2.4586	10.7233	0.0100	-14.9953	-5.5941	-11.5133	-4.4893	-4.5820

200.0	-9.1986	-2.0952	11.1758	0.0080	-15.1075	-5.3733	-10.4758	-1.7439	1.6689
250.0	-10.7136	-1.8097	10.7082	0.0058	-14.0435	-4.7072	-9.3924	-3.2869	3.2307
300.0	-12.1238	-1.5962	10.1419	0.0042	-12.9077	-4.0819	-8.5118	-3.2916	3.2481
400.0	-14.7375	-1.3210	8.9521	0.0023	-11.1503	-3.1625	-7.2265	-0.4258	0.3969
500.0	-17.1749	-1.1331	8.3207	0.0016	-10.3019	-2.7029	-6.4078	2.1612	-2.1831
600.0	-19.4980	-0.9603	8.5053	0.0014	-9.8691	-2.4641	-5.9044	2.3040	-2.3219
800.0	-23.8986	-0.6720	9.9646	0.0010	-9.0007	-2.0628	-5.4405	-0.1041	0.0908

8.3.3 Legendre function and its first and second derivatives to ψ

When calculating the load Green's function of the indirect influence to various geodetic variation, it is necessary to calculate the Legendre function $P_n(cos\psi)$ and its first and second derivatives to ψ. Here, let $t = cos\theta, u = sin\theta$, and give the fast recursive algorithm directly.

$$P_n(t) = \frac{2n-1}{n} tP_{n-1}(t) - \frac{n-1}{n} P_{n-2}(t) \tag{3.47}$$

$$P_1 = t, \quad P_2 = \frac{1}{2}(3t^2 - 1) \tag{3.48}$$

$$\frac{\partial}{\partial\psi} P_n(t) = \frac{2n-1}{n} t \frac{\partial}{\partial\psi} P_{n-1}(t) - \frac{2n-1}{n} uP_{n-1}(t) - \frac{n-1}{n} \frac{\partial}{\partial\psi} P_{n-2}(t) \tag{3.49}$$

$$\frac{\partial}{\partial\psi} P_1(t) = -u, \quad \frac{\partial}{\partial\psi} P_2(t) = -3ut \tag{3.50}$$

$$\frac{\partial^2}{\partial\psi^2} P_n(t) = \frac{2n-1}{n}\left(t \frac{\partial^2}{\partial\psi^2} P_{n-1} - 2u \frac{\partial}{\partial\psi} P_{n-1} - tP_{n-1}\right) - \frac{n-1}{n} \frac{\partial^2}{\partial\psi^2} P_{n-2} \tag{3.51}$$

$$\frac{\partial^2}{\partial\psi^2} P_1(t) = -t, \quad \frac{\partial^2}{\partial\psi^2} P_2(t) = 3(1 - 2t^2) \tag{3.52}$$

8.3.4 Calculation of load deformation field from the river-lake water variations

The changes of inland water bodies such as rivers, lakes, reservoirs, glaciers and snow mountains are represented by load equivalent water height variation grid. According to the load Green's function integral algorithm (the sum of load direct influence and indirect influence integral), the load effects on various geodetic variations at any point on the ground or in near-surface space can be calculated. The equivalent water height variation grids of multiple water bodies at the same sampling epoch can be merged into a grid directly, and then the load Green's function integral calculation is carried out.

Here, the river (lake) bottom topography is combined with the water-level observations on river (lake) surface, and the equivalent water height variation grid time series is constructed from the river (lake) water-level monitoring data. Then, according to the load Green's function integral algorithm, the load effect grid time series on all-element geodetic variations are calculated. Fig 3.1 is the calculation process of surface load deformation field at one sampling epoch time.

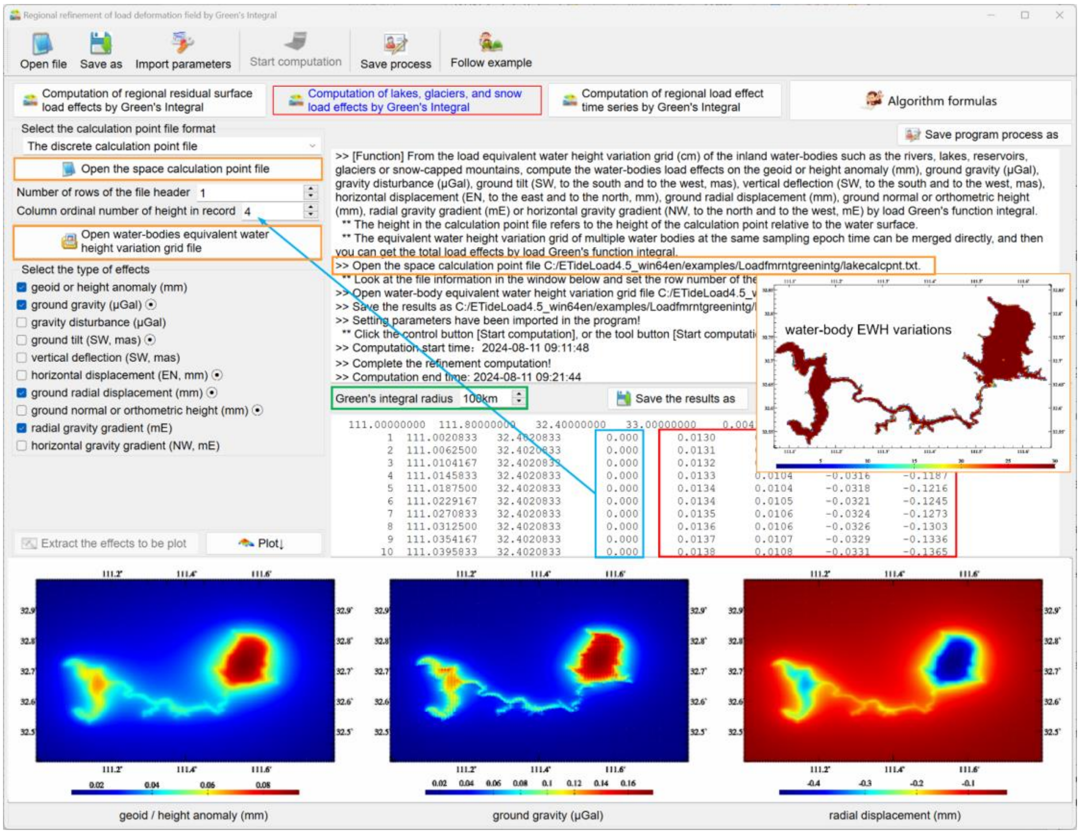


Fig 3.1 Calculation of load deformation field of water variations in rivers and lakes

8.3.5 Regional approach of load deformation field using remove-restore scheme

When the load effects are calculated by the load Green's function integral from the load equivalent water height variation grid, the domain of the independent variable ψ of the integral kernel function should be $[0,2\pi)$ or $[-\pi,\pi)$, which is global. Direct integration need globally continuously distributed surface equivalent water height variation data, even if calculating the load effect at some a point. This is very inconvenient for the calculation and application of load deformation effect in a local area such as a country or region, and it is also inconvenient to use the advantages of surface load observations in local areas to improve the regional load deformation field.

Similar to the local gravity field approach scheme in physical geodesy, we can let the global load spherical harmonic coefficient model as the reference field, calculate and remove the reference model value from the surface load variations in local areas to obtain the residual load variations, then employ the load Green's integral to the residual load variations to refine the load deformation field in this area. This scheme can be also called as the remove-restore scheme for regional approach of load deformation field.

The process of the scheme at one epoch time is as follows: (a) The reference model value of the equivalent water height (EWH) variations in local areas are calculated from the load spherical harmonic coefficient model at the epoch. (b) From the regional high-resolution EWH variation grid, remove the reference model value to obtain the regional EWH residual value grid. This step is called as 'Remove'. (c) Using a smaller integral radius, the residual value of high-resolution load deformation field grid is calculated by the load Green's function integral. (d) The high-resolution model value grid of regional load deformation field are calculated from the load spherical harmonic coefficient model. (e) The refined value of regional high-resolution load deformation field at the epoch time are obtained by adding the high-resolution reference model value grid to residual value grid. This step is called as 'Restore'. The whole process can be called as 'Remove - load Green's function integral - Restore' scheme.

In the following, taking the refinement of the surface atmosphere load deformation field in a certain area of southern China as an example, using the 'Remove-load Green's function integral-Restore' scheme, the surface atmosphere load deformation field variation grid weekly time series are refined from the 3.75′×3.75′ surface atmospheric pressure variation (hPa) grid weekly time series whose time span is January 2018 to December 2020 with a total of 157 sampling epochs. Of which 12 epochs of surface atmospheric pressure variation grid are shown in Fig 3.2, and the upper left corner of the graph is the sampling epoch date, such as 20180214, which means February 14, 2018.

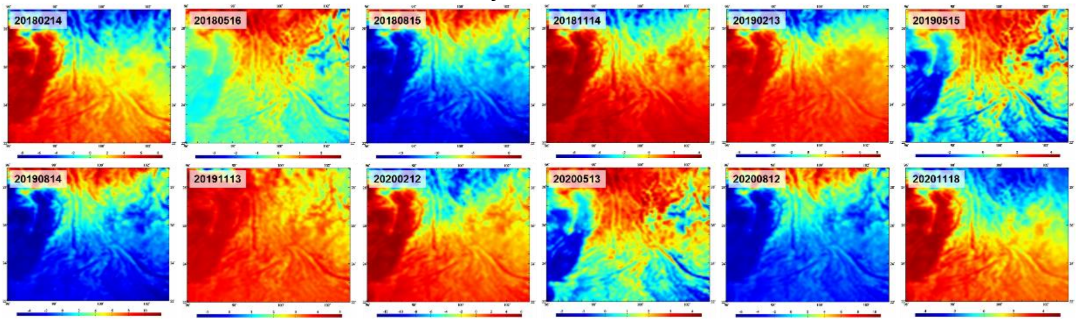


Fig 3.2 Regoinal 3.75′×3.75′ surface atmospheric pressure variation (hPa) grid weekly time series

Taking 157 180-degree global surface atmosphere load spherical harmonic coefficient model weekly time series calculated in section 8.2.5 from January 2018 to December 2020 as the load deformation field reference model time series, and similar to the regional geoid refinement scheme, the load equivalent water height grid data area (the calculation area) should be generally bigger than refine result area of the load deformation field to suppress the integral edge effect. In this case, the data area is 96°E ~ 103°E, 22°N ~ 29°N, and the result area is 98°E ~ 101°E, 24°N ~ 27°N.

Step 1: Input the 3.75′×3.75′ zero-value grid of the calculation area (zero-value means that the calculation point height relative to the ground is equal to zero), select the maximum calculation degree 180, and calculate the reference model value grid time series of the surface atmosphere variations in the calculation area from the global atmosphere load spherical harmonic coefficient model time series.

Step 2: The 3.75′×3.75′ surface atmosphere variation grid weekly time series, minus the corresponding reference model value grid weekly time series to generate the 3.75′×3.75′ surface atmosphere residual value grid weekly time series. Where, 12 epochs of residual value grid is shown in Fig 3.3.

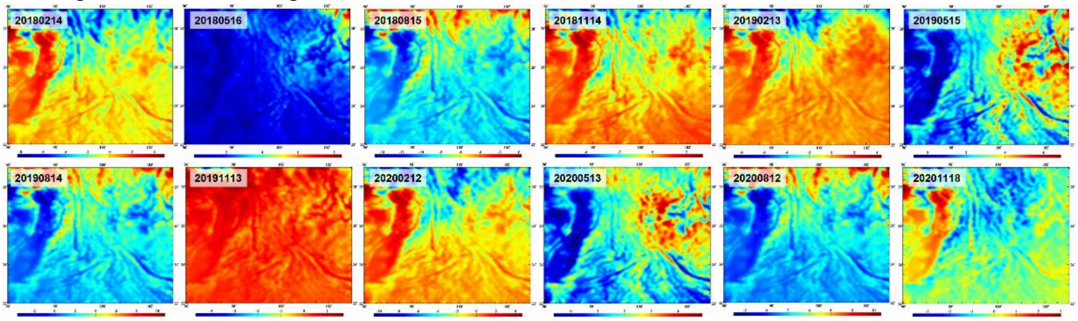


Fig 3.3 Regoinal 3.75′×3.75′ surface atmosphere residual variation (hPa) grid weekly time series

Step 3: Input the 3.75'×3.75' zero-value grid in the result area, select the integral radius of 200 km, and calculate the 3.75'×3.75' load deformation field residual value grid weekly time series from the 3.75'×3.75' surface atmosphere residual variation grid weekly time series using the load Green's function integral.

Step 4: Input the 3.75′×3.75′ zero-value grid in the result area, select the maximum calculation degree 180, and calculate the 3.75′×3.75′ reference model value grid weekly time series of the surface atmosphere load deformation field in the result area from the surface atmosphere load spherical harmonic coefficient model weekly time series.

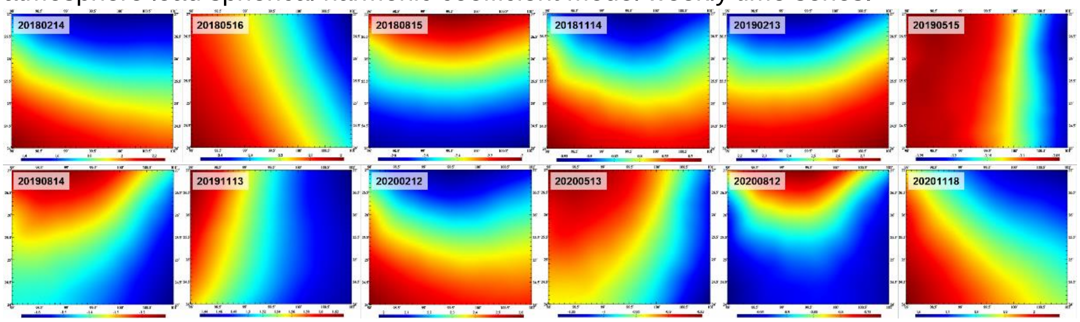


Fig 3.4 Regoinal 3.75′×3.75′ surface atmosphere load effect grid weekly time series on geoid (mm)

Step 5: The 3.75′×3.75′ residual value grid weekly time series of load deformation field in the result area is added to the 3.75′×3.75′ residual value grid weekly time series, and the 3.75′×3.75′ grid time series of surface atmosphere load deformation field in the result area are obtained. Where, 12 epochs of regional atmosphere load deformation field, including the load effects on geoid, ground gravity, ground tilt, ellipsoidal height and radial gravity gradient, are shown in Fig 3.4-Fig 3.8 respectively.

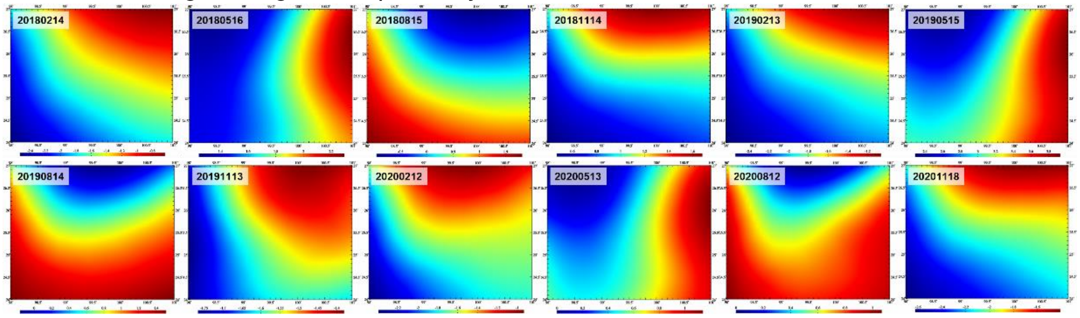


Fig 3.5 Regoinal 3.75′×3.75′ surface atmosphere load effect grid weekly time series on ground gravity (mGal)

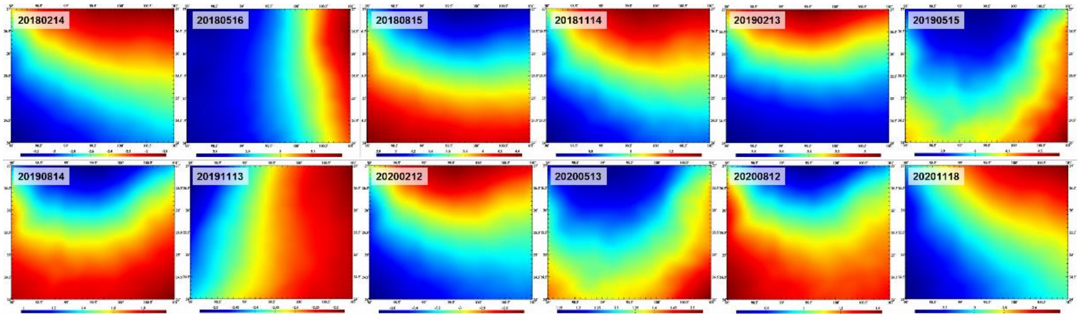


Fig 3.6 Regoinal 3.75′×3.75′ surface atmosphere load effect grid weekly time series on ellipsoidal height (mm)

In order to visually display the time-varying characteristics of surface atmosphere load effects on various geodetic variations and quantitative relationship between the surface atmosphere load effects on different types of geodetic variations in the region, the time series of surface atmosphere load effects on the geoid, ground gravity, ellipsoidal height and radial gravity gradient from January 2018 to December 2020 at the central ground point of the region are calculated using the 'Remove - load Green's function integral - Restore' scheme, as shown in Fig 3.9.

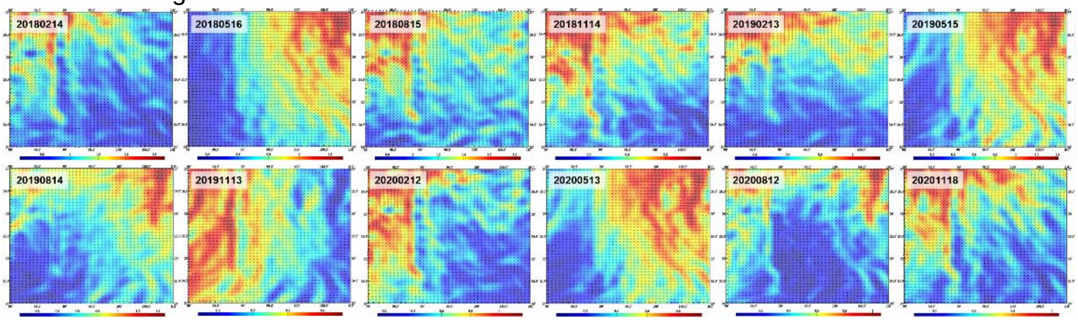


Fig 3.7 Regoinal 3.75′×3.75′ surface atmosphere load effect grid weekly time series on ground tilt (mas)

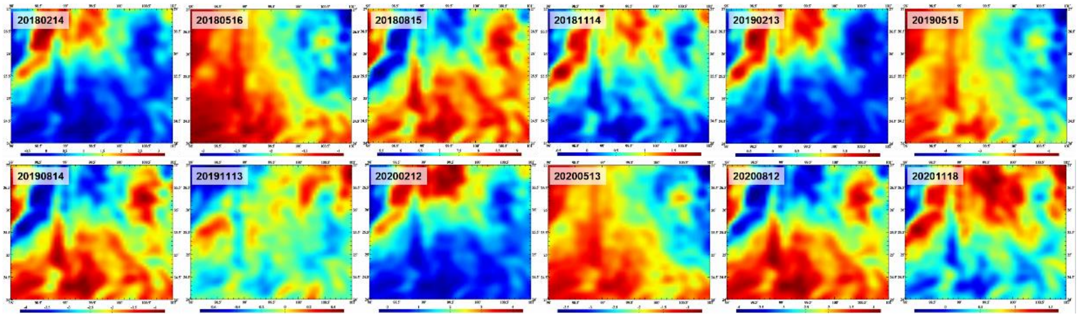


Fig 3.8 Regoinal 3.75′×3.75′ surface atmosphere load effect grid weekly time series on radial gravity gradient (mE)

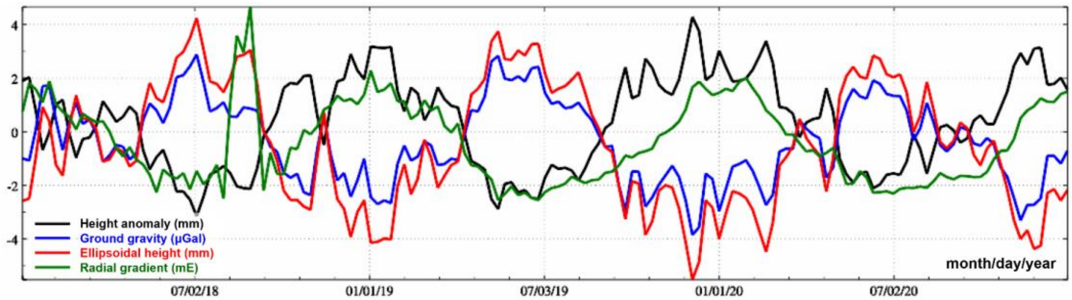


Fig 3.9 surface atmosphere load effect weekly time series on geodetic variations at the central ground point of the region

In the following, the 0.5°×0.5° global surface atmospheric pressure diurnal variations in the global reanalysis data ERA-40/ERA-Interim from the European Centre for Medium-Range Weather Forecasts (ECMWF) are employed to construct the 0.5°×0.5 surface atmosphere variation (hPa) grid weekly time series (157 sampling epochs) from January 2018 to December 2020 in chinese mainland and adjacent areas.

Taking 157 180-degree global surface atmosphere load spherical harmonic coefficient model weekly time series calculated in section 8.2.5 from January 2018 to December 2020 as the load deformation field reference model time series. Firstly, using the 'Remove-load Green's function integral - Restore' scheme and the integral radius 200 km, the surface atmosphere load effect weekly time series on the geoid, ground gravity, ellipsoidal height and radial gravity gradient are calculated at 6 CORS stations in mainland China. Then, the load Green's function integration method with the integral radius 800km is directly employed to directly calculate the surface atmosphere load effect weekly time series on the geoid, ground gravity, ellipsoidal height and radial gravity gradient are calculated at 6 CORS stations in mainland China. Finally, the two calculation results are compared.

The load effect weekly time series curves at 6 CORS stations in mainland China calculated by the two methods are shown in Fig 3.10 ~ Fig 3.13. The upper figure of each figure is the calculation result using the remove -restore method, and the lower figure is the calculation result uising the load Green's function integral (direct integral) method.

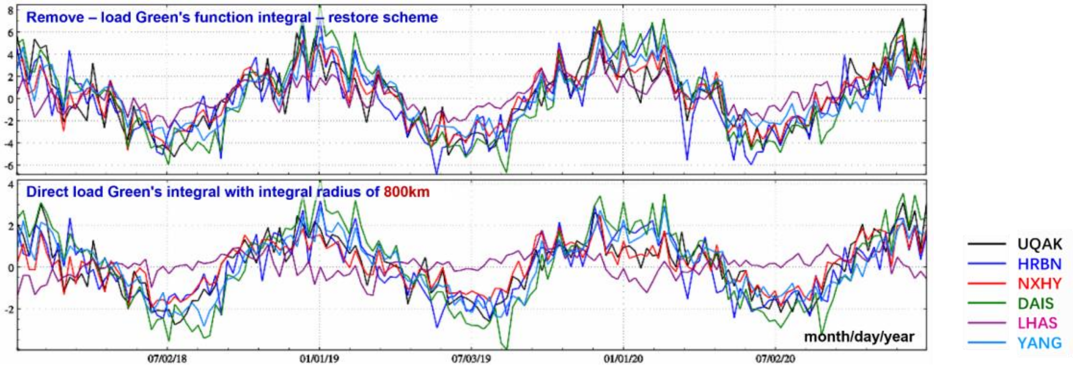


Fig 3.10 Surface atmosphere load effect time series on geoid (mm) at 6 CORS stations in mainland China using two scheme

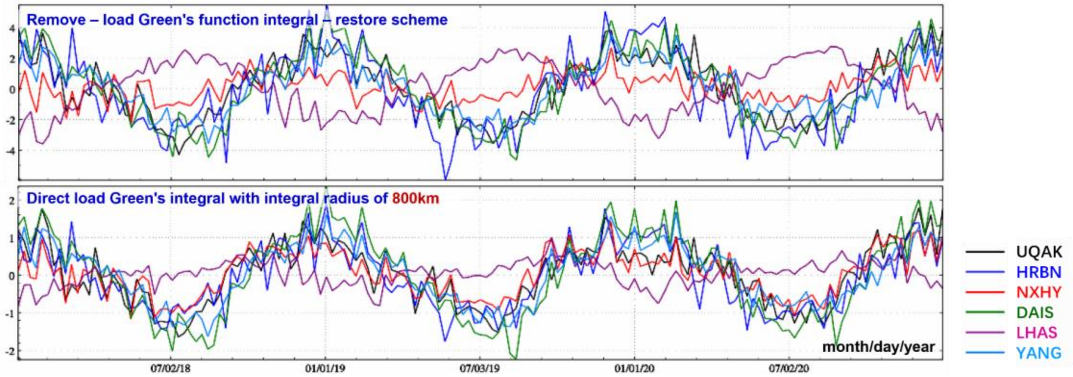


Fig 3.11 Surface atmosphere load effect time series on ground gravity (mGal) at 6 CORS stations in mainland China using two scheme

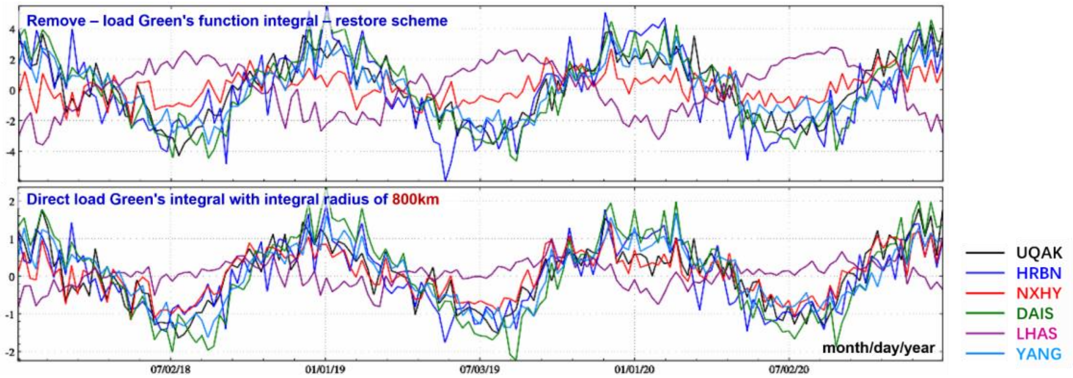


Fig 3.12 Surface atmosphere load effect time series on ellipsoidal height (mm) at 6 CORS stations in mainland China using two scheme

Tab 3.2 gives the differences statistics between the atmosphere load effects time series calculated by the two schemes at 6 CORS stations in mainland China. In Tab 3.2, ksi, gra, hgt and grr represent the atmosphere load effect time series on the geoid, ground gravity, ellipsoidal height and radial gravity gradient, respectively.

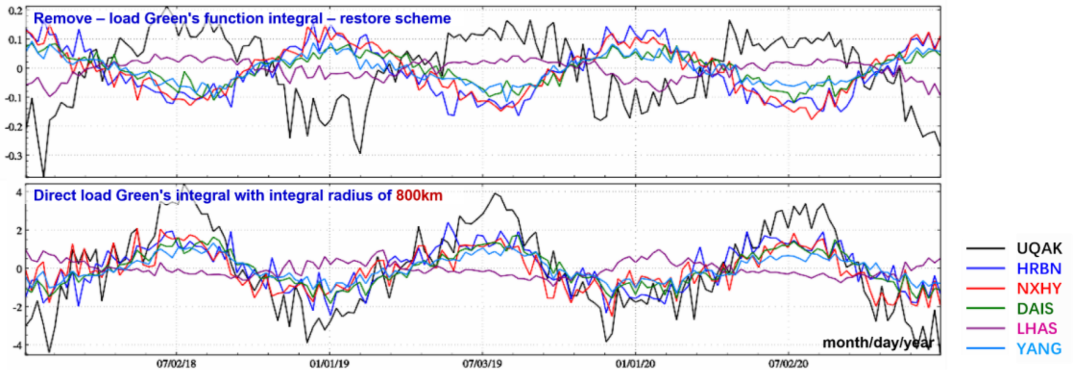


Fig 3.13 Surface atmosphere load effect time series on radial gravity gradient (mE) at 6 CORS stations in mainland China using two scheme

Tab 3.2 The differences statistics between the atmosphere load effects time series calculated by the two schemes at 6 CORS stations in mainland China

CORS station	Geodetic variation	unit	Remove-restore		Direct integral		Difference	
			mean	RSM	mean	RSM	mean	RSM
UQAK E87.97° N47.10°	ksi	mm	0.1618	2.9040	0.0669	1.3829	0.0949	1.6514
	gra	mGal	0.1363	2.0060	0.0382	0.8115	0.0982	1.2499
	hgt	mm	-0.2201	4.0837	-0.1053	2.2224	-0.1148	2.0922
	grr	mE	-0.0029	0.1224	-0.0582	2.1550	0.0553	2.0549
HRBN E126.62° N45.70°	ksi	mm	0.0335	3.1523	-0.0208	1.4114	0.0542	1.8525
	gra	mGal	-0.0628	2.5363	-0.0148	0.8494	-0.0479	1.7076
	hgt	mm	0.0053	4.3568	0.0411	2.3354	-0.0357	2.2104
	grr	mE	-0.0111	0.0883	0.0308	1.1439	-0.0419	1.2281
NXHY E105.63° N36.55°	ksi	mm	0.3069	2.6001	0.0994	1.0561	0.2075	1.6745
	gra	mGal	0.1592	0.9325	0.0576	0.6012	0.1016	0.4970
	hgt	mm	-0.3920	3.4507	-0.1589	1.6570	-0.2331	2.0348
	grr	mE	-0.0073	0.0818	-0.1068	1.0503	0.0995	1.1172
DAIS E122.20° N30.23°	ksi	mm	0.2630	3.5502	0.0907	1.9842	0.1722	1.6910
	gra	mGal	0.1284	2.4349	0.0522	1.1244	0.0763	1.3303
	hgt	mm	0.3564	4.7804	-0.1429	3.0833	-0.2135	1.9432
	grr	mE	-0.0026	0.0517	-0.0441	0.8954	0.0415	0.9444
LHAS E91.10° N29.65°	ksi	mm	0.3231	1.3847	0.0826	0.5451	0.2405	1.2880
	gra	mGal	0.1167	1.6451	0.0464	0.3234	0.0703	1.4307
	hgt	mm	-0.4104	1.8119	-0.1262	0.8929	-0.2842	1.6980
	grr	mE	-0.0043	0.0325	-0.0425	0.3946	0.0383	0.4219

YANG E109.22° N19.77°	ksi	mm	0.3055	2.4528	0.0904	1.3640	0.2151	1.2191
	gra	mGal	0.0748	1.6494	0.0499	0.7787	0.0249	0.8961
	hgt	mm	-0.4103	3.4732	-0.1365	2.1386	-0.2738	1.5685
	grr	mE	-0.0041	0.0461	-0.0344	0.6216	0.0303	0.6665

Fig 3.10 ~ Fig 3.13 show that the geometric shape of the time series curve of the load effects calculated by the two schemes is basically the same, but the numerical value is obviously different. Tab 3.2 shows that even if the integral radius reaches 800 km, the error of the direct integral of the load Green's function will exceed the magnitude of the calculated signal itself. This is because as long as the integral radius is less than $\sqrt{2}R$ (R is the mean radius of the Earth), the direct integral of the load Green's function fails to achieve global surface integral, and the calculated load effect signal is not sufficient. It can be seen that in most cases, the direct integral method of load Green's function is difficult to meet the high-precision geodesy. It is suggested to adopt the more rigorous the 'remove -load Green's function integral -Restore' scheme in theory.

8.4 Ocean and atmosphere load tidal effects outside the solid Earth

The ocean tides redistribute the mass of seawater and cause the geopotential variations, which excite the deformation of the solid Earth and generate the associted geopotential through the action of load Love numbers at the same time. Similarly, atmospheric tides redistribute the density of the atmosphere, cause the geopotential variations and then generate the associted geopotential.

8.4.1 Construction of tidal load spherical harmonic coefficient model

Both ocean tide and solid Earth tide are generated by the tidal force from the Moon and Sun, and have the same periodic variation characteristics, such as diurnal and semi-diurnal periodic variations. Therefore, it is generally difficult to find mathematical methods to completely separate the ocean tidal load effect on geodetic variations from their solid tidal effects. The method of load Green's function and sea surface tidal height convolution, or the method of spherical harmonic analysis of ocean tide and spherical harmonic synthesis of ocean tidal load effect, is usually employed to calculate the ocean tidal load effects on geodetic variations. The ocean tidal loads are located on the surface, and the load Green's functions need to be calculated by the high-degree load Love numbers. The load spherical harmonic coefficient model also need be expressed as a high-degree or ultrahigh-degree spherical harmonic function series.

(1) Spherical harmonic analysis method of global ocean tide

General construction procedure of the global ocean tidal load normalized spherical harmonic coefficient model (in FES2004 format) from global ocean tidal height harmonic constant grids is generally composed of the following three steps.

Step 1: From the global ocean tidal harmonic constant grid model of each tidal constituent, generate the normalized spherical harmonic coefficient model of each tidal constituent by spherical harmonic analysis method.

Step 2: According to the astronomical tide height algorithm, convert the normalized spherical harmonic coefficients based on the harmonic constant of the tidal constituent into the normalized spherical harmonic coefficient based on the tidal load of the tidal constituent.

Step 3: Merging the global normalized tidal load spherical harmonic coefficients of all tidal constituents, generate the global ocean tidal load normalized spherical harmonic coefficient model in FES2004 format.

Astronomical tidal height $T(t)$ of sea surface at the epoch time t, expressed as the height of the astronomical tidal level relative to the local mean sea surface, is equal to the sum of M tidal constituent heights

$$T(\theta,\lambda,t)=\sum_{i=1}^{M}T_i(\theta,\lambda,t)=\sum_{i=1}^{M}H_i(\theta,\lambda)cos[\phi_i(t)-g_i(\theta,\lambda)] \quad (4.1)$$

Where, M is the number of the ocean tidal constituents, θ_i, H_i, g_i are the astronomical argument, amplitude and phase at Greenwich of the tidal constituent σ_i, respectively. T_i is The astronomical tide height of the tidal constituent σ_i, which can be expanded as

$$T_i(\theta,\lambda,t)=H_i(\theta,\lambda)cosg_i(\theta,\lambda)cos\phi_i(t)+H_i(\theta,\lambda)sing_i(\theta,\lambda)sin\phi_i(t)$$
$$=H_i^{+}(\theta,\lambda)cos\phi_i(t)+H_i^{-}(\theta,\lambda)sin\phi_i(t)=H_i^{+}cos\phi_i+H_i^{-}sin\phi_i \quad (4.2)$$

On the other hand, through the spherical harmonic analysis, the tidal height T_i of the tidal constituent σ_i can be also expressed as the normalized spherical harmonic series

$$T_i(\theta,\lambda,t)=\sum_{n=1}^{N}\sum_{m=0}^{n}\left[T_{i,nm}^{+}(\lambda,t)+T_{i,nm}^{-}(\lambda,t)\right]\bar{P}_{nm}(cos\theta) \quad (4.3)$$

Where ,
$$T_{i,nm}^{+}(\lambda,t)=\bar{C}_{i,nm}^{+}cos(\phi_i+m\lambda)+\bar{S}_{i,nm}^{+}sin(\phi_i+m\lambda) \quad (4.4)$$

$$T_{i,nm}^{-}(\lambda,t)=\bar{C}_{i,nm}^{-}cos(\phi_i-m\lambda)+\bar{S}_{i,nm}^{-}sin(\phi_i-m\lambda) \quad (4.5)$$

In (4.4) and (4.5), the superscript + is the normalized spherical harmonic coefficient of the in-phase amplitude (H_icosg_i) of the tidal constituent σ_i, and the superscript − is the normalized spherical harmonic coefficient of the out-of-phase amplitude (H_ising_i) of the tidal constituent σ_i.

Expand the trigonometric functions in the formulas (4.4) and (4.5), we have

$$T_{i,nm}^{+}(\lambda,t)=\bar{C}^{+}[cos\phi_icosm\lambda-sin\phi_isinm\lambda]+\bar{S}^{+}[sin\phi_icosm\lambda+cos\phi_isinm\lambda]$$
$$=[\bar{C}^{+}cosm\lambda+\bar{S}^{+}sinm\lambda]cos\phi_i+[-\bar{C}^{+}sinm\lambda+\bar{S}^{+}cosm\lambda]sin\phi_i \quad (4.6)$$

$$T_{i,nm}^{-}(\lambda,t)=\bar{C}^{-}[cos\phi_icosm\lambda+sin\phi_isinm\lambda]+\bar{S}^{-}[sin\phi_icosm\lambda-cos\phi_isinm\lambda]$$
$$=[\bar{C}^{-}cosm\lambda-\bar{S}^{-}sinm\lambda]cos\phi_i+[\bar{C}^{-}sinm\lambda+\bar{S}^{-}cosm\lambda]sin\phi_i \quad (4.7)$$

Comparing the formula (4.2) and formula (4.3), for the tidal constituent σ_i, (the tidal constituent number i is omitted below), we have

$$H^{+}=\sum_{n=1}^{N}\sum_{m=0}^{n}\bar{P}_{nm}\,[(\bar{C}^{+}+\bar{C}^{-})cosm\lambda+(\bar{S}^{+}-\bar{S}^{-})sinm\lambda] \quad (4.8)$$

$$H^{-}=\sum_{n=1}^{N}\sum_{m=0}^{n}\bar{P}_{nm}\,[(\bar{S}^{+}+\bar{S}^{-})cosm\lambda+(-\bar{C}^{+}+\bar{C}^{-})sinm\lambda] \quad (4.9)$$

$$\bar{C}^{+}=\hat{C}^{+}sin\varepsilon^{+},\quad \bar{C}^{-}=\hat{C}^{-}sin\varepsilon^{-},\quad \bar{S}^{+}=\hat{C}^{+}cos\varepsilon^{+},\bar{S}^{-}=\hat{C}^{-}cos\varepsilon^{-} \quad (4.10)$$

Where, ε_i is the phase bias of constituent σ_i, which is defined by the sign of the maximum amplitude H_i of the equilibrium tidal height of the constituent σ_i (Cartwright & Eden, 1973), as shown in Tab 4.1. The value of H_i is shown in the last column of Tab 1.7 ~ Tab 1.12.

Table 4.1 Values of the phase bias ε_i according to the sign of H_i

		$H_i > 0$	$H_i < 0$
$m = 0$	long period	π	0
$m = 1$	diurnal	$\pi/2$	$-\pi/2$
$m = 2$	semi-diurnal	0	π

(2) The direct influences of ocean tidal loads to the geopotential coefficients

According to the universal gravitation theorem, the gopotential V^{ot} directly generated by ocean tidal load can be expressed as

$$V^{ot}(\theta, \lambda, t) = G\rho_w \iint_S \frac{H(\theta', \lambda', t)}{L} dS \tag{4.11}$$

where H is the instantaneous ocean tidal height, S represents the whole sea surface and L is the spatial distance between the calculation point $\boldsymbol{e} = (\theta, \lambda)$ and the sea surface moving area-element $\boldsymbol{e}' = (\theta', \lambda')$. L can be expressed by Legendre function series as

$$\frac{1}{L} = \frac{1}{R} \sum_{n=0}^{\infty} \left(\frac{R}{r}\right)^{n+1} P_n(cos\psi) \tag{4.12}$$

From the spherical harmonic function addition theorem, we have:

$$P_n(\psi_k) = P_n(\boldsymbol{e}, \boldsymbol{e}_k) = \frac{4\pi}{2n+1} \sum_{m=-n}^{n} \bar{Y}_{nm}(\boldsymbol{e}) \bar{Y}_{nm}(\boldsymbol{e}_k) \tag{4.13}$$

Substituting the formula (4.13) into (4.12), and then substituting the formula (4.12) into (4.11), the following integral relationship between the global instantaneous tidal height H and their direct influence to geopotential coefficient variations $(\Delta\bar{C}_{nm}, \Delta\bar{S}_{nm})$ can be obtained:

$$\begin{bmatrix} \Delta\bar{C}_{nm} \\ \Delta\bar{S}_{nm} \end{bmatrix} = \frac{G\rho_w}{g_0(2n+1)} \int_0^{2\pi} \int_0^{\pi} H \bar{P}_{nm}(cos\theta) \begin{bmatrix} cosm\lambda \\ sinm\lambda \end{bmatrix} sin\theta d\theta d\lambda \tag{4.14}$$

Here, $g_0 \approx GM/R^2$ is the mean gravity on the global sea surface.

The harmonic constants (amplitude H_i and phase g_i at Greenwich) of the tidal constituent σ_i are replaced by the spherical harmonic functions of the in-phase amplitude $H_i cosg_i$ and out-of-phase amplitude $H_i sing_i$, and substituted into formula (4.1), so that the instantaneous tidal height $H(\theta, \lambda, t)$ is expanded as follows:

$$H(\theta, \lambda, t) = \sum_{\sigma_i} \sum_{n=1}^{N} \sum_{m=0}^{n} \bar{P}_{nm}(sin\theta) \sum_{+}^{-} H_{i,nm}^{\pm}(\lambda, t) \tag{4.15}$$

$$H_{i,nm}^{\pm}(\lambda, t) = \bar{C}_{i,nm}^{\pm} cos(g_i + \varepsilon_i \pm m\lambda) + \bar{S}_{i,nm}^{\pm} sin(g_i + \varepsilon_i \pm m\lambda) \tag{4.16}$$

Where, $\left(\bar{C}_{i,nm}^{\pm}, \bar{S}_{i,nm}^{\pm}\right)$ are called as the prograde and retrograde normalized spherical harmonic coefficients of the tidal constituent σ_i with degree-n and order-m, which can be expressed in terms of the harmonic amplitude $\hat{C}_{i,nm}^{\pm}$ and phase bais $\varepsilon_{i,nm}^{\pm}$ as:

$$\bar{C}_{i,nm}^{\pm} = \hat{C}_{i,nm}^{\pm} sin\varepsilon_{i,nm}^{\pm}, \quad \bar{S}_{i,nm}^{\pm} = \hat{C}_{i,nm}^{\pm} cos\varepsilon_{i,nm}^{\pm} \tag{4.17}$$

Substituting the formula (4.15) into (4.14), considering the formulas (4.18) and (4.19), the geopotential coefficient variations can be expressed as:

$$\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm} = \sum_{\sigma_i}\left(C_{i,nm}^{\pm}, \mp iS_{i,nm}^{\pm}\right)e^{\pm i\phi_i} \tag{4.18}$$

Comparing the formulas (4.28) and (4.16), we have:

$$C_{i,nm}^{\pm} = \frac{4\pi G\rho_w}{g_0(2n+1)}\hat{C}_{i,nm}^{\pm}sin\left(\varepsilon_{i,nm}^{\pm} + \varepsilon_i\right) \tag{4.19}$$

$$S_{i,nm}^{\pm} = \frac{4\pi G\rho_w}{g_0(2n+1)}\hat{C}_{i,nm}^{\pm}cos\left(\varepsilon_{i,nm}^{\pm} + \varepsilon_i\right) \tag{4.20}$$

Using the formulas (4.19) and (4.20), the harmonic constant grid model represented by the tidal amplitude and phase at Greenwich of the tidal constituent can be transformed into the normalized spherical harmonic coefficients of the constituent, and the direct influence of ocean tidal load to geopotential coefficients $(\Delta\bar{C}_{nm}, \Delta\bar{S}_{nm})$ can be calculated according to the formula (4.18). Furthermore, the formulas (2.6) ~ (2.20) in which the surface load spherical harmonic coefficient variations $\{\Delta\bar{C}_{nm}^{w}, \Delta\bar{S}_{nm}^{w}\}$ replaced by the direct influence of ocean tidal load to the geopotential coefficients $(\Delta\bar{C}_{nm}, \Delta\bar{S}_{nm})$ are the algorithm formulas of the ocean tidal load effects on all-element geodetic variations.

8.4.2 Calculation and analysis of load effects of ocean and atmosphere tide

(1) Construction of ocean and surface atmosphere tidal load spherical harmonic coefficient model

Taking the global ocean tide model FES2014b-extrapolated (the harmonic constant models with 34 ocean tidal constituents) from Centre national d 'études spatiales (CNES) of France as an example, here introduces the harmonic analysis process of global ocean tidal harmonic constant models and construction of ocean tidal load spherical harmonic coefficient model.

FES2014 is a global ocean tide assimilation model based on fluid dynamics launched by CNES in 2016. FES2014 assimilates a variety of satellite altimetry and global gauge measured data since 1990. Among them, FES2014b-extrapolated optimizes the coverage of sea surface altimetric data in near-shore shallow waters through satellite altimetry waveform resampling. FES2014b consist of 34 tidal constituent harmonic constant models ($2N_2$, Eps_2, J_1, K_1, K_2, L_2, La_2, M_2, M_3, M_4, M_6, M_8, M_f, MKS_2, M_m, MN_4, MS_4, MSf, $MSqm$, Mtm, Mu_2, N_2, N_4, Nu_2, O_1, P_1, Q_1, R_2, T_2, S_1, S_2, S_4, Sa and Ssa) and the spatial resolution is 3.75′×3.75′.

Step 1: Gridate the 34 tidal harmonic constants of the FES2014b tidal harmonic constant models to generate 34 tidal constituents of 1°×1°, 30′×30′, 15′×15′ and 10′×10′ harmonic constant spherical coordinate grid models, respectively. Four spatial resolutions will be employed to analyze the maximum appropriate degree of the ocean tidal load spherical harmonic coefficient model.

Step 2: For any tidal harmonic constant spherical coordinate grid model, using the formulas (4.4) ~ (4.6), make spherical harmonic analysis on the global grid of in-phase amplitude and out-of-phase amplitude according to the FFT algorithm, and generate the 34 normalized spherical harmonic coefficient model of in-phase amplitude and out-of-phase amplitude of tidal constituent σ_i. Similar to the spherical harmonic analysis of surface load in Section 3.2, the cumulative iterative spherical harmonic analysis method can effectively improve the approach level.

Step 3: Using the formulas (4.20) ~ (4.22), 34 normalized spherical harmonic coefficient models $(\bar{C}_i^+, \bar{S}_i^+, \bar{C}_i^-, \bar{S}_i^-)$ of in-phase amplitude and out-of-phase amplitude of tidal constituent σ_i are combined to obtain a normalized spherical harmonic coefficient model of global ocean tide load according to the conventional format. The ocean tidal load spherical harmonic coefficient model FES2004 format in IERS conventions (2010) is adopted here.

Step 4: On the basis of the above 34 tidal load spherical harmonic coefficient models, the equilibrium tide Ω_1 and Ω_2 load spherical harmonic coefficients are selected from the FES2004S1.dat in IERS conventions (2010), and the FES2014b ocean tidal load spherical harmonic coefficient model composed of 36 tidal constituents is constructed.

The maximum degree of the load spherical harmonic model of some a tidal constituent is equal to the number of cell-grids in the latitude direction in the harmonic constant grid of the tidal constituent. The grid resolution of different constituents or the maximum degree of their spherical harmonic coefficient models need not be consistent. The maximum degree of the ocean tidal load spherical harmonic coefficient model is the maximum degree of all the tidal constituents. The unit for ocean tidal load spherical harmonic coefficient is consistent with that for ocean tidal height. The unit in this example is cm.

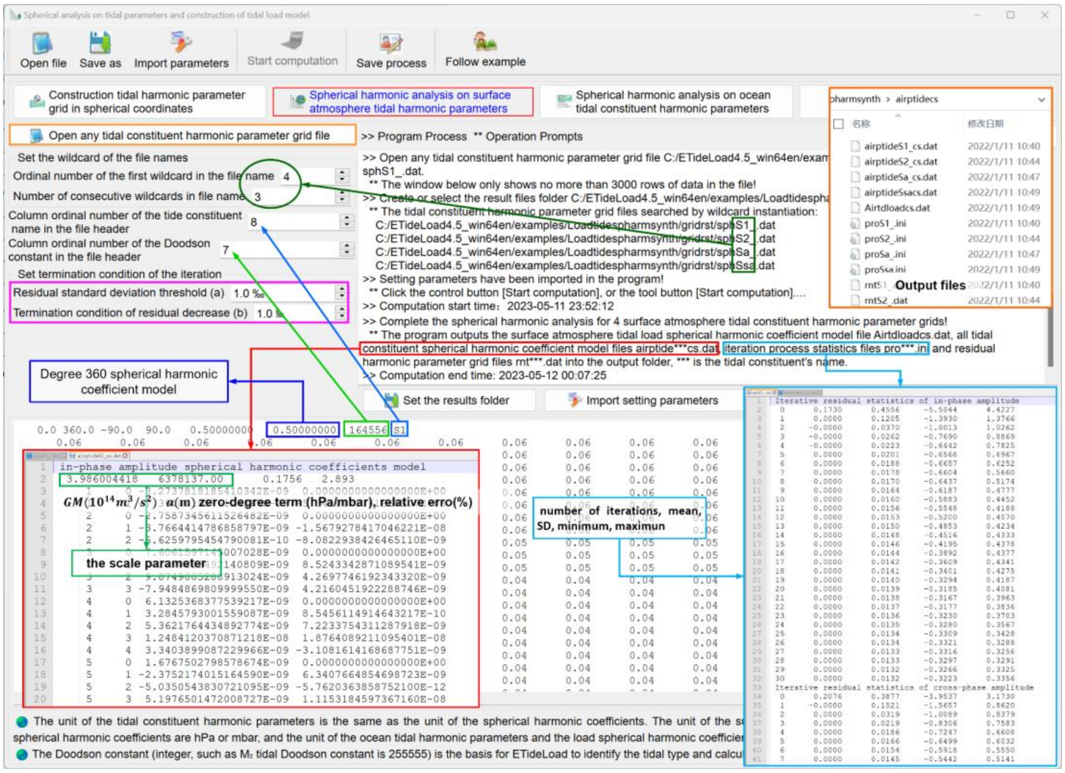


Fig 4.1 ETideLoad4.5 program for Spherical harmonic analysis on ocean tidal constituent harmonic constants

Fig.4.1 is the ETideLoad4.5 program for spherical harmonic analysis on ocean tidal constituent harmonic constant grid. The program reads 34 tidal 30'×30' harmonic constant spherical coordinate grids, and employs the cumulative approach method to perform spherical harmonic analysis on each tidal harmonic constant. Where, the M_2 tidal spherical harmonic coefficient model is shown in the lower left figure, and the iterative residual vaiations are shown in the lower right figure. The 360-degree FES2014b360cs global ocean tidal load spherical harmonic coefficient model is constructed, as shown in Fig 4.2.

The load spherical harmonic coefficient model for each tidal constituent is composed of the same in-phase amplitude spherical harmonic coefficient model and the out-of-phase amplitude spherical harmonic coefficient model. The file header are the geocentric gravitational constant GM ($\times 10^{14}$m^3/s^2), equatorial radius a (m) of the Earth, zero-degree term $a\Delta C_{00}$ (cm), relative error Θ (%).

```
Ocean tidal height load normalized spherical harmonic coefficient model in cm.
Created by ETideLoad, ZHANG Chuanyin, Chinese Academy of Surveying and Mapping.
 Doodson name    n    m        Csin+       Ccos+       Csin-       Ccos-        C+      eps+        C-      eps-
 247.455  2N2    1    0   0.00458562  0.00231038  0.00458562  0.00231038  0.005135   63.2596  0.005135   63.2596
 247.455  2N2    1    1  -0.00773380  0.00473565  0.01063946 -0.00152991  0.009069  301.4805  0.010749   98.1828
 247.455  2N2    2    0   0.01415077 -0.00470716  0.01415077 -0.00470716  0.014913  108.3994  0.014913  108.3994
 247.455  2N2    2    1  -0.01749377  0.01964053 -0.02057617  0.01244109  0.026302  318.3086  0.024045  301.1587
 247.455  2N2    2    2  -0.05076973  0.15409810  0.03408330 -0.00708020  0.162246  341.7648  0.034811  101.7353
 247.455  2N2    3    0  -0.00345932 -0.05402235 -0.00345932 -0.05402235  0.054133  183.6639  0.054133  183.6639
 247.455  2N2    3    1   0.00459468  0.02860553  0.08674509  0.04125120  0.028972    9.1250  0.096054   64.5668
 247.455  2N2    3    2  -0.01359111 -0.04803085  0.00043095  0.01917460  0.049917  195.7997  0.019179    1.2875
 247.455  2N2    3    3   0.11576000  0.04745531  0.10043379 -0.03897379  0.125109   67.7090  0.107731  111.2090
 247.455  2N2    4    0  -0.04607076  0.02579335 -0.04607076  0.02579335  0.052800  299.2429  0.052800  299.2429
 247.455  2N2    4    1   0.03322584  0.01467790  0.01394749  0.02945707  0.036324   66.1660  0.032592   25.3369
 247.455  2N2    4    2   0.06616682 -0.16308472  0.08023800  0.03608357  0.175996  157.9166  0.087978   65.7862
 247.455  2N2    4    3  -0.04323293 -0.08712246 -0.08031745  0.08908738  0.097259  206.3921  0.119948  317.9635
 247.455  2N2    4    4  -0.07108370  0.11911427 -0.03283587  0.04029420  0.138712  329.1726  0.051979  320.8233
 247.455  2N2    5    0   0.00423674  0.05025371  0.00423674  0.05025371  0.050432    4.8190  0.050432    4.8190
 247.455  2N2    5    1  -0.06599377  0.02863740 -0.06611923 -0.08775797  0.071939  293.4580  0.109878  216.9954
 247.455  2N2    5    2   0.03191636  0.09160043 -0.12292118  0.09809027  0.097002   19.2099  0.157262  308.5896
```

Fig 4.2 Ocean tidal load spherical harmonic coefficient model FES2014b360cs.dat

For high-precision geodesy, the short-wave component of the ocean tidal load effects cannot be ignored, and a high-degree spherical harmonic coefficient model is usually required. Tab 4.2 show the residual change of ocean tidal load spherical harmonic analysis with the resolution of tidal harmonic constant grid or the maximum degree of load spherical harmonic coefficient model.

Tab 4.2 Residual change of ocean tidal load spherical harmonic analysis with the resolution of tidal harmonic constant grid

<table>
<tr><th rowspan="2">Input grid resolution</th><th rowspan="2">Maximum degree</th><th rowspan="2">Name</th><th rowspan="2">Tidal constituent</th><th colspan="3">First-degree term×10⁻⁸</th><th rowspan="2">Relative error (%)</th></tr>
<tr><th>$\Delta\bar{C}_{10}$</th><th>$\Delta\bar{C}_{11}$</th><th>$\Delta\bar{S}_{11}$</th></tr>
<tr><td rowspan="4">1°×1°</td><td rowspan="4">180</td><td rowspan="2">K_1</td><td>in-phase</td><td>6.5903</td><td>15.2405</td><td>5.7951</td><td>15.109</td></tr>
<tr><td>out-of-phase</td><td>-23.6187</td><td>5.4510</td><td>9.1115</td><td>13.080</td></tr>
<tr><td rowspan="2">M_2</td><td>in-phase</td><td>6.4087</td><td>8.2092</td><td>-3.9331</td><td>16.593</td></tr>
<tr><td>out-of-phase</td><td>3.3741</td><td>0.7698</td><td>7.4235</td><td>14.206</td></tr>
<tr><td>30′×30′</td><td>360</td><td></td><td>in-phase</td><td>6.7466</td><td>14.4650</td><td>5.6522</td><td>10.522</td></tr>
</table>

		K_1	out-of-phase	-23.9366	5.5500	9.2329	9.785
		M_2	in-phase	6.3545	7.5901	-4.2676	11.266
			out-of-phase	4.3474	-0.2498	5.9033	10.673
15′×15′	720	K_1	in-phase	6.7290	14.1161	5.5337	7.549
			out-of-phase	-23.9978	5.5530	9.3081	7.069
		M_2	in-phase	6.3464	7.5080	-4.5272	7.980
			out-of-phase	4.7902	-0.6035	5.1936	7.687
10′×10′	1080	K_1	in-phase	6.6860	14.0149	5.4796	6.161
			out-of-phase	-23.9629	5.5763	9.3395	5.922
		M_2	in-phase	6.2795	7.5429	-4.6921	6.867
			out-of-phase	4.9361	-0.7832	4.9103	6.435

Tab 4.2 shows that the short and medium wave components of ocean tides are obvious. Considering both accuracy and computational efficiency, the appropriate maximum degree of the ocean tidal load spherical harmonic coefficient model can be selected as 720.

Similarly, from the global surface atmosphere tidal harmonic constant grid models, can construct the surface atmosphere tidal load spherical harmonic coefficient model by the spherical harmonic analysis. The 360-degree surface atmosphere tidal load spherical harmonic coefficient model ECMWF2006cs360.dat in ETideLoad4.5 were constructed according to the process above from the 0.5˚× 0.5˚ global harmonic constant grids of the four atmospheric pressure tidal constituents (S_1, S_2, Ss_a, Sa). The unit for surface atmosphere tidal load spherical harmonic coefficient is consistent with that surface atmospherical pressure tide. The unit in this example is hPa.

Tab 4.3 show the residual change of surface atmosphere tidal load spherical harmonic analysis with the resolution of surface atmosphere tidal harmonic constant grid or the maximum degree of load spherical harmonic coefficient model.

Tab 4.3 Residual change of surface atmosphere tidal load spherical harmonic analysis with the resolution of atmosphere tidal harmonic constant grid

Input grid resolution	Maximum degree	Name	Tidal constituent	First-degree term×10⁻⁸			Relative error (%)
				$\Delta\bar{C}_{10}$	$\Delta\bar{C}_{11}$	$\Delta\bar{S}_{11}$	
1˚×1˚	180	S_1	in-phase	-0.3276	-0.7396	-5.3411	4.378
			out-of-phase	0.1765	-4.3745	-0.1072	4.335
		S_2	in-phase	-0.0630	0.0080	0.3390	1.238
			out-of-phase	0.1374	0.5236	-0.1086	1.365
			in-phase	0.6526	-3.5846	1.2772	3.841

		S_{Sa}	out-of-phase	6.4837	-2.5040	2.4911	1.158
		S_a	in-phase	8.2106	-3.5243	3.5038	1.488
			out-of-phase	-16.1599	-0.8292	-12.1651	2.554
30′×30′	360	S_1	in-phase	-0.3274	-0.7396	-5.3408	2.927
			out-of-phase	0.1765	-4.3747	-0.1074	2.617
		S_2	in-phase	-0.0630	0.0077	0.3391	0.848
			out-of-phase	0.1374	0.5237	-0.1087	0.903
		S_{Sa}	in-phase	0.6528	-3.5850	1.2760	1.871
			out-of-phase	6.4837	-2.5041	2.4916	0.646
		S_a	in-phase	8.2104	-3.5242	3.5038	0.839
			out-of-phase	-16.1596	-0.8291	-12.1654	1.276

Tab 4.3 shows that the medium and long waves of the global surface atmospheric pressure tide are dominant. Considering both accuracy and computational efficiency, the appropriate maximum degree of the surface atmospherr tidal load spherical harmonic coefficient model can be selected from 180 to 360.

(2) Spherical harmonic synthesis and analysis of ocean tidal load effects

The ocean tidal loads are located on the sea surface, and the height of the calculation point relative to the sea surface is orthometric (or normal) height h.

In the following, three calculation points are selected: P_1(105°E, 32°N, h720m) in the inland area above 400km from the coastline, P_2(121.3°E, 28.8°N, h11m) in the coastal zone and P_3(123.47°E, 25.75°N, h3m) on the sea island 200km away from the coastline. The 720-degree global ocean tidal load spherical harmonic coefficient model FES2014b720cs.dat is employed to calculate the ocean tidal load effect time series on all-element variations at these three ground points. The time span of the time series is from January 1, 2020 to January 31, 2020, with a time interval of 30 minutes. Comparing the similarities and differences of the ocean tidal load effect time series at three points in different regions, the spatial and time-varying characteristics of the ocean tidal load effects are analyzed.

Fig 4.3 is the ocean tidal load effect time series on geodetic variations at P_1 point in the inland area. It is shown in the figure that even in the inland areas more than 400km away from the coastline, the difference between the maximum and minimum values of the ocean tidal load effects on geoid can reach 8mm, that on ellipsoidal height can reach 15mm, that on normal height can reach 22mm, that on horizontal displacement can reach 8mm, that on ground tilt can reach 4.5mas, that on radial gravity gradient can reach 2.6mE and that on horizontal gravity gradient can reach 1.9mE. It can be seen that even in inland areas, centimeter-level precision geodesy should also take into account the ocean tidal load effects.

Fig 4.4 is the ocean tidal load effect time series on geodetic variations at P_2 point on the coast. It is shown in the figure that the ocean tidal load effect in the coastal zone is generally 10 times that of the inland P_1 point. At P_2 point, the difference between the maximum and minimum values of the ocean tidal load effects on geoid can reach 6.8cm, that on normal height can reach 20cm, that on ground gravity can reach 250μGal, that on ground tilt can reach 110mas, that on horizontal displacement can reach 3.1cm, that on horizontal gravity gradient can reach 42mE and that on radial gravity gradient can reach 30.5mE.

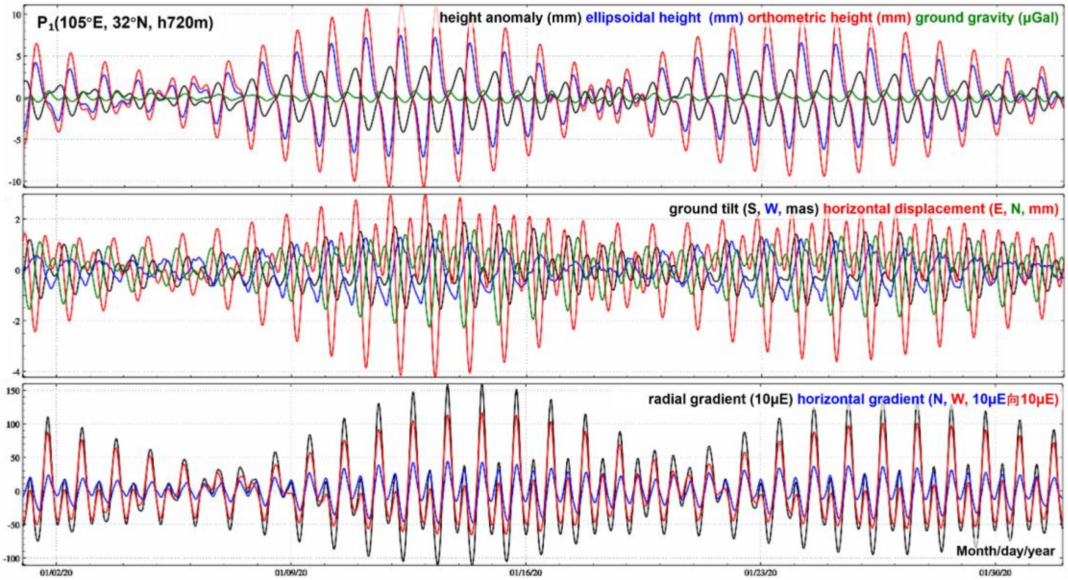


Fig 4.3 The ocean tidal load effect time series on geodetic variations at P_1 point in the inland area

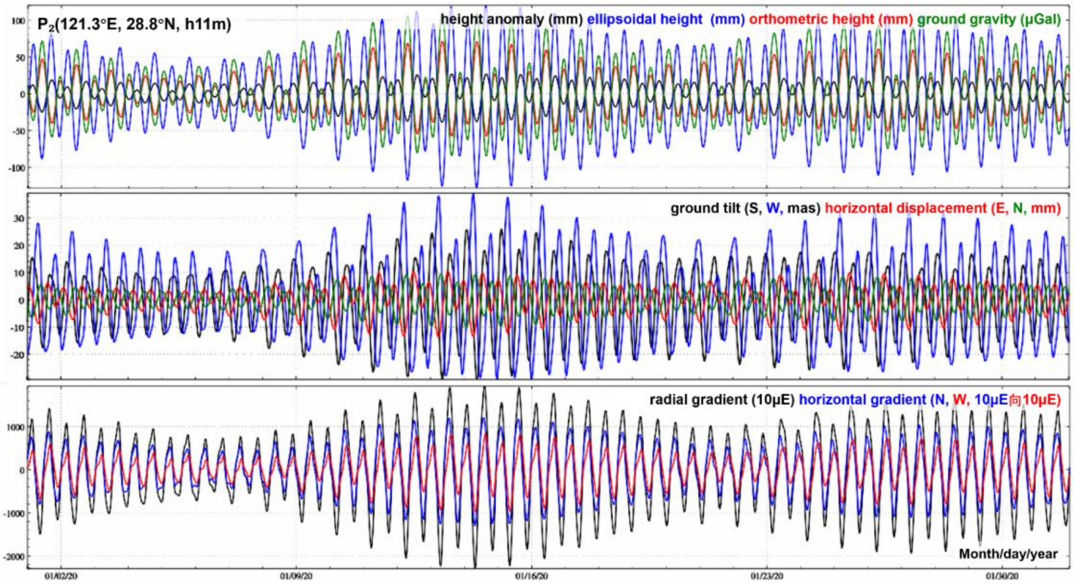


Fig 4.4 The ocean tidal load effect time series on geodetic variations at P_2 point on the coastal zone

Fig 4.5 is the ocean tidal load effect time series on geodetic variations at P_3 point on 200 km offshore island. It is shown in the figure that after a certain distance from the shore, the amplitude of the ocean tide becomes lower, the tidal wave structure is simpler than that of the near shore, and the medium and short wave parts of the ocean tidal load effect are weakened. At P_3 point, the difference between the maximum and minimum values of the ocean tidal load effects on geoid can reach 6.6cm, that on normal height can reach 16cm, that on ground gravity can reach 70μGal, that on ground tilt can reach 76mas, that on horizontal displacement can reach 2.8cm, that on horizontal gravity gradient can reach 2.3mE and that on radial gravity gradient can reach 3.5mE.

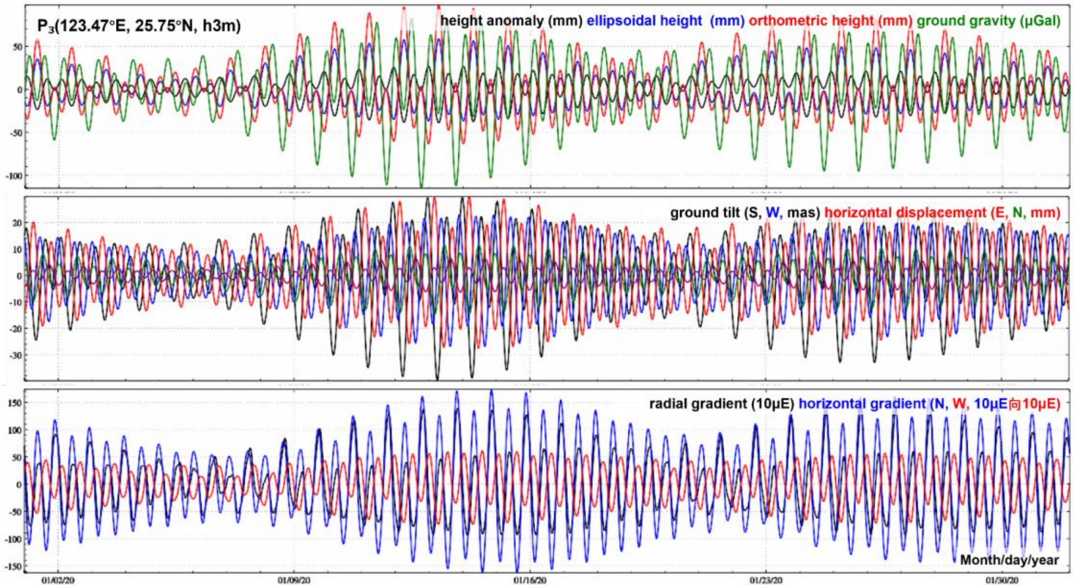


Fig 4.5 The ocean tidal load effect time series on geodetic variations at P_3 point on offshore island

Different from the solid Earth tidal effects, the ocean tidal load effect on ground normal height is in phase with that on ground ellipsoidal height (the two sign are consistent). In most areas, the amplitude of ocean tidal load effect on ground normal height is about 1.5 times that on ground ellipsoidal height. In coastal waters, the ocean tidal load effects on gravity gradient and ground tilt are generally much greater than the solid Earth tidal effects on that.

(2) Spherical harmonic synthesis and analysis of surface atmosphere tidal load effects

In the following, using the same calculation scheme as the ocean tidal load effects, the 360-degree global surface atmosphere tidal load spherical harmonic coefficient model ECMWF2006n360cs.dat is employed to calculate the atmosphere tidal load effect time series on all-element variations at the ground point P(105°N, 20°E). The time span of the time series is from January 1,2018 to December 31,2020 (3 years), with a time interval of 30 minutes, as shown in Fig 4.6.

It is slightly different from the spherical harmonic synthesis calculation of ocean tidal load effect. When calculating the indirect influence of atmosphere loads, it is assumed that the atmosphere loads are concentrated on the ground, and the calculation point height h is the height of the point relative to the ground. When calculating the direct influence of atmosphere loads to gravity and radial gravity gradient, it is assumed that there is the proportional relationship $(1-h/44330)^{5225}$ between atmospheric pressure P_h at ground height h and ground atmospheric pressure P_0.

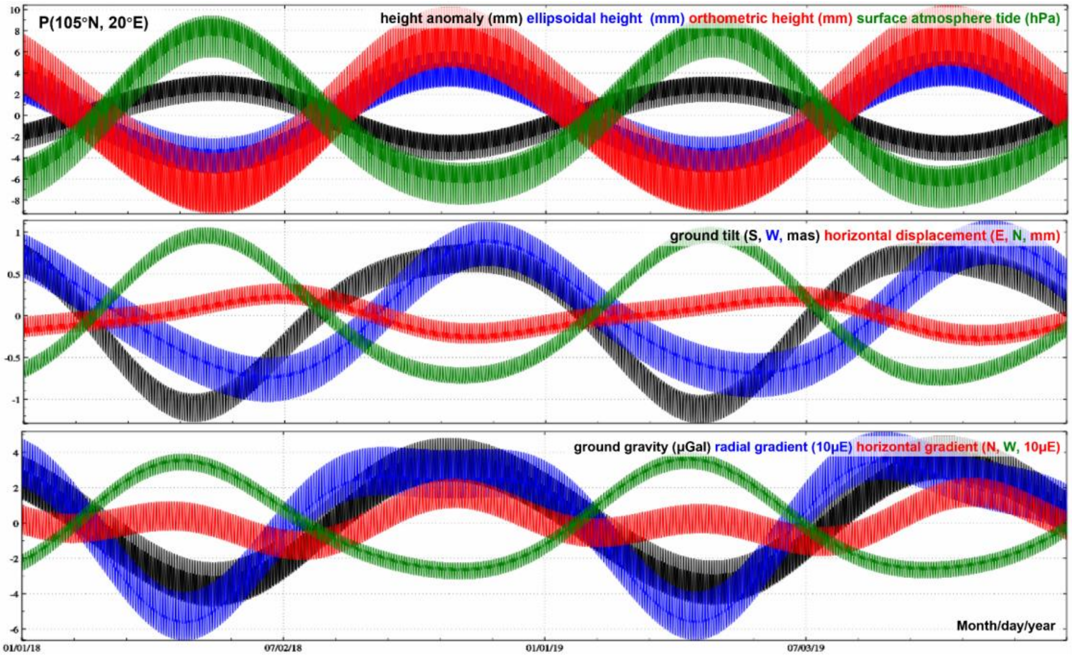


Fig 4.6 The surface atmosphere tidal load effect time series on geodetic variations

Fig 4.6 shows that the difference between the maximum and minimum of the atmosphere tidal load effect on ground normal height can reach 2cm. For every 1hPa increases in the ground atmospheric pressure, the ground normal height is reduced by about 1mm, that is, the atmosphere tidal load effect admittance of the ground normal height is close to −1.0mm/hPa. The annual periodic amplitude of surface atmospheric pressure tide is 3 ~ 5 times of the diurnal amplitude. In the inland area, the surace atmospheric pressure is high in winter and low in summer, which leads to the decrease of the ground in winter and the uplift in summer, resulting in the ground vertical deformation of the annual and semi-annual period, which should be taken into account in the centimeter-level geodesy.

8.4.3 Regional refinement of Green's integral method for ocean tidal load effects

Taking the global ocean tidal load spherical harmonic coefficient model as the reference field and using the remove-restore scheme, the accuracy of the ocean tidal load effect can be further improved by the regional high-precision and high-resolution harmonic constant grid models. The scheme can be generally composed of the following five steps, which can be called as the ‘Remove - load Green’s function integral - Restore’ scheme.

(a) The global ocean tidal load spherical harmonic coefficient model employed as the reference field and the regional high-resolution ocean tide harmonic constant grid models are selected, and the regional ocean tide harmonic constant reference value grids are calculated from the ocean tidal load spherical harmonic coefficient model.

(b) From the regional high-resolution ocean tide harmonic constant grid model, the reference model value are removed to obtain the regional ocean tide harmonic constant residual grids. This step is called 'Remove'.

(c) Using a smaller integral radius, the residual value of the ocean tidal load effect at the target point is calculated using the load Green's function integral.

(d) The reference model value of ocean tidal load effects at the target point is calculated from the global ocean tidal load spherical harmonic coefficient model.

(e) The refine value of the ocean tidal load effects at the target point is obtained by adding the residual value of the ocean tide load effects to the reference model value. This step is called 'Restore'.

In the following, we still select two calculation points of P_2 (121.3°E, 28.8°N, h11m) located in the coastal zone and P_3 (123.47°E, 25.75°N, h3m) on the sea island 200 km away from the coastline, and the 720-degree global ocean tide load spherical harmonic coefficient model FES2014b720cs.dat is employed as the ocean tidal load reference field. From the 1.2′×1.2′ high-precision ocean tide model TMchinaR1 (J. Xu, J. Y. Bao, 2008) composed of 10 tidal harmonic constant models, the residual value and refine value time series of the ocean tidal load effect on all-element geodetic variations at these two points are calculated according to the ‘Remove - load Green's function integral – Restore’ scheme. The time span of the time series is from 0 : 00 on January 1, 2020 to 24 : 00 on January 31, 2020, with a time interval of 30 minutes.

Firstly, the 1.2′×1.2′ ocean tide harmonic constant model value grids in China offshore are calculated from the 720-degree ocean tidal load spherical harmonic coefficient model FES2014b720cs.dat. The number of tidal constituents and the grid specification are the same as those of the high-precision ocean tide model TMchinaR1 in China offshore. Then, the 10 tidal harmonic constant grids of TMchinaR1 in China offshore are removed from the corresponding tidal harmonic constant model value grids to generate the harmonic constant residual value grids of 10 tidal constituents in China offshore.

Secondly, using the load Green’s function integral with the smaller integral radius (300km integral radius in this examples), the residual value time series of ocean tidal load effect on geodetic variations at P_2 and P_3 points are calculated from the 1.2′×1.2′ harmonic constant residual value grids of the 10 constituents in the coastal waters of China, as shown in Fig 4.7 and Fig 4.8.

Finally, the ocean tidal load effect model value time series at P_2 and P_3 are calculated from the global ocean tidal load spherical harmonic coefficient model, and added with the ocean tide load effect residual value time series calculated above to obtain the refine value time series of ocean tidal load effect on all-element geodetic variations at P_2 and P_3, respectively. Figure 4.9 is the refine value time series of ocean tidal load effects on geodetic variations at the P_2 on the coast.

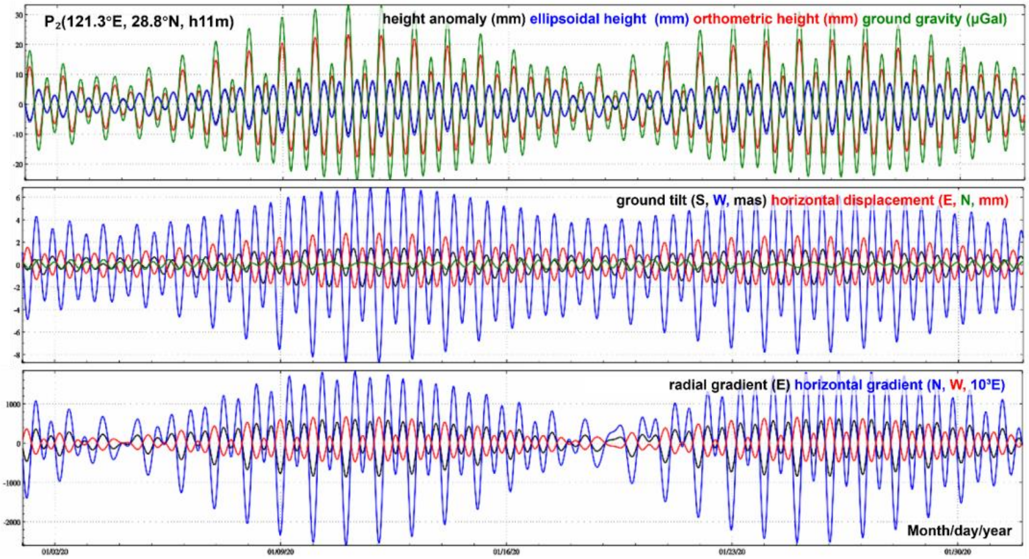


Figure 4.7 The residual value time series of ocean tidal load effects on geodetic variations at the P_2 in the coastal

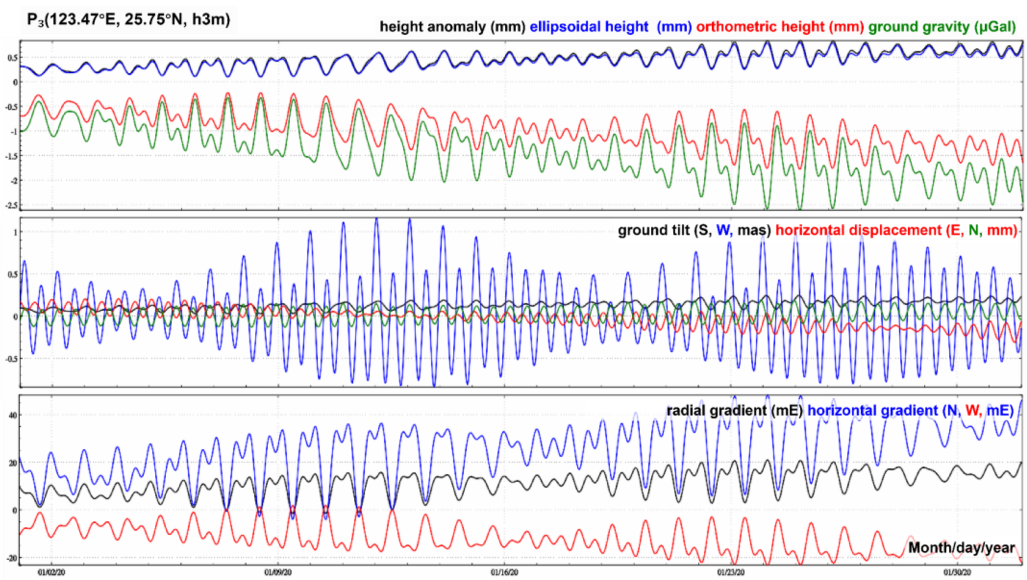


Figure 4.8 The residual value time series of ocean tidal load effects on geodetic variations at the P_3 on the sea island

It is not difficult to understand that the residual value time series of ocean tidal load effects on geodetic varitions also represents the error influence of the 720-degree FES2014b ocean tidal load spherical harmonic coefficient model to geodetic variations. Fig 4.7 and Fig 4.8 show that even for the high-degree ocean tidal load spherical harmonic coefficient model with good quality of 720-degree FES2014, the model error influence to the normal height in coastal areas can be as high as 5.9cm, and that to the geoid and ellipsoidal height can reach 1.7cm and 4.1cm, respectively, and that to the ground tilt and horizontal displacement can reach 23.6mas and 1.6cm, respectively.

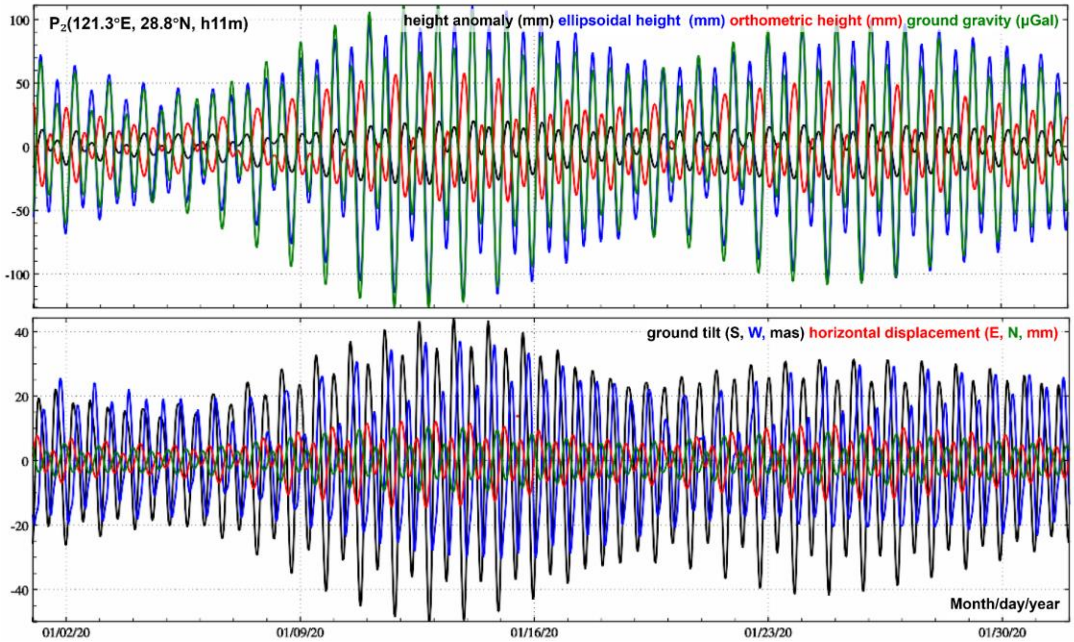


Figure 4.9 The refine value time series of ocean tidal load effects on geodetic variations at the P_2 in the coastal zone

In coastal areas, the error of ocean tidal spherical harmonic coefficient model has a great influence to gravity gradient, which is far beyond the magnitude of the ocean tidal load effect itself. The ocean tidal load effects on gravity gradient are dominant in the ultrashort wave parts, and the high-degree ocean tidal load spherical harmonic coefficient model FES2014b720cs cannot contain these ultrashort wave signals in coastal areas. The calculation results of the residual load effects on gravity gradient are divergent and not available using load Green's function integral.

8.5 The effects of the Earth's mass centric variations and figure polar shifts

The Earth's mass centric coordinates and figure polar coordinates are both important geodetic elements with both geometric and physical properties. In the Earth-fixed coordinate system with arbitrary positioning and orientation, the mass centric coordinates of the deforming Earth can be uniquely determined by the degree-1 geopotential coefficients

$(\bar{C}_{10}, \bar{C}_{11}, \bar{S}_{11})$, and the mechanical figure polar coordinates of the deforming Earth can be uniquely determined by the degree-2 tesseral harmonic geopotential coefficients $(\bar{C}_{21}, \bar{S}_{21})$. Therefore, the various tidal and non-tidal effects on Earth's center of mass and figure pole can be accurately obtained in geodesy.

8.5.1 The tidal effects on the Earth's center of mass and figure pole

The tidal effects on the Earth's center of mass and figure pole are determined by the tidal effects on the degree-1 and degree-2 order 1 geopotential coefficients, respectively. The figure polar tidal effect are the sum of the body and load tidal effect of all the degree-2 diurnal tidal wave.

(1) Tidal effect prediction calculation on the Earth's mass centric variations

The solid Earth tide is derived on basis of the mechanical balance theory of the celestial gravitation and centrifugal force. The Earth's tidal force from the celestial body at the Earth's center of mass is always equal to zero, so geodesy does not specifically study the solid tidal effect on the Earth's center of mass. Ocean tides and surface atmosphere tides lead to the redistribution of surface mass, causing periodic variations of Earth's center of mass.

Section 8.4 has introduced the spherical harmonic synthesis algorithm for ocean and surface atmosphere tidal load effects. From the degree-1 spherical harmonic coefficient of each tidal constituent σ_j in the load tidal spherical harmonic coefficient model, including the in-phase and out-of-phase amplitudes of the degree-1 term, the variations of Earth’s center of mass caused by the tidal constituent σ_j at any epoch time can be calculated. After that, the contributions of all the tidal constituents are superimposed, which is the tidal load effects on the Earth's mass centric variations at the epoch time.

Assuming that the in-phase and out-of-phase amplitudes of the degree-1 tidal load spherical harmonic coefficients of the tidal constituent σ_j are $\left(\Delta\bar{C}_{10}^{j+}, \Delta\bar{C}_{11}^{j+}, \Delta\bar{S}_{11}^{j+}\right)$ and $\left(\Delta\bar{C}_{10}^{j-}, \Delta\bar{C}_{11}^{j-}, \Delta\bar{S}_{11}^{j-}\right)$, respectively, and considering that the degree-1 load potential Love number is equal to zero, $k_1' \equiv 0$, then at any epoch time t, the tidal load effects on the Earth's mass centric variations can be expressed as:

$$\begin{cases}\Delta x_{cm}(t) = \sqrt{3}R\frac{\rho_w}{\rho_e}\sum_{j=1}^{n}\left[\Delta\bar{C}_{11}^{j+}cos\left(\phi_j(t)+\varepsilon_j\right) + \Delta\bar{C}_{11}^{j-}sin\left(\phi_j(t)+\varepsilon_j\right)\right] \\ \Delta y_{cm}(t) = \sqrt{3}R\frac{\rho_w}{\rho_e}\sum_{j=1}^{n}\left[\Delta\bar{S}_{11}^{j+}cos\left(\phi_j(t)+\varepsilon_j\right) + \Delta\bar{S}_{11}^{j-}sin\left(\phi_j(t)\right)+\varepsilon_j\right] \\ \Delta z_{cm}(t) = \sqrt{3}R\frac{\rho_w}{\rho_e}\sum_{j=1}^{n}\left[\Delta\bar{C}_{10}^{j+}cos\left(\phi_j(t)+\varepsilon_j\right) + \Delta\bar{C}_{10}^{j-}sin\left(\phi_j(t)+\varepsilon_j\right)\right]\end{cases} \quad (5.1)$$

Where, $\phi_j(t)$ is the astronomical argument of the tidal constituent σ_j at the epoch time t, ε_j is the phase bias of σ_j, n is the number of tidal constituents in the tidal load spherical harmonic coefficient model, e.g. the 720-degree ocean tidal load spherical harmonic coefficient model FES2014b720cs.dat constructed in Section 8.2.3 has 34 tidal constituents including degree-1 terms $(n = 34)$, and the 360-degree surface atmosphere tidal load spherical harmonic coefficient model ECMWF2006n360cs.dat constructed in Section 8.2.4 has 4 tidal constituents including degree-1 terms $(n = 4)$.

Here, from the in-phase and out-of-phase amplitudes of the degree-1 terms of 34 tidal constituents in the model FES2014b720cs.dat in section 8.2.3, the ocean tidal load effect time series on the Earth's mass centric variations are calculated as shown in Fig 5.1. The time span of the time series is from July 1, 2016 to July 15, 2016, with a time interval of 30 minutes.

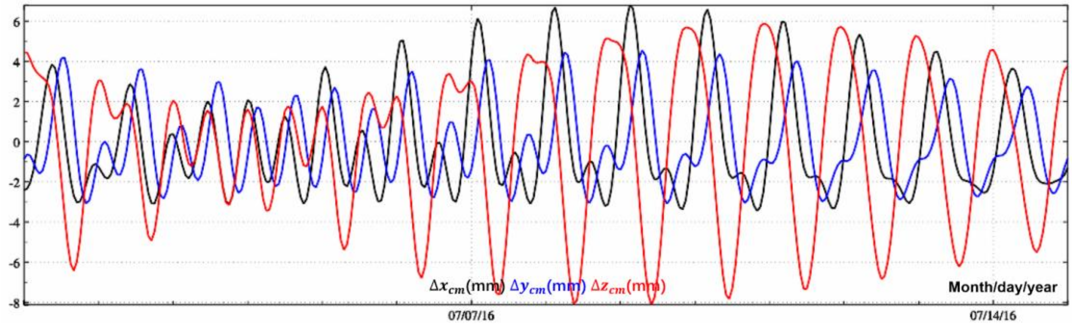


Fig 5.1 The ocean tidal load effect time series on the Earth's mass centric variations

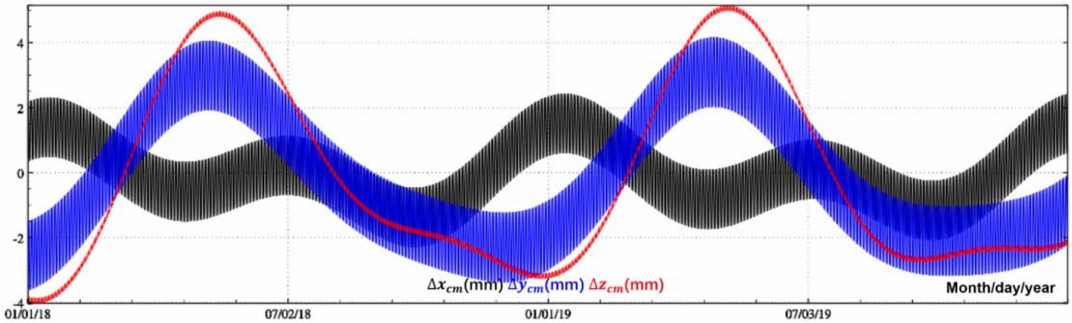


Fig 5.2 The surface atmosphere tidal load effect time series on the Earth's mass centric variations

Here, from the in-phase and out-of-phase amplitudes of the degree-1 terms of 4 tidal constituents in the model ECMWF2006n360cs.dat in section 8.2.4, the surface atmosphere tidal load effect time series on the Earth's mass centric variations are calculated as shown in Fig 5.2. The time span of the time series is from January 1, 2018 to December 31, 2019 (2 years), with a time interval of 4 hours.

For ocean or surface atmosphere tidal load models constructed from different sources of observations, the load tidal effects on the Earth's mass centric variations may have some small differences.

(2) Tidal effect prediction calculation on the Earth's figure polar shift

The rigorous algorithm formulas for determining the figure polar shift $(\Delta x_{sfp}, -\Delta y_{sfp})$ from the degree-2 tesseral harmonic geopotential coefficient variations $(\Delta\bar{C}_{21}, \Delta\bar{S}_{21})$ with the geopotential coefficients $(\bar{C}_{20}, \bar{S}_{22})$ are as follows:

$$\Delta x_{sfp} = -\frac{\sqrt{3}b}{\bar{C}_{20}}\Delta\bar{C}_{21} - \frac{6\bar{S}_{22}b}{(\bar{C}_{20})^2}\Delta\bar{S}_{21}, \quad \Delta y_{sfp} = +\frac{\sqrt{3}b}{\bar{C}_{20}}\Delta\bar{S}_{21} - \frac{6\bar{S}_{22}b}{(\bar{C}_{20})^2}\Delta\bar{C}_{21} \qquad (5.2)$$

Where, b is the short semi-axis of the Earth, $\Delta\bar{C}_{21}, \Delta\bar{S}_{21}$ take the approximate mean value.

The solid Earth tidal effects on the degree-2 tesseral harmonic geopotential coefficients in Section 8.1, namely the degree-2 diurnal body tide cluster $(\Delta\bar{C}_{21},\Delta\bar{S}_{21})$, characterize the solid tidal effects on the Earth's figure pole. There are frequency dependent in the degree-2 diurnal poetential Love number of the rotating microellipsoidal Earth (48 degree-2 diurnal tidal constituents are corrected for frequency, as shown in Tab 1.7). The solid Earth tidal effects on Earth's figure polar shift can be calculated from solid tidal effect on the degree-2 tesseral harmonic geopotential coefficients of 48 degree-2 diurnal tidal constituents.

Similar to the load tidal effects on the Earth's mass centric variations, the algorithm formulas for predicting the load tidal effects on the Earth's figure polar shift can be derived from the degree-2 tesseral load tidal spherical harmonic coefficients:

$$\Delta x_{sfp}=-\frac{3\rho_w}{5\rho_e}\frac{b(1+k_2')}{\bar{C}_{20}}\begin{pmatrix}\sqrt{3}\sum_{j=1}^{n}\left[\Delta\bar{C}_{21}^{j+}cos\left(\phi_j(t)+\varepsilon_j\right)+\Delta\bar{C}_{21}^{j-}sin\left(\phi_j(t)+\varepsilon_j\right)\right]\\+\frac{6\bar{S}_{22}}{\bar{C}_{20}}\sum_{j=1}^{n}\left[\Delta\bar{S}_{21}^{j+}cos\left(\phi_j(t)+\varepsilon_j\right)+\Delta\bar{S}_{21}^{j-}sin\left(\phi_j(t)+\varepsilon_j\right)\right]\end{pmatrix} \quad (5.3)$$

$$\Delta y_{sfp}=\frac{3\rho_w}{5\rho_e}\frac{b(1+k_2')}{\bar{C}_{20}}\begin{pmatrix}\sqrt{3}\sum_{j=1}^{n}\left[\Delta\bar{S}_{21}^{j+}cos\left(\phi_j(t)+\varepsilon_j\right)+\Delta\bar{S}_{21}^{j-}sin\left(\phi_j(t)+\varepsilon_j\right)\right]\\-\frac{6\bar{S}_{22}}{\bar{C}_{20}}\sum_{j=1}^{n}\left[\Delta\bar{C}_{21}^{j+}cos\left(\phi_j(t)+\varepsilon_j\right)+\Delta\bar{C}_{21}^{j-}sin\left(\phi_j(t)+\varepsilon_j\right)\right]\end{pmatrix} \quad (5.4)$$

Where, $\bar{C}_{21}^{j+},\Delta\bar{C}_{21}^{j-},\Delta\bar{S}_{21}^{j+},\Delta\bar{S}_{21}^{j-}$ are the in-phase and out-of-phase amplitudes of the degree-1 order-1 tidal load spherical harmonic coefficients of the tidal constituent σ_j, respectively, n is the number of tidal constituents in the tidal load spherical harmonic coefficient model, for the model FES2014b720cs.dat, $n=34$.

Using the same numerical standard and ocean tidal load effect algorithm on geopotential coefficients in section 8.2.3, the ocean tidal load effect (in unit of m) time series on the figure polar shift from January 1, 2019 to February 28, 2019 is calculated by the ocean tidal load model FES2014b720cs.dat (34 tidal constituents), with a time interval of 30 minutes, as shown in Fig 5.3.

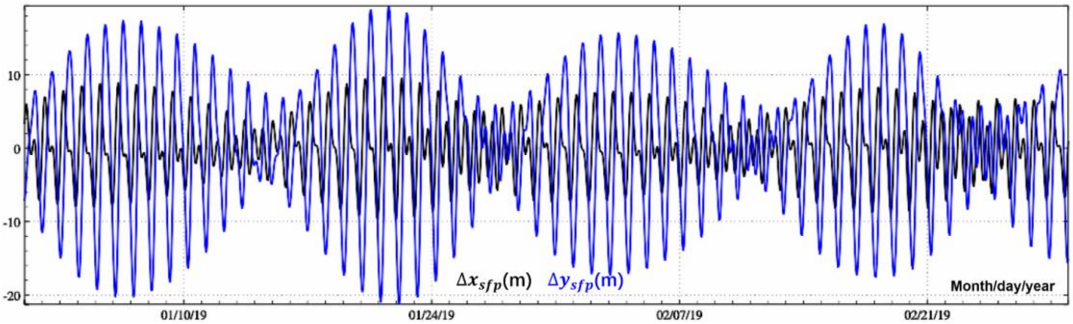


Fig 5.3 The ocean tidal load effect time series on Earth's figure polar shift in ITRS

Fig 5.3 shows that the ocean tidal load effects on the Earth's figure polar shift are dominated by diurnal variation, and the difference between the maximum and minimum values is more than 40m in one month.

Using the same numerical standard and surface atmosphere tidal load effect algorithm on geopotential coefficients in section 8.2.4, the surface atmosphere tidal load effect (in unit of m) time series on the figure polar shift from January 1, 2018 to January 31, 2019 is calculated by the surface atmosphere tidal load model ECMWF2006n360cs.dat (4 tidal constituents), with a time interval of 4 hours, as shown in Fig 5.4.

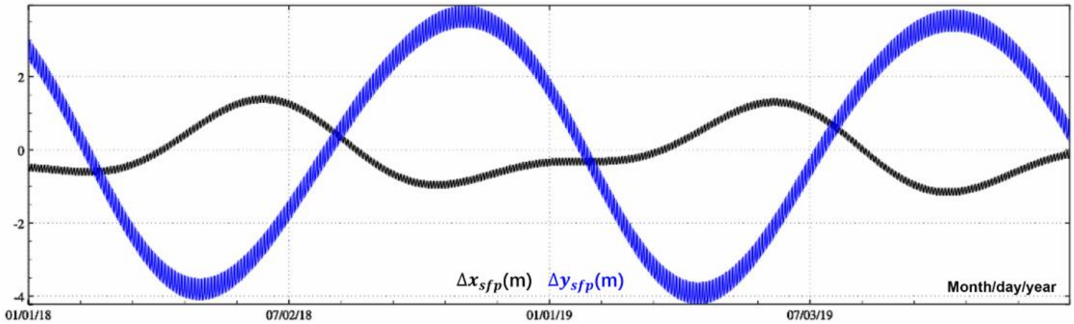


Fig 5.4 The surface atmosphere tidal load effect time series on Earth's figure polar shift in ITRS

Fig 5.4 shows that the surface atmosphere tidal load effect on the Earth's figure polar shift are dominated by diurnal variation in the short peroid, and the amplitude is decimeter level. The annual amplitude is large, and the difference between the maximum and minimum values is more than 7m.

8.5.2 The load effects on the Earth's center of mass and figure pole

Section 8.2 has introduced the non-tidal load spherical harmonic synthesis algorithm for the change of the Earth's gravity field, including the load effect calculation of the sea level variations, surface atmosphere variations and land water variations. Among them, the degree-1 load spherical harmonic coefficient variations in the non-tidal load spherical harmonic model can be employed to calculate the non-tidal load effects on Earth's mass centric variations, and the degree-2 tesseral load spherical harmonic coefficient variations can be employed to calculate the non-tidal load effects on Earth's figure polar shift.

(1) Calculation of non-tidal load effect on the Earth's mass centric variations

Assuming that the degree-1 non-tidal load spherical harmonic coefficient variations are $(\Delta\bar{C}_{10}^{w}, \Delta\bar{C}_{11}^{w}, \Delta\bar{S}_{11}^{w})$, considering $k_1' = 0$, the non-tidal load effect on Earth's mass centric variations can be obtained as follows:

$$\Delta x_{cm} = \sqrt{3}R\frac{\rho_w}{\rho_e}\Delta\bar{C}_{11}^{w}, \quad \Delta y_{cm} = \sqrt{3}R\frac{\rho_w}{\rho_e}\Delta\bar{S}_{11}^{w}, \quad \Delta z_{cm} = \sqrt{3}R\frac{\rho_w}{\rho_e}\Delta\bar{C}_{10}^{w} \tag{5.5}$$

From the degree-1 load spherical harmonic coefficient variation weekly time series $(\Delta\bar{C}_{10}^{sea}, \Delta\bar{C}_{11}^{sea}, \Delta\bar{S}_{11}^{sea})$ in the sea level variation load spherical harmonic coefficient model weekly time series constructed in 8.2.4 section, the Earth's mass centric variation weekly time series (in unit of mm, relative to the mean center of mass in 2018) are calculated according to formula (5.5), and the result is shown in figure 5.5. The time span of the time series is from January 2018 to December 2020.

From the degree-1 load spherical harmonic coefficient variation weekly time series

$(\Delta\bar{C}_{10}^{air}, \Delta\bar{C}_{11}^{air}, \Delta\bar{S}_{11}^{air})$ in the surface atmosphere variation load spherical harmonic coefficient model weekly time series constructed in 8.2.5 section, the Earth's mass centric variation weekly time series (in unit of mm, relative to the mean center of mass in 2018) are calculated according to formula (5.5), and the result is shown in figure 5.6. The time span of the time series is from January 2018 to December 2020.

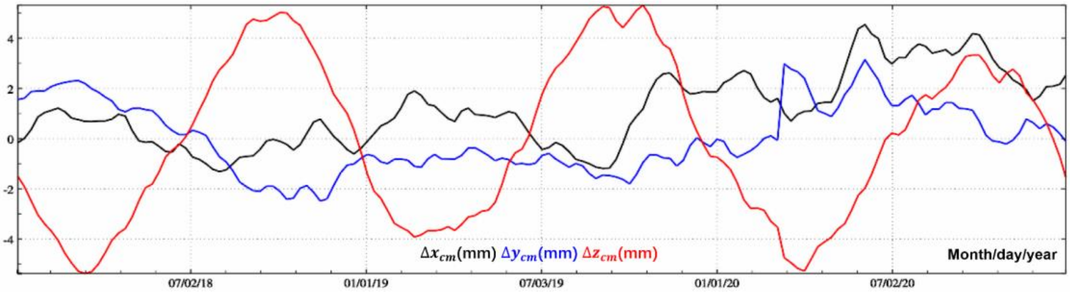


Fig 5.5 The sea level variation load effect time series on Earth's mass centric variation (relative to the mean center of mass in 2018)

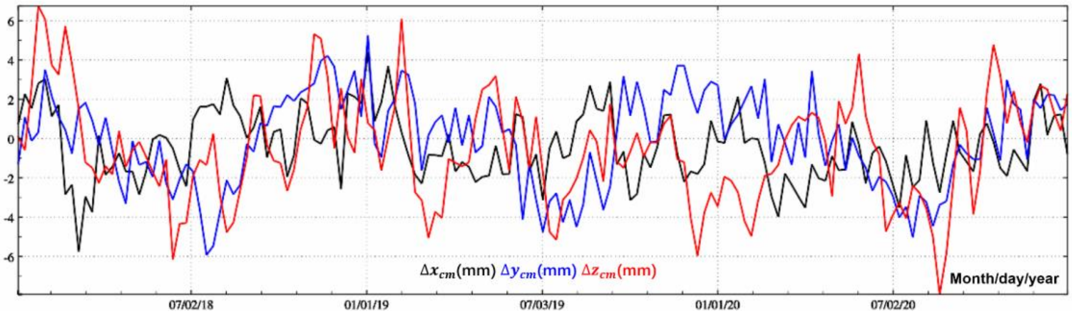


Fig 5.6 The surface atmosphere variation load effect time series on Earth's mass centric variation (relative to the mean center of mass in 2018)

From the degree-1 load spherical harmonic coefficient variation weekly time series $(\Delta\bar{C}_{10}^{lnd}, \Delta\bar{C}_{11}^{lnd}, \Delta\bar{S}_{11}^{lnd})$ in the global land water variation load spherical harmonic coefficient model weekly time series constructed in 8.2.6 section, the Earth's mass centric variation weekly time series (in unit of mm, relative to the mean center of mass in 2018) are calculated according to formula (5.5), and the result is shown in figure 5.7. The time span of the time series is from January 2018 to September 2020.

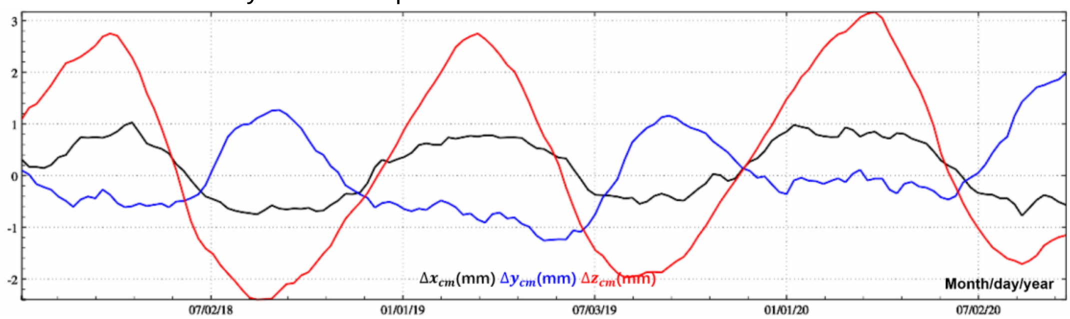


Fig 5.7 The global land water variation load effect time series on Earth's mass centric variation (relative to the mean center of mass in 2018)

Fig 5.5 ~ Fig 5.7 show that in the non-tidal load effects on Earth's mass centric variation, the difference between the maximum and minimum values of the load effect of sea level variations reaches 10mm, that of land water variations reaches 5mm, and that of surface atmosphere variations exceeds 10mm.

(2) Calculation of non-tidal load effect on the Earth's figure polar shift

The algorithm formula of the degree-2 tesseral non-tidal load spherical harmonic coefficient variations $(\Delta\bar{C}_{21}^{w}, \Delta\bar{S}_{21}^{w})$ expressed by the degree-2 tesseral harmonic geopotential coefficient variations $(\Delta\bar{C}_{21}, \Delta\bar{S}_{21})$ is:

$$\Delta\bar{C}_{21} = \frac{3\rho_w}{5\rho_e}(1+k_2')\Delta\bar{C}_{21}^{w}, \quad \Delta\bar{S}_{21} = \frac{3\rho_w}{5\rho_e}(1+k_2')\Delta\bar{S}_{21}^{w} \tag{5.6}$$

Substituting Formula (5.6) into Formula (5.2), the non-tidal load effect algorithm formulas on the figure polar shift can be obtained from the degree-2 order-1 non-tidal load spherical harmonic coefficient variations $(\Delta\bar{C}_{21}^{w}, \Delta\bar{S}_{21}^{w})$.

$$\Delta x_{sfp} = -\frac{3\rho_w}{5\rho_e}\frac{b}{\bar{C}_{20}}(1+k_2')\left(\sqrt{3}\Delta\bar{C}_{21}^{w} + \frac{6\bar{S}_{22}}{\bar{C}_{20}}\Delta\bar{S}_{21}^{w}\right) \tag{5.7}$$

$$\Delta y_{sfp} = +\frac{3\rho_w}{5\rho_e}\frac{b}{\bar{C}_{20}}(1+k_2')\left(\sqrt{3}\Delta\bar{S}_{21}^{w} - \frac{6\bar{S}_{22}}{\bar{C}_{20}}\Delta\bar{C}_{21}^{w}\right) \tag{5.8}$$

From the degree-2 order-1 sea level variation load spherical harmonic coefficient variation weekly time series $(\Delta\bar{C}_{21}^{sea}, \Delta\bar{S}_{21}^{sea})$ in 8.2.4 section, the Earth's figure polar shift weekly time series (in unit of m, relative to the mean figure pole in 2018) are calculated according to formulas (5.7) and (5.8), and the result is shown in figure 5.8. The time span of the time series is from January 2018 to December 2020.

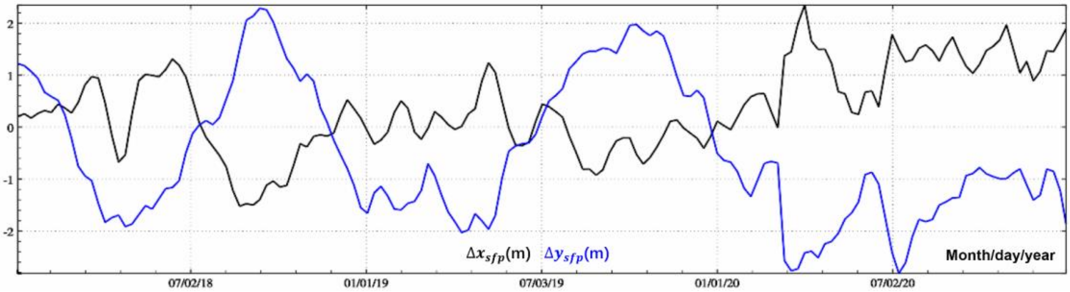


Fig 5.8 The sea level variation load effect time series on Earth's figure polar shift in ITRS (relative to the mean figure pole in 2018)

From the degree-2 order-1 surface atmosphere variation load spherical harmonic coefficient variation weekly time series $(\Delta\bar{C}_{21}^{air}, \Delta\bar{S}_{21}^{air})$ in 8.2.5 section, the Earth's figure polar shift weekly time series (in unit of m, relative to the mean figure pole in 2018) are calculated according to formulas (5.7) and (5.8), and the result is shown in figure 5.9. The time span of the time series is from January 2018 to December 2020.

From the degree-2 order-1 global land water variation load spherical harmonic coefficient variation weekly time series $(\Delta\bar{C}_{21}^{lnd}, \Delta\bar{S}_{21}^{lnd})$ in 8.2.6 section, the Earth's figure polar shift weekly time series (in unit of m, relative to the mean figure pole in 2018) are calculated according to formulas (5.7) and (5.8), and the result is shown in figure 5.10. The time span of the time series is from January 2018 to September 2020.

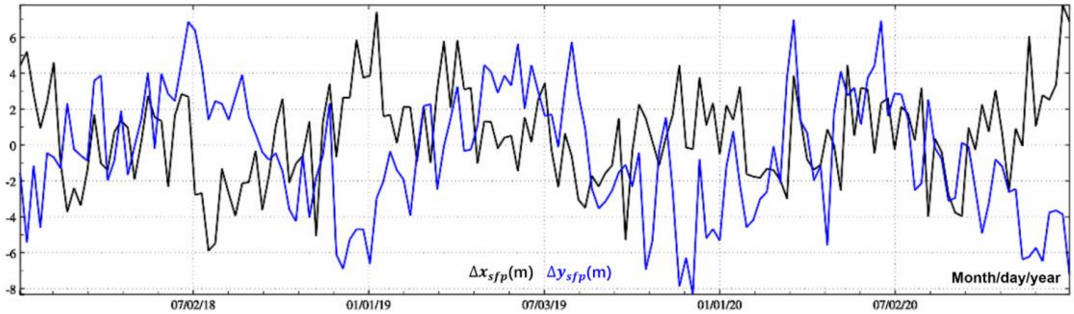


Fig 5.9 The surface atmosphere variation load effect time series on Earth's figure polar shift in ITRS (relative to the mean figure pole in 2018)

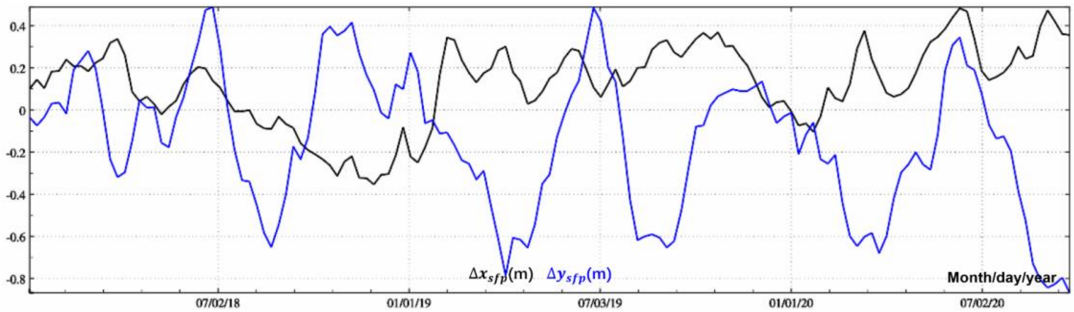


Fig 5.10 The global land water variation load effect time series on Earth's figure polar shift in ITRS (relative to the mean figure pole in 2018)

The statistical results show in the non-tidal load effects on Earth's figure polar shift, that the difference between the maximum and minimum values of the load effects of sea level variations is more than 4m, that of global land water variations is 1.2m, and that of surface atmosphere variations is more than 14m.

8.5.3 Earth's mass centric variation effects on all-element geodetic variations

The coordinates of the Earth's center of mass are geodetic elements with global spatial scale, and the Earth's center of mass is the degree-1 term of the mechanical equilibrium figure of the deforming Earth. The variations of the Earth's center of mass measured by the SLR have generally removed the ocean and atmosphere tidal load effects, which represent the deformation of whole Earth system excited by the non-tidal load variations, thus affecting various geometric and physical geodetic elements in the whole Earth space, rather than simply showing the ground site displacement of pure geometric elements.

The variations $(\Delta x_{cm}, \Delta y_{cm}, \Delta z_{cm})$ of the Earth's center of mass can be determined by measuring the degree-1 geopotential coefficient variations $(\Delta\bar{C}_{10}, \Delta\bar{C}_{11}, \Delta\bar{S}_{11})$ according to the following formula:

$$\Delta x_{cm} = \sqrt{3}R\Delta\bar{C}_{11}, \quad \Delta y_{cm} = \sqrt{3}R\Delta\bar{S}_{11}, \quad \Delta z_{cm} = \sqrt{3}R\Delta\bar{C}_{10} \tag{5.9}$$

In Formulas (2.8) ~ (2.20), let $n=1$, considering $\bar{P}_{10}(cos\theta)=\sqrt{3}cos\theta$, $\bar{P}_{11}(cos\theta)=\sqrt{3}\sin\theta$, we can obtain the algorithm formulas of the Earth's mass centric variation effects on geodetic variations from the SLR measured variations of the Earth's center of mass. Among them, the algorithm formula of the Earth's mass centric variation effect on height anomaly at (r,θ,λ) is:

$$\Delta\zeta(r,\theta,\lambda)=\frac{GM}{\gamma r^2}\frac{a}{R}(\Delta x_{cm}cos\lambda sin\theta+\Delta y_{cm}sin\lambda sin\theta+\Delta z_{cm}cos\theta) \quad (5.10)$$

The algorithm formula of the Earth's mass centric variation effect on ground gravity is⦿:

$$\Delta g^s(r,\theta,\lambda)=\frac{2GM}{r^3}\frac{a}{R}(1+2h_1')(\Delta x_{cm}cos\lambda sin\theta+\Delta y_{cm}sin\lambda sin\theta+\Delta z_{cm}cos\theta) \quad (5.11)$$

Where, h_1' is the degree-1 load radial Love number.

The algorithm formula of the Earth's mass centric variation effect on gravity (disturbance) outside the solid Earth is:

$$\Delta g^\delta(r,\theta,\lambda)=\frac{2GM}{r^3}\frac{a}{R}(\Delta x_{cm}cos\lambda sin\theta+\Delta y_{cm}sin\lambda sin\theta+\Delta z_{cm}cos\theta) \quad (5.12)$$

The algorithm formula of the Earth's mass centric variation effect on ground tilt is⦿:

South: $$\Delta\xi^s(r,\theta,\lambda)=\frac{GM}{\gamma r^3}\frac{a}{R}sin\,\theta\,(1-h_1')(\Delta x_{cm}cos\theta cos\lambda+\Delta y_{cm}\,sin\,\theta\,sin\lambda-\Delta z_{cm}sin\theta) \quad (5.13)$$

West: $$\Delta\eta^s(r,\theta,\lambda)=\frac{GM}{\gamma r^3}\frac{a}{R}(1-h_1')(\Delta x_{cm}sin\lambda-\Delta y_{cm}cos\lambda) \quad (5.14)$$

The algorithm formula of the Earth's mass centric variation effect on vertical deflection outside the solid Earth is:

South: $$\Delta\xi^s=\frac{GM}{\gamma r^3}\frac{a}{R}sin\,\theta\,(\Delta x_{cm}cos\theta cos\lambda+\Delta y_{cm}\,sin\,\theta\,sin\lambda-\Delta z_{cm}sin\theta) \quad (5.15)$$

West: $$\Delta\eta^s(r,\theta,\lambda)=\frac{GM}{\gamma r^3}\frac{a}{R}(\Delta x_{cm}sin\lambda-\Delta y_{cm}cos\lambda) \quad (5.16)$$

The algorithm formula of the Earth's mass centric variation effect on ground site displacement is⦿:

East: $$\Delta e(r,\theta,\lambda)=-\frac{GM}{r^2\gamma}\frac{a}{R}l_1'(\Delta x_{cm}sin\lambda-\Delta y_{cm}cos\lambda) \quad (5.17)$$

North: $$\Delta n=-\frac{GM}{r^2\gamma}\frac{a}{R}l_1'sin\theta(\Delta x_{cm}cos\theta cos\lambda+\Delta y_{cm}cos\theta sin\lambda-\Delta z_{cm}sin\theta) \quad (5.18)$$

Radial: $$\Delta r(r,\theta,\lambda)=\frac{GM}{r^2\gamma}\frac{a}{R}h_1'(\Delta x_{cm}cos\lambda sin\theta+\Delta y_{cm}sin\lambda sin\theta+\Delta z_{cm}cos\theta) \quad (5.19)$$

Where, l_1' is the degree-1 load horizontal Love number.

The algorithm formula of the Earth's mass centric variation effect on gravity gradient outside the solid Earth is:

Radial: $$\Delta T_{rr}(r,\theta,\lambda)=\frac{6GM}{r^4}\frac{a}{R}(\Delta x_{cm}cos\lambda sin\theta+\Delta y_{cm}sin\lambda sin\theta+\Delta z_{cm}cos\theta) \quad (5.20)$$

North: $$\Delta T_{NN}(r,\theta,\lambda)=\frac{GM}{r^4}\frac{a}{R}(\Delta x_{cm}cos\lambda sin\theta+\Delta y_{cm}sin\lambda sin\theta+\Delta z_{cm}cos\theta) \quad (5.21)$$

West: $$\Delta T_{WW}(r,\theta,\lambda)=\frac{GM}{r^4}\frac{a}{R}(\Delta x_{cm}sin\lambda+\Delta y_{cm}cos\lambda) \quad (5.22)$$

In the above expressions, the Earth's mass centric variation effects on the geodetic variations marked ⦿ are valid only when their sites are fixed with the solid Earth, and that on the remaining geodetic variations are suitable on the ground or outside the solid Earth.

In the following, using the Earth's mass centric variation time series from Center for Space Research in University of Texas in USA (UT/CSR) from LAGEOS-1/2, Stella, Starlette, AJISAI, BEC and LARES Satellite Laser Ranging (SLR) measured, the Earth's mass centric variation effect time series on various geodetic elements at the ground point P (105°E, 32°N, H720m) are calculated according to the formulas (5.10) ~ (5.22) as shown in Fig 5.11 and Fig 5.12, with the mean radius $R = 6371000$m of the Earth and h_1' =-0.2871, l_1' =0.1045. The time span of the time series is from January 2018 to December 2022 (5 years).

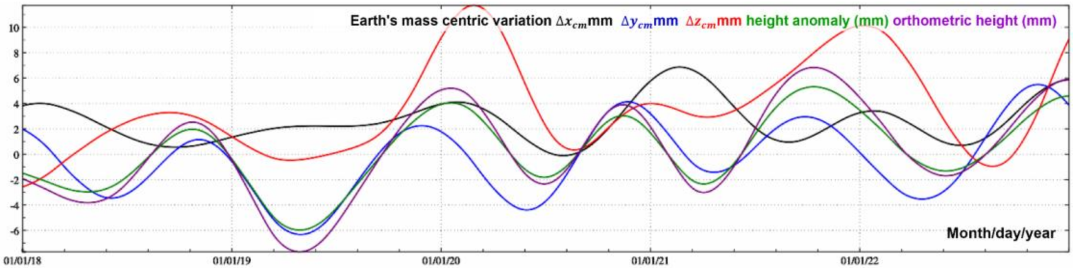


Fig 5.11 The Earth's mass centric variation and their effect time series on the geoid and normal height at the ground point P

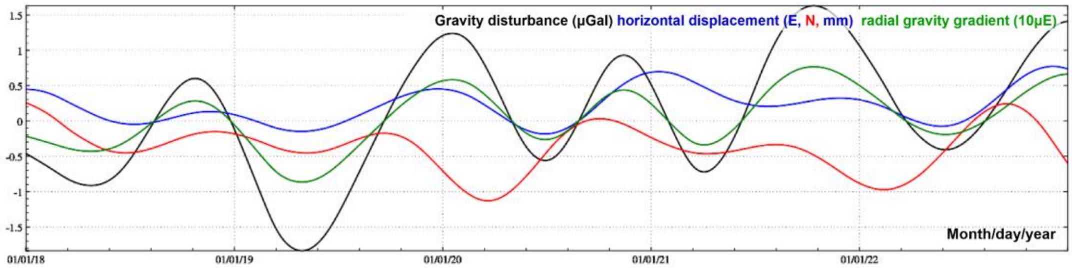


Fig 5.12 The Earth's mass centric variation effect time series on the geodetic variations at the ground point P

The Earth's center of mass is a typical geodetic element and its variation effects on various geodetic elements objectively exist, while the center of crustal shape is fictitious, and the geodetic elements are not affected by this fictitious center of crustal shape. So it is recommended to dilute the concepts of center of crustal shape and center of mass of solid Earth. In the calculation of the load Green's function integral and load spherical harmonic synthesis, only the Earth's mass centric load effects are considered, and the degree-1 load potential Love number is therefore always equal to zero, namely $k_1' \equiv 0$.

8.5.4 Earth's figure polar shift effects on all-element geodetic variations

The Earth's figure polar shift $\left(\Delta x_{sfp} = b\Delta\mu_1, \Delta y_{sfp} = -b\Delta\mu_2\right)$ can be determined by measuring the degree-2 tesseral harmonic geopotential coefficient variations $(\Delta\bar{C}_{20}, \Delta\bar{C}_{21})$ as follows:

$$\Delta\mu_1 = \frac{\Delta x_{sfp}}{b} = -\frac{\sqrt{3}}{\bar{C}_{20}}\Delta\bar{C}_{21}, \quad \Delta\mu_2 = -\Delta y_{sfp}/b = -\frac{\sqrt{3}}{\bar{C}_{20}}\Delta\bar{S}_{21} \tag{5.23}$$

Here, the geopotential coefficients $\bar{C}_{20}$ and $\bar{S}_{22}$ can be the mean values.

From Formula (5.23), we have:

$$\Delta\bar{C}_{21} = -\frac{\bar{C}_{20}}{\sqrt{3}b}\Delta x_{sfp}, \quad \Delta\bar{S}_{21} = \frac{\bar{C}_{20}}{\sqrt{3}b}\Delta y_{sfp} \tag{5.24}$$

Substitute Formula (5.24) into Formula (5.6), and have:

$$\Delta\bar{C}_{21}^{w} = -\frac{5\bar{C}_{20}}{3\sqrt{3}b}\frac{\rho_e}{(1+k_2')\rho_w}\Delta x_{sfp}, \quad \Delta\bar{S}_{21}^{w} = \frac{5\bar{C}_{20}}{3\sqrt{3}b}\frac{\rho_e}{(1+k_2')\rho_w}\Delta y_{sfp} \tag{5.25}$$

Then substitute the formula (5.25) into the formulas (2.8) ~ (2.20), and let n = 2, m = 1, considering $\bar{P}_{nm}(cos\theta) = \sqrt{15}sin\theta cos\theta$, we can obtain the algorithm formulas of the Earth's figure polar shift effects on geodetic variations from the measured figure polar shift. Among them, the algorithm formula of the Earth's figure polar shift effect on height anomaly at (r,θ,λ) is:

$$\Delta\zeta(r,\theta,\lambda) = -\frac{\sqrt{5}GMa^2}{2\gamma r^3}\frac{\bar{C}_{20}}{b}\left(\Delta x_{sfp}cos\lambda - \Delta y_{sfp}sin\lambda\right)sin2\theta \tag{5.26}$$

Where, $\bar{C}_{20}$ is the degree-2 zonal geopotential coefficient, and the approximate mean value is taken.

The algorithm formula of the Earth's figure polar shift effect on ground gravity is⦿:

$$\Delta g^{s}(r,\theta,\lambda) = -\frac{3\sqrt{5}GMa^2}{2r^4}\frac{\bar{C}_{20}}{b}\frac{1-\frac{3k_2'}{2}+h_2'}{1+k_2'}\left(\Delta x_{sfp}cos\lambda - \Delta y_{sfp}sin\lambda\right)sin2\theta \tag{5.27}$$

Where, k_2', h_2' are the degree-2 load potential and radial Love number, respectively.

The algorithm formula of the Earth's figure polar shift effect on gravity (disturbance) outside the solid Earth is:

$$\Delta g^{\delta}(r,\theta,\lambda) = -\frac{3\sqrt{5}GMa^2}{2r^4}\frac{\bar{C}_{20}}{b}\left(\Delta x_{sfp}cos\lambda - \Delta y_{sfp}sinm\lambda\right)sin2\theta \tag{5.28}$$

The algorithm formula of the Earth's figure polar shift effect on ground tilt is⦿:

South: $$\Delta\xi^{s}(r,\theta,\lambda) = -\frac{\sqrt{5}GMa^2}{\gamma r^4}\frac{\bar{C}_{20}}{b}\frac{1+k_2'-h_2'}{1+k_2'}\left(\Delta x_{sfp}sin\lambda + \Delta y_{sfp}cos\lambda\right)cos\theta \tag{5.29}$$

West: $$\Delta\eta^{s}(r,\theta,\lambda) = -\frac{\sqrt{5}GMa^2}{\gamma r^4}\frac{\bar{C}_{20}}{b}\frac{1+k_2'-h_2'}{1+k_2'}\left(\Delta x_{sfp}cos\lambda - \Delta y_{sfp}sin\lambda\right)sin\theta cos2\theta \tag{5.30}$$

The algorithm formula of the Earth's figure polar shift effect on vertical deflection outside the solid Earth is:

South: $$\Delta\xi(r,\theta,\lambda) = -\frac{\sqrt{5}GMa^2}{\gamma r^4}\frac{\bar{C}_{20}}{b}\left(\Delta x_{sfp}sin\lambda + \Delta y_{sfp}cos\lambda\right)cos\theta \tag{5.31}$$

West: $$\Delta\eta(r,\theta,\lambda) = -\frac{\sqrt{5}GMa^2}{\gamma r^4}\frac{\bar{C}_{20}}{b}\left(\Delta x_{sfp}cos\lambda - \Delta y_{sfp}sin\lambda\right)sin\theta cos2\theta \tag{5.32}$$

The algorithm formula of the Earth's figure polar shift effect on ground site displacement is⦿:

East: $$\Delta n(r,\theta,\lambda) = \frac{\sqrt{5}GMa^2}{\gamma r^3}\frac{\bar{C}_{20}}{b}\frac{l_2'}{1+k_2'}\left(\Delta x_{sfp}cos\lambda - \Delta y_{sfp}sin\lambda\right)sin\theta cos2\theta \tag{5.33}$$

North: $$\Delta e(r,\theta,\lambda) = \frac{\sqrt{5}GMa^2}{\gamma r^3}\frac{\bar{C}_{20}}{b}\frac{l_2'}{1+k_2'}\left(\Delta x_{sfp}sin\lambda + \Delta y_{sfp}cos\lambda\right)cos\theta \tag{5.34}$$

Radial: $$\Delta r(r,\theta,\lambda) = -\frac{\sqrt{5}GMa^2}{2\gamma r^3}\frac{\bar{C}_{20}}{b}\frac{h_2'}{1+k_2'}\left(\Delta x_{sfp}cos\lambda - \Delta y_{sfp}sin\lambda\right)sin2\theta \tag{5.35}$$

Where, l_2' is the degree-2 load horizontal Love number.

The algorithm formula of the Earth's figure polar shift effect on gravity gradient outside the solid Earth is:

$$\text{Radial:}\ \Delta T_{rr}(r,\theta,\lambda) = -\frac{6\sqrt{5}GMa^2}{r^5}\frac{\bar{C}_{20}}{b}\left(\Delta x_{sfp}cos\lambda - \Delta y_{sfp}sin\lambda\right)sin2\theta \tag{5.36}$$

$$\text{North:}\ \Delta T_{NN}(r,\theta,\lambda) = \frac{2\sqrt{5}GMa^2}{r^5}\frac{\bar{C}_{20}}{b}\left(\Delta x_{sfp}cos\lambda - \Delta y_{sfp}sin\lambda\right)sin2\theta \tag{5.37}$$

$$\text{West:}\ \Delta T_{WW}(r,\theta,\lambda) = -\frac{\sqrt{5}GMa^2}{r^5}\frac{\bar{C}_{20}}{b}\left(\Delta x_{sfp}cos\lambda - \Delta y_{sfp}sin\lambda\right)ctg\theta \tag{5.38}$$

In the following, using the the degree-2 tesseral harmonic geopotential coefficient variation monthly time series (the 5 years of mean removed) from Center for Space Research in University of Texas in USA (UT/CSR) from LAGEOS-1/2, Stella, Starlette, AJISAI, BEC and LARES Satellite Laser Ranging (SLR) measured, the Earth's figure polar coordinate variation time series are calculated as shown in Fig 5.13.

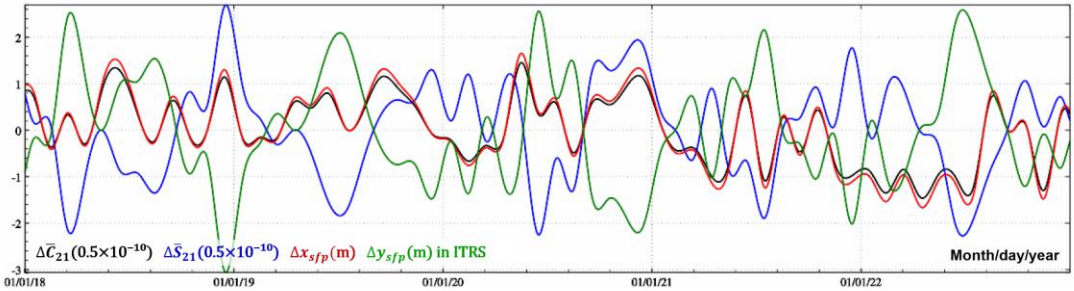


Fig 5.13 The degree-2 tesseral harmonic geopotential coefficient and figure polar coordinate variation time series in ITRS from SLR measured

Taking the degree-2 load Love number $k_2' = -0.3058$, $h_2' = -0.9946$ and $l_2' = 0.0241$, the Earth's short semi-axis $b = 6356751.655$m and the degree-2 zonal harmonic geopotential coefficient $\bar{C}_{20} = -4.84165 \times 10^{-4}$, the time series of the Earth's figure polar shift effects on various geodetic elements at the ground point P (105.0°E, 32.0°N, H720m) are calculated according to formulas (5.26) ~ (5.38) from the coordinate variation time series of Earth's figure pole, as shown in Fig 5.14. The time span of the time series is from January 2018 to December 2022 (5 years).

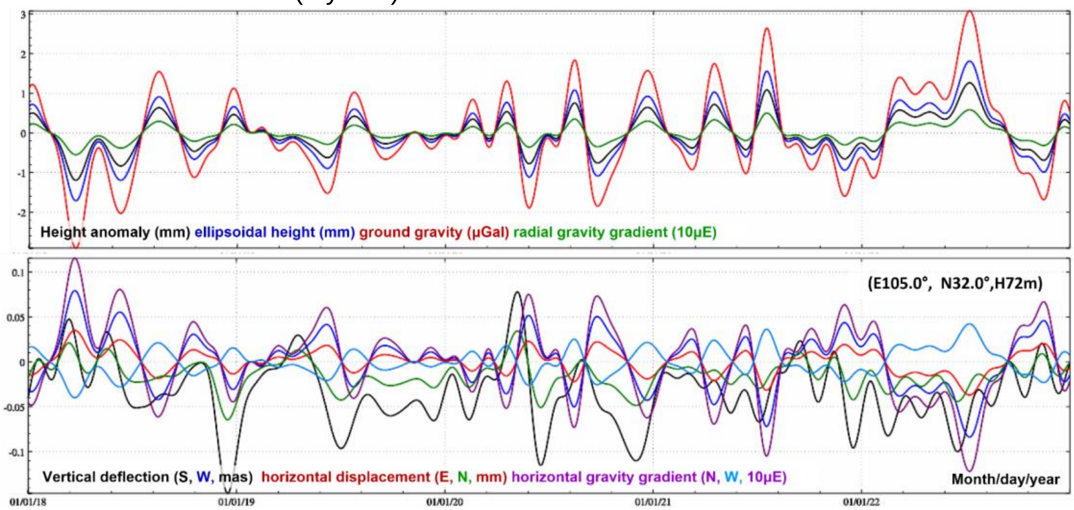


Fig 5.14 The time series of Earth's figure polar shift effects on various geodetic elements at the point P

Fig 5.14 shows that although the Earth's figure polar shift itself can reach the meter level, the resulting effect on geoid is not greater than 2mm. The Earth's figure polar shift effects on horizontal geodetic elements such as ground horizontal displacement, vertical deviation or horizontal gravity gradient are small and can be generally ignored.

8.6 Earth's rotation polar shift effects on geodetic variations and tidal effects on EPR

The instantaneous Earth's rotation axis is inconsistent with the mean Earth's figure axis, which leads to the change of the centrifugal force potential in Earth space with time. The variation of the centrifugal force potential excites solid Earth deformation, causing the redistribution of the mass inside the Earth and generating the associated geopotential.

8.6.1 Earth's rotation polar shift effects on geodetic variations

The variation of centrifugal force potential in the Earth space caused by the Earth's rotation polar shift is the direct influence of the Earth's rotation motion to geopotential variation, which can be expressed by the direct influence of the Earth's rotation polar shift on the degree-2 tesseral harmonic geopotential coefficient variation $\Delta\bar{C}_{21}^{dr}+i\Delta\bar{S}_{21}^{dr}$. Considering the relationship between the long-peroid Love number k_0 and the degree-2 zonal geopotential coefficient $\bar{C}_{20}$, we have:

$$\Delta\bar{C}_{21}^{dr}+i\Delta\bar{S}_{21}^{dr}=\frac{\sqrt{3}}{k_0}\bar{C}_{20}(m_1+im_2)=-\frac{1}{\sqrt{15}}\frac{\omega^2a^3}{GM}m \tag{6.1}$$

Where, $m=m_1+im_2$ is the complex form of Earth's rotation polar shift (in unit of radian), and ω is the angular rate of Earth's rotation.

The variation of centrifugal force potential caused by the Earth's rotation polar shift furtherly excites the deformation of the solid Earth and produces the associated geopotential, which is usually characterized by the degree-2 diurnal body tidal Love number k_{21}. The indirect influence of the centrifugal force potential to the degree-2 tesseral harmonic geopotential coefficient is as follows:

$$\Delta\bar{C}_{21}^{in}+i\bar{S}_{21}^{in}=\frac{\sqrt{3}k_{21}}{k_0}\bar{C}_{20}m=-\frac{1}{\sqrt{15}}\frac{\omega^2a^3}{GM}k_{21}m \tag{6.2}$$

The Earth's rotation polar shift effects on the degree-2 tesseral harmonic geopotential coefficient are equal to the sum of the direct effects (non-conservative) and indirect effects (conservative) of the centrifugal force potential, that is:

$$\Delta\bar{C}_{21}+i\Delta\bar{S}_{21}=\left(\Delta\bar{C}_{21}^{dr}+\Delta\bar{C}_{21}^{in}\right)+i\left(\bar{S}_{21}^{dr}+\bar{S}_{21}^{in}\right)=-\frac{1}{\sqrt{15}}\frac{\omega^2a^3}{GM}(1+k_{21})m \tag{6.3}$$

Similar to the solid Earth tidal effect algorithm formulas, The Earth's rotation polar shift effect algorithm formulas on various geodetic elements can be obtained. Considering $\bar{P}_{nm}(cos\theta)=\sqrt{15}sin\theta cos\theta$, the algorithm formula of the Earth's rotation polar shift effect on height anomaly at (r,θ,λ) is:

$$\Delta\zeta(r,\theta,\lambda)=\frac{GM}{\gamma r}\left(\frac{a}{r}\right)^2(1+k_{21})(\Delta\bar{C}_{21}^{dr}cos\lambda+\Delta\bar{S}_{21}^{dr}sin\lambda)\bar{P}_{21}(cos\theta)$$
$$=-\frac{\omega^2a^5}{2\gamma r^3}(1+k_{21})m^*e^{i\lambda}sin2\theta \quad (6.4)$$

Where, $e^{i\lambda}=cos\lambda+isin\lambda$ and $m^*=m_1-im_2$ is the complex conjugate of Earth's rotation polar shift m.

The algorithm formula of the Earth's rotation polar shift effect on ground gravity is⦿:

$$\Delta g^s=-\frac{3}{2}\frac{\omega^2a^5}{r^4}\left(1-\frac{3}{2}k_{21}+h_{21}\right)m^*e^{i\lambda}sin2\theta \quad (6.5)$$

The algorithm formula of the Earth's rotation polar shift effect on gravity (disturbance) outside the solid Earth is:

$$\Delta g^\delta=-\frac{3}{2}\frac{\omega^2a^5}{r^4}(1+k_{21})m^*e^{i\lambda}sin2\theta \quad (6.6)$$

The algorithm formula of the Earth's rotation polar shift effect on ground tilt is⦿:

South: $$\delta\xi^s=-\frac{\omega^2a^5}{\gamma r^4}(1+k_{21}-h_{21})m^*e^{i\lambda}sin\theta cos2\theta \quad (6.7)$$

West: $$\delta\eta^s=-\frac{\omega^2a^5}{\gamma r^4}(1+k_{21}-h_{21})m^*e^{i(\lambda-\pi/2)}cos\theta \quad (6.8)$$

The algorithm formula of the Earth's rotation polar shift effect on vertical deflection outside the solid Earth is:

South: $$\Delta\xi=-\frac{\omega^2a^5}{\gamma r^4}(1+k_{21})m^*e^{i\lambda}sin\theta cos2\theta \quad (6.9)$$

West: $$\Delta\eta=-\frac{\omega^2a^5}{\gamma r^4}(1+k_{21})m^*e^{i(\lambda-\pi/2)}cos\theta \quad (6.10)$$

The algorithm formula of the Earth's rotation polar shift effect on ground site displacement is⦿:

East: $$\Delta e=\frac{\omega^2a^5}{\gamma r^3}l_{21}m^*e^{i(\lambda-\pi/2)}cos\theta \quad (6.11)$$

North: $$\Delta n=\frac{\omega^2a^5}{\gamma r^3}l_{21}m^*e^{i\lambda}sin\theta cos2\theta \quad (6.12)$$

Radial: $$\Delta r=-\frac{\omega^2a^5}{2\gamma r^3}h_{21}m^*e^{i\lambda}sin2\theta \quad (6.13)$$

The algorithm formula of the Earth's rotation polar shift effect on gravity gradient outside the solid Earth is:

Radial: $$\Delta T_{rr}=-\frac{6\omega^2a^5}{r^5}(1+k_{21})m^*e^{i\lambda}sin2\theta \quad (6.14)$$

North: $$\Delta T_{NN}=\frac{2\omega^2a^5}{r^5}(1+k_{21})m^*e^{i\lambda}sin2\theta \quad (6.15)$$

West: $$\Delta T_{WW}=-\frac{\omega^2a^5}{r^5}(1+k_{21})m^*e^{i\lambda}ctg\theta \quad (6.16)$$

In the above expressions, the Earth's rotation polar shift effects on the geodetic variations marked ⦿ are valid only when their sites are fixed with the solid Earth, and that on the remaining geodetic variations are suitable on the ground or outside the solid Earth.

Taking the degree-2 tesseral body-tidal Love number $k_{21}=0.3077+0.0036i$, $h_{21}=0.6207$ and $l_{21}=0.0836$, the time series of Earth's rotation polar shift effects on various geodetic elements at the ground point P (105.0°E, 32.0°N, H720m) are calculated according to formulas (6.4) ~ (6.16) from the IERS Earth orientation parameters (EOP) time series, as shown in Fig 6.1. The time span of the time series is from January 2018 to December 2022 (5 years). In Fig 6.1, the 5-year mean value of the time series of Earth's rotation polar shift is removed, and the Earth's rotation polar shift (m_1, m_2) have been converted into the Earth's rotation polar coordinate variations $(\Delta x_p = m_1 b, \Delta y_p = -m_2 b)$ (in unit of m) in the ITRS (x and y axes in the Earth-fixed rectangular coordinate system).

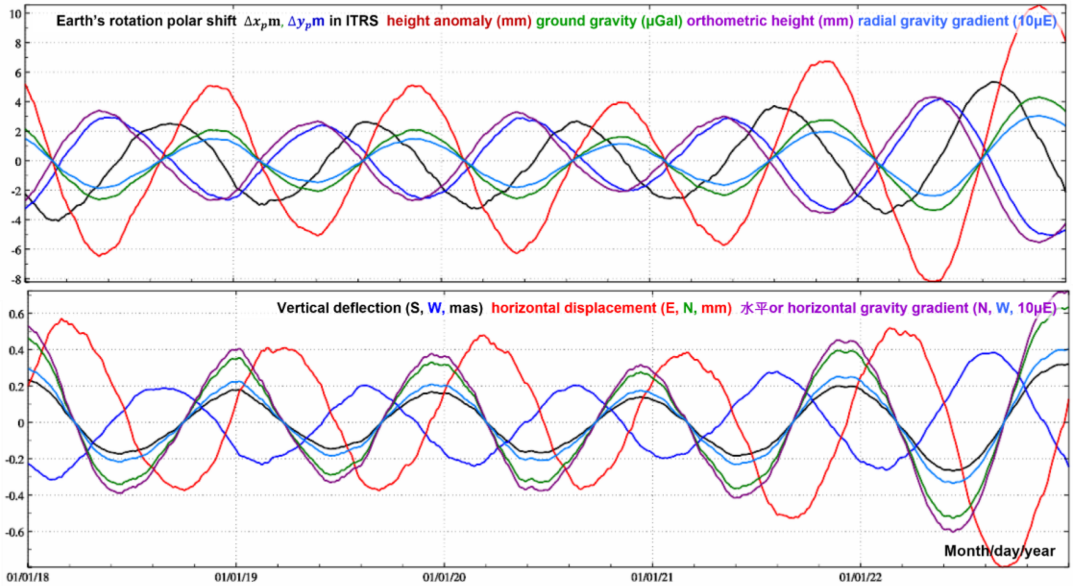


Fig 6.1 The time series of Earth's roration polar shift effects on various geodetic elements at the point P

Fig 6.1 shows that although the Earth's rotation polar shift itself can reach the meter level, the resulting effect on geoid or ground normal height is only in mm level, that on ground gravity is μGal level, that on radial gravity gradient is 10μE level, that on horizontal geodetic elements are small and can be generally ignored.

8.6.2 Self-consistent equilibrium ocean polar tide effects

It is generally believed that the ocean polar tide is the manifestation of the centrifugal force of rotation polar shift on the ocean, and its main periodic constituents are about 433 days of Chandler wobble and annual variation. In these long periods, the ocean polar tide load is expected to have an equilibrium response, that is, the displacement of the ocean surface is expected to be balanced with the equipotential surface acted by the centrifugal force.

(1) Radial displacement, sea surface height and the rotation polar shift effect on geopotential

Assuming that the centrifugal force of rotation polar shift is $\Delta\Psi_c$, the ground radial displacement is generated under the action of the radial Love number h_2, and then the Earth's rotation polar shift effect on the radial displacement can be expressed as:

$$r_p(\theta,\lambda,t)=\frac{h_2}{g_0}\Delta\Psi_c=H_pRe\left(m^*(t)h_2\bar{P}_{21}(cos\theta)e^{i\lambda}\right) \quad (6.17)$$

$$H_p=\sqrt{\frac{A}{\rho_eR}}\frac{\omega^2R^2}{GM}=\frac{\sqrt{8\pi}}{\sqrt{15}}\frac{\omega^2R^4}{GM}=\frac{\sqrt{8\pi}}{\sqrt{15}}\frac{\omega^2R^2}{g_0},\frac{8\pi}{5}R^4=\frac{3}{\rho_eR}A \quad (6.18)$$

Where, H_p is the scale factor of the Earth's rotation polar shift effect on radial displacement, $g_0=GM/R^2$ is the mean ground gravity, and $H_p=0.1385$m when the rotation polar shift parameter $m(t)$ is in unit of angular seconds (as or ″).

Similar to the expression of ocean tidal height, the Earth's rotation polar shift effect $\hbar_o(\theta,\lambda,t)$ on sea surface height can be expressed using the ocean spatial admittance function $Z(\theta,\lambda)$ as follows:

$$\hbar_o(\theta,\lambda,t)=H_pRe[m^*(t)Z(\theta,\lambda)] \quad (6.19)$$

After introducing the scale factor H_p, the ocean admittance function $Z(\theta,\lambda)$ becomes a normalized (dimensionless) spatial harmonic function, which can be decomposed into the spherical harmonic series as follows:

$$Z(\theta,\lambda)=\sum_{n=0}^{\infty}Z_n(\theta,\lambda) \quad (6.20)$$

The Earth's polar shift effect $\hbar_o(\theta,\lambda,t)$ on sea surface height also associates the re-adjustment of ocean mass and geopotential variations on sea surface. This is the direct influence of sea surface height variation induced by rotation polar shift to the geopotential, which can be expressed as:

$$U(\theta,\lambda,t)=\sum_{n=0}^{\infty}U_n(\theta,\lambda,t)=H_pg_0Re[m^*(t)\sum_{n=0}^{\infty}\alpha_nZ_n(\theta,\lambda)] \quad (6.21)$$

Here, $\alpha_n=\frac{3}{2n+1}\frac{\rho_w}{\rho_e}$.

The direct influence U_n of geopotential produces the associated potential under the action of the load potential Love number k_n', so we have:

$$U_o^a(\theta,\lambda,t)=\sum_{n=0}^{\infty}k_n'U_n(\theta,\lambda,t)=H_pg_0Re[m^*(t)\sum_{n=0}^{\infty}k_n'\alpha_nZ_n(\theta,\lambda)] \quad (6.22)$$

The ocean polar tide effect on the geopotential is equal to the sum of the direct influence of sea surface height variation induced by rotation polar shift to geopotential and the associated potential, namely

$$U_o(\theta,\lambda,t)=\sum_{n=0}^{\infty}(1+k_n')U_n(\theta,\lambda,t)$$
$$=H_pg_0Re[m^*(t)\sum_{n=0}^{\infty}(1+k_n')\alpha_nZ_n(\theta,\lambda)] \quad (6.23)$$

(2) Self-consistent equilibrium ocean polar tide effects on geopotential coefficients

On the two maximum long-period constituents of the solid Earth tide, the ocean is likely to have a long-wave response corresponding to the equilibrium response. As the period increases, the deviation of this response from the equilibrium state is smaller. The equilibrium ocean polar tide effect assumes that the instantaneous ocean surface is a gravity equipotential surface, that is, the instantaneous ocean surface and equipotential surface are in an equilibrium state, and then the equilibrium displacement of the ocean surface relative to the seabed is determined by subtracting the polar tide effect from the sea equipotential surface.

The equilibrium ocean polar tide admittance function $\bar{Z}^c$ is proportional to the ground tilt tidal factor (namely sea surface height tidal factor) $\gamma_2 = 1 + k_2 - h_2$, which can be expressed as the product of the normalized equilibrium admittance function $\bar{E}^c$ and ground tilt tidal factor γ_2.

$$\bar{Z}^c(\theta,\lambda) = \gamma_2 \bar{E}^c(\theta,\lambda) \tag{6.24}$$

$$\bar{E}^c(\theta,\lambda) = \sum_{n=0}^{\infty} \bar{E}_n^c(\theta,\lambda) = \mathcal{O}(\theta,\lambda)\left[\bar{P}_{21}(cos\theta)e^{i\lambda} + K^c\right] \tag{6.25}$$

Where, $\mathcal{O}(\theta,\lambda)$ is an ocean function, $\mathcal{O}(\theta,\lambda) = 1$ when (θ,λ) is located in the ocean area, and $\mathcal{O}(\theta,\lambda) = 0$ when (θ,λ) is located on land.

The complex constant K^c is employed in the equation (6.25) to maintain the mass conservation of the classical equilibrium ocean polar tide. Assuming that the ocean has a constant density, the zero-degree spherical harmonic component of the ocean polar tide should be equal to zero, namely $\bar{Z}_0^c = \bar{E}_0^c = 0$.

The self-consistent equilibrium ocean polar tide response function $\bar{Z}^s(\theta,\lambda)$ after considering the rotation polar shift centrifugal potential and its associated potential is also proportional to the ground tilt tidal factor $\gamma_2 = 1 + k_2 - h_2$, which can be expressed by the normalized self-consistent equilibrium admittance function $\bar{E}^s$ as:

$$\bar{Z}^s(\theta,\lambda) = \gamma_2 \bar{E}^s(\theta,\lambda) \tag{6.26}$$

$$\bar{E}^s(\theta,\lambda) = \sum_{n=0}^{\infty} \bar{E}_n^s = \mathcal{O}(\theta,\lambda)\left[\bar{P}_{21}(cos\theta)e^{i\lambda} + \sum_{n=0}^{\infty} \gamma'_n \alpha_n \bar{E}_n^s + K^s\right] \tag{6.27}$$

Where, K^s is a complex constant, which is employed to maintain the self-consistent balance of ocean polar tide mass conservation. $\gamma'_n = 1 + k'_n - h'_n$ is the degree-n load tidal factor of ground tilt.

The spherical harmonic components of the normalized admittance functions $\bar{E}_n^c$ and $\bar{E}_n^s$ are defined from the coefficients $(\bar{A}_{nm} + i\bar{B}_{nm})$ as the following spherical harmonic series:

$$\bar{E}(\theta,\lambda) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \bar{P}_{|n|m}(cos\theta)(\bar{A}_{nm} + i\bar{B}_{nm})e^{i\lambda} \tag{6.28}$$

The first and second terms of Equation (6.27) can be considered as the first and second terms of the self-consistent equilibrium pole tide, so the normalized admittance can be calculated by using the iterative scheme of $\bar{E}_n^s = \bar{E}_n^c$ in the first iteration.

Let $\bar{A}_{nm} = \bar{A}_{nm}^R + i\bar{A}_{nm}^I, \bar{B}_{nm} = \bar{B}_{nm}^R + i\bar{B}_{nm}^I$ be the degree-n order-m ocean polar tide load coefficients in self-consistent equilibrium state, then the direct influence of ocean polar tide loads to normalized geopotential coefficients can be expressed by the Earth's rotation polar shift parameters (m_1, m_2) (Desai, 2002) as follows:

$$\begin{bmatrix}\Delta\bar{C}_{nm}\\ \Delta\bar{S}_{nm}\end{bmatrix} = R_n\left\{\begin{bmatrix}\bar{A}_{nm}^R\\ \bar{B}_{nm}^R\end{bmatrix}(m_1\gamma_2^R + m_2\gamma_2^I) + \begin{bmatrix}\bar{A}_{nm}^I\\ \bar{B}_{nm}^I\end{bmatrix}(m_2\gamma_2^R - m_1\gamma_2^I)\right\} \tag{6.29}$$

$$R_n = \frac{\omega^2 R^4}{GM}\frac{4\pi G\rho_w}{g_0(2n+1)} = \frac{\omega^2 R^2}{g_0^2}\frac{4\pi G\rho_w}{2n+1}, \ \gamma_2 = \gamma_2^R + i\gamma_2^I \tag{6.30}$$

Here, from the IERS Earth orientation parameters (EOP) time series and the 360-degree self-consistent equilibrium ocean polar tide load coefficient model (Desai, 2002) in IERS conventions (2010), the time series of ocean polar tide effects on various geodetic elements are calculated at the point P (121.3°E, 28.8°N, H11m) in the coastal zone as shown in Fig 6.2. The time span of the time series is from January 1,2018 to December 31,2022 (4 years), with a time interval of 1 day. Where, the calculation subroutine for formula (6.29) can be obtained from the IERS website.

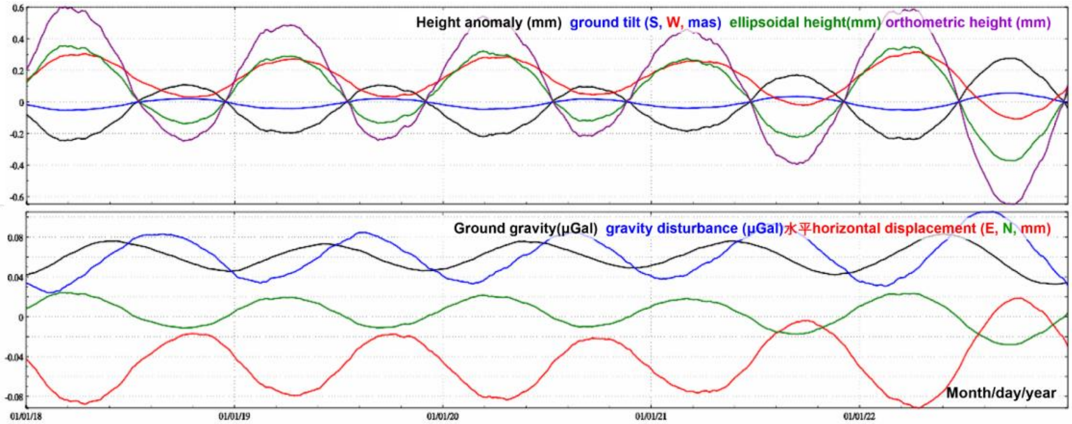


Fig 6.2 Ocean polar tide effect time series on geodetic variations at the point P in the coastal zone area

The ocean polar tide effects on geodetic variations are small, which can be ignored for general geodetic cases.

8.6.3 Various tidal effects on the Earth rotation parameters

(1) Zonal harmonic tidal effects on length of day and Earth's rotation rate

The response of the solid Earth to the zonal harmonic tidal potential causes the periodic variation of the principal moment of inertia, and then the amplitude of the Earth's rotation motion is amplified and the scale factor of the rotation rate is changed according to the conservation principle of angular momentum.

The Earth's tidal generating potential V_G from the celestial body at the ground point P (θ,λ) can be expanded into a spherical harmonic function series as:

$$V_G(P) = GM\sum_{n=2}^{\infty}\frac{a^n}{r^{n+1}}P_n(cos\psi) \tag{6.31}$$

Where, ψ is the geocentric angle distance between the ground point P (θ,λ) and celestial body (r,Θ,Λ), and (r,Θ,Λ) is the spherical coordinates of the celestial body in the Earth-fixed coordinate system, they all change with time. The degree-2 tidal potential can be decomposed into three groups of spherical harmonic functions as follows:

$$V_{G,20}(P) = GM\frac{a^2}{r^3}P_{20}(cos\theta)P_{20}(cos\Theta) \tag{6.32}$$

$$V_{G,21}(P) = \frac{1}{3}GM\frac{a^2}{r^3}P_{21}(cos\theta)P_{21}(cos\Theta)cos(\Lambda-\lambda) \tag{6.33}$$

$$V_{G,22}(P) = \frac{1}{12}GM\frac{a^2}{r^3}P_{22}(cos\theta)P_{22}(cos\Theta)cos2(\Lambda-\lambda) \tag{6.34}$$

The formulas (6.33) and (6.34) contain the tesseral and sector harmonic functions, respectively, to describe the semidiurnal and diurnal variations of short-period tides, while the formula (6.32) contains the zonal harmonic function, which only depends on the geocentric colatitude Θ of the celestial body and changes slowly, so it is employed to describe the medium and long-period tidal waves.

The main periods of the zonal harmonic tides on the Moon are 14 days M_f and 28 days M_m, and the main periods of the zonal harmonic tides from the Sun are semi-annual S_{sa} and annual S_a. These zonal harmonic tides are the largest terms that cause changes in the length of day (LOD).

Considering the frequency dependent corrections of long-period Love number $k_{20}(\sigma)$ from the mantle anelasticity, taking the scale factor $k/c_m = 0.94$ ($c_m = 0.293$ is the polar moment of inertia coefficient of the mantle), the Earth rotation long-period tidal change correction algorithm formulas (IERS conventions, 2010) with the periods of 5 days to 18.6 years are:

$$\Delta UT1 = m_3 \Lambda_0 = -\sum_{i=1}^{62}(A_i sin\phi_i - B_i cos\phi_i) \tag{6.35}$$

$$\Delta LOD = \sum_{i=1}^{62}(A_i' cos\phi_i - B_i' sin\phi_i) \tag{6.36}$$

$$\Delta\omega = \sum_{i=1}^{62}(A_i'' cos\phi_i - B_i'' sin\phi_i) \tag{6.37}$$

Where, A_i, B_i, A_i', B_i', A_i'', B_i'' are the in-phase amplitude and out-of-phase amplitude of the long-period tidal constituent with frequency σ_i, respectively, as shown in column 7-12 of tab 6.1 (omit the tidal waves with all 6 coefficients less than 1.0), and ϕ_i is the astronomical argument of the long-period constituent σ_i, which is calculated by the basic Delaunay variables (columns 1~5 in the tab 6.1) or the Doodson number.

Tab 6.1 Zonal harmonic tidal effect corrections on length of day and rotation rate

Delaunay variables					Period (day)	$\Delta UT1$		ΔLOD		$\Delta\omega$	
l	l'	F	D	Ω		A_i	B_i	A_i'	B_i'	A_i''	B_i''
0	0	2	2	2	7.10	-0.1231	0.0000	1.0904	0.0000	-0.9203	0.0000
1	0	2	0	1	9.12	-0.4108	0.0000	2.8298	0.0000	-2.3884	0.0000
1	0	2	0	2	9.13	-0.9926	0.0000	6.8291	0.0000	-5.7637	0.0000
-1	0	2	2	2	9.56	-0.1974	0.0000	1.2978	0.0000	-1.0953	0.0000
0	0	2	0	0	13.61	-0.2989	0.0000	1.3804	0.0000	-1.1650	0.0000
0	0	2	0	1	13.63	-3.1873	0.2010	14.6890	0.9266	-12.3974	-0.7820
0	0	2	0	2	13.66	-7.8468	0.5320	36.0910	2.4469	-30.4606	-2.0652
2	0	0	0	0	13.78	-0.3384	0.0000	1.5433	0.0000	-1.3025	0.0000
0	0	0	2	0	14.77	-0.7341	0.0000	3.1240	0.0000	-2.6367	0.0000
-1	0	2	0	2	27.09	0.4352	0.0000	-1.0093	0.0000	0.8519	0.0000
1	0	0	0	-1	27.44	0.5339	0.0000	-1.2224	0.0000	1.0317	0.0000
1	0	0	0	0	27.56	-8.4046	0.2500	19.1647	0.5701	-16.1749	-0.4811
1	0	0	0	1	27.67	0.5443	0.0000	-1.2360	0.0000	1.0432	0.0000
-1	0	0	2	0	31.81	-1.8236	0.0000	3.6018	0.0000	-3.0399	0.0000
0	1	2	-2	2	121.75	-1.8847	0.0000	0.9726	0.0000	-0.8209	0.0000
0	0	2	-2	1	177.84	1.1703	0.0000	-0.4135	0.0000	0.3490	0.0000
0	0	2	-2	2	182.62	-49.7174	0.4330	17.1056	0.1490	-14.4370	-0.1257
0	1	0	0	0	365.26	-15.8887	0.1530	2.7332	0.0263	-2.3068	-0.0222

0	0	0	0	2	-3399.19	7.8998	0.0000	0.1460	0.0000	-0.1232	0.0000
0	0	0	0	1	-6798.38	-1617.2681	0.0000	-14.9471	0.0000	12.6153	0.0000

(2) Long-period ocean tidal correction for the Earth's rotation polar shift and effective excitation

The long-period term of the rotation polar shift mainly includes the half-Chandler, semi-annual, seasonal, month and fortnight period, etc., as well as the quasi-two-year and 300-day period. The motion equations of the unforced rotation expressed by the effective angular momentum function are:

$$\chi(t) = m^*(t) + \frac{i}{\sigma_c}\dot{m}^*(t), \quad \psi_3(t) = -m_3(t) = \frac{\Delta LOD(t)}{\Lambda_0} \tag{6.38}$$

$$\chi(t) = \chi_1(t) + i\chi_2(t)\ , \ m(t) = m_1(t) + im_2(t) \tag{6.39}$$

$$\begin{cases} \chi_1(t) = \frac{1.608}{(C-A)\omega}[h_1(t) + (1+k_2')\omega I_{13}(t)] \\ \chi_2(t) = \frac{1.608}{(C-A)\omega}[h_2(t) + (1+k_2')\omega I_{23}(t)] \\ \chi_3(t) = \frac{0.997}{C\omega}[h_3(t) + 0.750\omega I_{33}(t)] \end{cases} \tag{6.40}$$

Here, $m^*(t)$ is the complex conjugate of $m(t)$, σ_c is the complex frequency of Chandler's wobble, $\Lambda_0 = 86400s$ is the mean day of length, $\chi(t)$ is the effective angular momentum (EAM) function of rotation polar shift, $\boldsymbol{h}(t) = [h_1(t), h_2(t), h_3(t)]$ is the relative angular momentum of matter motion in Earth's interior, C and A are the main moments of inertia of the Earth, and ω is the mean angular rate of rotation.

The effective angular momentum function $\boldsymbol{\chi}(t) = [\chi_1(t), \chi_2(t), \chi_3(t)]$ in Formula (6.40) mainly includes two parts: the change $\Delta\boldsymbol{I}$ of inertia tensor caused by the mass redistribution, and the change $\Delta\boldsymbol{h}$ of relative angular momentum caused by the velocity of matter movement in Earth's interior. Four coefficients are introduced in the formula (6.40), 1.608 is the amplitude amplification factor considering the mantle anelasticity and liquid core effect, 0.750 is the scale factor of the rotation rate change considering the ocean motion friction and drag effect of the viscosity of the mantle, 0.997 indicates that the rotation centrifugal force reduces the rotation rate by 0.3%.

Tidal correction algorithm formulas for Earth's rotation polar shift and effective angular momentum with the periods of 9 days to 18.6 years are as follows:

$$m^*(t) = m_1(t) - im_2(t) = A_p e^{i[\phi(t)+\varphi_p]} + A_r e^{i[-\phi(t)+\varphi_r]} \tag{6.41}$$

$$\chi(t) = \chi_1(t) + i\chi_2(t) = A_p e^{i[\phi(t)+\varphi_p]} + A_r e^{i[-\phi(t)+\varphi_r]} \tag{6.42}$$

Here, $\phi(t)$ is the astronomical argument, A_p, φ_p are the prograde harmonic amplitude and phase of the long-period ocean tidal effect excited by the rotation polar motion or effective angular momentum, respectively, and A_r, φ_r are the retrograde harmonic amplitude and phase of that, respectively.

Tab 6.2 Long-period ocean tidal correction for the Earth's rotation polar shift and effective excitation

	Delaunay variables		Correction for polar shift m	Correction for EAM χ

	l	l'	F	D	Ω	Period (day)	A_pμas	φ_p°	A_rμas	φ_r°	A_pμas	φ_p°	A_rμas	φ_r°
m_{tm}	1	0	2	0	1	9.12	4.43	-112.62	5.57	21.33	205.83	67.21	269.95	21.17
M_{tm}	1	0	2	0	2	9.13	10.72	-112.56	13.48	21.3	497.59	67.27	652.59	21.14
m_f	0	0	2	0	1	13.63	27.35	-91.42	30.59	13.31	841.32	88.42	1002.12	13.15
M_f	0	0	2	0	2	13.66	66.09	-91.31	73.86	13.27	2028.73	88.53	2414.94	13.11
M_{sf}	0	0	0	2	0	14.77	5.94	-87.13	6.42	11.75	168.13	92.7	194.74	11.6
M_m	1	0	0	0	0	27.56	43.74	-56.7	31.12	-0.91	643.61	123.13	520.16	-1.06
M_{sm}	-1	0	0	2	0	31.81	8.85	-51.11	5.42	-4.21	111.62	128.72	79.23	-4.36
S_{sa}	0	0	2	-2	2	182.62	86.48	-20.3	99.77	175.57	118.56	159.42	336.32	175.46
S_a	0	1	0	0	0	365.26	17.96	-17.38	152.15	170.6	3.33	161.6	332.53	170.51
M_n	0	0	0	0	1	-6798.38	208.17	166.89	186.98	166.67	221.43	166.88	175.07	166.68

Fig 6.3 long-period tidal effect time series for the Earth’s rotation motion from January 1, 2026 to December 31, 2028 (3years) are predicted according to the formulas (6.35) ~ (6.37), (6.41) and (6.42), and the time series sampling interval is 4 hours.

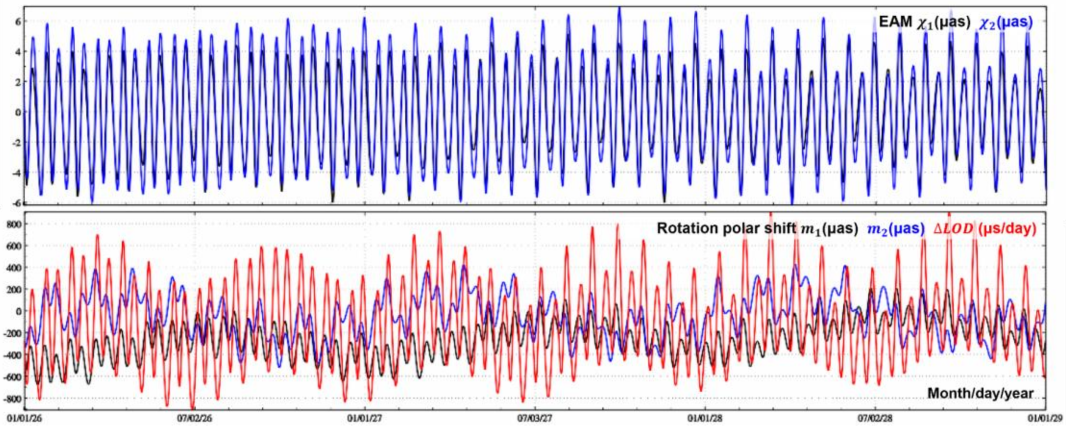


Fig 6.3 Long-period tidal effect time series for the Earth’s rotation motion

(3) Diurnal and semidiurnal ocean tidal effects on the Earth's rotation parameters

The current view is that the diurnal and semi-diurnal variations of the Earth's rotation are mainly the response of the solid Earth to the effects of ocean tides and ocean currents. The centrifugal potential variation excites the deformation of solid Earth, resulting in the change of Earth's inertia tensor. The Diurnal and semidiurnal moments are mainly from the principal inertia difference $B-A$ of the three-axis Earth (the principal inertial axis coordinate system), where B is the polar moment of inertia and A is the equatorial moment of inertia, then the degree-2 sector harmonic non-normalized geopotential coefficient C_{22} is:

$$C_{22}=\frac{1}{4}MR^2(B-A) \tag{6.43}$$

In general, the principal axis of the Earth-fixed coordinate system does not coincide with the principal axis of inertia of the Earth. At this case, the difference $B-A$ in the equatorial plane becomes:

$$B - A = 4MR^2\sqrt{C_{22}^2 + S_{22}^2} \tag{6.44}$$

and then leads to the rotation polar shift and UT1 variation as:

$$m(t) = -\frac{0.36GM}{\omega^2R^3}\frac{B-A}{A}sin2\Theta e^{-i(\Lambda-2\lambda)} \tag{6.45}$$

$$UT1(t) = -\frac{0.3GM}{8\omega^2R^3}\frac{B-A}{C_m}sin^2\Theta sin2(\Lambda - 2\lambda) \tag{6.46}$$

Where, Θ, Λ are the colatitude and longitude of the tidal celestial body in the Earth-fixed coordinate system, respectively.

It can be seen from the formulas (6.44) ~ (6.46) that C_{22} excites the semidiurnal variation of Earth's rotation. The theoretical calculation (Chao et al., 1996) shows that its magnitude is about 0.06 mas (1mas of geocentric angle distance is about 3cm away from the corresponding ground).

Similar to the excitation of the ocean tide to the Earth's rotation polar motion and rotation rate variation in Formula (6.40), the diurnal and semi-diurnal variations of the Earth's rotation excited by the ocean tide can be expressed by the harmonic function series as:

$$m_1 = \sum_{i=1}^{n}(-A_i^c cos\phi_i + A_i^s sin\phi_i) \tag{6.47}$$

$$m_2 = \sum_{i=1}^{n}(A_i^c sin\phi_i + A_i^s cos\phi_i) \tag{6.48}$$

$$\Delta UT1 = m_3\Lambda_0 = \sum_{i=1}^{n}(B_i^c cos\phi_i + B_i^s sin\phi_i) \tag{6.49}$$

At present, the Eanes2000 model and interp.f fortran code in the IERS conventions (2010) are widely employed, which can be obtained from the IERS website.

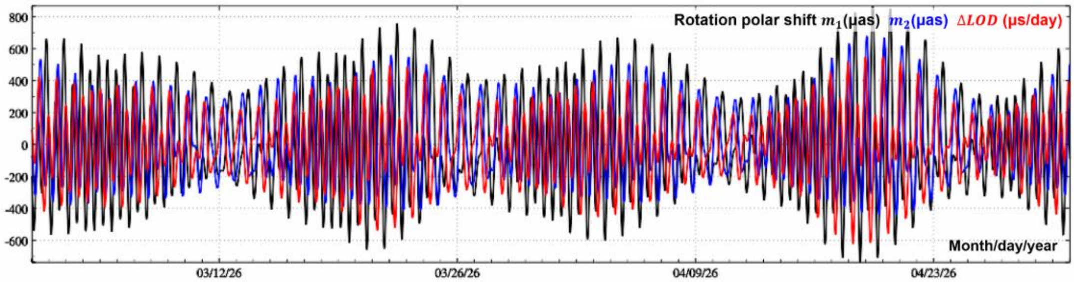


Fig 6.4 The time series of diurnal and semi-diurnal tidal effects on ERP

Fig 6.4 the time series of diurnal and semi-diurnal tidal effects on Earth's rotation parameters from March 1, 2026 to April 30, 2026 (2 months) are predicted according to the formulas (6.47) ~ (6.49), and the time series sampling interval is 15 minutes.

8.6.4 Calculation of CIP instantaneous polar coordinates in ITRS

According to the IERS conventions (2010), the instantaneous coordinates of the celestial intermediate polar (CIP) in the ITRS are expressed in the polar coordinate system, whose y-axis direction is opposite to the y-axis direction of the ITRS, denoted as (p_1, p_2), and its unit and direction are the same as the rotation polar shift (m_1, m_2). Since the forced nutation of external celestial bodies in GCRS with a period of less than 2 days is not included in the IAU2000/IAU2006 nutation model, it is necessary to consider the corresponding motion model of the Earth’s rotation pole in ITRS.

(p_1, p_2) is composed of $(m_1, m_2)_{IERS}$ provided by IERS Bulletin A and B, plus ocean tides and forced nutation correction terms of external celestial bodies with periods less than 2 days in GCRS, namely

$$(p_1, p_2) = (m_1, m_2)_{IERS} + (m_1, m_2)_{OT} + (m_1, m_2)_{LIB} \tag{6.50}$$

Where, $(m_1, m_2)_{OT}$ are the diurnal and semi-diurnal variations in the rotation polar coordinates caused by ocean tides, and $(m_1, m_2)_{LIB}$ are the variations in the rotation polar coordinates corresponding to motions with periods less than two days in space that are not part of the IAU 2000 nutation model.

The high-frequency polar shift term $(m_1, m_2)_{OT}$ mainly includes the diurnal and semi-diurnal variations caused by the ocean tide, which can be calculated according to the formulas (6.47) ~ (6.49). The non-zonal harmonic oscillation terms $(m_1, m_2)_{LIB}$, including the forced diurnal and semi-diurnal rotation polar shift terms, was previously regarded as nutation and is now classified as the rotation polar shift. The non-harmonic oscillation term is due to the diurnal and semi-diurnal terms of the tidal celestial bodies, resulting in the change of the Earth's inertial tensor $\Delta\boldsymbol{I}$ with time, which in turn produces the rotation polar shift according to formula (6.1).

The long-period terms and the long-term changes caused by the torque from tidal celestial bodies are generally considered to be included in the observed rotation polar shift and do not need to be added to the $(m_1, m_2)_{IERS}$.

8.7 Load deformation field approach from heterogeneous variations using SRBFs

When the considered variable of load effect is the geopotential differential or its linear combination, such as the load effect on gravity disturbance, vertical deflection, horizontal displacement or gravity gradient, these load Green's functions have serious high-degree oscillation and non-convergence troubles, and these load Green's function integrals have spectrum leakage and singularity problems, as shown in Fig 7.1.

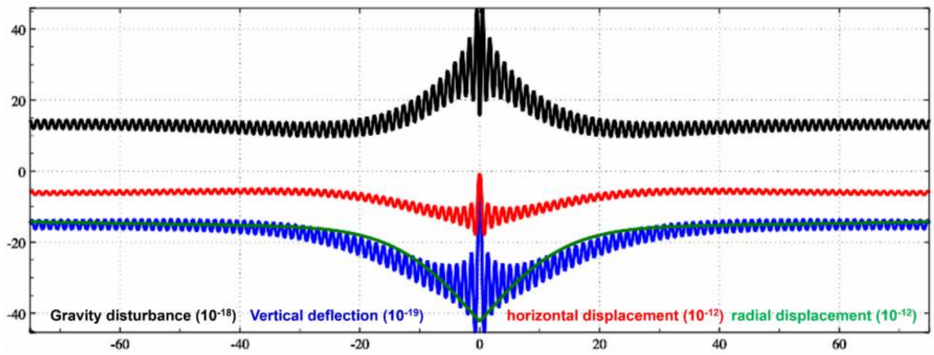


Fig 7.1 The near-zone characteristics of several load Green's functions (Indirect influence)

It is not difficult to find that when the Green's function integral is employed to calculate the load effect on the considered variable which is not dominant in the geopotential differential, such as the load effects on height anomaly, ground radial displacement or orthometric height, or load effects on ground gravity or tilt whose site has obvious vertical deformation, an acceptable integral result can be obtained. However, when being employed to calculate the load effect on gravity disturbance, vertical deflection or horizontal displacement, the integral result is very unstable, and its reliability is poor.

Similarly, for the monitoring to the land water and surface load deformation field, when the monitoring variable is GNSS ellipsoidal height variation, the regional land water variation and its load effect can be estimated by using load Green's function integral constraint. However, if the geopotential differential in the monitoring variable is dominant, such as the monitoring variable is gravity disturbance, vertical deflection, horizontal displacement or gravity gradient variations, it is difficult to obtain a stable solution by the load Green's function integral constraint method.

8.7.1 Spherical radial basis function representation of surface EWH

The surface load equivalent water height (EWH) $h_w(\boldsymbol{x})$ at the ground point $\boldsymbol{x}$ can be expressed as a linear combination of normalized surface harmonic basis functions:

$$h_w(\boldsymbol{x}) = r\sum_{n=2}^{N}\left(\frac{a}{r}\right)^n\sum_{m=-n}^{n}\bar{F}_{nm}\bar{Y}_{nm}(\boldsymbol{e}) \tag{7.1}$$

Where, $\boldsymbol{x} = r\boldsymbol{e} = r(sin\theta cos\lambda, sin\theta sin\lambda, cos\theta)$, r, λ, θ are the geocentric distance, longitude and colatitude of the surface point $\boldsymbol{x}$ respectively, $\bar{F}_{nm}$ is the fully normalized spherical harmonic coefficient, $\bar{Y}_{nm}$ is the normalized surface harmonic basis function, and a is the equatorial radius of the Earth, which means that $\bar{Y}_{nm}$ is defined on the spherical surface with the radius of a.

$$\bar{Y}_{nm}(\boldsymbol{e}) = \bar{P}_{nm}(cos\theta)cosm\lambda,\quad \bar{F}_{nm} = \delta\bar{C}_{nm},\quad m \geq 0$$
$$\bar{Y}_{nm}(\boldsymbol{e}) = \bar{P}_{n|m|}(cos\theta)sin|m|\lambda,\quad \bar{F}_{nm} = \bar{S}_{n|m|},\quad m < 0 \tag{7.2}$$

Here, $\bar{P}_{nm}(cos\theta)$ is the fully normalized associative Legendre function, n is called as the degree of the harmonic coefficient, and m is called as the order of harmonic coefficient.

If the domain of the surface spherical functions are transformed from the sphere surafce with radius a to the Bjerhammar spherical surface with radius $\mathcal{R}$, then the spherical harmonic coefficients $\bar{F}_{nm}$ should be converted into $\bar{E}_{nm}$, then $h_w(\boldsymbol{x})$ can also be expressed as the linear combination in the Bjerhammar spherical surface:

$$h_w(\boldsymbol{x}) = r\sum_{n=2}^{N}\left(\frac{\mathcal{R}}{r}\right)^n\sum_{m=-n}^{n}\bar{E}_{nm}\bar{Y}_{nm}(\boldsymbol{e}) \tag{7.3}$$

On the other hand, the surface load EWH $h_w(\boldsymbol{x})$ at any surface point $\boldsymbol{x}$ can also be expressed as a linear combination of K spherical radial basis functions (SRBFs) in the Bjerhammar spherical surface:

$$h_w(\boldsymbol{x}) = a\sum_{k=1}^{K}d_k\Phi_k(\boldsymbol{x}, \boldsymbol{x}_k) \tag{7.4}$$

Where, $\boldsymbol{x_k} = \mathcal{R}\boldsymbol{e_k}$ is the SRBF node defined on the Bjerhammar spherical surface, also known as the SRBF center or SRBF pole, d_k is the SRBF coefficient, K is the number of the SRBF nodes and equal to the number of SRBF coefficients, $\Phi_k(\boldsymbol{x}, \boldsymbol{x_k})$ is the spherical radial basis function of the EWH can be abbreviated as $\Phi_k(\boldsymbol{x}) = \Phi_k(\boldsymbol{x}, \boldsymbol{x_k})$.

The spherical radial basis function $\Phi_k(\boldsymbol{x}, \boldsymbol{x_k})$ can be furtherly expanded into the Legendre series:

$$\Phi_k(\boldsymbol{x}, \boldsymbol{x}_k) = \sum_{n=2}^{N} \phi_n P_n(\psi_k) = \sum_{n=2}^{N} \frac{2n+1}{4\pi} B_n \left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \tag{7.5}$$

Where, ϕ_n is the degree-n Legendre coefficient of SRBF, which characterizes the shape of SRBF and determines the spatial and spectral figures of SRBF, also known as the shape factor. When the spectral domain degree-n need be not emphasized, B_n is also called as the Legendre coefficient of SRBF. $\mu = R/r$ is also called as the bandwidth parameter since it is related to the spectral domain bandwidth of the sphericalradial basis function $\Phi_k(\boldsymbol{x})$.

Substituting formula (7.5) into (7.4), we have:

$$\begin{aligned} h_w(\boldsymbol{x}) &= \frac{r}{4\pi} \sum_{n=2}^{N} (2n+1) B_n \left(\frac{\mathcal{R}}{r}\right)^n \sum_{k=1}^{K} d_k P_n(\psi_k) \\ &= \frac{r}{4\pi} \sum_{k=1}^{K} d_k \sum_{n=2}^{N} (2n+1) B_n \left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \end{aligned} \tag{7.6}$$

Considering the addition theorem of spherical harmonics:

$$P_n(\psi_k) = P_n(\boldsymbol{e}, \boldsymbol{e}_k) = \frac{4\pi}{2n+1} \sum_{m=-n}^{n} \bar{Y}_{nm}(\boldsymbol{e}) \bar{Y}_{nm}(\boldsymbol{e}_k) \tag{7.7}$$

then the formula (7.5) can be written as

$$h_w(\boldsymbol{x}) = r \sum_{n=2}^{N} B_n \left(\frac{\mathcal{R}}{r}\right)^n \sum_{m=-n}^{n} \sum_{k=1}^{K} d_k \bar{Y}_{nm}(\boldsymbol{e}) \bar{Y}_{nm}(\boldsymbol{e}_k) \tag{7.8}$$

Comparing formulas (7.1), (7.3) and (7.8), we have:

$$\bar{F}_{nm} = \left(\frac{\mathcal{R}}{a}\right)^n \bar{E}_{nm} = B_n \left(\frac{\mathcal{R}}{a}\right)^n \sum_{k=1}^{K} d_k \bar{Y}_{nm}(\boldsymbol{e}_k) \tag{7.9}$$

Using formula (7.9), the spherical harmonic coefficient $\bar{F}_{nm}$ can be calculated from the spherical radial basis function coefficient d_k. After that, the spherical harmonic coefficient can be employed to calculate various load effects outside the solid Earth.

The location, distribution and amount of the SRBF nodes (centers) $\{\boldsymbol{x_k}\}$ on the Bjerhammar sphere surafce are the key indicators for load deformation field approach using spherical radial basis functions, which determine the spatial degrees of freedom (spatial resolution) and spatial feature of regional load deformation field, similar to the degree of the global surface load spherical harmonic coefficient model.

8.7.2 Spherical radial basis functions suitable for load deformation field

Some spherical radial basis functions such as the point mass kernel function, Poisson kernel function, radial multipole kernel function and Poisson wavelet kernel function are all harmonic, which are suitable for load deformation field approach.

Let $\boldsymbol{x}$ be the surface calculation point and $\boldsymbol{x_k}$ be the SRBF center on the Bjerhammar spherical surface $\Omega_{\mathcal{R}}$.

(1) The point mass kernel function

The point mass kernel function is an inverse multiquadric function (IMQ) proposed by Hardy (1971), which is the kernel function of gravitational potential integral formula $V = G \iiint \frac{dm}{L}$, and can be analytically expressed as:

$$\Phi_{IMQ}(\boldsymbol{x},\boldsymbol{x}_k)=\frac{1}{L}=\frac{1}{|x-x_k|} \tag{7.10}$$

Where, L is the space distance between $\boldsymbol{x}$ and $\boldsymbol{x_k}$. Since $\Delta(1/L)=0$, the point mass kernel function $\Phi_{IMQ}(\boldsymbol{x},\boldsymbol{x_k})$ satisfies the Laplace equation.

(2) The Poisson kernel function

The Poisson kernel function is derived from the Poisson integral formula, and can be analytically expressed as::

$$\Phi_P(\boldsymbol{x},\boldsymbol{x}_k)=-2r\frac{\partial}{\partial r}\left(\frac{1}{L}\right)-\frac{1}{L}=\frac{r^2-r_k^2}{L^3} \tag{7.11}$$

Here, r, r_k are the spherical radial distance (namely spherical center distance) of $\boldsymbol{x}$ and $\boldsymbol{x_k}$, respectively.

(3) The radial multipole kernel function

The analytical expression of the radial multipole kernel function is:

$$\Phi_{RM}^m(\boldsymbol{x},\boldsymbol{x}_k)=\frac{1}{m!}\left(\frac{\partial}{\partial r_k}\right)^m\frac{1}{L} \tag{7.12}$$

Where, m can be called as the order of the radial multipole kernel function, and the zero-order radial multipole kernel function is the point mass kernel function, namely $\Phi_{IMQ}(\boldsymbol{x},\boldsymbol{x}_k)=\Phi_{RM}^0(\boldsymbol{x},\boldsymbol{x}_k)$.

(4) The Poisson wavelet kernel function

The analytical expression of the Poisson wavelet kernel function is:

$$\Phi_{PW}^m(\boldsymbol{x},\boldsymbol{x}_k)=2(\chi_{m+1}-\chi_m),\quad \chi_m=\left(r_k\frac{\partial}{\partial r_k}\right)^m\frac{1}{L} \tag{7.13}$$

The zero-order Poisson wavelet kernel function is the Poisson kernel function $\Phi_P(\boldsymbol{x},\boldsymbol{x}_k)=\Phi_{PW}^0(\boldsymbol{x},\boldsymbol{x}_k)$.

(5) Calculation of spherical radial basis functions

The spherical radial basis function analytical expressions (7.10) ~ (7.13) are usually expressed in the Legendre series (7.5), and then calculated according to the Legendre expansion to highlight the spectral domain figures of the load deformation field.

ETideLoad4.5 normalizes the Legendre expansion of the spherical radial basis function $\Phi_k(\boldsymbol{x},x_k)$, and then calculates the spherical radial basis functions (SRBF) using the normalized Legendre expansion.

Let the spherical angular distance $\psi_k=0$ from $\boldsymbol{x}_k$ to $\boldsymbol{x}$, then $cos\psi_k=1$, $P_n(cos\psi_k)=P_n(1)=1$, substitute these into formula (7.5), we have the general expression of the normalization coefficient of SRBF:

$$\Phi^0=\sum_{n=2}^{N}\frac{2n+1}{4\pi}B_n\mu^n \tag{7.14}$$

The Legendre expansion of the normalized spherical radial basis function is:

$$\Phi_k(\boldsymbol{x},\boldsymbol{x}_k)=\frac{1}{\Phi^0}\sum_{n=2}^{N}\phi_nP_n(\psi_k)=\frac{1}{\Phi^0}\sum_{n=2}^{N}\frac{2n+1}{4\pi}B_n\mu^nP_n(\psi_k) \tag{7.15}$$

The mentioned four forms of SRBF and their corresponding Legendre coefficients are shown in Tab 7.1.

Tab 7.1 Spherical radial basis functions (SRBFs) and corresponding Legendre coefficients

SRBF	$\Phi_k(x,x_k)$	ϕ_n	B_n
Point mass kernel	$\frac{1}{L}=\frac{1}{\|x-x_k\|}$	μ^n	$\frac{4\pi}{2n+1}$
Poisson kernel function	$\frac{r^2-r_k^2}{L^3}$	$(2n+1)\mu^n$	4π
radial multipole kernel	$\frac{1}{m!}\left(\frac{\partial}{\partial r_k}\right)^m\frac{1}{L}$	$C_n^m\mu^{n-m}\ \ (n\geq m)$	$\frac{4\pi C_n^m}{2n+1}\mu^{-m}$
Poisson wavelet kernel	$2(\chi_{m+1}-\chi_m)$ $\chi_m=\left(r_k\frac{\partial}{\partial r_k}\right)^m\frac{1}{L}$	$(-nln\mu)^m(2n+1)\mu^n$	$4\pi(-nln\mu)^m$

8.7.3 Unit spherical Reuter grid construction algorithm

Given the Reuter grid level Q (even number), the geocentric latitude interval $d\varphi$ of the unit spherical Reuter grid in the spherical coordinate system and the geocentric latitude φ_i of the center of the cell-grid i are respectively

$$d\varphi=\frac{\pi}{Q},\quad \varphi_i=-\frac{\pi}{2}+\left(i-\frac{1}{2}\right)d\varphi,\quad 1\leq i<Q \tag{7.16}$$

The cell-grid number J_i in the prime vertical circle direction at latitude φ_i, the longitude interval $d\lambda_i$ and the side length dl_i are respectively

$$J_i=\left[\frac{2\pi cos\varphi_i}{d\varphi}\right],\quad d\lambda_i=\frac{2\pi}{J_i},\quad dl_i=d\lambda_i cos\varphi_i \tag{7.17}$$

It is not difficult to find that $dl_i\approx d\varphi$. Let

$$\varepsilon_i=\frac{ds_i-ds}{ds}=\frac{dl_i-d\varphi}{d\varphi}=\frac{d\lambda_i}{d\varphi}cos\varphi_i-1 \tag{7.18}$$

Where, ds is the cell-grid area near the equator, ds_i is the cell-grid area at the prime vertical circle φ_i, and ε_i is the relative bias of the parallel circle cell-grid area relative to the cell-grid area near the equator.

ε_i is generally small, about a few ten-thousandth, and the value is related to the Reuter grid level Q. Near the equator, we have $ds=d\varphi\cdot d\varphi$, $\varepsilon_{Q/2}=0$.

Given the range of longitude and latitude of the local area, you can directly determine the minimum and maximum value of i according to the formula (7.16), and then calculate the maximum J_i at each prime vertical circle according to the formula (7.17), to determine the regional Reuter grid whose level is Q without calculating the global grid.

8.7.4 SRBF representation for load effects on all-element geodetic variations

According to the surface load effect spherical harmonic series expansion (2.8)~(2.20), the spherical radial basis function parameterization of the load effects on various geodetic variations can be derived from surface EWH spherical radial basis function expansion (7.6) as follows. Where, the SRFB parameterization of surafce EWH is rewritten as:

$$\Delta h_w(\boldsymbol{x})=r\sum_{k=1}^K d_k\sum_{n=2}^N(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \tag{7.19}$$

SRFB parameterization of the load effect on height anomaly:

$$\Delta\zeta = \frac{3\rho_w}{\rho_e}\frac{GM}{\gamma r}\sum_{k=1}^{K} d_k \sum_{n=2}^{N} B_n(1+k_n')\left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \tag{7.20}$$

SRFB parameterization of the load effect on ground gravity◉:

$$\Delta g^s = \frac{3\rho_w}{\rho_e}\frac{GM}{r^2}\sum_{k=1}^{K} d_k \sum_n (n+1)\left(1+\frac{2}{n}h_n' - \frac{n+1}{n}k_n'\right) B_n\left(\frac{\mathcal{R}}{r}\right)^{n-1} P_n(\psi_k) \tag{7.21}$$

SRFB parameterization of the load effect on gravity (distrubance):

$$\Delta g^\delta = \frac{3\rho_w}{\rho_e}\frac{GM}{r^2}\sum_{k=1}^{K} d_k \sum_n (n+1)(1+k_n')B_n\left(\frac{\mathcal{R}}{r}\right)^{n-1} P_n(\psi_k) \tag{7.22}$$

SRFB parameterization of the load effect on ground tilt◉:

$$\text{South: } \Delta\xi^s = \frac{3\rho_w}{\rho_e}\frac{GM}{\gamma r^2}\sum_{k=1}^{K} d_k \cos\alpha_k \sum_n (1+k_n'-h_n')B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{7.23}$$

$$\text{West: } \Delta\eta^s = \frac{3\rho_w}{\rho_e}\frac{GM}{\gamma r^2}\sum_{k=1}^{K} d_k \sin\alpha_k \sum_n (1+k_n'-h_n')B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{7.24}$$

SRFB parameterization of the load effect on vertical deflection:

$$\text{South: } \Delta\xi = \frac{3\rho_w}{\rho_e}\frac{GM}{\gamma r^2}\sum_{k=1}^{K} d_k \cos\alpha_k \sum_n (1+k_n')B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{7.25}$$

$$\text{West: } \Delta\eta = \frac{3\rho_w}{\rho_e}\frac{GM}{\gamma r^2}\sum_{k=1}^{K} d_k \sin\alpha_k \sum_n (1+k_n')B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{7.26}$$

SRFB parameterization of the load effect on ground site displacement ◉:

$$\text{East: } \Delta e = -\frac{3\rho_w}{\rho_e}\frac{GM}{\gamma r}\sum_{k=1}^{K} d_k \cos\alpha_k \sum_n l_n' B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{7.27}$$

$$\text{North: } \Delta n = -\frac{3\rho_w}{\rho_e}\frac{GM}{\gamma r}\sum_{k=1}^{K} d_k \sin\alpha_k \sum_n l_n' B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{7.28}$$

$$\text{Radial: } \Delta r = \frac{3\rho_w}{\rho_e}\frac{GM}{r\gamma}\sum_{k=1}^{K} d_k \sum_{n=2}^{N} B_n h_n'\left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \tag{7.29}$$

SRFB parameterization of the load effect on normal (orthometric) height◉:

$$\Delta\hbar = \frac{3\rho_w}{\rho_e}\frac{GM}{\gamma r}\sum_{k=1}^{K} d_k \sum_{n=2}^{N} B_n(h_n'-k_n'-1)\left(\frac{R}{r}\right)^n P_n(\psi_k) \tag{7.30}$$

SRFB parameterization of the load effect on gravity gradient:

$$\text{Radial: } \Delta T_{rr} = \frac{3\rho_w}{\rho_e}\frac{GM}{r^3}\sum_{k=1}^{K} d_k \sum_n (n+1)(n+2)(1+k_n')B_n\left(\frac{\mathcal{R}}{r}\right)^{n-1} P_n(\psi_k) \tag{7.31}$$

$$\text{North: } \Delta T_{NN} = \frac{3\rho_w}{\rho_e}\frac{GM}{r^3}\sum_{k=1}^{K} d_k \frac{\partial^2\psi_k}{\partial\varphi_k^2}\sum_n (1+k_n')B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial^2 P_n(\psi_k)}{\partial\psi_k^2} \tag{7.32}$$

$$\text{West: } \Delta T_{WW} = -\frac{3\rho_w}{\rho_e}\frac{GM}{r^3\cos^2\varphi}\sum_{k=1}^{K} d_k \frac{\partial^2\psi_k}{\partial\lambda_k^2}\sum_n (1+k_n')B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial^2 P_n(\psi_k)}{\partial\psi_k^2} \tag{7.33}$$

Where, α_k is the geodetic azimuth of ψ_k.

In the mentioned expressions, the load effects on the geodetic variations marked ◉ are valid only when their sites are fixed with the solid Earth, and that on the remaining geodetic variations are suitable on the ground or outside the solid Earth.

Similar to the spatial load Green's function integral method, if the regional surface load equivalent water height h_w is known, the SRBF spectral domain analysis of the load equivalent water height h_w can be performed according to Formula (7.19) to estimate the SRBF coefficients. ETideLoad4.5 calls this process as the load SRBF approach. The regional all-element load deformation field can be calculated from SRBF coefficients according to Formulas (7.20) to (7.33). ETideLoad4.5 calls this process as the load effect

SRBF synthesis.

Here, the point mass kernel function is selected as SRBF, and the minimum degree and the maximum degree are set to be 90 and 1800 respectively. The buried depth of the Bjerhammar sphere is set to be 5 km, and the maximum action distance of SRBF center is 150 km. The SRBF spatial curves of the load effects on gravity disturbance, vertical deflection, horizontal displacement, and radial displacement are calculated, as shown in Fig 2.

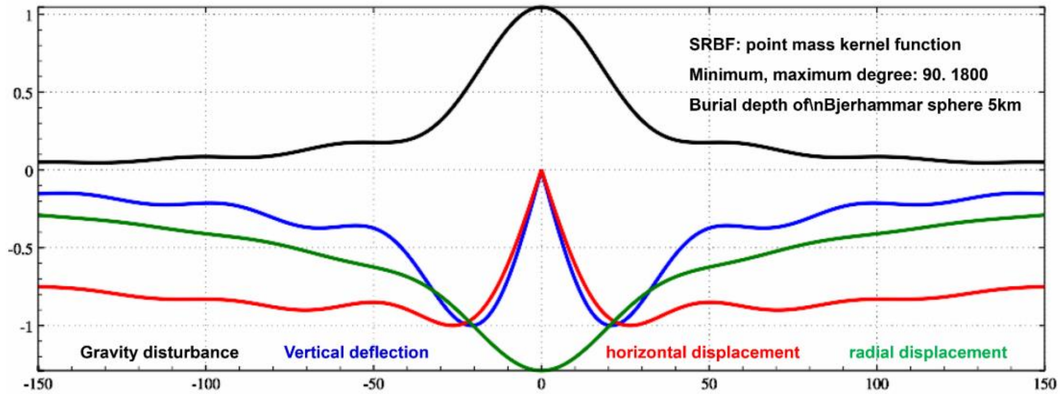


Fig 7.2 The near-zone characteristics of load effect SRBFs on several geodetic variations

Comparing Fig 7.1 and Fig 7.2, it can be seen that the convergence property of SRBF near the calculation point is obviously better than that of load Green's function even for the load effect on radial displacement. The SRBFs of load effects on gravity disturbance, vertical deflection and horizontal displacement are monotonically convergent within 20 km near the calculation point. It can be seen that the high-degree oscillation and non-convergence problems of load Green’s function can be effectively solved by using SRBF instead.

8.7.5 First and second order horizontal partial derivatives of ψ

The algorithm formulas of the SRBFs of the load effect need to employ the horizontal first-order and second-order partial derivatives of the spherical angle distance ψ, whose derivation process in the spherical coordinate system is given below.

$$\frac{\partial\psi}{\partial\varphi}=-cos\alpha,\quad \frac{\partial\psi}{\partial\lambda}=-cos\varphi sin\alpha \tag{7.34}$$

Where, α is the geodetic azimuth of ψ, which can be obtained from the spherical trigonometric formulas:

$$sin\psi cos\alpha=cos\varphi sin\varphi'-sin\varphi cos\varphi' cos(\lambda'-\lambda) \tag{7.35}$$

$$sin\psi sin\alpha=cos\varphi' sin(\lambda'-\lambda) \tag{7.36}$$

Taking the partial derivative with respect to φ on both sides of Equation (7.35), considering (7.34), we have:

$$-cos\psi cos^2\alpha+sin\psi\frac{\partial^2\psi}{\partial\varphi^2}=-\sin\varphi sin\varphi'-cos\varphi cos\varphi' cos(\lambda'-\lambda) \tag{7.37}$$

so that we have:

$$sin\psi \frac{\partial^2\psi}{\partial\varphi^2} = -\sin\varphi sin\varphi' - cos\varphi cos\varphi' cos(\lambda' - \lambda) + cos\psi cos^2\alpha \tag{7.38}$$

In the same way, taking the partial derivatives of both sides of Equation (7.36) with respect to λ, we have:

$$-cos\psi cos\varphi sin^2\alpha + sin\psi \frac{\partial^2\psi}{\partial\lambda^2} = -cos\varphi' sin(\lambda' - \lambda) \tag{7.39}$$

$$sin\psi \frac{\partial^2\psi}{\partial\lambda^2} = -cos\varphi' sin(\lambda' - \lambda) + cos\psi cos\varphi sin^2\alpha \tag{7.40}$$

8.7.6 Regional SRBF approach of surface loads and synthesis of load effects

Similar to the 'Remove - load Green's function integral - Restore' scheme of regional load deformation field approach using the load spherical harmonic coefficient model reference field, the regional high-resolution load deformation field SRBF approach can also adopt the 'remove- load SRBF approach-restore' scheme based on the load spherical harmonic coefficient model reference field. Here, the residual load SRBF approach is employed to instead of the residual load Green's function integral.

(1) The scheme for SRBF approach of residual loads and synthesis of load effects

Similar to the global load spherical harmonic analysis and the load effect spherical harmonic synthesis process, the residual load SRBF approach scheme also consists of two steps. In the first step, according to the regional surface load SRBF spectral domain expansion (7.19), the SRBF coefficients $\{d_k\}$ are estimated by the least square method from the regional residual EWHs. The step can be called as the regional load SRBF analysis. In the second step, according to the regional load effect SRBF synthesis algorithm formulas (7.20) ~ (7.33), the SRBF coefficients $\{d_k\}$ are employed to calculate the residual load effects on various geodetic variations. The second step can be called as the SRBF synthesis for regional load effects.

Similar to the global load spherical harmonic analysis method, the cumulative iterative SRBF analysis scheme can be employed to improve the SRBF approach level of regional residual EWHs.

(2) SRBF approach example of regional load deformation field using remove-restore scheme

The regional SRBF approach of the high-resolution load deformation field can also adopt the remove-restore scheme, that is, the 'load Green's function integral' in the 'remove-load Green's function integral-restore' scheme is replaced by 'load SRBF approach'. Where, the 'load SRBF approach' adopts a smaller SRBF center action distance (similar to the integral radius of the Green's function) to construct the residual EWH observation equation and estimate the SRBF coefficients. Then, the high-resolution residual grid of regional load deformation field is obtained by SRBF synthesis. The scheme can be called as the 'remove - load SRBF approach - restore' scheme.

Taking the 1′×1′ land water equivalent water height (EWH) variation grid (cm) at a sampling epoch time on May 30, 2018 in a region of southern China (taking the mean value of land water variation in the region in 2018 as the monitoring datum) as an example, the key steps of the SRBF approach of regional load deformation field are introduced. The land water here includes only 4m shallow soil water, wetland and vegetation water, but not including lakes, rivers and groundwater. The reference load deformation field adopts the 360-degree global terrestrial water variation load spherical harmonic coefficient model constructed in section 8.2.6 on May 30, 2018.

Similar to the regional refinement scheme of load Green's integral, the land water EWH variation grid area (the calculation area) should be generally bigger than refine result area of the load deformation field to suppress the edge effect. In this case, the calculation area is 97˚~103˚E, 24˚~29˚N, and the result area is 98.5˚~101.5˚E, 25.5˚~27.5˚N.

The step 1: Input the 1′×1′ zero value grid in the calculation area (zero value means that the height of calculation point relative to the ground is equal to zero), select the maximum calculation degree of 360, and calculate the 1′×1′ land water EWH variation reference model value grid in the calculation area from the global land water load spherical harmonic coefficient model, as shown in Fig 7.3 left.

The step 2: The 1′×1′ land water EWH residual value grid (Fig 7.3 right) is generated by subtracting the 1′×1′ land water EWH observation grid (Fig 7.3 left) from the 1′×1′ land water EWH model value grid (Fig 7.3 middle).

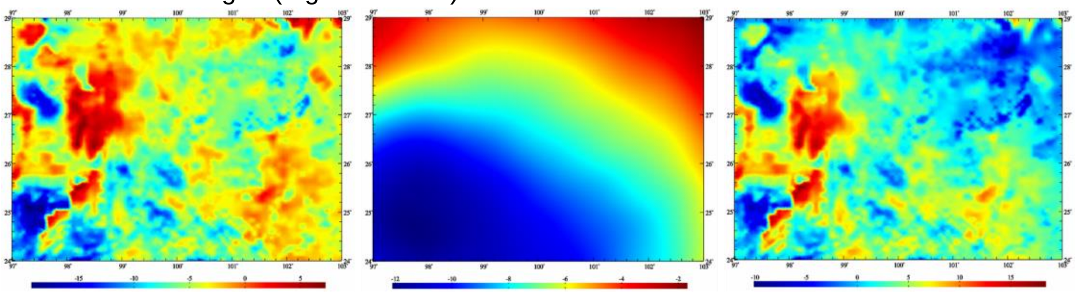


Fig 7.3 The 1′×1′ land water EWH variation obseration, model value and residual grid in the calculation area

The step 3: According to the load EWH SRBF expansion (7.19), the SRBF observation equations are constructed from the 1'×1' land water EWH variation residual grid, and the SRBF coefficients are estimated using the iterative least square method to cumulatively approach the EWH variation residual grid. Then, from the SRBF coefficients, according to load effect SRBF synthesis algorithm formulas (7.20) ~ (7.33), the 1′×1′ land water load deformation field residual grid are calculated, as shown in Fig 7.4.

The step 4: Input the 1′×1′ zero value grid in the result area, select the maximum calculation degree of 360, and calculate the 1′×1′ land water load deformation field model value grid from the global land water load spherical harmonic coefficient model.

The step 5: the 1′×1′ land water load deformation field residual value grid in the result area is added to the reference model value grid to obtain the 1′×1′ land water variation load deformation field grid refined in the result area, as shown in Fig 7.5.

Compared with the 8.5.5 section process, it is not difficult to find that the ‘remove - load SRBF approach - restore’ scheme and the 'remove - load Green's function integral - restore' scheme are only different in the step 3, while the step 1, 2, 4 and 5 are exactly the same.

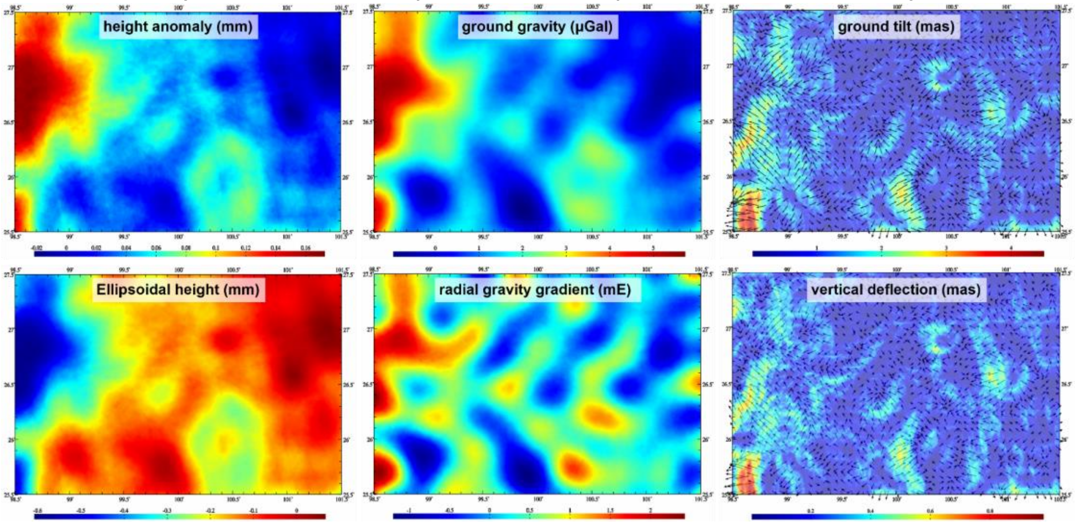


Fig 7.4 The 1′×1′ land water load deformation field residual value grid using load SRBF approach

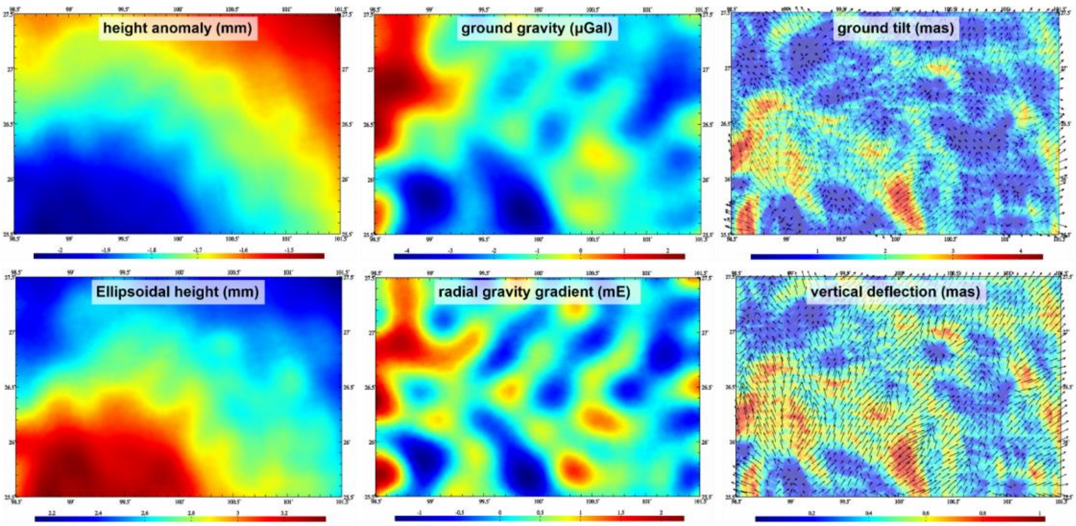


Fig 7.5 The 1′×1′ land water variation load deformation field grid refined using SRBFs in the result area

(3) SRBF coefficients estimation with edge effect suppression

Substituting the Legendre coefficient B_n in Tab 7.1 into the load EWH SRBF expansion (7.19), we can obtain the observation equations with the residual EWH variations $\Delta\tilde{h}_w(\boldsymbol{x}_i)$ as the observations and the SRBF coefficients $\{d_k\}$ as the unknowns.

$$L = \{\Delta\tilde{h}_w(\boldsymbol{x}_i)\}^T = \boldsymbol{A}\{d_k\}^T + \boldsymbol{\epsilon} \quad (i = 1, \cdots, M \; ; \; k = 1, \cdots, K) \tag{7.41}$$

Where, $\boldsymbol{A}$ is the $M \times K$ design matrix, $\boldsymbol{\epsilon}$ is the $M \times 1$ observation error vector, M is the number of observations, K is the number of RBF centers, that is, the number of unknowns $\{d_k\}$, and $\boldsymbol{x}_i$ is the location coordinates of the observations.

ETideLoad proposes an algorithm that can improve the performance of parameter estimation by suppressing edge effect. When the SRBF center v is located at the margin of the calculation area, its corresponding SRBF coefficient is set to zero, that is, $d_v = 0$ as the observation equation to suppress the edge effect. The normal equation with the additional suppression of edge effect constructed by ETideLoad is:

$$[\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A} + Q\Xi]\{d_k\}^T = \boldsymbol{A}^T\boldsymbol{P}\boldsymbol{L} \tag{7.42}$$

Here, Ξ is a diagonal matrix, whose element is equal to 1 only when the SRBF center corresponding to its subscript is in the margin of the area, and the others are zero. Q is equal to the root mean square of the non-zero diagonal elements of the coefficient matrix $\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A}$ of normal equation.

The action distance dr of SRBF centers is required to be equal to maintain the spatial consistency of the approach performance of load deformation field. Where dr corresponds to the argument domain of the SRBF coefficient, so any observation is a linear combination of the SRBFs whose centers only within the radius dr.

ETideLoad improves the ill-conditioned or singularity of $\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A}$ by adding some observation equations that can suppresses edge effect to improve the stability and reliability of parameter estimation.

(4) Comparative analysis of calculation results between the load Green's function integral and SRBF approach

The step 3 in the calculation process of the ‘remove - load SRBF approach - restore’ scheme above is replaced by the load Green's function integral, which becomes the 'remove - load Green's function integral - restore' process, namely,

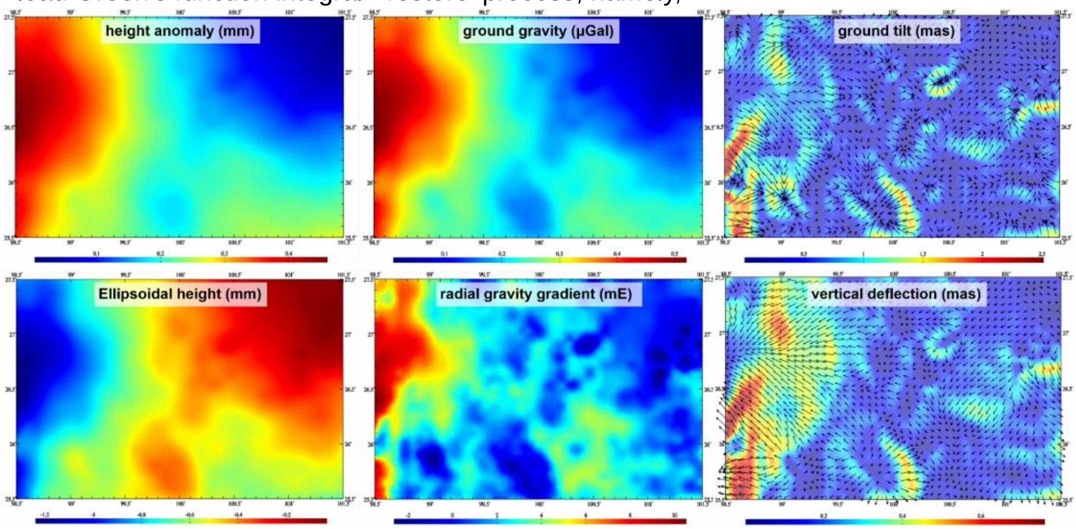


Fig 7.6 The 1′×1′ land water load deformation field residual value grid using load Green's integral

The step 3: From the 1'×1' land water EWH variation residual grid, the integral radius (equivalent to the center distance of SRBF) of 150 km selected, the 1′×1′ land water load deformation field residual grid are calculated according to load Green's function integral formulas, as shown in Fig 7.6.

Comparing Fig 7.4 and Fig 7.6, it can be seen that the spatial distribution characteristics of various geodetic variations of load effects calculated by the two schemes are all similar. The numerical results by the load Green's function integral are larger, and the spatial short-wave structure of numerical results by the load SRBF approach are rich.

8.7.7 Principle of cooperative monitoring from multiple heterogeneous geodetic techniques

ETideLoad4.5 proposes a deep fusion method for multi-source heterogeneous observation systems with good universality. Without losing generality, it is assumed that there are multiply heterogeneous geodetic monitoring techniques for a certain monitoring purpose. We can always express abstractly the monitoring purpose as a set of common parameters shared by various geodetic techniques, so as to transform the cooperative monitoring problem from multiply heterogeneous geodetic techniques into a mathematical problem on jointly solving of the common parameters.

The general scheme for solving such problems can be summarized as follows: (1) The observation equations with the common parameters as unknown parameters are composed of the monitoring variations from each group of geodetic techniques according to their respective geodetic system structures (namely the covariance structure based on respective geodetic principle), and then these normal equations are constructed according to the principle of least squares and normalized, respectively. (2) According to the monitoring quality of each geodetic technique (system), the normal equations are weighted respectively, and the combined normal equations are generated by weighted summation to solve the unknown common parameters.

For different geodetic techniques, the types and spatial distribution of monitoring variations and covariance properties of geodetic monitoring systems are generally different. It is necessary to normalize these different groups of normal equations (covariance structures) in order to effectively control the deep fusion of various types of monitoring variations using the respective covariance structure. The combined normal equation can be expressed as:

$$\sum_s \left(\frac{w_s}{Q_s}\boldsymbol{A}_s^T\boldsymbol{P}_s\boldsymbol{A}_s\right)\boldsymbol{X} = \sum_s \left(\frac{w_s}{Q_s}\boldsymbol{A}_s^T\boldsymbol{P}_s\boldsymbol{L}_s\right) \tag{7.43}$$

Where, $s = 1,\cdots,S$, S is the number of groups; $\boldsymbol{X}$ is the common parameter vector to be estimated ; $\boldsymbol{A}_S,\boldsymbol{L}_S$ and $\boldsymbol{P}_s$ are the design matrix, observation vector and observation weight matrix of the observation equation from the s group, respectively. The $\boldsymbol{P}_s$ of the observations from the s group is only employed to distinguish the difference between the group s of observation errors, which is completely independent with other group of observation errors. Q_s is the normalized parameter of the group s of normal equation, which can be the root mean square of the non-zero diagonal elements of the group s of normal equation coefficient matrix $\boldsymbol{A}_s^T\boldsymbol{P}_s\boldsymbol{A}_s$. w_s is the group s of weight only employed to distinguish the observation quality of different groups. Because the number S of the groups is generally not large (such as $S < 10$), it is easy to furtherly optimize w_s using the conventional statistical optimization scheme.

The combination parameter $\delta_s = Q_s/w_s$ in the normal equation (7.43) can completely separate the contributions of the observation system (covariance structure) from influences of observation quality w_s, so that the combination process (collaborative algorithm) is not affected by the observation errors. Therefore, it is conducive to the deep fusion of various geodetic variations from multiply monitoring systems with the diverse observation types, different spatial distribution and various observation system structure.

Index for ETideLoad4.5 scientific computing functions

Names table of the sample directories and executable files

No	Program name	Sample directory name / Executable program name
1	Computation of solid tidal effects on various geodetic variations outside solid Earth	Tideffectsolidearth
2	Spherical harmonic synthesis on ocean tidal load	OTideloadharmsynth

	effects outside solid Earth	
3	Spherical harmonic synthesis on atmosphere tidal load effects outside solid Earth	ATideloadharmsynth
4	Computation of Earth's rotation polar shift effects on geodetic variations and tidal effects on EPR	Poleshifteffectscalc
5	Computation of the permanent tidal and Earth's mass centric variation effects on geodetic variations	Permanentdgeocenter
6	Computation of solid Earth and load tidal effects on geodetic networks	Controlnetworktidef
7	The regional approach of load tidal effects by load Green's Integral	Tdloadgreenintegral
8	Forecast of various tidal effects on surface all-element geodetic variations	SolidLoadtidecalctl
9	Separation and processing of gross errors in geodetic variation time series	TmsrsErrorseppreproc
10	Low-pass filtering and signal reconstructing for irregular time series	Tmsrslowpfltrconstr
11	Weighted operation, difference, integral and interpolation on time series	TmsrsAddifferinterp
12	Normalized extraction from batch time series of geodetic monitoring network	Tmsrsbatchnormalize
13	Processing and analysis on batch time series of geodetic monitoring network	Tmsrsnetwkanalyspro
14	Construction and analysis on record time series from geodetic network	Tmrecordanalysproc
15	Processing and analysis on variation (vector) grid time series	Tmgridanalysisproc
16	Multi-form spatiotemporal interpolation from grid time series	Tmgrdinterpolation
17	Spherical harmonic analysis on global surface load time series	Loadspharmonanalys
18	Spherical analysis on tidal harmonic constants and construction of tidal load model	Loadtidespharmsynth
19	Computation of the load model value by spherical harmonic synthesis	Loadspharmsynthesis
20	Computation of load deformation field by spherical harmonic synthesis	Loadeformharmsynth
21	Regional refinement of load deformation field by Green's Integral	Loadfmrntgreenintg
22	Regional approach of load deformation field using SRBFs	LoadfmtewhSRBF
23	Load deformation field monitoring from heterogeneous variations with Green's integral constraints	LoadestimateGreen
24	Load deformation field monitoring from heterogeneous variations by spherical radial basis	LoadestimateSRBF

	functions	
25	Geodynamic calculation on geodetic field grid time series	Loadfmgridtmdyncalc
26	Complete computation processes of high-resolution regional load deformation field time series	Loadfmfdcalcdemo
27	Heterogeneous collaborative monitoring process of groundwater variations and load deformation field	Landwdfmonitordemo
28	Pseudo-stable adjustment of record time series for geodetic network variations	Tmrecordnetwkadjust
29	Gross error detection and spatial deformation analysis on InSAR variations	DynInSARsptmanalyse
30	Collaborative monitoring and processing of InSAR with CORS network	DynCORScntrtmInSAR
31	Deep fusion and time series analysis on multi-source InSAR variations	DynInSARfusiontmsqu
32	Computation of ground stability variation based on variation vectors	Dyngrndhgtstablility
33	Computation of ground stability variation based on gravity variations	Dyngrngravstablility
34	Computation of ground stability variation based on variation vectors	Dyndeflectstablility
35	Statistical synthesis and prediction of ground stability variations	Dynstabgrdintgrestm
36	Conversion of general ASCII data into ETideLoad format	EdPntrecordstandard
37	Data interpolation, extracting and land-sea area separation	Edatafsimpleprocess
38	Simple and direct calculation on geodetic data files	EdFlgeodatacalculate
39	Operations on variation time series with same specifications	Edtimeseriesfilescalc
40	Generating and constructing of regional geodetic grid	Edareageodeticdata
41	Constructing and transforming of vector grid file	EdVectorgridtransf
42	Statistical analysis on various geodetic data files	Tlstatisticanalysis
43	Gross error detection and weighted basis function gridding	AppGerrweighgridate
44	Visualization for multi-attributes in ground variation time series	Veiwtimesqu
45	Visualization for variation record time series on geodetic network	Viewtmrecords
46	Visualization for specified attribute in discrete point file	Viewpntdata
47	Visualization for geodetic grid and variation grid time series file	Viewgridata
48	Visualization for the geodetic vector grid file	Viewvectgrd

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