

Classroom Teaching, Self-Exercise, Science Research and Engineer Computing

Precise Approach of Earth Gravity Field and Geoid

PAGravf4.5 User Reference

- 🌐 Cross aliasing of heterogeneous observations in land-sea-space
- 🌐 Various terrain effects on all-element gravity field in whole space
- 🌐 Loop closed analytical operations on outer gravity field elements
- 🌐 All-element modeling on Earth gravity field in whole outer space
- 🌐 Index measurement of observation errors and computation control
- 🌐 Gravity prospecting modelling from heterogeneous observations



Chinese Academy of Surveying and Mapping

Chuanyin Zhang, October 2024, Beijing, China

Precise Approach of Earth Gravity Field and Geoid (PAGravf4.5) is a large Windows program package for geodetic scientific computation according to stationary gravity field theory, which includes five subsystems, namely, data analysis and preprocessing calculation of Earth gravity field, computation of various terrain effects on various field elements outside geoid, precision approach and all-element modelling on Earth gravity field, optimization, unification and application for regional height datum as well as editing, calculation and visualization tools for geodetic data files.

Strictly according to the approach theory of gravity field, PAGravf4.5 efficiently deals with various terrain effects on various gravity field elements outside geoid. Scientifically constructs the gravity field approach algorithm system with the spatial domain integrals based on boundary value theory and spectral domain radial basis function approaches to realize the all-element modelling on gravity field in whole Earth space on or outside the geoid and the fine gravity prospecting modelling from various heterogeneous observations. And develops some ingenious algorithms based on physical geodesy to improve and unify the regional height datum, so as to consolidate and expand the applications of Earth gravity field.

The basic principles, main methods and all the formulas in physical geodesy and Earth gravity field have been realized completely in PAGravf4.5 to improve high education environment. Many long-term puzzles such as various terrain effects on various observations, all-element analytical modelling on gravity field, fine gravity prospecting modelling from heterogeneous observations, external accuracy index measurement and computational performance control have been effectively solved to strengthen the application capacity of Earth gravity field.

PAGravf4.5 is suitable for senior undergraduates, graduate students, scientific researchers, and engineering technicians in geodesy and geophysics, geology and geoscience, geomatics and geographic information, seismic and geodynamics.

Key words: terrain effect, gravity field approach, geoid, spherical radial basis function, height datum, geodetic computation.

<https://www.zcyphygeodesy.com/en/>

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1 PAGravf4.5's features, strengths, concepts, and usage

Precise Approach of Earth Gravity Field and Geoid (PAGravf4.5) is a large Windows program package for scientific computation rigorously based on stationary gravity field theory. Strictly according to physical geodesy, PAGravf4.5 constructs the unified analytical algorithm system of various terrain effects on various gravity field elements on or outside the geoid to improve the gravity prospecting modelling and gravity field data processing. Scientifically constructs the gravity field approach algorithm system with the spatial domain integrals based on boundary value theory and spectral domain radial basis function approaches to realize the all-element modelling on gravity field in whole Earth space on or outside the geoid and fine gravity prospecting modelling from various heterogeneous observations. And develops some ingenious physical geodetic algorithms to improve and unify the regional height datum, so as to consolidate and expand the applications of Earth gravity field.

Precise Approach for Earth Gravity field and Geoid PAGravf4.5



Summary, parameter settings and visualization for PAGravf4.5



Data analysis and preprocessing calculation of Earth gravity field



Computation of various terrain effects on various field elements outside geoid

☆ The basic principles, main methods and all the formulas in physical geodesy and Earth gravity field have been realized completely in PAGravf4.5 to improve high education environment.

☆ Many long-term puzzles such as various terrain effects on various observations, all-element analytical modelling on gravity field, fine gravity prospecting modelling from heterogeneous observations, external accuracy index measurement and computational performance control have been effectively solved to strengthen the application capacity of Earth gravity field.

PAGravf4.5 scientific computation programs organization structure

www.zcphygeodesy.com/en/



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PAGravf4.5
Chinese Academy of Surveying & Mapping
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☆ PAGravf4.5 is suitable for senior under-graduates, graduate students, scientific researchers, and engineer technicians in geodesy and geophysics, geology and geoscience, geomatics and geographic information, seismic and geodynamics.

☆ There are the example files saved in the folder C:\PAGravf4.5_win64en\examples for each program, which includes the operation process file process.txt, some input-output data files and screenshots. It will take about 5 working days to complete all the example exercises. Thereafter, you can use PAGravf4.5 alone.

Geodetic data format and geodetic quantity convention in PAGravf4.5



Precise approach and all-element modeling on Earth gravity field



Optimization, unification, and application for regional height datum



Editing and calculation tools for geodetic data files

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- Cross aliasing of heterogeneous observations in land-sea-space
- Loop closed analytical operations on outer gravity field elements
- Index measurement of observation errors and computation control

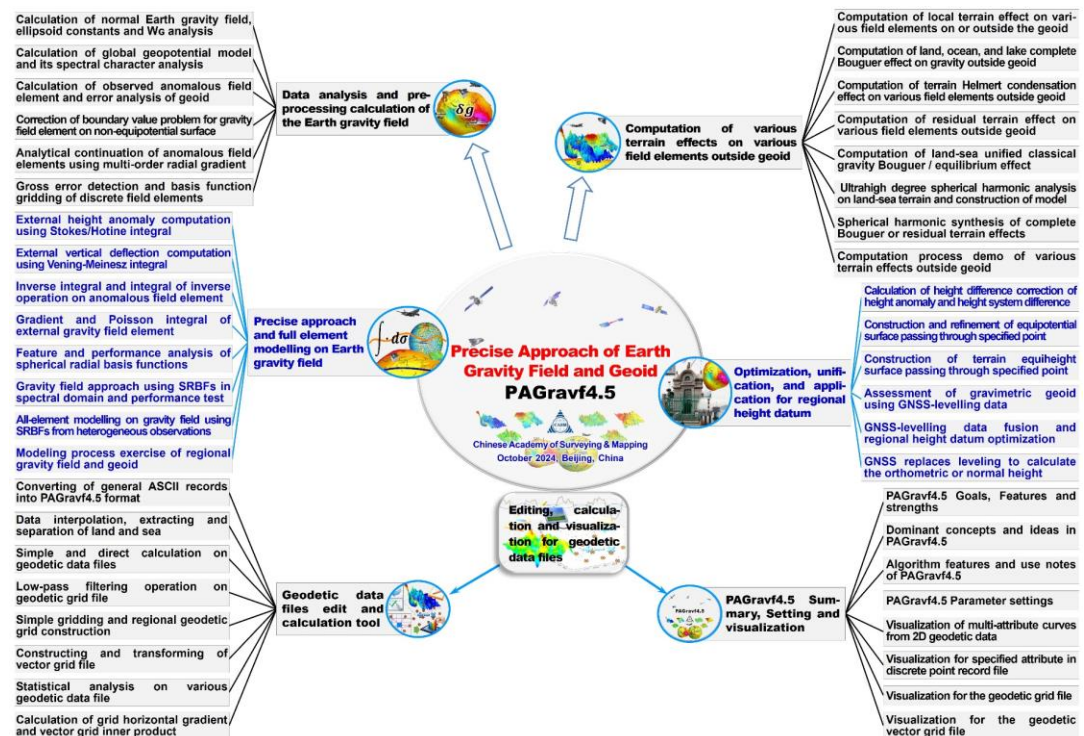
- Various terrain effects on all-element gravity field in whole space
- All-element modeling on Earth gravity field in whole outer space
- Gravity prospecting modelling from heterogeneous observations

1.1 PAGravf4.5 structure of computation functions

The basic principles, main methods and all the formulas in physical geodesy and Earth gravity field have been realized completely in PAGravf4.5 to popularize high

education. Many long-term puzzles such as various terrain effects on various observations, all-element analytical modelling on gravity field, fine gravity prospecting modelling from heterogeneous observations, external accuracy index measurement and computational performance control have been effectively solved to strengthen application capacity of Earth gravity field.

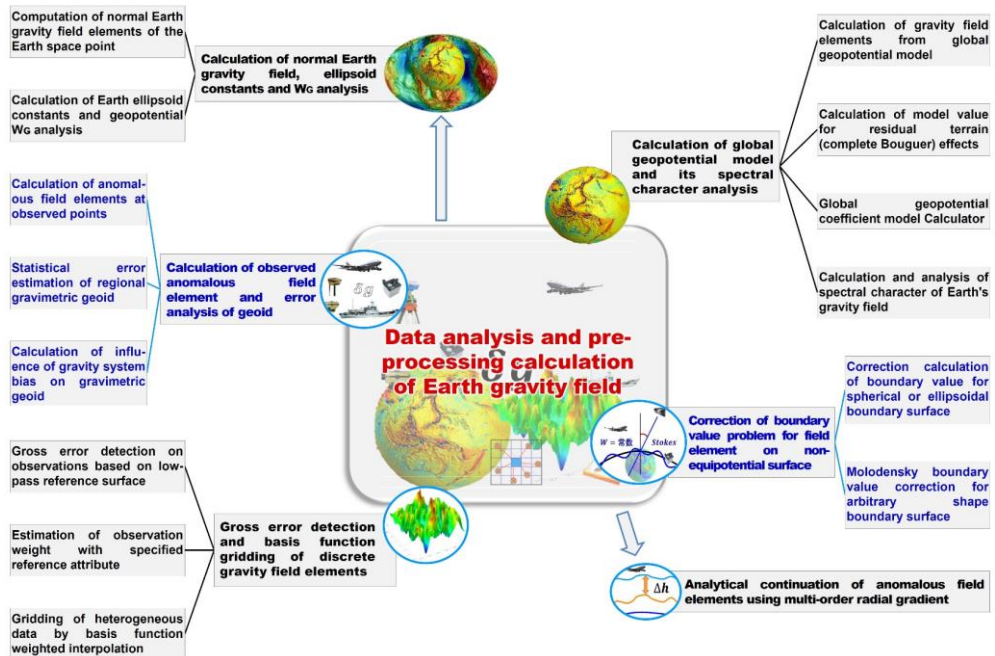
PAGrav4.5 has five subsystems, which includes data analysis and preprocessing calculation of Earth gravity field, computation of various terrain effects on various field elements outside geoid, precision approach and all-element modelling on Earth gravity field, optimization, unification and application for regional height datum as well as editing, calculation and visualization tools for geodetic data files.



You can design your own schemes and processes, then organize flexibly the related programs and functions in PAGrav4.5, perform some scientific computations for various terrain effects outside geoid, all-element modelling on gravity field, refinement of 1cm-level stationary geoid, fine gravity prospecting modelling from heterogeneous observations, improvement of height datum and application of Earth gravity field.

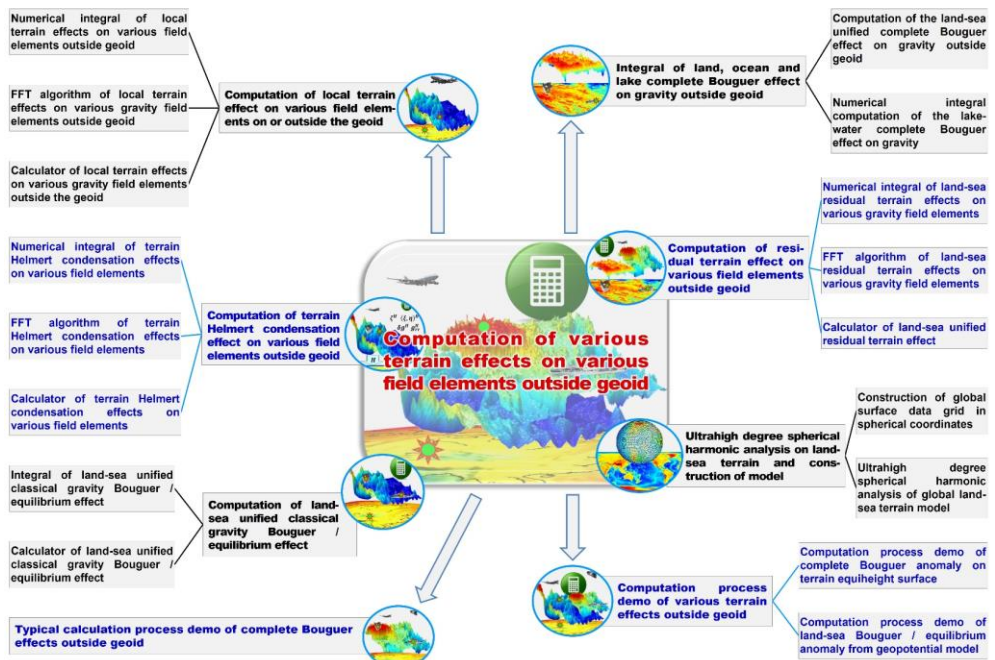
1.1.1 Data analysis and preprocessing calculation of Earth gravity field

The subsystem is mainly employed for the analysis and calculation of the normal Earth gravity field and Earth ellipsoid constants, calculation of the geopotential coefficient model and its spectral feature, correction of gravity field elements for the geodetic boundary value problem, and analytical continuation, gross error detection and gridding operation for gravity field observations.



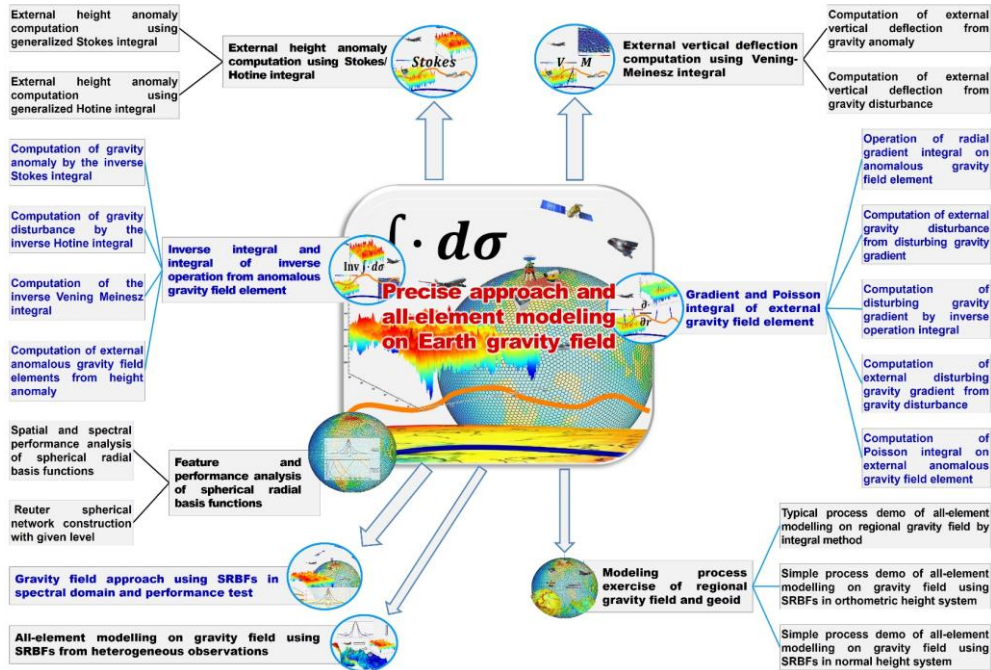
1.1.2 Computation of various terrain effects on various field elements outside geoid

Constructs the harmonic terrain influence field, and then develops the rigorous and unified analytical algorithm system for various modes of terrain effects on various types of gravity field elements in whole Earth space on or outside the geoid to synthesize effectively various complex geodetic observations for geophysical gravity prospecting modelling and gravity field data processing.

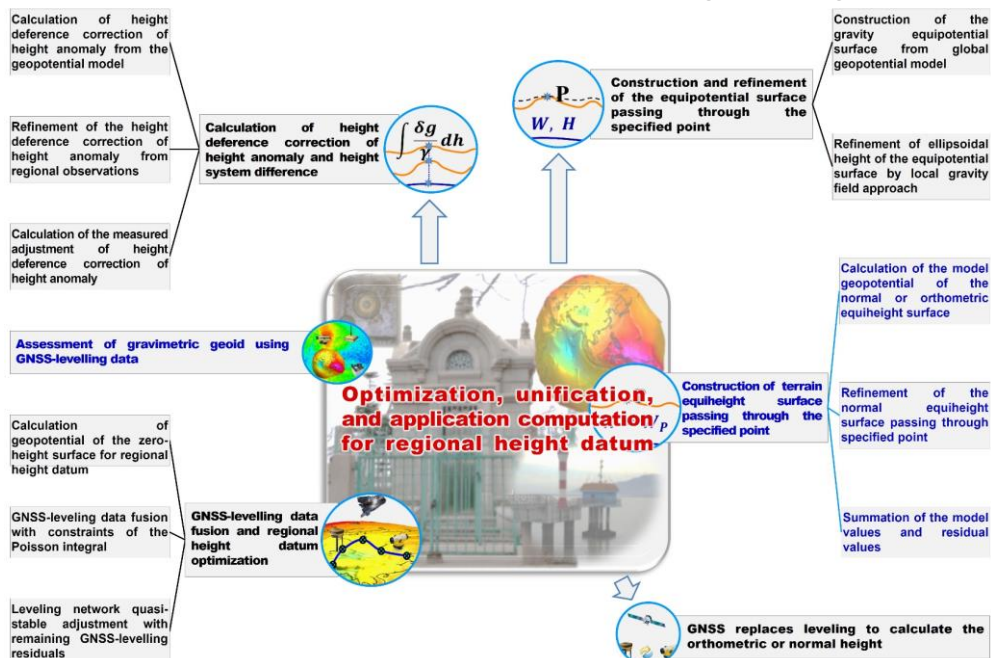


1.1.3 Precise approach and all-element modelling of Earth gravity field

PAGrav4.5 sets up the scientific gravity field approach system with the spatial domain integration algorithms based on boundary value theory and the spectral domain radial basis function approach algorithms to realize the all-element analytical modelling on gravity field in whole space on or outside the geoid from various heterogeneous observations in the different altitudes, cross-distribution and land-sea coexisting cases.

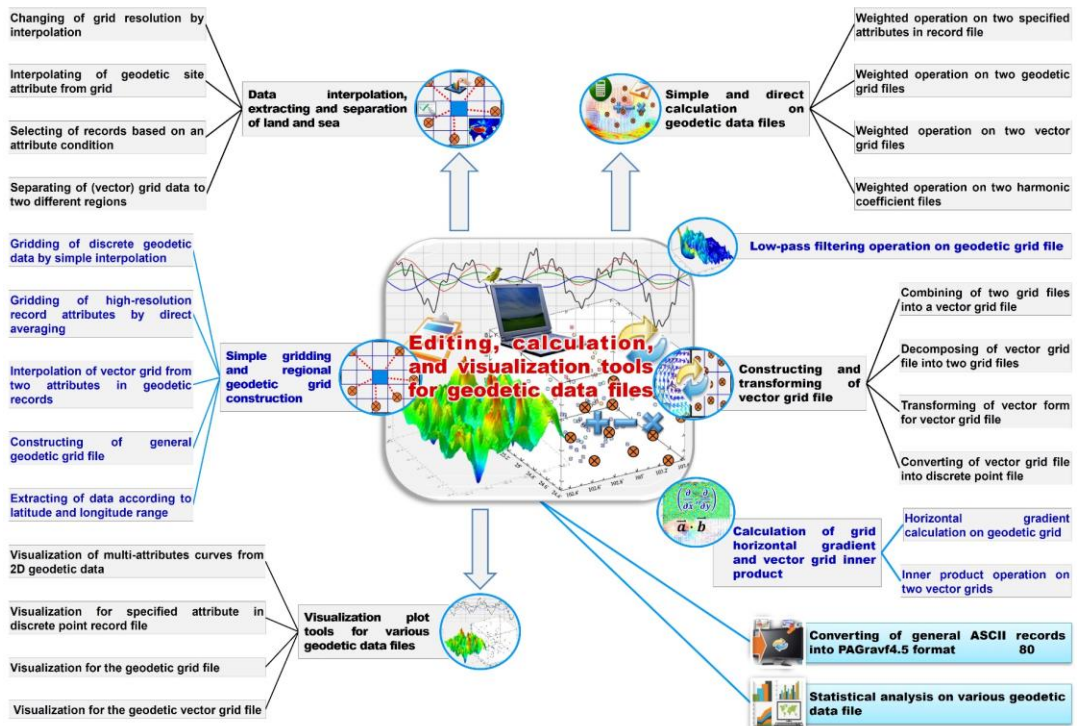


1.1.4 Optimization, unification, and application for regional height datum



Develops some ingenious physical geodetic algorithms to improve and unify the regional height datum, and then enhance the application capacity of Earth gravity field.

1.1.5 Editing, calculation, and visualization tools for geodetic data files



PAGravf4.5 were developed by QT C++ (Visual C++) for the user interface, intel Fortran (Fortran90, 132 Columns fixed format) for the core function modules and mathGL C++ for the geodetic data file visualization in the Visual Studio 2017 x64 integrated environment, which is composed of more than 50 win64 executable programs with nearly 600 function modules.

1.2 Scientific goals and strengths of PAGravf4.5

1.2.1 Scientific goals of PAGravf4.5

(1) Solves the analytical compatibility and rigorous unified computation problems of various modes of terrain effects on various types of field elements, to fulfill the requirements of gravity field data processing in various cases and comprehensively improve the gravity prospecting modelling.

(2) Sets up the scientific and complete gravity field approach algorithm system with the spatial integrals and SRBF spectral approaches, to realize the analytical loop closed operations and all-element modelling in whole Earth space on or outside the geoid from heterogeneous observations.

(3) Develops some ingenious physical geodetic algorithms based on the analytical relationship between Earth gravity field and height datum, to improve and unify the regional height datum, and consolidate and expand the applications of Earth gravity field.

1.2.2 Geodetic features and strengths

(1) Develops the unified analytical algorithm system for various modes of terrain effects on various gravity field elements outside the geoid, effectively to synthesize various heterogeneous observations for the gravity prospecting modelling and gravity field data processing.

(2) Sets up the gravity field approach algorithm system with the spatial domain integration algorithms based on boundary value theory and spectral domain radial basis function approach algorithms to realize the all-element analytical modelling in whole Earth space from various heterogeneous observations in the different altitudes, cross-distribution and land-sea coexisting cases.

(3) Presents the quantitative selection criteria for terrain effects according to physical geodesy, realizes the gravity prospecting modelling from heterogeneous observations, and develops some ingenious physical geodetic algorithms to improve and unify the regional height datum.

(4) Realizes the detection of observation gross errors, measurement of external accuracy indexes, control of the computational quality and modelling performance to strengthen application capacity of Earth gravity field.

1.3 Dominant concepts and ideas integrated into PAGravf4.5

1.3.1 Concepts and quantitative criterion for terrain effect

The terrain effect has always been a very tricky trouble in physical geodesy and gravity prospecting modelling. According to the basic requirements of physical geodesy, PAGravf4.5 puts forward the convenient and practical quantitative criterion for terrain effects, which can give a reliable basis for effectively improving the performance and role of terrain effects in physical geodesy and gravity prospecting modelling.

The terrain effect on gravity field element has three key elements: The adjustment mode of the terrain or crustal masses, type of gravity field element effected by the terrain, and location of gravity field element (positional relationship with the terrain masses).

According to the different adjustment modes of terrain masses, the terrain effects usually include the local terrain effect, terrain Bouguer effect, ocean Bouguer effect, crustal equilibrium effect, terrain Helmert condensation and residual terrain effect, etc.

In physical geodesy, computation of the terrain effects on gravity field elements serves two basic purposes. One is to improve the gridding performance of discrete field elements, and the other is to separate terrain ultrashort wave components for the gravity field approach. Accordingly, PAGravf4.5 defines the quantitative criterion for the selection of the terrain masses adjustment modes and algorithm parameters (such as integral radius, minimum degree of residual terrain model and spatial resolution, etc.).

(1) In order to improve the gridding performance of discrete field elements, it is expected to improve the smoothness of discrete field elements after the terrain effect removed. In this case, the optimal criterion for terrain effect is that the standard deviation

of discrete field elements would decrease after the terrain effect removed. This quantitative criterion is also applicable for gravity prospecting modelling.

(2) The terrain effect is expected to consist of only ultrashort wave components for gravity field approach purpose, so the optimal criterion should be that the standard deviation of field elements after the terrain effect removed would decrease, and the statistical mean of terrain effects in the range of tens of kilometers is small.

(3) The ratio D/ε of difference D between the maximum and minimum of terrain effects on a certain field element and its standard deviation ε , reflects the outlier of ultrashort wave signal in this mode of terrain effect. D/ε is large, which means that the proportion of ultrashort wave signals is small, but the signal is large. It is beneficial to improve the data processing performance to process this type of field element using this mode of terrain effect.

(4) When several modes of terrain effects are roughly same, the greater the ratio of the standard deviation of terrain effect on gravity disturbance to the standard deviation of terrain effect on height anomaly, the richer the ultrashort wave components of terrain effect, and the more favorable it is for geoid refinement.

Among the above four guideline criterion defined by PAGravf4.5, the first two are the binding regulations, which are globally applicable and need to be followed. The latter two can be the technical references and should be employed appropriately based on further analysis. These criterion will no longer be appropriate when gravity field observations are so scarce that their space statistical representation is severely underrepresented.

The statistical properties of terrain effects vary significantly with the terrain and gravity field nature in the target area. It is recommended to calculate, compare and analyze different modes of terrain effects on various field elements and their differences each other in advance, and then summarize the spectral domain character of the terrain effects and properties of the effect on different types of target field elements to design a calculation scheme with high adaptability based on these analysis results.

The performance of terrain effect is closely related to the complexity of the local terrain, short-wave figure of the gravity field and spatial distribution of the observations. When the terrain complexity is low, local gravity field structure is not complex and spatial distribution of the observations is dense, there is a possibility that the performance of the gridding or gravity field approach cannot be further improved using any mode of terrain effect.

1.3.2 The uniqueness of the geoid and PAGravf4.5's realization

The Stokes boundary value problem requires that there are no masses outside the geoid, and the terrain masses should be compressed into the geoid under the condition of keeping the disturbing geopotentials unchanging outside the Earth surface. PAGravf4.5 believes that there is some a way to compress the terrain masses that the

disturbing geopotential between ground and geoid are equal to the analytical continuation value of the disturbing geopotential outside the Earth surface, thereby the corresponding geoidal height is the analytical continuation solution of the geoid.

Various terrain effect on gravity disturbances and that on height anomalies computed by PAggrav4.5 satisfy the Hotine integral formula. For example, the terrain Helmert condensation effect on gravity disturbances (direct effects) and that on height anomalies (indirect effects) satisfy the Hotine integral formula. Therefore, no matter whether you choose the local terrain effect, terrain Helmert condensation or residual terrain effect, the regional geoid refined by PAggrav4.5 programs with the terrain effect remove-restore scheme is the analytical continuation solution of the geoid.

Obviously, the geoidal height determined from satellite gravity field or directly calculated from a global geopotential coefficients model are all the analytical continuation solution of the geoid. Maintaining the uniqueness of the geoid, PAggrav4.5 can deeply integrate satellite gravity, geopotential coefficient model and regional gravity field data, and then theoretically strictly approach the gravity field and geoid.

1.3.3 Classification and solution of gravity field boundary value problem

Most of the direct geodetic observations are based on the plumb line and level surface namely the natural coordinate system. For the sake of convenience, PAggrav4.5 divides the external boundary value problem into the Stokes problem and Molodensky problem according to whether the inner normal line of the boundary surface coincides with the plumb line, that is, whether the boundary surface is an equipotential surface.

(1) Stokes boundary value problem. The boundary value problem whose boundary surface is an equipotential surface, whose boundary value is a linear combination with the disturbing potential and its partial derivatives with respect to coordinates is called as the Stokes boundary value problem. The inner normal lines of the boundary surface in the Stokes problem coincide with the plumb lines.

(2) Molodensky boundary value problem. The boundary value problem whose boundary surface is not an equipotential surface, whose boundary value is a linear combination with the disturbing potential and its partial derivatives with respect to coordinates is called as the Molodensky boundary value problem. The inner normal lines of the boundary surface in the Molodensky boundary value problem do not coincide with the plumb lines.

When the boundary surface is not an equipotential surface, any one of the following three methods can be employed to solve the Molodensky boundary value problem.

(1) Analytically continue the gravity field elements on the boundary surface to the equipotential surface close to the boundary surface. In this case, the new boundary surface becomes the equipotential surface, and the boundary value problem into the Stokes problem, and then solve the Stokes problem.

(2) Correct the gravity field elements on the boundary surface from the direction of the inner normal line of boundary surface to the direction of plumb line, so that the

boundary value problem becomes a Stokes problem, and then solve the Stokes problem.

(3) Directly solve the Molodensky boundary value problem with the non-equipotential surface as the boundary surface. Which is not recommended by PAggrav4.5 due to the generally low accuracy of the solution.

PAGrav4.5 suggests that the geodetic boundary value problem is mainly solved according to the Stokes boundary value theory, and the Molodensky boundary value theory is mainly employed for the reduction of gravity field data into the equipotential surface, and for error analysis for processing of gravity field data on the non-equipotential surface.

1.3.4 Geopotential, geoid, and height datum

(1) The geoidal height is the solution of the geodetic boundary value problem, whose geopotential is a constant and equal to the normal potential of the ellipsoidal surface, and the ellipsoidal surface is also the starting surface of the geoidal height.

(2) Global geopotential W_0 is an appoint geopotential for the global height datum in IERS numerical standards, which can be calculated from a latest geopotential model and sea surface height observations according to the Gaussian geoid appoint.

(3) According to the define of height system, the zero normal height surface always coincides with the zero orthometric height surface everywhere, which is namely the geoid with the constant geopotential W_G or W_0 .

(4) Essentially, the gravimetric geoid determined according to the geodetic boundary value theory is the realization of the constant geopotential W_G in the Earth coordinate system, namely determining of the ellipsoidal height of the geoid.

(5) PAggrav4.5 recommends that the geoidal geopotential W_G or U_0 should replace the empirical appoint W_0 in the IERS numerical standard. The latter has no direct geodetic relationship with the geopotential of the gravimetric geoid.

Whether for realizing of global height datum or for refining of regional height datum, the geoidal geopotential W_G as the global geopotential can not only effectively reflect the unique invariance of geodetic datum, but also ensure the analytic rigor of gravity field approach in the realization and unification of height datum.

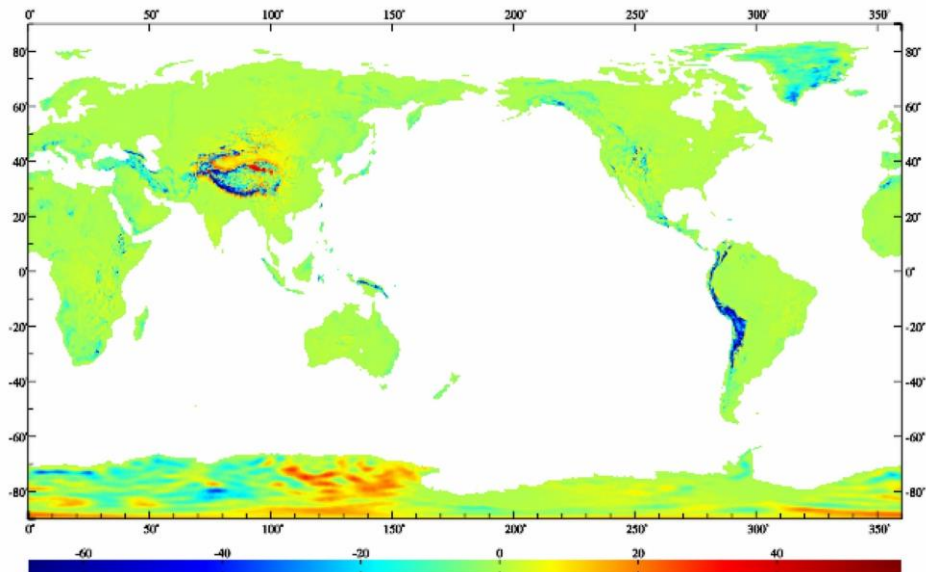
1.3.5 Analytical orthometric system is more suitable

(1) Let the gravity value of the move point between ground and geoid be equal to that analytically continued to the move point from the outer gravity field, and the resulting orthometric height is called as the analytical orthometric height.

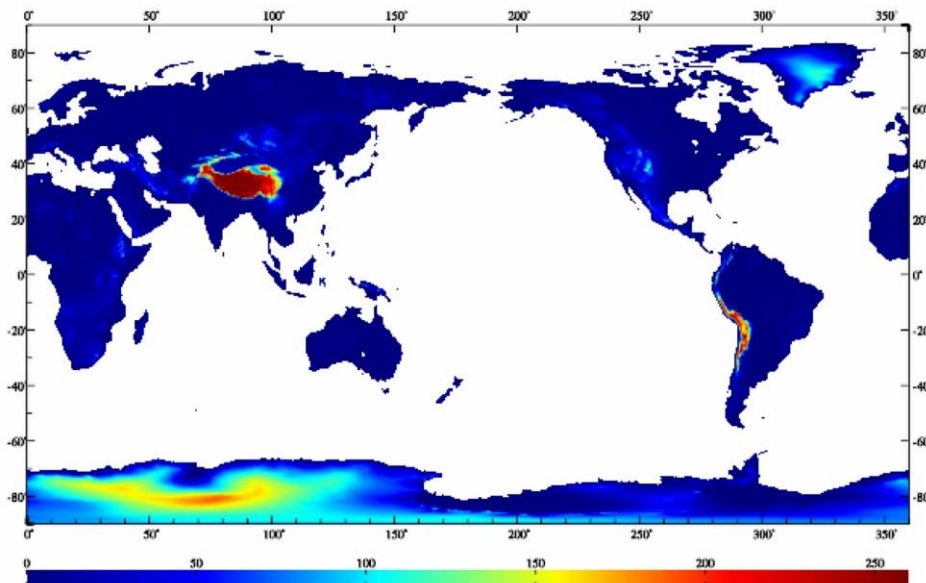
The analytic orthometric height is closer to the normal height, and the difference is about 60 cm from the Helmert orthometric height at 3000 m altitude.

(2) The geoidal height is the ellipsoidal height of the geoid, which is the solution of the Stokes boundary value problem in the Earth coordinate system. The measurement scale of the geoidal height is the geometric scale of the Earth coordinate system, while the measurement scale of the analytical orthometric height difference in the vertical direction is also strictly expressed by the geometric scale. Thus, the analytical

orthometric height is consistent with the geometric scales of the Earth coordinate system and the geoidal height.



The differences between the analytical orthometric and normal height of the global land ground (cm, 95% $<\pm 16$ cm)



The differences between the analytical orthometric and Helmert orthometric height of the global land ground (cm, 95% $<\pm 108$ cm)

(3) The analytical orthometric height is not directly related to the terrain density, which can be continuously refined with the latest gravity field data. On the view of uniqueness, repeatability and measurability of geodetic datum, the analytical orthometric height is more suitable for height datum purpose than other types of orthometric height.

Different from the Helmert orthometric system, the analytic orthometric system and normal height system are compatible and consistent with each other and supported by the rigorous gravity field theory, they can also be directly employed for the moon and Earth-like planets.

1.4 Data format, convention, and exercise in PAGravf4.5

1.4.1 Format convention for geodetic data file

PAGravf4.5 adopts 5 types of geodetic data files in own format. The program [Converting of general ASCII records into PAGravf4.5 format] is the important interface for PAGravf4.5 to accept external text data. Using the program [Simple gridding and regional geodetic grid construction], you can construct a numerical grid with the given grid specification. The other programs or functions only accept the format data generated by PAGravf4.5 own.

(1) The discrete geodetic record file

A discrete geodetic record data is represented by a one-dimensional array.

- Multiple rows of the file headers are allowed, whose content and format are not restricted.
- One record represents the geodetic data of one site. The attributes of each record include site number (name), longitude (degree decimal), latitude (degree decimal), attribute 4, ..., attribute n.
- The attribute convention is a numeric format, the number of the attributes (n) is not more than 80, and the attributes are separated by spaces.

(2) The geodetic network observation file

A geodetic network observation file can store the baseline component data for the CORS network, height differences for the levelling network or gravity differences for the gravity network.

- The file header occupies a row and includes the number of characters of the baseline or route name, number of characters of the site name,
- The file record includes the baseline or route name, starting site (longitude, latitude, height), ending site (longitude, latitude, height),, observations (default value is 9999).
- The relations between the baselines (or routes) and the sites in the geodetic network are reflected with the composition of the characters of their name. A baseline or route name is agreed to be composed of site names A and B at both ends (B***A), where the number of characters of all the sites names is required to be equal.

Therefore, the number of characters of the baseline or route name shall not be less than twice the number of characters of the site name.

(3) The geodetic numerical grid file

The geodetic numerical grid data is represented by a two-dimensional array.

- There is a row of file header at the beginning of the file. The file header contains

minimum longitude, maximum longitude, minimum latitude, maximum latitude, longitude interval of a cell grid, latitude interval of a cell grid. The units of all the attributes are decimal degrees.

- The grid elements are sequentially stored in an increasing manner of row latitude and column longitude until all data is stored. The elements are separated by spaces.

The cell-grid value represents the mean value of the cell-grid elements. In the numerical integral operation, the location of the center point of the cell-grid is employed to calculate the integral distance from the cell-grid to the calculation point.

(4) The geodetic vector grid file

A vector grid file is composed of the first component grid and the second component grid of the vector. The file header and the first component grid in the vector grid file are same as that in the geodetic grid file, and the second component grid follow the first component grid closely with the same way.

Vector grid such as vertical deflection and horizontal gradient vector grid in PAGravf4.5 are stored in the form of vector grid file.

(5) The spherical harmonic coefficient file

- The file header occupies a row and consists of two attributes for the scale parameters of the spherical harmonic coefficients model, namely the geocentric gravitational constant GM ($\times 10^{14} \text{m}^2/\text{s}^2$) and equatorial radius a (m) of the Earth.

- The geopotential coefficient model, and terrain masses spherical harmonic coefficient model and terrain geopotential coefficient model in PAGravf4.5 are stored in the form of spherical harmonic coefficient file.

- The spherical harmonic coefficients correspond to the scale parameters of GM and a . Here, the surface harmonic functions in the spherical harmonic coefficient model are defined on the spherical surface whose radius is equal to the equatorial radius a of the Earth.

- The degree n order m spherical harmonic coefficient is expressed by a record with the format 'degree n , order m , C_{nm} , S_{nm} (, C_{nm} error, S_{nm} error)'.
(Note: The original text contains a stray comma in the format string, which has been corrected.)

PAGravf4.5 does not require the degrees and orders of harmonic coefficients to be arranged and allows to exist insufficient orders.

1.4.2 The main geodeic quantity unit and direction

(1) Unit convention of anomalous gravity field elements: Height anomaly or geoidal height in the unit of m, gravity anomaly or gravity disturbance in the unit of mGal, vertical deflection in the unit of as namely ", and gravity gradient in the unit of E.

(2) The unit of terrain effect on some a gravity field element is the same as that of the gravity field element.

(3) Longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m), normal (orthometric) height / depth (m).

(4) Vertical deflection vector (SW). The first component points to the south direction,

and the second component points to the west direction, which forms a right-handed rectangular coordinate system with the gravity disturbance direction. This coordinate system is a natural coordinate system.

(5) Tangential gravity gradient vector (NW). The first component points to the north direction, and the second component points to the west direction, which forms a right-handed rectangular coordinate system with the disturbing gravity gradient direction (radial or zenith direction).

In PAGravf4.5, the disturbing gravity gradient T_{rr} and tangential gravity gradient are the second-order partial derivatives of the disturbing potential T to the coordinate, which are the main diagonal components of the gravity gradient tensor.

Please distinguish the concepts of the tangential gravity gradient and gravity horizontal gradient. The latter is the horizontal gradient of the vertical derivative of the disturbing potential, which is only the non-diagonal component of the gravity gradient tensor (cross item, such as T_{zx} or T_{zy}).

1.4.3 For classroom teaching and self-study exercise

To ease the classroom teaching and self-study, there are the example files saved in the directory C:\PAGravf4.5_win64en\examples for each Win64 program. Each example includes the operation process file process.txt, some input-output data files and screenshots. The directory name of the example files is the same as the name of the executable program.

Before using the PAGravf4.5 programs, it is recommended to perform completely the program example using the input-output example data files with comparing the screenshots and the process information in process.txt. It will take about 7 working days to complete all the example exercises. Thereafter, you can use PAGravf4.5 alone.

PAGravf4.5 is suitable for senior undergraduates, graduate students, scientific researchers, and engineering technicians in geodesy and geophysics, geology and geoscience, geomatics and geographic information, seismic and geodynamics, aerospace and satellite dynamics, which can be employed in the classroom teaching, self-exercise, science research and engineer computing.

You can design your own schemes and processes, then organize flexibly the related programs and functions in PAGravf4.5, perform some scientific computations for various terrain effects outside geoid, all-element modelling on gravity field, refinement of 1cm-level geoid, fine gravity prospecting modelling from heterogeneous observations, improvement of regional height datum and application of Earth gravity field.

1.4.4 Ignoring and expanding several classic concepts

(1) Ignoring the terrain correction and direct effect concepts

In the classic terrain correction, the correction object is only the terrestrial gravity. PAGravf4.5 need deal with various modes of terrain effects on various types of gravity field elements on the geoid or in its outer space. The classic direct effect is the effect of

terrain mass on gravity (gravity disturbance or gravity anomaly), while the indirect effect is the effect of terrain mass on geopotential (disturbing potential, height anomaly or geoid). PAGravf4.5 need deal with various terrain effect on all-element gravity field elements. The concept of terrain correction, direct and indirect effect can no longer meet the needs of PAGravf4.5.

PAGravf4.5 adopts the concept of terrain effect uniformly, and strictly distinguishes the adjustment mode of terrain masses, the type of field elements effected and the location of field elements. For example, the local terrain, terrain Helmert condensation and residual terrain effects on the ground gravity disturbance, external gravity disturbance and geoidal gravity disturbance include $3 \times 3 = 9$ different terrain effect quantities.

The terrain effect on various types of gravity field elements is equal to the negative value of its terrain correction. For example, the local terrain effect is equal to the negative value of classic local terrain correction.

The normal gravity field is the agreed starting datum for the anomalous gravity field, and there is no terrain influence on the normal gravity field. Therefore, the terrain effects on the gravity, gravity disturbance and gravity anomaly at the same point are exact equal, that on the geopotential and disturbing geopotential and that on the gravity gradient and disturbing gravity gradient are also equal, respectively. For example, the local terrain effects on the gravity, gravity disturbance and gravity anomaly are equal, and the terrain Helmert condensation, land-sea Bouguer, land-sea equilibrium and residual terrain effects on those are equal, respectively.

(2) Recommending $W_0=W_G$ as the global geopotential

PAGravf4.5 proposes that the scale parameters (GM , a) of global geopotential coefficient model, second-degree zonal harmonic coefficient \bar{C}_{20} and mean rotation angular velocity ω should be employed as the four basic parameters of the normal ellipsoid. In this case, the second-degree zonal harmonic term of anomalous gravity field is always zero, which is beneficial to improve the performance of gravity field approach.

The geoid determined according to the geodetic boundary value theory is essentially the realization of the constant geopotential W_G in the Earth coordinate system, namely determining of the ellipsoidal height of the geoid. The geoidal geopotential W_G is always equal to the normal potential U_0 of the normal ellipsoid. PAGravf4.5 suggests that the geoidal geopotential W_G replaces the appoint W_0 in the IERS numerical standard. The latter is usually calculated from the global geopotential model and satellite altimetry data according to the Gaussian geoid appoint.

(3) Recommending the analytical orthometric system

The analytic orthometric height is closer to the normal height, and the difference is about 60 cm from the Helmert orthometric height at 3000 m altitude. The analytical orthometric height is not directly related to the terrain density, which can be continuously refined with the latest gravity field data. The geometric scale of the analytical orthometric

height is consistent with that of the Earth coordinate system and the geoidal height.

On the view of uniqueness, repeatability and measurability of geodetic datum, analytical orthometric height is more suitable for height datum purpose than other types of orthometric height. Different from the Helmert orthometric system, the analytic orthometric system and normal height system are analytically consistent with each other and supported by the rigorous gravity field theory, they can also be directly employed for the moon and Earth-like planets.

1.5 Algorithm features and use notes of PAGravf4.5

1.5.1 Complete and analytical terrain effect algorithm system

PAGravf4.5 developed the complete algorithm set of terrain effects to realize various modes of terrain effects on different types of gravity field elements on the geoid or in its outer space.

(1) The set of algorithms are rigorous in theory, the numerical integral has no computation error, and the accuracy of the FFT algorithms is controllable.

(2) There are various modes of terrain masses adjustment, the type of gravity field element affected by terrain can be arbitrary and the field element can be located on or outside the geoid.

(3) Strictly follows the analytical relationship of gravity field between the terrain effects on different types of field elements.

(4) Makes full use of the analytical compatibility between different modes of terrain effects, so that the algorithm codes can be short and concise.

From the terrain effect formulas in sections 7.5 to 7.8, it is easy to see that many algorithm formulas are very similar. Some terrain effect algorithms were realized only by adjusting some parameters with the same codes.

1.5.2 Technical features of gravity field Integral algorithm

(1) The fixed integral radius for local gravity field refinement

Limiting the definition domain of kernel function, PAGravf4.5 executes the gravity field integral operation with the given radius, including numerical integral and FFT integral algorithm (kernel function windowing), to coordinate and unify various gravity field approach algorithms. Two-dimensional FFT adopts the modified two-dimensional planar kernel function, whose calculation accuracy is not significantly different from that of one-dimensional FFT in the range of latitude 10° .

(2) The calculation point and the move point (integral running areal element)

The coordinates of geodetic points are expressed as the latitude, longitude and ellipsoidal height. For example, the location of boundary surface, measurement point, calculation point and integral move point (areal element or volume element) are expressed by geodetic coordinates. The integral cell-grid location is the geodetic coordinates of the center of the cell-grid, and the integral radius is calculated by geodetic coordinates.

(3) The equipotential boundary surface

Most gravity field integral formulas are derived from Stokes boundary value theory, such as the Hotine integral, Vening Meinesz integral, radial gradient integral formula, etc. The solution of Stokes boundary value problem requires that the boundary surface is an equipotential surface, that is, the anomalous gravity field elements should be located on some an equipotential surface.

In PAGravf4.5, it can meet most requirements that the accuracy of the ellipsoidal height employed as the boundary surface is not less than 10 m. The boundary surface can be constructed from a 360-degree global geopotential coefficient model, which can also be replaced by the ellipsoidal height grid of normal or orthometric equiheight surface in near-surface space.

1.5.3 Performance of gravity field approach using spherical radial basis functions

(1) Using the edge effect suppression constraints to instead of normal equation regularization.

PAGravf4.5 proposes the algorithm to improve the estimation performance of spherical radial basis functions (SRBF) coefficients by suppressing edge effects. When the SRBF center is located at the margin of the calculation area, let that the SRBF coefficient is equal to zero as the observation equation to improve the stability and reliability of unknown SRBF coefficient estimation.

After the edge effect suppression method employed, the normal equation need not be regularized. Which can keep the analytical nature of the SRBF approach algorithm and local gravity field from being affected by the observation errors.

(2) Employing the cumulative SRBF approach method to achieve the best approach of local gravity field.

The target field elements are equal to the convolution of the observations and the filter SRBFs. When the target field elements and the observations are of different types, it is difficult for one SRBF to match the spectral center and bandwidth of the observations and target field elements at the same time, which would make the spectral leakage of the target field elements. In addition, the SRBF type, the minimum and maximum degree of Legendre expansion and the SRBF center distribution also all affect the approach performance of gravity field. Therefore, only the optimal estimation of the SRBF coefficient with the burial depth as the parameter is not enough to ensure the best approach of gravity field.

PAGravf4.5 proposes a cumulative SRBF approach scheme according to the linear additivity of gravity field to solve the key troubles mentioned. Using the multiple cumulative SRBF approach, it is not necessary to determine the optimal burial depth.

When each SRBF approach of gravity field employs a SRBF with different spectral figure, the cumulative SRBF approach can fully resolve the spectral domain signal of the target field elements by combining multiple SRBF spectral centers and bandwidths,

and then optimally restore the target field elements in space domain.

The character of cumulative SRBF approach scheme of gravity field: the essence of each SRBF approach is to employ the previous approach results as the reference field, and then refine the residual target field elements by remove -restore scheme.

(3) Proposing the normalization method of the normal equations for heterogeneous observation fusion.

PAGravf4.5 recommends a universal multi-source heterogeneous observation deep fusion method by the normalization of the normal equations, which can effectively control the deep fusion of different types of observations using covariance structure to approach gravity field from heterogeneous observation field elements. This method completely separates the contribution of the observation system model (covariance structure) and influences of observation quality (errors or gross errors), so that the fusions are away from the observation errors (gross error), observation types and spatial distribution differences of measurement points. Which is conducive to the fusion of multiple types of observation field elements with extreme differences in spatial distribution (such as very few astronomical vertical deflection or GNSS levelling data included), and is conducive to the exact detection of observation gross errors.

In this case, the normal equation does not also need to be iteratively calculated, which conducive to improve the analytical nature of the SRBF approach algorithm.

(4)The typical technical features of SRBF approach program in PAGrav4.5

① The analytical function relationships between gravity field elements are strict, and the SRBF approach performance has nothing to do with the observation errors. ② Various heterogeneous observations in the different altitudes, cross-distribution, and land-sea coexisting cases can be directly employed to model the all-element gravity field models on or outside the geoid without reduction, continuation and gridding. ③ Can integrate very few astronomical vertical deflection or GNSS-levelling data, and effectively absorb the edge effect. ④ Has the strong capacity in the detection of observation gross errors, measurement of external accuracy indexes and control of computational performance.

1.5.4 Computation scheme of gravity prospecting modelling from multi-source heterogeneous observations

PAGravf4.5 has the high-precision analytical computation capacity of various terrain effects on any type of field element. At the same time, it can model analytically the all-element gravity field in whole space outside geoid from various heterogeneous observations. The combination of the two can effectively solve the trouble of gravity prospecting modelling that can deeply fuse all the gravity field information in multi-source heterogeneous observations.

In any region of the world, you can accurately calculate the land-sea unified complete Bouguer gravity anomaly (disturbance), complete Bouguer vertical deflection, complete Bouguer gravity gradient, and classical Bouguer / isostatic gravity anomaly

(disturbance) from heterogeneous observations such as gravity, gravity gradient, satellite altimetry, (astronomical) vertical deflection, GNSS-leveling etc. in the different altitudes, cross-distribution and land-sea coexisting cases.

The general computation process for fine gravity prospecting from heterogeneous observations is as follows.

(1) Select the calculation area, target calculation surface (terrain equiheight surface is recommended here) for gravity prospecting modelling and obtain (or collect) all the gravity field and geodetic observations as much as possible.

(2) Call the related programs in the subsystem [Precision approach and all-element modelling on Earth gravity field] to determine the high-resolution grid of gravity field elements corresponding to the target prospecting model on the calculation surface.

(3) Call the related programs in the subsystem [Computation of various terrain effects on various field elements outside geoid] to determine the terrain effect grid on field element corresponding to the target prospecting model on the calculation surface.

(4) By subtracting the terrain effect grid from the gravity field element grid directly, you can obtain the target gravity prospecting model which have deeply fuse all the gravity field information in heterogeneous observations.

The computation of gravity prospecting modelling with the analytic relations of gravity field as the constraints by PAGravf4.5 can deeply fuse all the gravity field information from heterogeneous observations in the different altitudes, cross-distribution and land-sea coexisting cases, whose observation mode can be on terrestrial, marine, aviation and satellite.

1.5.5 Algorithm and computation scheme optimization

The algorithm system in PAGravf4.5 is scientific and rigorous, and there are several schemes to calculate the same terrain effect. Various field elements can be estimated from any type of field element. The field element type can be traditional or uncommon such as satellite tracking satellite and satellite orbit perturbation, and the applicable space can be on the geoid or in its outer space.

The approach of Earth gravity field is linear operation. The terrain effect and gravity field approach algorithms in PALGravf4.5 are all linear. Therefore, Any program in PAGravf4.5 can output the error distribution characters of the target field elements with the simulated spatial noises as observations. PAGravf4.5 has strong error analysis capacity, which can be the important means to optimize the gravity prospecting modelling and gravity field approach scheme.

One mode of terrain effect, there are also multiple computation schemes to be chose. For example, to compute the land-sea complete Bouguer effect, PAGravf4.5 has three schemes and programs to be chose.

For the specific data processing and modelling purpose of gravity field, there are multiple PAGravf4.5 programs, different parameter settings or multiple schemes to be chose. In the actual application, you should investigate the gravity data situations and

the nature of gravity field in the target area, carefully select, test and analyze the related PAGravf4.5 algorithms, parameters and schemes to design and optimize the computation technology route.

1.5.6 Performance testing and analysis of algorithm and parameter

(1) Performance test for terrain effect algorithm

The terrain effect optimization criterion proposed by PAGravf4.5 according to the basic principles of physical geodesy, can greatly reduce the complexity of terrain effect analysis, and provide a concrete and feasible technical route for effectively playing the key role of terrain effect in gravity prospecting modelling and gravity field approach.

The statistical properties of terrain effects vary significantly with the local terrain, gravity field nature and observations distribution in the computation area. PAGravf4.5 terrain effect computation subsystem presents some cases in a difficult mountain area, and the ratio of the maximum and minimum values to the standard deviation of various terrain effects on various field elements was statistically analyzed. The statistical analysis results of these cases show that, when ignoring the observation distribution and gravity field nature, the local terrain effect is favorable for the gravity data processing, the terrain Helmert condensation is favorable for the processing of gravity gradient data, and the residual terrain effect is more favorable for the refinement of geoid.

Before the computation, it is necessary to carefully test and analyze the technical route of terrain effects according to the local terrain, gravity field nature and available gravity resources in the target area according to the quantitative criterion of terrain effect, so as to ensure that the terrain effect algorithms and parameter settings can be based on some evidence. Only then can the applicability and technical level of the terrain effect processing scheme be significantly improved.

(2) Performance test for gravity field approach algorithm

The performance of most gravity field approach algorithms and their parameter settings can be tested and verified with an ultra-high degree global geopotential coefficient model. Many program samples of PAGravf4.5 take the 2 to 540th degree EGM2008 geopotential coefficient model as the reference field, and then employ the anomalous gravity field calculated by the 541 to 1800th degree model as the reference true values for testing and verification.

The test outline of the PAGravf4.5 program algorithm: Take some residual anomalous field elements calculated by the 541 to 1800th degree EGM2008 geopotential coefficients as the observations, call the PAGravf4.5 programs or functions to be verified, and obtain the calculated values of the target residual field elements. Then compare the difference between the calculated values and the target reference values calculated by the EGM2008 model, to evaluate the technical performance of the algorithm programs and their parameter settings in PAGravf4.5.

PAGravf4.5 can compute various types of field elements on or outside the geoid from some a type of field elements and can also cyclically calculate the same type of

field elements. Comparing the difference and similarity between the observed field elements and the calculated field elements obtained by the cyclical computation, the algorithm character and performance of the relevant programs and functions called in the computation process can be analyzed.

(3) Performance test for gravity field approach using SRBF

The best approach scheme of gravity field using SRBF is related to the observation situations, nature of gravity field and algorithm parameters. The program of [Gravity field approach using SRBFs in spectral domain and performance test] can be employed to comprehensively analyze the spectral center and bandwidth of the observation, target field element and SRBF in different parameter combination cases. According to the principle of fully resolving the spectrum of the target field element, design and optimize the scheme and relevant parameters for SRBF approach of gravity field in advance.

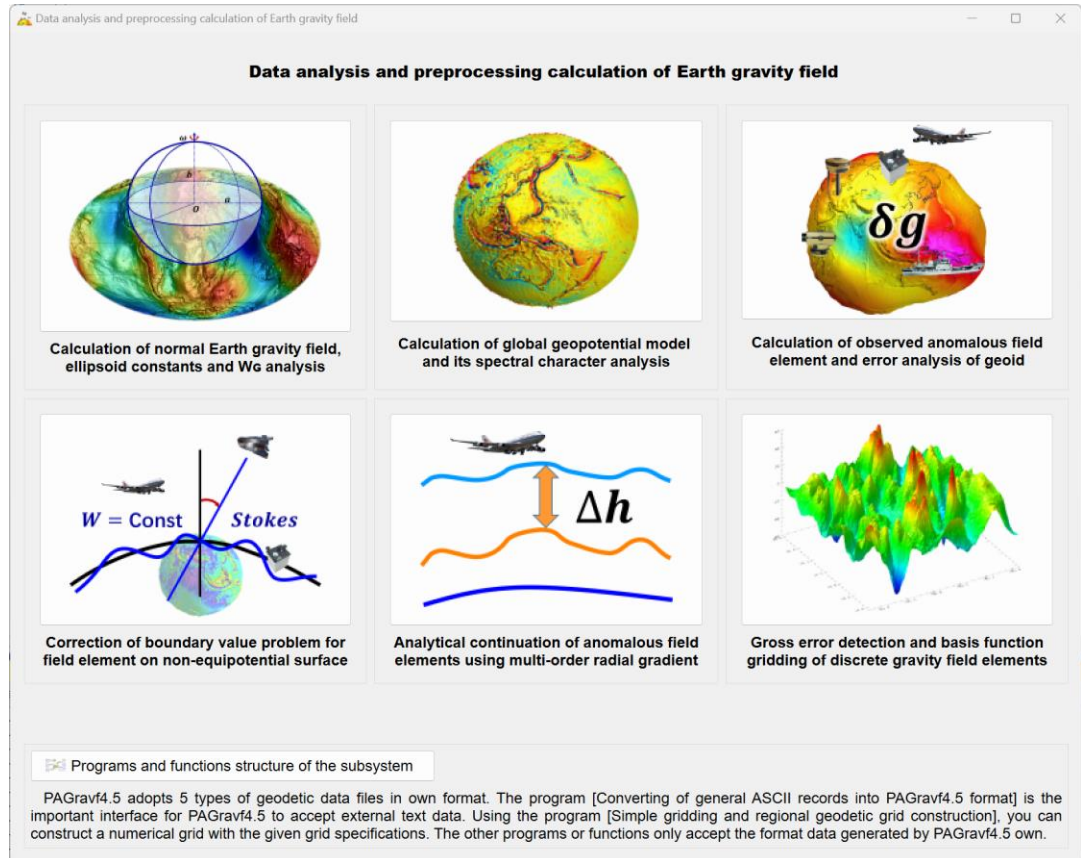
(4) Notes on the PAGravf4.5 program examples

In the examples of terrain effect programs of PAGravf4.5, a typical difficult mountainous area with an average altitude of 4000m and terrain relief of more than 3000m is selected to facilitate the display of the details of terrain effect and algorithm characters. Similarly, in the gravity field integral examples of PAGravf4.5, the region with typical complex features where the short-wave signal of the gravity field is rich (residual gravity disturbance after the 540-degree reference model value removed, and the spatial variation exceeds 300mGal) is selected to facilitate the display of the detailed features of the local gravity field and its approach algorithm.

The statistical results of these samples roughly show the basic performance of the corresponding algorithms. Since the main purpose of these samples is to introduce the computation process, there is no statistical analysis and optimization of the algorithm itself and parameter settings. Therefore, there is still greater potential, which need be further explored by the user in combination with the specific situations.

2 Data analysis and preprocessing calculation of Earth gravity field

The subsystem of the data analysis and preprocessing calculation of the Earth gravity field is mainly employed for the analysis and calculation of the normal Earth gravity field and Earth ellipsoid constants, calculation of the geopotential coefficient model and its spectral feature, correction of gravity field elements for the geodetic boundary value problem, and analytical continuation, gross error detection and gridding operation for gravity field observations.



2.1 Calculation of normal Earth gravity field, ellipsoid constants and W_g analysis

[Purpose] Calculate the normal gravity field elements at the Earth space point, geometrical and physical constants of the normal Earth ellipsoid and geoidal geopotential W_g according to the rigorous analytical algorithm of spherical harmonic expansion.

Essentially, the geoid determined according to the geodetic boundary value theory is the realization of the constant geopotential W_g in the Earth coordinate system, namely determining of the ellipsoidal height of the geoid. The geoidal geopotential W_g is always equal to the normal geopotential U_0 of the normal ellipsoid. PAGrav4.5 suggests that the geoidal geopotential W_g should replace the appoint empirical W_0 in the IERS

numerical standard. The latter is calculated from the global geopotential model and mean sea surface height model according to the Gaussian geoid appoint.

2.1.1 Computation of normal Earth gravity field elements of the Earth space point

[Function] Using the spherical harmonic expansion formula, calculate the normal geopotential (m^2/s^2), normal gravity (mGal), normal gravity gradient (E), normal gravity line direction ($'$, expressed by its north declination relative to the center of the Earth center of mass) or normal gravity gradient direction ($'$, expressed by its north declination relative to the Earth center of mass).

[Input file] The space calculation point file.

The record format: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m).....

[Parameter settings] Input the number of rows of the input file header, and column ordinal number of ellipsoidal height in the record, and select the normal gravity field elements to be calculated.

[Output file] The normal Earth gravity field element file.

Behind the record of the calculation point file, appends one or several columns of normal gravity field element calculation values, and keeps 4 significant figures.

Calculation of normal Earth gravity field, ellipsoid constant and W_s analysis

Open calculation points Import parameters Save as Start computation Ellipsoid calculation Follow example

Computation of normal Earth gravity field elements of the Earth space point Calculation of Earth ellipsoid constants and geopotential W_s analysis Calculation formulas of normal gravity field

Open space calculation point file

Set input point file format

Number of rows of file header 1

Column ordinal number of ellipsoidal height in the record 4

Select elements to be calculated

- ☒ Normal geopotential (m^2/s^2)
- ☒ Normal gravity (mGal)
- ☒ Normal gravity gradient (E)
- ☒ Normal gravity line direction ($'$)
- ☒ Normal gravity gradient direction ($'$)

Save the results as

Import setting parameters

Start Computation

Display of the input-output file

no	lon(deg)	lat(deg)	ellipheight(m)							
3248	103.671939	31.938051	2743.9394	62609994.0026	978633.0022	3074.9063	-10.3533	5.1183		
3249	103.696944	31.864721	2501.2449	62612369.3305	978701.8741	3075.2510	-10.3399	5.1446		
3250	103.718330	31.831114	2435.4206	62613013.6271	978719.4456	3075.3432	-10.3339	5.1567		
3251	103.735559	31.795280	2366.5700	62613687.5573	978737.7722	3075.4395	-10.3273	5.1696		
3252	103.777216	31.776390	2294.0304	62614397.5729	978758.6092	3075.5429	-10.3239	5.1764		
3253	103.822773	31.758333	2233.2317	62614992.6837	978775.8946	3075.6223	-10.3206	5.1828		
3254	103.849717	31.724168	2215.4606	62615164.7271	978778.5505	3075.6525	-10.3144	5.1951		
3255	103.816666	31.650003	2242.9951	62614897.3184	978764.1322	3075.6047	-10.3009	5.2217		
3256	103.783335	31.616667	2297.3654	62614365.2277	978744.6783	3075.5227	-10.2949	5.2337		
3257	103.740556	31.581110	2218.6104	62615136.1113	978766.0935	3075.6335	-10.2883	5.2464		
3258	103.703884	31.560833	2207.1173	62615248.6381	978768.0029	3075.6482	-10.2846	5.2537		
3259	103.682782	31.531391	2245.2634	62614875.3318	978753.8691	3075.5901	-10.2793	5.2643		

The geoidal geopotential W_s is always equal to the normal geopotential U_s of the normal ellipsoid. PAGrav4.5 suggests that the geoidal geopotential W_s should replace the appoint empirical W_s in the IERS numerical standard. The latter is calculated from the global geopotential model and mean sea surface height model according to the Gaussian geoid appoint.

2.1.2 Calculation of Earth ellipsoid constant and geopotential W_g analysis

[Function] From four basic parameters of the Earth ellipsoid, calculate the main

geometric and physical derived constants of the Earth ellipsoid.

[Parameter settings] Select the four basic parameters of the Earth ellipsoid.

The fourth basic parameter can be selected from the second-degree zonal harmonic coefficient \bar{C}_{20} from global geopotential model, dynamic form factor J_2 , reciprocal $1/f$ of the ellipsoid flattening and ellipsoid normal geopotential U_0 .

The coefficient \bar{C}_{20} from EGM2008 global geopotential model is selected currently as the fourth basic parameter.

[Output] Interactively output the summary of the calculation results of the Earth ellipsoid constants in the program interface.

The tide system of the normal ellipsoid is consistent with \bar{C}_{20} or J_2 .

Calculation of Earth ellipsoid constants and geopotential U_0 analysis

Set four basic parameters of Earth ellipsoid

Geocentric gravitational constant $GM(10^{-11}m^3/s^2)$ of the Earth 3.986004415 Mean angular velocity $\omega(10^{-5})$ of the Earth 7.292115 Major semi axis $a(m)$ of the Earth 6378136.3

Select the fourth basic parameter from $\bar{C}_{20}(10^{-6})$, $J_2(10^{-7})$, $1/f$ and U_0 Normal ellipsoid geopotential $U_0=Wo$ 62636858.3919

Enter the four basic parameters of Earth ellipsoid

Geometric derived constants of Earth ellipsoid

Reciprocal flattening $1/f$ 298.2577612300

Minor semi axis of the Earth $b(m)$ 6356751.5584

Radius of sphere of same volume $R(m)$ 6371000.0713

Linear eccentricity $E(m)$ 521853.4816

Square of first eccentricity e^2 0.006694367942498012

Square of second eccentricity e'^2 0.006739333137795320

Equatorial curvature radius $M(m)$ 6335438.7088

Polar radius of curvature $c(m)$ 6399592.8846

Physical derived constants of Earth ellipsoid

Dynamic form factor J_2 1.0826322774 Normal potential at ellipsoid $U_0=Wo(m^2/s^2)$ 62636858.3919 Gravity flattening reciprocal $1/f_0$ 517.6435137497

Geodetic parameter m 0.0034497853945 Normal gravity at equator $g_0(m/s^2)$ 9.7803275820 Normal gravity at pole $g_p(m/s^2)$ 9.8321870774

Computation Process ** Operation Prompts

Polar radius of curvature $c = 6399592.8846$

Normal potential at ellipsoid $U_0 = 62636858.3919$

Gravity flattening reciprocal $1/f_0 = 517.6435225016$

Geodetic parameter $m = 0.0034497853945$

Normal gravity at equator $g_0 = 9.7803274325$

Normal gravity at pole $g_p = 9.8321870775$

The reciprocal $1/f$ of the ellipsoid flattening selected as the fourth basic parameter.

The four basic parameters of the Earth ellipsoid have been entered into the system!

Click the [Calculation of the derived constants of Earth ellipsoid] control button, or the [Summary of the calculation results of the Earth ellipsoid constants] (see the interface for units).

Complete the calculation of the main geometric and physical derived constants of the Earth ellipsoid!

Summary of the calculation results of the Earth ellipsoid constants (see the interface for units):

Geocentric gravitational constant of the Earth (including the atmosphere) $GM = 3.986004415$

Major semi axis of the Earth $a = 6378136.3000$

Dynamical form factor of the Earth $J_2 = 1.0826322774$

Mean angular velocity of the Earth $\omega = 7.292115$

Reciprocal flattening $1/f = 298.2577612300$

Minor semi axis of the Earth $b = 6356751.5584$

Radius of sphere of same volume $R = 6371000.0713$

Linear eccentricity $E = 521853.4816$

Square of first eccentricity $e^2 = 0.006694367942498012$

Square of second eccentricity $e'^2 = 0.006739333137795320$

Equatorial curvature radius $M = 6335438.7088$

Polar radius of curvature $c = 6399592.8846$

Normal potential at ellipsoid $U_0 = 62636858.3919$

Gravity flattening reciprocal $1/f_0 = 517.6435225016$

Geodetic parameter $m = 0.0034497853945$

Normal gravity at equator $g_0 = 9.7803274325$

Normal gravity at pole $g_p = 9.8321870775$

The tide system of the normal ellipsoid is consistent with \bar{C}_{20} or J_2 .

PAGrav4.5 suggests that the scale parameters (GM , a) of global geopotential model, second-degree zonal harmonic coefficient \bar{C}_{20} and the mean rotation angular velocity ω should be employed as the four basic parameters of the normal ellipsoid. Using such a normal ellipsoid as the reference datum, the second-degree zonal harmonic term of anomalous gravity field is always zero, which is beneficial to improve the performance of the gravity field approach.

Geopotential model	Scale parameters		$\bar{C}_{20} \times 10^{-4}$	$W_0/U_0 (m^2/s^2)$	Tide system
	$GM \times 10^{14} m^2/s^3$	$a(m)$			
EGM2008	3.986004415	6378136.3	-4.84165143791	62636858.392	Tide free
EIGEN-6C4	3.986004415	6378136.46	-4.84165217061	62636856.834	zero tide
SGG-UGM2	3.986004415	6378136.3	-4.84168732275	62636858.644	zero tide

GOCO05c	3.986004415	6378136.3	-4.84169458843	62636858.694	zero tide
XGM2019	3.986004415	6378136.3	-4.84169494748	62636858.697	zero tide

2.2 Calculation of global geopotential model and its spectral character analysis

[Purpose] From the global geopotential coefficient model, calculate the model value of anomalous gravity field elements at any point in the Earth space, calculate the degree variance of the geopotential coefficients, error degree variance and cumulative error of anomalous field elements, then evaluate the performance of the geopotential model.

When the minimum and maximum degree n to be set is equal, the program calculates the contribution of the degree n geopotential coefficients to the anomalous gravity field element, which can be employed to analyze and evaluate the spectral and space properties of the geopotential coefficient model.

2.2.1 Calculation of gravity field elements from global geopotential model

[Function] From global geopotential coefficient model, calculate the model value of the (residual) height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection vector ("), south, west), disturbing gravity gradient (E, radial), tangential gravity gradient vector (E, north, west) or Laplace operator (E).

[Input files] The global geopotential coefficient model file, and the space calculation point location file.

The first row of the geopotential model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant GM ($10^{14}m^3/s^2$) and semi-major axis $a(m)$ of the Earth. Here, the surface harmonic functions in the geopotential model are defined on the spherical surface whose radius is equal to the semi-major axis a of the Earth.

The space calculation point location file may be a discrete calculation point file or an ellipsoidal height grid file of the calculation surface.

The record format of the calculation point file: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m).....

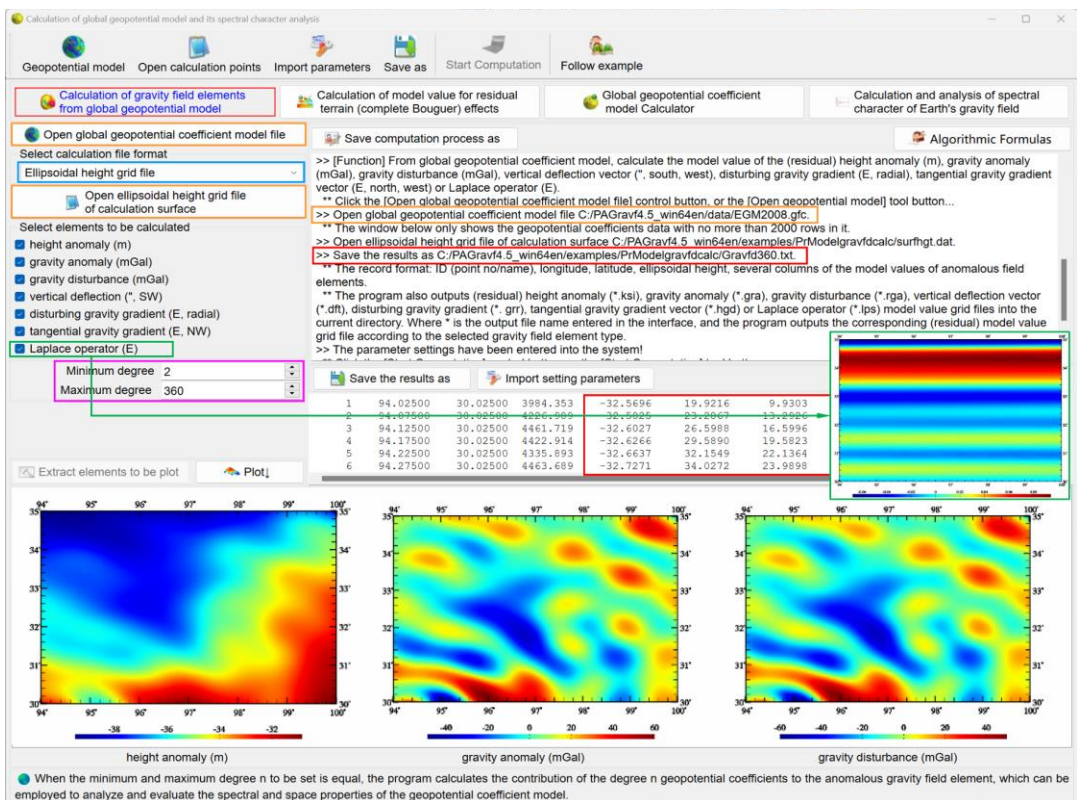
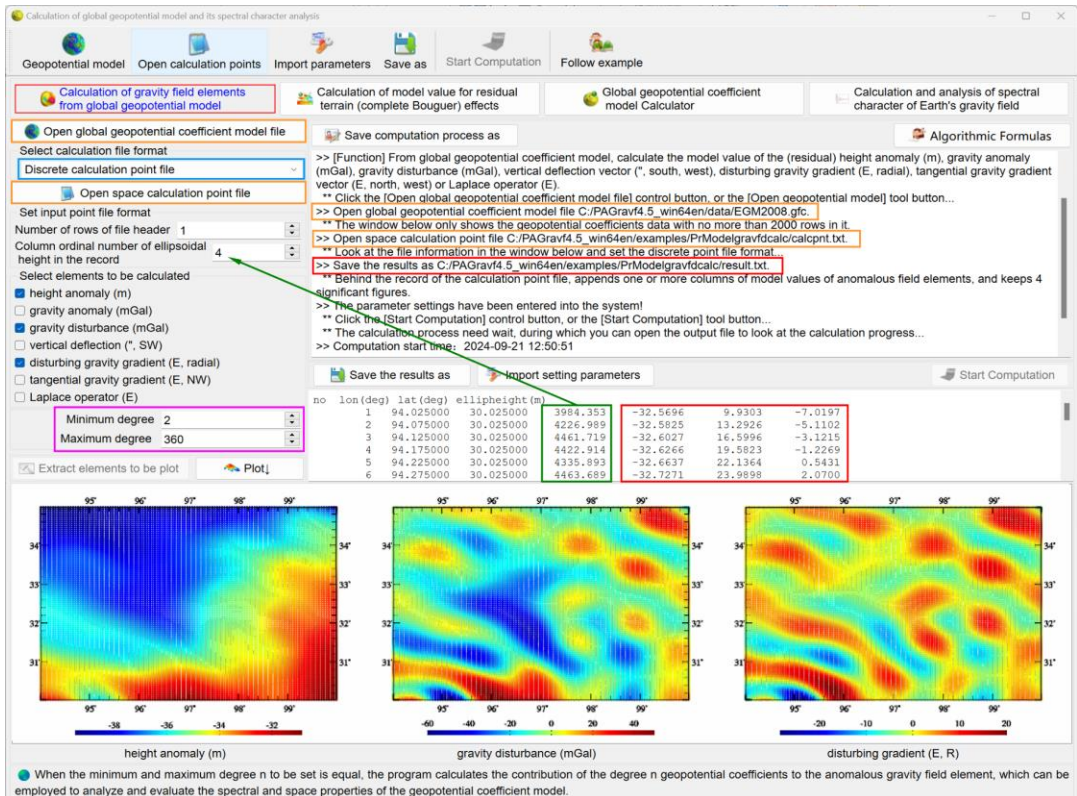
[Parameter settings] Input the number of rows of the file header, column ordinal number of ellipsoidal height in the record, enter the minimum and maximum calculation degree of the geopotential model, and select the type of model gravity field elements to be calculated.

When the minimum calculation degree is equal to 2, the program calculates the model value of anomalous gravity field elements. And when the minimum calculation degree is greater than 2, the program calculates the model value of residual anomalous gravity field elements.

The program selects the minimum of the maximum degree of the global geopotential model and the input maximum degree as the calculation degree.

The calculation process need wait, during which you can open the output file to look

at the calculation progress...



[Output file] The model value file of (residual) anomalous gravity field element.

When the discrete calculation point file input, the output file record format: Behind the record of the calculation point file, appends one or more columns of model values of anomalous field elements selected, and keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: point no/name, longitude, latitude, ellipsoidal height, several columns of the model values of anomalous field elements selected.

The program also outputs (residual) height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection vector (*.dft), disturbing gravity gradient (*.grr), tangential gravity gradient vector (*.hgd) or Laplace operator (*.lps) model value grid files into the current directory. Where * is the output file name entered in the interface, and the program outputs the corresponding (residual) model value grid file according to the selected gravity field element type.

The theoretical value of the Laplace operator (E) of any degree n, cumulative n degree or $n_1 \sim n_2$ degree are always equal to zero. By calculating the degree n, cumulative n degrees or $n_1 \sim n_2$ degrees of Laplace operators from some a global geopotential model, the spectral domain and spatial domain performance of the model can be observed and evaluated.

2.2.2 Calculation of model value for residual terrain (complete Bouguer) effects

[Function] From the global land-sea terrain geopotential coefficient model (m, the first degree term is set to zero), calculate the model value of residual terrain (complete Bouguer) effects on the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection vector (" , south, west), disturbing gravity gradient (radial, E), tangential gravity gradient vector (E, north, west) or Laplace operator.

The global land-sea terrain geopotential coefficient model can be constructed by the function [Ultrahigh degree land-sea terrain spherical harmonic analysis and model construction].

[Input files] The global land-sea terrain geopotential coefficient model file, and the space calculation point location file.

The first row of the model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant GM ($10^{14} \text{m}^3/\text{s}^2$) and semi-major axis a(m) of the Earth.

The space calculation point location file may be a discrete calculation point file or an ellipsoidal height grid file of the calculation surface.

The record format of the calculation point file: point no / point name, longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m).....

[Parameter settings] Input the number of rows of the file header, column ordinal number of ellipsoidal height in the record, input the minimum and the maximum calculation degree of the land-sea terrain geopotential model, and select the type of model residual terrain (complete Bouguer) effects to be calculated.

When the minimum calculation degree is equal to 2, the program calculates the model value of land-sea complete Bouguer effects. And when the minimum calculation degree is greater than 2, the program calculates the model value of land-sea residual terrain effects.

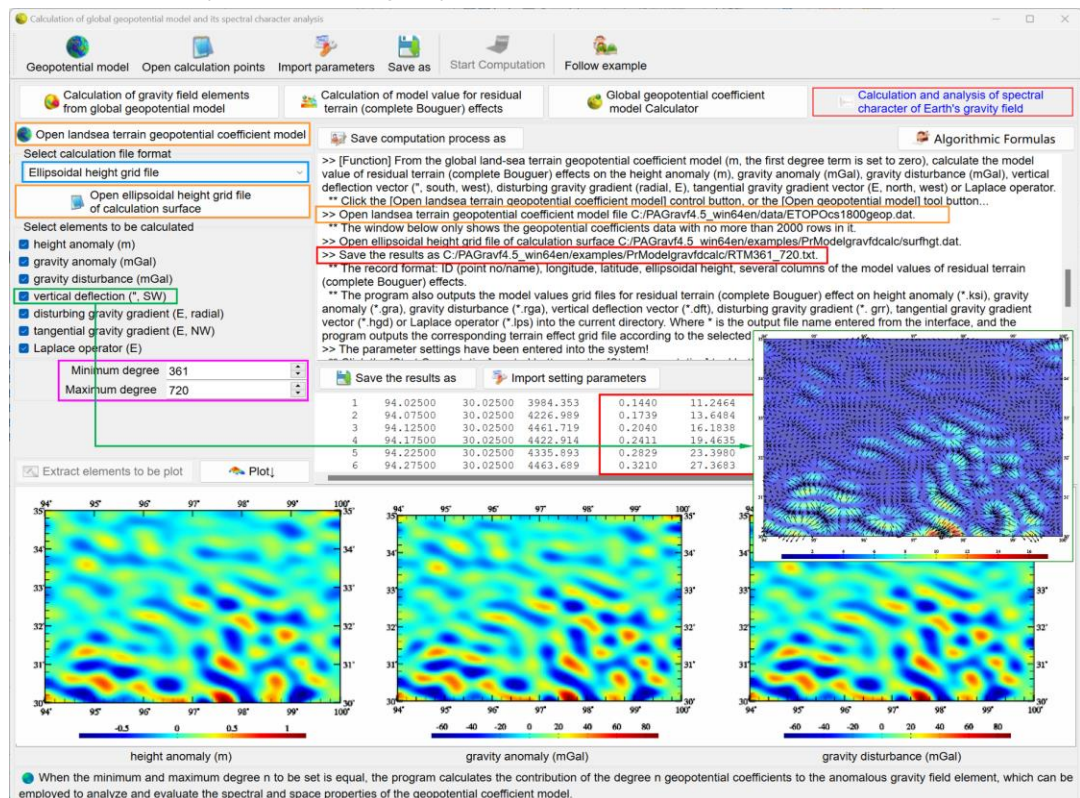
The program selects the minimum of the maximum degree of the model and the input maximum degree as the calculation degree.

The calculation process need wait, during which you can open the output file to look at the calculation progress...

[Output file] The model value file of residual terrain (complete Bouguer) effects.

When the discrete calculation point file input, the output file record format: Behind the record of the calculation point file, appends one or more columns of the model values of residual terrain (complete Bouguer) effects selected, and keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: point no/name, longitude, latitude, ellipsoidal height, several columns of the model values of residual terrain (complete Bouguer) effects selected.



The program also outputs the model value grid files for residual terrain (complete Bouguer) effect on height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection vector (*.dft), disturbing gravity gradient (*.grr), tangential gravity gradient vector (*.hgd) or Laplace operator (*.lps) into the current directory. Where * is the output file name entered from the interface, and the program outputs the

corresponding terrain effect grid file according to the selected gravity field element type.

Similarly, by calculating the degree n , cumulative n degrees or $n_1 \sim n_2$ degrees Laplace operators from some a global terrain geopotential model, the spectral domain and spatial domain performance of the model or that of residual terrain effect model values can be evaluated.

2.2.3 Global geopotential coefficient model calculator

[Function] Given the geodetic coordinates of the calculation point, from the global geopotential coefficient model file, calculate the model values of the height anomaly (m), gravity anomaly (mGal), gravity disturbance (mGal), vertical deflection vector (" , south, west), disturbing gravity gradient (radial, E), tangential gravity gradient vector (E, north, west), Laplace operator (E), gravity gradient (E) and geopotential (m^2/s^2). This program is suitable for classroom demonstrations.

When opening an ultrahigh degree geopotential coefficient model file, program need read and initialize, please wait...

After the geopotential model input, the geodetic coordinates of the calculation point can be entered repeatedly, and the model values of various field elements at the calculation point can be calculated and displayed in time.

From the Laplace equation, the sum of the disturbing gravity gradient (radial) and two components of the tangential gravity gradient should theoretically be equal to zero. Accordingly, the performance of the geopotential model can be evaluated.

Global geopotential coefficient model calculator

Input the geodetic coordinates of calculation point

Longitude 87.240000° Latitude 31.428100° Ellipsoidal height 5417.8300 m

Open global geopotential coefficient model file

When opening an ultrahigh-degree geopotential coefficient model file, program need read and initialize, please wait...

Set maximum calculation degree 1800

Start Computation

Model gravity field elements calculated

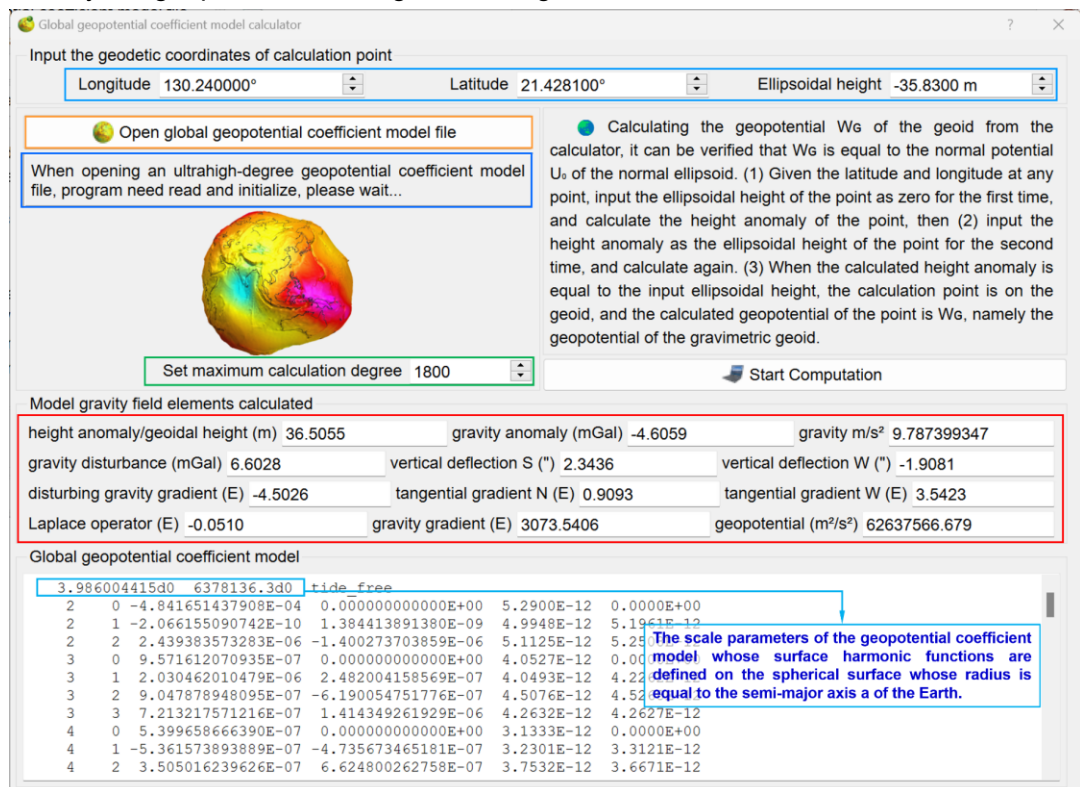
height anomaly/geoidal height (m)	-28.6109	gravity anomaly (mGal)	35.8449	gravity m/s^2	9.777951466
gravity disturbance (mGal)	27.0723	vertical deflection S (")	5.0582	vertical deflection W (")	-1.0667
disturbing gravity gradient (E)	1.9139	tangential gradient N (E)	-30.5760	tangential gradient W (E)	28.7014
Laplace operator (E)	0.0393	gravity gradient (E)	3072.8990	geopotential (m^2/s^2)	62583559.925

Global geopotential coefficient model

3.986004415d0 6378136.3d0 tide free

The scale parameters of the geopotential coefficient model whose surface harmonic functions are defined on the spherical surface whose radius is equal to the semi-major axis a of the Earth.

Calculating the geopotential W_G of the geoid from the calculator, it can be verified that W_G is equal to the normal potential U_0 of the normal ellipsoid. Calculation process: Given the latitude and longitude at any point, input the ellipsoidal height of the point as zero for the first time, and calculate the height anomaly of the point, then input the height anomaly as the ellipsoidal height of the point for the second time, and calculate again. When the calculated height anomaly is equal to the input ellipsoidal height, the calculation point is on the geoid, and the calculated geopotential of the point is W_G , namely the geopotential of the gravimetric geoid.



It is easy to verify that the gravity potential on the geoid at any point is always equal to W_G . Changing the latitude and longitude of the calculation point, adjust the ellipsoidal height of the calculation point according to the calculated value of the height anomaly, then you will find that the gravity potential of the geoid remains unchanged at any point all the word.

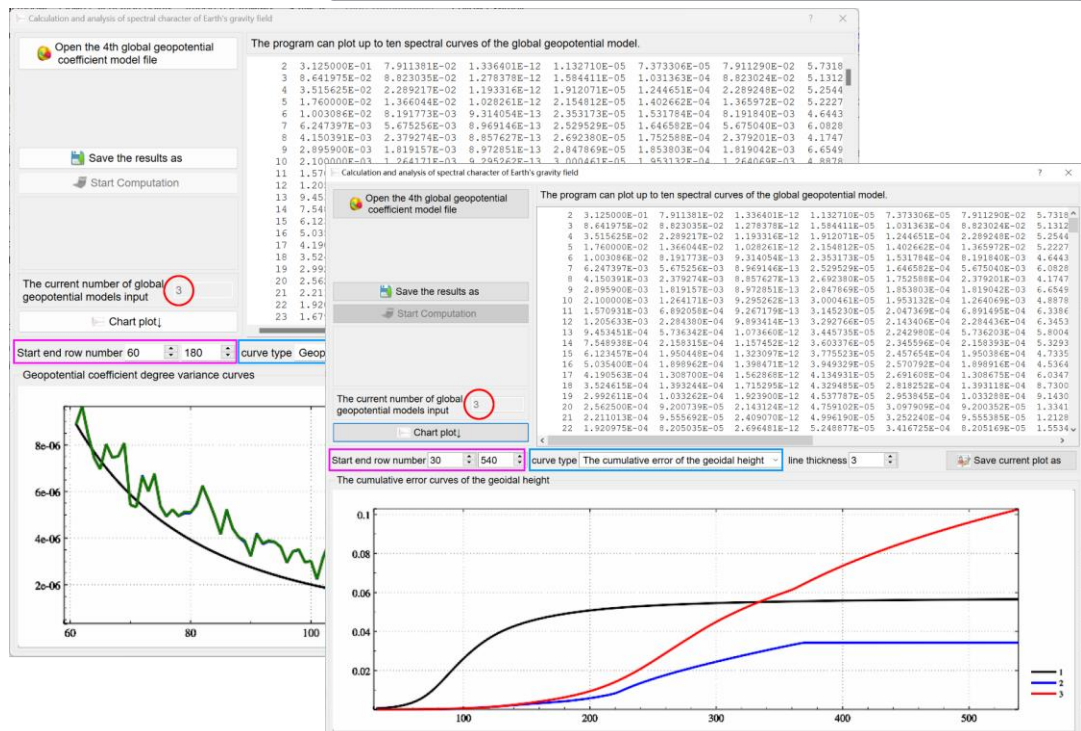
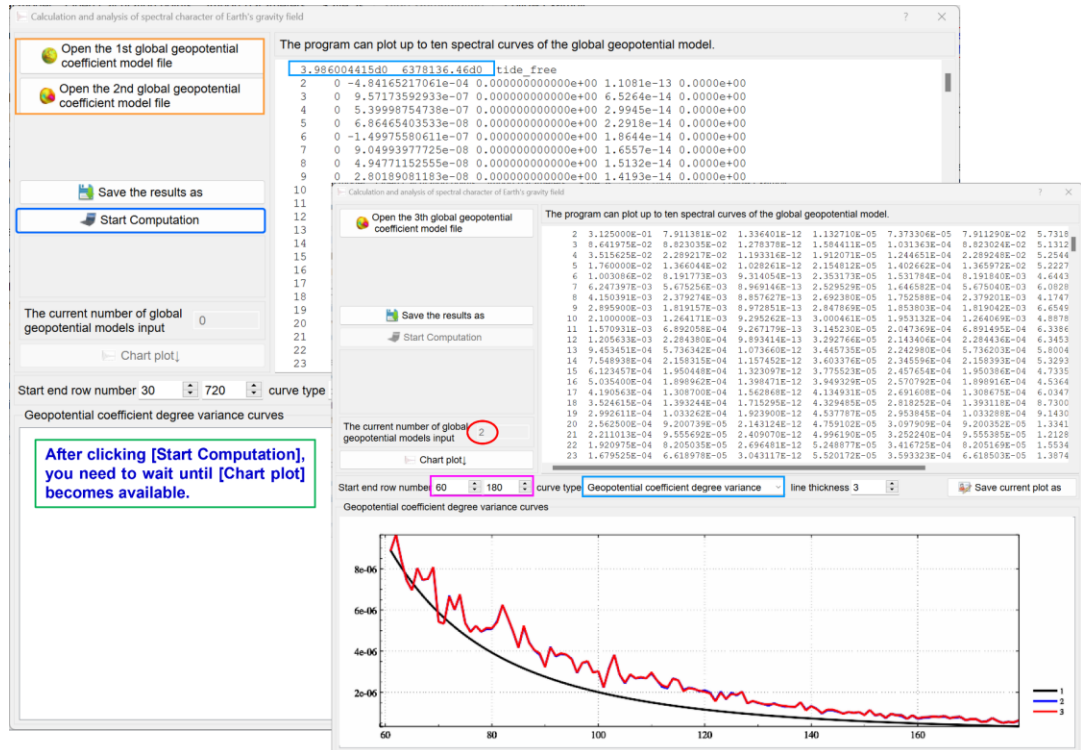
2.2.4 Calculation and analysis of spectral character of Earth's gravity field

[Function] From several global geopotential models, calculate the Kaula curve, geopotential coefficient degree variance, error degree variance and cumulative error of the geoid and gravity anomaly. Compare the spectral properties of different geopotential coefficient models.

Firstly, open two global geopotential coefficient model files, can then continue to open other geopotential coefficient model files as needed. The program can plot up to

ten spectral curves of the global geopotential model.

After clicking [Start Computation], you need to wait until [Chart plot] becomes available.



The interface shows the current number of global geopotential models input.

The geopotential coefficient degree variance curves, the error degree variance curves or the cumulative error curves of the geoid and gravity anomaly can be selected to plot.

2.3 Calculation of observed anomalous field element and error analysis of geoid

[Purpose] From the observed gravity data, calculate the gravity anomaly, gravity disturbance or disturbing gravity gradient at the measurement point, and estimate the influence of the grid mean gravity error and integral radius on regional gravimetric geoid.

2.3.1 Calculation of anomalous field elements at observed points

[Function] Using the rigorous spherical harmonic expansion formula, calculate the normal gravity field elements, and then calculate the gravity anomaly (mGal), gravity disturbance (mGal) or disturbing gravity gradient (E, radial) from the observed gravity (mGal) or gravity gradient (E).

[Input file] The observed gravity data file.

The record format: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), height (m), ..., observation,

[Parameter settings] Input the number of rows of the file header, column ordinal number of height attribute and observation in record.

Calculation of observed anomalous field element and error analysis of geoid

Open observations | Save as | Import parameters | Start Computation | Save process | Follow example

Calculation of anomalous field elements at observed points | Statistical error estimation of regional gravimetric geoid | Calculation of influence of gravity system bias on gravimetric geoid

Open the observed gravity data file

Set input point file format

Number of rows of file header: 1

Column ordinal number of height in the record: 4

Column ordinal number of observation in the record: 6

Select calculation type: gravity anomaly (mGal)

Save the results as

Import setting parameters

Start Computation

Computation Process ** Operation Prompts

>> [Function] Using the rigorous spherical harmonic expansion formula, calculate the normal gravity field elements, and then calculate the gravity anomaly (mGal), gravity disturbance (mGal) or disturbing gravity gradient (E, radial) from the observed gravity (mGal) or gravity gradient (E).

** Click the [Open the observed gravity data file] control button, or the [Open observation file] tool button...

>> Open the observed gravity data file C:\PAGrav4.5_win64en/examples/ProbsAnomousgrav/obsgrav.txt.

** Look at the file information in the window below, set the input file format parameters, select the anomalous field element, and click the [Import setting parameters] button to input the parameters into the system...

** When the height is normal (orthometric) height, the program calculates the gravity anomaly at the observed point, and when the height is the ellipsoidal height, the program calculates the gravity disturbance or disturbing gravity gradient at the observed point.

>> Save the results as C:\PAGrav4.5_win64en/examples/ProbsAnomousgrav/result.txt.

** Behind the input file record, appends a column of gravity anomaly, gravity disturbance or disturbing gravity gradient calculated value at the observed point, and keeps 4 significant figures.

>> The parameter settings have been entered into the system!

** Prepare to calculate gravity anomaly (mGal)...

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-21 17:09:51

>> Complete the calculation of the observed anomalous field element!

no	lon(degree/decimal)	lat	ellipHeight(m)	normalHeight(m)	obsGrav(mGal)
3248	103.671939	31.938051	2743.9394	2774.9485	978685.3537
3249	103.696944	31.864721	2501.2449	2532.6723	978711.5370
3250	103.718330	31.831114	2435.4206	2467.0248	978720.1941
3251	103.735559	31.795280	2366.5700	2398.2238	978757.9613
3252	103.777216	31.776390	2294.0304	2325.9778	978784.3962
3253	103.822773	31.758333	2233.2317	2265.6064	978787.9523
3254	103.849717	31.724168	2215.6606	2248.2924	978780.1624
3255	103.816666	31.650003	2242.9951	2275.1975	978785.6408
3256	103.783335	31.616667	2297.3654	2329.3256	978762.6285
3257	103.740556	31.581110	2218.6104	2250.3209	978787.4579
3258	103.703884	31.560833	2207.1173	2238.6600	978797.9046
3259	103.682782	31.531391	2245.2634	2276.8015	978786.2913
3260	103.651939	31.510554	2219.9076	2251.4497	978780.8011
3261	103.613883	31.491662	2080.5161	2112.0781	978799.7103
3262	103.545003	31.444998	2051.8797	2083.3689	978803.4042
3263	103.516664	31.386384	2187.5689	2219.2172	978769.9887
3264	103.499170	31.362780	2175.1817	2206.9690	978758.0577
3265	103.474724	31.325836	2075.9238	2107.9969	978756.2234
3266	103.469164	31.282503	1931.3869	1963.8325	978794.8112
3267	103.486671	31.251384	1935.5737	1968.3811	978806.5352

Extract elements to be plot | Plot

gravity anomaly (mGal)

When the height is normal (orthometric) height, the program calculates the gravity

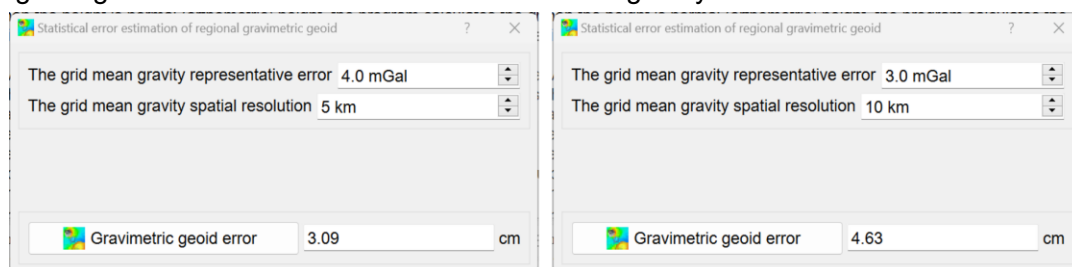
anomaly at the observed point, and when the height is the ellipsoidal height, the program calculates the gravity disturbance or disturbing gravity gradient at the observed point.

[Output file] The observed anomalous field element file.

Behind the input file record, appends a column of gravity anomaly, gravity disturbance or disturbing gravity gradient calculated value at the observed point, and keeps 4 significant figures.

2.3.2 Statistical error estimation of regional gravimetric geoid

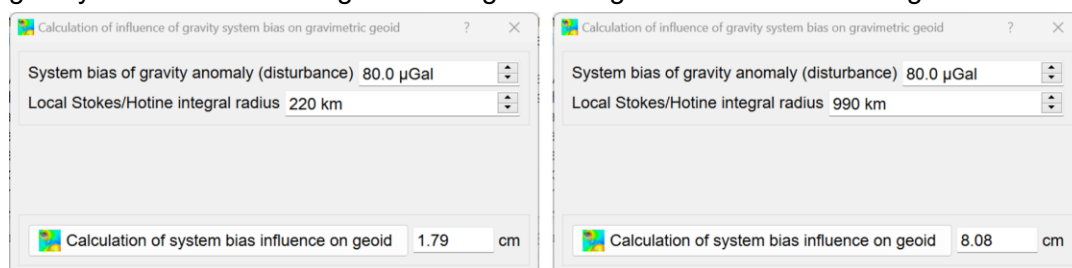
[Function] Estimate the error of the regional gravimetric geoid from the spatial resolution and error of the grid mean gravity. The regional gravimetric geoid error analysis is based on the Stokes/Hotine integral formula with a finite integral radius, ignoring the far-area effect of residual anomalous gravity field.



The grid mean gravity representative error here mainly refers to the terrain representative error, because in general, the gravity observation error during the gridding process is much smaller than the terrain representative error.

2.3.3 Calculation of influence of gravity system bias on gravimetric geoid

[Function] Calculate the influence of the systematic bias of the gravity anomaly or gravity disturbance on the gravimetric geoid using the Stokes/Hotine integral formula.



The system bias of gravity anomaly or gravity disturbance. Whose possible causes are the inconsistency in normal gravity field, nonuniform in height datum, improper value of global geopotential W_0 or gravity datum error etc.

2.4 Correction of boundary value problem for gravity field element on non-equipotential surface

[Purpose] Correct the anomalous gravity field elements located on the spherical surface, ellipsoidal surface or other shape of non-equipotential surface, to convert the Molodensky boundary value problem on non-equipotential boundary surface into the

Stokes boundary value problem.

2.4.1 Correction calculation of boundary value for spherical or ellipsoidal boundary surface

[Function] With the geopotential model, calculate the correction value (mGal) of the gravity disturbance or gravity anomaly from the non-equipotential spherical or ellipsoidal boundary surface into the equipotential surface, thereby converting the Molodensky boundary value problem into the Stokes problem.

The ellipsoidal boundary value correction is mainly employed for ellipsoidal harmonic analysis of the global gravity field by FFT integral method. Spherical boundary value correction is commonly employed for the Bjerhammar boundary value problem.

[Input files] The global geopotential coefficient model file, the calculation point file on the boundary surface.

The first row of the geopotential model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant GM ($10^{14}m^3/s^2$) and semi-major axis $a(m)$ of the Earth.

The record format of the calculation point file: ID (point no / point name), longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m).....

Correction of boundary value problem for gravity field element on non-equipotential surface

Open calculation points Import parameters Save as Start Computation Save process Follow example

Correction calculation of boundary value for spherical or ellipsoidal boundary surface Molodensky boundary value correction for arbitrary shape boundary surface Boundary value correction formula

Open global geopotential coefficient model file

Open the calculation point file on non-equipotential surface

Set input point file format

Number of rows of file header 1

Column ordinal number of ellipsoidal height in the record 4

Select type of field element

gravity disturbance (mGal)

Select type of boundary surface

ellipsoidal surface

Maximum calculation degree 360

Computation Process ** Operation Prompts

Save computation process as

surface, thereby converting the Molodensky boundary value problem into the Stokes problem.

>> The ellipsoidal boundary value correction is mainly employed for ellipsoidal harmonic analysis of the global gravity field by FFT integral method. Spherical boundary value correction is commonly employed for the Bjerhammar boundary value problem.

** Input the global geopotential coefficient model file and the calculation point file on the boundary surface...

>> Open global geopotential coefficient model file C:/PAGrav4.5_win64en/data/EGM2008.gfc.

** The window below only shows the geopotential coefficients data with no more than 2000 rows in it.

>> Open the calculation point file on the boundary surface C:/PAGrav4.5_win64en/examples/PrBoundaryvalueAdj/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters, select the field element and boundary surface, enter the output file name, click the [Import setting parameters] button to input the parameters into the system...

>> Save the results as C:/PAGrav4.5_win64en/examples/PrBoundaryvalueAdj/result.txt.

** The boundary surface is an ellipsoidal surface. The program appends a column of the vertical deflection correction value of gravity behind the input file record.

>> The parameter settings have been entered into the system!

Save the results as Import setting parameters Start Computation

number (value or str)	long (degree/decimal)	lat (degree/decimal)	ellipHeight (m)
3248	103.671939	31.938051	2743.9394
3249	103.696944	31.864721	2501.2449
3250	103.718330	31.831114	2435.4206
3251	103.735559	31.795280	2366.5700
3252	103.777216	31.776390	2294.0304
3253	103.822773	31.758333	2233.2317
3254	103.849717	31.724168	2215.6606
3255	103.816666	31.650003	2242.9951
3256	103.783335	31.616667	2297.3654
3257	103.740556	31.581110	2218.6104
3258	103.703884	31.560833	2207.1173
3259	103.682782	31.531391	2245.2634
3260	103.651939	31.510554	2219.9076
3261	103.613883	31.491662	2080.5161
3262	103.545003	31.444998	2051.8797
3263	103.516664	31.386384	2187.5689
3264	103.499170	31.362780	2175.1817
3265	103.474724	31.325836	2075.9238

Extract corrections Plot

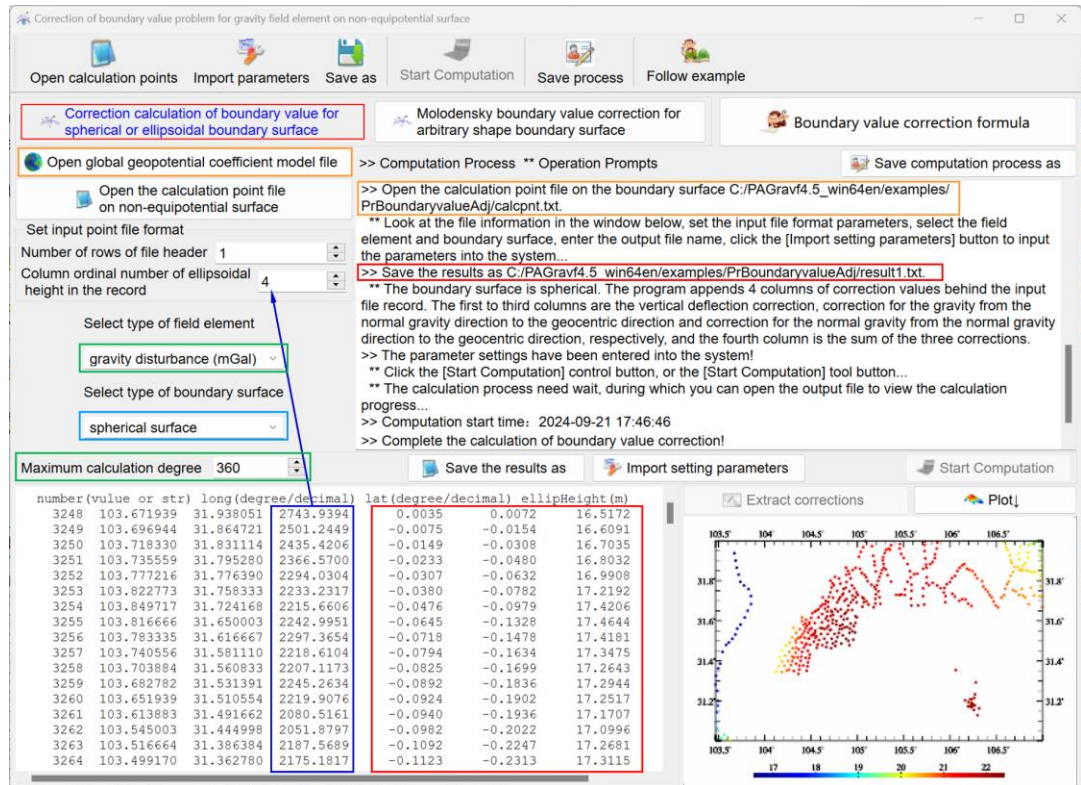
[Parameter settings] Input the number of rows of the calculation point file header, and column ordinal number of height and observation attribute in record, select the type of field element and boundary surface, and enter the maximum calculation degree of

geopotential coefficient model.

The program selects the minimum of the maximum degree of the global geopotential model and the input maximum degree as the calculation degree.

[Output file] The calculation result file.

When the boundary surface is an ellipsoidal surface, the program appends a column of the vertical deflection correction value of gravity behind the input calculation point file record.



When the boundary surface is spherical, the program appends 4 columns of correction values behind the input calculation point file record. The first to third columns are the vertical deflection correction, correction for the gravity from the normal gravity direction to the geocentric direction and correction for the normal gravity from the normal gravity direction to the geocentric direction, respectively, and the fourth column is the sum of the three corrections.

2.4.2 Molodensky boundary value correction for arbitrary shape boundary surface

[Function] From the gravity anomaly or gravity disturbance (mGal) model grid, height anomaly model grid, ellipsoidal height grid of the boundary surface and ellipsoidal height grid of the reference equipotential surface (for Stokes boundary problem), calculate the Molodensky I corrections of the gravity anomalies or gravity disturbances on the non-equipotential boundary surface on the ground or outside the Earth, thereby

converting the Molodensky boundary value problem into a Stokes problem.

The boundary surface can be located at any altitude outside the geoid, and the shape of the boundary surface can be irregular.

[Input files] The calculation point file on non-equipotential surface, the ellipsoidal height grid file of the boundary surface, the field element grid file on the boundary surface, the height anomaly grid file on the boundary surface, and the ellipsoidal height grid file of the equipotential surface.

The grid specifications of the four input grid files are required to be identical, and the field element type is required to be the same as that selected in the selection box below.

The ellipsoidal height grid file of the boundary surface is employed to indicate the location of the non-equipotential boundary surface.

The ellipsoidal height grid file of the equipotential surface is employed to indicate the location of the reference equipotential surface.

[Parameter settings] Input the number of rows of the calculation point file header, column ordinal number of ellipsoidal height attribute in record, select the type of field element, and enter the Molodensky I integral radius.

Open the calculation point file on non-equipotential surface

Set input point file format

Number of rows of file header 1

Column ordinal number of ellipsoidal height in the record 4

Input grid data on non-equipotential surface

Open the ellipsoidal height grid file of the boundary surface

Open the field element grid file on the boundary surface

Open the height anomaly grid file on the boundary surface

Open the ellipsoidal height grid file of the equipotential surface

Select type of field element

gravity disturbance (mGal)

Molodensky I integral radius 120 km

Computation Process ** Operation Prompts

Save computation process as

files...

>> Open the calculation point file on the boundary surface C:/PAGrav4.5_win64en/examples/PrBoundaryvalueAdj/dbmrga.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Open the ellipsoidal height grid file of the boundary surface C:/PAGrav4.5_win64en/examples/PrBoundaryvalueAdj/dbmght150s.dat.

>> Open the field element grid file on the boundary surface C:/PAGrav4.5_win64en/examples/PrBoundaryvalueAdj/dbmGM1800150srga.dat.

>> Open the height anomaly grid file on the boundary surface C:/PAGrav4.5_win64en/examples/PrBoundaryvalueAdj/dbmGM1800150sksi.dat.

>> Open the ellipsoidal height grid file of the equipotential surface C:/PAGrav4.5_win64en/examples/PrBoundaryvalueAdj/dwmhgt150s.dat.

>> Save the results as C:/PAGrav4.5_win64en/examples/PrBoundaryvalueAdj/result2.txt.

** Behind the record of the source calculation point file, appends a column of the Molodensky boundary value correction, and keeps 4 significant figures.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-21 18:28:22

>> Complete calculation of Molodensky boundary value correction!

>> Computation end time: 2024-09-21 18:34:06

Save the results as

Import setting parameters

Start Computation

number	lon(degree/decimal)	ellipHeight(m)	disturbGrav(mGal)
11569	106.020833	27.020833	1217.221 -31.0162 -12.3972
11570	106.062500	27.020833	1201.227 -33.8392 -13.1861
11571	106.104167	27.020833	1185.247 -33.9853 -15.2362
11572	106.145833	27.020833	1210.287 -30.7623 -3.0684
11573	106.187500	27.020833	1228.340 -25.5689 3.5451
11574	106.229167	27.020833	1247.396 -21.2304 9.0750
11575	106.270833	27.020833	1244.440 -20.7500 10.4302
11576	106.312500	27.020833	1199.469 -25.2967 3.1198
11577	106.354167	27.020833	1183.494 -32.6787 8.8636
11578	106.395833	27.020833	1109.535 -37.6863 -10.3428
11579	106.437500	27.020833	1000.613 -35.4965 -55.1332
11580	106.479167	27.020833	1135.735 -25.6242 -8.7070
11581	106.520833	27.020833	1249.869 -14.5582 11.1296
11582	106.562500	27.020833	1251.986 -8.4721 4.7083
11583	106.604167	27.020833	1289.077 -9.8491 7.5174
11584	106.645833	27.020833	1292.154 -14.3500 8.7262
11585	106.687500	27.020833	1228.242 -15.3480 -0.9306
11586	106.729167	27.020833	1211.352 -9.6375 -3.4317

Extract corrections

Plot

[Output file] The Molodensky boundary value correction file.

Behind the record of the source calculation point file, appends a column of the Molodensky boundary value correction, and keeps 4 significant figures.

When the boundary surface is the ground surface and the reference equipotential

surface is the geoid, the program calculates the classical Molodensky I (mGal).

2.5 Analytical continuation of anomalous field elements using multi-order radial gradient

[Function] From the ellipsoidal height grid of the current altitude surface and anomalous field element grid on the surface, calculate the 1st to 3rd order radial gradients using the rigorous radial gradient integral formula, and then calculate the continuation corrections of the field elements from the current altitude to the target altitude.

[Input files] The anomalous field element grid file on the current altitude surface, the ellipsoidal height grid file of the current altitude surface, and the space position file of anomalous field elements.

The space position file record format: ID (Point no./point name), longitude (decimal degrees), latitude (decimal degrees),

[Parameter settings] Input the number of rows of the calculation point file header, column ordinal number of ellipsoidal height attribute in record, and enter the order number and integral radius of the radial gradient.

The larger the integral radius, the greater the edge effect. The integral radius should not be less than three times of the continuation height difference.

Open calculation points Import parameters Save as Start Computation Save process Follow example

Open the space position file of anomalous field elements

Set input point file format

Number of rows of file header 1

Column ordinal number of ellipsoidal height for current field element 4

Column ordinal number of ellipsoidal height for target field element 5

Open the field element grid file on the current altitude surface

Open the ellipsoidal height grid file of the current altitude surface

Order number of the radial gradient continuation 1

Radial gradient integral radius 30 km

no lon (degree/decimal) lat ellipsoid height (m) geoid height (m) disturb Grav (mGal)

11569	106.020833	27.020833	1217.221	-30.8052	-14.8212	-1.6832
11570	106.062500	27.020833	1201.227	-30.8052	-16.1491	-1.9187
11571	106.104167	27.020833	1185.247	-30.7849	-14.8039	-1.6058
11572	106.145833	27.020833	1210.287	-30.7411	-10.1454	-0.6859
11573	106.187500	27.020833	1228.340	-30.6802	-3.6100	0.6175
11574	106.229167	27.020833	1247.396	-30.6193	1.9480	1.7425
11575	106.270833	27.020833	1244.440	-30.5729	3.5017	2.0024
11576	106.312500	27.020833	1199.469	-30.5503	-0.1382	1.0932
11577	106.354167	27.020833	1183.494	-30.5360	-6.8208	-0.4436
11578	106.395833	27.020833	1109.535	-30.4998	-11.3515	-1.4961
11579	106.437500	27.020833	1000.613	-30.4157	-8.9350	-1.1580
11580	106.479167	27.020833	1135.735	-30.2841	0.8626	0.4426
11581	106.520833	27.020833	1249.869	-30.1357	11.6378	2.4377
11582	106.562500	27.020833	1251.986	-30.0096	17.2750	3.1842
11583	106.604167	27.020833	1289.077	-29.9216	15.2947	2.2168
11584	106.645833	27.020833	1292.154	-29.8523	10.0757	0.5266
11585	106.687500	27.020833	1228.242	-29.7662	8.2597	-0.1705
11586	106.729167	27.020833	1211.352	-29.6471	13.0764	0.8541
11587	106.770833	27.020833	1339.471	-29.5138	20.7468	2.8368

Save the results as Import setting parameters Start Computation

Extract corrections Plot

The radial gradient continuation method adopts the gradient of the observed field element to make analytical continuation, which is suitable for the upward and downward analytical continuation of ground, flight altitude and near range (10km).

The main components of the field element gradient are short-wave and ultra-short-wave, and the integral radius for radial gradient calculation generally does not need to be large. Therefore, the analytical continuation by the radial gradient method is conducive to the efficient usage of gravity field observation resources.

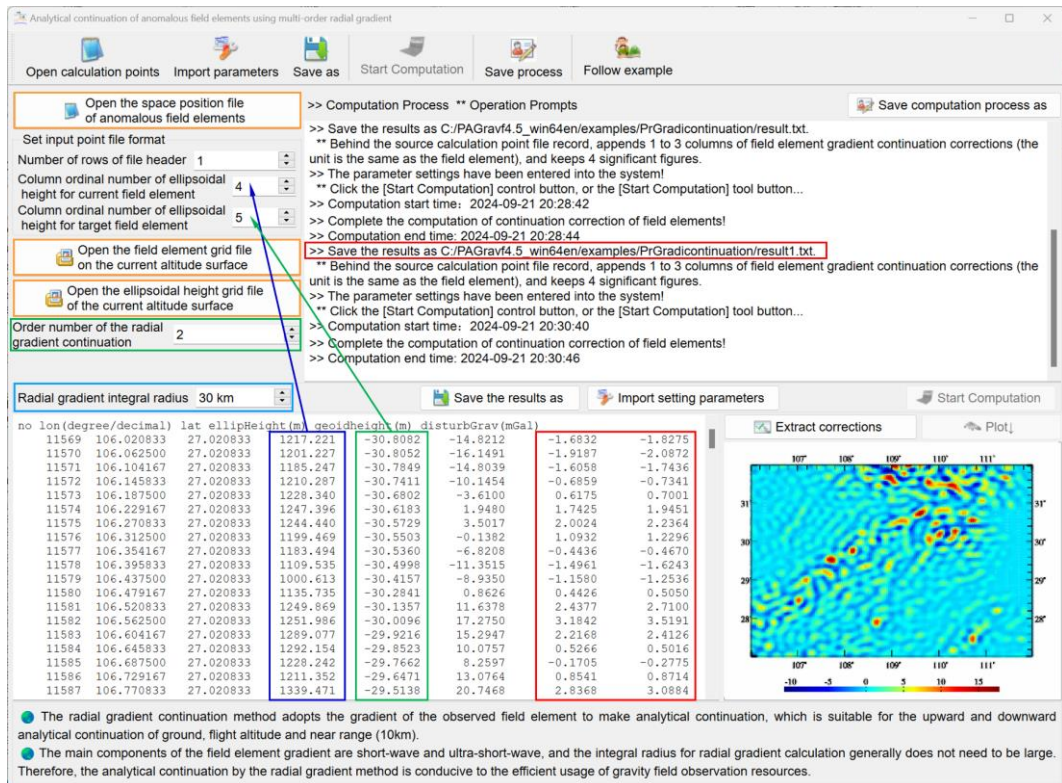
[Output file] The analytical continuation result file.

Behind the source calculation point file record, appends 1 to 3 columns of field

element gradient continuation corrections (the unit is the same as the field element), and keeps 4 significant figures.

The radial gradient continuation method adopts the gradient of the observed field element to make analytical continuation, which is suitable for the upward and downward analytical continuation of ground, flight altitude and near range (10km).

The main components of the field element gradient are short-wave and ultra-short-wave, and the integral radius for radial gradient calculation generally does not need to be large. Therefore, the analytical continuation by the radial gradient method is conducive to the efficient usage of gravity field observation resources.



PAGravf4.5 recommends a remove-continuation-restore scheme combined with ultra-high-degree geopotential model and residual field element radial gradient continuation. Firstly, remove the ultra-high-degree model values of field elements at the current altitude to obtain the residual field elements, and then conduct analytical continuation of residual field elements, and finally restore the ultra-high-degree model values of field elements at target altitude.

2.6 Gross error detection and basis function gridding of discrete field elements

2.6.1 Gross error detection on observations based on low-pass reference surface

[Function] Select the low-pass grid as the reference surface, interpolate the

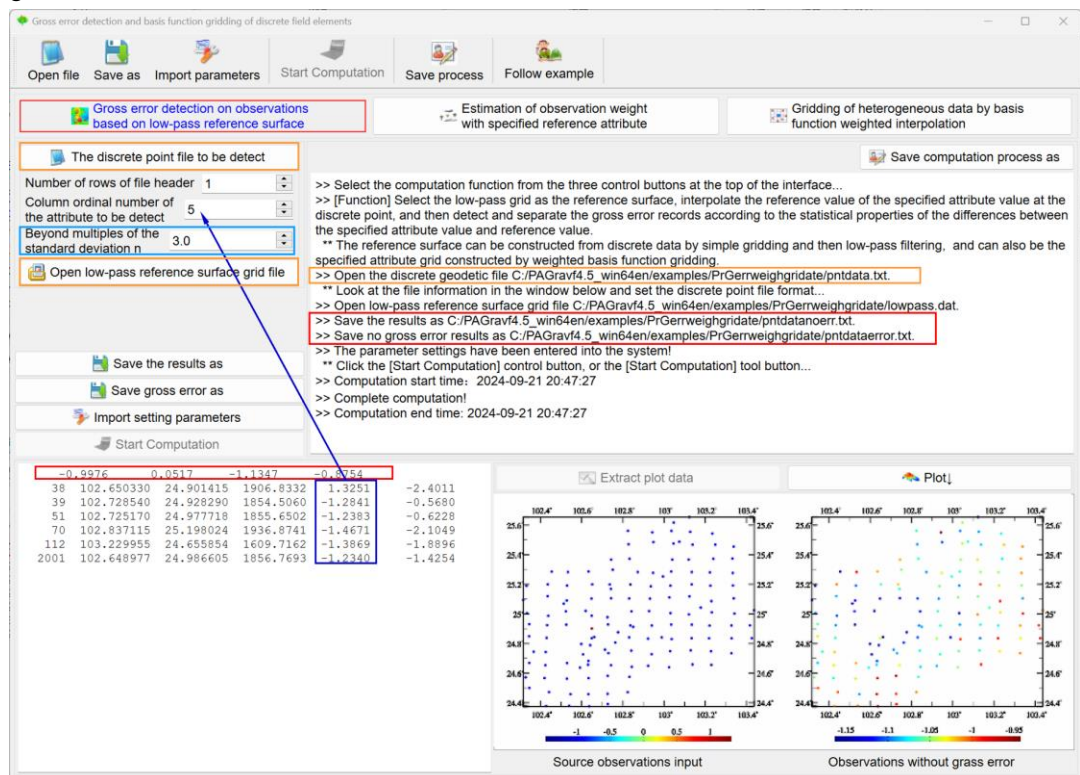
reference value of the specified attribute value at the discrete point, and then detect and separate the gross error records according to the statistical properties of the differences between the specified attribute value and reference value.

[Input files] The discrete geodetic point file to be detect, the low-pass reference surface grid file.

The reference surface can be constructed from discrete data by simple gridding and then low-pass filtering, and can also be the specified attribute grid constructed by weighted basis function gridding. When the zero-value grid of the observation area is employed as the reference surface, that is, no reference surface support, the program performs simple gross error detection.

[Parameter settings] Enter number of rows of the discrete geodetic point file header, column ordinal number of the attribute to be detect in the record, and beyond multiples of the standard deviation.

When the absolute value of the difference between the attribute and its mean is greater than n times the attribute standard deviation, the record in which attribute is a gross error record.



[Output files] The discrete geodetic point file without gross error, whose format is the same with the input discrete point file. The gross error point file, whose file header include the mean, standard deviation, minimum and maximum of the differences.

2.6.2 Estimation of observation weight with specified reference attribute

[Function] Using the weight function defined by PAGravf4.5, estimate the

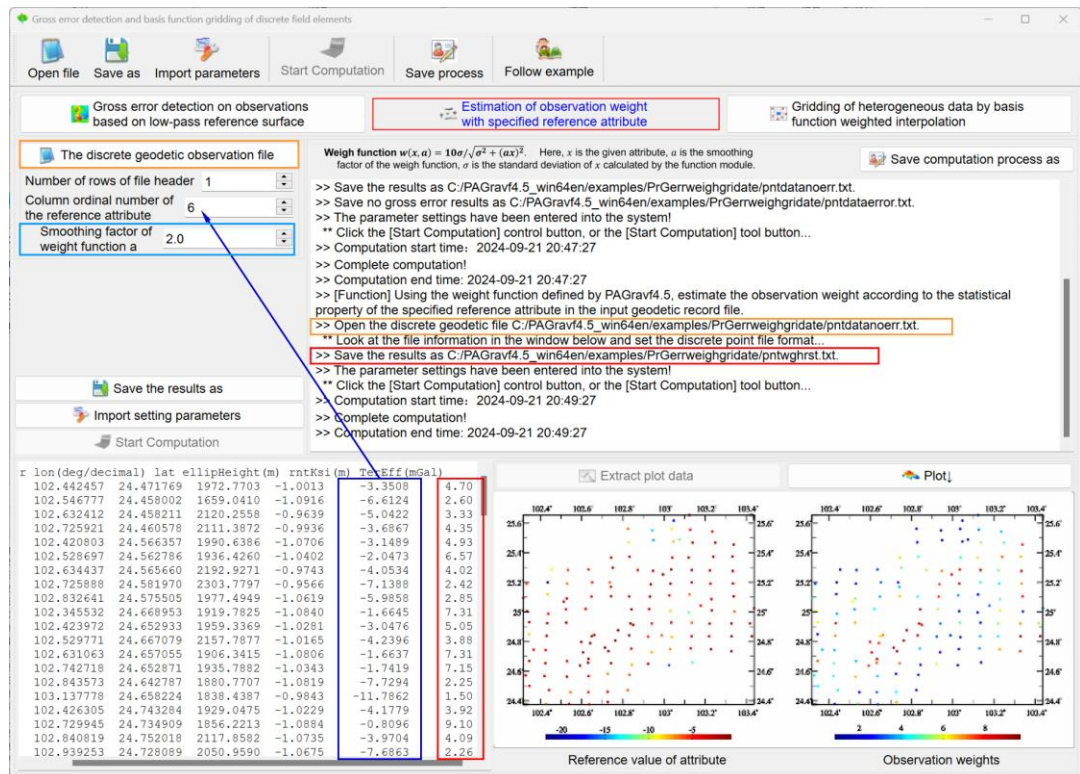
observation weight according to the statistical property of the specified reference attribute in the input geodetic record file.

[Input file] The discrete geodetic observation file.

Weight function defined by PAggrav4.5 is $w(x, a) = 10\sigma\sqrt{\sigma^2 + (ax)^2}$, here x is the reference attribute, a is the given smoothing factor of the weight function, and σ is the standard deviation of x calculated automatically by the program.

[Parameter settings] Enter number of rows of the discrete geodetic observation file header, column ordinal number of the reference attribute in the record, and the smoothing factor of the weight function.

The larger the weight function smoothing factor a , the slower the weight function w decays with distance.



2.6.3 Gridding of heterogeneous data by basis function weighted interpolation

[Function] According to the given grid specification (grid range and spatial resolution), and specified basis function, grid the specified attribute in the input discrete geodetic point file by the weighted basis function interpolation method.

[Input file] The discrete geodetic observation file.

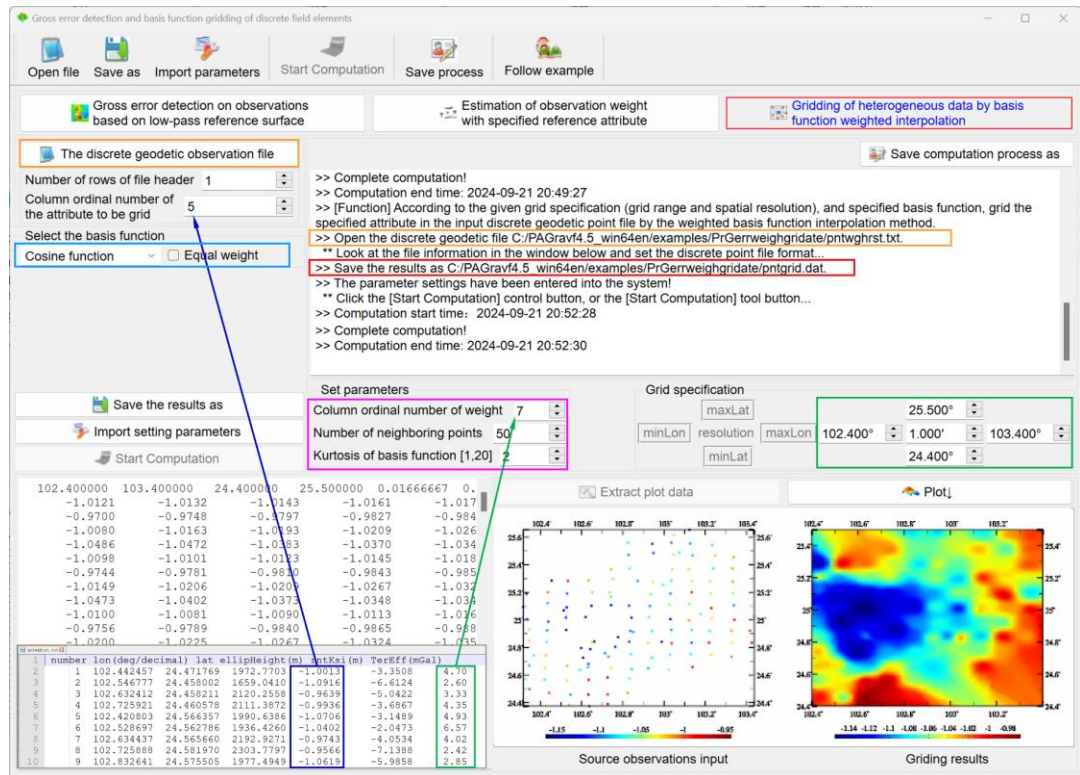
[Parameter settings] Enter number of rows of the discrete point file header, column ordinal number of the attribute to be grid in the file record, and grid specification parameters. Select the base function and set its parameters.

The smaller the kurtosis is (the slower the basis function value reduces with

distance), the larger the number of neighboring points for the interpolation, the smoother the interpolation result, the weaker the edge effect and the stronger the interpolation capacity for sparse data.

The interpolation weight is equal to the product of the attribute weight and base function value.

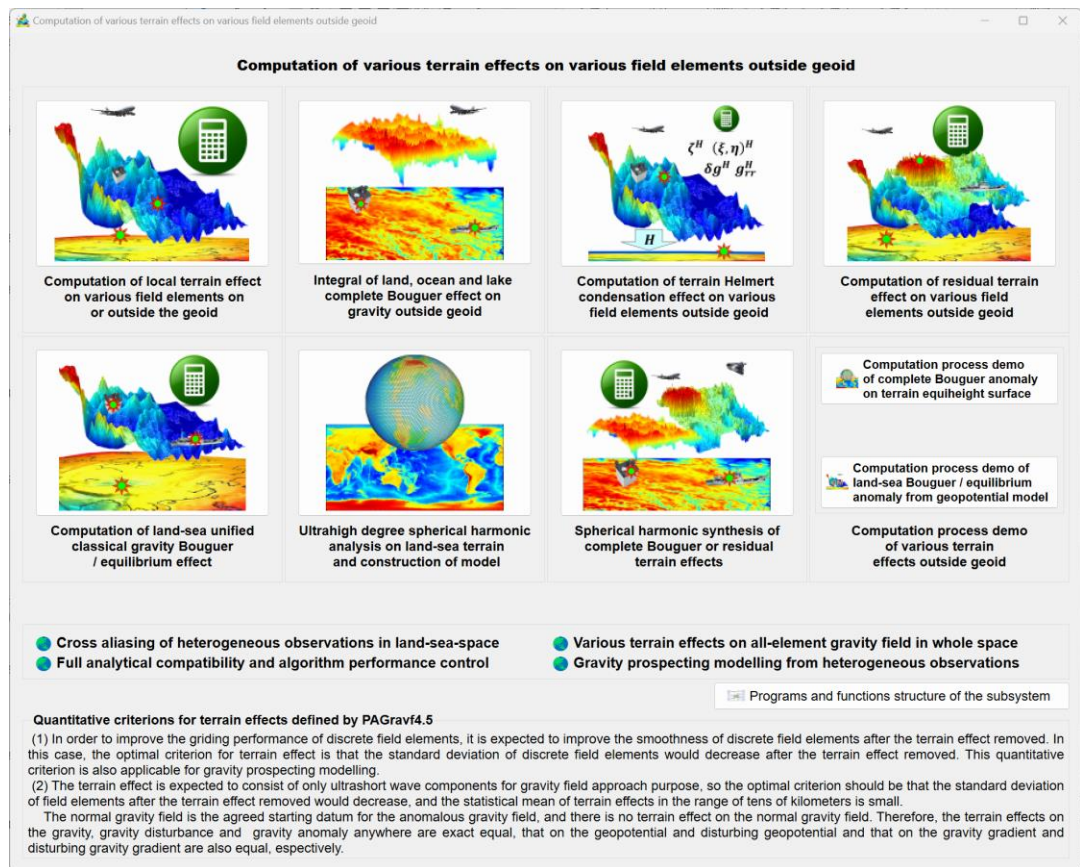
[Output file] The geodetic grid file.



The program is specially designed by PAGravf4.5 based on the properties of general geophysical field, and it is suitable for gridding of single types of multi-source heterogeneous geophysical field.

3 Computation of various terrain effects on various field elements outside geoid

PAGrav4.5 develops the unified analytical algorithm system for various modes of terrain effects on different types of gravity field elements on or outside the geoid to improve the gravity prospecting modelling and gravity field data processing, which can be employed uniformly to deal with various terrestrial, marine and airborne and satellite gravity field data.



The normal gravity field is the agreed starting datum for the anomalous gravity field, and there is no terrain effect on the normal gravity field. Therefore, the terrain effects on the gravity, gravity disturbance and gravity anomaly anywhere are exact equal, that on the geopotential and disturbing geopotential and that on the gravity gradient and disturbing gravity gradient are also equal, respectively.

From the terrain data of typical difficult mountainous areas with a mean altitude of 4000m and terrain undulation of more than 3000m, we verify and analyze the performance of various terrain effect algorithms in this group of programs to facilitate and quickly grasp the characteristics and usage of terrain effects.

3.1 Computation of local terrain effect on various field elements on or outside the geoid

[Purpose] Using the rigorous numerical integral method or FFT algorithm, from the ground digital elevation model and ground ellipsoidal height grid, compute the local terrain effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (" , to south, to west) or (disturbing) gravity gradient (E, radial) on or outside the geoid.

The terrain effect on field element is equal to the negative value of the classic terrain correction, such as the local terrain effect is equal to the negative local terrain correction. Since the normal gravity field keeps unchanged, the terrain effect on the gravity disturbance and gravity anomaly is always equal to the terrain effect on gravity.

3.1.1 Numerical integral of local terrain effects on various field elements outside geoid

[Function] Using the rigorous numerical integral algorithm, from the ground digital elevation model and ground ellipsoidal height grid, compute the local terrain effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (" , to south, to west) or (disturbing) gravity gradient (E, radial) on or outside the geoid.

[Input files] The ground digital elevation model file and ground ellipsoidal height grid file with the same grid specification, and the calculation point position file or ellipsoidal height grid file of the calculation surface.

The ground digital elevation model (normal /orthometric height) is employed to indicate terrain relief, and the ground ellipsoidal height grid represents the terrain surface position employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters, select the gravity field element type, and enter the integral radius.

[Output file] The local terrain effect result file.

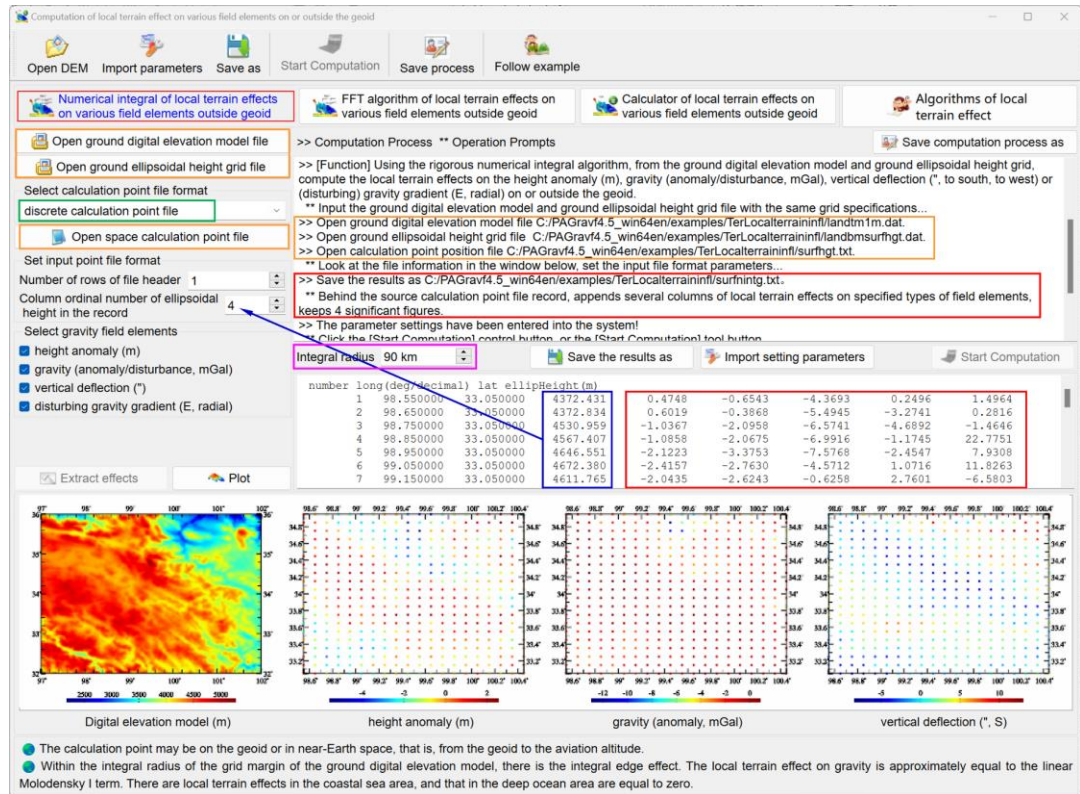
When the discrete calculation point file input, the output file record format: Behind the source calculation point file record, appends several columns of local terrain effects on specified types of field elements, keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: Point no, longitude, latitude, ellipsoidal height, several columns of local terrain effects on specified types of field elements, keeps 4 significant figures.

At the same time, the program also outputs the local terrain effect grid files on height anomaly (*.ksi), gravity (anomaly/disturbance, *.gra), vertical deflection (*.dft) or (disturbing) gravity gradient (*.grr) into the current directory. Where * is the output file name entered from the interface. The program outputs the local terrain effect grid files on the specified types of elements.

In this example, the 1' digital elevation model is used, the integral radius is 90km, the terrain surface (ground) is selected as the calculation surface which is represented by the ground ellipsoidal height grid, and the local terrain effects on various ground field

elements are calculated. After the 1° area of the grid margin with integral edge effect deducted, the statistical analysis on the local terrain effects on various field elements are shown in the table below.

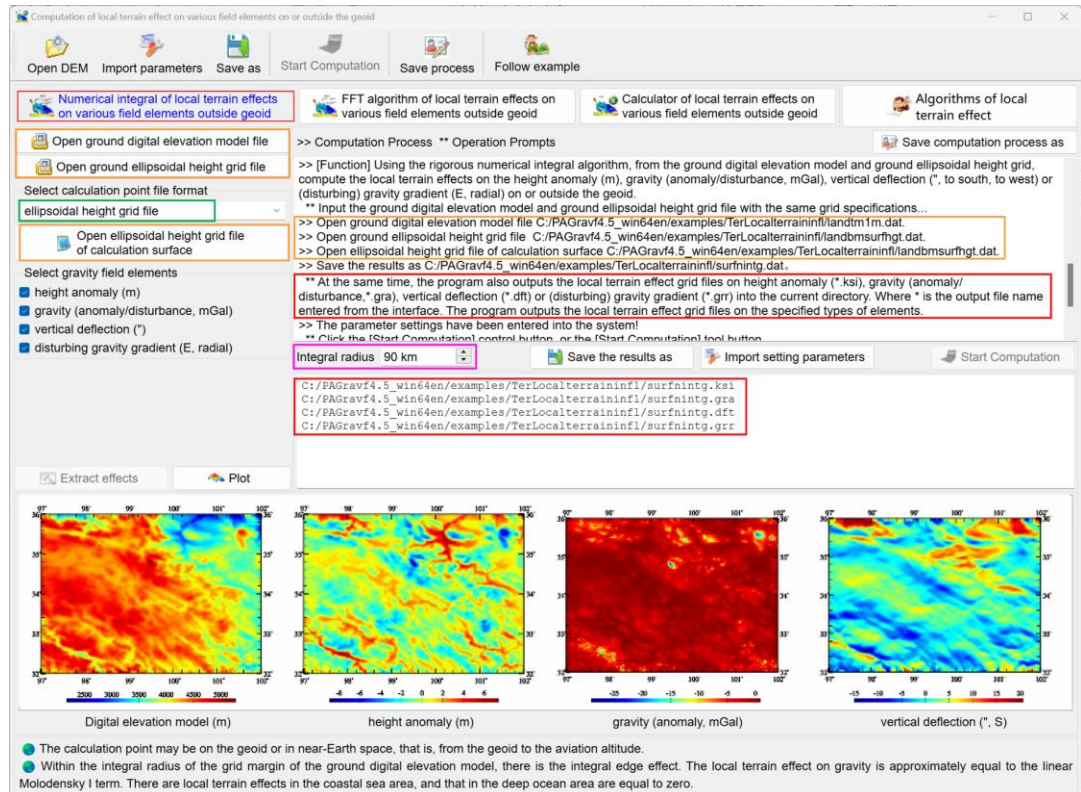


Let D be the difference between the maximum value and the minimum value, ϵ is the standard deviation, and a large D/ϵ indicates that the number of ultrashort wave signals accounts for a small proportion but that the amplitude is prominent. In general, if a certain type of terrain effect on a certain type of field element is larger, the more favorable it is to process the data of this type of field element.

Field element type /unit	mean	standard deviation ϵ	minimum	maximum	D/ϵ
height anomaly /m	-0.2233	1.4995	-8.9709	4.9249	9.3
gravity (disturbance) /mGal	-0.5364	0.7599	-14.3238	0.5061	19.5
vertical deflection /S"	1.0139	3.2809	-8.7789	13.2512	6.7
vertical deflection /W"	1.7161	3.2445	-6.8166	14.4212	6.5
gravity gradient /E	1.3980	83.1923	-368.3504	493.4792	10.4

In this example, the D/ϵ of the local terrain effect on gravity disturbance is large, showing that the local terrain effect is suitable to process the gravity disturbance data. However, the D/ϵ of the local terrain effect on the vertical deflection is the smallest, which shows that it is not a good choice to employ the local terrain effect to process the vertical

deflection data.



3.1.2 FFT algorithm of local terrain effects on various gravity field elements outside geoid

[Function] Using the FFT integral algorithm, from the ground digital elevation model and ground ellipsoidal height grid, compute the local terrain effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (" , to south, to west) or (disturbing) gravity gradient (E, radial) on or outside the geoid.

[Input files] The ground digital elevation model file, the ground ellipsoidal height grid file and the ellipsoidal height grid file of the calculation surface with the same grid specification.

The ground digital elevation model (normal /orthometric height) is employed to indicate terrain relief. The ground ellipsoidal height grid represents the terrain surface position employed to calculate the integral distance.

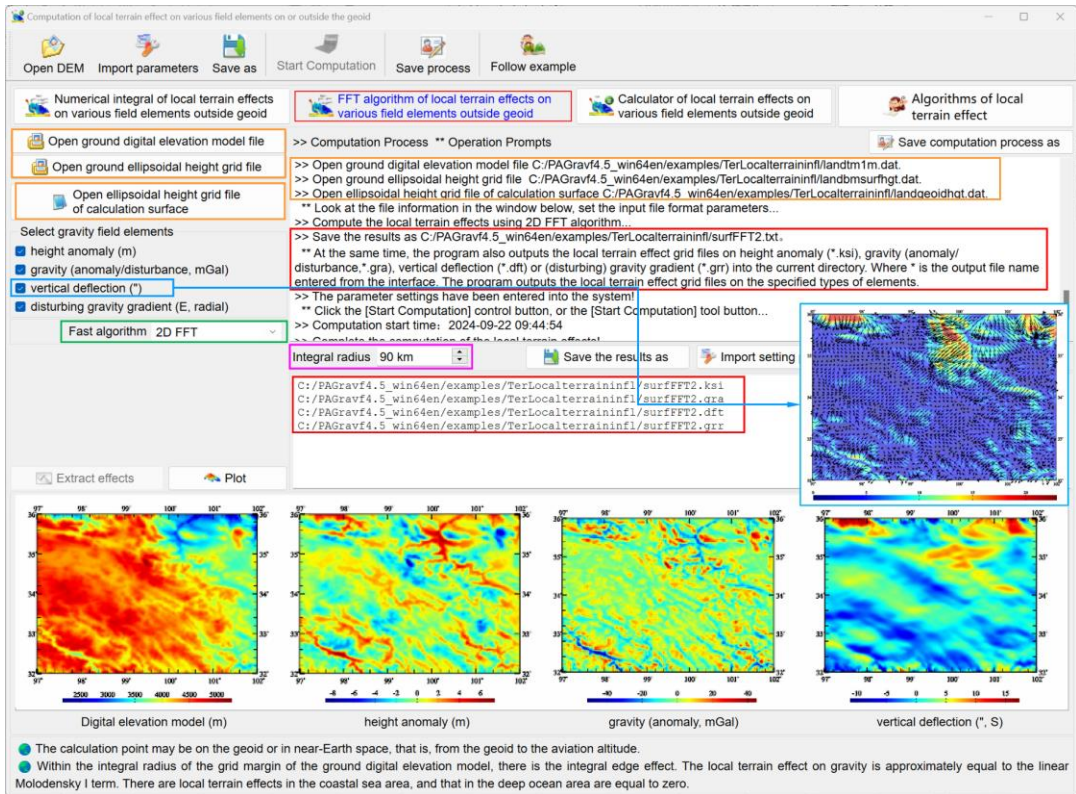
[Parameter settings] Select the gravity field element type and the integral fast algorithm and enter the integral radius.

[Output file] The local terrain effect file.

Record format: Point no, longitude, latitude, ellipsoidal height, several columns of local terrain effects on specified types of field elements, keeps 4 significant figures.

At the same time, the program also outputs the local terrain effect grid files on height anomaly (*.ksl), gravity (anomaly/disturbance, *.gra), vertical deflection (*.dft) or

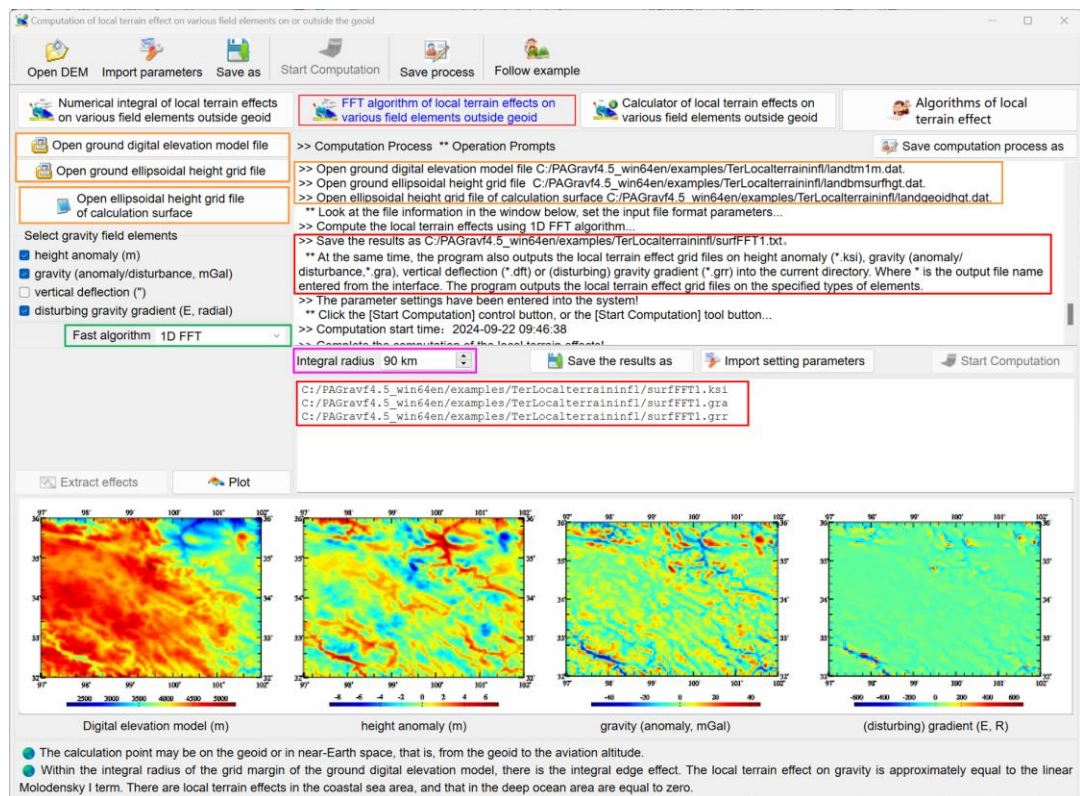
(disturbing) gravity gradient (*.grr) into the current directory. Where * is the output file name entered from the interface. The program outputs the local terrain effect grid files on the specified types of elements.



Using the exact same parameters as the numerical integral, compute the local terrain effects on various field elements by the FFT algorithm, and statistically analyze the difference between the FFT result and the numerical integral result.

FFT – numerical integral		mean	standard deviation	minimum	maximum
height anomaly /m	FFT2	-0.0001	0.0138	-0.0686	0.0883
	FFT1	0.0012	0.0027	-0.0115	0.0110
gravity (anomaly, disturbance) /mGal	FFT2	0.0352	0.2316	-0.7410	1.4406
	FFT1	0.0341	0.2298	-0.7602	1.3356
vertical deflection /S"	FFT2	-0.0082	0.0197	-0.1176	0.1275
	FFT1	-0.0026	0.0138	-0.1616	0.1538
vertical deflection /W"	FFT2	0.0018	0.0400	-0.3873	0.1555
	FFT1	0.0051	0.0198	-0.4444	0.2844
(disturbing) gravity gradient /E	FFT2	0.0042	0.5176	-19.6242	10.5354
	FFT1	0.0036	0.5859	-21.2847	12.1181

There is little difference between the computation results of the rigorous numerical integral and the fast FFT algorithm, and there is also no obvious difference between the results of the one-dimensional FFT and the two-dimensional FFT.



3.1.3 Calculator of local terrain effects on various gravity field elements outside the geoid

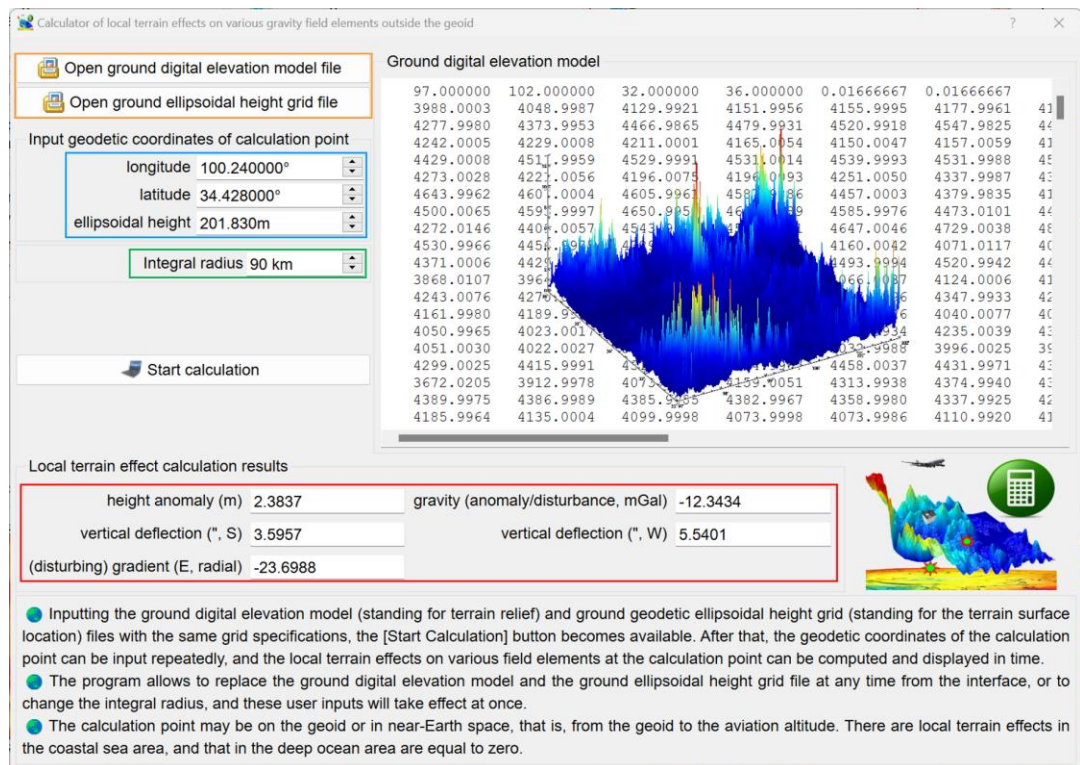
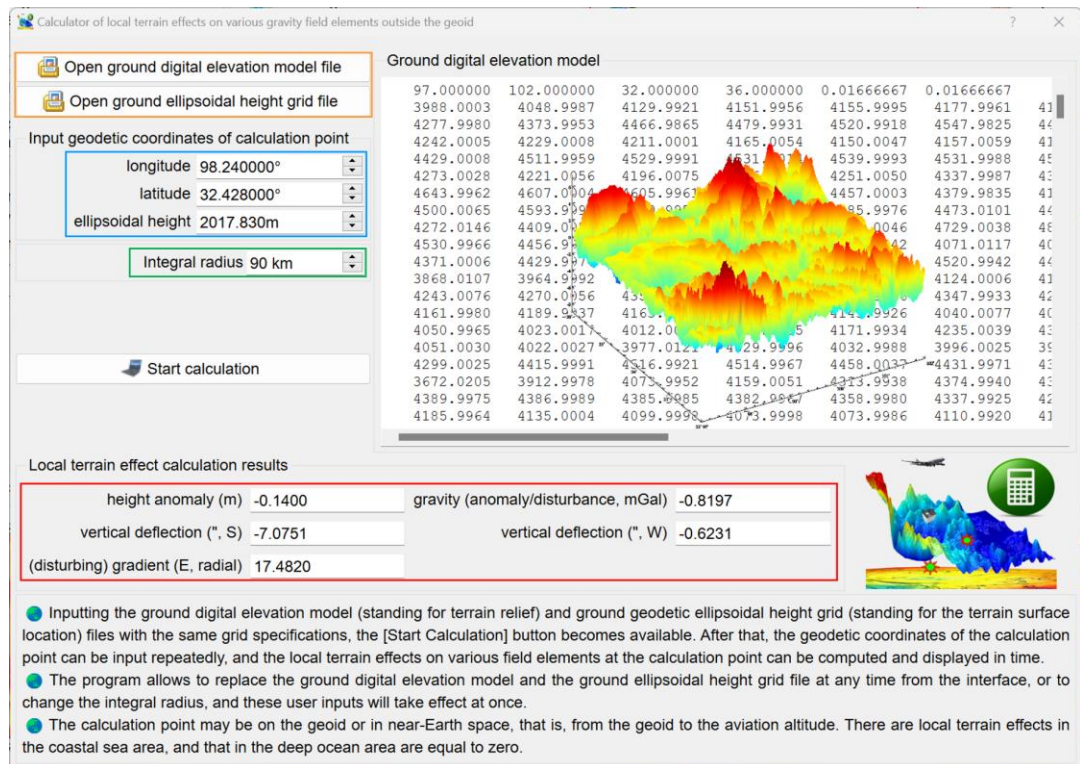
[Function] From the ground digital elevation model and ground ellipsoidal height grid file, given the geodetic coordinates of the calculation point on or outside the geoid, compute the local terrain effects on the height anomaly (m), gravity (anomaly / disturbance, mGal), vertical deflection (", to south, to west) and (disturbing) gravity gradient (E, radial).

Inputting the ground digital elevation model (standing for terrain relief) and ground geodetic ellipsoidal height grid (standing for the terrain surface location) files with the same grid specification, the [Start Calculation] button becomes available. After that, the geodetic coordinates of the calculation point can be input repeatedly, and the local terrain effects on various field elements at the calculation point can be computed and displayed in time.

The program allows to replace the ground digital elevation model and the ground ellipsoidal height grid file at any time from the interface, or to change the integral radius, and these user inputs will take effect at once.

The calculation point may be on the geoid or in near-Earth space, that is, from the

geoid to the aviation altitude.



Within the integral radius of the grid margin of the ground digital elevation model,

there is the integral edge effect. The local terrain effect on gravity is approximately equal to the linear Molodensky I term. There are local terrain effects in the coastal sea area, and that in the deep ocean area are equal to zero.

3.2 Computation of land, ocean, and lake complete Bouguer effect on gravity outside geoid

[Purpose] Using the rigorous numerical integral method or FFT algorithm, from the land-sea terrain model and ellipsoidal height grid file of the land ground and sea surface, compute the land-sea unified complete Bouguer effect on the gravity anomaly (mGal) or gravity disturbance (mGal) on the geoid or in near-Earth space.

The complete Bouguer effect on the anomalous field element is much larger than the element itself, which is usually employed in geophysics to detect the geometric structure of the gravity field. Physical geodesy pays attention to the requirement of quantitative accuracy, and would not directly use the complete Bouguer effect, which is mainly employed to compute the residual terrain effect.

Since the normal gravity field keeps unchanged, the complete Bouguer effect on the gravity disturbance and gravity anomaly is always equal to the complete Bouguer effect on gravity.

The program is suitable for the unified computation of the complete Bouguer effect on gravity, gravity anomaly and gravity disturbance in land, land-sea junction and sea area. The calculation point may be on the geoid or in near-Earth space.

If the ocean water depth in the land-sea terrain model is set to zero, the program automatically computes the land complete Bouguer effect in the near-Earth space. If the terrain height in the land-sea terrain model is set to zero, the program automatically computes the seawater complete Bouguer effect in the near-Earth space.

The complete Bouguer effect here is defined as the variation of Earth gravity field because of the terrain masses above the geoid removed and the seawater density compensated to the terrain density. There is the sea water Bouguer effect in the offshore land area, while there is the local terrain effect in the coastal sea area.

3.2.1 Computation of the land-sea unified complete Bouguer effect on gravity outside geoid

[Function] Using the rigorous numerical integral method or FFT algorithm, from the land-sea terrain model and ellipsoidal height grid file of the land-sea surface, compute the land-sea unified complete Bouguer effect on the gravity (mGal) on the geoid or in near-Earth space.

[Input files] The land-sea terrain model file and the ellipsoidal height grid file of the land-sea surface with the same grid specification, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

In the land-sea terrain model, the land terrain height is greater than zero while the seafloor water depth is less than zero.

The ellipsoidal height grid of the land-sea surface stands for the land-sea surface position employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters, select the integral algorithm, and enter the land integral radius and sea integral radius.

Computation of land, ocean, and lake complete Bouguer effect on gravity outside geoid

Open DTM Import parameters Save as Start Computation Save process Follow example

Computation of the land-sea unified complete Bouguer effect on gravity outside geoid

Numerical integral computation of the lake-water complete Bouguer effect on gravity

Formulas of land-sea unified complete Bouguer effect

Save computation process as

Open the land-sea terrain model file

Open the ellipsoidal height grid file of the land-sea surface

Select calculation point file format

discrete calculation point file

Open the calculation point location file

Set input point file format

Number of rows of file header 1

Column ordinal number of ellipsoidal height in the record 4

Land integral radius 90 km

Sea integral radius 300 km

Save the results as

Import setting parameters

Start Computation

Extract effects

Plot

land-sea terrain model (m)

complete Bouguer effects (mGal)

The program is suitable for the unified computation of the complete Bouguer effect on gravity, gravity anomaly and gravity disturbance in land, land-sea junction and sea area. The calculation point may be on the geoid or in near-Earth space.

If the ocean water depth in the land-sea terrain model is set to zero, the program automatically computes the land complete Bouguer effect in the near-Earth space. If the terrain height in the land-sea terrain model is set to zero, the program automatically computes the seawater complete Bouguer effect in the near-Earth space.

The complete Bouguer effect here is defined as the variation of Earth gravity field because of the terrain masses above the geoid removed and the seawater density compensated to the terrain density. There is the sea water Bouguer effect in the offshore land area, while there is the local terrain effect in the coastal sea area.

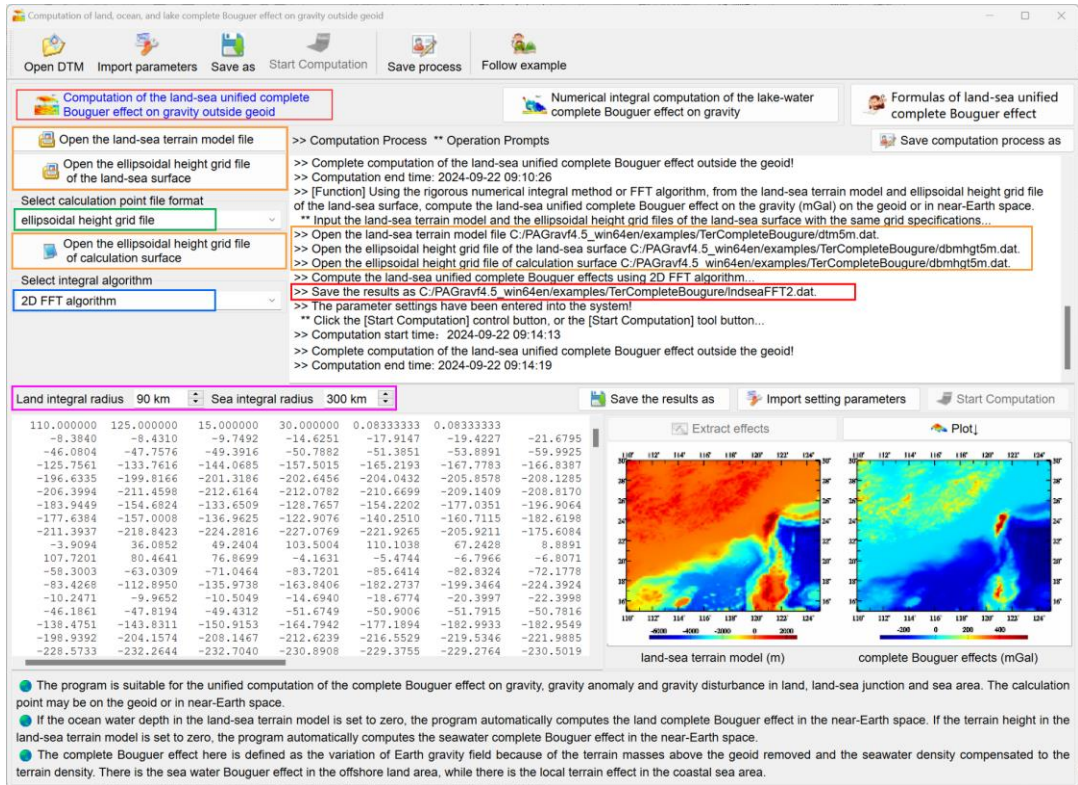
[Output file] The land-sea unified complete Bouguer effect file.

When the discrete calculation point file input, the output file record format: Behind the source calculation point file record, appends the local terrain effect, spherical shell Bouguer effect and sear-water complete Bouguer effect and landsea complete Bouguer effect, and keep 4 significant figures.

In this example, the 5' land-sea terrain model is employed, the integral radius for local terrain effect is 90km and that for sear-water complete Bouguer effect is 300km, the land-sea surface is the calculation surface which is represented by the land-sea surface ellipsoidal height grid, and the land-sea unified complete Bouguer effects on the gravity are computed. After the 3° area of the grid margin with integral edge effect deducted, the statistical analysis on the complete Bouguer effects are shown in the table below.

mGal	mean	standard deviation	minimum	maximum
numerical integral	-17.2622	110.3575	-260.5460	562.1404
FFT2 – numerical integral	1.7056	1.8637	0.0265	19.7512

FFT1 – numerical integral	2.0864	2.2786	0.0268	19.3779
FFT2 – FFT1	-0.3808	0.7884	-3.3353	0.8730



Due to the large the complete Bouguer effect on various field elements, the error caused by the third-order approximation of terrain relief sometimes even exceeds the anomalous field element itself. PAGrav4.5 therefore recommends that, in addition to the complete Bouguer effect on gravity, the effects on other types of field elements should be computed by the remove-restore scheme with the global land-sea terrain mass spherical harmonic coefficient model as the reference terrain field. For the computation process, please refer to [Computation process demo of complete Bouguer anomaly outside geoid].

3.2.2 Numerical integral computation of the lake-water complete Bouguer effect on gravity

[Function] Using the rigorous numerical integral algorithm, from the lake water-depth grid (value on land is zero) and ellipsoidal height grid file of the lake surface, compute the lake-water complete Bouguer effect on the gravity (mGal).

[Input files] The lake water-depth grid file and ellipsoidal height grid file of the lake surface with the same grid specifications, and the calculation point position file or ellipsoidal height grid file of the calculation surface.

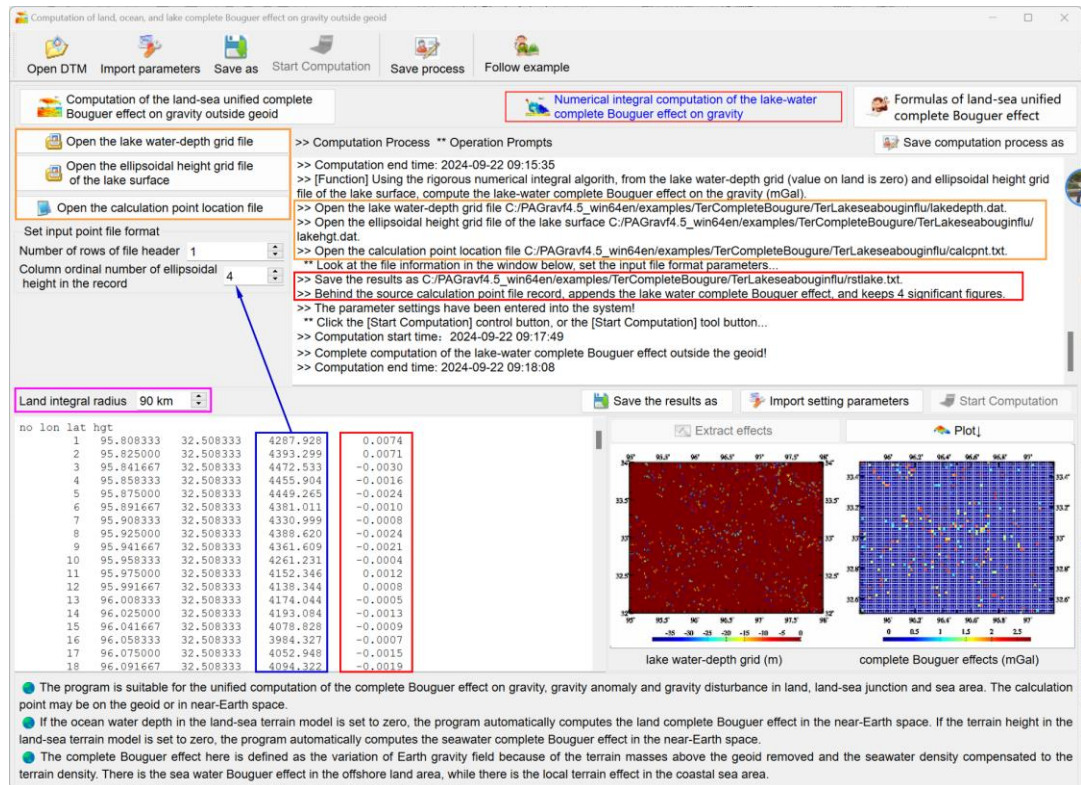
The ellipsoidal height grid of the lake surface stands for the lake surface position

employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters and integral radius.

[Output file] The lake water complete Bouguer effect file.

When the discrete calculation point file input, the output file record format: Behind the source calculation point file record, appends the lake-water complete Bouguer effect, and keeps 4 significant figures.



3.3 Computation of terrain Helmert condensation effect on various field elements outside geoid

[Purpose] Using the rigorous numerical integral or FFT algorithm, from the ground digital elevation model and ground ellipsoidal height grid, compute the terrain Helmert condensation effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (" to south, to west) or (disturbing) gravity gradient (E, radial) on or outside the geoid.

Since the normal gravity field keeps unchanged, the terrain Helmert condensation effect on the gravity disturbance and gravity anomaly is always equal to the terrain Helmert condensation effect on gravity.

The calculation point may be on the geoid or in near-Earth space.

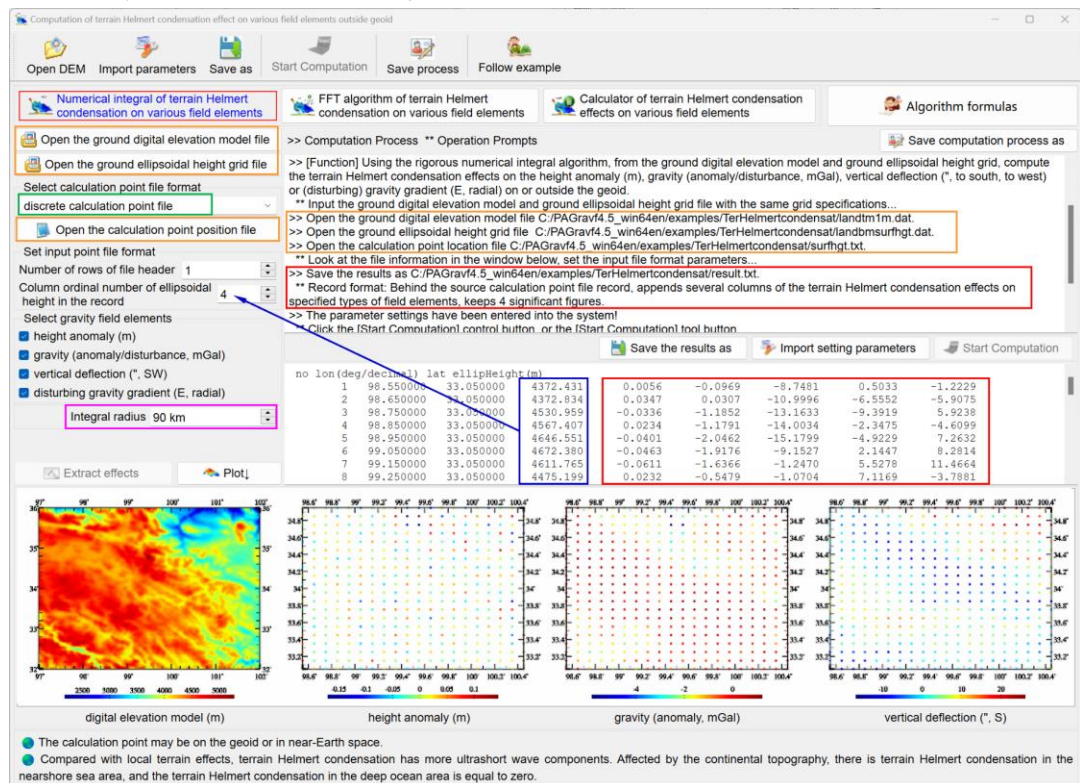
3.3.1 Numerical integral of terrain Helmert condensation effects on various field elements

[Function] Using the rigorous numerical integral algorithm, from the ground digital elevation model and ground ellipsoidal height grid, compute the terrain Helmert condensation effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (" , to south, to west) or (disturbing) gravity gradient (E, radial) on or outside the geoid.

[Input files] The ground digital elevation model file and ground ellipsoidal height grid file with the same grid specifications, and the calculation point location file or the ellipsoidal height grid file of the calculation surface.

The ground digital elevation model (normal /orthometric height) is employed to indicate terrain relief, and the ground ellipsoidal height grid stands for the ground position employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters, select the gravity field element type, and enter the integral radius.



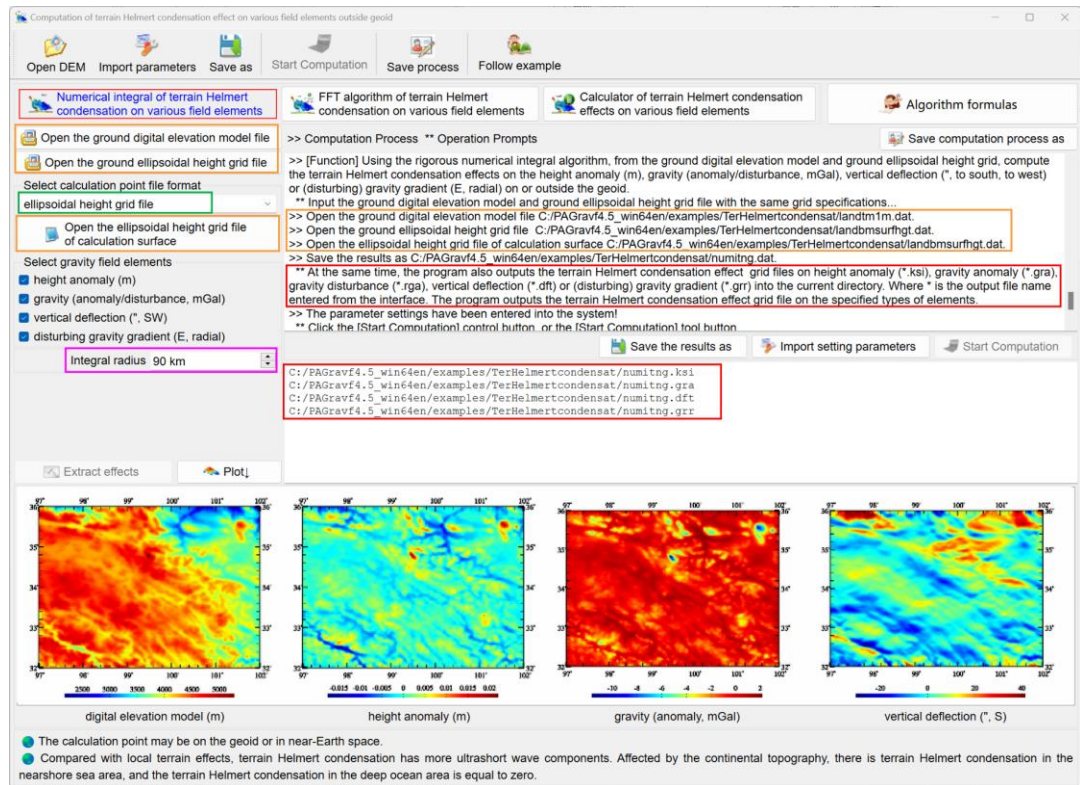
[Output file] The terrain Helmert condensation effect result file.

When the discrete calculation point file input, the output file record format: Behind the source calculation point file record, appends several columns of terrain Helmert condensation effects on specified types of field elements, keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: Point no, longitude, latitude, ellipsoidal height, several columns of terrain Helmert condensation

effects on specified types of field elements, keeps 4 significant figures.

At the same time, the program also outputs the terrain Helmert condensation effect grid files on height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection (*.dft) or (disturbing) gravity gradient (*.grr) into the current directory. Where * is the output file name entered from the interface. The program outputs the terrain Helmert condensation effect grid file on the specified types of elements.



In this example, the 1' digital elevation model is employed, the integral radius is 90km, the ground is selected as the calculation surface which is represented by the ground ellipsoidal height grid, and the terrain Helmert condensation effects on various ground field elements are computed. After the 1° area of the grid margin with integral edge effect deducted, the statistical analysis on the terrain Helmert condensation effects on various field elements are shown in the table below.

The terrain Helmert condensation has more ultrashort wave components. If the actual gravity field structure and the spatial distribution of gravity observations are not considered, only by comparing D/ϵ , the terrain Helmert condensation is more suitable for processing (disturbing) gravity gradient data than the local terrain effect.

Field element type /unit	mean	standard deviation ϵ	minimum	maximum	D/ϵ
height anomaly /m	-0.0005	0.0029	-0.0134	0.0180	1.6
gravity (anomaly, disturbance) /mGal	0.2932	0.4168	-0.4313	3.1693	8.6

vertical deflection /S"	2.0301	6.5718	-17.5982	26.6152	6.8
vertical deflection /W"	3.4358	6.5000	-13.6625	28.9665	6.8
(disturbing) gravity gradient /E	-0.3677	16.5856	-257.5543	112.1249	22.3

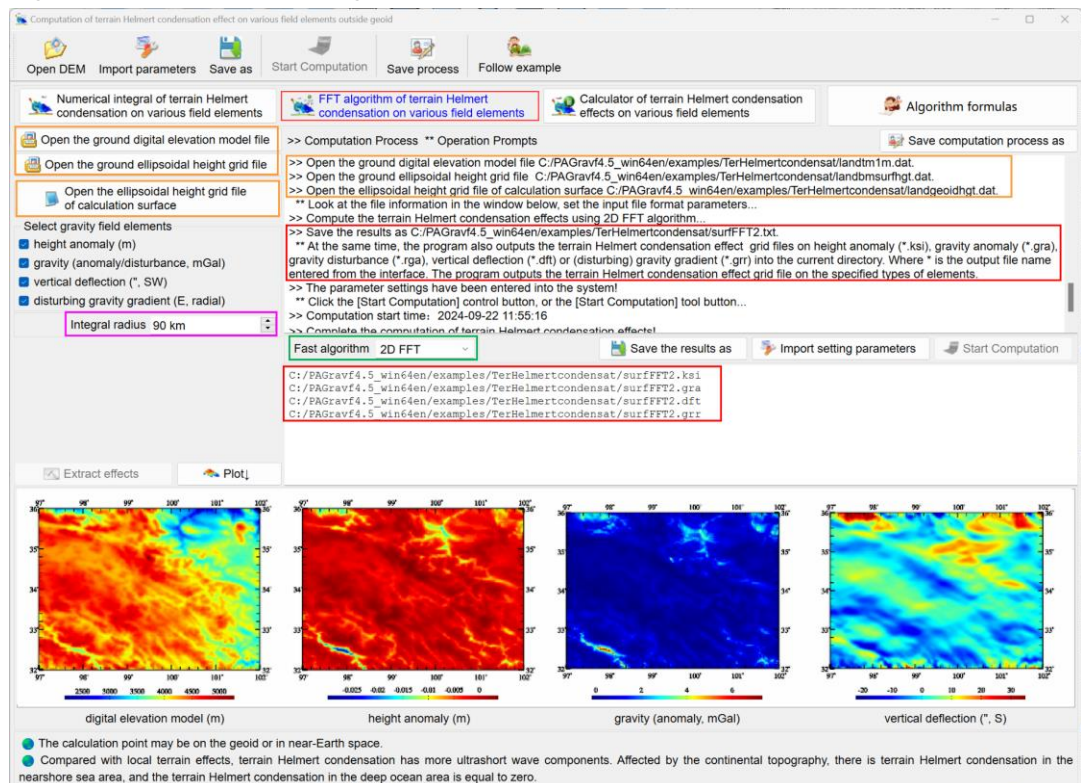
3.3.2 FFT algorithm of terrain Helmert condensation effects on various field elements

[Function] Using the FFT integral algorithm, from the ground digital elevation model and ground ellipsoidal height grid, compute the terrain Helmert condensation effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (" to south, to west) or (disturbing) gravity gradient (E, radial) on or outside the geoid.

[Input files] The ground digital elevation model file, ground ellipsoidal height grid file, and ellipsoidal height grid file of the calculation surface with the same grid specifications.

The ground digital elevation model (normal /orthometric height) is employed to indicate terrain relief and the ground ellipsoidal height grid represents the terrain surface location employed to calculate the integral distance.

[Parameter settings] Select the gravity field element type and the integral fast algorithm and enter the integral radius.

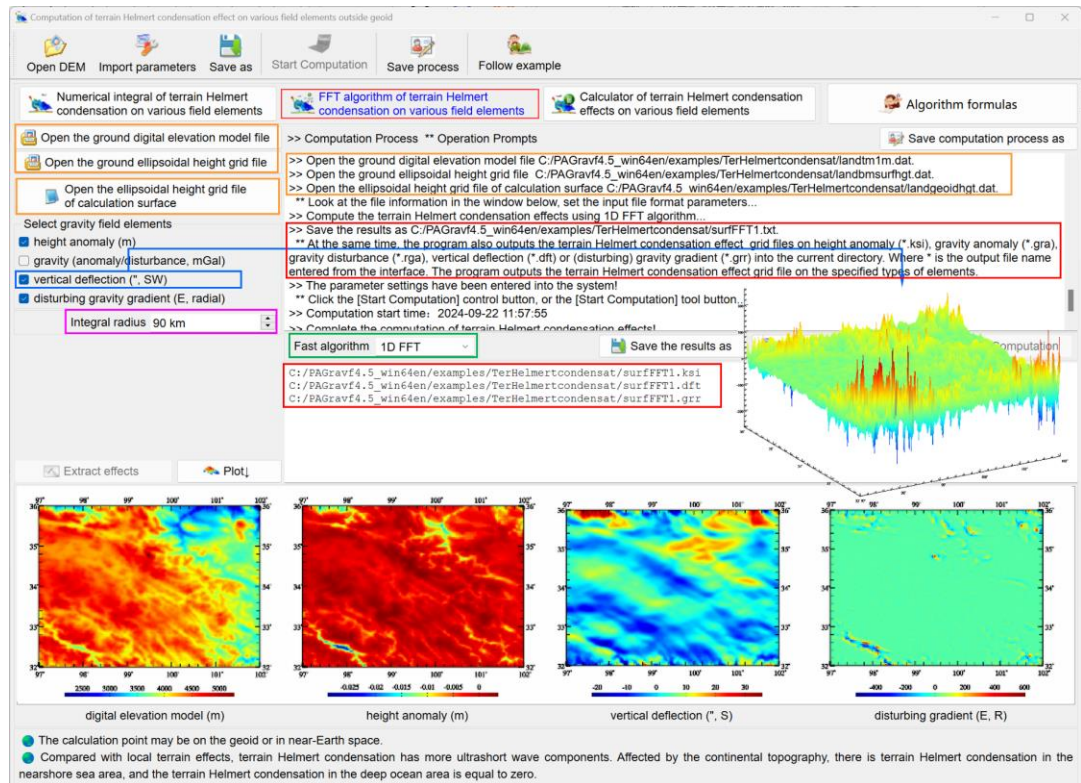


[Output file] The local terrain effects file.

Record format: Point no, longitude, latitude, ellipsoidal height, several columns of

terrain Helmert condensation effects on specified types of field elements, keeps 4 significant figures.

At the same time, the program also outputs the terrain Helmert condensation effect grid files on the height anomaly (*.ksi), gravity anomaly (*.gra), gravity disturbance (*.rga), vertical deflection (*.dft) or (disturbing) gravity gradient (*.grr) into the current directory, where * is the output file name entered from the interface. The program outputs the terrain Helmert condensation effect grid file on the specified types of elements.



Using the exact same parameters as the numerical integral, compute the terrain Helmert condensation effects on various field elements by the FFT algorithm, and statistically analyze the difference between the FFT result and the numerical integral result. Compared with local terrain effects, terrain Helmert condensation has more ultrashort wave components. Affected by the continental topography, there is terrain Helmert condensation in the nearshore sea area, and the terrain Helmert condensation in the deep ocean area is equal to zero.

FFT – numerical integral		mean	standard deviation	minimum	maximum
height anomaly /m	FFT2	0.0008	0.0015	-0.0026	0.0064
	FFT1	0.0007	0.0015	-0.0027	0.0063
gravity (anomaly, disturbance) /mGal	FFT2	0.0009	0.1262	-0.5535	0.6116
	FFT1	0.0025	0.1255	-0.5085	0.6768

vertical deflection /S"	FFT2	-0.0174	0.0404	-0.2220	0.1438
	FFT1	-0.0062	0.0240	-0.2359	0.2179
vertical deflection /W"	FFT2	0.0025	0.0804	-0.5046	0.2902
	FFT1	0.0093	0.0306	-0.6211	0.4129
(disturbing) gravity gradient /E	FFT2	0.0062	0.4810	-17.1773	9.8987
	FFT1	0.0062	0.4799	-16.8811	9.8312

3.3.3 Calculator of terrain Helmert condensation effects on various field elements

[Function] From the ground digital elevation model and ground ellipsoidal height grid file, given the calculation point geodetic coordinates on or outside the geoid, compute the terrain Helmert condensation effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (" , to south, to west) and (disturbing) gravity gradient (E, radial).

Inputting the ground digital elevation model (standing for terrain relief) and ground geodetic ellipsoidal height grid (standing for the terrain surface location) files with the same grid specifications, the button [Start Calculation] becomes available. After that, the geodetic coordinates of the calculation point can be input repeatedly, and the terrain Helmert condensation effects on various field elements at the calculation point can be computed and displayed in time.

Calculator of terrain Helmert condensation effects on various field elements

Open the ground digital elevation model file

Open the ground ellipsoidal height grid file

Input geodetic coordinates of calculation point

longitude 98.240000°

latitude 32.428000°

ellipsoidal height 2017.830m

Integral radius 90 km

Start calculation

Ground digital elevation model

97.000000	102.000000	32.000000	36.000000	0.01666667	0.01666667
3988.0003	4048.9987	4129.9921	4151.9956	4155.9995	4177.9961
4277.9980	4373.9953	4466.9865	4479.9911	4520.9918	4547.9825
4242.0005	4229.0008	4211.0001	4165.0004	4150.0047	4157.0059
4429.0008	4511.9959	4529.9991	4431.0004	4539.9993	4531.9988
4273.0028	4221.0056	4197.0004	4251.0050	4337.9987	
4643.9962	4604.0004	457.0003	457.0003	4379.9835	
4500.0065	4593.0004	4597.0004	4473.0101		
4272.0146	4409.0004	4729.0038			
4530.9966	4450.9974	4071.0117			
4371.0006	4424.9974	4520.9942			
3868.0107	3964.9992	4124.0006			
4243.0076	4270.0056	4347.9933			
4161.9980	4189.9935	4040.0077			
4050.9965	4023.0017	4235.0039			
4051.0030	4022.0027	3996.0025			
4299.0025	4415.9991	4431.9971			
3672.0205	3912.9978	4374.9940			

Terrain Helmert condensation effect calculation results

height anomaly (m)	-0.0207	gravity (anomaly/disturbance, mGal)	-0.6315
vertical deflection (", S)	-14.1486	vertical deflection (", W)	-1.2267
(disturbing) gradient (E, radial)	9.7229		

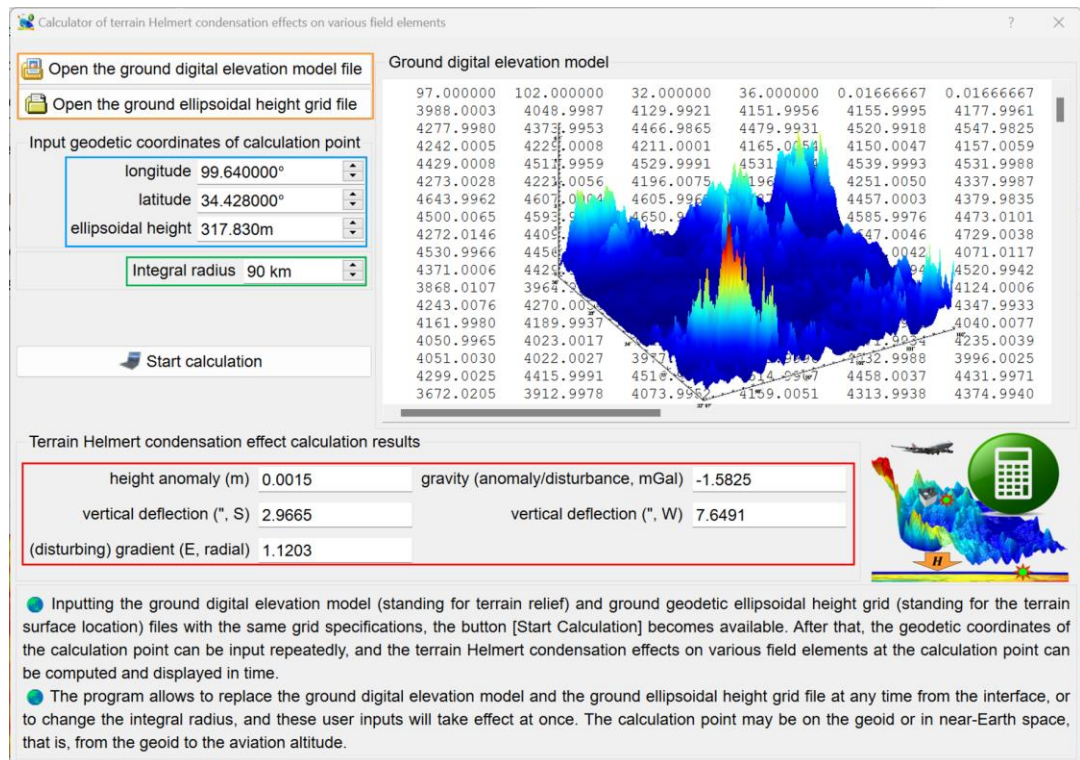
Inputting the ground digital elevation model (standing for terrain relief) and ground geodetic ellipsoidal height grid (standing for the terrain surface location) files with the same grid specifications, the button [Start Calculation] becomes available. After that, the geodetic coordinates of the calculation point can be input repeatedly, and the terrain Helmert condensation effects on various field elements at the calculation point can be computed and displayed in time.

The program allows to replace the ground digital elevation model and the ground ellipsoidal height grid file at any time from the interface, or to change the integral radius, and these user inputs will take effect at once. The calculation point may be on the geoid or in near-Earth space, that is, from the geoid to the aviation altitude.

The program allows to replace the ground digital elevation model and the ground ellipsoidal height grid file at any time from the interface, or to change the integral radius,

and these user inputs will take effect at once.

The calculation point may be on the geoid or outside the geoid in near-Earth space, that is, from the geoid to the aviation altitude.



3.4 Computation of residual terrain effect on various field elements outside geoid

[Purpose] Using the rigorous numerical integral or FFT algorithm, from the high-resolution land-sea terrain model, low-pass land-sea terrain model and ellipsoidal height grid of the land-sea surface, compute the residual terrain effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (\", to south, to west) or (disturbing) gravity gradient (E, radial) on or outside the geoid.

The land-sea residual terrain effect here is defined as the short-wave and ultra-short-wave components of the land-sea complete Bouguer effect.

Since the normal gravity field keeps unchanged, the residual terrain effect on the gravity disturbance and gravity anomaly is always equal to the residual terrain effect on gravity.

The low-pass land-sea terrain model can be generated by low-pass filtering the high-resolution land-sea terrain model, or also calculated from a global land-sea terrain mass spherical harmonic coefficient model by the spherical harmonic synthesis.

It is recommended that the land-sea low-pass terrain model is constructed by the spherical harmonic synthesis from a global land-sea terrain mass spherical harmonic

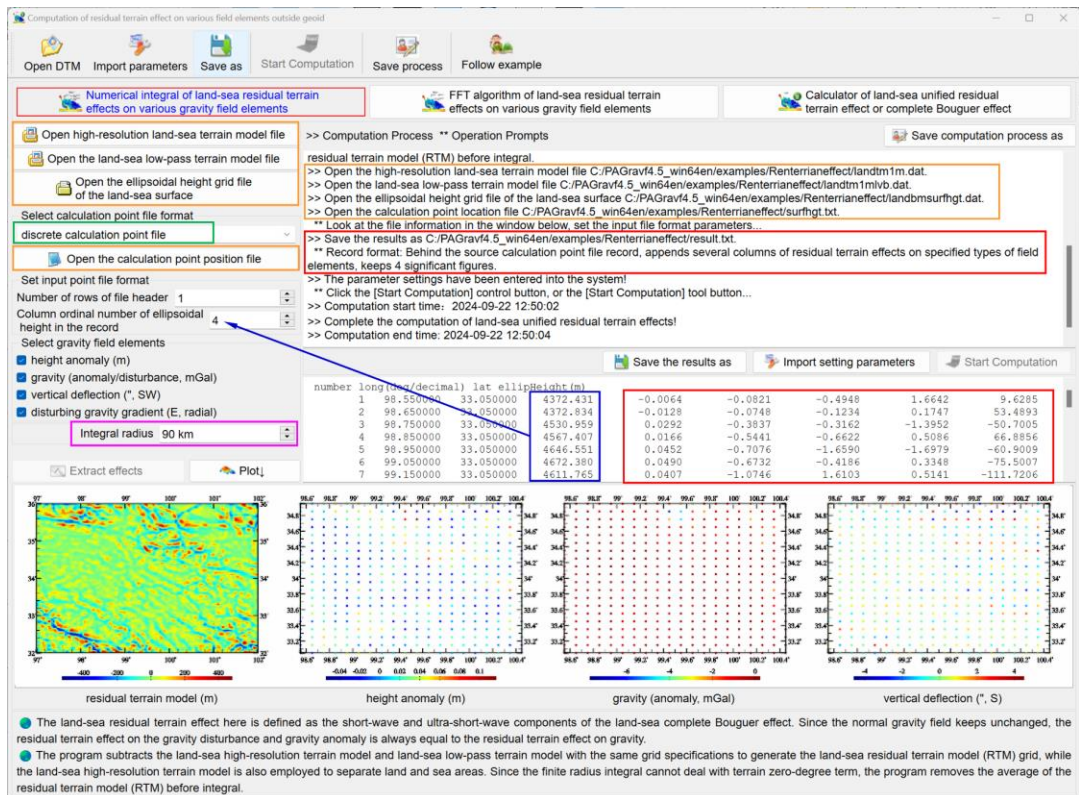
coefficient model to effectively improve the approach performance of gravity field.

3.4.1 Numerical integral of land-sea residual terrain effects on various gravity field elements

[Function] Using the rigorous numerical integral algorithm, from the high-resolution land-sea terrain model, low-pass land-sea terrain model and ellipsoidal height grid of the land-sea surface, compute the residual terrain effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (" to south, to west) or (disturbing) gravity gradient (E, radial) on or outside the geoid.

The program subtracts the land-sea high-resolution terrain model and land-sea low-pass terrain model with the same grid specification to generate the land-sea residual terrain model (RTM) grid, while the land-sea high-resolution terrain model is also employed to separate land and sea areas. Since the finite radius integral cannot deal with terrain zero-degree term, the program removes the average of the residual terrain model (RTM) before integral.

[Input files] The high-resolution land-sea terrain model file, land-sea low-pass terrain model file and ellipsoidal height grid file of land-sea surface with the same grid specifications, and the calculation point position file or ellipsoidal height grid file of the calculation surface.



In the land-sea terrain model, the land terrain height is greater than zero while the seafloor water depth is smaller than zero.

The ellipsoidal height grid of the land-sea surface stands for the land-sea surface location employed to calculate the integral distance.

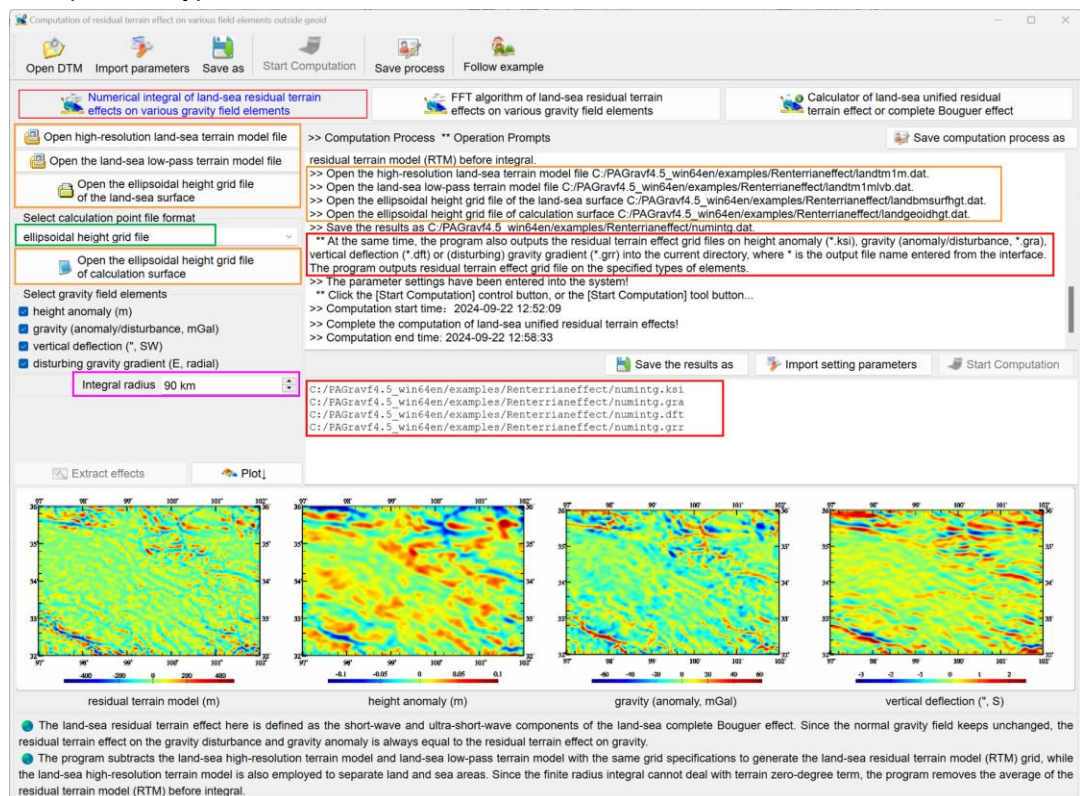
[Parameter settings] Set the input file format parameters, select the gravity field element type, and enter the integral radius.

[Output file] The residual terrain effect result file.

When the discrete calculation point file input, the output file record format: Behind the source calculation point file record, appends several columns of residual terrain effects on specified types of field elements, keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: Point no, longitude, latitude, ellipsoidal height, several columns of residual terrain effects on specified types of field elements, keeps 4 significant figures.

At the same time, the program also outputs the residual terrain effect grid files on height anomaly (*.ksi), gravity (anomaly/disturbance, *.gra), vertical deflection (*.dft) or (disturbing) gravity gradient (*.grr) into the current directory, where * is the output file name entered from the interface. The program outputs residual terrain effect grid file on the specified types of elements.



In this example, the 1' land-sea terrain model is employed, the low-pass terrain model is constructed from the 1440-degree global land-sea terrain mass spherical harmonic coefficient model, the integral radius is 90km, and the residual terrain effects on various ground field elements are computed. After the 1° area of the grid margin with integral edge effect deducted, the statistical analysis on the residual terrain effects on

various field elements are shown in the table below.

Field element type /unit	mean	standard deviation ϵ	minimum	maximum	D/ ϵ
height anomaly /m	0.0188	0.0566	-0.2151	0.3835	10.2
gravity(anomaly, disturbance) /mGal	0.5342	1.0570	-1.6605	13.2024	14.1
vertical deflection /S"	-0.0000	0.8730	-4.7838	4.6030	10.8
vertical deflection /W"	-0.0075	0.8157	-6.2512	5.1511	14.0
(disturbing) gravity gradient /E	-7.0310	300.1898	-928.6079	884.6364	6.0

When the zero-value grid is employed as the low-pass land-sea terrain model, the program computes the land-sea unified complete Bouguer effect on various field elements but without the integral far-zone effect. In this case, the integral radius is generally not less than 300km.

The spectral domain properties of the residual terrain model (RTM) can be regulated by the land-sea low-pass terrain models.

If the gravity field structure and the spatial distribution of gravity observations are not considered, only compared the D/ ϵ with the local terrain effect and terrain Helmert condensation, the residual terrain effect is more conducive to modelling of geoid (height anomaly), suitable for processing of vertical deflection data such as for satellite altimetry data, but not conducive to processing of (disturbing) gravity gradient data.

It is not difficult to see that whether in the land area or in the sea area, the residual terrain could be positive or negative.

3.4.2 FFT algorithm of land-sea residual terrain effects on various gravity field elements

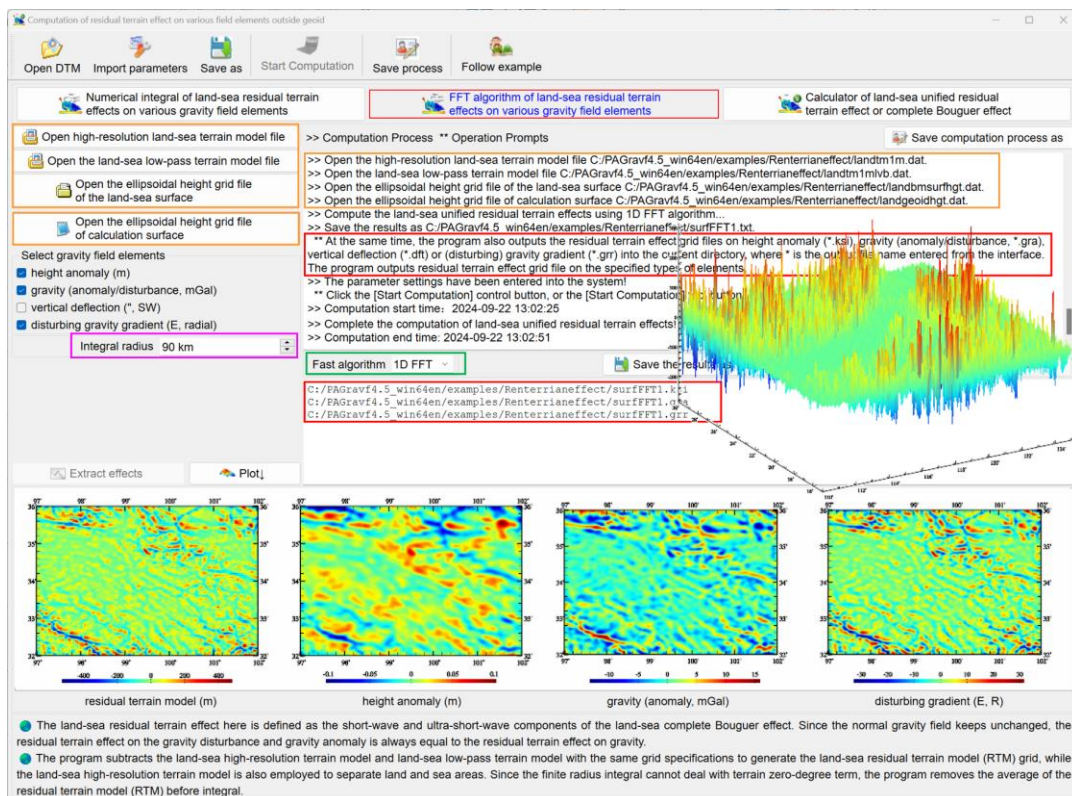
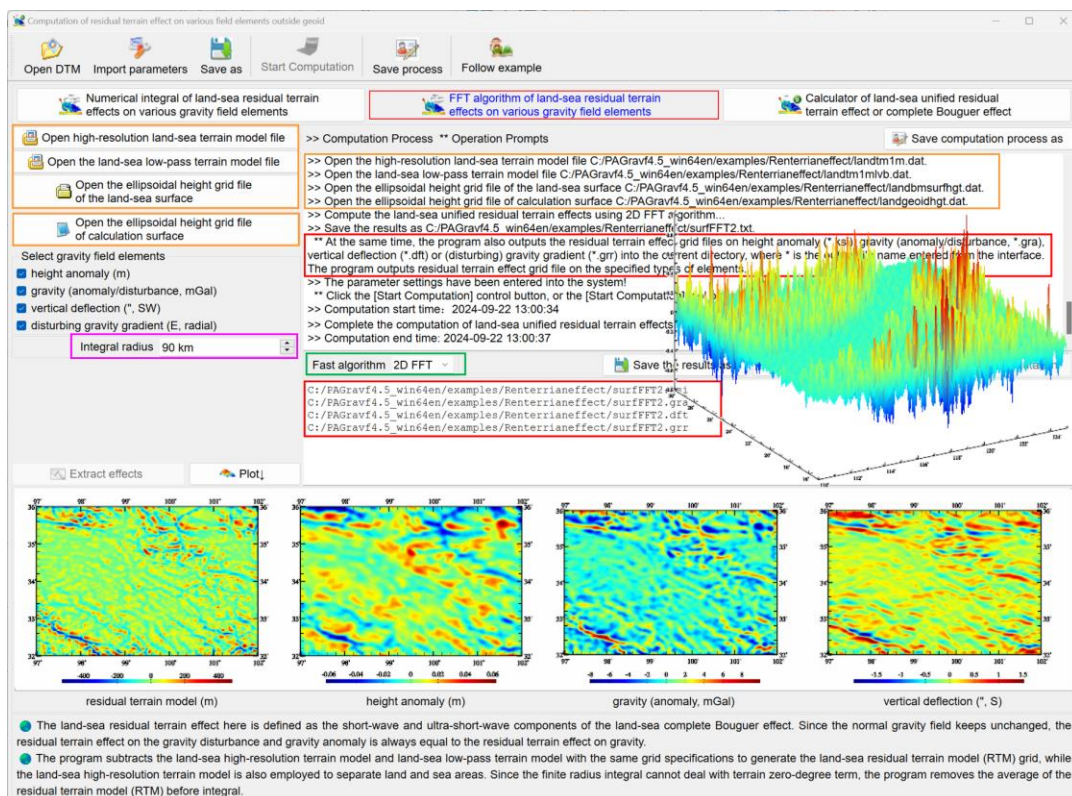
[Function] Using the FFT integral algorithm, from the high-resolution land-sea terrain model, low-pass land-sea terrain model and ellipsoidal height grid of the land-sea surface, compute the residual terrain effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (" , to south, to west) or (disturbing) gravity gradient (E, radial) on or outside the geoid.

[Input files] The high-resolution land-sea terrain model file, land-sea low-pass terrain model file, ellipsoidal height grid file of land-sea surface, and ellipsoidal height grid file of the calculation surface with the same grid specifications.

The ellipsoidal height grid of the land-sea surface stands for the land-sea surface position employed to calculate the integral distance.

[Parameter settings] Select the gravity field element type and the integral fast algorithm and enter the integral radius.

[Output file] The land-sea residual terrain effect file.



The program also outputs the residual terrain effect grid files on height anomaly (*.ksi), gravity (anomaly,disturbance, *.gra), vertical deflection (*.dft) or (disturbing) gravity gradient (*.grr) into the current directory, where * is the output file name entered from the interface. The program outputs residual terrain effect grid file on the specified types of elements.

Using the exact same parameters as the numerical integral, compute the residual terrain effects on various field elements according to the FFT algorithm, and statistically analyze the difference between the FFT result and the numerical integral result.

FFT – numerical integral		mean	standard deviation	minimum	maximum
height anomaly /m	FFT2	-0.0129	0.0335	-0.2051	0.1237
	FFT1	-0.0096	0.0242	-0.1514	0.0870
gravity(anomaly, disturbance) /mGal	FFT2	0.1031	0.2585	-0.4320	3.0334
	FFT1	0.0328	0.2266	-0.7472	1.3431
vertical deflection /S"	FFT2	0.0001	0.3380	-1.7813	1.8500
	FFT1	0.0002	0.0021	-0.0248	0.0334
vertical deflection /W"	FFT2	0.0029	0.3145	-1.9819	2.3883
	FFT1	0.0001	0.0040	-0.1178	0.0822
(disturbing) gravity gradient /E	FFT2	0.4968	15.1962	-67.4822	97.0052
	FFT1	-0.0239	0.3995	-10.3352	4.9832

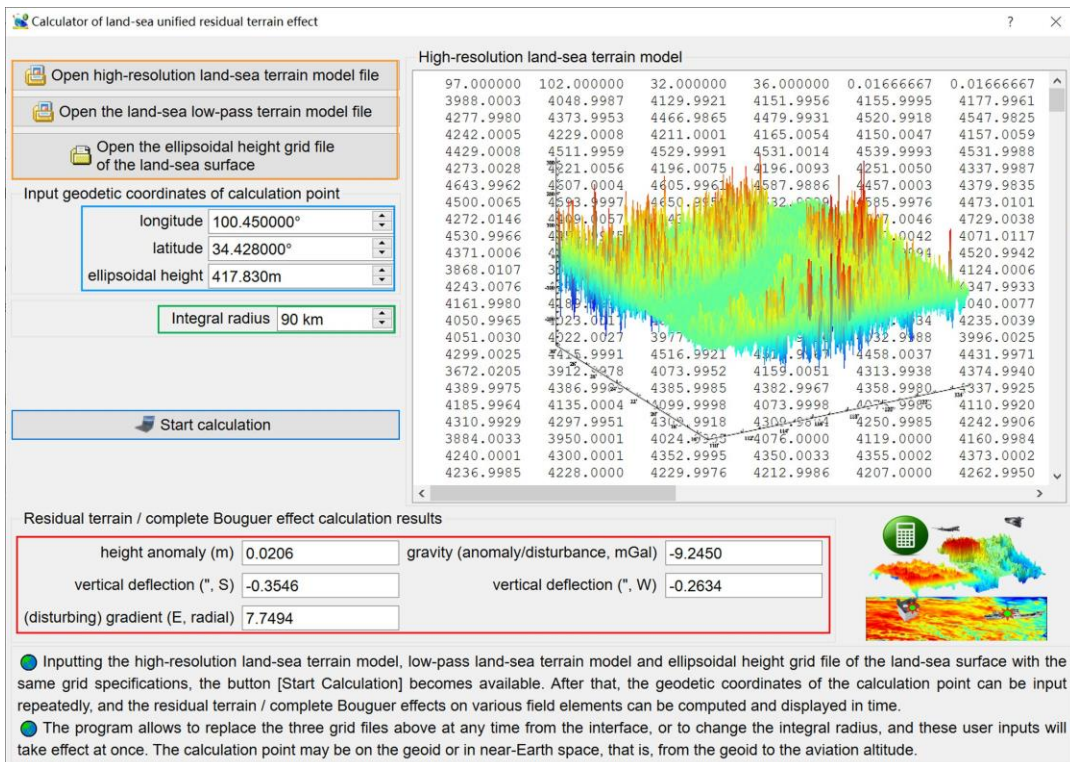
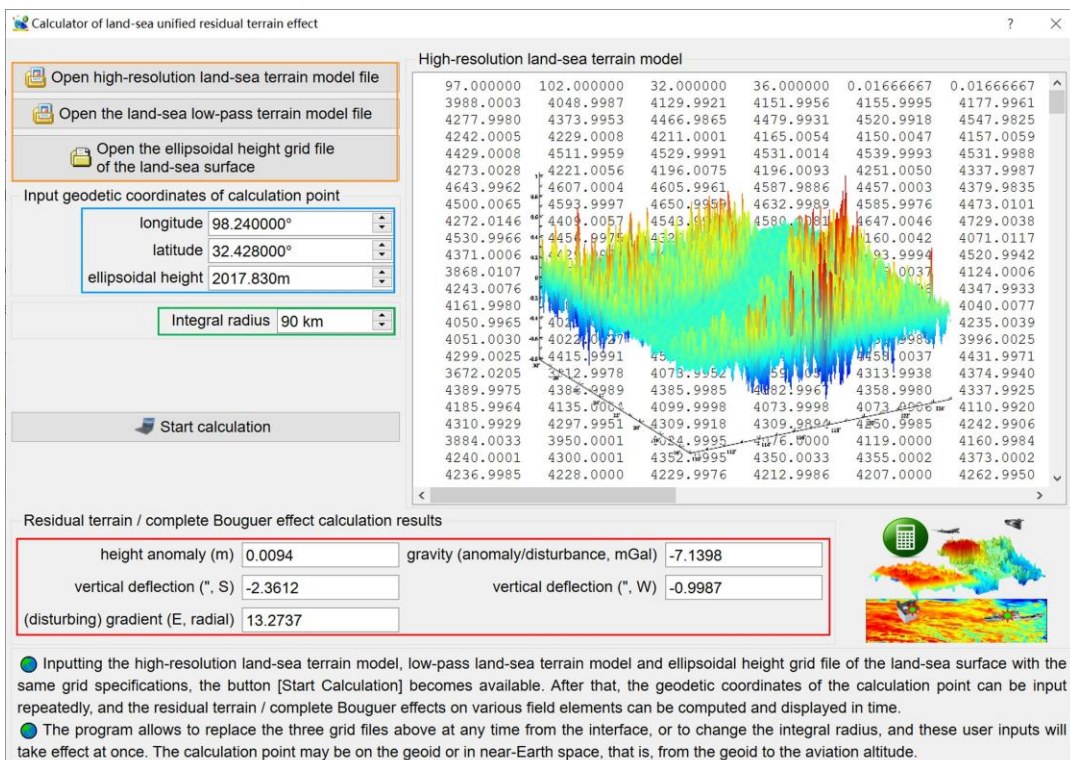
3.4.3 Calculator of land-sea unified residual terrain effect

[Function] From the high-resolution land-sea terrain model, low-pass land-sea terrain model and ellipsoidal height grid file of the land-sea surface with the same grid specifications, given the calculation point geodetic coordinates on or outside the geoid, compute the land-sea unified residual terrain effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection (" , to south, to west) and (disturbing) gravity gradient (E, radial).

Inputting the high-resolution land-sea terrain model, low-pass land-sea terrain model and ellipsoidal height grid file of the land-sea surface with the same grid specifications, the button [Start Calculation] becomes available. After that, the geodetic coordinates of the calculation point can be input repeatedly, and the residual terrain / complete Bouguer effects on various field elements at the calculation point can be computed and displayed in time.

The program allows to replace the three grid files mentioned at any time from the interface, or to change the integral radius, and these user inputs will take effect at once.

The calculation point may be on the geoid or in near-Earth space, that is, from the geoid to the aviation altitude.



3.5 Computation of land-sea unified classical gravity Bouguer / equilibrium effect

[Purpose] From the land-sea terrain model and ellipsoidal height grid of the land-sea surface, compute the land-sea unified classical Bouguer / equilibrium effects on land-sea surface gravity.

The terrain effect is equal to the negative value of the classical terrain correction. For example, the plane layer effect is equal to the negative layer correction, and the seawater Bouguer effect is equal to the negative seawater Bouguer correction.

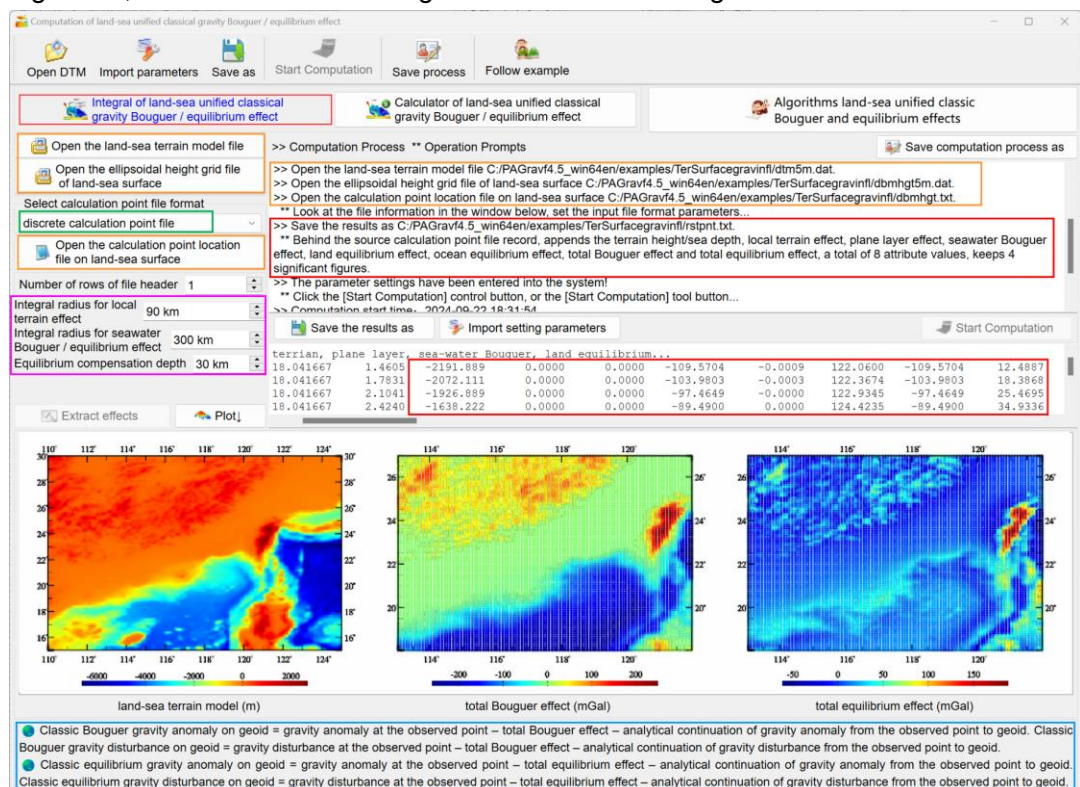
3.5.1 Integral of land-sea unified classical gravity Bouguer / equilibrium effect

[Function] From the land-sea terrain model and ellipsoidal height grid of the land-sea surface, compute the land-sea unified classical Bouguer / equilibrium effect on land-sea surface gravity (mGal).

In the land-sea terrain model, the land terrain height is greater than zero while the seafloor water depth is smaller than zero.

The ellipsoidal height grid of the land-sea surface stands for the land-sea surface position employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters, select the integral algorithm, and enter the land integral radius and sea integral radius.



The local terrain effect is the ultrashort wave effect of the terrain, and the integral radius of about 100km is suitable.

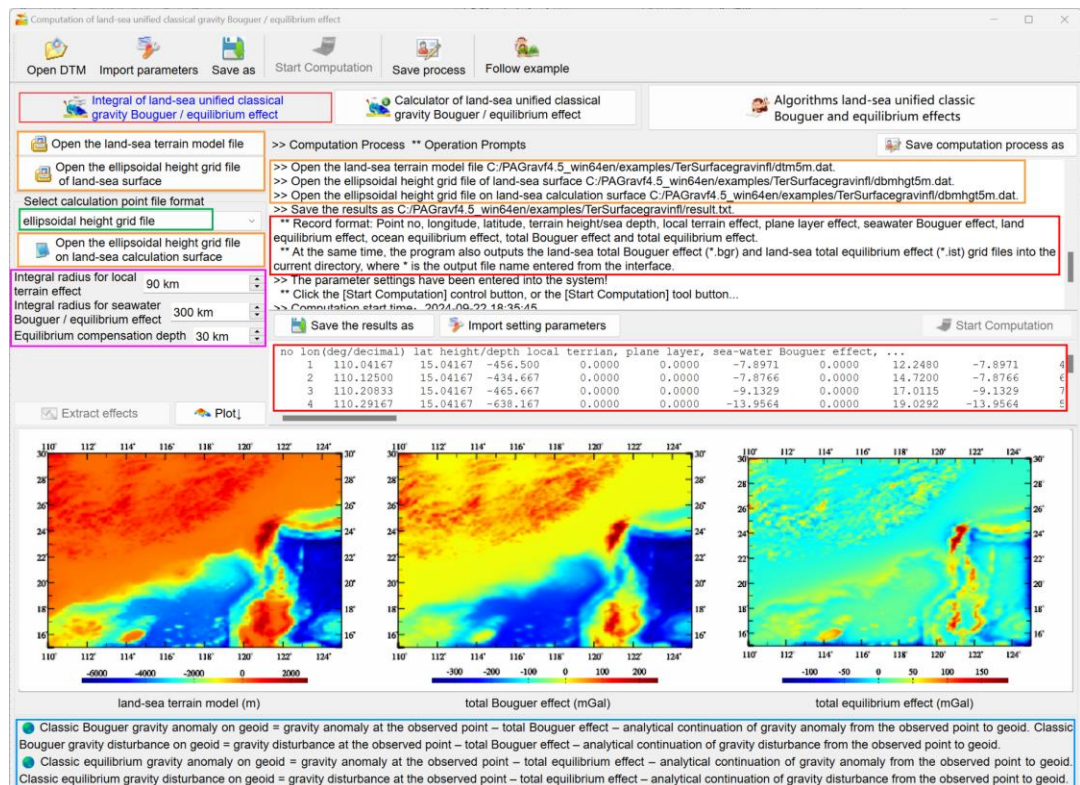
The seawater Bouguer and equilibrium effects include the medium and long wave effects of the terrain, and the integral radius should be much larger than that of local terrain effect.

[Output file] The gravity Bouguer / equilibrium effect file.

When the discrete calculation point file input, the output file record format: Behind the source calculation point file record, appends the local terrain effect, plane layer effect, seawater Bouguer effect, land equilibrium effect, ocean equilibrium effect, total Bouguer effect and total equilibrium effect, a total of 7 attribute values, keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: Point no, longitude, latitude, ellipsoidal height, local terrain effect, plane layer effect, seawater Bouguer effect, land equilibrium effect, ocean equilibrium effect, total Bouguer effect and total equilibrium effect.

At the same time, the program also outputs the land-sea total Bouguer effect (*.bgr) and land-sea total equilibrium effect (*.ist) grid files into the current directory, where * is the output file name entered from the interface.



The program is suitable for the unified computation of the classical Bouguer / equilibrium effect on gravity in land, land-sea junction and sea area. The analytical continuation need be computed by calling the programs in the subsystem [Data analysis and preprocessing calculation of the Earth gravity field].

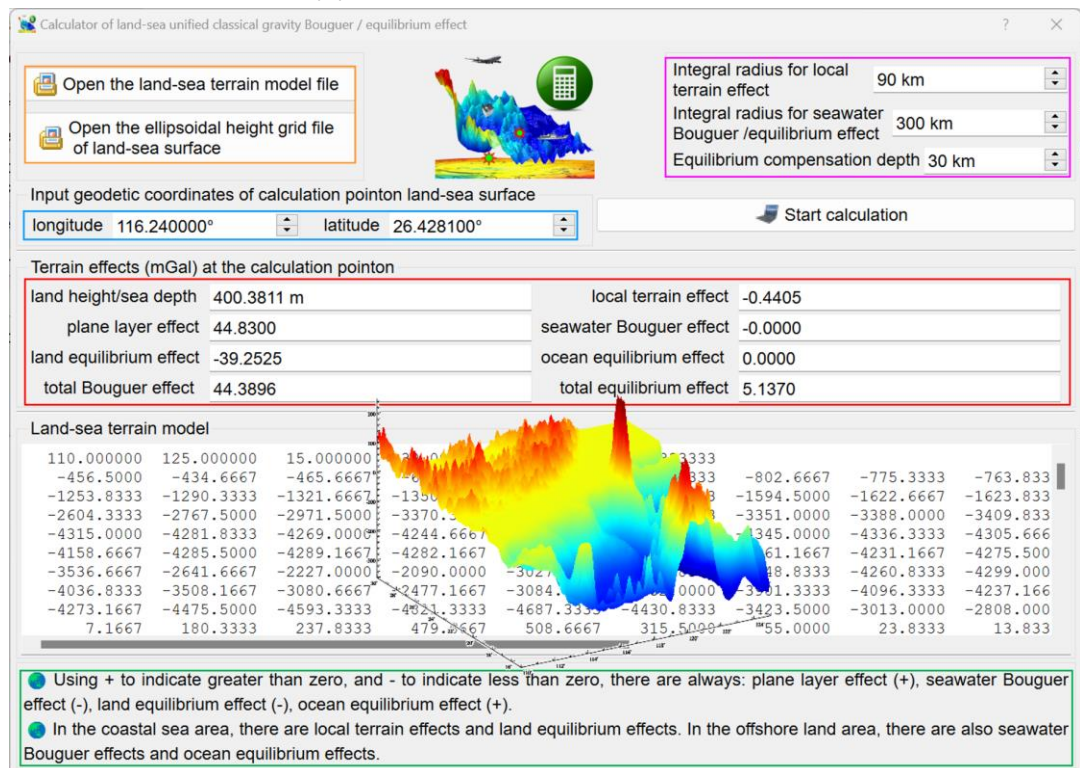
In any case, the classical Bouguer/equilibrium gravity anomaly (disturbance) can be achieved with a simple two-step calculation. The first step is to obtain the gravity

anomaly (disturbed gravity) on the geoid from terrestrial, marine and airborne observed gravity (or from a geopotential coefficient model), the second step is to call this program to obtain the total Bouguer/equilibrium effect, and the two-step results subtraction is the classical Bouguer /equilibrium gravity anomaly (disturbance).

3.5.2 Calculator of land-sea unified classical gravity Bouguer / equilibrium effect

[Function] From the land-sea terrain model file and ellipsoidal height grid file of the land-sea surface with the same grid specifications, given the longitude and latitude of the calculation point on land-sea surface, calculate the land-sea unified classical Bouguer / equilibrium effect on land-sea surface gravity (mGal).

Using + to indicate greater than zero, and - to indicate less than zero, there are always: plane layer effect (+), seawater Bouguer effect (-), land equilibrium effect (-), ocean equilibrium effect (+).

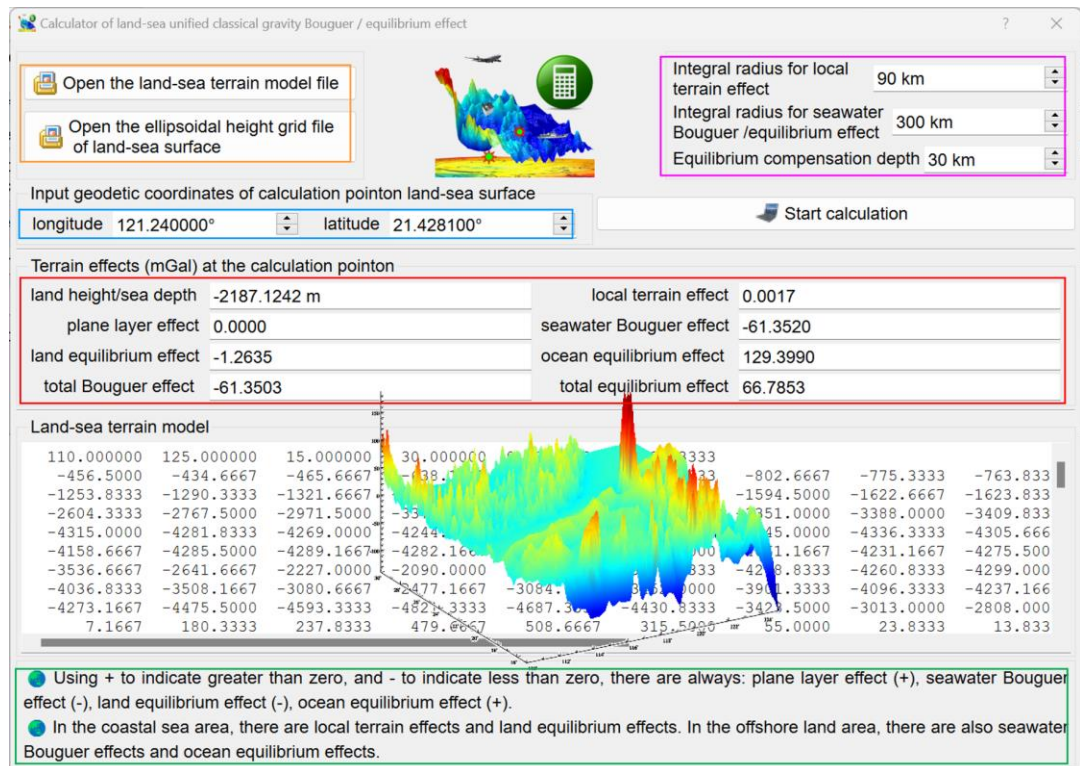


In the coastal sea area, there are local terrain effects and land equilibrium effects. In the offshore land area, there are also seawater Bouguer effects and ocean equilibrium effects.

Classic Bouguer gravity anomaly on geoid = gravity anomaly at the measurement point – total Bouguer effect – analytical continuation of gravity anomaly from the measurement point to the geoid.

Classic Bouguer gravity disturbance on geoid = gravity disturbance at the measurement point – total Bouguer effect – analytical continuation of gravity disturbance

from the measurement point to the geoid.



Classic equilibrium gravity anomaly on geoid = gravity anomaly at the measurement point – total equilibrium effect – analytical continuation of gravity anomaly from the measurement point to the geoid.

Classic equilibrium gravity disturbance on geoid = gravity disturbance at the measurement point – total equilibrium effect – analytical continuation of gravity disturbance from the measurement point to the geoid.

3.6 Ultrahigh degree spherical harmonic analysis on land-sea terrain and construction of model

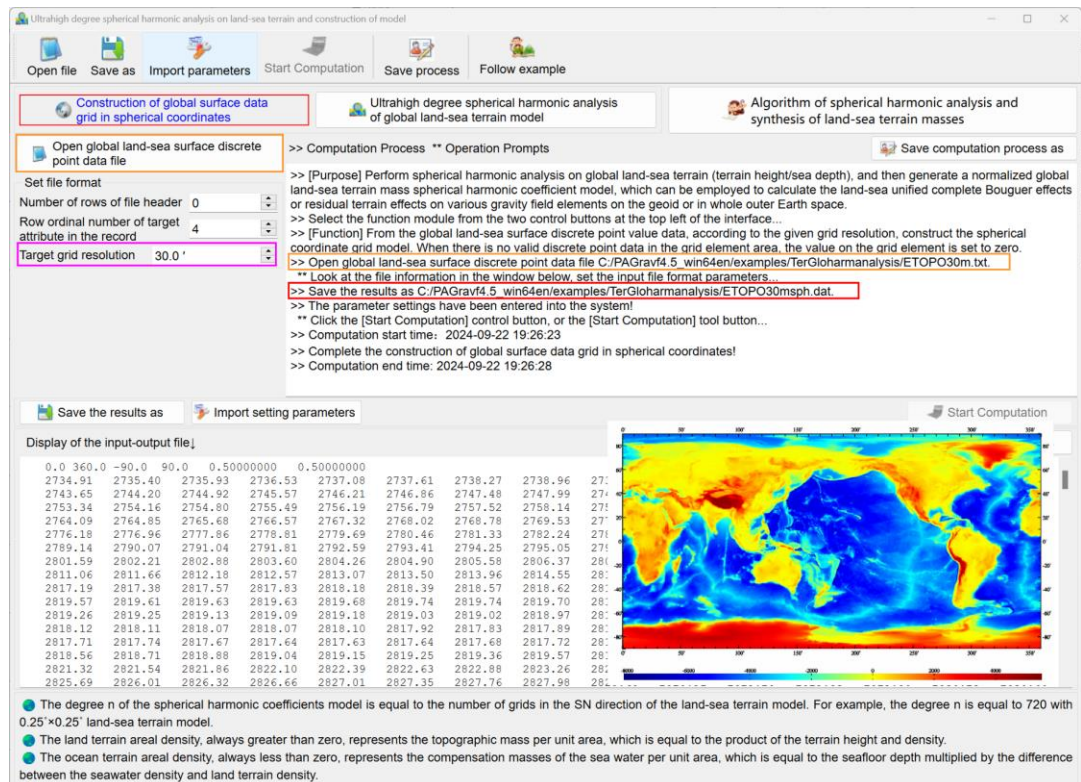
[Purpose] Perform spherical harmonic analysis on global land-sea terrain (terrain height/seafloor depth), and then generate a normalized global land-sea terrain mass spherical harmonic coefficient model, which can be employed to calculate the land-sea unified complete Bouguer effects or residual terrain effects on various gravity field elements on the geoid or in whole outer Earth space.

3.6.1 Construction of global surface data grid in spherical coordinates

[Function] From the global land-sea surface discrete point value data, according to the given grid resolution, construct the spherical coordinate grid model. When there is no valid discrete point data in the grid element area, the value on the grid element is set to zero.

[Input file] A global land-sea surface discrete point file.

[Output file] The spherical coordinate grid file.



[Input file] The global land-sea terrain model grid file in the spherical coordinate

[Parameter settings] Enter the iteration condition parameters.

Ultrahigh degree spherical harmonic analysis of global land-sea terrain model

Construction of global surface data grid in spherical coordinates

Open global land-sea terrain model grid in spherical coordinate system

Set iteration termination condition

Residual standard deviation threshold (a) 1.0 %

Termination condition of residual decrease (b) 1.0 %

☒ Simultaneously output terrain geopotential coefficient model

Computation Process ** Operation Prompts

** 16th iteration, the residual standard deviation = 7.243e+04

** 17th iteration, the residual standard deviation = 7.169e+04

** 18th iteration, the residual standard deviation = 7.103e+04

** 19th iteration, the residual standard deviation = 7.045e+04

** 20th iteration, the residual standard deviation = 6.993e+04

** 21th iteration, the residual standard deviation = 6.947e+04

** standard deviation of global land-sea terrain = 6.4176m.

File header of the spherical harmonic coefficient model: the geocentric gravitational constant $\gamma_0 \times 10^{-14} \text{ m}^3/\text{s}^2$, equatorial radius a (m) of the Earth, zero-degree term ΔC_{20} (kg/m^2), relative error Θ (%), where Θ is the residual standard deviation of the last step iteration, as a percentage of the standard deviation of the source grid values, and GM, a are also known as the scale parameters of the spherical harmonic coefficient model.

The program also outputs the global land-sea terrain geopotential coefficient model file "geop.dat" in the current directory, where "i" is the file name of the global land-sea terrain mass spherical harmonic coefficient model.

Complete the ultrahigh degree spherical harmonic analysis of global land-sea terrain model!

Computation end time: 2024-09-22 19:30:22

Save the results as **Save residual DTM as** **Import setting parameters**

Display of the input-output file:

3.986004415	6378136.30	-3667855.301	2.521
0	1.713661746628344E-01	0.000000000000000E+00	
1	1.666283046094737E-01	1.1455495760110544E-01	
2	1.633627492305205E-01	0.000000000000000E+00	
3	8.479043366221993E-02	9.1248955791892794E-02	
4	-1.1820159432578198E-01	-1.6730453760116247E-02	
5	-6.4915287439358635E-02	0.000000000000000E+00	
6	-4.4601487637124751E-02	4.0150215900756667E-02	
7	-1.3058410613568169E-01	1.2619038289781818E-01	
8	3.736351282940550E-02	1.5252441007074926E-01	
9	0.1006094799382276E-01	0.000000000000000E+00	
10	-1.5986576603564118E-02	-8.3251300057077774E-02	
11	-1.1442732934894125E-01	1.9235264166930027E-02	
12	3.10098478651494894E-01	-4.2950380278059655E-02	
13	-1.1646562236291021E-02	1.2919605986189289E-01	
14	0.14659492084399357E-01	0.000000000000000E+00	
15	-7.2051327049100396E-03	-1.9203743552770599E-02	

Algorithm of spherical harmonic analysis and synthesis of land-sea terrain masses

Computation process as

0.0 360.0 -90.0 0.0 5.000000000 0.500000000

2734.91	2735.40	2735.93	2736.53	2737.08	2737.61
2743.65	2744.20	2744.92	2745.57	2746.21	2746.86
2753.34	2754.16	2754.80	2755.49	2756.19	2756.79
2764.09	2764.85	2765.68	2766.57	2767.32	2768.02
2776.18	2776.96	2777.86	2778.81	2779.69	2780.46
2789.14	2790.07	2791.04	2791.81	2792.59	2793.41
2801.59	2802.21	2802.88	2803.60	2804.26	2804.90
2811.66	2811.66	2812.18	2812.57	2813.07	2813.50
2817.19	2817.38	2817.57	2817.83	2818.18	2818.39
2819.57	2819.61	2819.63	2819.63	2819.68	2819.74
2819.26	2819.25	2819.13	2819.09	2819.18	2819.03

The file header of the spherical harmonic coefficient model : the geocentric gravitational constant GM ($\times 10^{14} \text{m}^3/\text{s}^2$), equatorial radius a (m) of the Earth, zero-degree term $a\Delta C_{00}$ (kg/m^2), relative error Θ (%), where Θ is the residual standard deviation of the last step iteration as a percentage of the standard deviation of the source grid values, and GM, a are also known as the scale parameters of the spherical harmonic coefficient model.

69

The program also outputs the global land-sea terrain geopotential coefficient model file *geop.dat into the current directory, where * is the file name of the global land-sea terrain mass spherical harmonic coefficient model.

The program employs an iterative algorithm, and the calculation process need wait... During the period, you can open the file Harminf.txt in the current directory to look at the iterative process...

Harminf.txt record format: number of iterations, mean, standard deviation, minimum, maximum (of the residual terrain area density).

3.7 Spherical harmonic synthesis of complete Bouguer or residual terrain effects

[Purpose] From global land-sea terrain mass spherical harmonic coefficient model (kg/m^2), calculate the model value of terrain height/seafloor depth, as well as the land-sea unified complete Bouguer or residual terrain effects on various gravity field elements on the geoid or in whole outer Earth space, and analyzes the spectral and spatial domain properties of the global land-sea terrain effect.

Since the normal gravity field keeps unchanged, the terrain effect on the gravity disturbance and gravity anomaly is always equal to that on gravity, and the terrain effect on disturbing geopotential is always equal to that on geopotential.

The program is suitable for the unified computation of the complete Bouguer and residual terrain effects on various gravity field elements in land, land-sea junction and sea area. The calculation point may be on the geoid or in whole outer Earth space.

Using the equal minimum and maximum degree n , the program can calculate the contribution of the degree n land-sea terrain coefficients to various gravity field element. From the degree n , cumulative n degrees or $n_1 \sim n_2$ -degrees terrain effects calculated from a global land-sea terrain model, the spectral domain and spatial domain performance of the model can be observed and evaluated.

3.7.1 Calculation of model value for complete Bouguer or residual terrain effects

[Function] From global land-sea terrain mass spherical harmonic coefficient model (kg/m^2), calculate the model value of terrain height/seafloor depth, as well as the land-sea unified complete Bouguer or residual terrain effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection vector (" , south, west), (disturbing) gravity gradient (E, radial), tangential gravity gradient vector (E, north, west), or (disturbing) geopotential (m^2/s^2) on the geoid or in whole outer Earth space.

The global land-sea terrain mass spherical harmonic coefficient model can be constructed by the function [Ultrahigh degree land-sea terrain spherical harmonic analysis and model construction].

[Input files] The global land-sea terrain mass spherical harmonic coefficient model file, and the space calculation point location file.

The model file header : the geocentric gravitational constant GM ($\times 10^{14} \text{m}^3/\text{s}^2$), equatorial radius a (m) of the Earth, zero-degree term $a\Delta C_{00}$ (kg/m^2), relative error Θ (%). (GM, a) are also known as the scale parameters of the spherical harmonic coefficient model.

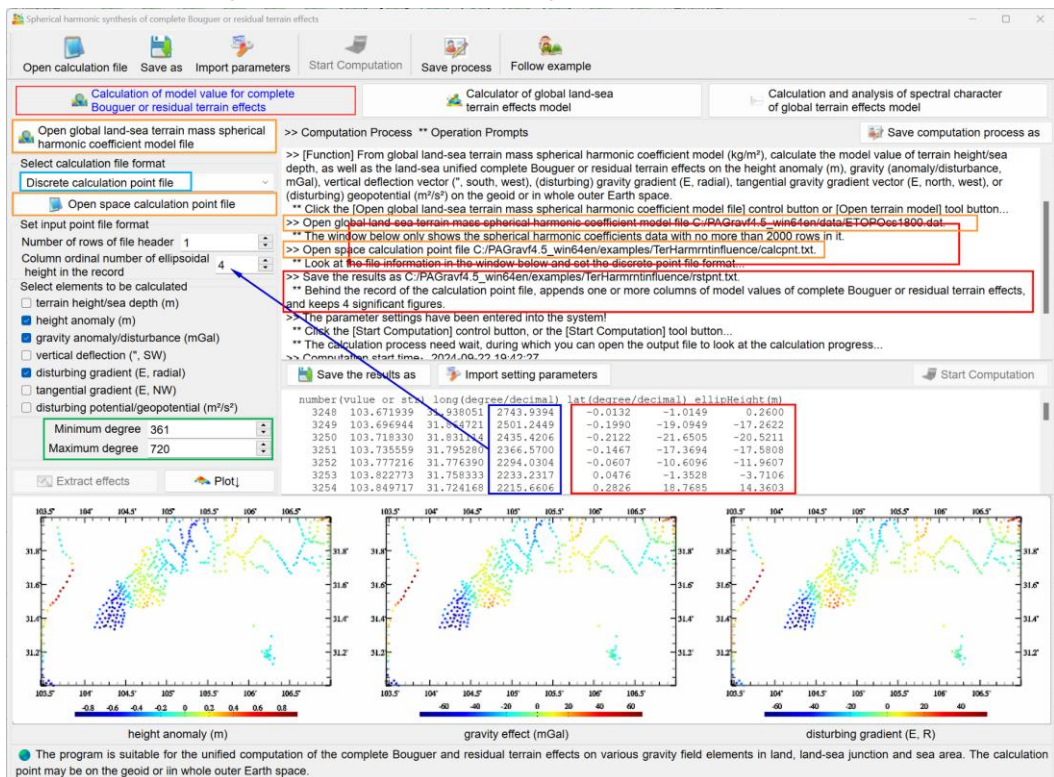
The space calculation point location file may be a discrete calculation point file or an ellipsoidal height grid file of the calculation surface.

The record format of the calculation point file: point no / point name, longitude (decimal degrees), latitude (decimal degrees), ellipsoidal height (m).....

[Parameter settings] Set the calculation point file format parameters, input the minimum and the maximum calculation degree of the land-sea terrain mass spherical harmonic coefficient model, and select the type of model residual terrain (complete Bouguer) effects to be calculated.

When the minimum calculation degree is less to 3, the program calculates the model value of land-sea complete Bouguer effects. And when the minimum calculation degree is greater than 2, the program calculates the model value of land-sea residual terrain effects.

The program selects the minimum of the maximum degree of the model and the input maximum degree as the calculation degree.

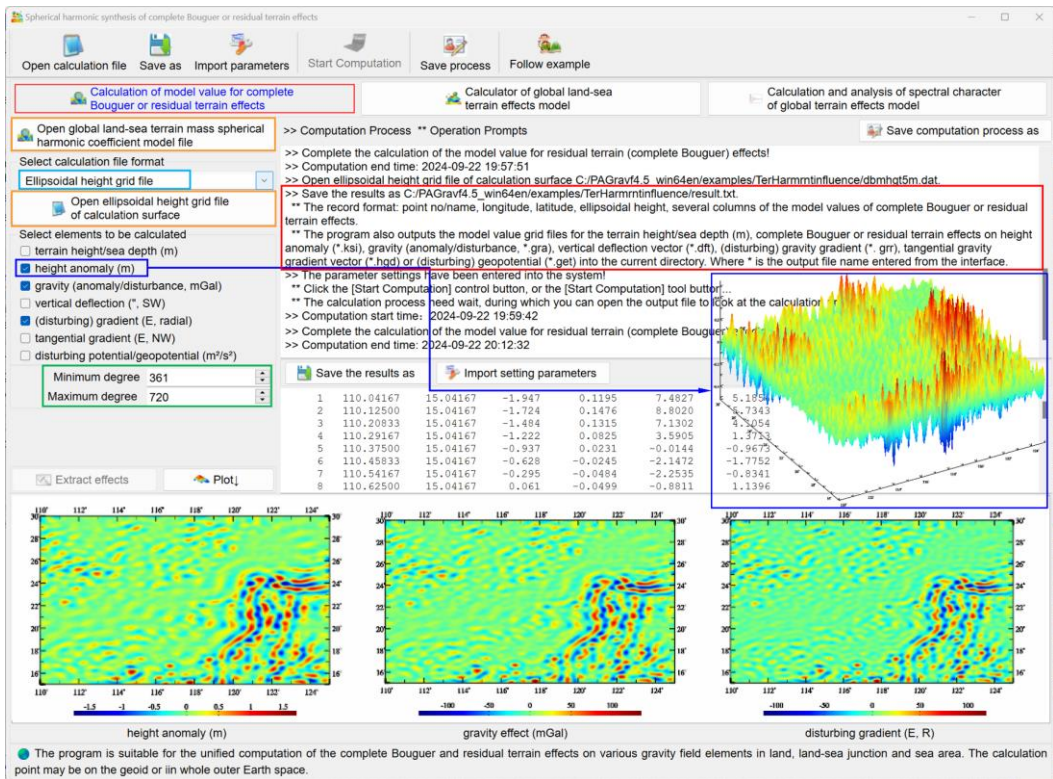


The calculation process need wait, during which you can open the output file to look at the calculation progress...

[Output file] The model value file of residual terrain (complete Bouguer) effects.

When the discrete calculation point file input, the output file record format: Behind the record of the calculation point file, appends one or more columns of the model values of residual terrain (complete Bouguer) effects selected, and keeps 4 significant figures.

When the ellipsoidal height grid file input, the output file record format: point no/name, longitude, latitude, ellipsoidal height, several columns of the model values of residual terrain (complete Bouguer) effects selected.



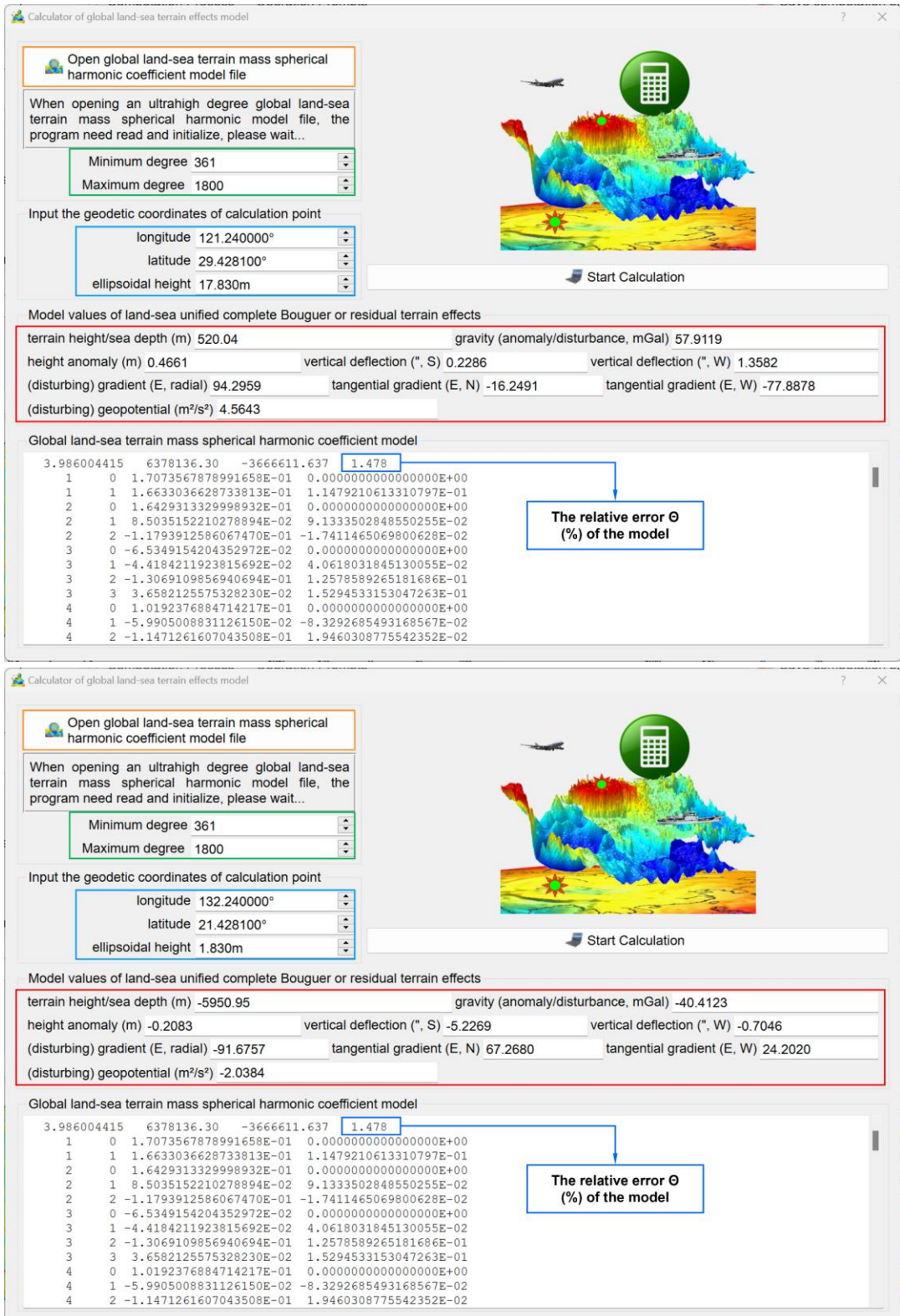
The program also outputs the model value grid files for the terrain height/seafloor depth (m), complete Bouguer or residual terrain effects on height anomaly (*.ksi), gravity (anomaly/disturbance, *.gra), vertical deflection vector (*.dft), (disturbing) gravity gradient (*.grr), tangential gravity gradient vector (*.hgd) or (disturbing) geopotential (*.get) into the current directory. Where * is the output file name entered from the interface.

When calculating the model value of terrain height/seafloor depth, the program ignores the ellipsoidal height of the calculation point.

3.7.2 Calculator of global land-sea terrain effects model

[Function] From the global land-sea terrain mass spherical harmonic coefficient model (kg/m^2) file, given the geodetic coordinates of space calculation point, calculate the model values of terrain height/seafloor depth and the complete Bouguer or residual terrain effects on the height anomaly (m), gravity (anomaly/disturbance, mGal), vertical deflection vector (" south, west), (disturbing) gravity gradient (E, radial), tangential gravity gradient vector (E, north, west) and (disturbing) geopotential (m^2/s^2). This

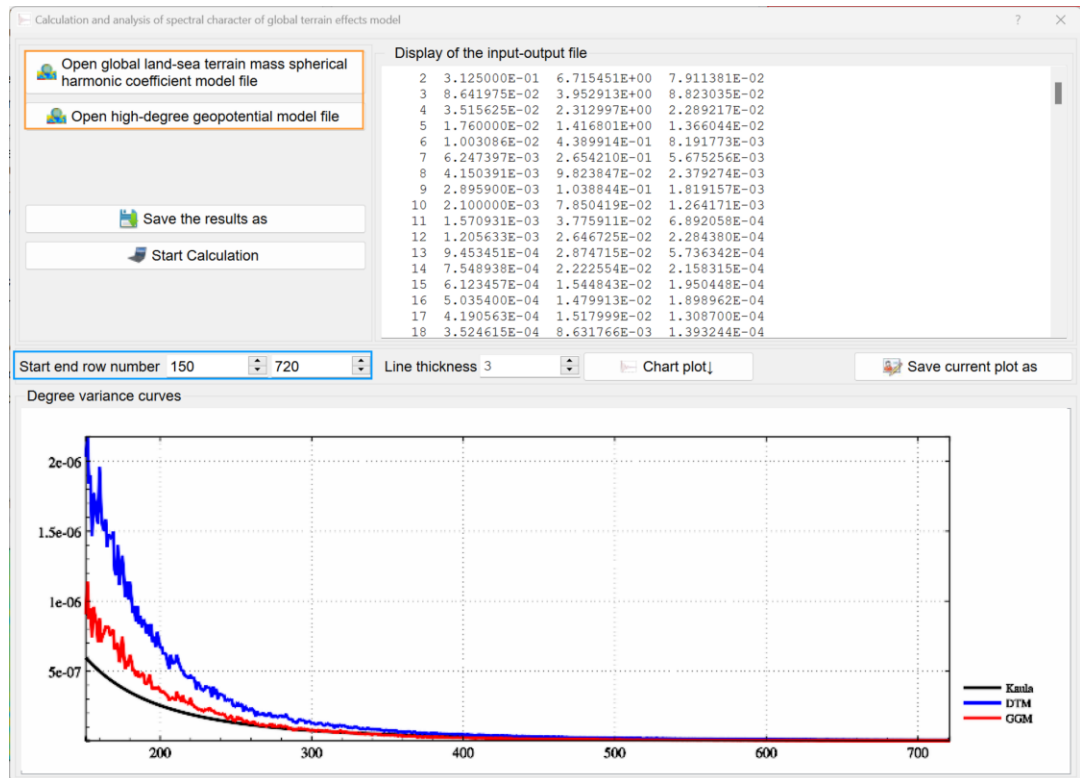
function is suitable for classroom demonstrations.



When an ultrahigh degree global land-sea terrain mass spherical harmonic model file opened, the program need read and initialize, please wait...

3.7.3 Calculation and analysis of spectral character of global terrain effects model

[Function] Calculate the degree variance of the land-sea terrain geopotential coefficient model and global geopotential coefficient model, and then analyze the spectral or spatial domain properties of the land-sea complete Bouguer and residual terrain effects from the degree variance curves.

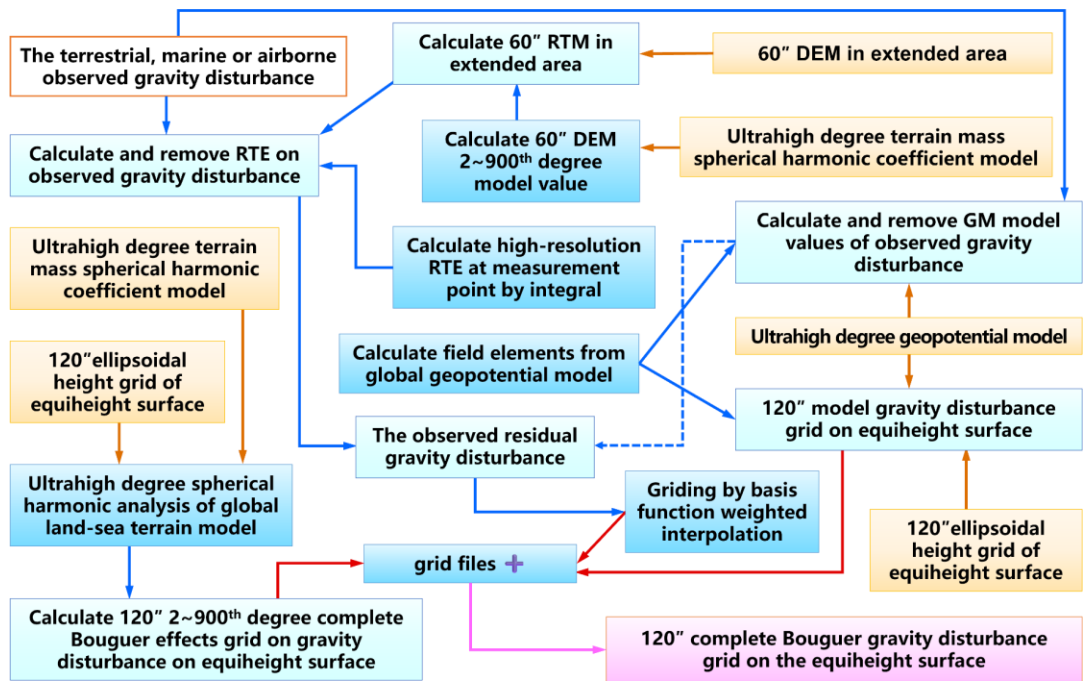


3.8 Computation process demo of various terrain effects outside geoid

3.8.1 Computation process demo of complete Bouguer anomaly on terrain equiheight surface

From the ground digital elevation model and discrete observed gravity disturbance calculated from EGM2008 geopotential model, a remove-restore scheme with the residual terrain effects employed, calculate the complete Bouguer gravity disturbance grid on an equipotential surface which is also the observation reduction surface, to show the basic computation scheme and process of the land-sea unified complete Bouguer effects on various gravity field elements near-Earth space.

The complete Bouguer effect is defined as the variation of Earth gravity field because of the terrain mass above the geoid removed and the seawater density compensated to the terrain density.



Computation process demo of complete Bouguer anomaly on or outside geoid

● Input and output data and related terrain models

Let terrain data range (extended area, E94.5~99.5°, N30.5~34.5°) \supset result range (observed point distribution range / observation reduction surface range, E95.0 ~ 99.0°, N31.0 ~ 34.0°) to suppress the edge effect of integral.

(1) The observed gravity disturbance and disturbing gravity gradient file Obsgrav.txt.

The gravity disturbances are simulated from the 2~1800th degree EGM2008 model. PAGravf4.5 employs the exact same algorithms to process various terrestrial, marine, and airborne gravity data in a unified way, and there is no need to distinguish whether the observed point is on the ground, at the air altitude or in the sea area.

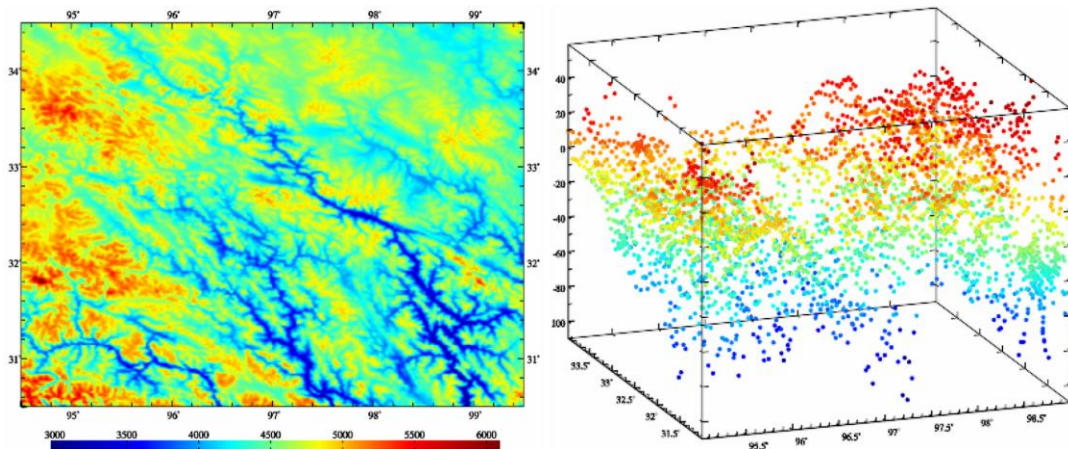
The format of the file record: ID, longitude (°), latitude (°), ellipsoidal height (m), gravity disturbance (mGal). The distribution of the observed points is shown in Fig.

(2) The 1800-degree terrain mass spherical harmonic coefficient model file ETOPOcs1800.dat and the 2190-degree EGM2008 geopotential coefficient model file EGM2008.gfc.

The two model files are stored in the directory C:\PAGravf4.5_win64en\data. The 1800-degree global land-sea terrain mass spherical harmonic coefficient model ETOPOcs1800.dat is generated by the PAGravf4.5 function [Ultrahigh degree spherical harmonic analysis of global land-sea terrain model] from the global 2'×2' land-sea terrain model ETOPO2v2g.

(3) The ground digital elevation model (DEM)

It is required that the DEM grid range (extended area) be larger than the calculation area to eliminate the integral edge effect.



Ground digital elevation model (m) and gravity measurement point distribution

Two kinds of DEM resolutions are required. The high-resolution is employed for the observation data reduction, that is, to calculate and remove the residual terrain effects on the observations. The other resolution is consistent with the calculation result resolution and is employed to restore the residual terrain effects on the results. In this example, they are 60" and 120" respectively, and the corresponding files are extdtm60s.dat and extdtm120s.dat.

Simple and direct calculation on geodetic data files

● Construct the ground ellipsoidal height grid and ellipsoidal height grid of the terrain equiheight surface.

Weighted operation on two specified attributes in record file | Weighted operation on two geodetic grid files | Weighted operation on two vector grid files | Weighted operation on two harmonic coefficient files

Open geodetic grid file 1 | Open geodetic grid file 2 | Select operation mode | Set weight | The first weight 1.00 | The second weight 1.00 | Vector grid operation

Program Process ** Operation Prompts

```
>> [Function] Perform weighted plus, minus, or multiply operation on grid elements in two (vector) grid files with the same specifications.
>> Open geodetic grid file 1 C:\PAGrav4.5_win64en\examples\Terrain\exercise\TerComp\process\extdtm60s.dat
>> Open geodetic grid file 2 C:\PAGrav4.5_win64en\examples\Terrain\exercise\TerComp\process\EGM180ks120s.dat
>> Save the results as C:\PAGrav4.5_win64en\examples\Terrain\exercise\TerComp\process\surfhgt60s.dat
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2023-03-18 12:37:44
>> Complete the computation!
>> Computation end time: 2023-03-18 12:37:44
>> Open geodetic grid file 1 C:\PAGrav4.5_win64en\examples\Terrain\exercise\TerComp\process\const339.5.dat
>> Open geodetic grid file 2 C:\PAGrav4.5_win64en\examples\Terrain\exercise\TerComp\process\EGM180ks120s.dat
>> Save the results as C:\PAGrav4.5_win64en\examples\Terrain\exercise\TerComp\process\equihgt120s.dat
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2023-03-18 12:44:42
>> Complete the computation!
>> Computation end time: 2023-03-18 12:44:42
```

Save the results as | Import setting parameters | Start computation

Display of the input-output file

94.500000	99.500000	30.500000
304.9759	304.9250	304.8757
304.4461	304.4353	304.4261
304.6977	304.7340	304.7837
305.6230	305.6941	305.7651
306.5466	306.5795	306.6058
306.6824	306.6657	306.6451
306.1619	306.1215	306.0821
305.6821	305.6697	305.6585
305.8508	305.8859	305.9240
306.5752	306.6304	306.6856
304.9252	304.8780	304.8321
304.4372	304.4266	304.4191
304.6815	304.7264	304.7740
305.5747	305.6442	305.7135
306.4293	306.4679	306.4925
306.5528	306.5377	306.5189
306.0610	306.0205	305.9836
305.6288	305.6193	305.6156
305.8399	305.8782	305.9189
306.5802	306.6355	306.6912

60" ground ellipsoidal height grid | 120" ellipsoidal height grid of the terrain equiheight surface

(4) 60" ground ellipsoidal height grid file surfhgt60s.dat

The ground ellipsoidal height grid is employed to give the space location of the

residual terrain mass (the integral move cell) which is indispensable for high-precision calculation. In this example, the 60" ground ellipsoidal height grid file `surfhgt60s.dat` is employed for the 60" residual terrain model.

(5) 120" ellipsoidal height grid file `equihgt120s.dat` of terrain equiheight surface.

The terrain equiheight surface is the reduction surface of the observations and calculation surface of the result grid, which is regarded as an equipotential surface. The ellipsoidal height of the terrain equiheight surface here is equal to the sum of the 2~180th degree EGM2008 model geoidal height and mean of DEM.

The gridding operation is not analytical, which is easy to weaken the analytical nature of the gravity field. Non-analytical operations are required to be performed on some an equipotential surface to minimize the negative effects on gravity field. In this example, the terrain equiheight surface is regarded as an equipotential surface.

When the normal (orthometric) heights of the surface are zero, namely whose ellipsoidal height are the geoidal height, the reduction surface and calculation surface are the geoid in the traditional sense.

(6) The result products: 120"×120" complete Bouguer gravity disturbance grid file on the terrain equiheight surface.

● Called functions and input-output data flow

(1) Calculate and remove the model terrain height value, and then construct 60" residual terrain model (RTM) grid.

Call the function [Calculation of model value for complete Bouguer or residual terrain effects] with the minimum degree 1 and the maximum degree 900, select the calculation type 'terrain height/sea depth (m)', input the land-sea terrain mass spherical harmonic coefficient model file `ETOPOcs1800.dat` and ground ellipsoidal height grid file `surfhgt60s.dat`, and generate 60" model terrain height grid files `mdltdtm60s.dtm`.

Let `extdtm60s.dat` minus `mdltdtm60s.dat`, the residual terrain models (RTM) `resdtm60s.dat` in the extended area are obtained, as shown in the figure.

	mean	standard deviation	minimum	maximum
30" RTM (m)	0.0053	175.5869	-959.5450	59.0160
60" RTM (m)	0.0053	175.5869	-959.5450	886.2500

(2) Calculate and remove the ultrahigh-degree model gravity disturbances at the observed points.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and the maximum degree 720, input the file `EGM2008.gfc` and the observation file `Obsgrav.txt`, select the type 'gravity disturbance', and generate the model gravity disturbance file `Obsgravmdl.txt` (columns 6) at the observed points.

Subtract the observed gravity disturbance (column 5) and model gravity disturbance (column 6) to generate the model residual gravity disturbance (column 7)

file Obsgravmdlresd.txt.

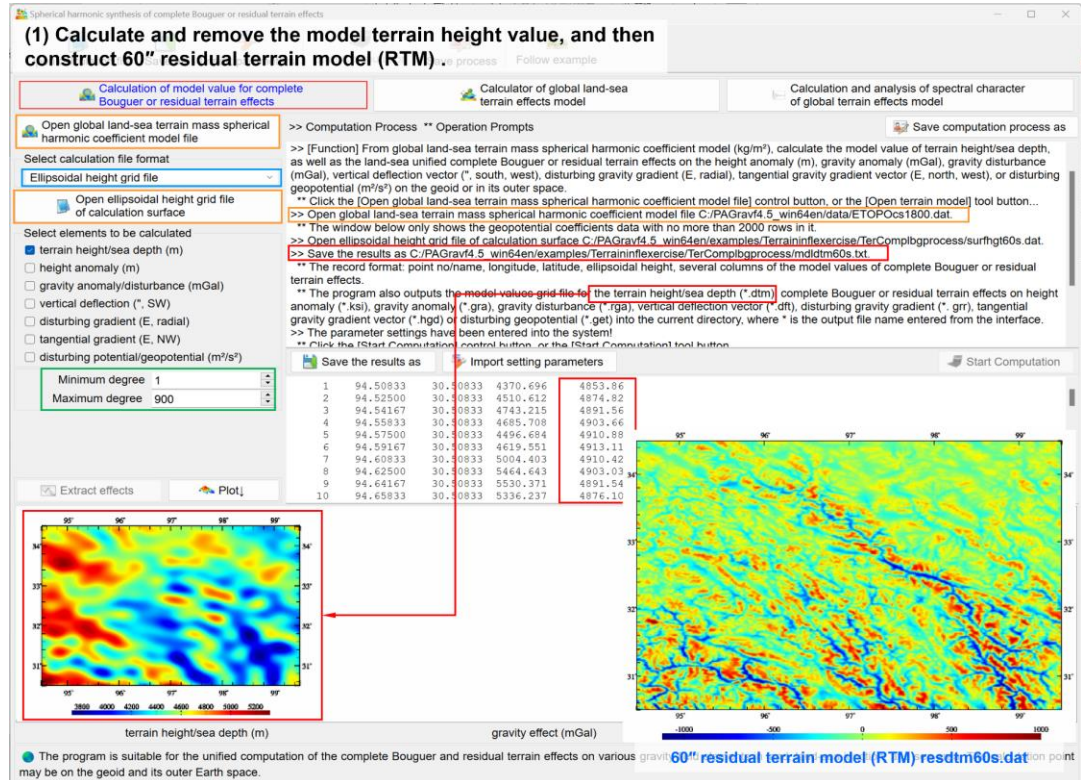


Table 2 shows the statistical results on the gravity disturbances after the 2~720th degree model values removed.

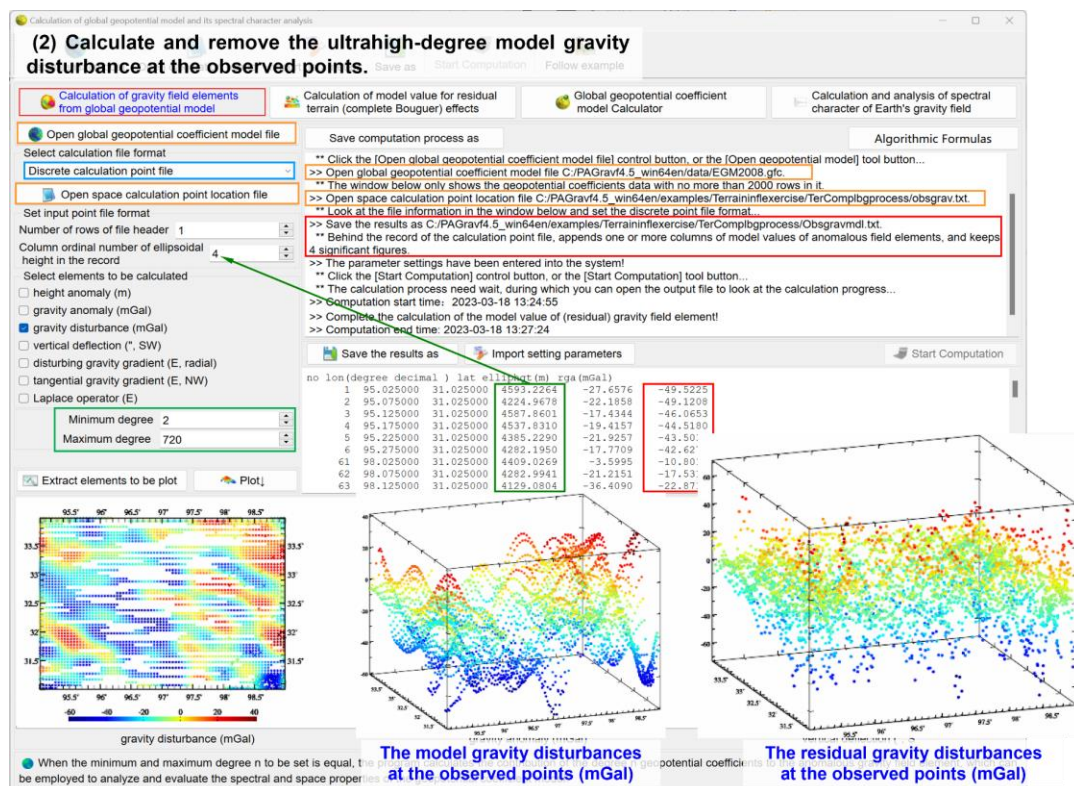
Observed points	mean	standard deviation	minimum	maximum
Observed gravity disturbance (mGal)	-15.6106	25.5080	-110.7251	59.0160
Residual of gravity disturbance (mGal)	-0.4881	17.4588	-74.6129	71.5003

(3) Calculate and remove the residual terrain effects on the gravity disturbances at the observed points.

Call the function [Numerical integral of land-sea residual terrain effects on various gravity field elements], input the file Obsgravmdlresd.txt, the high-resolution DEM extdtm60s.dat, low-pass DEM mlddtm60s.dtm and the ground ellipsoidal height grid file surfhgt60s.dat, set the integral radius 90km, and generate the residual terrain effects (RTE) file Obsgravresdtm.txt (column 8) on the gravity disturbances.

Subtract the residual gravity disturbance (column 7) and its residual terrain effect (column 8), to generate the remaining residual gravity disturbance file Obsgravresidual.txt (column 9).

After the residual terrain effects removed, the statistical results on the residual gravity disturbances are shown in Table 3.



Observed points	mean	standard deviation	minimum	maximum
RTE on gravity disturbance (mGal)	4.8843	7.2038	-73.7901	118.6158
Remaining residual gravity disturbance (mGal)	-5.3034	19.7638	-144.5444	92.4782

In this example, the continuation correction of residual radial gradient (within a height difference of 1000m, it is small) is ignore. In this case, the remaining residual gravity disturbance at the observed point is equal to that on the equipotential surface.

The basic purpose of the statistics in Tables 1 to 3 is to improve the residual terrain effect algorithm and relative parameters according to the gridding optimization criteria. Since the simulated data lack sufficient ultrashort wave information of the real gravity field, the optimization criterion analysis process is omitted in this example.

So far, the reduction processing of the gravity disturbances from the observed points to the terrain equiheight surface has been completed.

(4) Gridding on the remaining residual gravity disturbance into 120"×120" grids on the terrain equiheight surface.

Call the function [Gridding of heterogeneous data by basis function weighted interpolation], select 'equal weights of observations' (the weights can be estimated with the residual terrain effects as the reference attribute in advance), and grid on the 9th column of attributes (from the file Obsgravresidual.txt), to generate 120" remaining

(3) Calculate and remove the residual terrain effect on the gravity disturbance at the observed points.

The figure displays a screenshot of the 'Gravity Disturbance' software interface, showing the calculation process and the resulting gravity disturbance map.

Software Interface Components:

- Top Bar:** 'Computation of residual terrain effect on various field elements outside geoid'.
- Left Panel (Input/Output):**
 - Open high-resolution land-sea terrain model file
 - Open land-sea low-pass terrain model file
 - Open the ellipsoidal height grid file of the land-sea surface
 - Select calculation point file format: **discrete calculation point file**
 - Open the calculation point position file
 - Set input point file format: Number of rows of file header: 1
 - Column ordinal number of ellipsoidal height in the record: 4
 - Select gravity field elements:
 - ☐ height anomaly (m)
 - ☐ gravity anomaly (mGal)
 - ☒ gravity disturbance (mGal)
 - ☐ vertical deflection (", SW)
 - ☐ disturbing gravity gradient (E, radial)
 - Integral radius: 90 km
 - Extract effects
 - Plot
- Top Center Panel (Computation Process):**
 - >> Computation Process ** Operation Prompts
 - ** The program subtracts the land-sea high-resolution terrain model and land-sea low-pass terrain model with the same grid specifications to generate the land-sea residual terrain model (RTM) grid, while the land-sea high-resolution terrain model is also used to separate land and sea areas. Since the finite radius integral cannot cut with terrain zero-degree term, the program removes the average of the residual terrain model (RTM) before integral.
 - >> Open the high-resolution land-sea terrain model file C:\PAGrav4.5_win64en\examples\Terrain\infxercise\TerComblpprocess\extdmt60s.dat.
 - >> Open the land-sea low-pass terrain model file C:\PAGrav4.5_win64en\examples\Terrain\infxercise\TerComblpprocess\mddmt60s.dtm.
 - >> Open the ellipsoidal height grid file of the land-sea surface C:\PAGrav4.5_win64en\examples\Terrain\infxercise\TerComblpprocess\surfhtg60s.dat.
 - >> Open the calculation point location file C:\PAGrav4.5_win64en\examples\Terrain\infxercise\TerComblpprocess\Obsgravdmsd.txt.
 - ** Look at the file information in the window below, set the input file format parameters.
 - ** Record format: Behind the source calculation points file record, appends several columns of residual terrain effects on specified types of field elements, keeps 4 significant figures.
 - >> The parameter settings have been entered into the system!
 - ** Click the [Start Computation] control button, or the [Start-Computation] tool button...
 - >> Computation start time: 2023-03-18 16:18:50
- Right Panel (File Information):**
 - Save the results as
 - Import setting parameters
 - Start Computation
- Bottom Panel (Results):**
 - no longitude decimal | lat ellipshgt(m) r50
 - 1 95.025000 31.025000 4593.2264
 - 2 95.075000 31.025000 4224.9678
 - 3 95.125000 31.025000 4587.8601
 - 4 95.175000 31.025000 4537.8310
 - 5 95.225000 31.025000 4385.2290
 - 6 95.275000 31.025000 4282.1950
 - 61 95.025000 31.025000 4409.0269
 - 62 96.075000 31.025000 4282.9941
 - 63 96.125000 31.025000 4129.0804
 - 64 96.175000 31.025000 4372.1236

Gravity Disturbance Map:

The map shows the gravity disturbance (mGal) on a grid. The x-axis represents longitude (95° to 96°) and the y-axis represents latitude (31° to 33°). The color scale ranges from -40 to 100 mGal. The map is labeled 'residual terrain model (m)' and 'gravity disturbance (mGal)'.

Legend:

- The calculation point may be on the geoid and its outer near-Earth space. Since the normal gravity field keeps unchanged, the residual terrain effect on the gravity disturbance and gravity anomaly is always equal to the residual terrain effect on gravity.
- The program subtracts the land-sea high-resolution terrain model and land-sea low-pass terrain model with the same grid specifications to generate the land-sea residual terrain model (RTM) grid, while the land-sea high-resolution terrain model is also employed to separate land and sea areas.

Remaining Gravity Disturbance:

The remaining residual gravity disturbance (mGal) is shown on the map.



The range and resolution of the grid is the same as the result grid.

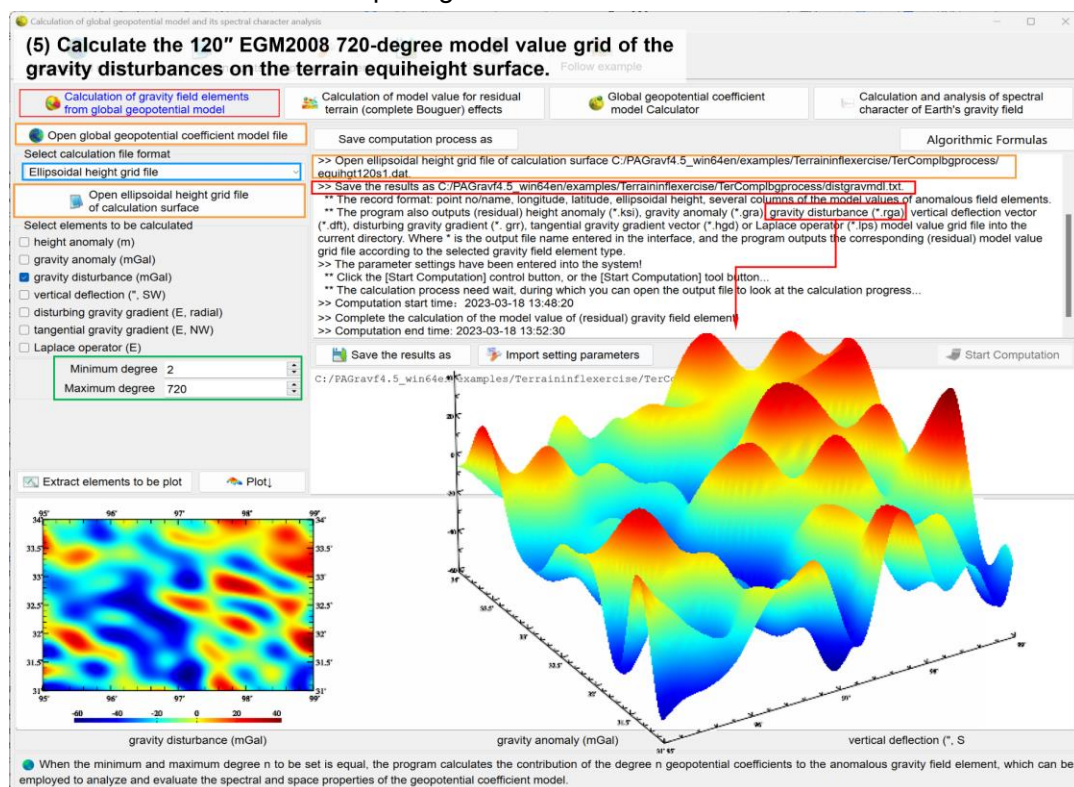
(5) Calculate the 120" EGM2008 720-degree model value grid of the gravity disturbance on the terrain equiheight surface.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and the maximum degree 720, input the file EGM2008.gfc and the ellipsoidal height grid file equihgt120s1.dat (from equihgt120s.dat with grid edge removed) of the terrain equiheight surface, and select the calculation type 'gravity disturbance', to generate 120" model gravity disturbance grid file distgravmdl.rga on the terrain equiheight surface.

Here, the geopotential model, the minimum and maximum degree are required to be the same as in step (2).

(6) Calculate the 120" model complete Bouguer effect grid on the gravity disturbance on the terrain equiheight surface.

Call the function [Calculation of model value for complete Bouguer or residual terrain effects] with the minimum degree 2 and the maximum degree 900, select the type 'gravity disturbance' input the land-sea terrain mass spherical harmonic coefficient model file ETOPOcs1800.dat and 120" ground ellipsoidal height grid file surfhgt120s1.dat of the terrain equiheight surface and, and then generate the 120" model complete Bouguer effect grid file distgravmdlcmpbg.gra on the gravity disturbances on the terrain equiheight surface.

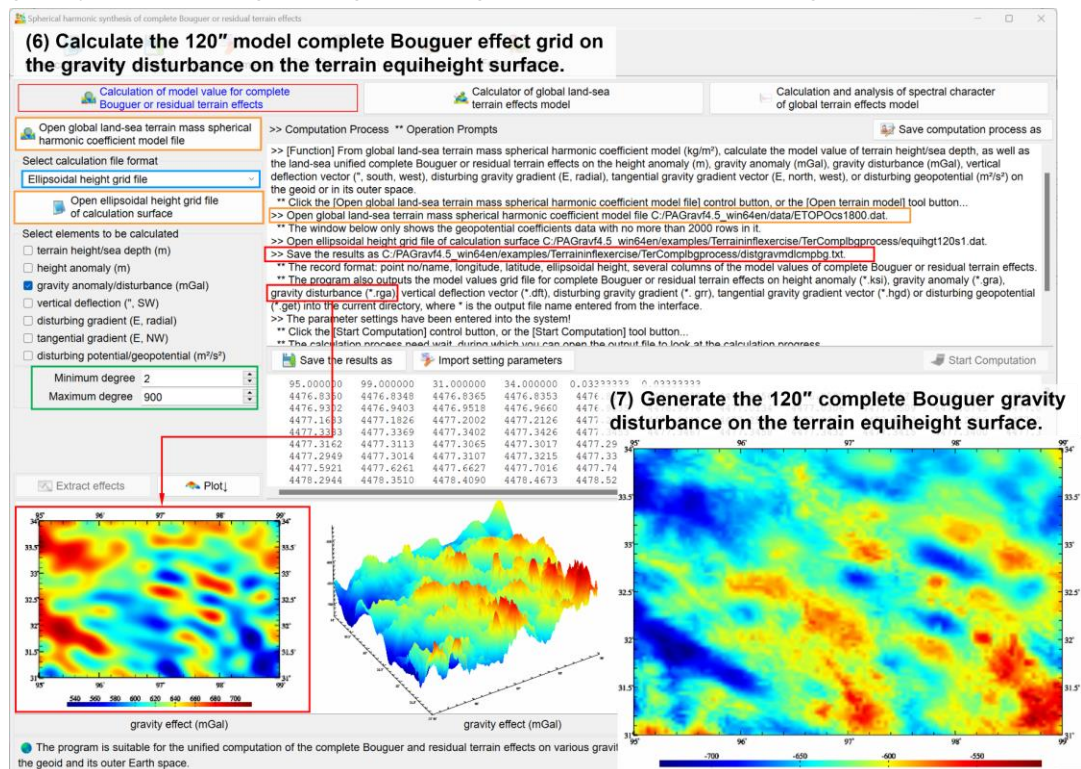


Here, the terrain mass spherical harmonic coefficient model and the maximum

degree are required to be the same as in step (1).

(7) Generate the 120" complete Bouguer gravity disturbance on the terrain equiheight surface.

Sum up the remaining residual grid `distgravresidual.dat`, the ultrahigh degree model value grid `distgravmdl.dat`, the residual terrain effect grid `distgravresidtm.dat` and the model complete Bouguer effect grid `distgravmdlcmpbg.gra` four grid files of the gravity disturbance with the same grid specifications, to generate the 120" complete Bouguer gravity disturbance grid `distgravcmpbg.dat` on the terrain equiheight surface.



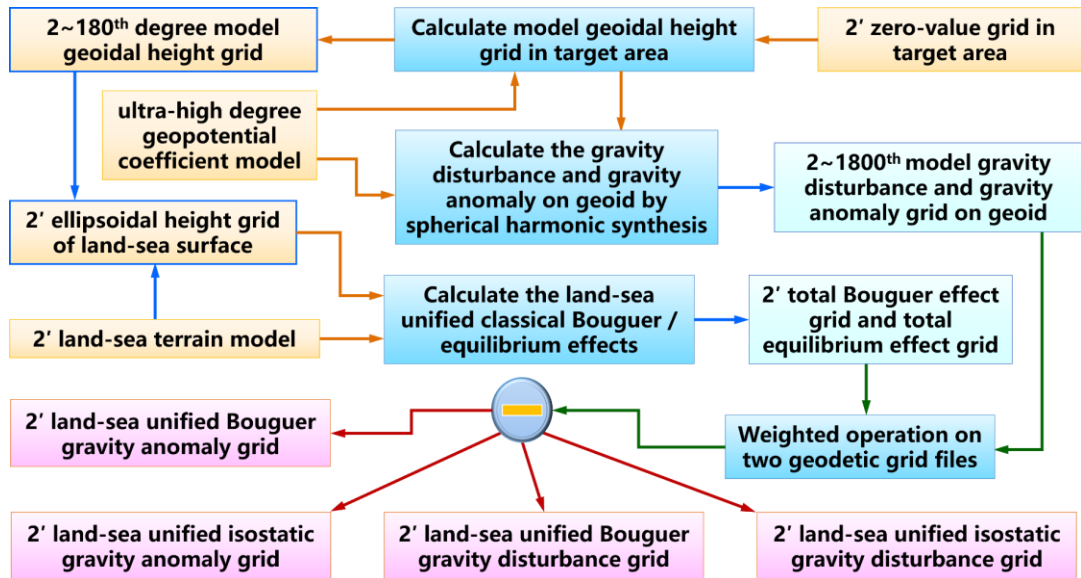
3.8.2 Computation process demo of land-sea Bouguer / equilibrium anomaly from geopotential model

From the Earth geopotential coefficient model and land-sea terrain model, the classical Bouguer gravity anomaly (disturbance) and isostatic gravity anomaly (disturbance) are calculated synchronously in four steps in any region of the world to demonstrate the fast and convenient computation process of the land-sea unified classical Bouguer / isostatic anomaly.

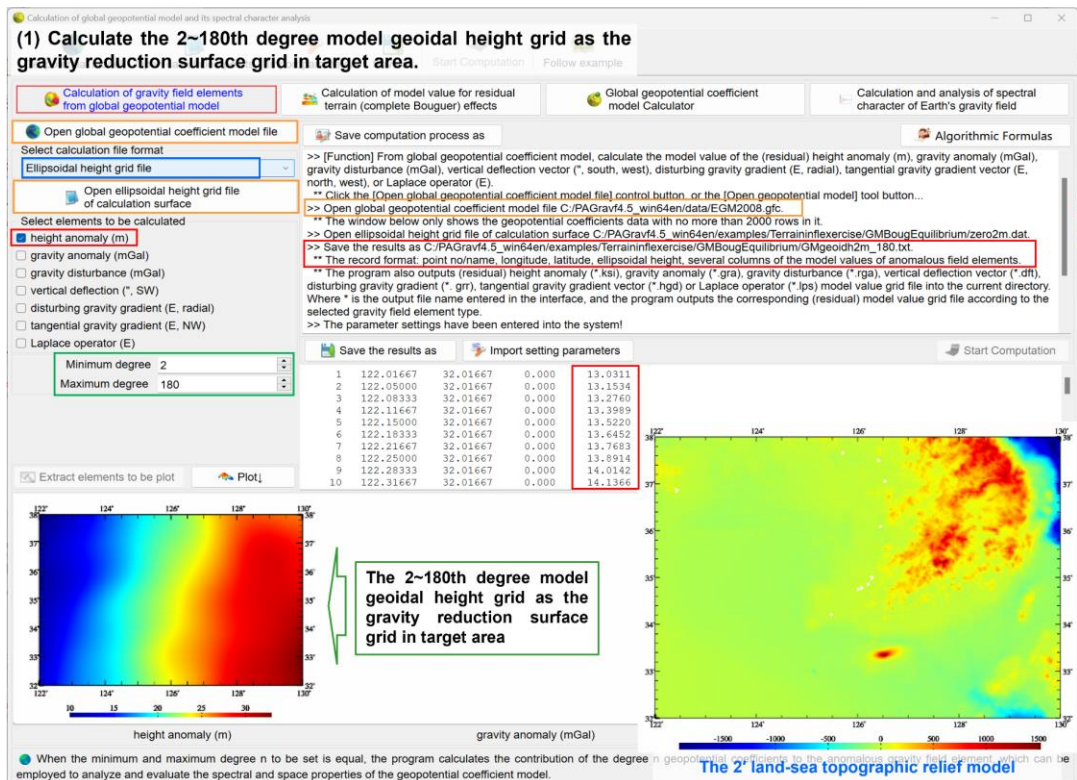
(1) Calculate the 2~180th degree model geoidal height grid as the gravity reduction surface grid in target area.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and the maximum degree 180, input the file `EGM2008.gfc` and the zero-value grid file `zero2m.dat` of the target area, and select the calculation type 'height anomaly', to generate 2'×2' model geoidal height grid file

GMgeoidh2m_180.ksi.



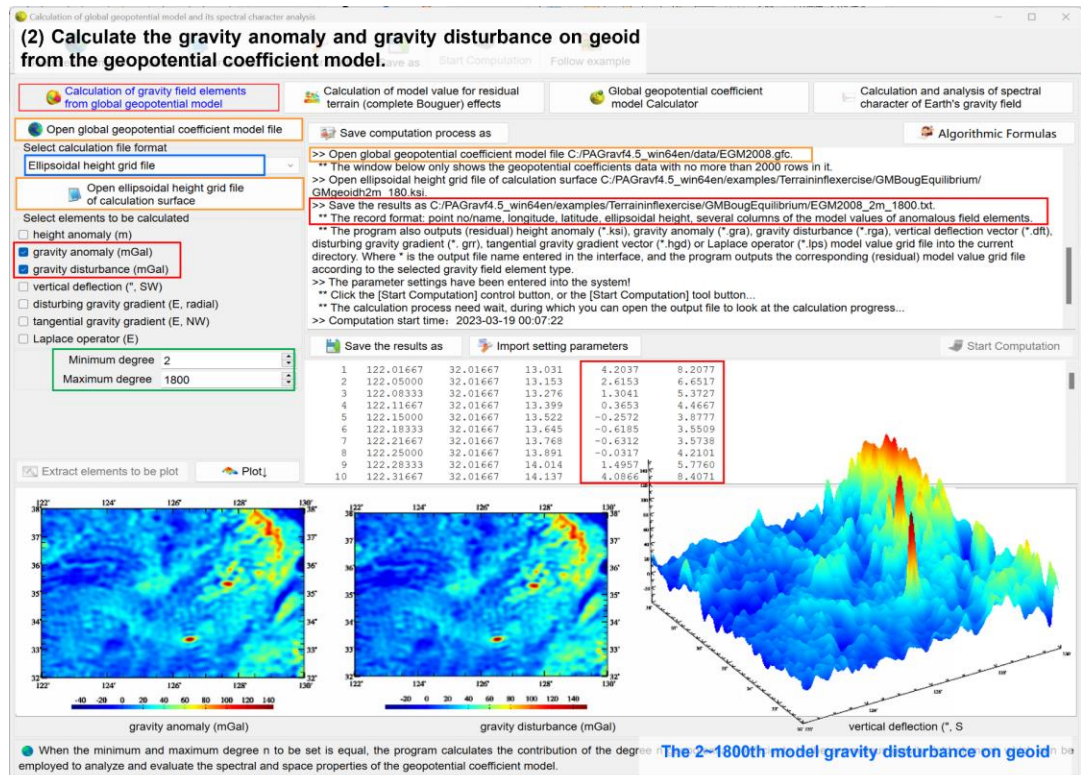
Computation process demo of land-sea Bouguer/equilibrium anomaly from geopotential model



The 2~180th degree model geoidal height grid here is employed as the reduction surface and location for the classical Bouguer / equilibrium anomaly.

(2) Calculate the gravity anomaly and gravity disturbance on geoid from the geopotential coefficient model.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and the maximum degree 1800, input the file EGM2008.gfc and model geoidal height grid file GMgeoidh2m_180.ksi, and select the calculation type 'gravity anomaly' and 'gravity disturbance', to generate the 2'×2' gravity anomaly grid file EGM2008_2m_1800.gra and gravity disturbance grid file EGM2008_2m_1800.rga in the target area.



(3) Calculate the total Bouguer effects and total equilibrium effects on gravity.

Call the function [Computation of land-sea unified classical gravity Bouguer / equilibrium effect], input the land-sea terrain model file extlandseadtm2m.dat and land-sea surface ellipsoidal height grid file extlandseahgt2m.dat, and set the land integral radius 90km, sea integral radius 200km and equilibrium compensation depth 30km to generate the 2'×2' total Bouguer effect grid file BougEquinfl2m.bgr and total equilibrium effect grid file BougEquinfl2m.ist.

Because the normal gravity field has nothing to do with the terrain effect, the Bouguer / equilibrium effect on the gravity anomaly, gravity disturbance and gravity is equal everywhere and does not need to be distinguished.

(4) Generate the 2'×2' land-sea unified classical Bouguer / isostatic anomaly grid model.

(3) Calculate the total Bouguer effects and total equilibrium effects on gravity.

port parameters Save as Start Computation Save process Follow example

Integral of land-sea unified classical gravity Bouguer / equilibrium effect Calculator of land-sea unified classical gravity Bouguer / equilibrium effect Algorithms land-sea unified classical Bouguer and equilibrium effects

Open the land-sea terrain model file
Open the ellipsoidal height grid file of land-sea surface
Select calculation points file format
ellipsoidal height grid file
Open the ellipsoidal height grid file on land-sea calculation surface

Integral radius for local terrain effect 90 km
Integral radius for seawater Bouguer / equilibrium effect 300 km
Equilibrium compensation depth 30 km

Save computation process as

Computation Process ** Operation Prompts

>> Open the land-sea terrain model file C:\PAGrav4.5_win64en/examples/TerrainInflexercise/GMBougEquilibrium/xtlandseadm2m.dat.
>> Open the ellipsoidal height grid file of land-sea surface C:\PAGrav4.5_win64en/examples/TerrainInflexercise/GMBougEquilibrium/xtlandseahg2m.dat.
>> Open the ellipsoidal height grid file on land-sea calculation surface C:\PAGrav4.5_win64en/examples/TerrainInflexercise/GMBougEquilibrium/xtlandseahg2m.dat.
>> Save the results as C:\PAGrav4.5_win64en/examples/TerrainInflexercise/GMBougEquilibrium/BougEquinft2m.txt
** Record format: Point no, longitude, latitude, ellipsoidal height, terrain height/sea depth, local terrain effect, plane layer effect, seawater Bouguer effect, land equilibrium effect, ocean equilibrium effect, total Bouguer effect and total equilibrium effect.
** At the same time, the program also outputs the land-sea total Bouguer effect (*.bgr) and land-sea total equilibrium effect (*.ist) grid file into the current directory, where * is the output file name entered from the interface.

Save the results as Import setting parameters Start Computation

no	lon(deg/decimal)	lat	height/depth	local terrain	plane layer	sea-water Bouguer effect	...
1	121.01667	30.01667	43.360	-0.0930	4.8550	-0.0052	-0.5258 0.0729
2	121.05000	30.01667	20.550	-0.0329	2.3010	-0.0053	-0.5820 0.0774
3	121.08333	30.01667	45.640	-0.1658	5.1102	-0.0056	-0.6299 0.0821
4	121.11667	30.01667	7.880	-0.0164	0.8823	-0.0057	-0.6957 0.0870
5	121.15000	30.01667	6.400	-0.0072	0.7166	-0.0058	-0.7545 0.0922
6	121.18333	30.01667	5.000	-0.0311	0.5598	-0.0060	-0.8137 0.0977

Extract effects Plot

land-sea terrain model (m) total Bouguer effect (mGal) total equilibrium effect (mGal)

Classic Bouguer gravity anomaly on geoid = gravity anomaly at the measurement point - total Bouguer effect - analytical continuation of gravity anomaly from the measurement point to the geoid. Classic Bouguer gravity disturbance on geoid = gravity disturbance at the measurement point - total Bouguer effect - analytical continuation of gravity disturbance from the measurement point to the geoid.
Classic equilibrium gravity anomaly on geoid = gravity anomaly at the measurement point - total equilibrium effect - analytical continuation of gravity anomaly from the measurement point to the geoid. Classic equilibrium gravity disturbance on geoid = gravity disturbance at the measurement point - total equilibrium effect - analytical continuation of gravity disturbance from the measurement point to the geoid.

(4) Generate the 2'x2' land-sea unified classical Bouguer / isostatic anomaly grid model.

computation Save process Follow example

Weighted operation on two specified attributes in record file Weighted operation on two geotid grid files Weighted operation on two vector grid files Weighted operation on two harmonic coefficient files

Open geotid grid file 1
Open geotid grid file 2
Select operation mode
Plus +
Set weight
The first weight 1.00
The second weight 1.00
Vector grid operation

Program Process ** Operation Prompts

>> Select the function module from the four control buttons at the top of the interface...
>> [Function] Perform weighted plus, minus, or multiply operation on grid elements in two (vector) grid files with the same specifications.
>> Open geotid grid file 1 C:\PAGrav4.5_win64en/examples/TerrainInflexercise/GMBougEquilibrium/EGM2008_2m_1800.gra.
>> Open geotid grid file 2 C:\PAGrav4.5_win64en/examples/TerrainInflexercise/GMBougEquilibrium/BougEquinft2m0.bgr.
>> Save the results as C:\PAGrav4.5_win64en/examples/TerrainInflexercise/GMBougEquilibrium/Istbgravanom2m.dat.
>> The parameter settings have been entered into the system!

Click the [Start] button
>> Computation s
>> Complete the t
>> Computation e
>> Open geotid
>> Save the result
>> The parameter
Click the [Start] button
>> Computation s
>> Complete the t
>> Computation e

Save the res

Display of the input-output file:

122.000000	130.000000	32.000000	38.000000	0.03333333	0.0
8.3775	6.8966	5.6899	4.8540	4.3325	14.7983
19.2133	17.4019	15.8291	14.7983	14.7983	14.7983
24.3218	26.4713	28.0518	28.6086	27.1	27.1
12.9158	10.3765	10.0539	11.7974	14.7	14.7
15.3356	13.0657	11.4755	11.0485	11.7	11.7
22.3411	22.1708	21.4036	20.3436	19.7	19.7
12.8327	12.3341	11.1957	10.0041	9.7	9.7
22.0979	22.6739	23.1295	23.1236	22.7	22.7
23.9972	26.5997	28.6183	30.3440	31.7	31.7
27.7904	27.6035	26.8185	26.0919	25.7	25.7
36.7723	32.2543	27.9404	25.2177	24.7	24.7
23.0243	24.8093	26.4821	28.4630	31.7	31.7
60.4889	61.1129	60.1925	58.4199	56.7	56.7
52.1456	47.6924	45.4768	45.1435	45.7	45.7
9.4949	11.3991	12.7729	13.7259	14.7	14.7
0.4051	-4.5601	-10.9319	-16.3954	-17.7	-17.7
14.7698	13.3947	11.4517	9.3666	7.7	7.7
22.1548	20.7063	19.4075	18.5611	18.7	18.7
22.9153	25.1115	26.7739	27.3524	26.7	26.7
16.1274	12.9445	12.2021	13.7359	16.7	16.7
16.1741	13.6392	11.3091	9.8888	9.5055	9.5055

The 2' land-sea unified classical Bouguer gravity anomaly and disturbance

The 2' land-sea unified classical isostatic gravity anomaly and disturbance

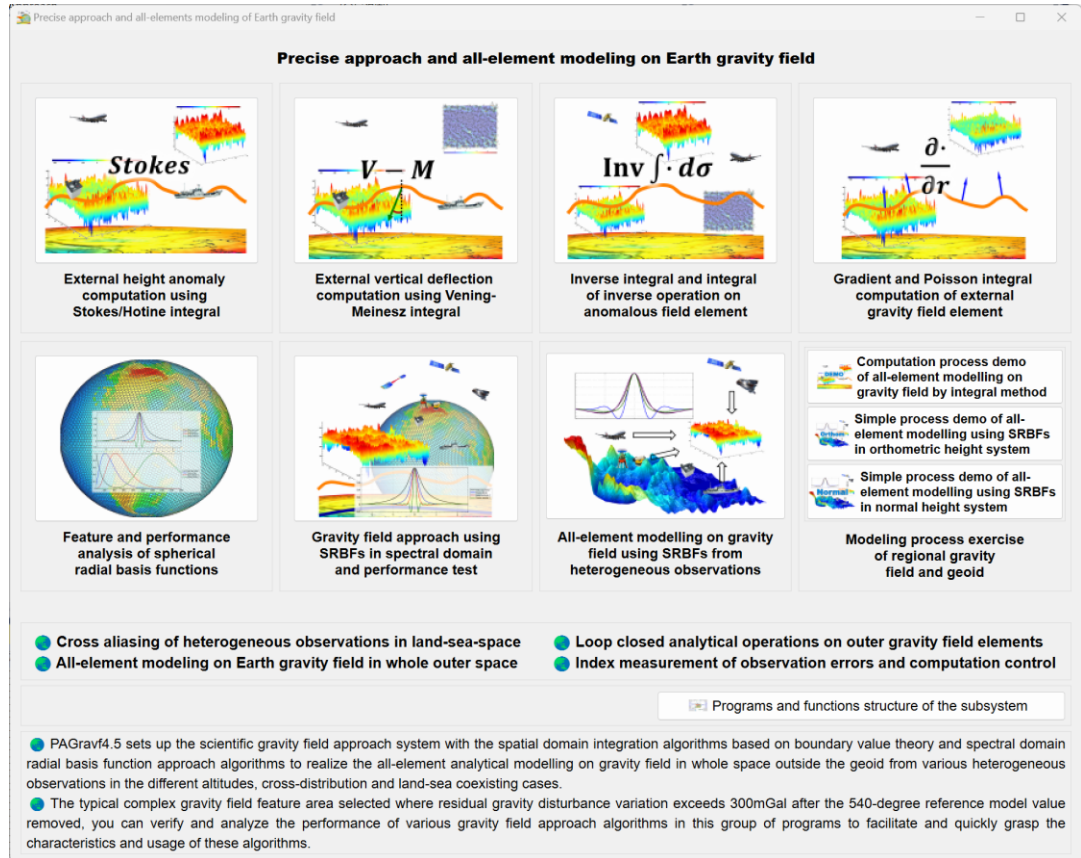
Subtract the gravity anomaly grid EGM2008_2m_1800.gra and gravity disturbance

grid EGM2008_2m_1800.rga on geoid from the total Bouguer effect grid (the grid edge removed) BougEquinfl2m0.bgr respectively to get the classical Bouguer gravity anomaly grid model Clsbggravanom2m.dat and classical Bouguer gravity disturbance grid model Clsbgdistgrav2m.dat.

Subtract the gravity anomaly grid EGM2008_2m_1800.gra and gravity disturbance grid EGM2008_2m_1800.rga on geoid from the total isostatic effect grid (the grid edge removed) BougEquinfl2m0.ist respectively to get the classical isostatic gravity anomaly grid model Istbggravanom2m.dat and classical isostatic gravity disturbance grid model Istbgdistgrav2m.dat.

4 Precise approach and all-element modelling on Earth gravity field

PAGravf4.5 sets up the scientific gravity field approach system with the spatial domain integration algorithms based on boundary value theory and spectral domain radial basis function approach algorithms to realize the all-element analytical modelling on gravity field in whole space on or outside the geoid from various heterogeneous observations in the different altitudes, cross-distribution and land-sea coexisting cases.



The typical complex gravity field feature area selected where residual gravity disturbance variation exceeds 300mGal after the 540-degree reference model value removed, you can verify and analyze the performance of various gravity field approach algorithms in this group of programs to facilitate and quickly grasp the characteristics and usage of these algorithms.

4.1 External height anomaly computation using Stokes/Hotine integral

[Purpose] Using the generalized Stokes/Hotine rigorous numerical integral or FFT algorithm, from the ellipsoidal height grid of the equipotential surface and gravity anomaly or disturbance (mGal) grid on the surface, compute the height anomaly (m) on or outside the geoid.

Height anomaly on the geoid is equal to the geoid undulation, that is, the geoidal (ellipsoidal) height.

The Stokes boundary value theory requires that the boundary surface should be an equipotential surface, that is, the gravity anomaly/disturbance should be on the equipotential surface.

It is usually necessary to employ the remove-restore scheme with a reference geopotential model to use the finite radius for gravity field integral. Firstly, remove model gravity anomaly/disturbance on the boundary surface, then integrate to obtain the residual height anomaly at the calculation point, and finally restore the model height anomaly at the calculation point.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

4.1.1 External height anomaly computation using generalized Stokes integral

[Function] From the ellipsoidal height grid of the equipotential surface and gravity anomaly (mGal) grid on the surface, compute the external residual height anomaly (m) by the Stokes integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and the residual gravity anomaly grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

External height anomaly computation using Stokes/Hotine integral

Import parameters Save as Start Computation Save process Follow example

External height anomaly computation using generalized Stokes integral

Open the ellipsoidal height grid file of the equipotential surface

Open the residual gravity anomaly grid file on the equipotential surface

Select calculation point file format
discrete calculation point file

Open the calculation point position file

Set input point file format
number of rows of file header 1
column ordinal number of ellipsoidal height in the record 4

Integral radius 180 km

no lon(degree/decimal) lat ellipHeight(m)

1	97.008333	33.008333	3942.764	-0.0294
2	97.025000	33.008333	3989.787	-0.0340
3	97.041667	33.008333	4034.817	-0.0404
4	97.058333	33.008333	4070.847	-0.0485
5	97.075000	33.008333	4106.877	-0.0582
6	97.091667	33.008333	4119.913	-0.0693
7	97.108333	33.008333	4115.946	-0.0817
8	97.125000	33.008333	4090.977	-0.0952
9	97.141667	33.008333	4070.007	-0.1090
10	97.158333	33.008333	3991.047	-0.1235
11	97.175000	33.008333	3985.070	-0.1362
12	97.191667	33.008333	3956.107	-0.1475
13	97.208333	33.008333	3965.137	-0.1552
14	97.225000	33.008333	3964.173	-0.1592
15	97.241667	33.008333	3983.205	-0.1581
16	97.258333	33.008333	3953.251	-0.1526

Stokes boundary value theory requires that the boundary surface should be an equipotential surface, that is, the gravity anomaly/disturbance should be on the equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

The ellipsoidal height grid of the equipotential surface stands for the space position

of the equipotential surface, employed to calculate the integral distance.

External height anomaly computation using Stokes/Hotine integral

Import parameters Save as Start Computation Save process Follow example

External height anomaly computation using generalized Stokes integral

Open the ellipsoidal height grid file of the equipotential surface

Open the residual gravity anomaly grid file on the equipotential surface

Select calculation point file format

ellipsoidal height grid file

Open the ellipsoidal height grid file of the calculation surface

Select integral algorithm

numerical integral

Save computation process as

Computation Process ** Operation Prompts

>> Complete the computation of the height anomaly outside the geoid!

>> [Function] From the ellipsoidal height grid of the equipotential surface and gravity anomaly (mGal) grid on the surface, compute the external residual height anomaly (m) by the Stokes integral.

>> Open the ellipsoidal height grid file of the equipotential surface C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landgeoidhgt.dat.

>> Open residual gravity anomaly grid file on equipotential surface C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/resGMgeoid541_1800.gra.

>> Open the ellipsoidal height grid file of the calculation surface C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landbmsurfhtg.dat.

>> Compute external residual height anomaly by numerical integral...

>> Save the results as C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/stokesnintq.dat.

>> The parameter settings have been entered into the system!

>> Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Complete the computation of the height anomaly outside the geoid!

>> Computation start time: 2024-09-23 11:04:56

>> Computation end time: 2024-09-23 11:34:09

Integral radius 180 km

Save the results as Import setting parameters Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667	-0.1641
-0.0985	-0.0918	-0.0929	-0.1025	-0.1184	-0.1397	-0.4160
-0.3691	-0.3790	-0.3864	-0.3924	-0.3988	-0.4062	-0.9039
-0.7265	-0.7631	-0.8020	-0.8354	-0.8626	-0.8929	-0.8092
-1.0120	-1.0246	-1.0271	-1.0019	-0.9528	-0.8930	-0.8092
0.1001	0.1635	0.2066	0.2352	0.2495	0.2417	0.2169
-0.2054	-0.3400	-0.3818	-0.4330	-0.4706	-0.4958	-0.5033
0.1690	0.2420	0.2994	0.3331	0.3416	0.3352	0.3029
-0.2588	-0.2453	-0.2122	-0.1673	-0.1158	-0.0692	-0.0300
-0.0422	-0.0655	-0.0880	-0.1094	-0.1292	-0.1468	-0.1629
-0.2292	-0.2297	-0.2287	-0.2252	-0.2187	-0.2084	-0.1941
0.0855	0.0932	0.0876	0.0708	0.0422	0.0045	-0.0408
-0.2274	-0.1866	-0.1405	-0.0919	-0.0407	0.0079	0.0533
0.4118	0.4248	0.4271	0.4192	0.3997	0.3698	0.3315
0.0589	0.0536	0.0500	0.0468	0.0435	0.0399	0.0360
0.1401	0.1539	0.1599	0.1580	0.1466	0.1258	0.0957

Stokes boundary value theory requires that the boundary surface should be an equipotential surface, that is, the gravity anomaly/disturbance should be on the equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

gravity anomaly (mGal) height anomaly (m)

External height anomaly computation using Stokes/Hotine integral

Import parameters Save as Start Computation Save process Follow example

External height anomaly computation using generalized Stokes integral

Open the ellipsoidal height grid file of the equipotential surface

Open the residual gravity anomaly grid file on the equipotential surface

Select calculation point file format

ellipsoidal height grid file

Open the ellipsoidal height grid file of the calculation surface

Select integral algorithm

2D FFT algorithm

Save computation process as

Computation Process ** Operation Prompts

resGMgeoid541_1800.gra.

>> Open the ellipsoidal height grid file of the calculation surface C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landbmsurfhtg.dat.

>> Compute external residual height anomaly by numerical integral...

>> Save the results as C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/stokesnintq.dat.

>> The parameter settings have been entered into the system!

>> Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Complete the computation of the height anomaly outside the geoid!

>> Computation start time: 2024-09-23 11:04:56

>> Computation end time: 2024-09-23 11:34:09

>> Compute external residual height anomaly by 2D FFT algorithm...

>> Save the results as C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/stokesFFT2.dat.

>> The parameter settings have been entered into the system!

>> Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Complete the computation of the height anomaly outside the geoid!

>> Computation start time: 2024-09-23 11:39:25

>> Computation end time: 2024-09-23 11:39:27

Integral radius 180 km

Save the results as Import setting parameters Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667	-0.1667
-0.0801	-0.0775	-0.0825	-0.0952	-0.1146	-0.1390	-0.4362
-0.3914	-0.4036	-0.4126	-0.4191	-0.4241	-0.4292	-0.8971
-0.7545	-0.7992	-0.8366	-0.8651	-0.8842	-0.8944	-0.7000
-0.8988	-0.8660	-0.8037	-0.8596	-0.8213	-0.7479	-0.1697
0.0897	0.1378	0.1713	0.1904	0.1958	0.1884	-0.4217
-0.2694	-0.3182	-0.3601	-0.3932	-0.4158	-0.4257	0.2763
0.1558	0.2243	0.2769	0.3098	0.3208	0.3092	-0.0479
-0.2183	-0.2149	-0.1962	-0.1656	-0.1276	-0.0868	-0.1642
-0.0505	-0.0730	-0.0945	-0.1144	-0.1326	-0.1492	-0.1861
-0.2305	-0.2288	-0.2252	-0.2196	-0.2115	-0.2005	-0.0306
-0.0563	-0.0666	-0.0703	-0.0607	-0.0397	0.0086	0.0978
-0.2157	-0.1802	-0.1347	-0.0816	-0.0234	0.0371	0.2862
0.3944	0.3889	0.3769	0.3592	0.3374	0.3126	0.0116
0.0878	0.0701	0.0532	0.0379	0.0251	0.0160	0.0807
0.1639	0.1731	0.1734	0.1640	0.1448	0.1166	

Stokes boundary value theory requires that the boundary surface should be an equipotential surface, that is, the gravity anomaly/disturbance should be on the equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

gravity anomaly (mGal) height anomaly (m)

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m),

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The residual height anomaly result file.

When the discrete calculation point file input, the output file record format: Behind the source calculation point file record, appends a column of residual height anomaly calculated, keeps 4 significant figures.

When the ellipsoidal height grid file of the calculation surface input, the program outputs the residual height anomaly grid file with the same grid specification as the input grid file.

In this example, the residual ground height anomaly is calculated by the Stokes integral from the residual gravity anomaly on the geoid, and the integral radius is 180km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual height anomalies (regarded as the true reference value), and the difference between the results of the Stokes integral and model reference value, which can be employed to examine the performance of the algorithm.

The true reference model value (m)	mean	standard deviation	minimum	maximum
541~1800 th degree ground height anomalies	0.0010	0.1182	-0.6745	0.4760

Differences between integral and reference value (m)	mean	standard deviation	minimum	maximum
numerical integral	-0.0002	0.0324	-0.1159	0.1211
FFT2	0.0018	0.0326	-0.1150	0.1280
FFT1	0.0018	0.0327	-0.1178	0.1251

Furtherly, statistically analyze the differences between the results of Stokes numerical integral and FFT algorithm.

Differences between algorithms (m)	mean	standard deviation	minimum	maximum
FFT1- numerical integral	0.0021	0.0059	-0.0150	0.0683
FFT2- FFT1	0.0021	0.0059	-0.0150	0.0683

4.1.2 External height anomaly computation using generalized Hotine integral

[Function] From the ellipsoidal height grid of the equipotential surface and residual gravity disturbance (mGal) grid on the surface, compute the external residual height anomaly (m) by the Hotine integral.

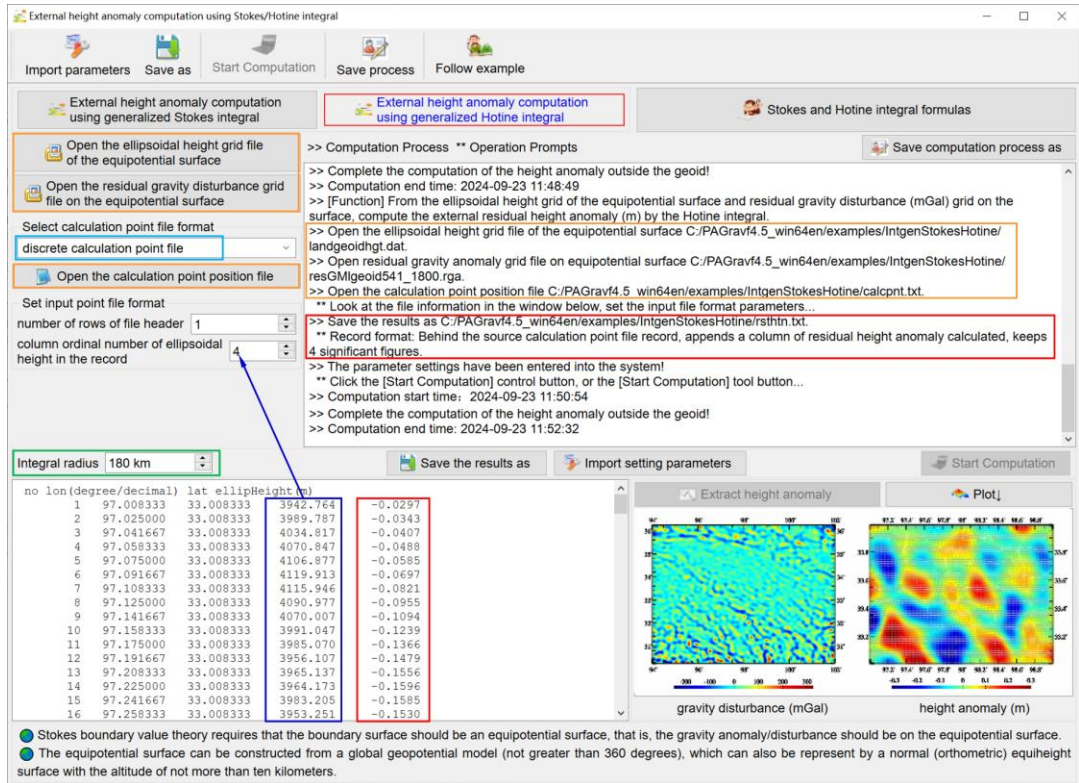
[Input files] The ellipsoidal height grid file of the equipotential surface and the residual gravity disturbance grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation

surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The residual height anomaly result file.



When the discrete calculation point file input, the output file record format: Behind the source calculation point file record, appends a column of residual height anomaly calculated, keeps 4 significant figures.

When the ellipsoidal height grid file of the calculation surface input, the program outputs the residual height anomaly grid file with the same grid specification as the input grid file.

As the Stokes integral algorithm analysis example, statistically analyze the difference between the results of the Hotine integral and model reference value, which can be employed to examine the performance of the algorithm.

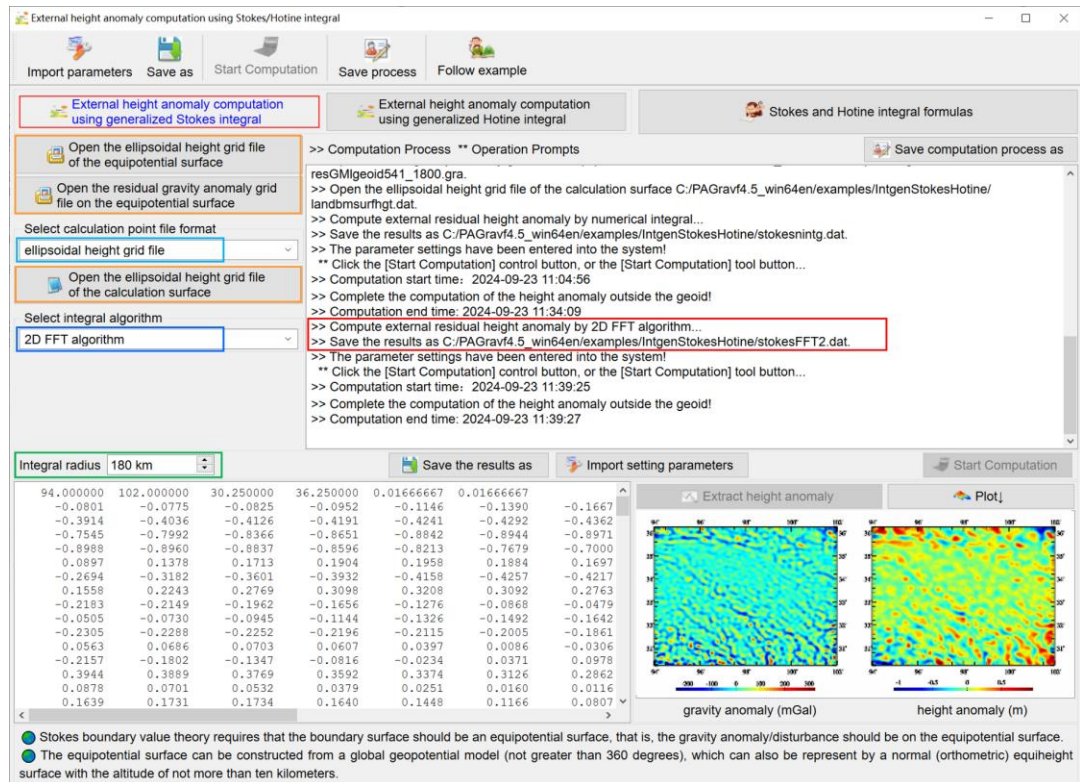
Differences between integral and reference value (m)	mean	standard deviation	minimum	maximum
numerical integral	-0.0001	0.0256	-0.0915	0.0957
FFT2	0.0019	0.0259	-0.0911	0.1065
FFT1	0.0018	0.0261	-0.0934	0.1036

Furtherly, statistically analyze the differences between the results of the Stokes and

Hotine integral.

Stokes-Hotine(m)	mean	standard deviation	minimum	maximum
numerical integral	0.0020	0.0090	-0.0264	0.0658
FFT2	-0.0000	0.0067	-0.0239	0.0258
FFT1	-0.0000	0.0068	-0.0245	0.0250

This example shows that the performance of the Hotine integral is slightly better than that of the Stokes integral.



4.2 External vertical deflection computation using Vening-Meinesz integral

[Purpose] Using the generalized Vening-Meinesz rigorous numerical integral or FFT algorithm, compute the vertical deflection (η , SW, to south, to west) on or outside the geoid from the ellipsoidal height grid of the equipotential surface and its gravity anomaly or disturbance (mGal) grid.

The generalized Vening-Meinesz formula is derived from the generalized Stokes/Hotine formula and belongs to the solution of the Stokes boundary value problem. Which requires the integrand gravity anomaly/disturbance to be on the equipotential surface.

4.2.1 Computation of external vertical deflection from gravity anomaly

[Function] From the ellipsoidal height grid of the equipotential surface and residual

gravity anomaly (mGal) grid on the surface, compute the external residual vertical deflection (" , SW, to south, to west) by the generalized Vening-Meinesz integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and the residual gravity anomaly grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m),

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The residual vertical deflection result file.

When the discrete calculation point file input, the output file record format Behind the source calculation point file record, appends two columns of residual vertical deflection southward and westward calculated, keeps 4 significant figures.

When the ellipsoidal height grid file of the calculation surface input, the program outputs the residual vertical deflection vector grid file with the same grid specification as the input grid file.

External height anomaly computation using Stokes/Hotine integral

Import parameters Save as Start Computation Save process Follow example

External height anomaly computation using generalized Stokes integral External height anomaly computation using generalized Hotine integral Stokes and Hotine integral formulas

Open the ellipsoidal height grid file of the equipotential surface
Open the residual gravity disturbance grid file on the equipotential surface
Select calculation point file format
discrete calculation point file
Open the calculation point position file
Set input point file format
number of rows of file header 1
column ordinal number of ellipsoidal height in the record 4
Integral radius 180 km

Computation Process ** Operation Prompts

>> Complete the computation of the height anomaly outside the geoid!
>> Computation end time: 2024-09-23 11:48:49
>> [Function] From the ellipsoidal height grid of the equipotential surface and residual gravity disturbance (mGal) grid on the surface, compute the external residual height anomaly (m) by the Hotine integral.
>> Open the ellipsoidal height grid file of the equipotential surface C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landgeoidht.dat.
>> Open residual gravity anomaly grid file on equipotential surface C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/resGMgeoid541_1800.rga.
>> Open the calculation point position file C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/calcpnt.txt.
>> Look at the file information in the window below, set the input file format parameters...
>> Save the results as C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/rsthtn.txt.
** Record format: Behind the source calculation point file record, appends a column of residual height anomaly calculated, keeps 4 significant figures.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2024-09-23 11:50:54
>> Complete the computation of the height anomaly outside the geoid!
>> Computation end time: 2024-09-23 11:52:32

Save the results as Import setting parameters Start Computation

no	lon(degree/decimal)	lat	ellipHeight(m)	
1	97.008333	33.008333	3942.764	-0.0297
2	97.025000	33.008333	3989.787	-0.0343
3	97.041667	33.008333	4034.817	-0.0407
4	97.058333	33.008333	4070.847	-0.0488
5	97.075000	33.008333	4106.877	-0.0585
6	97.091667	33.008333	4119.913	-0.0697
7	97.108333	33.008333	4115.946	-0.0821
8	97.125000	33.008333	4090.977	-0.0955
9	97.141667	33.008333	4070.007	-0.1094
10	97.158333	33.008333	3991.047	-0.1239
11	97.175000	33.008333	3985.070	-0.1366
12	97.191667	33.008333	3956.107	-0.1479
13	97.208333	33.008333	3965.137	-0.1556
14	97.225000	33.008333	3964.173	-0.1596
15	97.241667	33.008333	3983.205	-0.1585
16	97.258333	33.008333	3953.251	-0.1530

Extract height anomaly Plot

gravity disturbance (mGal) height anomaly (m)

Stokes boundary value theory requires that the boundary surface should be an equipotential surface, that is, the gravity anomaly/disturbance should be on the equipotential surface.
The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

In this example, the residual ground vertical deflection is calculated by the generalized Vening-Meinesz integral from the residual gravity anomaly on the geoid,

and the integral radius is 180km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual vertical deflections (regarded as the true reference value), and the difference between the results of the Vening-Meinesz integral and model reference value, which can be employed to examine the performance of the algorithm.

The true reference model value (")	mean	standard deviation	minimum	maximum
Ground vertical deflection (S)	0.0014	2.4951	-12.7789	14.2346
Ground vertical deflection (W)	0.0097	2.1772	-9.1577	10.4499

Differences between integral and reference value (m)		mean	standard deviation	minimum	maximum
Numerical integral	S"	0.0003	0.0380	-0.1061	0.1387
	W"	0.0011	0.0289	-0.0830	0.1091
FFT2	S"	0.0002	0.1107	-0.8974	0.7395
	W"	0.0011	0.1003	-0.6887	1.0743
FFT1	S"	0.0007	0.1090	-0.8189	0.6581
	W"	0.0011	0.0984	-0.6078	1.0866

External height anomaly computation using Stokes/Hotine integral

Import parameters Save as Start Computation Save process Follow example

External height anomaly computation using generalized Stokes integral

External height anomaly computation using generalized Hotine integral

Stokes and Hotine integral formulas

Open the ellipsoidal height grid file of the equipotential surface

Open the residual gravity disturbance grid file on the equipotential surface

Select calculation point file format

ellipsoidal height grid file

Open the ellipsoidal height grid file of the calculation surface

Select integral algorithm

1D FFT algorithm

Computation Process ** Operation Prompts

Save computation process as

>> Computation start time: 2024-09-23 11:54:42

>> Complete the computation of the height anomaly outside the geoid!

>> Computation end time: 2024-09-23 12:24:27

>> Compute external residual height anomaly by 2D FFT algorithm...

>> Save the results as C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/HotineFFT2.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-23 12:45:58

>> Complete the computation of the height anomaly outside the geoid!

>> Computation end time: 2024-09-23 12:45:59

>> Compute external residual height anomaly by 1D FFT algorithm...

>> Save the results as C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/HotineFFT1.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-23 12:48:07

>> Complete the computation of the height anomaly outside the geoid!

>> Computation end time: 2024-09-23 12:48:33

Integral radius 180 km

Save the results as Import setting parameters Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667	-0.1605	-0.1605	-0.1605	-0.1605
-0.0955	-0.0983	-0.0985	-0.0969	-0.1127	-0.1345	-0.4236	-0.4236	-0.4236	-0.4236
-0.3743	-0.3849	-0.3932	-0.4000	-0.4065	-0.4139	-0.8824	-0.8824	-0.8824	-0.8824
-0.7228	-0.7642	-0.8007	-0.8311	-0.8547	-0.8715	-0.6824	-0.6824	-0.6824	-0.6824
-0.8943	-0.8844	-0.8664	-0.8384	-0.7988	-0.7467	-0.1880	-0.1880	-0.1880	-0.1880
-0.0943	-0.1478	-0.1862	-0.2091	-0.2163	-0.2088	-0.4489	-0.4489	-0.4489	-0.4489
-0.2832	-0.3327	-0.3759	-0.4109	-0.4360	-0.4493	-0.3167	-0.3167	-0.3167	-0.3167
-0.1640	-0.2435	-0.3057	-0.3460	-0.3617	-0.3515	-0.0304	-0.0304	-0.0304	-0.0304
-0.2180	-0.2103	-0.1968	-0.1517	-0.1102	-0.0684	-0.1646	-0.1646	-0.1646	-0.1646
-0.0416	-0.0654	-0.0884	-0.1102	-0.1302	-0.1483	-0.1932	-0.1932	-0.1932	-0.1932
-0.2327	-0.2331	-0.2316	-0.2278	-0.2207	-0.2095	-0.0406	-0.0406	-0.0406	-0.0406
-0.0938	-0.1024	-0.0977	-0.0794	-0.0485	-0.0074	-0.0716	-0.0716	-0.0716	-0.0716
-0.2011	-0.1642	-0.1211	-0.0743	-0.0257	-0.0232	-0.3303	-0.3303	-0.3303	-0.3303
-0.4191	-0.4302	-0.4307	-0.4201	-0.3998	-0.3682	-0.0363	-0.0363	-0.0363	-0.0363
-0.0570	-0.0513	-0.0475	-0.0446	-0.0418	-0.0388	-0.1013	-0.1013	-0.1013	-0.1013
-0.1424	-0.1576	-0.1660	-0.1655	-0.1547	-0.1332				

Stokes boundary value theory requires that the boundary surface should be an equipotential surface, that is, the gravity anomaly/disturbance should be on the equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

4.2.2 Computation of external vertical deflection from gravity disturbance

[Function] From the ellipsoidal height grid of the equipotential surface and residual

gravity disturbance (mGal) grid on the surface, compute the external residual vertical deflection (" , SW, to south, to west) by the generalized Vening-Meinesz integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and the residual gravity disturbance grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

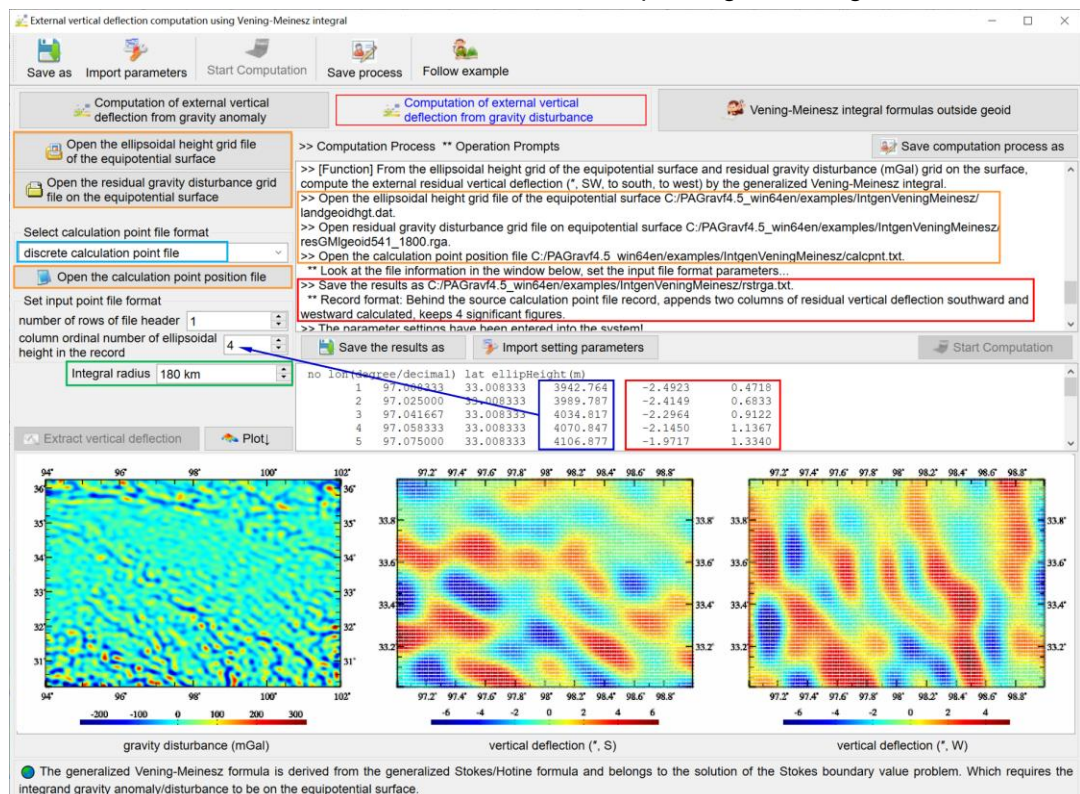
The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m),

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The residual vertical deflection result file.

When the discrete calculation point file input, the output file record format Behind the source calculation point file record, appends two columns of residual vertical deflection southward and westward calculated, keeps 4 significant figures.



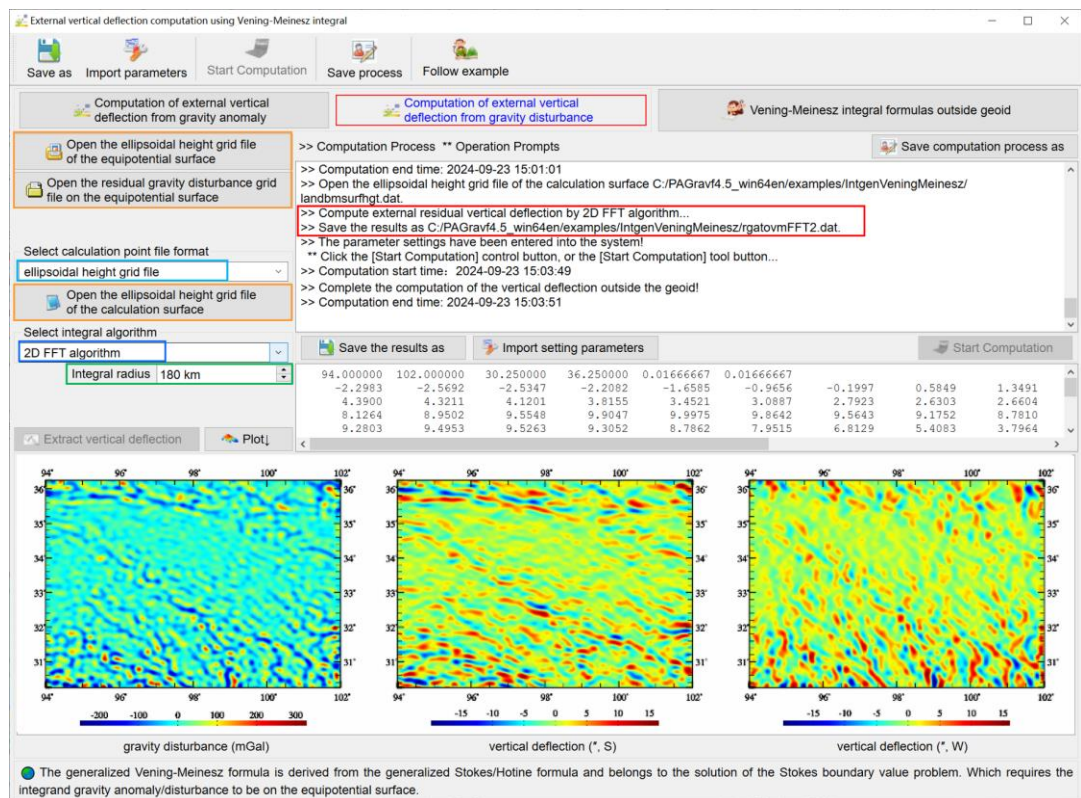
When the ellipsoidal height grid file of the calculation surface input, the program outputs the residual vertical deflection vector grid file with the same grid specification as the input grid file.

Similarly, statistically analyze the difference between the results of integral and model reference value, which can be employed to examine the performance of the

algorithm.

Differences between integral and reference value		mean	standard deviation	minimum	maximum
Numerical integral	S''	-0.0003	0.0274	-0.0872	0.0994
	W''	0.0005	0.0200	-0.0646	0.0836
FFT2	S''	0.0002	0.1072	-0.8692	0.7094
	W''	0.0010	0.0959	-0.6204	1.0981
FFT1	S''	0.0007	0.1058	-0.7910	0.6481
	W''	0.0010	0.0959	-0.6204	1.0981

Furtherly, statistically analyze the differences between the results of Vening-Meinesz integral from gravity anomaly and gravity disturbance.



Differences from gravity anomaly and disturbance		mean	standard deviation	minimum	maximum
Numerical integral	S''	-0.0000	0.0062	-0.0286	0.0329
	W''	0.0003	0.0059	-0.0206	0.0261
FFT2	S''	-0.0000	0.0062	-0.0286	0.0329
	W''	0.0001	0.0050	-0.0189	0.0244
FFT1	S''	-0.0000	0.0062	-0.0284	0.0325
	W''	0.0001	0.0050	-0.0189	0.0241

4.3 Inverse integral and integral of inverse operation on anomalous field element

[Purpose] Using the inverse integral or integral of inverse operation method, compute the anomalous gravity field element from the ellipsoidal height grid of the equipotential surface and height anomaly (m) or vertical deflection vector (") grid on the surface.

The integral of inverse operation formula belongs to the solution of the Stokes boundary value problem, which requires the integrand height anomaly or vertical deflection to be on the equipotential surface.

The inverse integral adopts the combination algorithm with Poisson integral and differentiation of anomalous field element, which does not require that the boundary surface should be a gravity equipotential surface.

It is usually necessary to employ the remove-restore scheme with a reference geopotential model to use the finite radius for gravity field integral. Firstly, remove the model values of source field element on the boundary surface, then compute the residual values of target field element at the calculation point by inverse integral or integral of inverse operation, and finally restore the model values of target field element at the calculation point.

4.3.1 Computation of gravity anomaly by the inverse Stokes integral

[Function] From the ellipsoidal height grid of the equipotential boundary surface and residual height anomaly (m) grid on the surface, compute the residual gravity anomaly (mGal) on the equipotential surface by the inverse Stokes integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and height anomaly grid file on the surface with the same grid specifications, and the calculation point position file on the surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, which is employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude,

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

When the ellipsoidal height grid file selected, the program let the ellipsoidal height grid of the equipotential surface as the calculation surface.

[Output file] The residual gravity anomaly result file.

When the discrete calculation point file input, the output file record format: Behind the input calculation point file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and a column of integral value of the residual gravity anomaly.

When the ellipsoidal height grid file selected, the program outputs the residual vertical deflection vector grid file with the same grid specification as the input grid file.

Inverse integral and integral of inverse operation on anomalous field element

Save as Import parameters Start Computation Save process Follow example

Computation of gravity anomaly by the inverse Stokes integral Computation of gravity disturbance by the inverse Hotine integral Computation of the inverse Vening Meinesz integral Computation of anomalous field elements from height anomaly Integral formula of inverse operation

Open the ellipsoid height grid file of the equipotential surface
Open the height anomaly grid file on the equipotential surface
Select calculation point file format
discrete calculation point file
Open the calculation point file on the equipotential surface
Set input point file format
number of rows of file header 1

Integral radius 150 km

no lon(degree/decimal) lat ellipHeight(m)

1	97.008333	33.008333	3942.764	-37.2501	24.7224
2	97.025000	33.008333	3989.787	-37.2203	24.6842
3	97.041667	33.008333	4034.817	-37.1899	22.9058
4	97.058333	33.008333	4070.847	-37.1590	19.2599
5	97.075000	33.008333	4106.877	-37.1276	13.9076
6	97.091667	33.008333	4119.913	-37.0959	7.1243
7	97.108333	33.008333	4115.946	-37.0640	-0.9416
8	97.125000	33.008333	4090.977	-37.0318	-9.7023
9	97.141667	33.008333	4070.007	-36.9990	-18.9075
10	97.158333	33.008333	3991.047	-36.9665	-27.8771
11	97.175000	33.008333	3985.070	-36.9327	-36.2732
12	97.191667	33.008333	3956.107	-36.8988	-43.4193
13	97.208333	33.008333	3965.137	-36.8642	-49.0686
14	97.225000	33.008333	3964.173	-36.8295	-52.4761
15	97.241667	33.008333	3953.205	-36.7943	-53.5072
16	97.258333	33.008333	3953.251	-36.7595	-51.6556
17	97.275000	33.008333	4016.279	-36.7238	-46.8428
18	97.291667	33.008333	4054.318	-36.6883	-39.1123
19	97.308333	33.008333	4090.360	-36.6528	-28.6690

Ignore the ellipsoid height

height anomaly (m) gravity anomaly (mGal)

The integral of inverse operation formula belongs to the solution of the Stokes boundary value problem, which requires the integrand height anomaly or vertical deflection to be on the equipotential surface.
The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

Inverse integral and integral of inverse operation on anomalous field element

Save as Import parameters Start Computation Save process Follow example

Computation of gravity anomaly by the inverse Stokes integral Computation of gravity disturbance by the inverse Hotine integral Computation of the inverse Vening Meinesz integral Computation of anomalous field elements from height anomaly Integral formula of inverse operation

Open the ellipsoid height grid file of the equipotential surface
Open the height anomaly grid file on the equipotential surface
Select calculation point file format
ellipsoid height grid file
Select integral algorithm
numerical integral

Integral radius 150 km

no lon(degree/decimal) lat ellipHeight(m)

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
-6.7082	-3.5451	-0.1911	3.0391	6.6519	10.3730
36.7580	35.8638	35.2066	34.1624	32.4529	32.1184
43.8609	47.1039	49.4559	52.2163	55.5724	56.0212
4.5287	-8.9848	-23.7249	-30.1206	-31.7801	-39.6205
5.7532	11.3936	12.7210	14.9511	18.9953	19.5816
28.0887	27.6742	24.2507	20.7547	12.5472	2.4261
24.3313	32.0133	41.5405	46.6957	45.8446	49.2392
-58.8703	-52.9381	-43.6688	-32.7235	-18.4676	-7.3091
13.7746	11.6956	10.1260	9.3898	9.3222	10.0556
28.0073	27.1485	25.4340	22.3360	18.5072	14.2898
17.4349	22.5400	26.0993	29.6066	31.5147	32.3806
-12.9644	-15.9979	-19.6681	-24.2073	-28.0318	-33.9067
-60.1987	-59.1808	-55.8066	-52.7477	-48.8362	-44.7585
5.6019	9.5282	9.6246	8.9249	6.3753	3.6689
4.2451	11.7662	18.6738	25.5670	31.5455	36.9296
26.1350	18.6380	10.9680	3.6098	-7.6106	-18.3577
-30.1325	-19.4527	-10.3495	-2.6276	3.8415	9.0357
13.3615	14.0104	14.6722	15.2175	15.2282	14.6317

height anomaly (m) gravity anomaly (mGal)

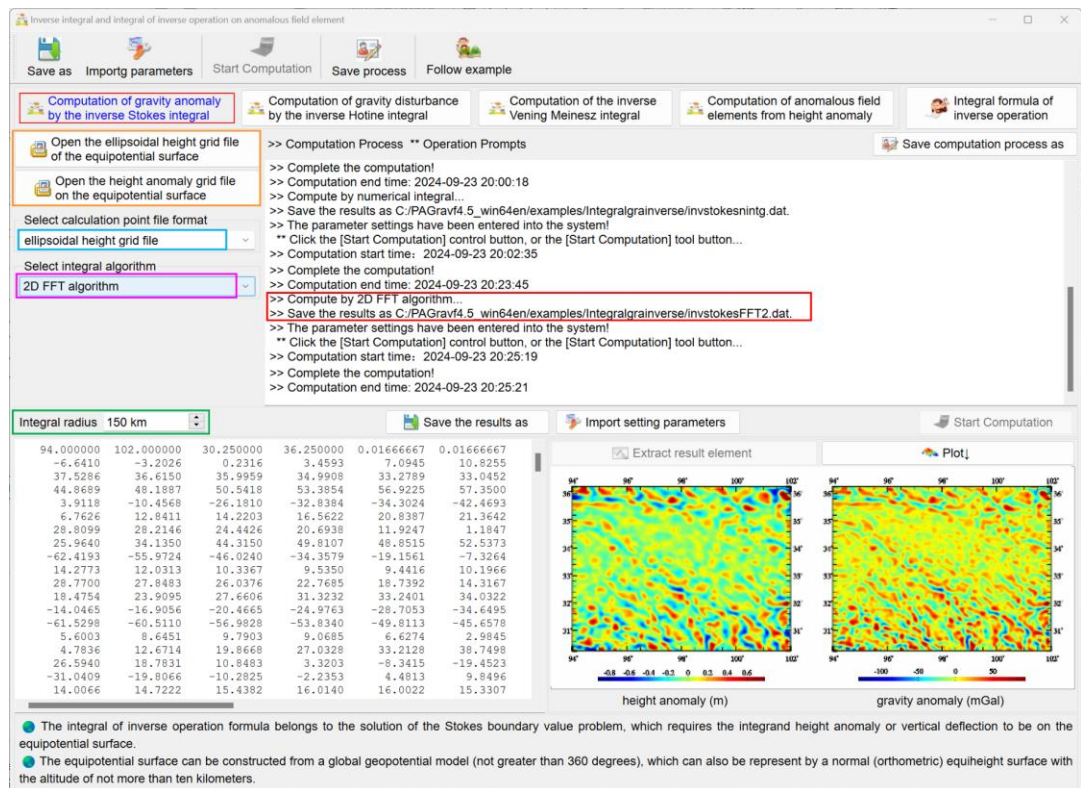
The integral of inverse operation formula belongs to the solution of the Stokes boundary value problem, which requires the integrand height anomaly or vertical deflection to be on the equipotential surface.
The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

In this example, the residual gravity anomaly on the geoid is calculated by the

inverse Stokes integral from the residual height anomaly on the geoid, and the integral radius is 150km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual gravity anomalies (regarded as the true reference value) on the geoid, and the differences between the results of the integral and model reference values, which can be employed to examine the performance of the algorithm.

The true reference model value (mGal)	mean	standard deviation	minimum	maximum
541~1800th degree residual gravity anomalies	0.4334	34.0852	-170.9556	177.8780

Differences between integral and reference value (mGal)	mean	standard deviation	minimum	maximum
Numerical integral	-0.0415	2.9285	-15.2734	14.6970
FFT2	-0.0433	2.8208	-11.9201	13.3607
FFT1	-0.0444	3.0509	-15.7950	15.5242



4.3.2 Computation of gravity disturbance by the inverse Hotine integral

[Function] From the ellipsoidal height grid of the equipotential boundary surface and residual height anomaly (m) grid on the surface, compute the residual gravity disturbance on the surface by the inverse Hotine integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and height

anomaly grid file on the surface with the same grid specifications, and the calculation point position file on the surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, which is employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude,

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

When the ellipsoidal height grid file selected, the program let the ellipsoidal height grid of the equipotential surface as the calculation surface.

[Output file] The residual gravity disturbance result file.

When the discrete calculation point file input, the output file record format: Behind the input calculation point file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and a column of integral value of the residual gravity disturbance.

When calculated on the calculation surface, the program outputs the residual vertical deflection vector grid file with the same grid specification as the input grid file.

Inverse integral and integral of inverse operation on anomalous field element

Save as Import parameters Start Computation Save process Follow example

Computation of gravity anomaly by the inverse Stokes integral Computation of gravity disturbance by the inverse Hotine integral Computation of the inverse Vening Meinesz integral Computation of anomalous field elements from height anomaly Integral formula of inverse operation

Save computation process as

Operation Prompts

>> [Function] From the ellipsoidal height grid of the equipotential boundary surface and residual height anomaly (m) grid on the surface, compute the residual gravity disturbance on the surface by the inverse Hotine integral.

Input the ellipsoidal height grid file of the equipotential surface and height anomaly grid file on the surface with the same grid specification...

>> Open the ellipsoidal height grid file of the equipotential surface C:/PAGrav4.5_win64en/examples/integralgrainverse/landgeoidhgt.dat.

>> Open the height anomaly grid file on the equipotential surface C:/PAGrav4.5_win64en/examples/integralgrainverse/resGMlandbm541_1800.ksl.

>> Open the calculation point file on the equipotential surface C:/PAGrav4.5_win64en/examples/integralgrainverse/calcpnt.txt.

>> Look at the file information in the window below, set the input file format parameters...

>> Save the results as C:/PAGrav4.5_win64en/examples/integralgrainverse/invhfn.txt.

>> Behind the input calculation point file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and a column of integral value of the residual gravity disturbance.

>> The parameter settings have been entered into the system!

>> Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-23 20:30:46

>> Complete the computation!

>> Computation end time: 2024-09-23 20:31:41

Integral radius 150 km Save the results as Import setting parameters Start Computation

no	lon(degree/decimal)	lat	ellipHeight(m)		
1	97.008333	33.008333	3942.764	-37.2501	6.2142
2	97.025000	33.008333	3989.787	-37.2203	5.8380
3	97.041667	33.008333	4034.817	-37.1899	4.9738
4	97.058333	33.008333	4070.847	-37.1590	3.6134
5	97.075000	33.008333	4106.877	-37.1276	1.6840
6	97.091667	33.008333	4119.913	-37.0959	-0.6950
7	97.108333	33.008333	4115.946	-37.0640	-3.4436
8	97.125000	33.008333	4090.977	-37.0318	-6.5390
9	97.141667	33.008333	4070.007	-36.9990	-9.6002
10	97.158333	33.008333	3991.047	-36.9665	-13.3681
11	97.175000	33.008333	3985.070	-36.9327	-16.1738
12	97.191667	33.008333	3956.107	-36.8988	-19.0284
13	97.208333	33.008333	3965.137	-36.8642	-20.6898
14	97.225000	33.008333	3964.173	-36.8295	-21.9527
15	97.241667	33.008333	3983.205	-36.7943	-21.8169
16	97.258333	33.008333	3953.251	-36.7595	-21.7681
17	97.275000	33.008333	4016.279	-36.7238	-18.7898
18	97.291667	33.008333	4054.318	-36.6883	-15.3468
19	97.308333	33.008333	4090.360	-36.6528	-10.9020

Ignore the ellipsoidal height

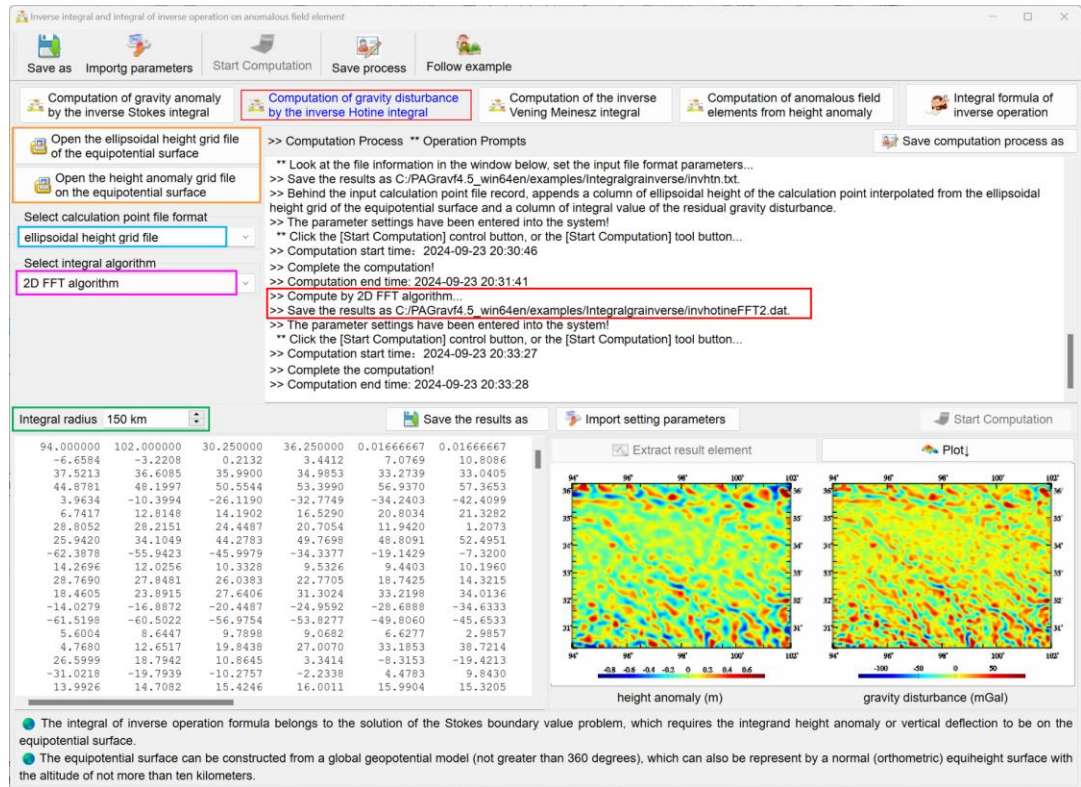
height anomaly (m) gravity disturbance (mGal)

1. The integral of inverse operation formula belongs to the solution of the Stokes boundary value problem, which requires the integrand height anomaly or vertical deflection to be on the equipotential surface.

2. The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

In this example, the residual gravity disturbance on the geoid is calculated by Hotine integral of inverse operation from the residual height anomaly on the geoid, and the integral radius is 150km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual gravity

disturbances (regarded as the true reference value) on the geoid, and the differences between the results of the integral and model reference values, which can be employed to examine the performance of the algorithm.



The true reference model value (mGal)	mean	standard deviation	minimum	maximum
541~1800th degree residual gravity disturbances	0.4348	34.1479	-171.3088	178.1561

Differences between integral and reference value (mGal)	mean	standard deviation	minimum	maximum
Numerical integral	0.1277	2.9277	-9.6849	9.6267
FFT2	-0.0451	2.8982	-12.2656	13.7990
FFT1	0.1326	2.9671	-9.8383	9.7441

4.3.3 Computation of the inverse Vening Meinesz integral

[Function] From the ellipsoidal height grid of the equipotential boundary surface and residual vertical deflection vector (" , SW) grid on the surface, compute the residual height anomaly, gravity anomaly and gravity disturbance on the surface by the inverse Vening Meinesz integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and vertical deflection vector grid file on the surface with the same grid specifications, and the calculation point position file on the surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, which is employed to calculate the integral distance.

The record format of the discrete calculation point file: point no / point name, longitude (decimal degrees), latitude,

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

When the ellipsoidal height grid file selected, the program let the ellipsoidal height grid of the equipotential surface as the calculation surface.

[Output file] The inverse Vening Meinesz integral result file.

Integral radius 150 km

Save the results as

Plot

height anomaly (m)

gravity disturbance (mGal)

point no	point name	longitude (decimal degrees)	latitude	ellipsoidal height (m)	residual height anomaly (m)	gravity anomaly (m/s²)	gravity disturbance (mGal)
3333	33.008333	3942.764	-37.2501	0.0986	23.0001	22.9698	
5000	33.008333	3989.787	-37.2203	0.0846	22.9339	22.9078	
1667	33.008333	4034.817	-37.1899	0.0639	21.1547	21.1350	
3333	33.008333	4070.847	-37.1590	0.0366	17.6405	17.6293	
5000	33.008333	4106.877	-37.1276	0.0033	12.4966	12.4956	
1667	33.008333	4119.913	-37.0959	-0.0351	5.9362	5.9470	
3333	33.008333	4115.946	-37.0640	-0.0772	-1.7394	-1.7156	
5000	33.008333	4090.977	-37.0318	-0.1213	-10.1584	-10.1211	
1667	33.008333	4070.007	-36.9990	-0.1655	-18.9011	-18.8502	
3333	33.008333	3991.047	-36.9665	-0.2077	-27.5122	-27.4484	
5000	33.008333	3985.070	-36.9327	-0.2458	-35.5120	-35.4365	
1667	33.008333	3956.107	-36.8988	-0.2773	-42.4147	-42.3294	
3333	33.008333	3965.137	-36.8642	-0.2999	-47.7421	-47.6499	
5000	33.008333	3964.173	-36.8295	-0.3115	-51.0471	-50.9514	
1667	33.008333	3983.205	-36.7943	-0.3100	-51.9444	-51.8491	
3333	33.008333	3953.251	-36.7595	-0.2941	-50.1432	-50.0528	
5000	33.008333	4016.279	-36.7238	-0.2629	-45.4879	-45.4071	
1667	33.008333	4054.318	-36.6883	-0.2165	-37.9888	-37.9222	

Ignore the ellipsoidal height

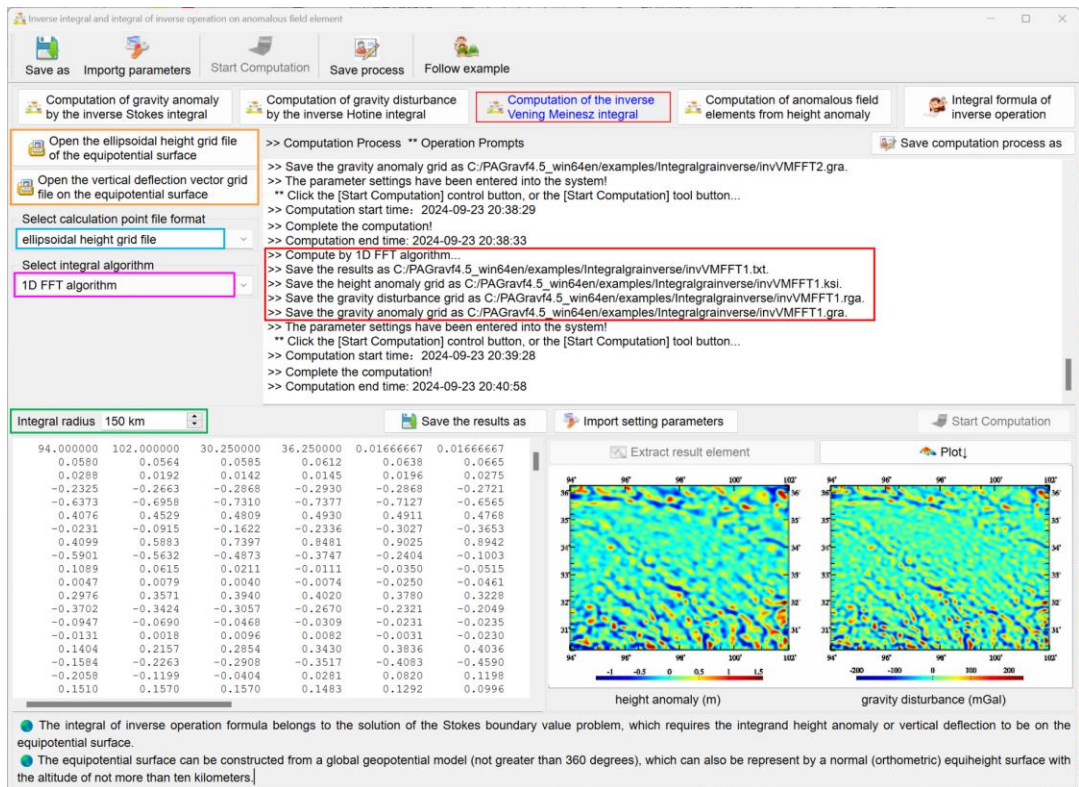
The integral of inverse operation formula belongs to the solution of the Stokes boundary value problem, which requires the integrand height anomaly or vertical deflection to be on the equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represented by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

When the discrete calculation point file input, the output file record format: Behind the input calculation point file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and 3 columns of attributes including the residual height anomaly, gravity anomaly and gravity disturbance.

When the ellipsoidal height grid file selected, the output file record format: point no/name, longitude, latitude, ellipsoidal height, residual height anomaly, gravity anomaly and gravity disturbance.

The program also outputs (residual) height anomaly (*.ksi), gravity disturbance (*.rga) and gravity anomaly (*.gra) grid files into the current directory, where * is the output file name entered from the interface.



In this example, the residual geoidal height, gravity anomaly and gravity disturbance on the geoid are calculated by the inverse Vening Meinesz integral from the residual vertical deflection vector on the geoid, and the integral radius is 150km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual geoidal heights, gravity anomalies and gravity disturbances (regarded as the true reference value) on the geoid, and the differences between the results of the integral and model reference values, which can be employed to examine the performance of the algorithm.

The true reference model value	mean	standard deviation	minimum	maximum
residual geoidal heights (m)	0.0045	0.2172	-1.1490	0.9110
residual gravity disturbances (mGal)	0.4348	34.1479	-171.3088	178.1561
residual gravity anomalies (mGal)	0.4334	34.0852	-170.9556	177.8780

Differences between integral and reference value		mean	standard deviation	minimum	maximum
Numerical integral	residual geoidal height (m)	-0.0002	0.0331	-0.1066	0.1266
	residual gravity disturbances (mGal)	-0.0281	3.5304	-19.8306	15.5799
	residual gravity anomalies (mGal)	-0.0280	3.5293	-19.8233	15.5701

FFT2	residual geoidal height (m)	0.0001	0.0337	-0.1057	0.1271
	residual gravity disturbances (mGal)	-0.0131	2.4732	-12.3026	9.8919
	residual gravity anomalies (mGal)	-0.0131	2.4729	-12.2973	9.8831
FFT1	residual geoidal height (m)	-0.0001	0.0332	-0.1068	0.1263
	residual gravity disturbances (mGal)	-0.0283	3.5527	-19.9907	15.7164
	residual gravity anomalies (mGal)	-0.0283	3.5516	-19.9830	15.7072

4.3.4 Computation of external anomalous gravity field elements from height anomaly

[Function] From the ellipsoidal height grid of the boundary surface and residual height anomaly grid (m) on the surface, compute the residual gravity anomaly (mGal), gravity disturbance (mGal) and vertical deflection vector (" , SW) on or outside the geoid. The inverse operation of height anomaly adopts the combination algorithm with Poisson integral and differentiation, which does not require that the boundary surface should be a gravity equipotential surface.

[Input files] The ellipsoidal height grid file of the boundary surface and height anomaly grid file on the surface with the same grid specifications, and the calculation point position file or ellipsoidal height grid file of the calculation surface.

The ellipsoidal height grid of the boundary surface stands for the space position of the boundary surface, which employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m),

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The external anomalous field elements result file.

When the discrete calculation point file input, the output file record format: Behind the input calculation point file record, appends 4 columns of attributes including residual gravity anomaly, residual gravity disturbance and residual vertical deflection southward and westward.

When the ellipsoidal height grid file selected, the output file record format: point no/name, longitude, latitude, ellipsoidal height, residual gravity anomaly, residual gravity disturbance and residual vertical deflection southward and westward.

The program also outputs (residual) gravity anomaly (*.gra), gravity disturbance (*.rga) and vertical deflection vector(*.dft) grid files into the current directory, where * is the output file name entered from the interface.

Inverse integral and integral of inverse operation on anomalous field element

Save as Import parameters Start Computation Save process Follow example

Computation of gravity anomaly by the inverse Stokes integral Computation of gravity disturbance by the inverse Hotine integral Computation of the inverse Vening Meinesz integral **Computation of anomalous field elements from height anomaly** Integral formula of inverse operation

Open the ellipsoidal height grid file of the boundary surface
Open the height anomaly grid file on the boundary surface
Select calculation point file format
discrete calculation point file
Open the calculation point position file
Set input point file format
number of rows of file header 1

>> Computation Process ** Operation Prompts

[Function] From the ellipsoidal height grid of the boundary surface and residual height anomaly grid (m) on the surface, compute the residual gravity anomaly (mGal), gravity disturbance (mGal) and vertical deflection vector (η , ξ) on or outside the geoid. The inverse operation of height anomaly adopts the combination algorithm with Poisson integral and differentiation, which does not require that the boundary surface should be a gravity equipotential surface.
** Input the ellipsoidal height grid of the boundary surface and height anomaly grid file on the surface with the same grid specification...
Open the ellipsoidal height grid file of the boundary surface C:/PA/Grav4.5_win64en/examples/Integralgrainverse/landgeoidht.dat.
Open the height anomaly grid file on the boundary surface C:/PA/Grav4.5_win64en/examples/Integralgrainverse/resGMgeoid541_1800.ksl.
Open the calculation point position file C:/PA/Grav4.5_win64en/examples/Integralgrainverse/calcpnt.txt.
** Look at the file information in the window below, set the input file format parameters...
Save the results as C:/PA/Grav4.5_win64en/examples/Integralgrainverse/invksioter.txt.
Behind the input calculation point file record, appends 4 columns of attributes including residual gravity anomaly, residual gravity disturbance and residual vertical deflection southward and westward.
The parameter settings have been entered into the system!
Click the [Start Computation] control button, or the [Start Computation] tool button...
Computation start time: 2024-09-23 20:46:10
Complete the computation!
Computation end time: 2024-09-23 20:54:09

Integral radius 150 km Save the results as Import setting parameters Start Computation

Extract result element Plot

gravity disturbance (mGal) vertical deflection S (η)

imal)	lat	ellipHeight (m)	2.6212	6.2592	-2.5007	0.4847
3333	33.008333	3942.764	6.0388	6.0365	-2.4248	0.6987
5000	33.008333	3989.787	5.2179	5.2151	-2.3076	0.9304
1667	33.008333	4034.817	3.8190	3.8156	-2.1573	1.1571
3333	33.008333	4070.847	1.8631	1.8588	-1.9846	1.3566
5000	33.008333	4106.877	-0.5421	-0.5472	-1.8034	1.5177
1667	33.008333	4119.913	-3.3350	-3.3411	-1.6284	1.6250
3333	33.008333	4115.946	-6.4301	-6.4372	-1.4752	1.6670
5000	33.008333	4090.977	-9.6925	-9.7007	-1.3639	1.6181
1667	33.008333	4070.007	-13.0955	-13.1048	-1.3008	1.4939
3333	33.008333	3991.047	-16.1623	-16.1724	-1.2320	1.2209
5000	33.008333	3985.070	-18.9050	-18.9158	-1.4222	0.8360
1667	33.008333	3956.107	-20.8767	-20.8879	-1.6097	0.3179
3333	33.008333	3965.137	-22.0755	-22.0867	-1.8711	-0.3092
5000	33.008333	3964.173	-22.1895	-22.2002	-2.1941	-1.0304
1667	33.008333	3989.205	-21.4492	-21.4591	-2.5624	-1.8234
3333	33.008333	3953.251	-19.0783	-19.0868	-2.9355	-2.6206
5000	33.008333	4016.279	-15.7055	-15.7122	-3.2973	-3.3943
1667	33.008333	4054.318				

The integral of inverse operation formula belongs to the solution of the Stokes boundary value problem, which requires the integrand height anomaly or vertical deflection to be on the equipotential surface.
The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

Inverse integral and integral of inverse operation on anomalous field element

Save as Import parameters Start Computation Save process Follow example

Computation of gravity anomaly by the inverse Stokes integral Computation of gravity disturbance by the inverse Hotine integral Computation of the inverse Vening Meinesz integral **Computation of anomalous field elements from height anomaly** Integral formula of inverse operation

Open the ellipsoidal height grid file of the boundary surface
Open the height anomaly grid file on the boundary surface
Select calculation point file format
ellipsoidal height grid file
Open the ellipsoidal height grid file of the calculation surface

>> Computation Process ** Operation Prompts

and residual vertical deflection southward and westward.
The parameter settings have been entered into the system!
Click the [Start Computation] control button, or the [Start Computation] tool button...
Computation start time: 2024-09-23 20:46:10
Complete the computation!
Computation end time: 2024-09-23 20:54:09
Open the ellipsoidal height grid file of the calculation surface C:/PA/Grav4.5_win64en/examples/Integralgrainverse/landmsurfhtg.dat.
Save the results as C:/PA/Grav4.5_win64en/examples/Integralgrainverse/surfgravfd.dat.
Save the gravity anomaly grid as C:/PA/Grav4.5_win64en/examples/Integralgrainverse/surfgravfd.gra.
Save the gravity disturbance grid as C:/PA/Grav4.5_win64en/examples/Integralgrainverse/surfgravfd.rga.
Save vertical deflection vector grid as C:/PA/Grav4.5_win64en/examples/Integralgrainverse/surfgravfd.dff.
The parameter settings have been entered into the system!
Click the [Start Computation] control button, or the [Start Computation] tool button...
Computation start time: 2024-09-23 21:01:44
Complete the computation!
Computation end time: 2024-09-23 21:13:21

Integral radius 150 km Save the results as Import setting parameters Start Computation

Extract result element Plot

gravity disturbance (mGal) vertical deflection S (η)

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
11.4128	13.0112	13.5037	13.3752	12.9300	12.3297
12.2286	12.9126	13.7712	14.4755	14.8649	15.6350
-0.5715	-4.2236	-6.0262	-6.7989	-6.5922	-4.3403
-37.2156	-44.7613	-51.0366	-54.3760	-54.4024	-52.2276
27.4578	31.0473	33.1427	34.1140	34.0444	32.8437
-0.8411	-5.0104	-9.3308	-14.1952	-19.0398	-23.9456
28.3579	38.0082	45.8366	50.5764	51.8258	51.0678
-41.3629	-38.2614	-31.7155	-23.1777	-13.4434	-4.5923
7.0217	4.5984	2.5959	1.1491	0.2233	-0.2655
5.2099	5.1750	4.9190	4.0159	2.8697	1.6782
23.9381	27.0111	28.5656	28.6058	26.8087	23.3214
-30.2443	-29.8047	-28.8801	-27.9116	-26.9503	-26.7764
-14.4973	-12.8509	-10.7049	-9.1703	-7.6347	-6.3731
-0.6123	-0.1993	-0.4046	-1.2041	-2.8020	-4.6923
10.4837	15.6332	19.9038	23.1644	25.2077	26.0062
-9.3028	-13.0914	-16.8779	-20.5369	-24.4111	-28.3535
-14.6855	-8.5257	-3.2954	0.8040	3.7755	5.6646
6.8612	8.0384	8.9141	9.2855	8.9443	7.8440

The integral of inverse operation formula belongs to the solution of the Stokes boundary value problem, which requires the integrand height anomaly or vertical deflection to be on the equipotential surface.
The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

In this example, the ground residual gravity anomaly, gravity disturbance and

vertical deflection are computed from the residual geoidal height, and the integral radius is 150km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual gravity anomalies, residual gravity disturbances and residual vertical deflections (regarded as the true reference value) on the ground, and the differences between the results of the integral and model reference values, which can be employed to examine the performance of the algorithm.

The 541 to 1800 th degree true reference model value	mean	standard deviation	minimum	maximum
Ground residual gravity anomalies (mGal)	-0.0349	15.7184	-93.7784	66.5507
Ground residual gravity disturbances (mGal)	-0.0346	15.7527	-93.9854	66.6638
Ground vertical deflection (" , S)	0.0014	2.4951	-12.7789	14.2346
Ground vertical deflection (" , W)	0.0097	2.1772	-9.1577	10.4499

Differences between integral and reference value	mean	standard deviation	minimum	maximum
Ground residual gravity anomalies (mGal)	-0.0104	2.0577	-8.2097	10.7064
Ground residual gravity disturbances (mGal)	-0.0100	2.0929	-8.3217	10.8982
Ground vertical deflection (" , S)	0.0004	0.0075	-0.0357	0.0388
Ground vertical deflection (" , W)	0.0003	0.0076	-0.0362	0.0358

4.4 Gradient and Poisson integral computation of external gravity field element

[Purpose] Using rigorous numerical integral method, carry out the radial gradient integral, integral inverse, inverse operation integral and Poisson integral operation on the anomalous gravity field element.

The radial gradient integral algorithm of the anomalous field element is derived from the solution of the Stokes boundary value problem, which requires the integrand field elements to be on the equipotential surface. The Poisson integral is the solution of the first boundary value problem in the mathematical sense, and the boundary surface is not required to be an equipotential surface.

4.4.1 Operation of radial gradient integral on anomalous gravity field element

[Function] From the ellipsoidal height grid of the equipotential boundary surface and anomalous gravity field element grid on the surface, compute the radial gradient (/km) of the field element on the surface by the numerical integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and anomalous field element grid file on the surface with the same grid specifications, and the calculation point position file on the surface.

The ellipsoidal height grid of the equipotential surface stands for the space position

of the equipotential surface, employed to calculate the integral distance.

Gradient and Poisson integral computation of external gravity field element

Save as Import parameters Start Computation Save process Follow example

Operation of radial gradient integral on anomalous gravity field element

Computation of external gravity disturbance from disturbing gravity gradient

Computation of disturbing gravity gradient by inverse operation integral

Computation of external disturbing gravity gradient from gravity disturbance

Computation of Poisson integral on external anomalous field element

Open the ellipsoidal height grid file of the boundary surface

Open the anomalous field element grid file on the boundary surface

Select calculation point file format

discrete calculation point file

Open the calculation points file on the equipotential surface

Set input point file format

number of rows of file header 1

Integration radius 120 km

Save the results as

Import setting parameters

Start Computation

Extraction of radial gradient of element

Plot

anomalous field element on surface

radial gradient (/km) of element

no lon (degree/decimal) lat ellipHeight (m)

1	97.008333	33.008333	3942.764	-37.2501	-0.0252
2	97.025000	33.008333	3989.787	-37.2203	-0.0252
3	97.041667	33.008333	4034.817	-37.1899	-0.0234
4	97.058333	33.008333	4070.847	-37.1590	-0.0197
5	97.075000	33.008333	4106.877	-37.1276	-0.0142
6	97.091667	33.008333	4119.913	-37.0959	-0.0074
7	97.108333	33.008333	4115.946	-37.0640	0.0008
8	97.125000	33.008333	4090.977	-37.0318	0.0097
9	97.141667	33.008333	4070.007	-36.9990	0.0190
10	97.158333	33.008333	3991.047	-36.9665	0.0281
11	97.175000	33.008333	3985.070	-36.9327	0.0366
12	97.191667	33.008333	3956.107	-36.8988	0.0439
13	97.208333	33.008333	3965.137	-36.8642	0.0496
14	97.225000	33.008333	3964.173	-36.8295	0.0531
15	97.241667	33.008333	3983.205	-36.7943	0.0541
16	97.258333	33.008333	3993.251	-36.7595	0.0523
17	97.275000	33.008333	4016.279	-36.7238	0.0474
18	97.291667	33.008333	4054.318	-36.6883	0.0396

Ignore the ellipsoidal height

The radial gradient integral algorithm of anomalous field element is derived from the solution of Stokes boundary value problem, which requires the integrand field elements to be on the equipotential surface. The Poisson integral is the solution of the first boundary value problem, and the boundary surface is not required to be an equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

Gradient and Poisson integral computation of external gravity field element

Save as Import parameters Start Computation Save process Follow example

Operation of radial gradient integral on anomalous gravity field element

Computation of external gravity disturbance from disturbing gravity gradient

Computation of disturbing gravity gradient by inverse operation integral

Computation of external disturbing gravity gradient from gravity disturbance

Computation of Poisson integral on external anomalous field element

Open the ellipsoidal height grid file of the boundary surface

Open the anomalous field element grid file on the boundary surface

Select calculation point file format

ellipsoidal height grid file

Integration radius 120 km

Save the results as

Import setting parameters

Start Computation

Extraction of radial gradient of element

Plot

anomalous field element on surface

radial gradient (/km) of element

94.000000 102.000000 30.250000 36.250000 0.01666667 0.01666667

0.0400	0.0328	0.0240	0.0142	0.0038	-0.0069
-0.0688	-0.0663	-0.0637	-0.0617	-0.0608	-0.0612
-0.0857	-0.0891	-0.0935	-0.0989	-0.1046	-0.1097
0.0182	0.0393	0.0565	0.0688	0.0758	0.0774
0.0011	-0.0004	-0.0007	-0.0002	0.0005	0.0010
-0.0504	-0.0517	-0.0487	-0.0409	-0.0283	-0.0119
-0.0393	-0.0745	-0.1071	-0.1333	-0.1497	-0.1542
0.1109	0.1033	0.0857	0.0610	0.0323	0.0029
-0.0338	-0.0252	-0.0187	-0.0142	-0.0119	-0.0114
-0.0577	-0.0573	-0.0535	-0.0458	-0.0346	-0.0207
-0.0108	-0.0273	-0.0417	-0.0522	-0.0578	-0.0580
0.0330	0.0372	0.0423	0.0493	0.0586	0.0704
0.1409	0.1330	0.1239	0.1145	0.1051	0.0958
-0.0367	-0.0444	-0.0469	-0.0438	-0.0353	-0.0222
0.0168	-0.0040	-0.0257	-0.0469	-0.0657	-0.0814
-0.0545	-0.0390	-0.0212	-0.0016	0.0194	0.0411
0.0536	0.0318	0.0109	-0.0075	-0.0218	-0.0315

The radial gradient integral algorithm of anomalous field element is derived from the solution of Stokes boundary value problem, which requires the integrand field elements to be on the equipotential surface. The Poisson integral is the solution of the first boundary value problem, and the boundary surface is not required to be an equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude,

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

When the ellipsoidal height grid file selected, the program let the ellipsoidal height grid of the equipotential surface as the calculation surface.

[Output file] The radial gradient result file.

When the discrete calculation point file input, the output file record format: Behind the input calculation point file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and a column of calculated radial gradient.

When calculated on the calculation surface, the program outputs the radial gradient (/km) grid file with the same grid specification as the input grid file.

4.4.2 Computation of external gravity disturbance from disturbing gravity gradient

[Function] From the ellipsoidal height grid of the equipotential boundary surface and residual disturbing gravity gradient (E, radial) grid on the surface, compute the residual gravity disturbance (mGal) on or outside the geoid by the numerical integral.

Gradient and Poisson integral computation of external gravity field element

Save as Import parameters Start Computation Save process Follow example

Operation of radial gradient integral on anomalous gravity field element

Computation of external gravity disturbance from disturbing gravity gradient

Computation of disturbing gravity gradient by inverse operation integral

Computation of external disturbing gravity gradient from gravity disturbance

Computation of Poisson integral on external anomalous field element

Open the ellipsoidal height grid file of the equipotential surface

Open residual disturbing gradient grid file on the equipotential surface

Select calculation point file format

discrete calculation point file

Open the calculation point file on the equipotential surface

Set input point file format

number of rows of file header 1

Integral radius 120 km

Save the results as Import setting parameters Start Computation

Computation Process ** Operation Prompts

>> [Function] From the ellipsoidal height grid of the equipotential boundary surface and residual disturbing gravity gradient (E, radial) grid on the surface, compute the residual gravity disturbance (mGal) on or outside the geoid by the numerical integral.

** Input the ellipsoidal height grid file of the equipotential surface and residual disturbing gravity gradient grid file on the surface with the same grid specification...

>> Open the ellipsoidal height grid file of the equipotential surface C:\PAGrav4.5_win64en\examples\Intgendistgradient\landgeoidhgt.dat.

>> Open disturbing gravity gradient grid file on the equipotential surface C:\PAGrav4.5_win64en\examples\Intgendistgradient\resGMgeoid541_1800.grr.

>> Open the calculation point position file C:\PAGrav4.5_win64en\examples\Intgendistgradient\calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save the gravity disturbance as C:\PAGrav4.5_win64en\examples\Intgendistgradient\grtorgadm0.txt.

>> Behind the input calculation point file record, appends a column of residual gravity disturbance, and keeps four significant digits.

** The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-24 11:16:45

>> Complete the computation!

>> Computation end time: 2024-09-24 11:17:30

no	lon(degree/decimal)	lat	ellipHeight(m)	
1	97.008333	33.008333	3942.764	22.6149
2	97.025000	33.008333	3989.787	22.6235
3	97.041667	33.008333	4034.817	20.9473
4	97.058333	33.008333	4070.847	17.5408
5	97.075000	33.008333	4106.877	12.4745
6	97.091667	33.008333	4119.913	5.5244
7	97.108333	33.008333	4115.946	-1.8431
8	97.125000	33.008333	4090.977	-10.4866
9	97.141667	33.008333	4070.007	-19.5993
10	97.158333	33.008333	3991.047	-28.7201
11	97.175000	33.008333	3985.070	-37.3472
12	97.191667	33.008333	3956.107	-44.9479
13	97.208333	33.008333	3965.137	-50.9812
14	97.225000	33.008333	3964.173	-54.9269
15	97.241667	33.008333	3983.205	-56.3203
16	97.258333	33.008333	3953.251	-54.7987
17	97.275000	33.008333	4016.279	-50.1460
18	97.291667	33.008333	4054.318	-42.3364

Extract external gravity disturbance

Plot

disturbing gravity gradient (E, radial)

external gravity disturbance (mGal)

The radial gradient integral algorithm of anomalous field element is derived from the solution of Stokes boundary value problem, which requires the integrand field elements to be on the equipotential surface. The Poisson integral is the solution of the first boundary value problem, and the boundary surface is not required to be an equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represented by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

[Input files] The ellipsoidal height grid file of the equipotential surface and residual disturbing gravity gradient grid file on the surface with the same grid specifications, and

the calculation point position file or ellipsoidal height grid file of the calculation surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m),

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The external residual gravity disturbance result file.

When the discrete calculation point file input, the output file record format: Behind the input calculation point file record, appends a column of residual gravity disturbance, and keeps four significant digits.

When the ellipsoidal height grid file input, the program outputs the residual gravity disturbance grid file with the same grid specification as the input grid file.

Gradient and Poisson integral computation of external gravity field element

Save as Import parameters Start Computation Save process Follow example

Operation of radial gradient integral on anomalous gravity field element

Computation of external gravity disturbance from disturbing gravity gradient

Computation of disturbing gravity gradient by inverse operation integral

Computation of external disturbing gravity gradient from gravity disturbance

Computation of Poisson integral on external anomalous field element

Open the ellipsoidal height grid file of the equipotential surface

Open residual disturbing gradient grid file on the equipotential surface

Select calculation point file format

ellipsoidal height grid file

Open the ellipsoidal height grid file of the calculation surface

>> Computation Process ** Operation Prompts

Save computation process as

** Look at the file information in the window below, set the input file format parameters...

>> Save the gravity disturbance as C:/PAGrav4.5_win64en/examples/Intgendistgradient/grtorgadm.txt.

>> Behind the input calculation point file record, appends a column of residual gravity disturbance, and keeps four significant digits.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-24 11:16:45

>> Complete the computation!

>> Computation end time: 2024-09-24 11:17:30

>> Open the ellipsoidal height grid file of the calculation surface C:/PAGrav4.5_win64en/examples/Intgendistgradient/landbmsurfht.dat.

>> Save the gravity disturbance as C:/PAGrav4.5_win64en/examples/Intgendistgradient/grtorgadm.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-24 11:18:55

>> Complete the computation!

>> Computation end time: 2024-09-24 11:33:37

Integral radius 120 km

Save the results as

Import setting parameters

Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
-5.4909	-6.4631	-8.5821	-11.7865	-15.6321	-19.85
-48.0048	-47.6351	-46.4747	-44.8593	-43.2903	-41.71
-101.2762	-109.7248	-117.7312	-123.8692	-127.9094	-131.70
-148.8079	-151.8567	-152.3336	-146.9276	-136.2635	-123.03
41.7051	49.2578	53.6431	56.2174	57.0286	54.74
-41.2681	-51.9621	-61.8403	-69.4929	-76.4285	-81.21
23.8710	36.8403	47.6062	54.4733	57.0010	57.05
-51.5814	-47.2451	-38.7297	-27.8056	-15.6015	-4.54
-7.7964	-12.6661	-16.4324	-18.9854	-20.3489	-20.49
-2.8810	-2.4568	-2.4804	-2.9259	-3.5347	-4.01
35.0115	37.7838	38.0756	36.0699	31.3534	24.23
-52.4301	-47.7452	-41.8967	-35.5431	-28.7790	-22.76
34.0699	39.2253	43.5772	46.8551	48.5496	48.48
1.9419	-1.9419	-5.4733	-8.6848	-11.5458	-13.95
11.3998	16.6768	20.7352	23.4826	24.5368	23.73
-58.0468	-64.7971	-69.1522	-71.2812	-72.4016	-71.62
-12.4080	-8.9217	-7.0164	-6.4471	-6.9306	-8.11

Extract external gravity disturbance

Plot

disturbing gravity gradient (E, radial)

external gravity disturbance (mGal)

The radial gradient integral algorithm of anomalous field element is derived from the solution of Stokes boundary value problem, which requires the integrand field elements to be on the equipotential surface. The Poisson integral is the solution of the first boundary value problem, and the boundary surface is not required to be an equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

In this example, the residual gravity disturbance on the ground is computed from the residual disturbing gravity gradient on the geoid, and the integral radius is 120km. After the 2° area of the grid margin with integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual gravity disturbance (regarded as the true reference value) on the ground, and the differences between the results of the integral and model reference values, which can be employed to examine the performance of the algorithm.

The 541 to 1800 th degree true reference model value	mean	standard deviation	minimum	maximum
Ground residual gravity disturbances (mGal)	-0.0346	15.7527	-93.9854	66.6638

Differences between integral and reference value	mean	standard deviation	minimum	maximum
Ground residual gravity disturbances (mGal)	0.0071	4.2456	-18.5325	16.5266

4.4.3 Computation of disturbing gravity gradient by inverse operation integral

[Function] From the ellipsoidal height grid of the equipotential surface and residual gravity disturbance (mGal) grid on the surface, compute the residual disturbing gravity gradient (E, radial) on the surface by the inverse operation integral.

[Input files] The ellipsoidal height grid file of the equipotential surface and residual gravity disturbance grid file on the surface with the same grid specifications, and the calculation point position file on the surface.

The ellipsoidal height grid of the equipotential surface stands for the space position of the equipotential surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude,

The screenshot displays the software interface for computing gravity field elements. The main window is titled "Gradient and Poisson integral computation of external gravity field element". It features several tabs at the top: "Operation of radial gradient integral on anomalous gravity field element", "Computation of external gravity disturbance from disturbing gravity gradient", "Computation of disturbing gravity gradient by inverse operation integral" (which is highlighted with a red box), "Computation of external disturbing gravity gradient from gravity disturbance", and "Computation of Poisson integral on external anomalous field element".

Below the tabs, there are input fields and buttons for file selection and computation. A "Computation Process" section contains a list of prompts and instructions, including file paths and parameters. A table of calculation points is shown, with columns for ID, longitude, latitude, and ellipsoidal height. The table is partially highlighted with a red box. Below the table, there are two heatmaps: "gravity disturbance (mGal) on surface" and "disturbing gravity gradient (E, radial)".

At the bottom, there are buttons for "Save the results as", "Import setting parameters", and "Start Computation". A status bar at the very bottom provides additional information about the computation process.

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

When the ellipsoidal height grid file selected, the program let the ellipsoidal height grid of the equipotential surface as the calculation surface.

[Output file] The residual disturbing gravity gradient file.

When the discrete calculation point file input, the output file record format: Behind the input calculation point file record, appends a column of ellipsoidal height of the calculation point interpolated from the ellipsoidal height grid of the equipotential surface and a column of integral value of the residual disturbing gravity gradient.

When the ellipsoidal height grid file selected, the program outputs the residual disturbing gravity gradient grid file with the same grid specification as the input grid file.

Integral radius 120 km

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
-112.9508	-100.5689	-86.2807	-70.0350	-52.3943	-33.13
100.5612	94.8312	89.0615	84.9566	83.7303	85.91
148.3945	156.5703	167.9025	181.6957	196.9110	211.24
-82.3655	-130.7107	-168.9259	-194.7092	-207.1051	-206.56
-19.2439	-19.3484	-22.6509	-27.9468	-33.9816	-39.49
78.8567	89.5537	90.5251	79.9201	57.2490	23.71
63.1468	147.8592	228.7359	296.4236	342.6878	361.58
-204.1974	-185.9357	-148.8509	-90.3037	-26.9057	36.56
62.7685	40.9181	23.6119	10.8818	2.2801	-2.78
88.2194	92.8901	89.1706	75.8932	53.1062	22.23
-40.4884	-4.3659	27.5326	51.2541	64.1525	65.23
-79.6335	-78.5558	-80.3205	-87.2888	-100.8419	-121.12
-246.8885	-228.8089	-209.8435	-191.5598	-174.7083	-159.11
116.3989	134.4200	139.7425	130.9599	108.2413	73.42
-77.8705	-33.6482	13.0742	58.2786	98.6038	131.66
103.9993	78.5124	48.1730	13.0325	-26.0951	-67.53
-74.0066	-28.4254	13.5757	47.9650	71.7441	83.31

● The radial gradient integral algorithm of anomalous field element is derived from the solution of Stokes boundary value problem, which requires the integrand field elements to be on the equipotential surface. The Poisson integral is the solution of the first boundary value problem, and the boundary surface is not required to be an equipotential surface.

● The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represent by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

In this example, the residual disturbing gravity gradient on the geoid is calculated by inverse operation integral from the residual gravity disturbance on the geoid, and the integral radius is 120km. After the 2° area of the grid margin with the integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual disturbing gravity gradient (regarded as the true reference value) on the geoid, and the differences between the results of the integral and model reference values, which can be employed to examine the performance of the algorithm.

The 541 to 1800 th degree true reference model value	mean	standard deviation	minimum	maximum
Residual disturbing gravity gradients (E, radial) on geoid	0.4763	68.2499	-288.1750	387.7286

Differences between integral and reference value	mean	standard deviation	minimum	maximum
Residual disturbing gravity gradients (E, radial) on geoid	-0.0562	6.6405	-37.1465	28.3557

4.4.4 Computation of external disturbing gravity gradient from gravity disturbance

[Function] From the ellipsoidal height grid of the boundary surface and residual gravity disturbance grid (mGal) on the surface, compute the residual disturbing gravity gradient (E, radial) on or outside the geoid. The inverse integral of gravity disturbance adopts the combination algorithm with Poisson integral and differentiation, which does not require that the boundary surface should be a gravity equipotential surface.

[Input files] The ellipsoidal height grid file of the boundary surface and residual gravity disturbance grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

The ellipsoidal height grid of the boundary surface stands for the space position of the boundary surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: ID (point no / point name), longitude (decimal degrees), latitude, ellipsoidal height (m),

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

[Output file] The external residual disturbing gravity gradient result file.

Gradient and Poisson integral computation of external gravity field element

Save as Import parameters Start Computation Save process Follow example

Operation of radial gradient integral on anomalous gravity field element Computation of external gravity disturbance from disturbing gravity gradient Computation of disturbing gravity gradient by inverse operation integral **Computation of external disturbing gravity gradient from gravity disturbance** Computation of Poisson integral on external anomalous field element

Open the ellipsoidal height grid file of the boundary surface Open the gravity disturbance grid file on the boundary surface Select calculation point file format discrete calculation point file Open the calculation point position file Set input point file format number of rows of file header 1

Integral radius 120 km

no lon (degree/decimal) lat ellipHeight (m)

1	97.008333	33.008333	3942.764	25.9295
2	97.025000	33.008333	3989.787	25.7579
3	97.041667	33.008333	4034.817	23.9132
4	97.058333	33.008333	4070.847	20.4859
5	97.075000	33.008333	4106.877	15.5549
6	97.091667	33.008333	4119.913	9.4470
7	97.108333	33.008333	4115.946	2.3353
8	97.125000	33.008333	4090.977	-5.5390
9	97.141667	33.008333	4070.007	-13.9461
10	97.158333	33.008333	3991.047	-22.4698
11	97.175000	33.008333	3985.070	-30.3218
12	97.191667	33.008333	3956.107	-37.3873
13	97.208333	33.008333	3965.137	-42.6330
14	97.225000	33.008333	3964.173	-46.0654
15	97.241667	33.008333	3983.205	-46.9850
16	97.258333	33.008333	3953.251	-45.9626
17	97.275000	33.008333	4016.279	-41.2274
18	97.291667	33.008333	4054.318	-34.2891

Save the results as Import setting parameters Start Computation

Extract external disturbing gradient Plot

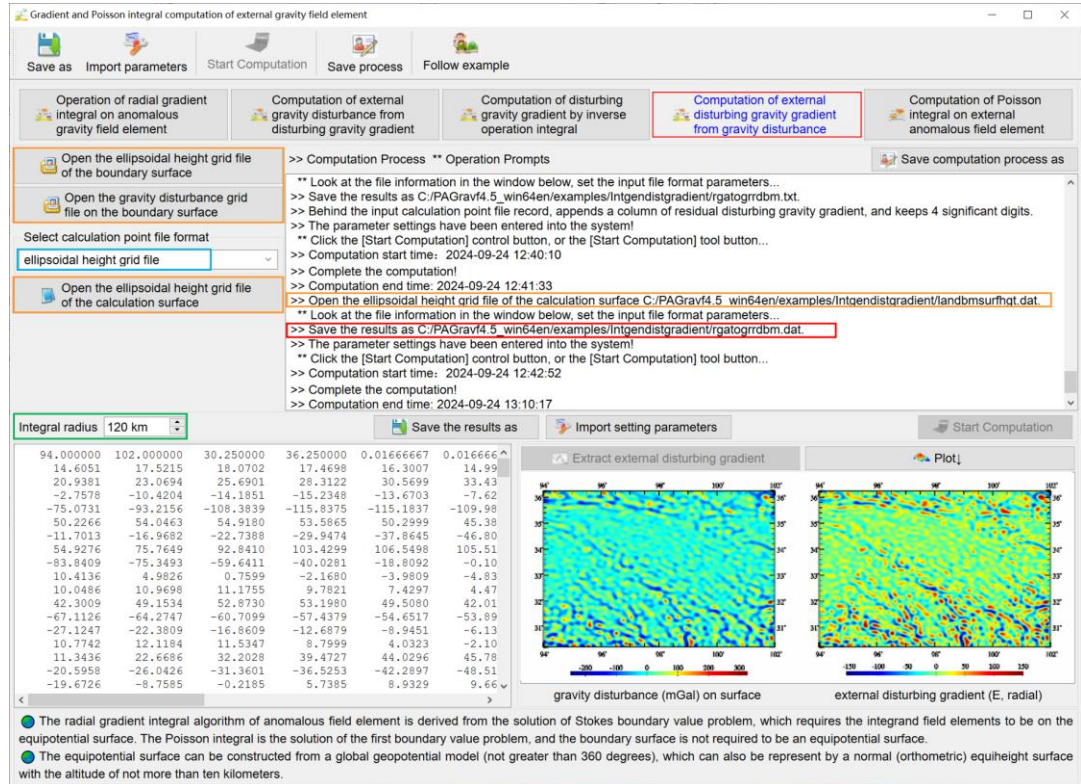
gravity disturbance (mGal) on surface external disturbing gradient (E, radial)

The radial gradient integral algorithm of anomalous field element is derived from the solution of Stokes boundary value problem, which requires the integrand field elements to be on the equipotential surface. The Poisson integral is the solution of the first boundary value problem, and the boundary surface is not required to be an equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represented by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

When the discrete calculation point file input, the output file record format: Behind the input calculation point file record, appends a column of residual disturbing gravity gradient, and keeps four significant digits.

When the ellipsoidal height grid file input, the program outputs the residual disturbing gravity gradient grid file with the same grid specification as the input grid file.



In this example, the ground residual disturbing gravity gradients are computed from the residual gravity disturbance on the geoid, and the integral radius is 120km. After the 2° area of the grid margin with the integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual disturbing gravity gradients (regarded as the true reference value) on the ground, and the differences between the results of the integral and model reference values, which can be employed to examine the performance of the algorithm.

The 541 to 1800 th degree true reference model value	mean	standard deviation	minimum	maximum
Ground residual disturbing gravity gradients (E, radial)	-0.2872	26.9448	-148.0282	136.7864

Differences between integral and reference value	mean	standard deviation	minimum	maximum
Ground residual disturbing gravity gradients (E, radial)	-0.0361	2.7361	-14.2089	13.6802

Compared with the 4.4.3 inverse operation integral of gravity disturbance, the

performance and accuracy of the inverse integral of gravity disturbance are comparable to those of the inverse operation integral.

4.4.5 Computation of Poisson integral on external anomalous gravity field element

[Function] From the ellipsoidal height grid of the boundary surface and the residual anomalous gravity field element grid on the surface, compute the residual anomalous gravity field element on or outside the geoid by the Poisson integral. The Poisson integral is the solution of the first boundary value problem in the mathematical sense, and the boundary surface need be not an equipotential surface.

[Input files] The ellipsoidal height grid file of the boundary surface and the residual anomalous field element grid file on the surface with the same grid specifications, and the calculation point position file or the ellipsoidal height grid file of the calculation surface.

The ellipsoidal height grid of the boundary surface stands for the space position of the boundary surface, employed to calculate the integral distance.

The record format of the discrete calculation point file: point no / point name, longitude (decimal degrees), latitude, ellipsoidal height (m),

[Parameter settings] Set the input file format parameters, enter the integral radius, and select algorithm.

Computation of Poisson integral on external anomalous field element

Operation of radial gradient integral on anomalous gravity field element

Computation of external gravity disturbance from disturbing gravity gradient

Computation of disturbing gravity gradient by inverse operation integral

Computation of external disturbing gravity gradient from gravity disturbance

Computation of Poisson integral on external anomalous field element

Operation Prompts

>> [Function] From the ellipsoidal height grid of the boundary surface and residual gravity disturbance grid (mGal) on the surface, compute the residual disturbing gravity gradient (E, radial) on or outside the geoid. The inverse integral of gravity disturbance adopts the combination algorithm with Poisson integral and differentiation, which does not require that the boundary surface should be a gravity equipotential surface.

** Input the ellipsoidal height grid of the boundary surface and residual anomalous gravity field element grid file on the surface with the same grid specification...

>> Open the ellipsoidal height grid file of the boundary surface C:/PAGrav4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.

>> Open anomalous field element grid file on the boundary surface C:/PAGrav4.5_win64en/examples/Intgendistgradient/resGMgeoid541_1800.ksl.

>> Open the calculation point position file C:/PAGrav4.5_win64en/examples/Intgendistgradient/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save Poisson integral results as C:/PAGrav4.5_win64en/examples/Intgendistgradient/possonksi.txt.

>> Behind the input calculation point file record, appends a column of Poisson integral value, and keeps 4 significant digits.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-24 11:57:56

Input Data Table:

no	lon (degree/decimal)	lat	ellipHeight (m)	Poisson integral value
1	97.008333	33.008333	3942.764	-0.0249
2	97.025000	33.008333	3989.787	-0.0297
3	97.041667	33.008333	4034.817	-0.0365
4	97.058333	33.008333	4070.847	-0.0451
5	97.075000	33.008333	4106.877	-0.0555
6	97.091667	33.008333	4119.913	-0.0673
7	97.108333	33.008333	4115.946	-0.0803
8	97.125000	33.008333	4090.977	-0.0941
9	97.141667	33.008333	4070.007	-0.1079
10	97.158333	33.008333	3991.047	-0.1218
11	97.175000	33.008333	3985.070	-0.1332
12	97.191667	33.008333	3956.107	-0.1424
13	97.208333	33.008333	3965.137	-0.1472
14	97.225000	33.008333	3964.173	-0.1477
15	97.241667	33.008333	3983.205	-0.1421
16	97.258333	33.008333	3953.251	-0.1316
17	97.275000	33.008333	4016.279	-0.1126
18	97.291667	33.008333	4054.318	-0.0878

field element on surface

external field element

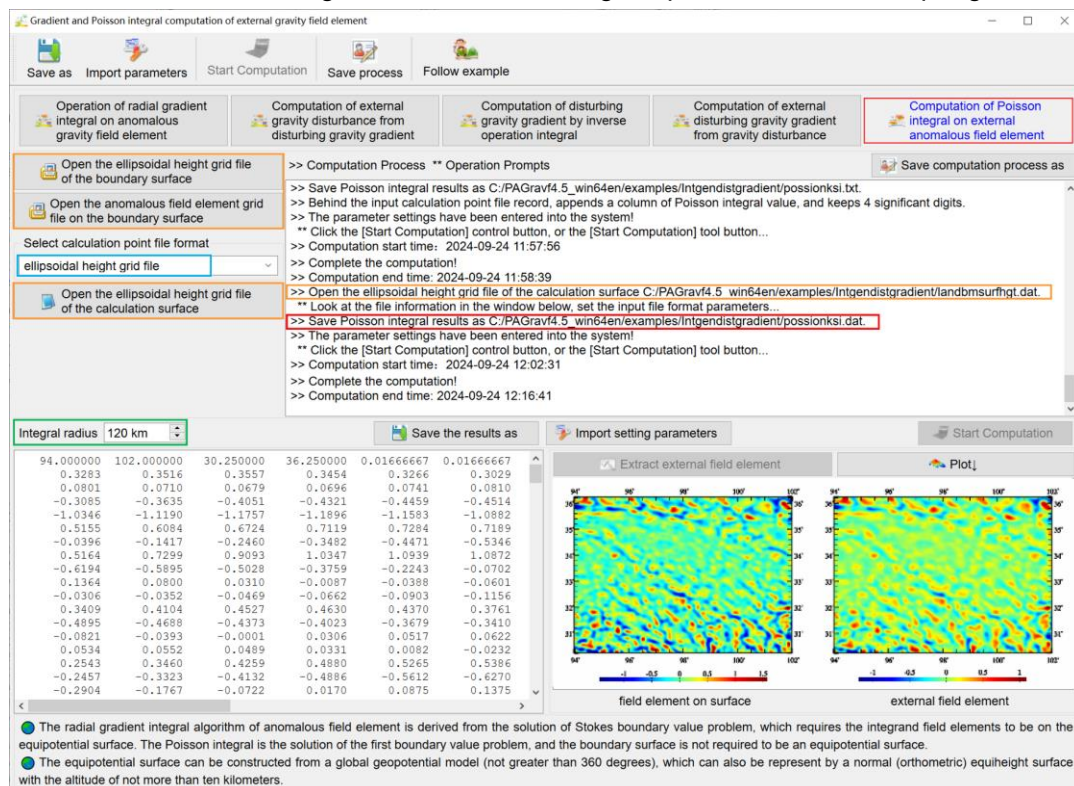
The radial gradient integral algorithm of anomalous field element is derived from the solution of Stokes boundary value problem, which requires the integrand field elements to be on the equipotential surface. The Poisson integral is the solution of the first boundary value problem, and the boundary surface is not required to be an equipotential surface.

The equipotential surface can be constructed from a global geopotential model (not greater than 360 degrees), which can also be represented by a normal (orthometric) equiheight surface with the altitude of not more than ten kilometers.

[Output file] The external residual anomalous field element result file.

When the discrete calculation point file input, the output file record format: Behind the input calculation point file record, appends a column of residual anomalous field element, and keeps four significant digits.

When the ellipsoidal height grid file selected, the program outputs the residual anomalous field element grid file with the same grid specification as the input grid file.



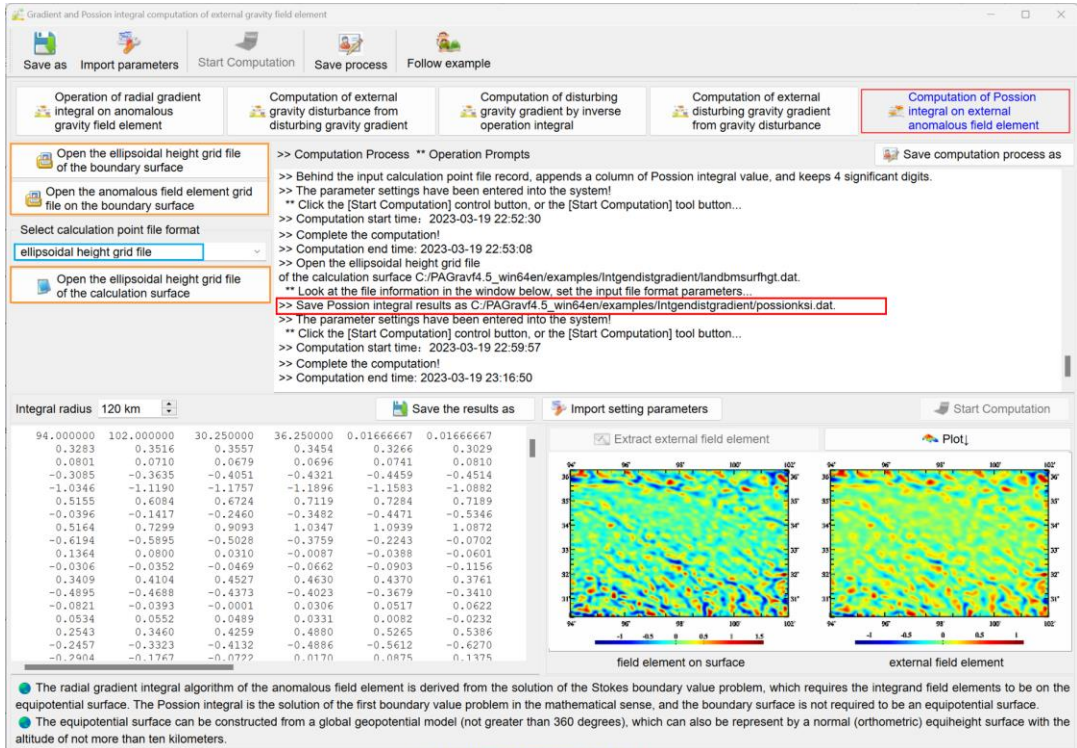
In this example, the ground residual height anomalies and ground gravity disturbances are computed respectively from the residual geoidal height and gravity disturbance on the geoid, and the integral radius is 120km. After the 2° area of the grid margin with the integral edge effect deducted, statistically analyze the 541 to 1800th degree model residual height anomalies and gravity disturbances (regarded as the true reference value) on the ground, and the differences between the results of the integral and model reference values, which can be employed to examine the performance of the algorithm.

The 541 to 1800 th degree true reference model value	mean	standard deviation	minimum	maximum
Ground residual height anomalies (m)	0.0010	0.1182	-0.6745	0.4760
Ground residual gravity disturbances (mGal)	-0.0346	15.7527	-93.9854	66.6638

Differences between integral and reference value	mean	standard deviation	minimum	maximum
--	------	--------------------	---------	---------

Ground residual height anomalies (m)	0.0004	0.0074	-0.0326	0.0334
Ground residual gravity disturbances (mGal)	0.0476	1.1919	-5.0173	6.5138

The edge effect of the Poisson integral here is small, which can effectively suppress the attenuation of short-wave signals, and is suitable for upward and downward analytical continuation.



You can call this function repeatedly to iteratively perform the Poisson integral operation. In general, once iteration and at most 3 iterations are sufficient to meet the accuracy requirements in most cases.

4.5 Feature and performance analysis of spherical radial basis functions

[Purpose] Calculate the spatial and spectral curves of four kinds of spherical radial basis functions (SRBF) for various types of anomalous gravity field elements, and then construct an equal area spherical grid according to the specified latitude and longitude range, to design the SRBF network and the corresponding approach algorithm of gravity field. The program does not require an input file.

4.5.1 Spatial and spectral performance analysis of spherical radial basis functions

[Function] Given the action distance (influence radius) and sampling interval of spherical radial basis functions, calculate and display the spatial and spectral curves of

spherical radial basis function of the gravity disturbance, height anomaly (disturbing potential), total vertical deflection or disturbing gravity gradient. Continuously adjusting the minimum and maximum degree of SRBF Legendre expansion, buried depth of the reference surface and order number of multipolar SRBF, the spatial and spectral performance and feature for the SRBF of various gravity field elements can be analyzed and revealed.

[Parameter settings] Select the type of anomalous gravity field element, set the spherical radial basis function parameters and plot parameters.

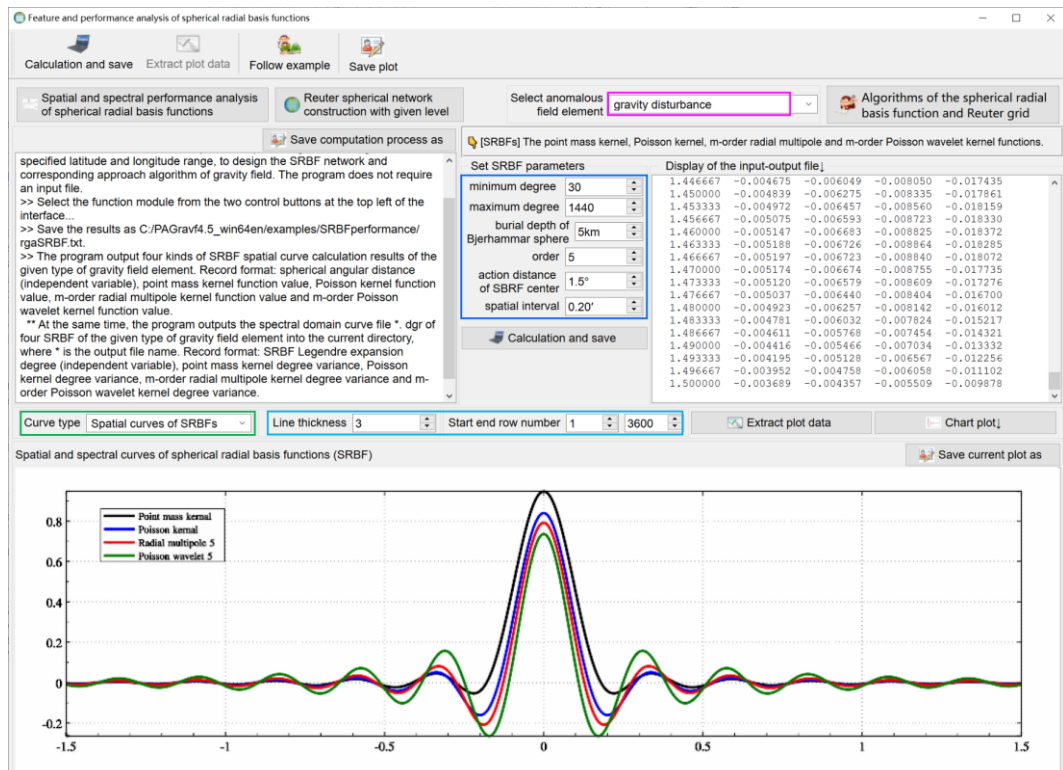
Enter the minimum and maximum degree of SRBF Legendre expansion. Minimum and maximum degree can be employed to adjust SRBF bandwidth.

Enter the burial depth of Bjerhammar sphere: The depth of the Bjerhammar sphere relative to the mean height surface of the observations. Combined with the degree of SRBF Legendre expansion, can be employed to adjust the spectral center and bandwidth of spherical radial basis function. The greater the burial depth, the smoother the SRBF, the smaller the kurtosis namely the wider the spectral bandwidth.

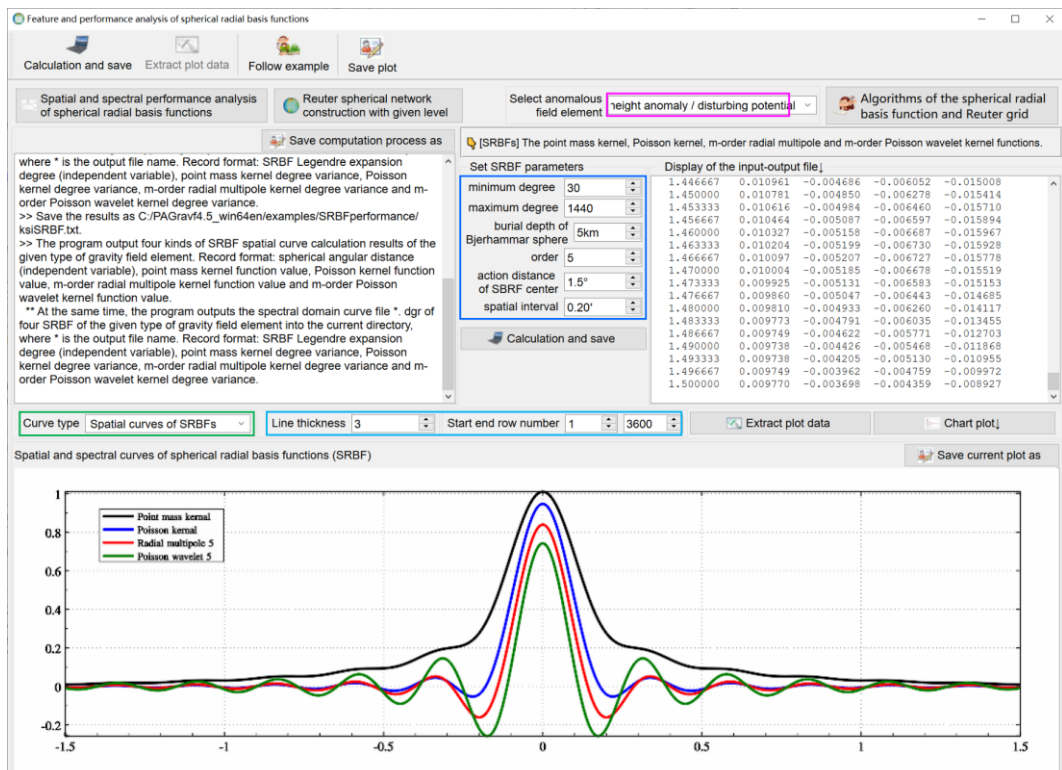
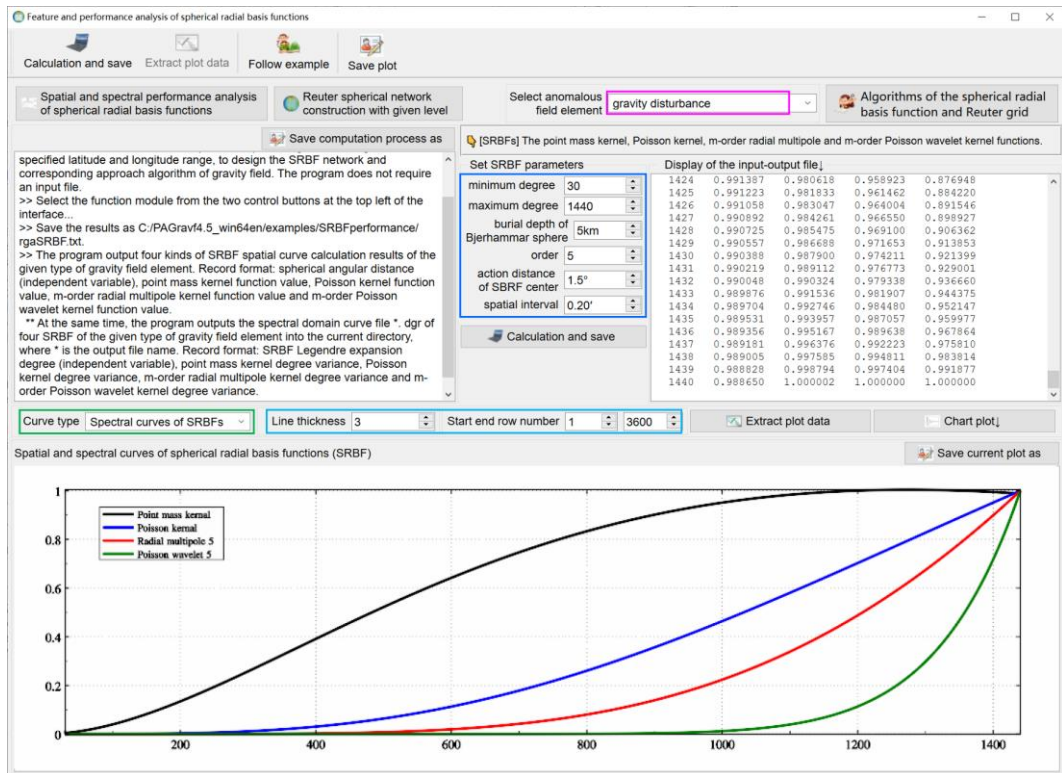
Set the order number m of radial multipole kernel function or Poisson wavelet kernel function. The greater the m , the bigger the kurtosis of SRBF.

Input the SRBF spatial definition domain represented by spherical angular distance, which is also known as the influence radius, is equivalent to the integral radius of regional gravity field.

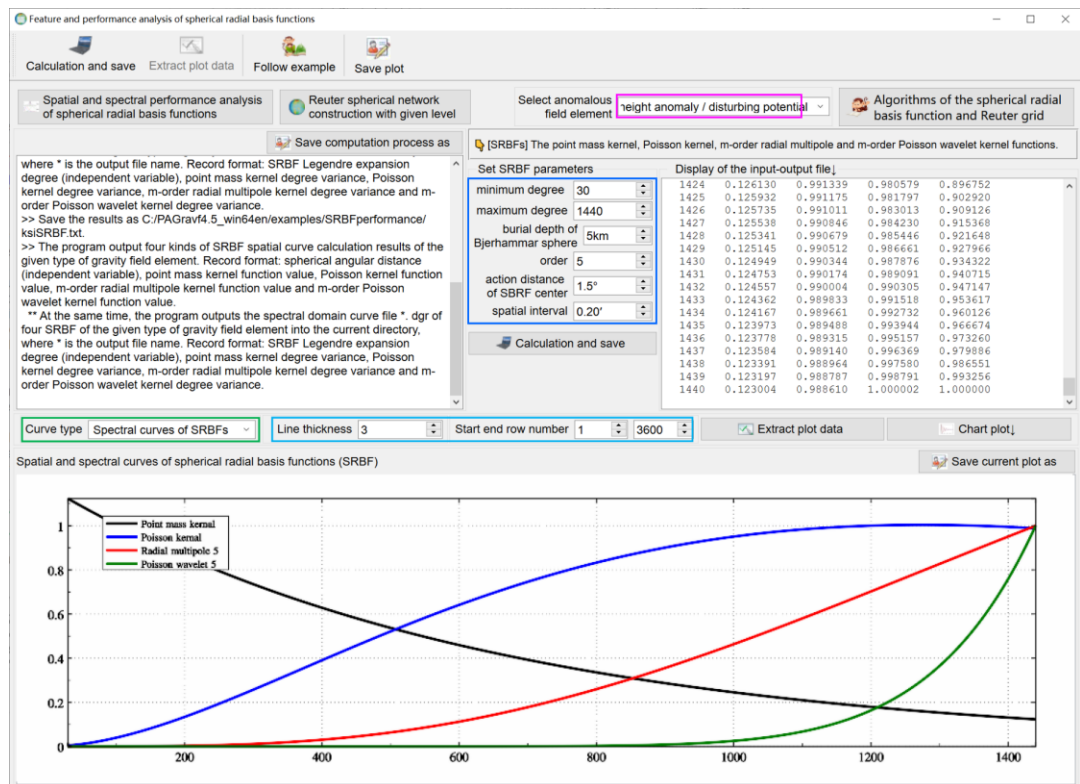
Enter the sampling interval of the SRBF spatial curve.



[Output files] The spatial and spectral curves of spherical radial basis functions curves files.



The program output four kinds of SRBF spatial curve calculation results of the given type of gravity field element. Record format: spherical angular distance (independent variable), point mass kernel function value, Poisson kernel function value, m-order radial multipole kernel function value and m-order Poisson wavelet kernel function value.



At the same time, the program outputs the spectral domain curve file *.dgr of four SRBF of the given type of gravity field element into the current directory, where * is the output file name. Record format: SRBF Legendre expansion degree (independent variable), point mass kernel degree variance, Poisson kernel degree variance, m-order radial multipole kernel degree variance and m-order Poisson wavelet kernel degree variance.

4.5.2 Reuter spherical network construction with given level

[Function] Given the Reuter network level K and regional latitude and longitude range, construct the global or regional Reuter spherical coordinate grid. The spatial resolution of the grid on the unit sphere is about π/K , the cell-grid area at the equator is $A = \pi^2/K^2$, and the cell-grid area at the poles is equal to $\pi A/4$.

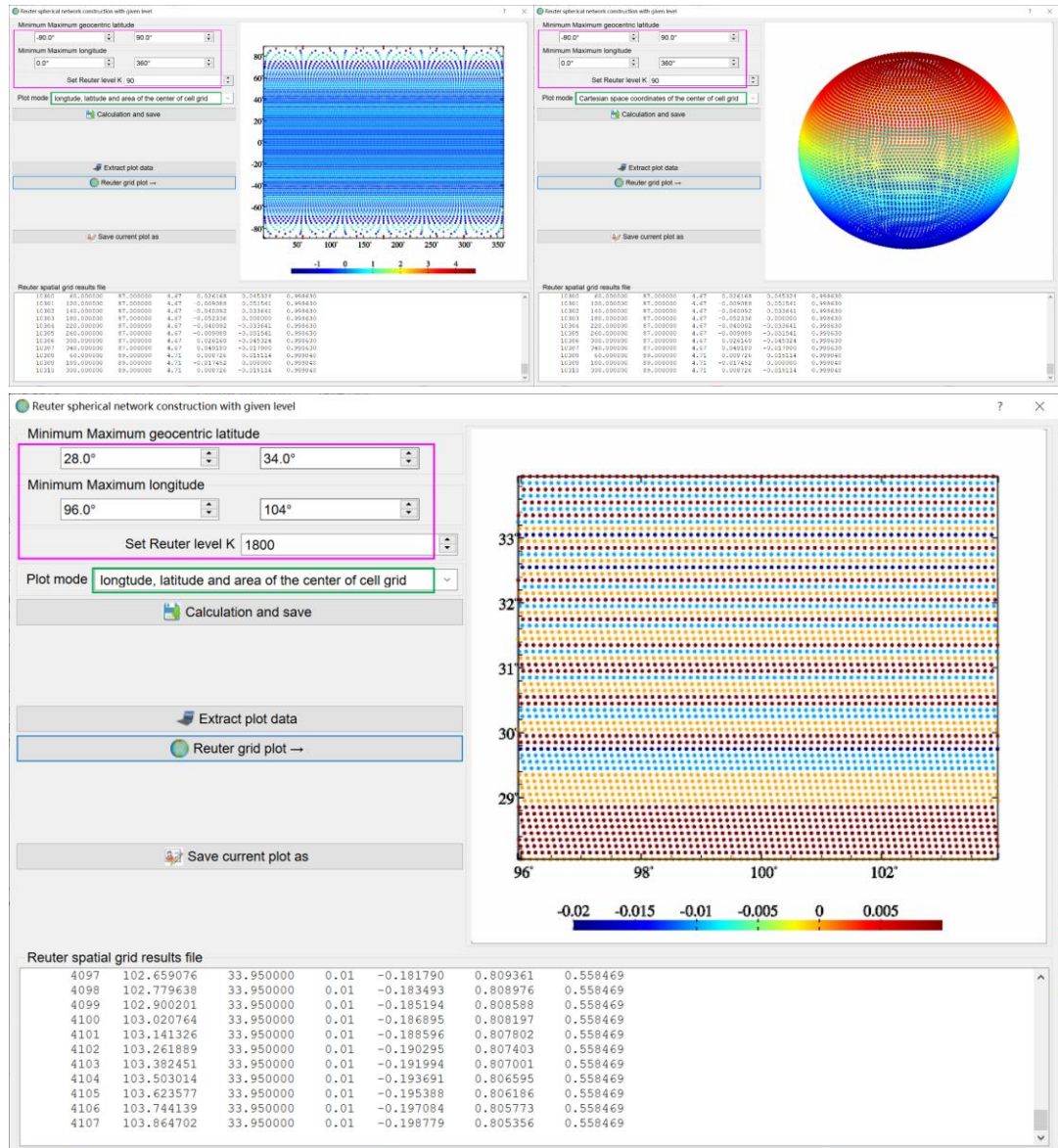
Reuter network level K: the spherical surface is divided into K prime vertical circles, and the latitude interval is π/K . The larger the K value, the greater the spatial resolution of the spherical Reuter network. When $K = 360$, the spatial resolution is 30', and when $K = 1800$, the spatial resolution is 6'.

[Parameter settings] Enter the latitude and longitude range and Reuter grid level K

and set plot parameters.

[Output file] The spherical coordinate grid file of the Reuter unit spherical.

Record format of the result file: cell-grid no, central longitude, geocentric latitude, percentage of area deviation, rectangular coordinates X, Y, Z. Global Reuter network does not contain bipolar cell grids. The percentage of cell-grid area deviation is equal to the difference between the cell-grid area and the cell-grid area A at the equator, divided by the cell-grid area A at the equator, multiplied by 100.



The program output Reuter network parameter file *.par into the current directory, where * is the output file name. The file header format: the latitude and longitude range, Reuter grid level, and total number of points. The record format: point no, prime vertical circle direction grid center latitude (°), prime vertical circle direction grid number,

longitude interval ('), cell-grid area deviation percentage.

4.6 Gravity field approach using SRBFs in spectral domain and performance test

[Function] From a single type of observations selected from the residual gravity disturbance (mGal), height anomaly (m), gravity anomaly (mGal), disturbing gravity gradient (E, radial) or vertical deflection ("), and a kind of spherical radial basis function (SRBF) selected from the point mass Kernel function, Poisson kernel function, m-order radial multipole kernel or m-order wavelet kernel function, estimate the residual gravity disturbance, height anomaly, gravity anomaly, disturbing gravity gradient or vertical deflection on or outside the geoid.

Selecting different type of observations and constructing different figure of SRBF, we can calculate different type of target field elements, and then fully verify and analyze the spatial and spectral properties of gravity field approach algorithms using SRBFs.

Setting the observation weight of the target field element to be evaluated to zero, or directly taking the observed point of the target field element to be evaluated as the calculation point, we can effectively detect the gross error of the target observations and measure their external accuracy indexes.

The program itself can be employed for analytical continuation, gridding, type conversion and all-element modelling on gravity field from various single type of observed field elements.

[Input files] The discrete residual anomalous field element observation file and ellipsoidal height grid file of the calculation surface.

The record agreed format of the observation file: ID, longitude (decimal degrees), latitude, ..., ellipsoidal height (m), ..., observation, ..., weight, ...

There is no limit to the grid resolution of the calculation surface.

When selecting 'Synchronous calculation of elements at discrete points', the program requires input the calculation point space location file. The header file occupies 1 row, and the agreed format of the file record: ID, longitude (degree decimal), latitude, ellipsoidal height (m),

If the observed points are taken as the calculation points to be evaluated, the field elements at the observed points can be estimated from the observations input by the program, and then we can effectively detect the gross error of the target observations to be evaluated and measure their external accuracy indexes.

[Parameter settings] Select the type of the observations and type of unknown target field element, set the input observation file format parameters, SRBF parameters and algorithm parameters.

When the column ordinal number of the weight attribute is less than 1, exceeds the column number of the record, or the weight is less than zero, the program makes the weight equal to 1.

When the weight in the file record is equal to zero, the observation will not

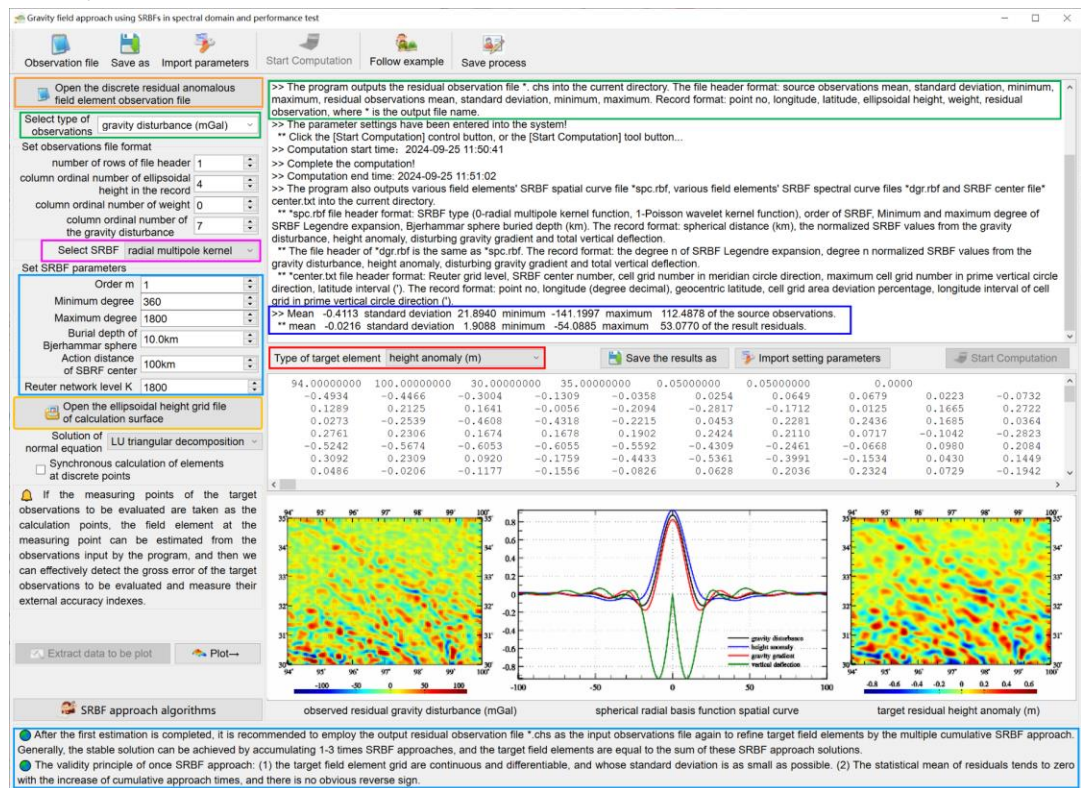
participate in the estimation of the SRBF coefficient, and the program can be employed to measure the external accuracy index of the observations.

Select spherical radial basis function: radial multipole kernel function, Poisson wavelet kernel function. The zero-order radial multipole kernel function is the point mass kernel function, and the zero-order Poisson wavelet kernel function is the Poisson kernel function.

Enter the order m. The order number m of radial multipole kernel function and Poisson wavelet kernel function. The greater the m, the bigger the kurtosis of SRBF.

Input the Bjerhammar sphere burial depth: The depth of the Bjerhammar sphere relative to the mean height surface of the observations, which can be employed to adjust the spectral center and bandwidth of SRBF when combined with the degree of SRBF Legendre expansion.

The greater the burial depth, the smoother the SRBF, the smaller the kurtosis namely the wider the spectral bandwidth.



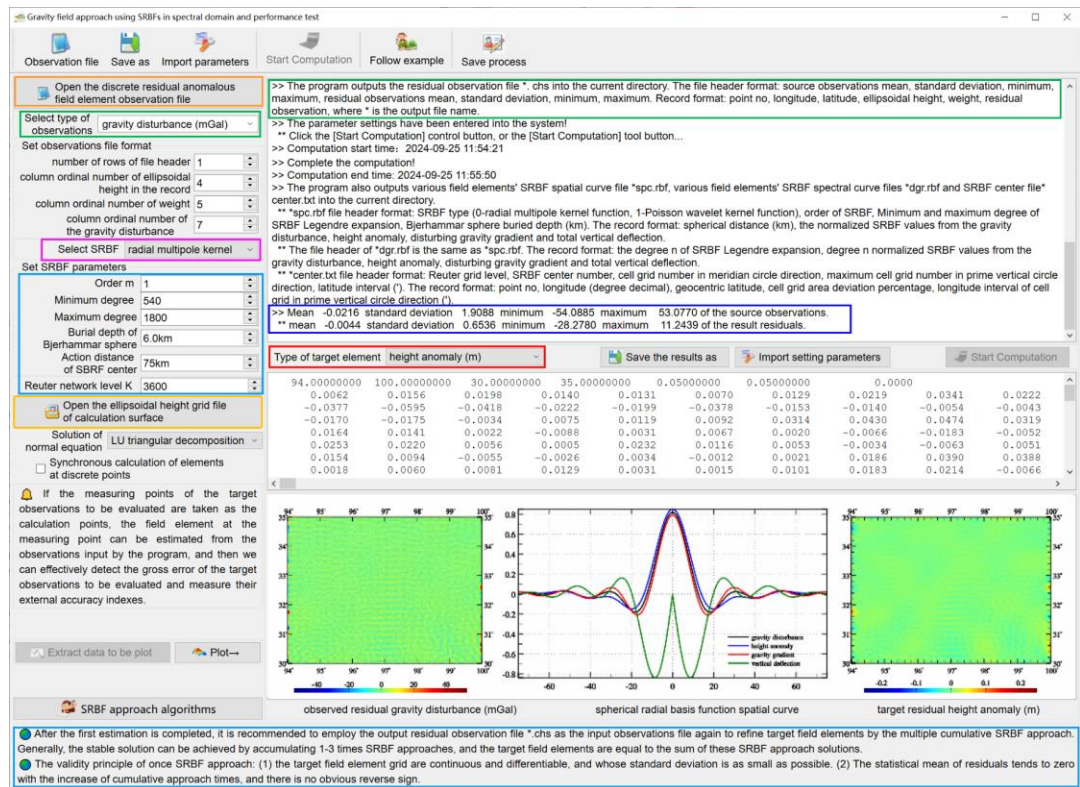
Enter the action distance of SRBF center. The action distance is also called as the influence radius = spherical angular distance × the Bjerhammar spherical radius, which is equivalent to the integral radius for local gravity field.

A fixed action distance is adopted to ensure the coordination and consistency of the spatial and spectral figure of regional gravity field.

The suitable burial depth is about 1/20~1/5 of the action distance of SRBF center.

Set the Reuter network level K: The spherical surface is divided into K prime vertical

circles, and the latitude interval is $180^\circ/K$. The larger the K value, the greater the spatial resolution of the spherical Reuter network. The suitable $180^\circ/K$ is approximately equal to the average distance between observation points.



PAGrav4.5 adopts the Reuter grid fitting algorithm to quickly determine the effective number J of observations in the cell-grid where each SRBF center (node) is located. When J is less than 1, the SRBF center is eliminated to ensure that the spatial distribution of observations is consistent with the spatial distribution of SRBF centers everywhere.

If the distribution of observations is uniform, the SRBF centers will also be uniformly distributed, and if the distribution of observations is irregular, the distribution of SRBF centers will also be irregular.

Enter minimum and maximum degree of SRBF Legendre expansion. Minimum and maximum degree can be employed to adjust the SRBF bandwidth.

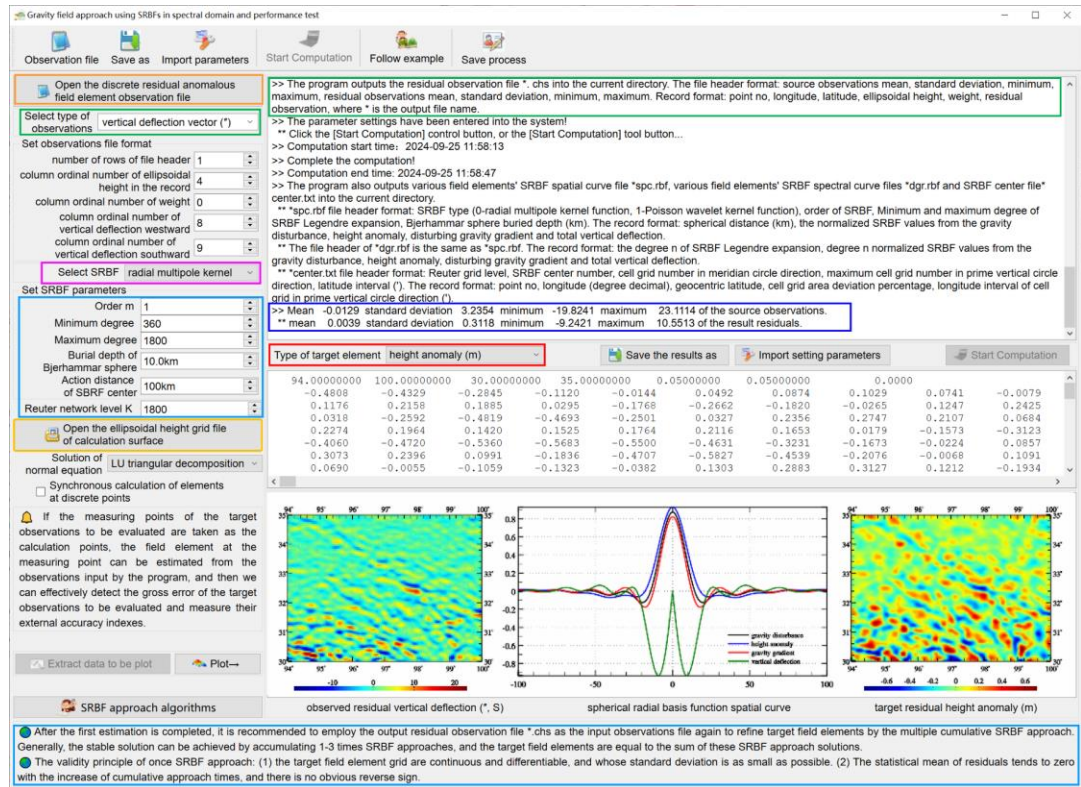
The minimum degree has a great influence on the statistical mean of the residual observations from the calculation results. Maximum degree, Reuter grid level K have some influence on the statistical standard deviation of the residuals. The influence of burial depth D has no obvious regularity, and the sensitivity is not large.

Select the solution of normal equation. LU triangular decomposition method, Cholesky decomposition, smallest norm solution.

The normal equation no longer needs regularization and iterative computation.

[Output file] The approached target field element grid file.

When 'synchronous calculation of the field elements at discrete points' selected, the program outputs the target type of field element file *.tgt at the calculation points into the current directory, where & is the output file name.



The program outputs the residual observation file *.chs into the current directory. The file header format: source observations mean, standard deviation, minimum, maximum, residual observations mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, weight, source observation, residual observation, where * is the output file name.

The program also outputs the various field elements' SRBF spatial curve file *spc.rbf, various field elements' SRBF spectral curve files *dgr.rbf and SRBF center file* center.txt into the current directory.

*spc.rbf file header format: SRBF type (0-radial multipole kernel function, 1-Poisson wavelet kernel function), order of SRBF, Minimum and maximum degree of SRBF Legendre expansion, Bjerhammar sphere buried depth (km). The record format: spherical distance (km), the normalized SRBF values from the gravity disturbance, height anomaly, disturbing gravity gradient and total vertical deflection.

The file header of *dgr.rbf is the same as *spc.rbf. The record format: the degree n of SRBF Legendre expansion, degree n normalized SRBF values from the gravity disturbance, height anomaly, disturbing gravity gradient and total vertical deflection.

*center.txt file header format: Reuter grid level, SRBF center number, cell-grid number in meridian circle direction, maximum cell-grid number in prime vertical circle

Gravely field approach using SBRF in spectral domain and performance test

Open file

Save as

Import parameters

Start Computation

Follow example

Save process

Open the discrete residual anomalous field element observation file

Select type of observation: height anomaly (m)

Set observations file format

number of rows of file header: 1

column ordinal number of ellipsoid height in the record: 4

column ordinal number of weight 0: 4

column ordinal number of the height anomaly: 5

Select SBRF: radial multipole kernel

Set SBRF parameters

Order m: 1

Minimum degree: 360

Maximum degree: 1800

Burial depth of Bjerrhammer sphere: 10.0km

Action distance of SBRF center: 100km

Reuter network level K: 1800

Open the ellipsoidal height grid file of calculation surface

Solution of LU triangular decomposition normal equation

☐ Synchronous calculation of elements at discrete points

If the measuring points of the target observations to be evaluated are taken as the calculation points, the field element at the measuring point can be estimated from the observations input by the program, and then we can effectively detect the gross error of the target observations to be evaluated and measure their external accuracy indexes.

Extract data to be plot

Plot

SBRF approach algorithms

>> The program outputs the residual observation file *.chs into the current directory. The file header format: source observations mean, standard deviation, minimum, maximum, residual observations mean, standard deviation, minimum, maximum. Record format: point no, longitude, latitude, ellipsoidal height, weight, residual observation, where "n" is the output file name.

>> The parameter settings have been entered into the system!

>> Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-25 12:02:43

>> Complete the computation!

>> Computation end time: 2024-09-25 12:03:01

>> The program also outputs various field elements' SBRF spatial curve file "spc.rbf, various field elements' SBRF spectral curve files "dgr.rbf and SBRF center file" center.txt into the current directory.

** spc.rbf file header format: SBRF type (0-radiar buried depth (km), 1-Poisson wavelet kernel function), order of SBRF, Minimum and maximum degree of SBRF Legendre expansion, Bjerrhammer sphere buried depth (km). The record format: spherical distance (km), the normalized SBRF values from the gravity disturbance, height anomaly, disturbing gravity gradient and total vertical deflection.

** The file header of "dgr.rbf" is the same as "spc.rbf". The record format: the degree n of SBRF Legendre expansion, degree n normalized SBRF values from the gravity disturbance, height anomaly, disturbing gravity gradient and total vertical deflection.

** center.txt file header format: Reuter grid level, SBRF center point, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval (°). The record format: point no, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (°).

>> Mean -0.0020 standard deviation 0.1590 minimum -0.8621 maximum 0.6548 of the source observations.

>> mean -0.0011 standard deviation 0.0135 minimum -0.3763 maximum 0.4258 of the result residuals.

Type of target element	gravity disturbance (mGal)
94.00000000	100.00000000
-65.3591	-58.7649
6.4548	29.7980
6.6296	-41.4182
29.0963	26.0823
-22.4518	-22.7403
39.4309	43.6354
14.0937	-0.1259
30.00000000	-35.2841
2.0592	29.7832
-66.0062	-73.4118
17.9708	-17.9708
-54.9616	-54.9616
-17.0100	-17.0100
-28.6960	-28.6960
0.05000000	7.2438
-53.6309	-53.6309
24.2028	24.2028
32.7800	32.7800
-54.9616	-54.9616
-64.3688	-64.3688
17.8789	17.8789
0.05000000	0.05000000
13.5926	13.5926
-13.1329	-13.1329
56.5887	56.5887
-1.1940	-1.1940
-19.3994	-19.3994
-21.0537	-21.0537
51.0941	51.0941
61.0445	61.0445
12.0766	12.0766
4.8602	4.8602
36.5839	36.5839
-30.9331	-30.9331
7.7557	7.7557
7.6201	7.6201
16.2492	16.2492
-19.7100	-19.7100

observed residual height anomaly (m)

spherical radial basis function spatial curve

target residual gravity disturbance (mGal)

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observations file again to refine target field elements by the multiple cumulative SBRF approach. Generally, the stable solution can be achieved by accumulating 1-3 times SBRF approaches, and the target field elements are equal to the sum of these SBRF approach solutions.

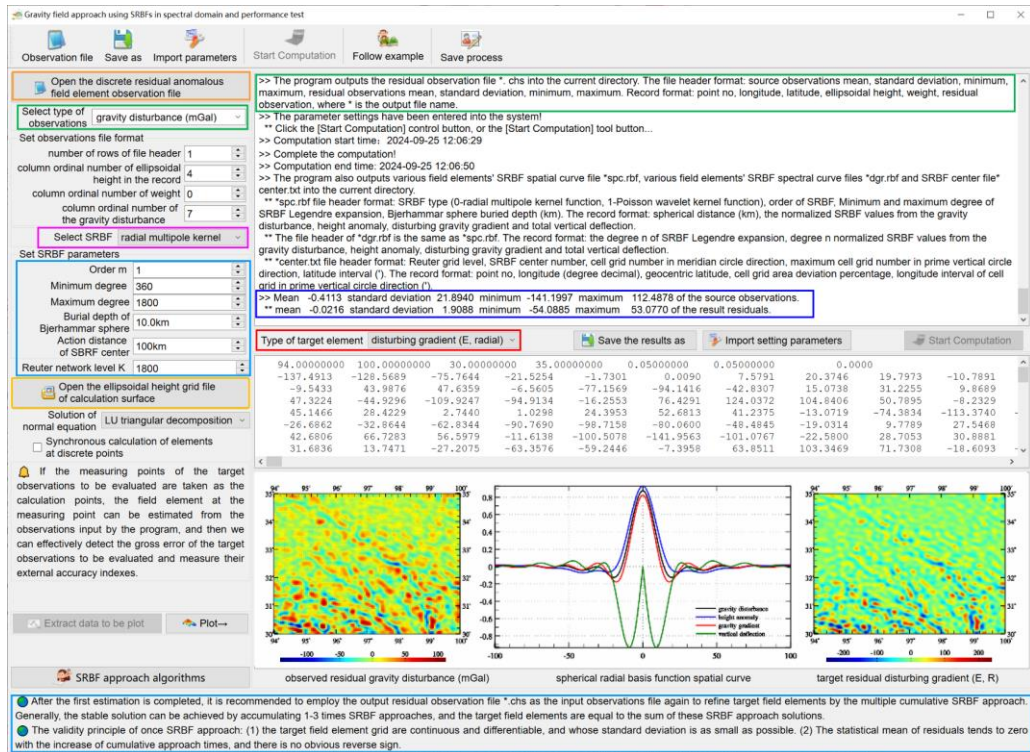
The validity principle of once SBRF approach: (1) the target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

The target field elements are equal to the convolution of the observations and the filter SRBF. When the target field elements and the observations are of different types, it is difficult for one SRBF to effectively match the spectral center and bandwidth of the observations and the target field elements at the same time, which would make the spectral leakage of the target field element. In addition, the SRBF type, minimum and maximum degree of Legendre expansion and SRBF center distribution also all affect the approach performance of gravity field. Therefore, only the optimal estimation of the SRBF coefficients with the burial depth as the parameter is not enough to ensure the best approach of the gravity field.

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of gravity field adopts the SRBF with different spectral figure, the cumulative SRBF approach can fully resolve the spectral domain signal of the target field elements by combining multiple SRBF spectral centers and bandwidths, and then optimally restore the target field elements in space domain.

In the cumulative SRBF approach scheme, it is not necessary to determine the optimal Bjerhammar sphere burial depth. Generally, the 1 to 3 cumulative SRBF approach of gravity field can obtain the stable results.

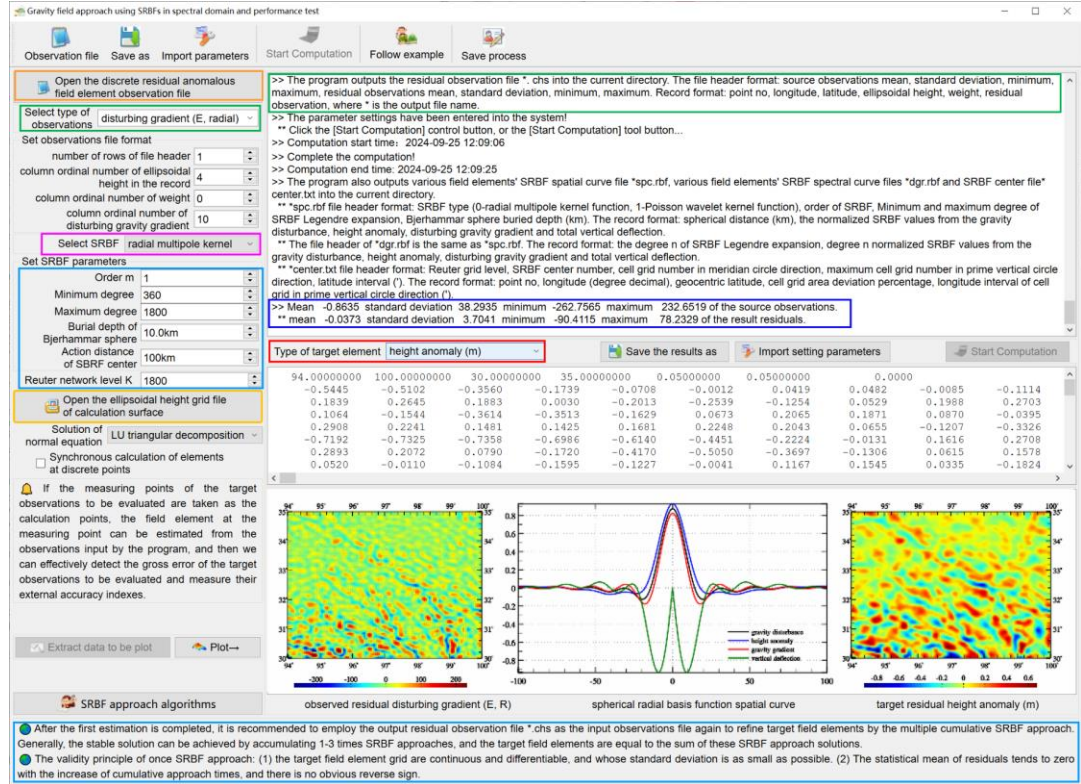


PAGrav4.5 gives the validity principle of once SRBF approach of gravity field: (1) the target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observations file again to refine target field elements by the multiple cumulative SRBF approach. Generally, the stable solution can be achieved by accumulating 1-3 times SRBF approaches, and the target field elements are equal to the sum of these SRBF approach solutions.

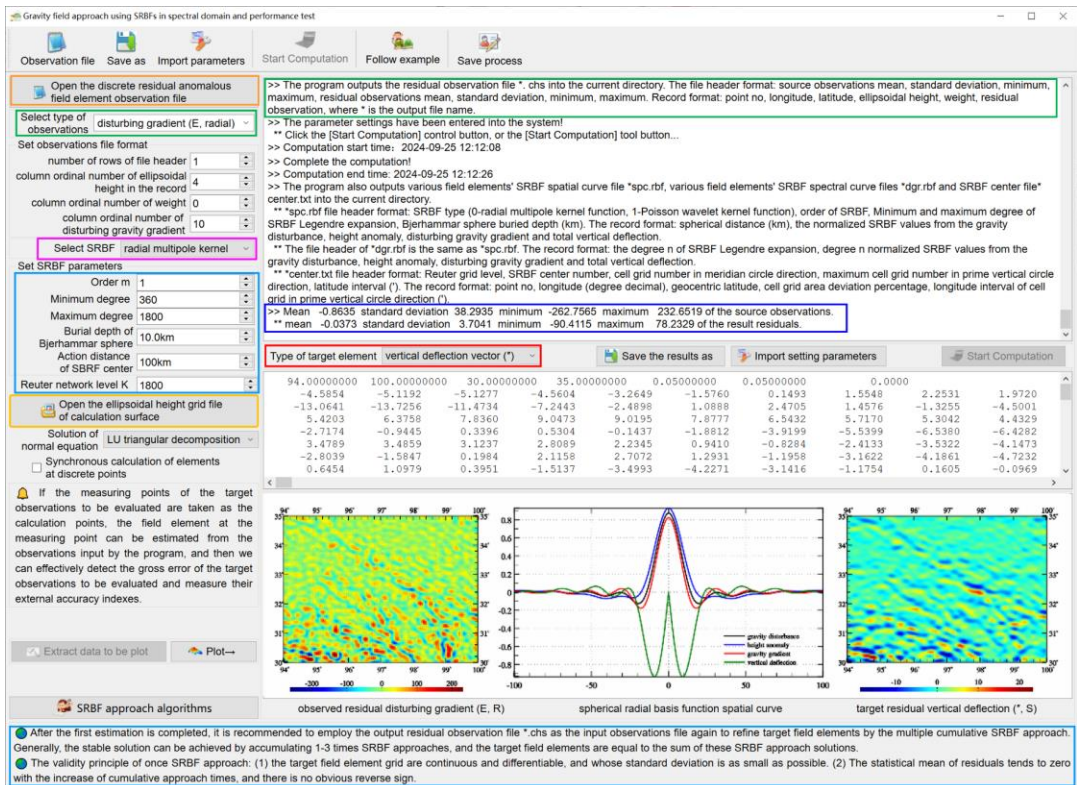
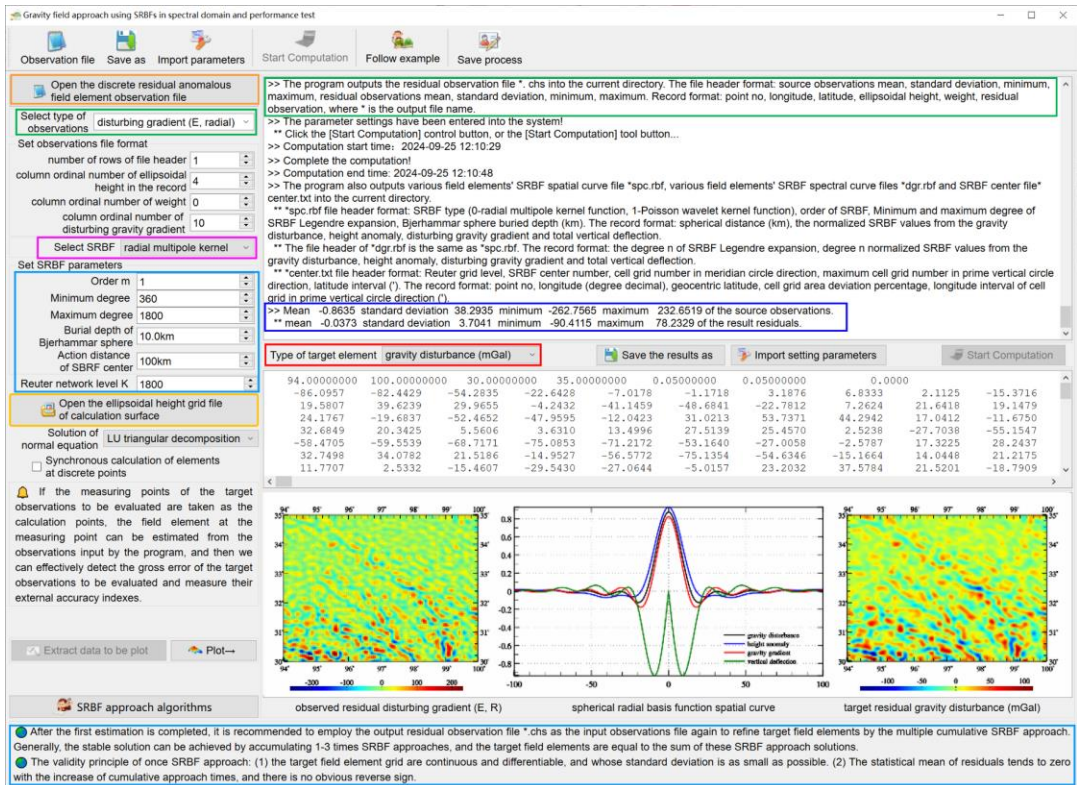
In this example, from one of the residual ground height anomaly, gravity disturbance, vertical deflection and disturbing gravity gradient, one type of residual gravity field elements on ground are estimated using SRBFs with the action distance 120~150km of SRBF center. After the 1° area of the grid margin with edge effect deducted, statistically analyze the 541 to 1800th degree model residuals (regarded as the true reference value), and the difference between the results of the SRBF approach and model reference value,

which can be employed to examine the performance of the algorithm.



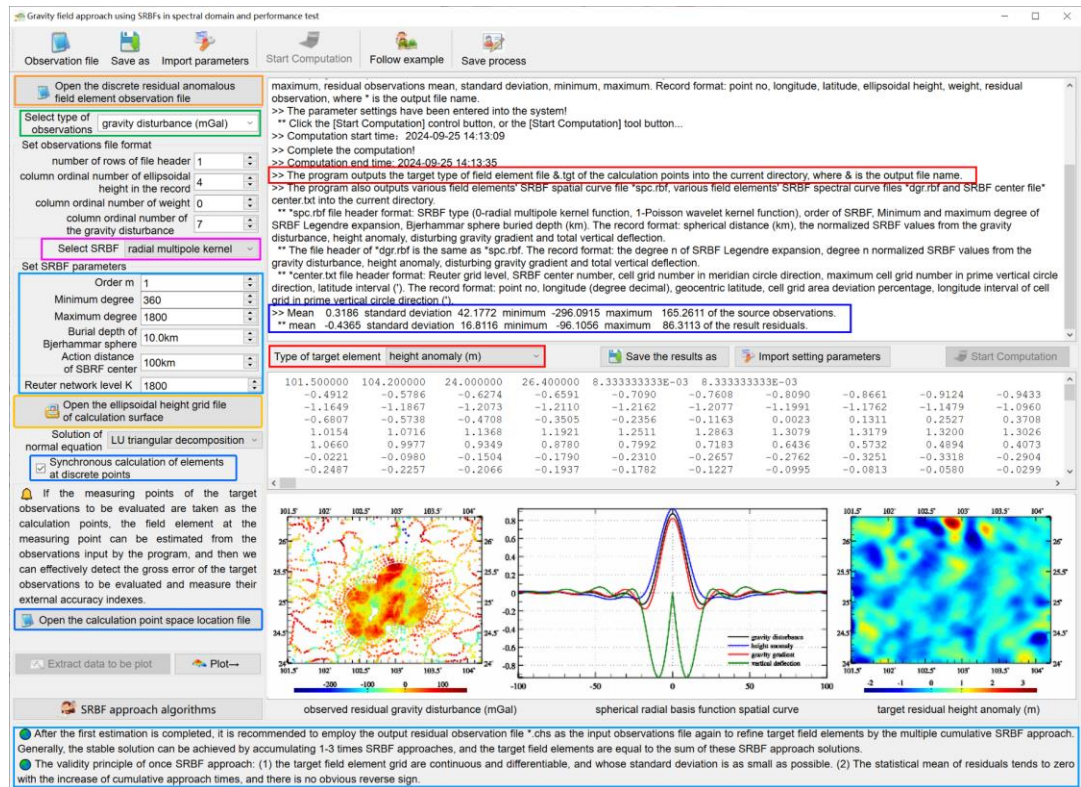
The 541~1800 th degree true reference model value	mean	standard deviation	minimum	maximum
residual geoidal heights (m)	-0.0020	0.1590	-0.8621	0.6546
residual gravity disturbances (mGal)	-0.4113	21.8940	-141.1997	112.4878

Difference between SRBF approach and reference model value		First SRBF approach		Second SRBF approach	
residual observations	target residuals	mean	standard deviation	mean	standard deviation
gravity disturbance	ground height anomaly (m)	0.0201	0.0185	0.0189	0.0162
vertical deflection		0.0058	0.0147	0.0058	0.0072
disturbing gravity gradient		-0.0055	0.0202	-0.0056	0.0185
height anomaly	ground gravity disturbance (mGal)	0.1731	1.4421	0.2075	1.4382
vertical deflection		0.3550	1.6905	0.3566	1.6845
disturbing gravity gradient		0.0840	1.5780	0.0906	1.5298



It can be found that the performance of the primary SRBF approach is better than that of gravity field integral, and the secondary cumulative SRBF approach is significantly better than gravity field integral.

The program itself has a strong capacity to detect the gross errors of discrete observations and directly measure the external accuracy indexes. In this example, only from discrete disturbance gravity observations, the gross errors of observed GNSS-leveilling geoidal heights are detected, and the external accuracy index can be measured directly using SRBFs.

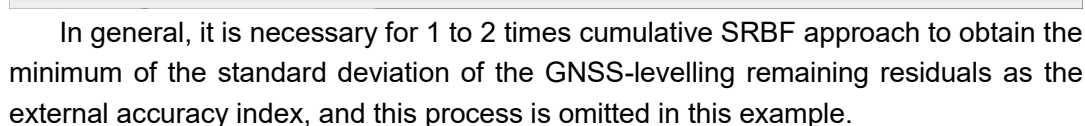


Firstly, calculate and remove the 2~540th degree EGM2008 model gravity field respectively from the observed gravity disturbances and observed GNSS-leveilling geoidal heights to get the observed residual gravity disturbance file rntobsdistgrav.txt as the input observation file and get the observed residual GNSS-leveilling geoidal height file rntGNSSlgeoidh.txt as the input calculation point space location file. Calculate the 2~180th degree EGM2008 model geoidal height grid in target area as the input ellipsoidal height grid file mdlgeoidh30s.dat of the calculation surface. Then call the program to output the residual GNSS-leveilling geoidal height file rntGNSSlgeoidh.tgt into the current directory.

Finally, subtract the observed geoidal height from the estimated geoidal height in record of the file rntGNSSlgeoidh.tgt to obtain the GNSS-leveilling remaining residuals, which is the statistical attribute, and then detect and remove the gross error points beyond 3 times of standard deviation range of the GNSS-leveilling remaining residuals.

Observed GNSS-levelling residuals (m)	number of points	mean	standard deviation	minimum	maximum
Original residuals	125	-0.3510	0.2774	-0.9982	0.3435
Residuals without error	119	-0.3540	0.2647	-0.9982	0.3136
Remaining residuals	119	-0.2651	0.0557 ^①	-0.3951	-0.1154

In the table, 0.0557^①m is the external accuracy index of GNSS-levelling expressed as standard deviation, that is, 5.57 cm, which indicates that the external accuracy of GNSS-leveling is not bad than 5.57 cm (standard deviation).



[Function] From various heterogeneous observations which can be the residual gravity disturbance, height anomaly, gravity anomaly, disturbing gravity gradient, or vertical deflection, determinate the residual gravity disturbance, height anomaly, gravity anomaly, disturbing gravity gradient, and vertical deflection outside geoid using spherical radial basis functions (SRBFs), to realize the unified modelling on regional gravity field and geoid.

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field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

[Input files] The heterogeneous observation file and ellipsoidal height grid file of the calculation surface.

The agreed format of the observation file record: ID (point no / station name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual height anomaly (residual geoidal height, m), 2 - residual gravity anomaly (mGal), 3 - residual disturbing gravity gradient (E, radial), 4 - residual vertical deflection southward component ("), 5 - residual vertical deflection westward component (").

The weights are only employed to distinguish the the same type element of observation errors at different observed points.

There is no limit to the grid resolution of the calculation surface.

[Parameter settings] Set the input observation file format parameters, SRBF parameters and algorithm parameters.

When the column ordinal number of the weight attribute is less than 1, exceeds the column number of the record, or the weight is less than zero, the program makes the weight equal to 1.

When the weight in the file record is equal to zero, the observation will not participate in the estimation of the SRBF coefficients, and the program can be employed to measure the external accuracy index of the observations.

Select the spherical radial basis function: the radial multipole kernel function or Poisson wavelet kernel function. The zero-order radial multipole kernel function is the point mass kernel function, and the zero-order Poisson wavelet kernel function is the Poisson kernel function.

Enter the order m . The order number m of radial multipole kernel function and Poisson wavelet kernel function. The greater the m , the bigger the kurtosis of SRBF.

Input the Bjerhammar sphere burial depth: The depth of the Bjerhammar sphere relative to the mean height surface of the observations, which can be employed to adjust the spectral center and bandwidth of SRBF when combined with the degree of SRBF Legendre expansion.

The greater the burial depth, the smoother the SRBF, the smaller the kurtosis namely the wider the spectral bandwidth.

Enter the action distance of SRBF center. The action distance is also called as the influence radius = spherical angular distance \times the Bjerhammar spherical radius, which is equivalent to the integral radius for local gravity field.

A fixed action distance is adopted to ensure the coordination and consistency of the spatial and spectral figure of regional gravity field.

Set the Reuter network level K: The spherical surface is divided into K prime vertical circles, and the latitude interval is $180^\circ/K$. The larger the K, the greater the spatial resolution of the spherical Reuter network. The suitable $180^\circ/K$ is approximately equal to the average distance between observation points.

Enter minimum and maximum degree of SRBF Legendre expansion. Minimum and maximum degree can be employed to adjust SRBF bandwidth.

Select the type of the adjustable observations and set the contribution rate κ of the adjustable observations.

The program multiplies the normal equation coefficient matrix and constant matrix of the adjustable observations by κ , respectively, to increase ($\kappa > 1$) or decrease ($\kappa < 1$) the contribution of the adjustable observations. When $\kappa = 1$, it means that there are not any adjustable observations selected. When $\kappa = 0$, the adjustable observations do not participate in the estimation of SBRF coefficients.

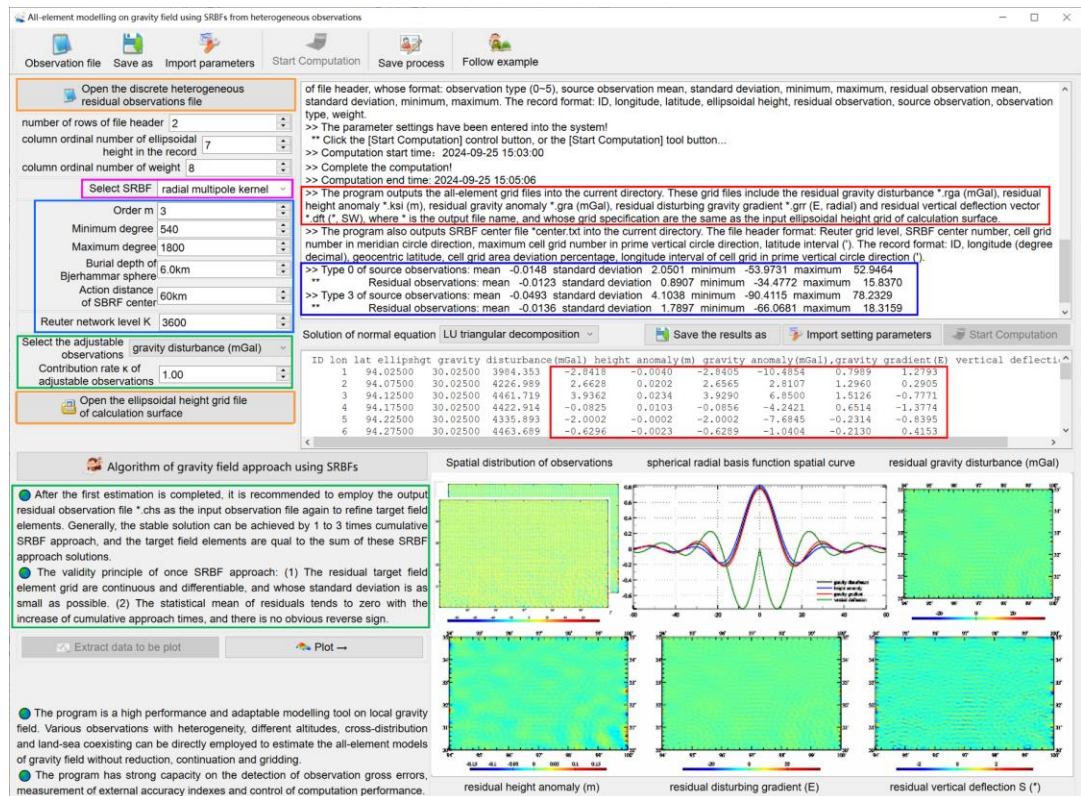
Select the solution of normal equation. LU triangular decomposition method, Cholesky decomposition, smallest norm solution.

The normal equation no longer needs regularization and iterative computation.

[Output file] The all-element gravity field file.

The file record: ID, longitude (degree decimal), latitude, ellipsoidal height (m) of

calculated point, residual gravity disturbance (mGal), residual height anomaly (m), residual gravity anomaly (mGal), residual disturbing gravity gradient (E, radial) and residual vertical deflection ($"$, SW).

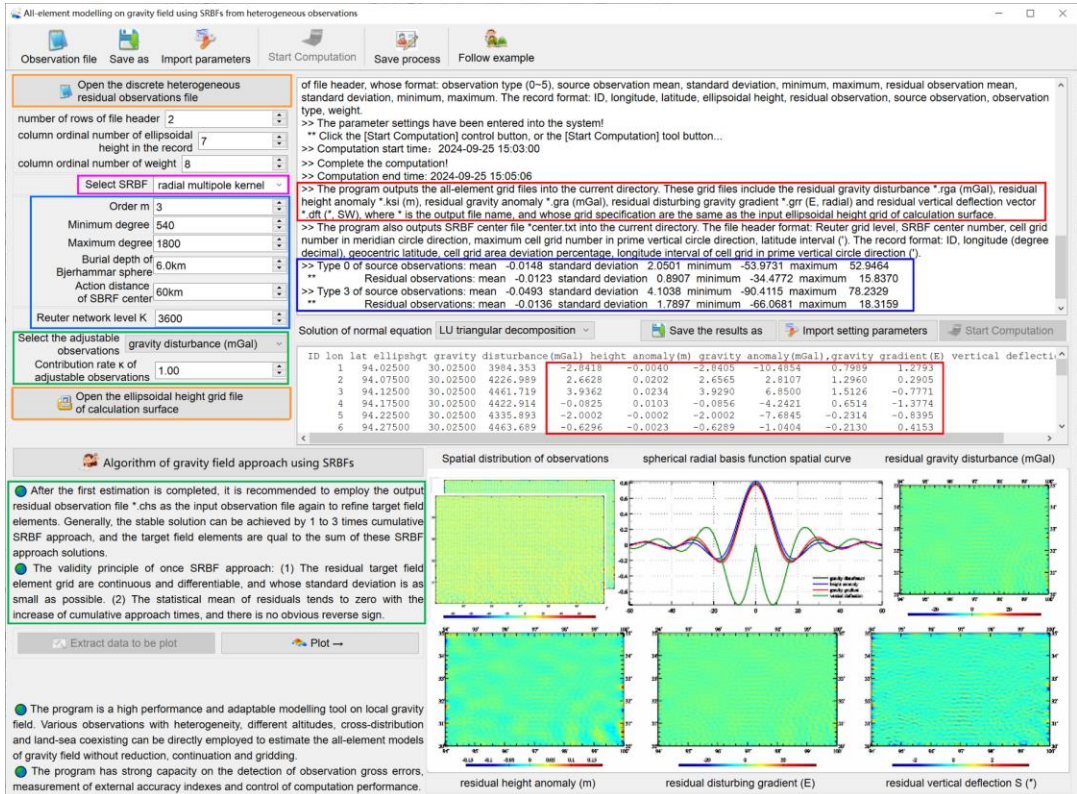


The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *.rga (mGal), residual height anomaly *.ksi (m), residual gravity anomaly *.gra (mGal), residual disturbing gravity gradient *.grr (E, radial) and residual vertical deflection vector *.dft ($"$, SW), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.

The program also outputs the residual observation file *.chs into the current directory. The statistical results of each type of observations occupies a row of file header, whose format: observation type (0~5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.

The program also outputs SRBF center file *.center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell-grid number in meridian circle direction, maximum cell-grid number in prime vertical circle direction, latitude interval ($"$). The record format: ID, longitude (degree decimal), geocentric latitude, cell-grid area deviation percentage, longitude interval of cell-grid in prime vertical circle

direction (').



PAGrav4.5 proposes a normalized method for normal equations to combine different types of heterogeneous observations for estimation of the SRBF coefficients in stead of the usual iterative scheme of the variance component estimation, so that the properties of the parameter estimation solution are only related to the spatial distribution of the observations without influence of observation errors. Which is conducive to combination of various types of observations with extreme differences in spatial distribution, such as a very small number of astronomical vertical deflections or GNSS-levelling data.

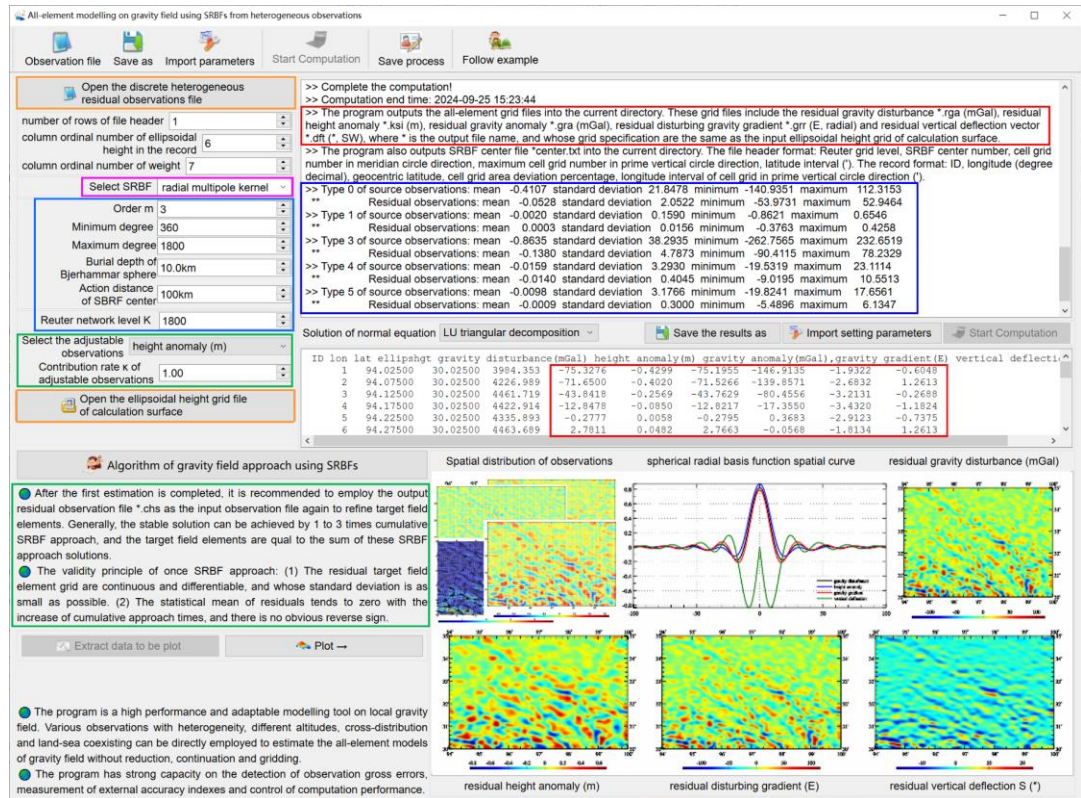
In this case, the normal equation does not also need to be iteratively resolved, which conducive to improve the analytical nature of SRBF approach algorithm.

PAGrav4.5 gives the validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Generally, 1 to 3 cumulative SRBF approaches on gravity field can obtain the stable approach results.

Selecting the adjustable observation and its contribution rate κ , we can effectively deal with the problem of high-precision gravity field approach from heterogeneous observations with extreme differences in spatial distribution, quality and accuracy. With

$\kappa = 0$ for some a type of observations, we can effectively detect the gross errors and evaluate the quality and external accuracy. With $\kappa > 1$ for several high-precision observations, we can effectively improve the contribution of the observations such as the astronomical vertical deflection or GNSS-leveilling observations.



In this example, from the residual ground height anomaly, gravity disturbance, vertical deflection or disturbing gravity gradient, the residual ground height anomaly is calculated using SRBFs with the action distance 150 km of SRBF center. After the 1° area of the grid margin with edge effect deducted, statistically analyze the 541~1800th degree model residuals (regarded as the true reference value), and the difference between the results of the SRBF approach and the model reference value, which can be employed to examine the performance of the algorithm.

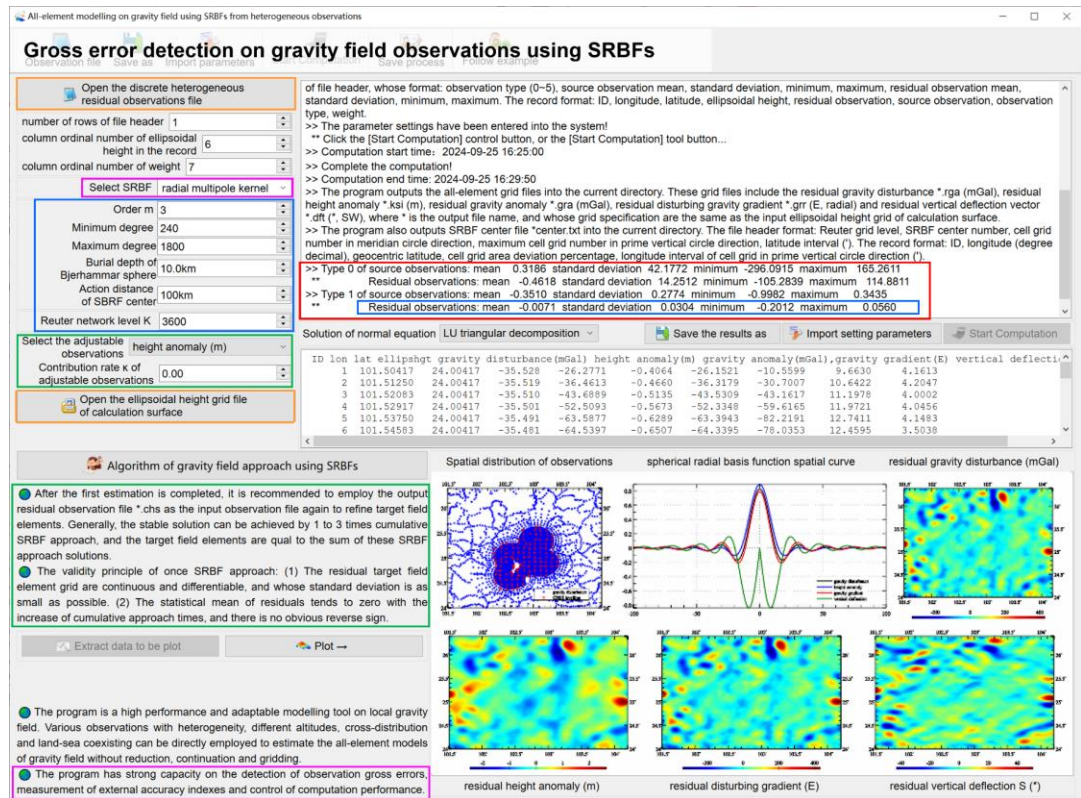
The 541~1800 th degree true reference model value	mean	standard deviation	minimum	maximum
residual geoidal heights (m)	-0.0020	0.1590	-0.8621	0.6546
residual gravity disturbances (mGal)	-0.4113	21.8940	-141.1997	112.4878
residual disturbing gravity gradients (E)	-0.8635	38.2935	-262.7565	232.6519

Difference between SRBF approach and reference model value		First SRBF approach		Second SRBF approach	
residual observations	target residuals	mean	standard	mean	standard

			deviation		deviation
gravity disturbance	ground height anomaly (m)	0.0023	0.0095	0.0005	0.0093
height anomaly					
gravity disturbance	ground height anomaly (m)	0.0036	0.0190	0.0031	0.0158
disturbing gravity gradient					

In addition, by setting the zero contribution rate ($\kappa = 0$) or zero weight ($p = 0$) of the observations to be investigated, and making full usage of the residual observation files *.chs output during the operation of the program, we can effectively deal with the typical troubles such as the observation gross error detection, measurement of external accuracy indexes, computation performance control and result quality assessment in various complex cases.

In this example, only from discrete disturbance gravity observations, the gross error of observed GNSS-leveilling geoidal heights is detected, and its external accuracy index can be measured directly using SRBFs. The process is shown in process2.txt.



Firstly, calculate and remove the 2~540th degree EGM2008 model gravity field respectively from the observed gravity disturbances and observed GNSS-leveilling geoidal heights, and generate the heterogeneous residual observation file obsresiduals0.txt. Calculate the 2~180th degree EGM2008 model geoidal height grid as the ellipsoidal height grid file mdlgeoidh30s.dat of calculation surface. Then call the

program, select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$ to get the remaining residual file GNSSlerrpk0.chs.

Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbance from the remaining residual file GNSSlerrpk0.chs, detect and remove the observation gross error points beyond 3 times of standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 of times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresidnoerr.txt.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresidnoerr.txt and call the program again to get the new remaining residual file GNSSlerrpk02.chs.

Since the contribution rate $\kappa = 0$ of GNSS-levelling observations is set in advance, it is essentially here to directly determine the external accuracy index of the GNSS-levelling observations using only discrete gravity disturbance observations. Before and after gross error removed, the statistical results on the observation residuals are as follows.

		number of points	mean	standard deviation	minimum	maximum
Gravity disturbance (mGal)	Original residuals	4219	0.3186	42.1772	-296.0915	165.2611
	Residuals without error	4213	0.3071	42.0482	-296.0915	165.2611
	Remaining residuals	4213	-0.4584	13.6071	-61.1040	64.8276
GNSS levelling geoidal height (m)	Original residuals	125	-0.3510	0.2774	-0.9982	0.3435
	Residuals without error	123	-0.3443 ^①	0.2745 ^③	-0.9982	0.3435
	Remaining residuals	123	-0.0070 ^②	0.0214 ^④	-0.0729	0.0577

The statistical mean ① minus ② of the remaining GNSS-levelling residuals in the table, that is, $-0.3443^{①} - (-0.0070^{②}) = -0.3373\text{m}$, is the difference between the regional height datum and global height datum (gravimetric geoid). Here provides the SRBF measurement method for regional height datum difference.

In the table, $0.2745^{③}\text{m} = 27.45\text{cm}$ can represent the accuracy index of the model geoidal height from the 2~540th degrees EGM2008 model.

$0.0214^{④}\text{m}$ is the external accuracy index of GNSS-levelling observations expressed as standard deviation, that is, 2.14 cm. Here provides the SRBF measurement method for the external accuracy index of GNSS-leveling observations. The result indicates that the external accuracy of GNSS-leveling observations is not bad than 2.14 cm (standard deviation).

Measurement of height datum difference using SRBFs

Open the discrete heterogeneous residual observations file

number of rows of file header: 1
column ordinal number of ellipsoid height in the record: 6
column ordinal number of weight: 7

Select SRBF: radial multipole kernel

Order m: 3
Minimum degree: 240
Maximum degree: 1800
Burial depth of Bjerrum sphere: 10.0km
Action distance of SBRF center: 100km
Reuter network level K: 3600

Select the adjustable observations: height anomaly (m)
Contribution rate k of adjustable observations: 0.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.

>> The parameter settings have been entered into the system!
>> Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Complete the computation!
>> Computation start time: 2024-09-25 16:34:12
>> Computation end time: 2024-09-25 16:39:29

>> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *rga (mGal), residual height anomaly *ksa (m), residual gravity anomaly *gra (mGal), residual disturbing gravity gradient *gr (E, radial) and residual vertical deflection vector *dft (*, SV), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.

>> The program also outputs SRBF center file *center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval (*). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (*).

>> Type 0 of source observations: mean 0.3071 standard deviation 42.0482 minimum -296.0915 maximum 165.2611
Residual observations: mean -0.4584 standard deviation 13.6071 minimum -61.1040 maximum 64.8276
>> Type 1 of source observations: mean -0.3443 standard deviation 0.2745 minimum -0.9982 maximum 0.3435
Residual observations: mean -0.0070 standard deviation 0.0214 minimum -0.0729 maximum 0.0577

Solution of normal equation LU triangular decomposition

Save the results as Import setting parameters Start Computation

ID	lon	lat	ellipshgt	gravity disturbance (mGal)	height anomaly (m)	gravity anomaly (mGal)	gravity gradient (E)	vertical deflection
1	101.50417	24.00417	-35.528	-25.0111	-0.4050	-25.6965	-10.5496	9.1444
2	101.51250	24.00417	-35.519	-34.2343	-0.4580	-34.0934	-25.9194	10.0077
3	101.52083	24.00417	-35.510	-41.6971	-0.5069	-41.5412	-38.8251	10.6429
4	101.52917	24.00417	-35.501	-50.3166	-0.5602	-50.1443	-54.5962	11.4401
5	101.53750	24.00417	-35.491	-61.0024	-0.6207	-60.8115	-75.9916	12.2335
6	101.54583	24.00417	-35.481	-62.1031	-0.6435	-61.9052	-72.0511	12.0208

Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.

The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

All-element modelling on gravity filed From observed gravity disturbance and GNSS-leveilling geoidal height using SRBF

Open the discrete heterogeneous residual observations file

number of rows of file header: 1
column ordinal number of ellipsoid height in the record: 6
column ordinal number of weight: 7

Select SRBF: radial multipole kernel

Order m: 3
Minimum degree: 240
Maximum degree: 1800
Burial depth of Bjerrum sphere: 10.0km
Action distance of SBRF center: 100km
Reuter network level K: 3600

Select the adjustable observations: height anomaly (m)
Contribution rate k of adjustable observations: 1.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.

>> The parameter settings have been entered into the system!
>> Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Complete the computation!
>> Computation start time: 2024-09-25 16:42:57
>> Computation end time: 2024-09-25 16:48:19

>> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *rga (mGal), residual height anomaly *ksa (m), residual gravity anomaly *gra (mGal), residual disturbing gravity gradient *gr (E, radial) and residual vertical deflection vector *dft (*, SV), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.

>> The program also outputs SRBF center file *center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval (*). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (*).

>> Type 0 of source observations: mean 0.3071 standard deviation 42.0482 minimum -296.0915 maximum 165.2611
Residual observations: mean -0.2139 standard deviation 12.7187 minimum -60.1001 maximum 64.8276
>> Type 1 of source observations: mean -0.0070 standard deviation 0.2745 minimum -0.6609 maximum 0.6808
Residual observations: mean -0.0003 standard deviation 0.0232 minimum -0.0794 maximum 0.0535

Solution of normal equation LU triangular decomposition

Save the results as Import setting parameters Start Computation

ID	lon	lat	ellipshgt	gravity disturbance (mGal)	height anomaly (m)	gravity anomaly (mGal)	gravity gradient (E)	vertical deflection
1	101.50417	24.00417	-35.528	-36.3117	-0.3491	-36.2043	-45.0018	7.8888
2	101.51250	24.00417	-35.519	-43.6862	-0.3963	-43.5642	-57.7869	8.6648
3	101.52083	24.00417	-35.510	-50.4192	-0.4407	-50.2837	-68.8737	9.2166
4	101.52917	24.00417	-35.501	-58.7040	-0.4911	-58.5529	-81.8445	9.9607
5	101.53750	24.00417	-35.491	-68.0315	-0.5449	-67.8639	-101.6327	10.8599
6	101.54583	24.00417	-35.481	-69.9342	-0.5694	-69.7590	-99.7716	10.4463

Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.

The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

The typical technical features of the program

- ① The analytical function relationships between gravity field elements are strict, and the SRBF approach performance has nothing to do with the observation errors.
- ② Various heterogeneous observations in the different altitudes, cross-distribution and land-sea coexisting cases can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.
- ③ Can integrate very few astronomical vertical deflection or GNSS-levelling observations, and effectively absorb the edge effect.
- ④ Has the strong capacity in detection of observation gross errors, measurement of external accuracy indexes and control of computational performance.

4.8. Modelling process exercise of regional gravity field and geoid

4.8.1 Computation process demo of all-element modelling on gravity field by integral method

From the ground digital elevation model and observed gravity disturbances simulated from EGM2008 geopotential model, using a remove-integral-restore scheme combination with residual terrain effects and EGM2008 model, compute the ground height anomaly, ground gravity disturbance and ground disturbing gravity gradient as well as the geoidal height, gravity disturbance and disturbing gravity gradient on the geoid, to show the key problem and computation process of all-element modelling on regional gravity field by integral method in space domain.

• Input and output data and related terrain models

Let terrain data range (extended area, E104.0~111.0°, N24.0~29.0°) \Rightarrow Calculation area (observed gravity point distribution / the boundary surface range, E104.5~110.5°, N24.5~28.5°) \Rightarrow Results area (regional gravity field model results range, E105.0~110.0°, N25.0~28.0°) to absorb the edge effect of integral.

(1) The observed gravity disturbance file Obsgrav.txt.

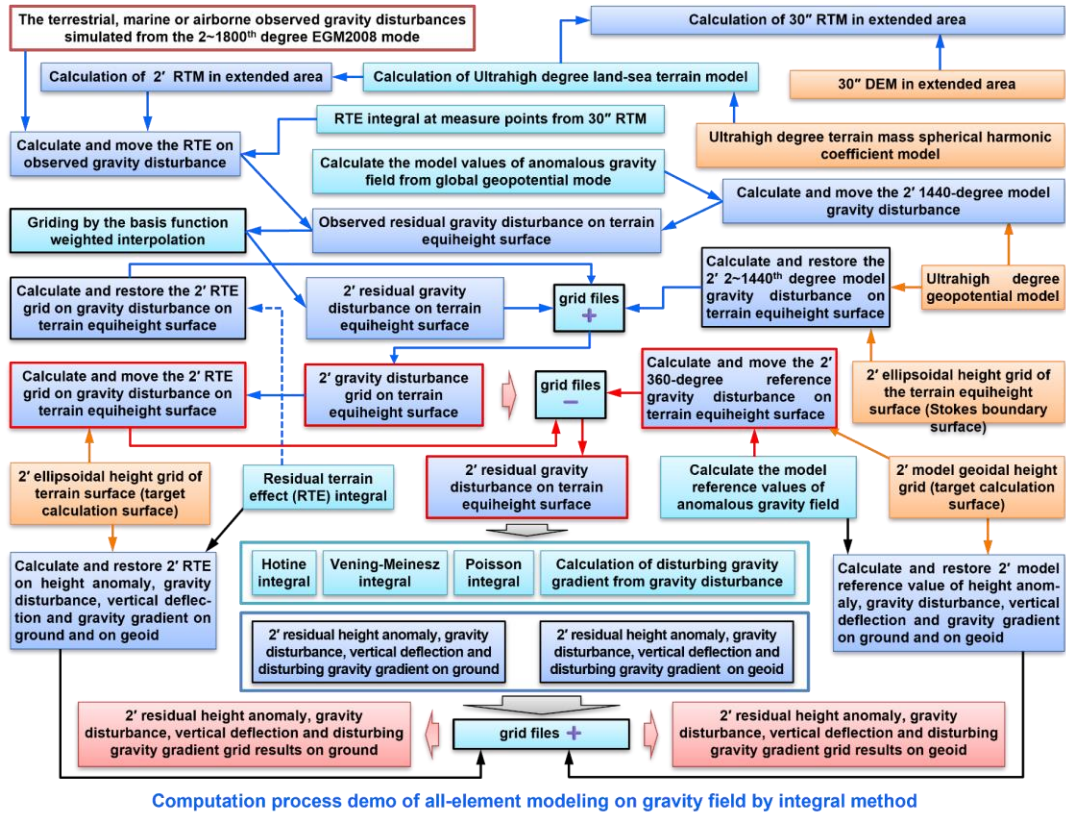
The gravity disturbances at the measurement points are simulated from the 2~1800th degree EGM2008 model. PAGravf4.5 employs the exact same algorithms to process all kinds of terrestrial, marine, and airborne gravity data in an unified way, and there is no need to distinguish whether the measurement point is on the ground, at the air altitude or in the sea area.

The format of the file record: ID (point no/name), longitude (°), latitude (°), ellipsoidal height (m), gravity disturbance (mGal). The distribution of observations is shown in Fig.

(2) The 3600-degree terrain mass spherical harmonic coefficient model file ETOPOcs3600.dat and 2190-degree geopotential coefficient model file EGM2008.gfc.

The two model files are stored in the directory C:\PAGravf4.5_win64en\data. The 3600-degree global land-sea terrain mass spherical harmonic coefficient model ETOPOcs3600.dat is generated by the PAGravf4.5 function [Ultrahigh degree spherical harmonic analysis of global land-sea terrain model] from the global 2'×2' land-sea terrain

model ETOPO2v2g.



(3) The ground digital elevation model (DEM)

Two resolutions of DEM are required. The high-resolution is employed for observation reduction, that is, to calculate and remove the residual terrain effects on the observations. The other resolution is consistent with the target result resolution and is employed to restore the residual terrain effects on the target field elements. In this example, they are 30" and 2' respectively, and the corresponding files are extdtm30s.dat and extdtm2m.dat.

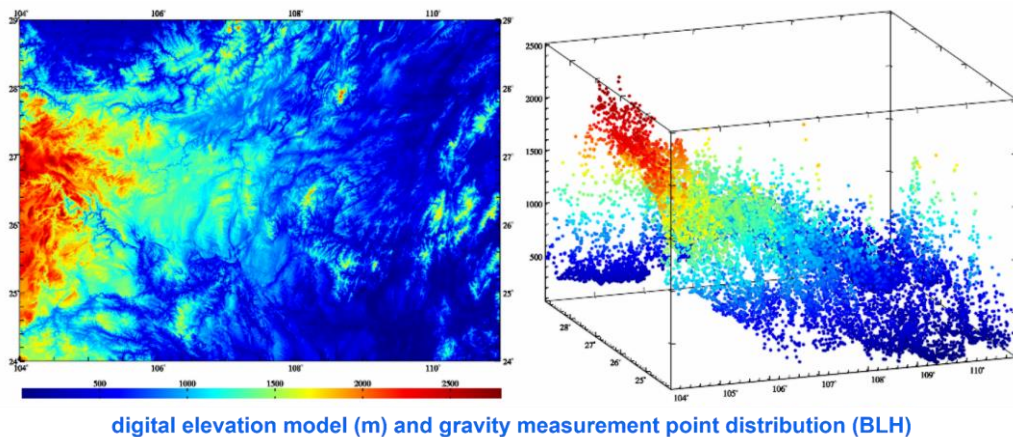
(4) The 30"/2' ellipsoidal height grid files equihgt30s.dat/equihgt2m.dat of terrain equiheight surface.

The ellipsoidal height of the terrain equiheight surface is equal to the sum of the 2~360th degree EGM2008 model height anomaly and mean of the terrain surface normal heights. In this example, the terrain equiheight surface is the reduction surface of the ground observed gravity disturbances, which is also the equipotential boundary surface for solving the Stokes boundary value problem.

When the normal (orthometric) heights of the surface are zero, namely whose ellipsoidal height are geoidal height, the reduction surface and the boundary surface are the geoid in the traditional sense.

One of the main purposes of using the terrain equiheight surface as the boundary surface of the Stokes boundary value problem is to make the ground measurement

points as close as possible to the boundary surface to suppress the attenuation of the ultrashort-wave signal of gravity field.



(5) The 2' geoidal height grid file geoidhgt2m.dat and 2' ground ellipsoidal height grid file surfhgt2m.dat.

The geoidal height grid is employed to stand for the space position of the gravity field element grid on the geoid, calculated from the 2~360th degree EGM2008 model.

The ground ellipsoidal height grid is the sum of the 2~360th degree model ground height anomaly grid and ground digital elevation model grid, which is employed to stand for the space position of the ground gravity field element grids.

(6) Output a series of result grid models of the regional gravity field.

Various anomalous gravity field element grid models on the geoid. The 2'×2' geoidal height, gravity disturbance, vertical deflection vector and disturbing gravity gradient grid models. The geoidal height grid model here also stands for the space position of the anomalous gravity field elements.

Various anomalous gravity field element grid models on the ground. The 2'×2' ground height anomaly, gravity disturbance, vertical deflection vector and disturbing gravity gradient grid models, as well as DEM or ground ellipsoidal height grid model which is employed to stand for the space position of the ground gravity field elements.

● Called functions and input-output data flow

(1) Calculate and remove the model terrain height value, and then construct the 30" and 2' residual terrain model (RTM) grids.

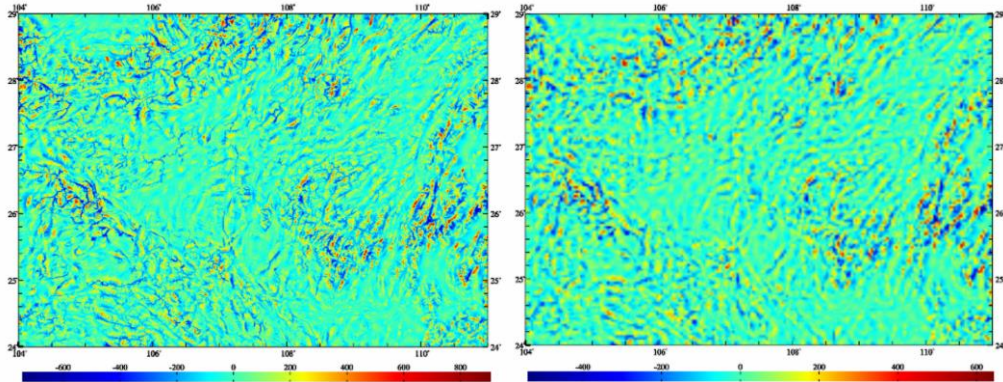
Call the function [Calculation of model value for complete Bouguer or residual terrain effects] with the minimum degree 1 and maximum degree 1800, select the calculation type 'terrain height/sea depth (m)', and input the land-sea terrain mass spherical harmonic coefficient model file ETOPOcs3600.dat, respectively generate 30" and 2' model terrain height grid files mldltm30s.dat and mldltm2m.dat from extdtm30s.dat and extdtm2m.dat.

After extdtm30s.dat minus mldltm30s.dat and extdtm2m.dat minus mldltm2m.dat, the residual terrain models (RTM) resdtm30s.dat and resdtm2m.dat in the extended

area are obtained, as shown in the figure.

When the program output file name is inconsistent with the file name given here, the program output file name need be renamed, and the subsequent ones are the same.

	mean	standard deviation	minimum	maximum
30" RTM (m)	-0.4626	137.2485	-746.0400	908.8900
2' RTM (m)	-0.8250	97.5569	-541.2900	645.0400



30" and 2' residual terrain model (RTM, m)

(2) Calculate and remove the ultrahigh-degree model gravity disturbances at the measurement points.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and maximum degree 1440, input the file EGM2008.gfc and observation file Obsgrav.txt, select the type 'gravity disturbance', and generate the model gravity disturbance file Obsgravmdl.txt (columns 6) at the measurement points.

Subtract the observed gravity disturbance (column 5) and model gravity disturbance (column 6) in Obsgravmdl.txt, to generate the model residual gravity disturbances (column 7) file Obsgravmdlresd.txt at the measurement points.

Table 2 shows the statistical results of the gravity disturbances at the measurement points before and after the 1440-degree model values removed.

Measurement points (mGal)	mean	standard deviation	minimum	maximum
Observed gravity disturbance	-27.7853	28.7143	-147.4878	74.7074
Model residual of gravity disturbance	-0.3743	6.4755	-35.8263	27.4789

(3) Calculate and remove the residual terrain effects on the gravity disturbances at the measurement points.

Call the function [Numerical integral of land-sea residual terrain effects on various gravity field elements], input the observation file (for the convenience of calculation, here adopts the geodetic coordinates of the measurement points in Obsgravmdlresd.txt),

high-resolution DEM extdtm30s.dat, low-pass DEM mdltdtm30s.dat and ground ellipsoidal height grid file surfhgt30s.dat, set the integral radius 60km, and generate the residual terrain effect (RTE) file Obsgravresdtm.txt (column 8).on the gravity disturbances.

Subtract the model residual gravity disturbance (column 7) and its residual terrain effect (column 8) in Obsgravresdtm.txt, to generate the residual gravity disturbances (column 9) file Obsgravresidual.txt at the measurement points.

After the residual terrain effects removed, the residual gravity disturbance statistical results at the measurement points are shown in Table 3.

Measurement points (mGal)	mean	standard deviation	minimum	maximum
Residual gravity disturbance	6.4474	9.7051	-28.6215	79.6853

In this example, the analytical continuation step using the residual radial gradient (within a height difference of 1000m, the value is small) is omitted. In this case, the residual gravity disturbance at the measurement point is equal to that on the equipotential surface.

So far, the reduction processing of the gravity disturbance from the measurement points to the terrain equiheight surface has been complete.

The basic purpose of the statistics in Tables 1 to 3 is to improve the residual terrain effect algorithm and relative parameters according to the gridding optimization criteria. Since the simulated data lacks sufficient ultrashort wave information of the real gravity field, the optimization criterion analysis process is omitted in this example.

(4) Grid the residual gravity disturbance into 2'×2' grid on terrain equiheight surface.

Call the function [Gridding of heterogeneous data by basis function weighted interpolation], select 'equal weights of observations' (the weights can be estimated with the residual terrain effect as the reference attribute in advance), and grid the 9th column of attributes (from the file Obsgravresidual.txt), to generate 2' residual gravity disturbance grid file distgravresidual.dat on the terrain equiheight surface.

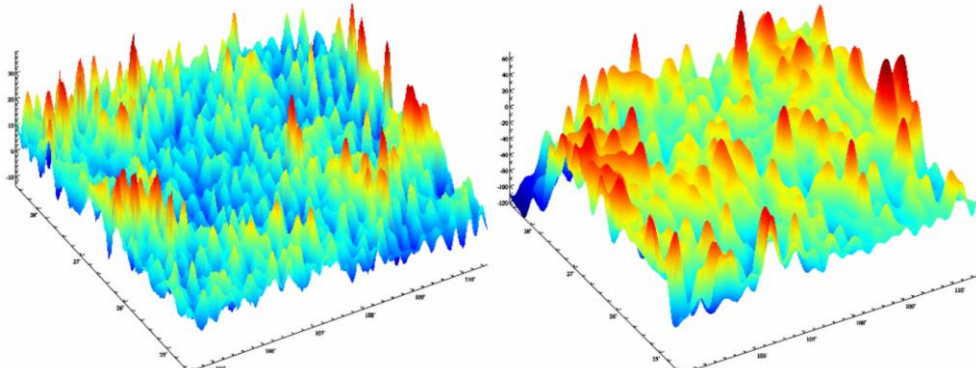
(5) Calculate the 2' EGM2008 1440-degree model value grid of the gravity disturbances on the terrain equiheight surface.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and maximum degree 1440, input the file EGM2008.gfc and ellipsoidal height grid file (equihgt2m01.dat) of the terrain equiheight surface, select the calculation type 'gravity disturbance', to generate 2' model gravity disturbance grid file distgravmdl.dat on the terrain equiheight surface.

Here, the geopotential model and the minimum and maximum degree are required to be the same as in step (2).

Step (2) removes the model gravity disturbances at the ground measurement points, and step (5) restores the model gravity disturbance grid on the reduction surface. The

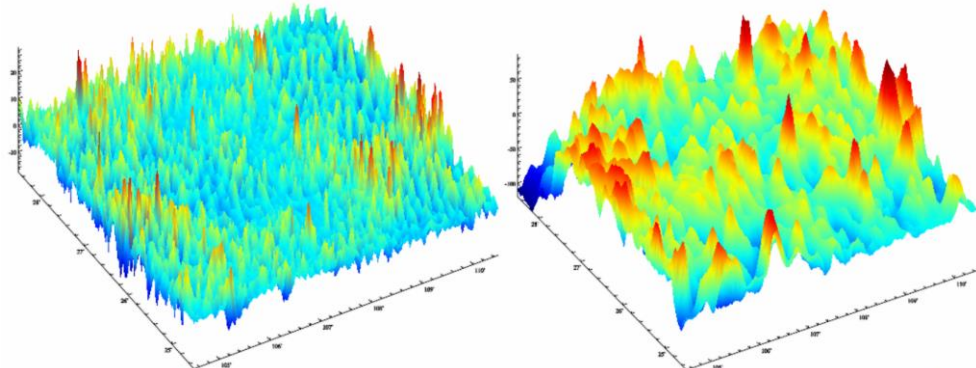
purpose of this remove-store process is to carry out analytical continuation with the help of the ultrahigh-degree geopotential model.



2' residual gravity disturbance (mGal) and 1440-degree model gravity disturbance (mGal) on terrain equiheight surface

(6) Calculate the 2' residual terrain effect grid on the gravity disturbances on the terrain equiheight surface.

Call the function [Numerical integral of land-sea residual terrain effects on various gravity field elements] with `extdtm2m.dat` as the high-resolution DTM and the 2' terrain model grid `mdl2m.dat` as low-pass DTM, input the 2' ground ellipsoidal height grid file `surfhgt2m.dat` (representing the 2' residual terrain mass position) and the ellipsoidal height grid file `equihgt2m01.dat` of the terrain equiheight surface, select the type 'gravity disturbance', set the integral radius 60km, and generate the 2' residual terrain effect grid file `distgravresidtm.dat` of the gravity disturbances on the terrain equiheight surface.



2' residual terrain effect on gravity disturbance and 1440-degree model gravity disturbance (mGal) on terrain equiheight surface

Here, the integral radius is required to be the same as in step (3).

(7) Generate the 2' gravity disturbance grid model on the terrain equiheight surface.

Sum up the residual grid `distgravresidual.dat`, the ultrahigh degree model value grid `distgravmdl.dat` and the residual terrain effect grid `distgravresidtm.dat` three grid files of the gravity disturbance with the same grid specifications, to generate the 2' gravity disturbance grid model on the terrain equiheight surface.

Steps (1) to (7) are the process of regional gravity field data reduction and processing, and the subsequent steps are the gravity field approach and modelling process according to the geodetic boundary value theory.

(8) According to the requirements of the gravity field approach, select the residual terrain model bandwidth (in this example, the maximum degree of the model terrain is 1440), recalculate the 2' residual terrain model `resdtm2m1.dat`, and then call the [Numerical integral of land-sea residual terrain effects on various gravity field elements] program, set the integral radius 60km, and calculate the 2' residual terrain effects on gravity disturbance on the terrain equiheight surface `equalsgravrtm.dat`.

The terrain effect optimization criteria for gravity field approach are different from that for gridding of anomalous gravity field elements, so those residual terrain models are generally constructed differently. However, it is difficult to process and analyze these problems from the simulated data, and the maximum degree of the model terrain here is directly taken as 1440.

(9) Calculate the 2~720th degree model reference value grid of the gravity disturbance on the terrain equiheight surface.

Call the function [Calculation of gravity field elements from global geopotential model] with the minimum degree 2 and maximum degree 720, input the file `EGM2008.gfc` and the ellipsoidal height grid file (`equihgt2m01.dat`) of the terrain equiheight surface, select the calculation type 'gravity disturbance', and generate 2' model gravity disturbance grid file `equdisgravmdl.dat` on the terrain equiheight surface.

The ultrahigh-degree geopotential model (1440 degree) in steps (2) and (5) is employed for analytical continuation, and the 2~720th degree geopotential model here is employed as the reference gravity field for regional gravity field integral.

(10) Calculate the 2' residual gravity disturbance grid on the terrain equiheight surface (remove the residual terrain effect and model reference value from gravity disturbance grid).

From the 2' gravity disturbance grid `equdistgrav.dat` on the terrain equiheight surface, subtract the 2' residual terrain effects `equdisgravrtm.dat`, and then subtract the 2~720th degree model reference value grid `equdisgravmdl.dat`, to generate the 2' residual gravity disturbance grid `equgravresidual.dat` on the terrain equiheight surface.

Terrain equiheight surface (mGal)	mean	standard deviation	minimum	maximum
2' gravity disturbance grid	-17.7675	23.3340	-119.4421	86.6984
Residual terrain effects	-1.6478	4.5234	-31.5185	22.3032
720-degree model values	-23.5523	19.5560	-112.3400	31.4088
Residual gravity disturbances	7.4326	13.1966	-56.9378	71.9857

The basic purpose of statistics here is to improve the residual terrain effect algorithm and parameters according to the gravity field approach optimization criterion (the optimization goal: the standard deviation of residual gravity disturbances in Table 4 is

the smallest, and the statistical mean is close to zero). The simulated data lacks sufficient ultrashort wave information of the real gravity field, so the optimization criterion analysis process is omitted in this example.

(11) Call the relevant gravity field integral function to calculate various types of residual anomalous gravity field elements on the ground and on the geoid, respectively.

When calculating the ground residual field elements, input the ground ellipsoidal height grid `surfhgt2m2.dat`, while when calculating the residual field elements on the geoid, input the model geoidal height grid `geoidhgt2m2.dat` calculated from the 360-degree EGM2008 model. The integral radius is 90km.

- Call the function [External height anomaly computation using generalized Hotine integral] to calculate the ground residual height anomaly and residual geoidal height grid `surfksiresidual.dat` and `geoidhgtresidual.dat`, respectively.

- Call the function [Computation of external vertical deflection from gravity disturbance] to calculate the ground residual vertical deflection grid `surfdftresidual.dat` and geoidal residual vertical deflection grid `geoiddftresidual.dat`, respectively.

- Call the function [Computation of Poisson integral on external anomalous gravity field element] to calculate the ground residual gravity disturbance grid `surfgraresidual.dat` and geoidal residual gravity disturbance grid `geoidrgaresidual.dat`, respectively.

- Call the function [Computation of external disturbing gravity gradient from gravity disturbance] to calculate the ground residual disturbing gravity gradient grid `surfgrresidual.dat` and geoidal residual disturbing gravity gradient grid `geoidgrresidual.dat`, respectively.

(12) Calculate the residual terrain effects of various anomalous field elements on the ground and on the geoid, respectively (to restore the residual terrain effects of various field elements).

Call the function [Numerical integral of land-sea residual terrain effects on various gravity field elements] with simultaneously selecting the height anomaly, gravity disturbance, vertical deflection and disturbing gravity gradient as the field element types. When calculating the residual terrain effect on the ground field elements, input the ground ellipsoidal height grid `surfhgt2m.dat`, and when calculating the residual terrain effect on the geoidal field elements, input the model geoidal height grid `geoidhgt2m.dat`.

Here, the integral radius and RTM `resdtm2m.dat` are required to be the same as in step (8).

The output grid files of the residual terrain effect (RTE) on the ground field elements include the RTE on height anomaly `surfhgt2mrtm.ksi`, RTE on gravity disturbance `surfhgt2mrtm.rga`, RTE on vertical deflection vector `surfhgt2mrtm.dft`, and RTE on disturbing gravity gradient `surfhgt2mrtm.grr`.

The output grid files of the residual terrain effect (RTE) on the geoidal field elements

include the RTE on geoidal height geoidhgrtm.ksi, RTE on gravity disturbance geoidhgrtm.rga, RTE on vertical deflection vector geoidhgrtm.dft, and RTE on disturbing gravity gradient geoidhgrtm.grr.

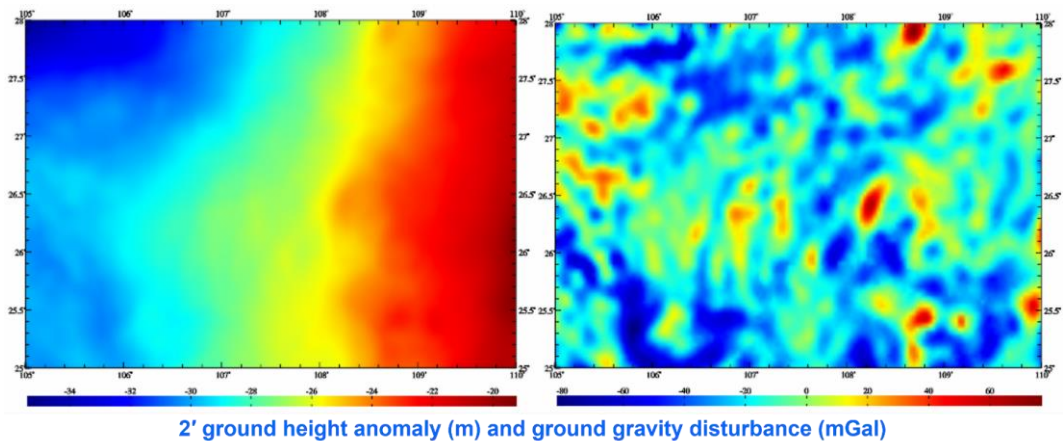
(13) Calculate the 2~720th degree model reference value of various anomalous field elements on the ground and on the geoid, respectively (to restore the model reference value of various field elements).

Call the function [Calculation of gravity field elements from global geopotential model] with simultaneously selecting the height anomaly, gravity disturbance, vertical deflection and disturbing gravity gradient as the field element types. When calculating the model reference value of the ground field elements, input the ground ellipsoidal height grid surfhgt2m.dat, and when calculating the model reference value of the geoidal field elements, input the model geoidal height grid geoidhgt2m.dat.

Here, the minimum degree is 2 and the maximum degree is 720, which to be the same as in step (9).

The output grid files of the model reference value (MRV) of the ground field elements include the MRV of height anomaly surfhgt2mgm720.ksi, MRV of gravity disturbance surfhgt2mgm720.rga, MRV of vertical deflection vector surfhgt2mgm720.dft, and MRV of disturbing gravity gradient surfhgt2mgm720.grr.

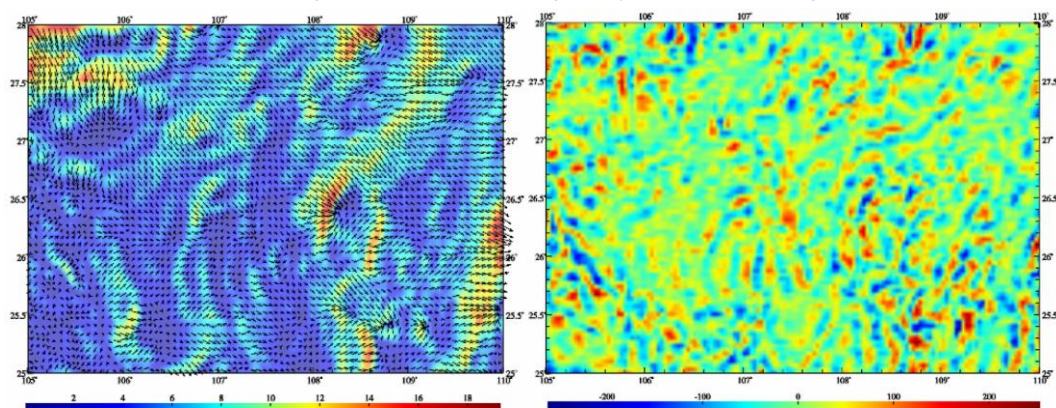
The output grid files of the model reference value (MRV) of the geoidal field elements include the MRV of geoidal height geoidh2mgm720.ksi, MRV of gravity disturbance geoidh2mgm720.rga, MRV of vertical deflection vector geoidh2mgm720.dft, and MRV of disturbing gravity gradient geoidh2mgm720.grr.



(14) Generate the target result grid of various anomalous gravity field elements on the ground.

The residual field element grid, residual terrain effect grid and model reference value grid of various gravity field elements on the ground are respectively summed together to generate the 2' ground height anomaly grid surfhgtksi2m.dat, ground gravity disturbance grid surfhgtgrga2m.dat, ground vertical deflection vector grid surfhgtgdf2m.dat and ground disturbing gravity gradient grid surfhgtgrr2m.dat grid, as

well as the ground ellipsoidal height grid (indispensable) which is employed to specify the space position of the ground anomalous gravity field element grid.



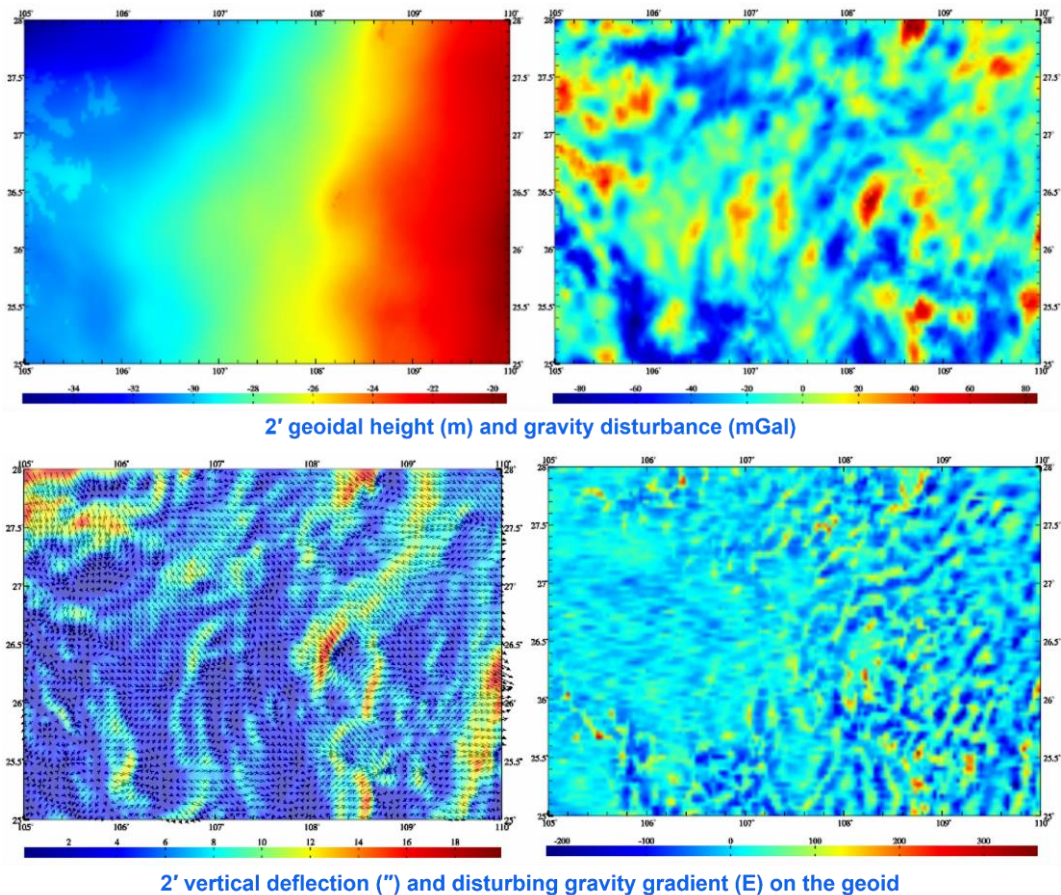
2' ground vertical deflection (") and ground disturbing gravity gradient (E)

Element type	Component	mean	standard deviation	minimum	maximum
Ground height anomaly (m)	MRV	-27.4261	3.2927	-36.0901	-20.1879
	RTE	-0.0008	0.0314	-0.1219	0.1363
	Residual	0.6136	0.1608	0.2218	1.2427
	Total	-26.8133	3.3149	-35.4331	-19.1927
Ground gravity disturbance (mGal)	MRV	-22.4136	15.7881	-76.2475	18.5218
	RTE	-0.7256	0.6856	-7.8832	1.8187
	Residual	6.8912	11.4588	-35.1321	65.7856
	Total	-16.2480	18.7702	-81.8505	77.2212
Ground vertical deflection (", S)	MRV	1.5543	3.0688	-5.7396	14.0662
	RTE	0.1340	0.7770	-3.5615	3.6938
	Residual	-0.0232	1.4285	-7.3171	6.1334
	Total	1.6649	3.2346	-9.5143	16.4299
Ground vertical deflection (", W)	MRV	-4.3753	2.7632	-12.4916	5.1844
	RTE	-0.0525	0.9824	-4.3215	5.2168
	Residual	0.0222	1.6118	-7.6123	6.2307
	Total	-4.4050	3.2895	-18.4035	6.0986
Ground disturbing gravity gradient (E)	MRV	-0.2116	9.6634	-34.2497	32.3422
	RTE	-0.0986	49.3177	-269.7075	232.3091
	Residual	-0.0622	15.1553	-62.7129	112.4539

	Total	-0.3736	52.1555	-262.5373	253.7105
--	-------	---------	---------	-----------	----------

(15) Generate the target result grid of various anomalous field elements on geoid.

The residual field element grid, residual terrain effect grid and model reference value grid of various gravity field elements on the geoid are respectively summed together to generate the 2' geoidal height grid geoidhksi2m.dat, geoidal gravity disturbance grid geoidhrga2m.dat, geoidal vertical deflection vector grid geoidhfft2m.dat and geoidal disturbing gravity gradient grid geoidhgrr2m.dat.



4.8.2 Simple process demo of all-element modelling on gravity field using SRBFs in orthometric height system

Exercise purpose: From the observed terrestrial, marine and airborne gravity disturbances and GNSS-leveling geoidal heights in orthometric height system, make the all-element models on gravity field using spherical radial basis functions (SRBFs) in six steps, in which all the terrain effects are not processed, to quickly master the essentials in observation analysis, computation quality control and all-element modelling on regional gravity field.

After the terrain effect processing omitted, SRBF approach process of gravity field is very simple because there is no need for additional continuation reduction, gridding and

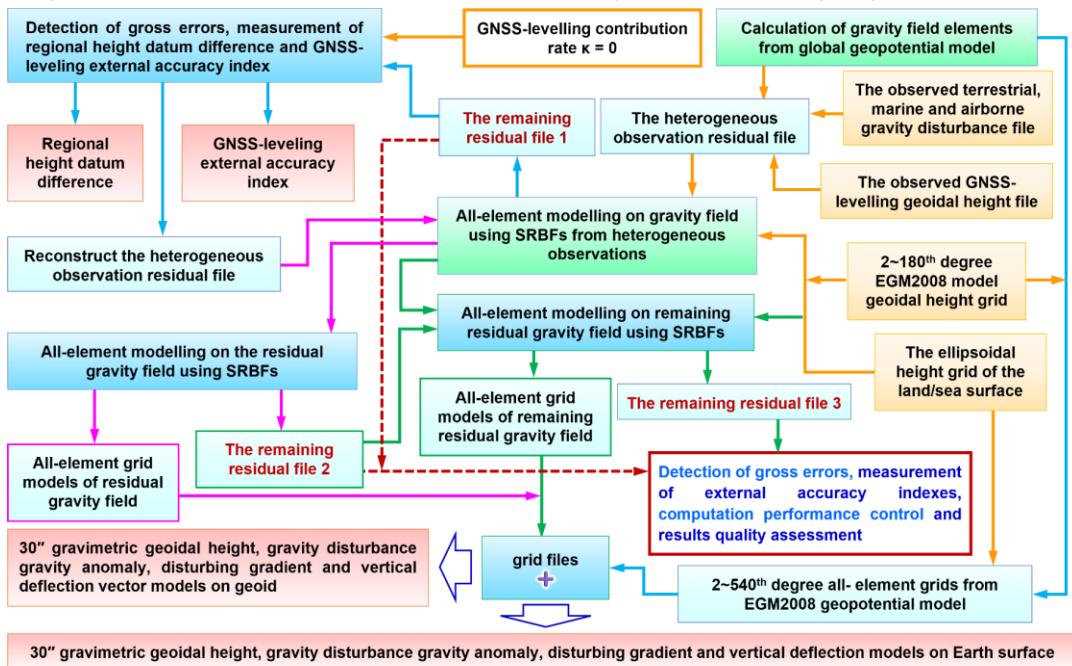
GNSS leveling fusion process.

- **The observed gravity disturbance and GNSS-leveling data**

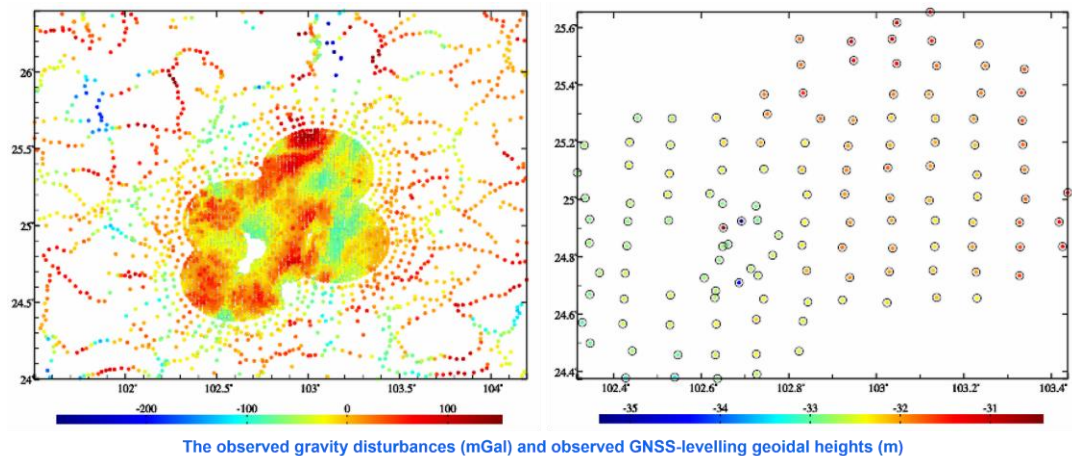
The observed terrestrial, marine and airborne gravity disturbance file obsdistgrav.txt. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed gravity disturbance (mGal), ...

The observed GNSS-leveling geoidal height file obsGNSSlgeoid.txt in orthometric height system. The file record format: ID, longitude (degree decimal), latitude, observed geoidal height (m), ...

In the example, the observed gravity disturbance and GNSS-leveling geoidal heights are simulated from the EGM2008 model (the 2~1800th degree) in advance.

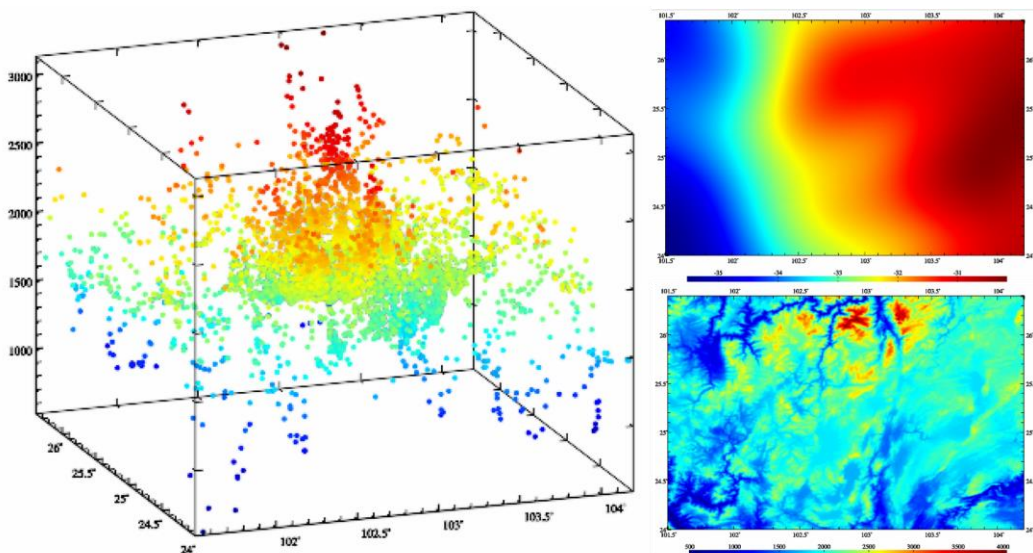


Simple process demo of All-element modelling on gravity filed using SRBFs in orthometric height system



It should be noted that since the observed geoidal height by GNSS-leveling is

essentially the height anomaly on the geoid in orthometric height system, the ellipsoidal height at GNSS-leveling sites must be the geoidal height, which can be employed by the observed GNSS-leveling geoidal height or the model geoidal height from the EGM2008 model (the 2~180th degree).



The distribution of gravity points, 2~180th degree model geoidal height and ellipsoidal height of the terrain surface

- **The ellipsoidal height grid of calculation surface:**

The model geoidal height grid file mdlgeoidh30s.dat calculated from the 2~180th degree geopotential model, which is employed for modelling on gravity field on geoid.

The ellipsoidal height grid file surfhgt30s.dat of the land/sea surface equal to the sum of the digital elevation model grid DEM30s.dat and the model geoidal height grid mdlgeoidh30s.dat, which is employed for modelling on ground gravity field.

Here, it is required that the grid range of the calculation surface is larger than the range of the target area to absorb edge effects.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, and input the file EGM2008.gfc, observed gravity disturbance file obsdistgrav.txt and observed GNSS-leveling geoidal height file obsGNSSlgeoid.txt, calculate and remove the 2~540th degree model value of these observations to generate the heterogeneous observation file obsresiduals0.txt according to the agreed format.

The agreed format of the heterogeneous observation file record: ID (point no / name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual geoidal height (m).

It should be noted that the ellipsoidal height of GNSS-leveling site must be the geoidal height and not the observed ellipsoidal height at GNSS-levelling site.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Calculation of global geopotential model and its spectral character analysis

Calculation of gravity field elements from global geopotential model | Calculation of model value for residual terrain (complete Bouguer) effects | Global geopotential coefficient model Calculator | Calculation and analysis of spectral character of Earth's gravity field

Open global geopotential coefficient model file | Save computation process as | Algorithmic Formulas

Select calculation file format | Discrete calculation points file | Open space calculation points file

Set input point file format | Number of rows of file header: 1 | Column ordinal number of ellipsoidal height in the record: 5 | Select elements to be calculated: ☒ height anomaly (m) | ☐ gravity anomaly (mGal) | ☐ gravity disturbance (mGal) | ☐ vertical deflection (" SW) | ☐ disturbing gravity gradient (E, radial) | ☐ tangential gravity gradient (E, NW) | ☐ Laplace operator (E)

Minimum degree: 2 | Maximum degree: 540

Save the results as | Import setting parameters | Start Computation

The observed gravity disturbances | The observed GNSS-leveling geoidal heights

The heterogeneous observation residuals

The ellipsoidal height here at GNSS-leveling point is the observed or model geoidal height, not the observed ellipsoidal height.

The model geoidal height (m) at the GNSS-levelling points

gravity anomaly (mGal)

ID	lon(degree decimal)	lat	ellip(m)	rent	kind	weight
1	102.4424	24.4717	1973.56	-32.7581	-32.6520	
2	102.5467	24.4580	1973.56	-32.9577	-32.5340	
3	102.6324	24.4582	1973.56	-32.5792	-32.4433	
4	102.7259	24.4605	2113.20	-32.3917	-32.3324	
5	102.4208	24.5663	1991.56	-32.6038	-32.5734	
6	102.5286	24.5627	1937.23	-32.5636	-32.4239	
7	102.4344	24.5656	2191.72	-32.3822	-32.3128	
8	102.7258	24.5919	2304.57	-32.2197	-32.2069	
9	102.8326	24.5755	1978.11	-32.5408	-32.0934	

ID	lon(degree decimal)	lat	ellip(m)	rent	kind	weight
1	102.3929	24.4944	2228.19	54.9765	0	1
2	102.3959	24.5089	2170.20	50.0971	0	1
3	102.3927	24.5296	2013.33	28.3652	0	1
4	102.3966	24.5113	1932.50	38.3822	0	1
5	102.3952	24.5034	1965.58	15.5784	0	1
6	102.3931	24.6178	1997.72	14.9731	0	1
7	102.3935	24.6384	1916.15	7.4068	0	1
4221	102.4424	24.4717	-32.6525	-0.1056	1	1
4222	102.5467	24.4580	-32.5340	-0.4237	1	1
4223	102.6324	24.4582	-32.4433	-0.1359	1	1
4224	102.7259	24.4605	-32.3324	-0.0593	1	1
4225	102.4208	24.5663	-32.5734	-0.0304	1	1
4226	102.5286	24.5627	-32.4239	-0.1397	1	1
4227	102.4344	24.5656	-32.3128	-0.0694	1	1
4228	102.7258	24.5919	-32.2197	-0.0128	1	1
4229	102.8326	24.5755	-32.5408	-0.4474	1	1
4230	102.3455	24.6889	-32.6394	-0.2903	1	1
4231	102.4239	24.6529	-32.4801	-0.0740	1	1
4232	102.5297	24.6670	-32.3057	-0.2186	1	1

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file.

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$, and input the heterogeneous residual file obsresiduals0.txt and model geoidal height grid file mdlgeoidh30s.dat to estimate the residual gravity field grid rntSRBFgeoidh30s0.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s0.chs.

Where, xxx=ksi stands for residual geoidal height (m), xxx=rga stands for residual gravity disturbance (mGal), xxx=gra stands for residual gravity anomaly (mGal), xxx=grr stands for residual disturbing gravity gradient (radial, E) and xx=dft stands for residual vertical deflection (SW, ").

Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbances from the remaining residual file rntSRBFgeoidh30s0.chs, detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresiduals01.txt.

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresiduals01.txt and repeat the step (2) to re-estimate the residual gravity field grid rntSRBFdatum30s.xxx on geoid and get the new remaining residual file rntSRBFdatum30s.chs.

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file. Follow example

Open the discrete heterogeneous residual observations file

number of rows of file header: 1
column ordinal number of ellipsoidal height in the record: 6
column ordinal number of weight: 7

Select SRBF radial multipole kernel
Order m: 5
Minimum degree: 360
Maximum degree: 1800
Burial depth of Bjerrhammar sphere: 10.0km
Action distance of SRBF center: 100km
Reuter network level K: 3600

Select the adjustable height anomaly (m):
Contribution rate k of adjustable observations: 0.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.

>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-28 18:03:50
>> Complete the computation!
>> Computation end time: 2024-09-28 18:10:25

>> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance "rga (mGal)", residual height anomaly "sa (m)", residual gravity anomaly "gra (mGal)", residual disturbing gradient "gr (E, radial) and residual vertical deflection vector "dft" (", SW), where "s" is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.

>> The program also outputs SRBF center file "center.txt" into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval ("). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (").

>> Type 0 of source observations: mean -0.3186 standard deviation 42.1772 minimum -296.0915 maximum 165.2611
Residual observations: mean 0.7368 standard deviation 16.9838 minimum -105.2839 maximum 114.8811
>> Type 1 of source observations: mean -0.3510 standard deviation 0.2774 minimum -0.9582 maximum 0.3435
Residual observations: mean -0.0410 standard deviation 0.0287 minimum -0.1943 maximum 0.0132

Solution of normal equation LU

ID	lon	lat	ellipshgt	rga	sa	gra	gr	dft
1	101.50417	24.1	101.50417	0.7368	16.9838	-105.2839	114.8811	
2	101.51250	24.1	101.51250	0.7368	16.9838	-105.2839	114.8811	
3	101.52083	24.1	101.52083	0.7368	16.9838	-105.2839	114.8811	
4	101.52917	24.1	101.52917	0.7368	16.9838	-105.2839	114.8811	
5	101.53750	24.1	101.53750	0.7368	16.9838	-105.2839	114.8811	
6	101.54583	24.1	101.54583	0.7368	16.9838	-105.2839	114.8811	

Select the remaining residuals (column 5) as the statistical reference.

Algorithm of gravity field approach using SRBs

After the first estimation is completed, it is recommended to employ the output residual observation file "chs" as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.

The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot

Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Separate the remaining residuals of the observed GNSS-leveling and observed gravity disturbances from rntSRBFgeoidh30s0.chs.

Gross error detection and basis function gridding of discrete field elements

Open file Save as Import parameters Start Computation Save process Follow example

Gross error detection on observations based on low-pass reference surface

The discrete point file to be detected

Number of rows of file header: 1
Column ordinal number of the attribute to be detected: 5
Beyond multiples of the standard deviation n: 3.0

Open low-pass reference surface grid file

Save the results as

Estimation of observation weight with specified reference attribute

Gridding of heterogeneous data by basis function weighted interpolation

Save computation process as

[Function] Select the low-pass grid as the reference surface, interpolate the reference value of the specified attribute value at the discrete point, and then detect and separate the gross error records according to the statistical properties of the differences between the specified attribute value and reference value.

** The reference surface can be constructed from discrete data by simple gridding and then low-pass filtering, and can also be the specified attribute grid constructed by weighted basis function gridding.

>> Open the discrete geoid file C:\Program Files\Examples\GravIndExercise\SRBFAppgeoidExercise\mbobid30sgeoid.txt

>> Look at the file information in the window below and set the discrete point file format...

>> Open low-pass reference surface grid file C:\Program Files\Examples\GravIndExercise\SRBFAppgeoidExercise\zero.dat

>> Save the results as C:\Program Files\Examples\GravIndExercise\SRBFAppgeoidExercise\mbobid30sgeoid.txt

>> Save no gross error results as C:\Program Files\Examples\GravIndExercise\SRBFAppgeoidExercise\mbobid30sgeoid.txt

>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Complete computation!
>> Computation start time: 2023-03-21 14:48:43
>> Computation end time: 2023-03-21 14:48:43

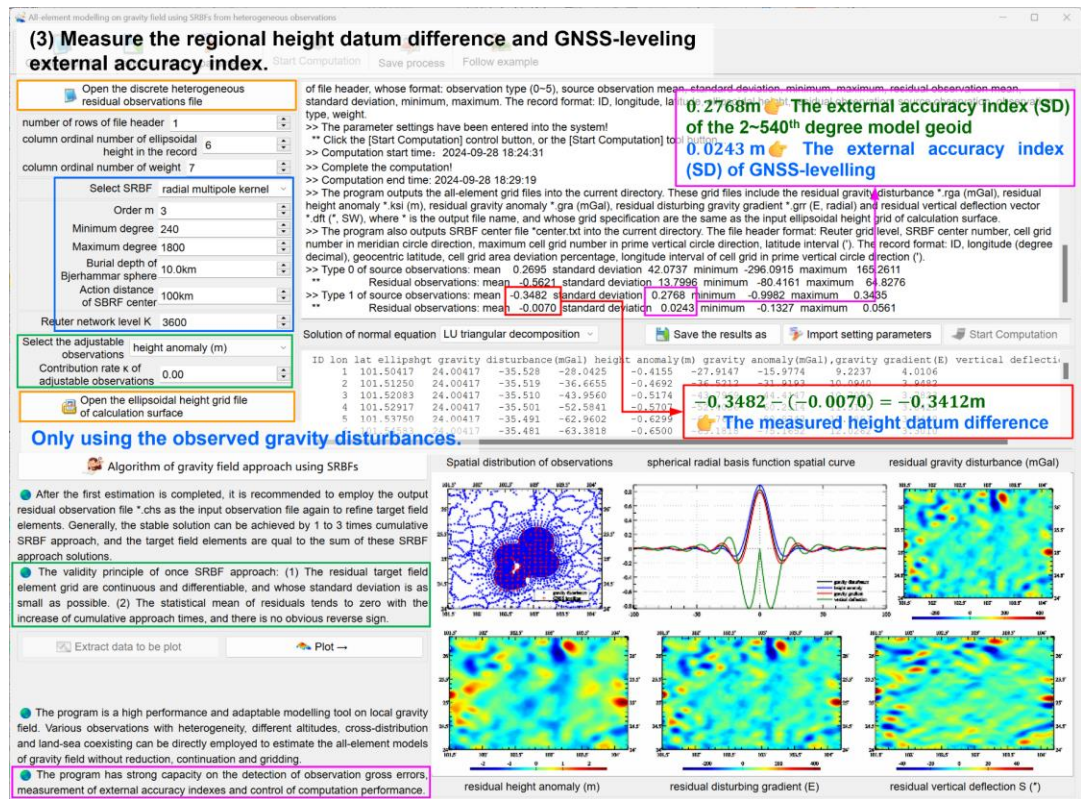
Extract plot data

Plot

Source observations input

Observations without gross error

Reconstruct the heterogeneous observation residual file obsresiduals01.txt.



Since the contribution rate of GNSS-levelling $\kappa = 0$ is set in advance, it is essentially here directly to measure the external accuracy index of the observed GNSS levelling only using the observed gravity disturbances. Before and after gross error removed, the statistical results on the observation residuals are as follows.

		number of points	mean	standard deviation	minimum	maximum
Gravity disturbance (mGal)	Original residuals	4219	0.3186	42.1772	-296.0915	165.2611
	Residuals without error	4215	0.2695	42.0737	-296.0915	165.2611
	Remaining residuals	4215	-0.4584	13.6071	-61.1040	64.8276
GNSS levelling geoidal height (m)	Original residuals	125	-0.3510	0.2774	-0.9982	0.3435
	Residuals without error	124	-0.3482 ^①	0.2768	-0.9982	0.3435
	Remaining residuals	124	-0.0070 ^②	0.0243 ^③	-0.1328	0.0561

The statistical mean ① minus ② of the GNSS-levelling remaining residuals in the table, that is, $-0.3482^{①} - (-0.0070^{②}) = -0.3412$ m, is the difference between the regional height datum and global height datum (gravimetric geoid). Here provides the SRBF

measurement method for regional height datum difference.

In the table, 0.0243^③m is the external accuracy index of the observed GNSS-leveilling expressed as standard deviation, that is, 2.43 cm. Here provides the SRBF measurement method for the external accuracy index of GNSS-leveling. The result indicates that the external accuracy of the observed GNSS-leveling is not bad than 2.43 cm (standard deviation).

In general, it is necessary to make 1 to 2 cumulative SRBF approach with *.chs as the input file to obtain the minimum of the standard deviation of the GNSS-leveilling remaining residuals as the external accuracy index, and this process is omitted in this example.

After removing the regional height datum difference of -0.3412m from the GNSS-leveilling residuals, the new heterogeneous observation residual file obsresiduals1.txt is reconstructed again.

(4) All-element modelling on the residual gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the heterogeneous residual file obsresiduals1.txt and the model geoidal height grid file mdlgeoidh30s.dat to estimate the 30" residual gravity field grid rntSRBFgeoidh30s1.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s1.chs.

(4) All-element modelling on the residual gravity field using SRBFs

Open the discrete heterogeneous residual observations file

number of rows of file header 1
column ordinal number of ellipsoidal height in the record 6
column ordinal number of weight 7

Select SRBF radial multipole kernel
Order m 3
Minimum degree 360
Maximum degree 1800
Burial depth of 10.0km
Action distance of SRBF center 100km
Reuter network level K 3600

Select the adjustable observations
height anomaly (m)
Contribution rate κ of adjustable observations 1.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.
 >> The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] tool button...
 >> Computation start time: 2024-09-28 18:36:27
 >> Complete the computation!
 >> Computation end time: 2024-09-28 18:41:35
 >> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *.rga (mGal), residual height anomaly *.ksa (m), residual gravity anomaly *.gra (mGal), residual disturbing gravity gradient *.gr (E, radial) and residual vertical deflection vector *.dft (*, SV), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.
 >> The program also outputs SRBF center file *.center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval (*). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (*).
 >> Type 0 of source observations: mean 0.2695 standard deviation 42.0737 minimum -296.0915 maximum 165.2611
 ** Residual observations: mean 0.0196 standard deviation 12.9866 minimum -80.4161 maximum 64.8276
 >> Type 1 of source observations: mean -0.0071 standard deviation 0.2768 minimum -0.6571 maximum 0.6846
 ** Residual observations: mean -0.0002 standard deviation 0.0276 minimum -0.1059 maximum 0.0768

Solution of normal equation LU triangular decomposition

Save the results as Import setting parameters Start Computation

ID lon lat ellipsoid gravity disturbance (mGal) height anomaly (m) residual gravity disturbance (mGal) residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S (*)

1 101.50417 24.00417 -35.528 -40.8686 -0.3641 -4-
 2 101.51250 24.00417 -35.519 -47.9108 -0.4135 -4-
 3 101.52083 24.00417 -35.510 -55.2656 -0.4640 -5-
 4 101.52917 24.00417 -35.501 -64.0905 -0.5229 -6-
 5 101.53750 24.00417 -35.491 -73.4952 -0.5848 -7-
 6 101.54583 24.00417 -35.481 -72.3357 -0.5786 -72.1577 -106.5435 10.3890 2.3703

Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.

The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations spherical radial basis function spatial curve residual gravity disturbance (mGal)

All-element models rntSRBFgeoidh30s1.xxx of the residual gravity field

residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S (*)

[The quality control scheme] You can further detect and remove the observation

gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file rntSRBFgeoidh30s1.chs, and then repeat the step (4). This process is omitted in this example.

(5) All-element modelling on the remaining residual gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the remaining residual file rntSRBFgeoidh30s1.chs and model geoidal height grid file mdlgeoidh30s.dat to estimate the 30" remaining residual gravity field grid rntSRBFgeoidh30s1.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s1.chs.

(5) All-element modelling on the remaining residual gravity field using SRBFs

Open the discrete heterogeneous residual observations file

number of rows of file header: 2
column ordinal number of ellipsoidal height in the record: 7
column ordinal number of weight: 8

Select SRBF: Poisson wavelet kernel
Order m: 5
Minimum degree: 540
Maximum degree: 5400
Burial depth of Bjerhammar sphere: 6.0km
Action distance of SRBF center: 60km
Reuter network level K: 5400

Select the adjustable observations: height anomaly (m)
Contribution rate κ of adjustable observations: 1.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoid height, residual observation, source observation, observation type, weight.
 >> The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] control button.
 >> Computation start time: 2024-09-28 19:56:11
 >> Complete the computation!
 >> Computation end time: 2024-09-28 20:03:11

Input the file rntSRBFgeoidh30s1.chs output from the previous step.

The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance "rga (mGal)", residual height anomaly "rsa (m)", residual gravity anomaly "gra (mGal)", residual disturbing gravity gradient "grt (E, radial) and residual vertical deflection vector "drt (" SW), where " is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.
 >> The program also outputs SRBF center file "center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval ("). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (").
 >> Type 0 of source observations: mean 0.0196 standard deviation 12.9866 minimum -80.4161 maximum 64.8276
 ** Residual observations: mean 0.0200 standard deviation 8.4565 minimum -54.9649 maximum 58.6241
 >> Type 1 of source observations: mean -0.0002 standard deviation 0.0276 minimum -0.1059 maximum 0.0768
 ** Residual observations: mean 0.0008 standard deviation 0.0147 minimum -0.0511 maximum 0.0345

Solution of normal equation LU triangular decomposition

Save the results as Import setting parameters Start Computation

ID	lon	lat	ellipsht	gravity disturbance (mGal)	height anomaly (m)	gravity anomaly (mGal)	gravity gradient (E)	vertical deflection
1	101.50417	24.00417	-35.528	-12.7117	-0.0168	-12.7045	-97.1597	-0.6515
2	101.51250	24.00417	-35.519	-6.4208	-0.0077	-6.4234	-54.3695	-1.2702
3	101.52083	24.00417	-35.510	2.3531	0.0053	2.35	11.5569	1.0240
4	101.52917	24.00417	-35.501	11.0246	0.0174	11.01	11.01	11.01
5	101.53750	24.00417	-35.491	16.0356	0.0255	16.02	16.02	16.02
6	101.54583	24.00417	-35.481	17.2077	0.0259	17.19	17.19	17.19

0.0147m \approx 1.5 cm. The accuracy index (SD) of geoid modeling.

Algorithm of gravity field approach using SRBFs

- After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.
- The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations spherical radial basis function spatial curve residual gravity disturbance (mGal)

All-element models rntSRBFgeoidh30s2.xxx of the remaining residual gravity field

residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S ("

		mean	standard deviation	minimum	maximum
Residual gravity disturbance (mGal)	Residuals	0.3523	42.1561	-296.0915	165.2611
	First SRBF	0.0196	12.9866	-80.4161	64.8276
	Second SRBF	0.0200	8.4565	-54.9649	58.6241
Residual GNSS-levelling geoidal height (m)	Residuals	-0.0071	0.2768	-0.6571	0.6846
	First SRBF	-0.0002	0.0276	-0.1059	0.0768
	Second SRBF	0.0008	0.0147 ^④	-0.0511	0.0345

In the table, 0.0147^④m = 1.5cm can be considered as the accuracy index of geoid

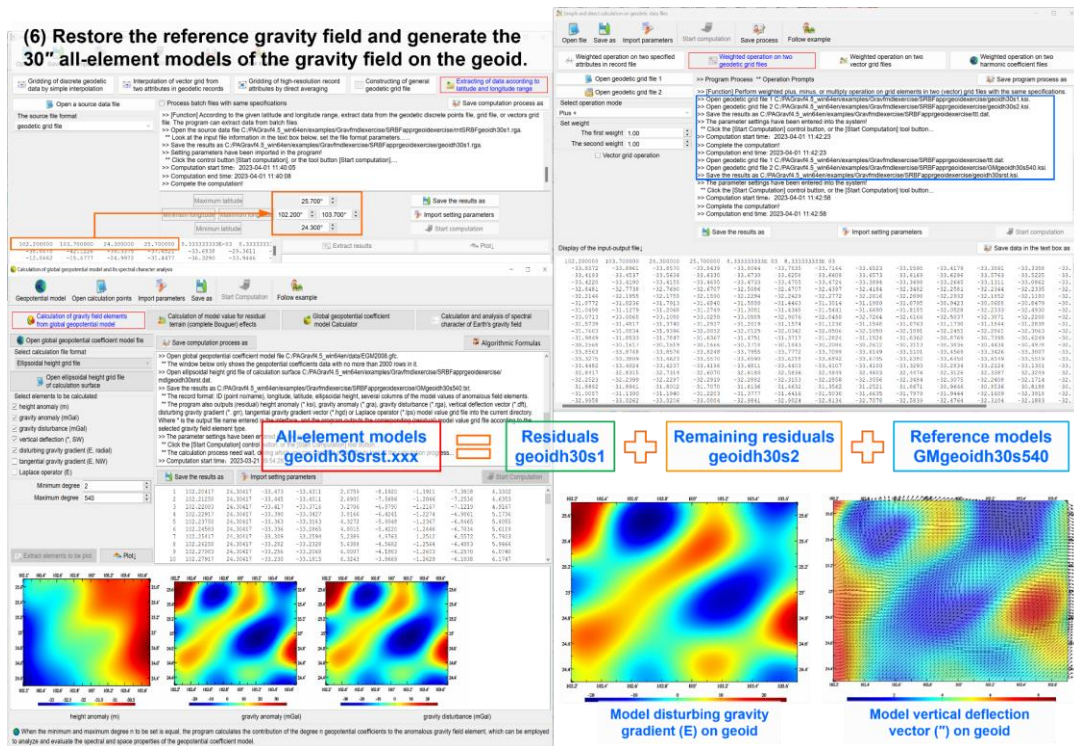
modelling.

[The quality control scheme] You can furtherly detect and remove again the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFgeoidheight30s2.chs, and then repeat from step (4). This process is omitted in this example.

You can also do further cumulative SRBF approach to improve the results. This example omits this process.

(6) Restore the reference gravity field and generate the 30" all-element models of the gravity field on the geoid.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, input the file EGM2008.gfc, and the model geoidal height grid file mdlgeoidh30srst.dat (from mdlgeoidh30s.dat with grid edge removed), to calculate the all-element grid 5Mgeoidh30s540.xxx of the reference gravity field on geoid.

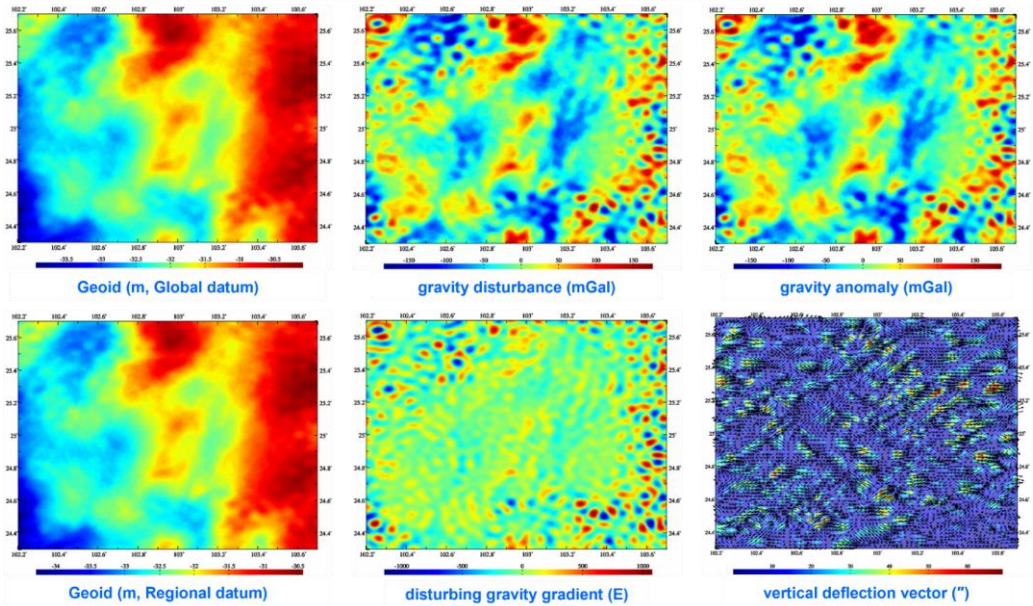


Add the residual gravity field grid geoidh30s1.xxx (from SRBFgeoidheight30s1.xxx with grid edge removed) and remaining residual gravity field grid geoidh30s2.xxx (from SRBFgeoidheight30s2.xxx with grid edge removed) to the reference gravity field grid GMgeoidh30s540.xxx, the 30" all-element gravity field models geoidh30srst.xxx on the geoid are obtained, which include the 30" gravimetric geoidal height grid (geoidh30srst.ksi, m), gravity disturbance grid (geoidh30srst.rga, mGal), gravity

anomaly grid (geoidh30srst.gra, mGal), disturbing gravity gradient grid (geoidh30srst.grr, radial, E) and vertical deflection vector grid (geoidh30srst.dft, SW, ").

Add the regional height datum difference -0.3411m to the 30" gravimetric geoidal height grid geoidh30srst.ksi in global height datum, the 30" gravimetric geoidal height grid geoidh30srgn.ksi in regional height datum can be obtained.

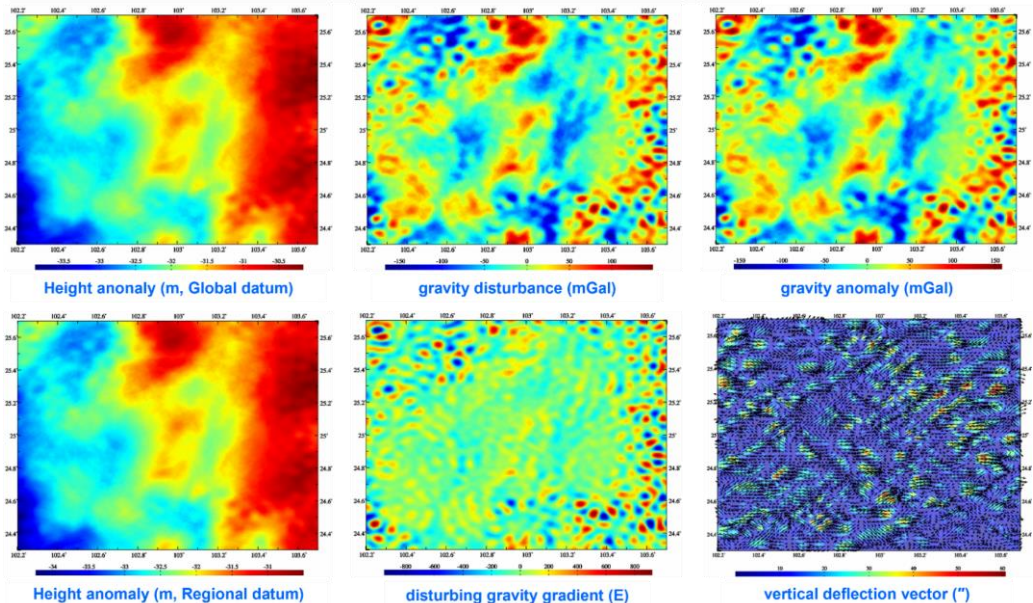
30"×30" full element models of gravity field on geoid



So far, the all-element modelling on gravity field on the geoid have been completed.

- Let the terrain surface as the calculation surface, and directly generate the 30" all-element models of the gravity field on the terrain surface.

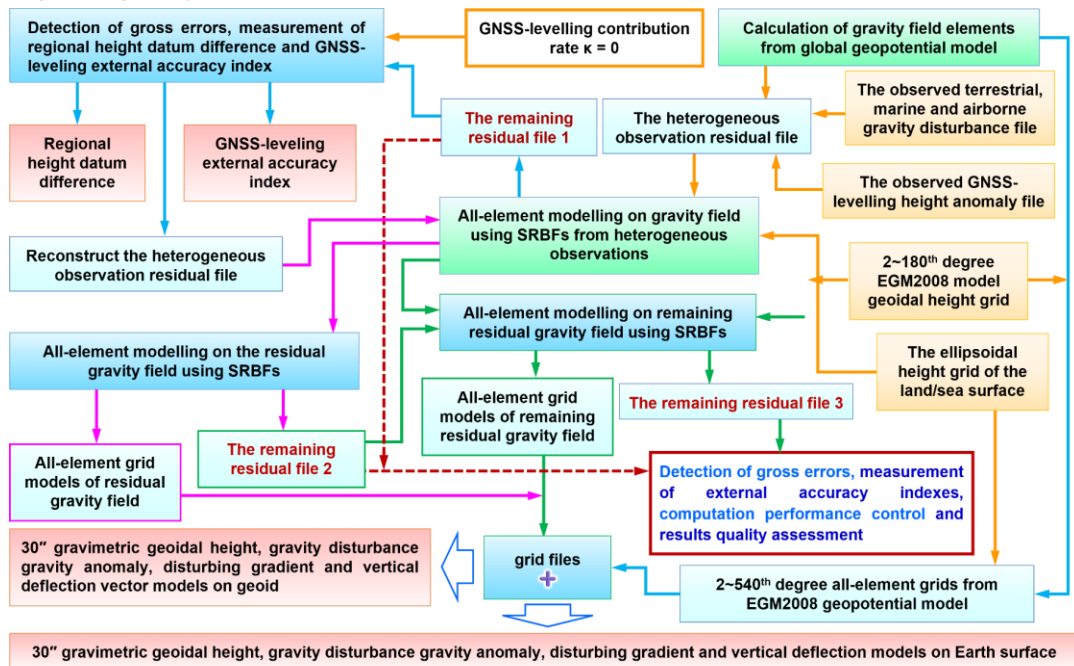
30"×30" full element models of gravity field on terrain surface



In step (3) to step (6) above, the input data file and all the parameter settings are kept the same, and only the calculation surface is changed to the terrain surface `surfhgt30s.dat`. Using the same computation process, you can synchronously obtain the 30" all-element models `surfhgt30srst.xxx` of the gravity field on the terrain surface, which include the 30" gravimetric ground height anomaly grid (`surfhgt30srst.ksi`, m, in global height datum), ground gravity disturbance grid (`surfhgt30srst.rga`, mGal), ground gravity anomaly grid (`surfhgt30srst.gra`, mGal), ground disturbing gravity gradient grid (`surfhgt30srst.grr`, radial, E), ground vertical deflection vector grid (`surfhgt30srst.dft`, SW, ") and ground height anomaly grid (`surfhgt30srgn.ksi`, m, in regional height datum).

4.8.3 Simple process demo of all-element modelling on gravity field using SRBFs in normal height system

Exercise purpose: From the observed terrestrial, marine and airborne gravity disturbances and GNSS-leveling height anomalies in normal height system, make the all-element models on gravity field using spherical radial basis functions (SRBFs) in six steps, in which all the terrain effects are not processed, to quickly master the essentials in observation analysis, computation performance control and all-element modelling on regional gravity field.



Simple process demo of all-element modelling on gravity field using SRBFs in normal height system

In this section, the observed GNSS-leveling height anomaly in the normal height system is employed to replace the observed GNSS-leveling geoidal height in orthometric height system in the 4.8.2, and the simple process of all-element modelling on gravity field using SRBFs is introduced. In the both cases, there is only a slight difference in the processing of the observed GNSS-leveling data, and the other

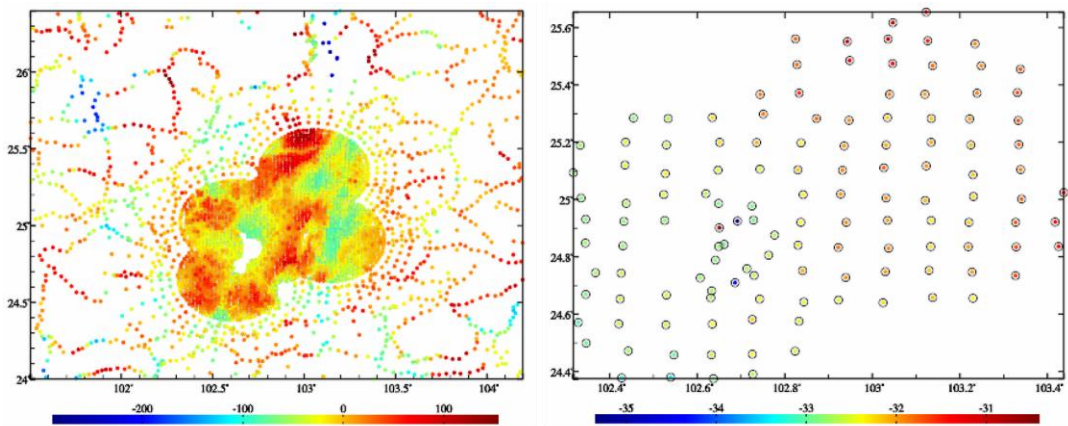
modelling processes are the same. For the convenience, here gives the complete quick process.

After the terrain effect processing omitted, SRBF approach process of gravity field is very simple because there is no need for additional continuation reduction, gridding and GNSS-leveling fusion process.

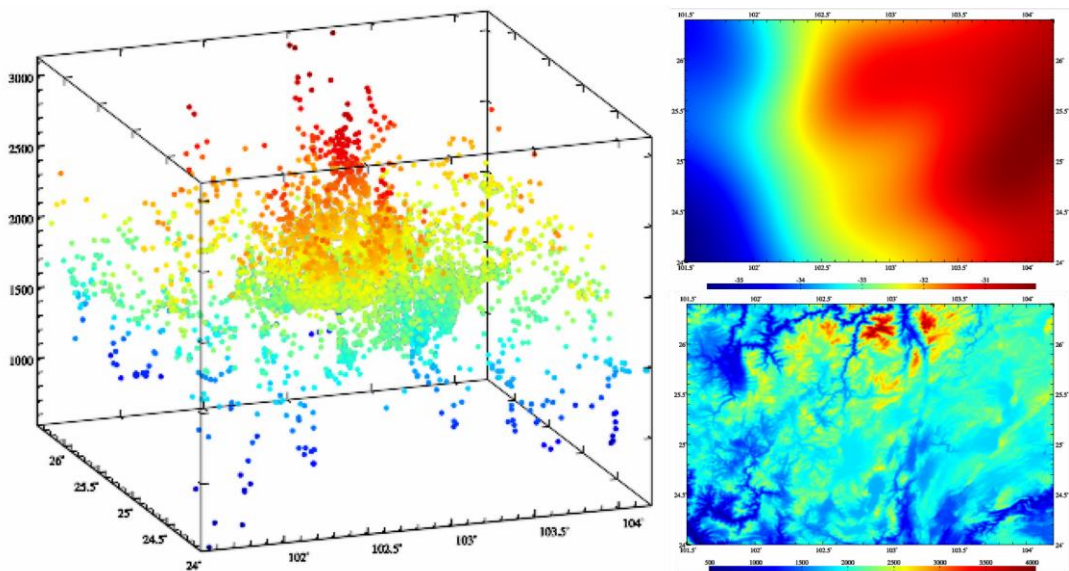
• The observed gravity disturbance and GNSS-leveling data

The observed terrestrial, marine and airborne gravity disturbance file obsdistgrav.txt. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed gravity disturbance (mGal), ...

The observed GNSS-leveling height anomaly file obsGNSSlksi.txt in normal height system. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed height anomaly (m), ...



The observed gravity disturbances (mGal) and observed GNSS-leveling height anomalies (m)



The distribution of gravity points, 2~180th degree model geoidal height and ellipsoidal height of the terrain surface

In the example, the observed gravity disturbances and GNSS-leveling anomalies

are simulated from the EGM2008 model (the 2~1800th degree) in advance.

- **The ellipsoidal height grid of calculation surface:**

The model geoidal height grid file mdlgeoidh30s.dat calculated from the 2~180th degree geopotential model, which is employed for modelling on gravity field on geoid.

The ellipsoidal height grid file surfhgt30s.dat of the land/sea surface equal to the sum of the digital elevation model grid DEM30s.dat and model geoidal height grid mdlgeoidh30s.dat, which is employed for modelling on ground gravity field.

Here, it is required that the grid range of the calculation surface is larger than the range of the target area to absorb edge effects.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, and input the file EGM2008.gfc, observed gravity disturbance file obsdistgrav.txt and observed GNSS-leveilling height anomaly file obsGNSSlksi.txt, calculate and remove the 2~540th degree model value of these observations to generate the heterogeneous observation file obsresiduals0.txt according to the agreed format.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Calculation of gravity field elements from global geopotential model

Open global geopotential coefficient model file

Select calculation file format

Discrete calculation points file

Open space calculation points file

Set input point file format

Number of rows of file header 1

Column ordinal number of ellipsoidal height in the record 5

Select elements to be calculated

☒ height anomaly (m)

☐ gravity anomaly (mGal)

☐ gravity disturbance (mGal)

☐ vertical deflection (" , SW)

☐ disturbing gravity gradient (E, radial)

☐ tangential gravity gradient (E, NW)

☐ Laplace operator (E)

Minimum degree 2

Maximum degree 540

Start Computation

Save the results as

Import setting parameters

Algorithmic Formulas

Click the [Open global geopotential coefficient model file] control button, or the [Open geopotential model] tool button...

Open global geopotential coefficient model file C:\PAGrav4.5\win64en\data\EGM2008.gfc

The window below only shows the geopotential coefficients data with no more than 2000 rows in it.

Open space calculation points file C:\PAGrav4.5\win64en/examples\Gravmdlexercise\SRBFAppwithGNSSlksi\obsGNSSlksi.txt

Look at the file information in the window below and set the discrete point file format.

Save the results as C:\PAGrav4.5\win64en/examples\Gravmdlexercise\SRBFAppwithGNSSlksi\obsGNSSlksi\GM540.txt

Behind the record of the calculation point file, appends one or more columns of model values of anomalous field elements, and keeps 4 significant figures.

The parameter settings have been entered into the system

Click the [Start Computation] control button, or the [Start Computation] tool button...

The calculation process need wait, during which you can open the output file to look at the calculation results.

Computation start time: 2023-03-21 15:28:18

Complete the calculation of the model value of (residual) gravity field element

Computation end time: 2023-03-21 15:29:04

The observed gravity disturbances

The observed GNSS-leveilling height anomalies

The heterogeneous observation residuals

The ellipsoidal height here at GNSS-leveilling point is the observed ellipsoidal height.

The model height anomaly (m) at the GNSS-leveilling points

gravity anomaly (mGal)

When the model height anomaly (m) at the GNSS-leveilling points is equal, the program calculates the contribution of the degree n geopotential to the gravity anomaly (mGal) and the gravity anomaly (mGal) can be employed to analyze and estimate the spectral and space properties of the geopotential coefficient model.

ID	lon(degree decimal)	lat	ellipht(m)	rent	kind	weight
1	102.3929	24.4944	2228.19	54.9765	0	1
2	102.3959	24.5089	2170.20	50.0971	0	1
3	102.3927	24.5296	2013.33	28.3652	0	1
4	102.3966	24.5453	2122.50	38.3822	0	1
5	102.3951	24.5600	1971.28	20.6411	0	1
6	102.4208	24.5663	1991.56	15.5784	0	1
7	102.4286	24.5627	1937.23	14.5045	0	1
8	102.3931	24.6178	1997.72	14.9731	0	1
9	102.3935	24.6384	1916.15	7.4068	0	1
4221	102.4424	24.4717	1973.56	-0.0882	1	1
4222	102.5467	24.4580	1659.69	-0.4184	1	1
4223	102.6324	24.4582	2120.99	-0.1378	1	1
4224	102.7259	24.4605	2112.20	-0.0659	1	1
4225	102.4208	24.5663	1991.56	-0.0029	1	1
4226	102.5286	24.5627	1937.23	-0.1219	1	1
4227	102.6344	24.5656	2193.72	-0.0607	1	1
4228	102.7435	24.5659	1920.60	-0.0100	1	1
4229	102.8435	24.5659	1920.60	-0.4484	1	1
4230	102.9435	24.5659	1920.60	-0.2580	1	1
4231	102.4239	24.6529	1960.26	-0.0416	1	1
4232	102.5297	24.6670	2158.55	-0.1896	1	1

The agreed format of the heterogeneous observation file record: ID (point no/name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual height anomaly (m).

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file.

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$, and input the heterogeneous residual file obsresiduals0.txt and terrain surface ellipsoidal height grid file surfhgt30s.dat to estimate the residual gravity field grid SRBFsurfhgt30s0.xxx on geoid, and get the remaining residual file SRBFsurfhgt30s0.chs.

Where, xxx=ksi stands for residual height anomaly (m), xxx=rga stands for residual gravity disturbance (mGal), xxx=gra stands for residual gravity anomaly (mGal), xxx=grr stands for residual disturbing gravity gradient (radial, E) and xx=dft stands for residual vertical deflection (SW, ").

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file.

Open the discrete heterogeneous residual observations file

number of rows of file header: 1
column ordinal number of ellipsoidal height in the record: 6
column ordinal number of weight: 7

Select SRBF: radial multipole kernel
Order m: 5
Minimum degree: 360
Maximum degree: 1800
Burial depth of Bjerrhammer sphere: 10.0km
Action distance of SRBF center: 100km
Reuter network level K: 3600

Select the adjustable observations: height anomaly (m)
Contribution rate κ of adjustable observations: 0.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.

>> The parameter settings have been entered into the system!
>> Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2024-09-28 21:09:13
>> Complete the computation!
>> Computation end time: 2024-09-28 21:14:56

>> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance "rga (mGal)", residual height anomaly "ksi (m)", residual gravity anomaly "gra (mGal)", residual disturbing gravity gradient "grr (E, radial) and residual vertical deflection vector "dft" (SW, "). where " is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.

>> The program also outputs SRBF center file "center.txt" into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval ("). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (").

>> Type 0 of source observations: mean 0.3186 standard deviation 42.1772 minimum -296.0915 maximum 165.2611
Residual observations: mean 0.7856 standard deviation 17.5917 minimum -105.2839 maximum 114.8811
>> Type 1 of source observations: mean -0.3452 standard deviation 0.2739 minimum -0.9755 maximum 0.3702
Residual observations: mean -0.0405 standard deviation 0.0271 minimum -0.1876 maximum 0.0099

Solution of normal equation LU tria

ID	lon	lat	ellipshgt	gravit	ksi	rga	gra	grr	dft
1	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750
2	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750
3	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750
4	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750
5	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750
6	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750
7	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750
8	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750
9	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750
10	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750
11	101.50417	24.0041	101.51250	24.0041	101.52083	24.0041	101.52917	24.0041	101.53750

Select the remaining residuals (column 5) as the statistical reference.

Algorithm of gravity field approach using SRBFs

- After the first estimation is completed, it is recommended to employ the output residual observation file "chs" as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.
- The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot

Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations

spherical radial basis function spatial curve

residual gravity disturbance (mGal)

residual height anomaly (m)

residual disturbing gradient (E)

residual vertical deflection S (")

Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbances from the remaining residual file SRBFsurfhgt30s0.chs, detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-leveling sites and beyond 5 times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresiduals01.txt.

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresiduals01.txt and repeat the step (2) to re-estimate the residual gravity field grid rntSRBFdatum30s.xxx on terrain surface and get the new remaining residual file rntSRBFdatum30s.chs.

Since the contribution rate of GNSS-levelling $\kappa = 0$ is set in advance, it is essentially here directly to measure the external accuracy index of the observed GNSS levelling only using the observed gravity disturbances.

Separate the remaining residuals of the observed GNSS-levelling and observed gravity disturbance from rntSRBFgeoidh30s0.chs.

Reconstruct the heterogeneous observation residual file obsresiduals01.txt.

Before and after gross error removed, the statistical results on the observation residuals are as follows.

		number of points	mean	standard deviation	minimum	maximum
Gravity disturbance (mGal)	Original residuals	4219	0.3186	42.1772	-296.0915	165.2611
	Residuals without error	4215	0.2695	42.0737	-296.0915	165.2611
	Remaining residuals	4215	-0.5677	13.8957	-80.4161	64.8276
GNSS levelling height anomaly (m)	Original residuals	125	-0.3452	0.2739	-0.9755	0.3702
	Residuals without error	123	-0.3404 ^①	0.2735	-0.9755	0.3702
	Remaining residuals	123	-0.0069 ^②	0.0233 ^③	-0.1295	0.0528

The statistical mean ① minus ② of the GNSS-levelling remaining residuals in the

table, that is, $-0.3404^{①} - (-0.0069^{②}) = -0.3335\text{m}$, is the difference between the regional height datum and the global height datum (gravimetric geoid). Here provides the SRBF measurement method for regional height datum difference.

In the table, $0.0233^{③}\text{m}$ is the external accuracy index of the observed GNSS-leveilling expressed as standard deviation, that is, 2.33cm . Here provides the SRBF measurement method for the external accuracy index of GNSS- leveling. The result indicates that the external accuracy of GNSS-leveilling is not bad than 2.33 cm (SD).

(3) Measure the regional height datum difference and GNSS-leveilling external accuracy index.

Open the discrete heterogeneous residual observations file

number of rows of file header: 1
column ordinal number of ellipsoidal height in the record: 6
column ordinal number of weight: 7

Select SRBF: radial multipole kernel
Order m: 3
Minimum degree: 240
Maximum degree: 1800
Burial depth of Bjerrhammar sphere: 10.0km
Action distance of SRBF center: 100km
Reuter network level K: 3600

Select the adjustable observations
height anomaly (m)
Contribution rate κ of adjustable observations: 0.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, type, weight.
 >> The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] button.
 >> Computation start time: 2024-09-28 21:17:31
 >> Complete the computation!
 >> Computation end time: 2024-09-28 21:22:53
 >> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance "rga (mGal)", residual height anomaly "ksa (m)", residual gravity anomaly "gra (mGal)", residual disturbing gravity gradient "gr (E, radial)" and residual vertical deflection vector "dft (" SW), where " is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.
 >> The program also outputs SRBF center file "center.txt" into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval ("). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (").
 >> Type 0 of source observations: mean 0.2695 standard deviation 42.0737 minimum -296.0915 maximum 169.2611
 >> Type 1 of source observations: mean -0.5677 standard deviation 13.8957 minimum -80.4161 maximum 44.8276
 >> Type 1 of source observations: mean -0.3404 standard deviation 0.2735 minimum -0.9755 maximum 0.3702
 Residual observations: mean -0.0069 standard deviation 0.0233 minimum -0.1295 maximum 0.0528

Solution of normal equation: LU triangular decomposition

ID lon lat ellipshgt gravity disturbance (mGal) height anomaly (m) residual gravity anomaly (mGal) residual disturbing gravity gradient (E, radial) residual vertical deflection vector (SW)

1 101.50417 24.00417 2427.222 -25.2759 0.2735 0.0233 0.0528
 2 101.51250 24.00417 2480.981 -33.0112 0.2735 0.0233 0.0528
 3 101.52083 24.00417 2435.157 -39.4282 0.2735 0.0233 0.0528
 4 101.52917 24.00417 2229.999 -47.4915 0.2735 0.0233 0.0528
 5 101.53750 24.00417 2032.509 -57.3974 0.2735 0.0233 0.0528

Only using the observed gravity disturbances.

Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.

The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot

Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations spherical radial basis function spatial curve residual gravity disturbance (mGal)

residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S (")

In general, it is necessary to make 1 to 2 cumulative SRBF approach with *.chs as the input file to obtain the minimum of standard deviation of GNSS-leveilling remaining residuals as the external accuracy index, and this process is omitted in this example.

After removing the regional height datum difference of -0.3345m from GNSS-leveilling residuals, the new heterogeneous observation residual file obsresiduals1.txt is reconstructed again.

(4) All-element modelling on the residual gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the heterogeneous residual file obsresiduals1.txt and terrain surface ellipsoidal height grid file surfhgt30s.dat to estimate the 30" residual gravity field grid SRBFsurfhgt30s1.xxx on terrain surface, and get the remaining residual file SRBFsurfhgt30s1.chs.

[The quality control scheme] You can furtherly detect and remove the observation

gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFsurfhtg30s1.chs, and then repeat the step (4). This process is omitted in this example.

(4) All-element modelling on the residual gravity field using SRBFs

Open the discrete heterogeneous residual observations file

number of rows of file header 1
column ordinal number of ellipsoidal height in the record 6
column ordinal number of weight 7

Select SRBF radial multipole kernel
Order m 3
Minimum degree 360
Maximum degree 1800
Burial depth of Bjerhammar sphere 10.0km
Action distance of SRBF center 100km
Reuter network level K 3600

Select the adjustable observations height anomaly (m)
Contribution rate κ of adjustable observations 1.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.
 >> The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] tool button...
 >> Complete the computation!
 >> Computation end time: 2024-09-28 21:43:45
 >> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *rga (mGal), residual height anomaly *ksi (m), residual gravity anomaly *gra (mGal), residual disturbing gravity gradient *gr (E, radial) and residual vertical deflection vector *dft (*, SW), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.
 >> The program also outputs SRBF center file *center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval (*). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (*).
 >> Type 0 of source observations: mean 0.2695 standard deviation 42.0737 minimum -296.0915 maximum 165.2611
 Residual observations: mean 0.0620 standard deviation 12.9866 minimum -80.4161 maximum 64.8276
 >> Type 1 of source observations: mean -0.0107 standard deviation 0.2759 minimum -0.6410 maximum 0.7047
 Residual observations: mean -0.0014 standard deviation 0.0291 minimum -0.1886 maximum 0.0595

Solution of normal equation LU triangular decomposition

Save the results as Import setting parameters Start Computation

ID lon lat ellipshgt gravity disturbance (mGal) height anomaly (m)
 1 101.50417 24.00417 2427.222 -33.8930 -0.3067
 2 101.51250 24.00417 2480.981 -41.3359 -0.3579
 3 101.52083 24.00417 2435.157 -47.3401 -0.3988
 4 101.52917 24.00417 2229.999 -55.4958 -0.4544
 5 101.53750 24.00417 2032.509 -65.0026 -0.5171
 6 101.54583 24.00417 1906.019 -65.4479 -0.5213

Can furtherly detect and remove the observation gross errors from *.chs, and then repeat the step (4).

Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.

The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations spherical radial basis function spatial curve residual gravity disturbance (mGal)

All-element models SRBFsurfhtg30s1.xxx of the residual gravity field

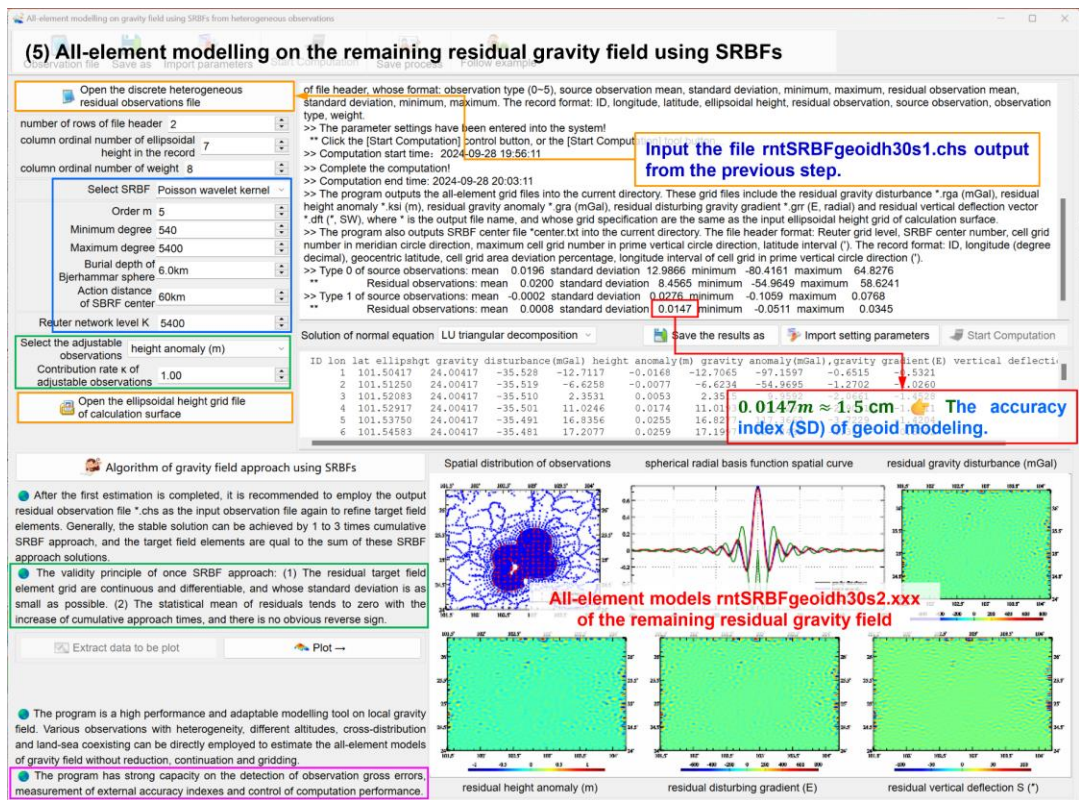
residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S (*)

(5) All-element modelling on the remaining residual gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the remaining residual file SRBFsurfhtg30s1.chs and terrain surface ellipsoidal height grid file surfhtg30s.dat to estimate the 30" remaining residual field grid SRBFsurfhtg30s2.xxx on the terrain surface, and get the remaining residual file SRBFsurfhtg30s2.chs.

In the table below, $0.0154^{(4)}\text{m} = 1.5\text{cm}$ can be considered as the accuracy index of ground height anomaly (quasigeoid) modelling.

		mean	standard deviation	minimum	maximum
Residual gravity disturbance (mGal)	Residuals	0.2695	42.0737	-296.0915	165.2611
	First SRBF	0.0620	12.9866	-80.4161	64.8276
	Second SRBF	0.1309	8.5135	-50.6030	57.3920
Residual GNSS-levelling height anomaly (m)	Residuals	-0.0071	0.2768	-0.6571	0.6846
	First SRBF	-0.0014	0.0291	-0.1886	0.0595
	Second SRBF	-0.0013	0.0154 ⁽⁴⁾	-0.0708	0.0315



[The quality control scheme] You can furtherly detect and remove again the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-leveilling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFsurfhtg30s2.chs, and then repeat from step (4). This process is omitted in this example.

You can also do further cumulative SRBF approach to improve the results. This example omits this process.

(6) Restore the reference gravity field and generate the 30" all-element models of the gravity field on the terrain surface.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, input the file EGM2008.gfc, and the terrain surface ellipsoidal height grid file surfhtg30srst.dat (from surfhtg30s.dat with grid edge removed), to calculate the all-element grid GMsurfhtg30s540.xxx of the reference gravity field on the terrain surface.

Add the residual gravity field grid surfhtg30s1.xxx (from SRBFsurfhtg30s0.xxx with grid edge removed) and remaining residual gravity field grid surfhtg30s2.xxx (from SRBFsurfhtg30s1.xxx with grid edge removed) to the reference gravity field grid GMsurfhtg30s540.xxx, the 30" all-element gravity field models surfhtg30srst.xxx on the terrain surface are obtained, which include the 30" gravimetric ground height anomaly grid (surfhtg30srst.ksi, m), ground gravity disturbance grid (surfhtg30srst.rga, mGal),

ground gravity anomaly grid (surfhgt30srst.gra, mGal), ground disturbing gravity gradient grid (surfhgt30srst.grr, radial, E) and ground vertical deflection vector grid (surfhgt30srst.dft, SW, ").

(6) Restore the reference gravity field and generate the 30" all-element models of the gravity field on terrain surface.

The software interface shows the process of restoring the reference gravity field and generating the 30" all-element models. The main window displays the 'Operation Properties' and 'Operation Progress' tabs. The 'Operation Properties' tab shows the selected operation mode, the source file, and the output file. The 'Operation Progress' tab shows the progress of the operation, including the calculation of the reference gravity field and the generation of the 30" all-element models.

The 'Operation Properties' tab shows the following information:

- Operation mode: **Weighted operation on two specified attributes in record file**
- Source file: **C:\Program Files (x86)\Surfhgt30srst\surfhgt30srst.gra**
- Output file: **C:\Program Files (x86)\Surfhgt30srst\surfhgt30srst.dft**
- Weight: **1.00**
- Second weight: **1.00**
- Operation start time: **2023-04-01 10:48:54**
- Operation end time: **2023-04-01 10:49:30**

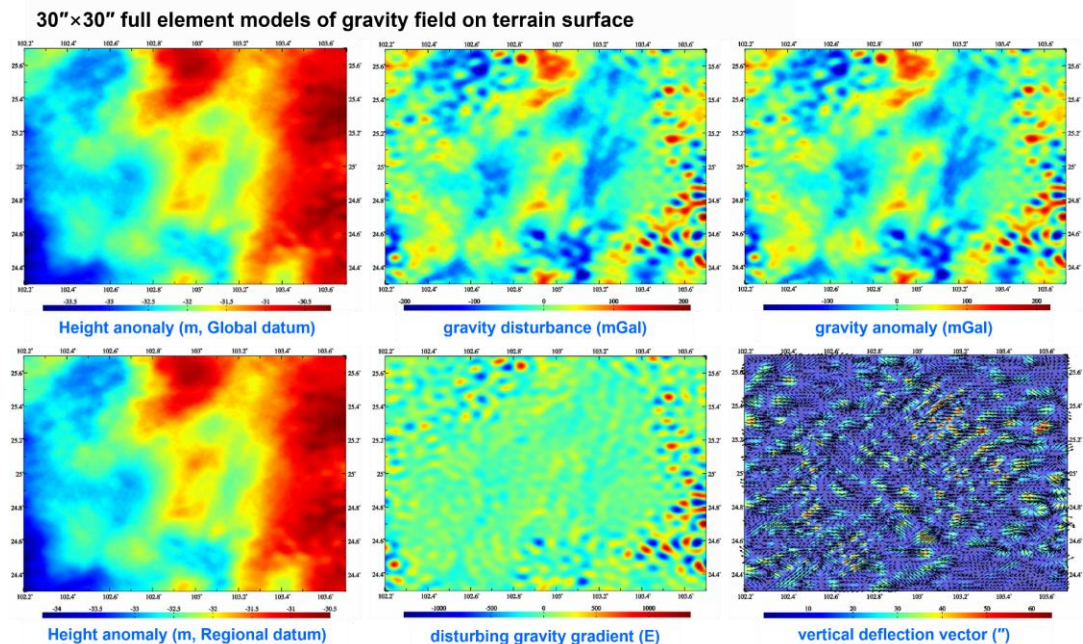
The 'Operation Progress' tab shows the progress of the operation, including the calculation of the reference gravity field and the generation of the 30" all-element models. The progress bar indicates that the operation is 100% complete.

The 'Full element models' section shows the results of the calculation, including the height anomaly, gravity disturbance, gravity anomaly, disturbing gravity gradient, and vertical deflection vector. The results are displayed in a grid format, with the x-axis representing longitude and the y-axis representing latitude.

The 'Residuals' section shows the residuals of the calculation, including the height anomaly, gravity disturbance, gravity anomaly, disturbing gravity gradient, and vertical deflection vector. The residuals are displayed in a grid format, with the x-axis representing longitude and the y-axis representing latitude.

The 'Remaining residuals' section shows the remaining residuals of the calculation, including the height anomaly, gravity disturbance, gravity anomaly, disturbing gravity gradient, and vertical deflection vector. The remaining residuals are displayed in a grid format, with the x-axis representing longitude and the y-axis representing latitude.

The 'Reference models' section shows the reference models of the calculation, including the height anomaly, gravity disturbance, gravity anomaly, disturbing gravity gradient, and vertical deflection vector. The reference models are displayed in a grid format, with the x-axis representing longitude and the y-axis representing latitude.



Add the regional height datum difference -0.3411m to the 30" gravimetric height anomaly grid surfhgt30srst.ksi in global height datum, the 30" gravimetric height

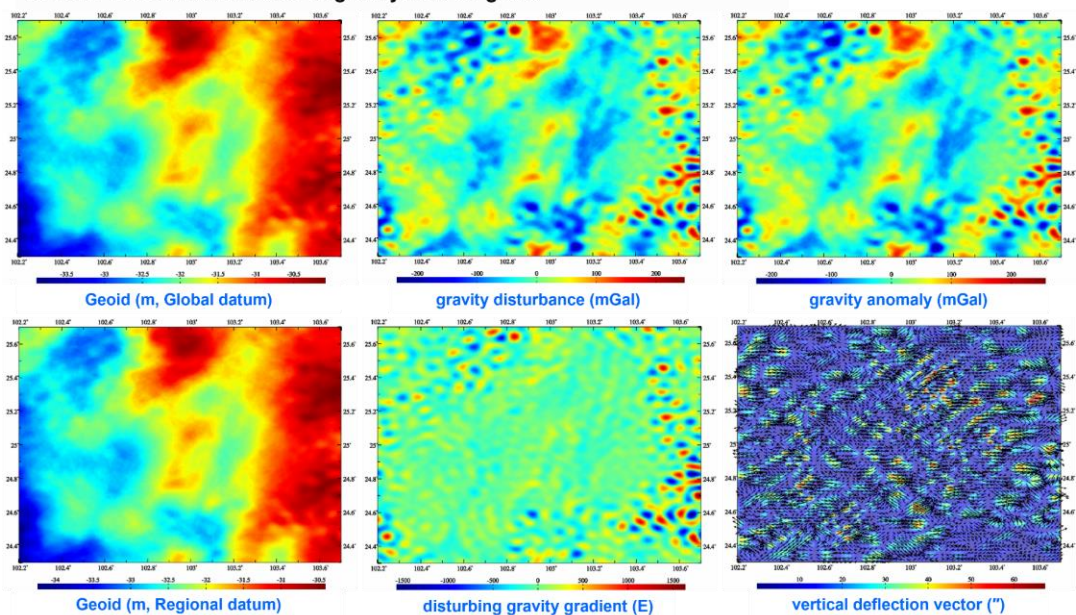
anomaly grid surfhgt30srgn.ksi in regional height datum can be obtained.

So far, the all-element modelling on gravity field on the terrain surface have been completed.

○ **Let the geoid as the calculation surface, and directly generate the 30" all-element models of the gravity field on the geoid.**

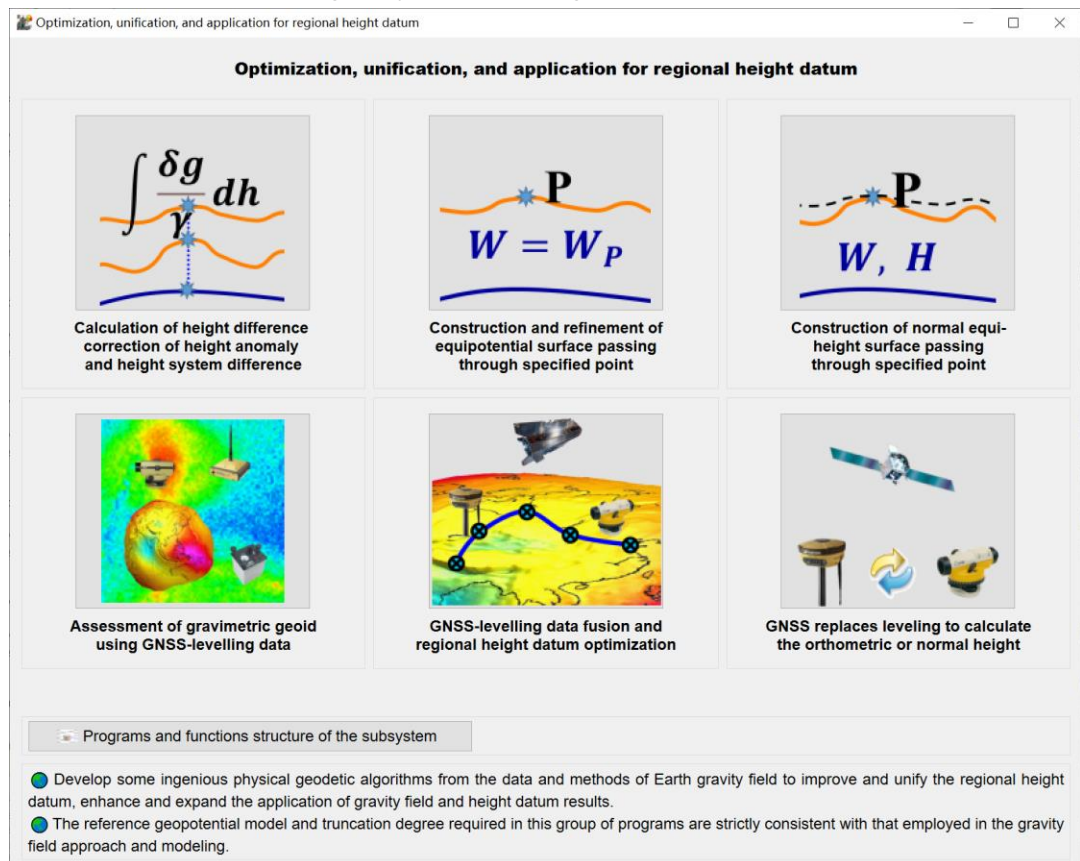
In step (3) to step (6) above, the input data file and all the parameter settings are kept same, and only the calculation surface is changed to the geoid. Using the same process, you can synchronously obtain the 30" all-element models geoidh30srst.xxx of the gravity field on the geoid, which include the 30" gravimetric geoidal height grid (geoidh30srst.ksi, m, in global height datum), gravity disturbance grid (geoidh30srst.rga, mGal), gravity anomaly grid (surfhgt30srst.gra, mGal), disturbing gravity gradient grid (geoidh30srst.grr, radial, E), vertical deflection vector grid (geoidh30srst.dft, SW, ") and geoidal height grid (geoidh30srgn.ksi, m) in regional height datum.

30"×30" full element models of gravity field on geoid



5 Optimization, unification, and application for regional height datum

Develop some ingenious physical geodetic algorithms from the data and methods of Earth gravity field to improve and unify the regional height datum, enhance and expand the application of gravity field and height datum results.



The reference geopotential model and truncation degree in this group of programs should be strictly consistent with that employed in gravity field approach and modelling.

5.1 Calculation of height difference correction of height anomaly and height system difference

[Calculation scheme] (a) Firstly, calculate the model height difference correction of height anomaly from the reference geopotential model. (b) Secondly, from the regional gravity field data, refine the residual height difference correction of height anomaly by the remove-restore scheme. (c) Finally, calculate the measured adjustment of height difference correction furtherly from the measured gravity at the measurement point. You can take one of the following three as the calculation result: (a), (a)+(b) or (a)+(b)+(c).

5.1.1 Calculation of height difference correction of height anomaly from the geopotential model

[Function] From the ellipsoidal height of the calculation point and target point in

near-Earth space, calculate the model value of radial gradient (cm/km) and height difference correction (m) of height anomaly using the reference geopotential model.

[Input files] The calculation point position file and the global geopotential coefficient model file.

The record format of the calculation point file: ID (point no/name), longitude (decimal degrees), latitude (decimal degrees),

The first row of the geopotential model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant GM ($10^{14}\text{m}^3/\text{s}^2$) and semi-major axis a(m) of the Earth.

[Parameter settings] Set number of rows of the calculation point file header, enter column ordinal number of ellipsoidal heights of the current point and target point in the file record, and input the maximum calculation degree for the geopotential model.

When the calculation point is on the ground and the target point is on the geoid, the program calculates the difference between the normal height and orthometric height, that is, the difference between the geoidal height and height anomaly.

The program selects the minimum of the maximum degree of the geopotential model and input maximum degree as the calculation degree.

Calculation of height difference correction of height anomaly and height system difference

Calculation points Save as Import parameters Start Computation Save process Follow example

Calculation of height difference correction of height anomaly from the geopotential model Refinement of the height difference correction of height anomaly from regional observations Calculation of the measured adjustment of height difference correction of height anomaly Analytic relationship between height systems

Open the calculation point position file Computation Process Operation Prompts Save computation process as

Set input point file format
 number of rows of file header 1
 column ordinal number of ellipsoidal height in the record 4
 column ordinal number of target ellipsoidal height in the record 5
 Maximum calculation degree for the geopotential model 360

Open the geopotential coefficient model file

Computation Process
 ** Select the calculation mode from the three control buttons at the top of the interface.
 ** [Function] From the ellipsoidal height of the calculation point and target point in near-Earth space, calculate the model value of radial gradient (cm/km) and height difference correction (m) of height anomaly using the reference geopotential model.
 ** Open the calculation point position file C:/PAGrav4.5_win64en/examples/AppHgtsysdifferent/calcpnt.txt.
 ** Look at the file information in the window below, set the input file format parameters.
 ** Open the geopotential coefficient model file C:/PAGrav4.5_win64en/data/EGM2008.gfc.
 ** The window below only shows the geopotential coefficients data with no more than 2000 rows in it
 ** Save the results as C:/PAGrav4.5_win64en/examples/AppHgtsysdifferent/mdldiffst.txt.
 ** Behind the source calculation point file record, appends a column of radial gradient of height anomaly and a column of model height difference correction of height anomaly, and keeps 4 significant figures.
 ** The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] tool button...
 >> Computation start time: 2024-09-29 14:40:55
 >> Complete to Calculate the model height difference correction of height anomaly!
 >> Computation end time: 2024-09-29 14:43:24

Save the results as Import setting parameters Start Computation

cn(degree/decimal)	lat	ellipHeight(m)	geoidHeight(m)	model radial gradient (cm/km)	model height difference correction (m)
11569	106.020833	27.020833	1217.221	-30.8082	0.6721 -0.0084
11570	106.062500	27.020833	1201.227	-30.8052	0.8284 -0.0102
11571	106.104167	27.020833	1185.247	-30.7849	0.9847 -0.0120
11572	106.145833	27.020833	1210.287	-30.7411	1.1368 -0.0141
11573	106.187500	27.020833	1228.340	-30.6802	1.2800 -0.0161
11574	106.229167	27.020833	1247.396	-30.6183	1.4102 -0.0180
11575	106.270833	27.020833	1244.440	-30.5729	1.5240 -0.0194
11576	106.312500	27.020833	1199.469	-30.5503	1.6184 -0.0199
11577	106.354167	27.020833	1193.494	-30.5360	1.6906 -0.0205
11578	106.395833	27.020833	1109.535	-30.4998	1.7396 -0.0198
11579	106.437500	27.020833	1000.613	-30.4157	1.7646 -0.0182
11580	106.479167	27.020833	1135.735	-30.2841	1.7631 -0.0206
11581	106.520833	27.020833	1249.869	-30.1357	1.7393 -0.0223
11582	106.562500	27.020833	1251.986	-30.0096	1.6959 -0.0217
11583	106.604167	27.020833	1289.077	-29.9216	1.6347 -0.0216
11584	106.645833	27.020833	1292.154	-29.8523	1.5599 -0.0206
11585	106.687500	27.020833	1228.242	-29.7662	1.4754 -0.0186
11586	106.729167	27.020833	1211.352	-29.6471	1.3857 -0.0172
11587	106.770833	27.020833	1339.471	-29.5138	1.2962 -0.0177

When the calculation point is on the ground and the target point is on the geoid, the program calculates the difference between the normal height and orthometric height, that is, the difference between the geoidal height and height anomaly.
 Height difference correction of height anomaly (m) = model correction, or model correction + residual correction, or model correction + residual correction + measured adjustment.
 Radial gradient of height anomaly (cm/km) = model radial gradient, or model radial gradient+ residual radial gradient, or model radial gradient+ residual radial gradient + measured adjustment.

[Output file] The model height difference correction file.

Behind the source calculation point file record, appends a column of radial gradient of height anomaly and a column of model height difference correction of height anomaly.

5.1.2 Refinement of the height difference correction of height anomaly from regional observations

[Function] From the ellipsoidal height of the equipotential surface and residual gravity disturbance grid on the surface, calculate the residual radial gradient (cm/km) of the height anomaly at the current calculation point and residual height difference correction (m) of height anomaly at the target point relative to the current point.

[Input files] The calculation point position file, ellipsoidal height grid file of the equipotential surface and residual gravity disturbance grid file on the surface with the same grid specifications.

The record format of the calculation point file: ID (point no/name), longitude (decimal degrees), latitude (decimal degrees),

The residual gravity disturbance should be calculated using the remove- restore scheme with the geopotential model and terrain effects in advance.

[Parameter settings] Set number of rows of the calculation point file header, enter column ordinal number of ellipsoidal heights of current point and target point in the file record, and input the residual integral radius.

Calculation of height difference correction of height anomaly and height system difference

Calculation points Save as Import parameters Start Computation Save process Follow example

Calculation of height difference correction of height anomaly from the geopotential model

Refinement of the height difference correction of height anomaly from regional observations

Calculation of the measured adjustment of height difference correction of height anomaly

Analytic relationship between height systems

Save computation process as

Open the calculation point position file

Set input point file format

number of rows of file header: 1

column ordinal number of ellipsoidal height in the record: 4

column ordinal number of target ellipsoidal height in the record: 5

Open residual gravity disturbance grid file on the equipotential surface

Open the ellipsoidal height grid file of the equipotential surface

Residual integral radius: 150 km

Computation Process ** Operation Prompts

>> [Function] From the ellipsoidal height of the equipotential surface and residual gravity disturbance grid on the surface, calculate the residual radial gradient (cm/km) of the height anomaly at the current calculation point and residual height difference correction (m) of height anomaly at the target point relative to the current point.

** The residual gravity disturbance should be calculated using the remove- restore scheme with the geopotential model and terrain effects in advance.

>> Open the calculation point position file C:/PAGrav4.5_win64en/examples/AppHgtssysdifferent/midfirst.txt.

** Look at the file information in the window below, set the input file format parameters.

>> Open residual gravity disturbance grid file on equipotential surface C:/PAGrav4.5_win64en/examples/AppHgtssysdifferent/dwmchrga.dat.

>> Open the ellipsoidal height grid file of the equipotential surface C:/PAGrav4.5_win64en/examples/AppHgtssysdifferent/dwmhgt150s.dat.

>> Save the results as C:/PAGrav4.5_win64en/examples/AppHgtssysdifferent/midfirst.txt.

** Behind the record of the source calculation point file, appends a column of the residual radial gradient and a column of the residual height difference correction, and keeps 4 significant figures.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-29 14:49:23

>> Complete to Calculate residual height difference correction of height anomaly!

Save the results as Import setting parameters Start Computation

ID	Longitude	Latitude	Height	Radial gradient	Height anomaly correction
0833	121.7221	-30.8082	0.6721	-0.0084	1.0239
0833	1201.227	-30.8052	0.8284	-0.0102	1.1014
0833	1185.247	-30.7849	0.9847	-0.0120	1.0145
0833	1210.287	-30.7411	1.1368	-0.0141	0.7312
0833	1228.340	-30.6802	1.2800	-0.0161	0.3232
0833	1247.396	-30.6183	1.4102	-0.0180	-0.0278
0833	1244.440	-30.5729	1.5240	-0.0194	-0.1268
0833	1199.469	-30.5503	1.6184	-0.0199	0.0985
0833	1183.494	-30.5360	1.6906	-0.0205	0.4995
0833	1109.535	-30.4998	1.7396	-0.0198	0.7431
0833	1000.613	-30.4157	1.7646	-0.0182	0.5545
0833	1135.735	-30.2841	1.7631	-0.0206	-0.0312
0833	1249.869	-30.1357	1.7393	-0.0223	-0.7351
0833	1251.986	-30.0096	1.6959	-0.0217	-1.1215
0833	1289.077	-29.9216	1.6347	-0.0216	-1.0465
0833	1292.154	-29.8523	1.5599	-0.0206	-0.7523
0833	1228.242	-29.7662	1.4754	-0.0186	-0.6524
0833	1211.352	-29.6471	1.3857	-0.0172	-0.9549
0833	1339.471	-29.5138	1.2962	-0.0177	-1.4728

residual radial gradient (km/cm)

height anomaly correction residual (m)

When the calculation point is on the ground and the target point is on the geoid, the program calculates the difference between the normal height and orthometric height, that is, the difference between the geoidal height and height anomaly.

Height difference correction of height anomaly (m) = model correction, or model correction + residual correction, or model correction + residual correction + measured adjustment.

Radial gradient of height anomaly (cm/km) = model radial gradient, or model radial gradient+ residual radial gradient, or model radial gradient+ residual radial gradient + measured adjustment.

[Output file] The residual height difference correction file.

Behind the record of the source calculation point file, appends a column of the residual radial gradient and a column of the residual height difference correction, and keeps 4 significant figures.

5.1.3 Calculation of the measured adjustment of height difference correction of height anomaly

[Function] Call the two upper left functions in turn, remove the reference model value and the refined residual value from the measured gravity disturbance at the calculation point, respectively, to obtain the remaining residual measured gravity disturbance, and then calculate the measured adjustment (cm/km) for radial gradient correction at the calculation point and measured adjustment (m) for the height difference correction at the target point relative to the current calculation point.

When there is the measured gravity data at the calculation point, this function can further refine the radial gradient and the height difference correction of height anomaly.

[Input file] The calculation point position file with the remaining residual gravity disturbance attribute in the record.

The record format of the calculation point file: ID (point no/name), longitude (decimal degrees), latitude (decimal degrees), ..., remaining residual gravity disturbance, ...

The remaining residual gravity disturbance = the measured disturbance gravity – model gravity disturbance - residual gravity disturbance.

[Parameter settings] Set number of rows of the calculation point file header, enter column ordinal number of ellipsoidal heights of current point and target point in the file record, and column ordinal number of remaining residual measured gravity disturbance.

Calculation of height difference correction of height anomaly and height system difference

Calculation points Save as Import parameters Start Computation Save process Follow example

Calculation of height difference correction of height anomaly from the geopotential model Refinement of the height difference correction of height anomaly from regional observations Calculation of the measured adjustment of height difference correction of height anomaly Analytic relationship between height systems

Open the calculation point position file

Set input point file format

number of rows of file header 1

column ordinal number of ellipsoidal height in the record 4

column ordinal number of target ellipsoidal height in the record 5

column ordinal number of remaining residual gravity disturbance 6

Computation Process ** Operation Prompts

disturbance at the calculation point, respectively, to obtain the remaining residual measured gravity disturbance, and then calculate the measured adjustment (cm/km) for radial gradient correction at the calculation point and measured adjustment (m) for the height difference correction at the target point relative to the current calculation point.

** When there is the measured gravity data at the calculation point, this function can further refine the radial gradient and the height difference correction of height anomaly.

** Open the calculation point position file C:/PAGrav4.5_win64en/examples/AppHgtysysdifferent/calcpnt1.txt.

** Look at the file information in the window below, set the input file format parameters.

** Save the results as C:/PAGrav4.5_win64en/examples/AppHgtysysdifferent/msrdiffad.txt.

** Behind the record of the source calculation point file, appends a column of the adjustment of radial gradient of the height anomaly (cm/km) and a column of the adjustment of residual height anomaly difference (m), and keeps 4 significant figures.

** The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-29 14:55:34

>> Complete to Calculate measured adjustment for height anomaly difference correction!

>> Computation end time: 2024-09-29 14:55:35

Save the results as Import setting parameters Start Computation

lat ellipHeight (m) geoidHeight (m)

lat	ellipHeight (m)	geoidHeight (m)
0833	27.020833	1217.221
2500	27.020833	1201.227
4167	27.020833	1185.247
5833	27.020833	1210.287
7500	27.020833	1228.340
9167	27.020833	1247.396
0833	27.020833	1244.440
2500	27.020833	1199.469
4167	27.020833	1183.494
5833	27.020833	1109.535
7500	27.020833	1000.613
9167	27.020833	1135.735
0833	27.020833	1249.569
2500	27.020833	1251.986
4167	27.020833	1289.077
5833	27.020833	1292.154
7500	27.020833	1228.242
9167	27.020833	1211.352
0833	27.020833	1339.471

measured adjustment (km/cm) for radial gradient measured adjustment (m) for height anomaly correction

measured adjustment (km/cm) for radial gradient

measured adjustment (m) for height anomaly correction

When the calculation point is on the ground and the target point is on the geoid, the program calculates the difference between the normal height and orthometric height, that is, the difference between the geoidal height and height anomaly.

Height difference correction of height anomaly (m) = model correction, or model correction + residual correction, or model correction + residual correction + measured adjustment.

Radial gradient of height anomaly (cm/km) = model radial gradient, or model radial gradient+ residual radial gradient, or model radial gradient+ residual radial gradient+ measured adjustment.

[Output file] The measured adjustment file of residual height difference.

Behind the record of the source calculation point file, appends a column of the adjustment of radial gradient of the height anomaly (cm/km) and a column of the adjustment of residual height anomaly difference (m), and keeps 4 significant figures.

Height difference correction of height anomaly (m) = model correction, or model correction + residual correction, or model correction + residual correction + measured adjustment.

Radial gradient of height anomaly (cm/km) = model radial gradient, or model radial gradient+ residual radial gradient, or model radial gradient+ residual radial gradient + measured adjustment.

5.2 Construction and refinement of equipotential surface passing through specified point

[Purpose] Firstly, calculate the model ellipsoidal height value of the gravity equipotential surface from the reference geopotential model, and then furtherly refine the ellipsoidal height grid of the equipotential surface from the anomalous field element grid using the remove-restore scheme.

5.2.1 Construction of the gravity equipotential surface from global geopotential model

[Function] From the geopotential coefficient model, calculate the model gravity (mGal) and model ellipsoidal height (m) grid of the gravity equipotential surface passing through the specified points (B, L, H).

[Input files] The equipotential surface range grid file and global geopotential coefficient model file.

The equipotential surface range grid file is only employed to give the latitude and longitude range and resolution of the surface grid.

The first row of the geopotential model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant GM ($10^{14}\text{m}^3/\text{s}^2$) and semi-major axis $a(\text{m})$ of the Earth.

[Parameter settings] Input geodetic coordinates of the specified point and the maximum calculation degree for the geopotential model.

The program requires the specified point is within the range of the equipotential surface grid.

The program selects the minimum of the maximum degree of the global geopotential model and the input maximum degree as the calculation degree. The degree of the model should not be too large, since the program adopts the iterative approach method, which need take a long time.

The reference geopotential model and truncation degree should be consistent with that employed in the gravity field approach and modelling.

[Output file] The model ellipsoidal height grid file and model gravity grid file of model equipotential surface.

The ellipsoidal height on the equipotential surface = ellipsoidal height cell-grid value + the 9th number of the file header.

Construction and refinement of equipotential surface passing through specified point

Grid range Save as Import parameters Start Computation Save process Follow example

Construction of the gravity equipotential surface from global geopotential model Refinement of ellipsoidal height of the equipotential surface by local gravity field approach Save computation process as

Open the equipotential surface range grid file Open the geopotential coefficient model file

Input geodetic coordinates of the specified point

longitude 110.24560000° latitude 27.46720000° ellipsoidal height 1346.0240 m

Maximum calculation degree for the geopotential model 360

Save the model gravity grid as Save model ellipsoidal height as

Import setting parameters Start Computation Extract results Plot

106.000000 112.000000 27.000000 32.000000 0.04166667 0.04166667 110.245600 27.467200 1346.0240^

-10.0529 -10.0392 -10.0236 -10.0051 -9.9929 -9.9563 -9.9245 -9.8870 -9.8433

-9.3656 -9.2779 -9.1876 -9.0958 -9.0034 -8.9112 -8.8202 -8.7311 -8.6444

-8.1236 -8.0587 -7.9944 -7.9300 -7.8645 -7.7974 -7.7277 -7.6550 -7.5785

-6.9202 -6.8099 -6.6966 -6.5808 -6.4632 -6.3442 -6.2243 -6.1039 -5.9834

-5.1467 -5.0271 -4.9068 -4.7855 -4.6630 -4.5390 -4.4135 -4.2863 -4.1574

-3.2228 -3.0881 -2.9540 -2.8210 -2.6891 -2.5586 -2.4296 -2.3020 -2.1759

-1.3199 -1.1990 -1.0782 -0.9572 -0.8364 -0.7157 -0.5956 -0.4763 -0.3582

0.3964 0.4892 0.5776 0.6616 0.7415 0.8176 0.8903 0.9602 1.0279

1.5026 1.5788 1.6585 1.7421 1.8295 1.9208 2.0157 2.1139 2.2152

2.9668 3.0750 3.1826 3.2896 3.3960 3.5020 3.6078 3.7138 3.8203

-10.1508 -10.1392 -10.1255 -10.1090 -10.0889 -10.0642 -10.0344 -9.9988 -9.9570

-9.4888 -9.4016 -9.3115 -9.2195 -9.1264 -9.0333 -8.9409 -8.8500 -8.7611

-8.2166 -8.1476 -8.0794 -8.0110 -7.9419 -7.8712 -7.7983 -7.7226 -7.6435

-6.9784 -6.8686 -6.7561 -6.6413 -6.5249 -6.4072 -6.2886 -6.1696 -6.0504

-5.2172 -5.0969 -4.9755 -4.8528 -4.7286 -4.6026 -4.4747 -4.3449 -4.2131

-3.2540 -3.1156 -2.9780 -2.8415 -2.7063 -2.5726 -2.4406 -2.3102 -2.1816

-1.3138 -1.1921 -1.0706 -0.9492 -0.8281 -0.7073 -0.5871 -0.4679 -0.3500^

The ellipsoidal height on the equipotential surface = ellipsoidal height cell grid value + the 9th number of the file header.

The reference geopotential model and truncation degree should be consistent with that employed in the gravity field approach and modeling. The degree of the model should not be too large, since the program adopts the iterative approach method, which need take a long time.

5.2.2 Refinement of ellipsoidal height of the equipotential surface by local gravity field approach

[Function] From the ellipsoidal height of some an equipotential boundary surface and residual gravity disturbance as well as the calculated model gravity and model ellipsoidal height grid, refine the ellipsoidal height grid of the equipotential surface passing through the specified points (B, L, H) by the Hotine integral.

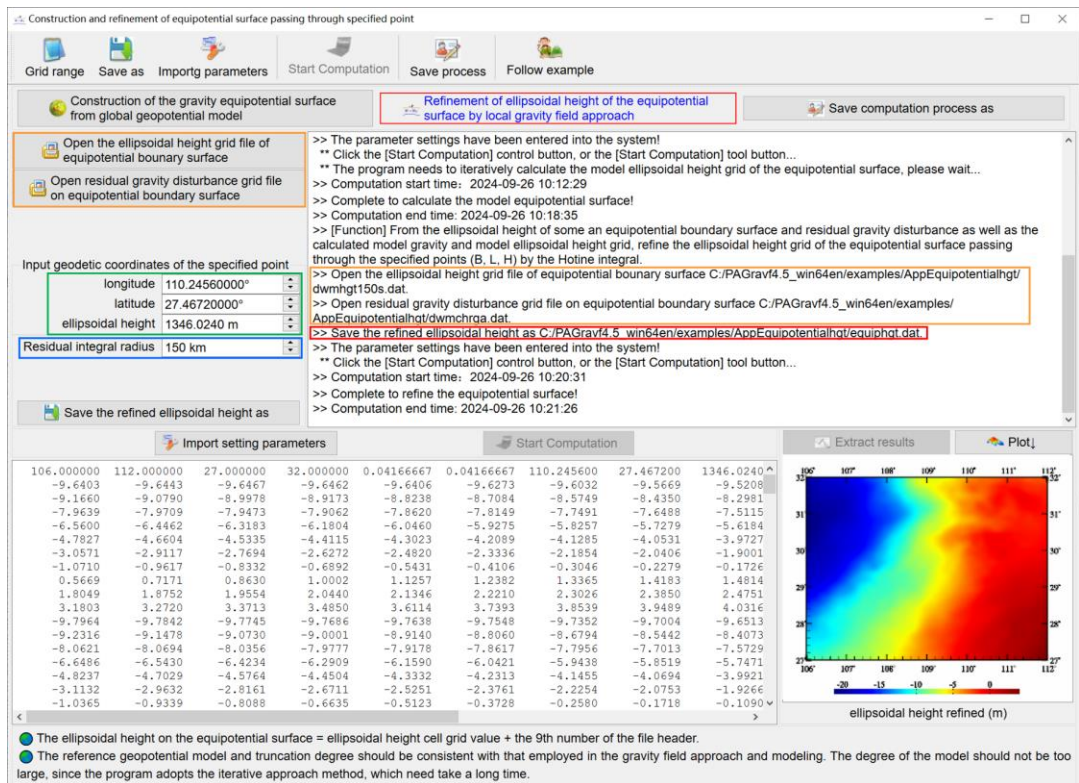
[Input files] The ellipsoidal height grid file of the equipotential boundary surface and residual gravity disturbance grid file on the surface, the model ellipsoidal height grid file and model gravity grid file on the model equipotential surface.

The equipotential boundary surface here refers specifically to the Stokes boundary equipotential surface where the residual gravity disturbance is located, different with the equipotential surface to be determined.

[Parameter settings] Input geodetic coordinates of the specified point and the residual integral radius.

[Output file] The refined ellipsoidal height grid file of the equipotential surface passing through the specified points (B, L, H)

The ellipsoidal height on the equipotential surface = ellipsoidal height cell-grid value + the 9th number of the file header.



5.3 Construction of terrain equiheight surface passing through specified point

[Purpose] Firstly, calculate the model geopotential of the normal or orthometric equiheight surface from the reference gravity field model, and then furtherly refine the geopotential from the anomalous field element grid using the remove-restore scheme.

5.3.1 Calculation of the model geopotential of the normal or orthometric equiheight surface

[Function] From the geopotential coefficient model, calculate the model geopotential, model ellipsoidal height (m) and model gravity (mGal) grid of the normal or orthometric equiheight surface passing through the specified point (B, L).

[Input files] The equiheight surface range grid file and global geopotential coefficient model file.

The equiheight surface range grid file is only employed to give the latitude and longitude range and resolution of the surface grid.

The first row of the geopotential model file is agreed to be the scale parameters of the geopotential coefficient model: the geocentric gravitational constant GM ($10^{14}m^3/s^2$) and semi-major axis $a(m)$ of the Earth.

[Parameter settings] Input geodetic coordinates of the specified point and the maximum calculation degree for the geopotential coefficient model and select the type of height system.

The program requires the specified point is within the range of the equiheight surface grid.

The program selects the minimum of the maximum degree of the global geopotential model and input maximum degree as the calculation degree. The degree of the model should not be too large, since the program adopts the iterative approach method, which need take a long time.

The reference geopotential model and truncation degree should be consistent with that employed in the gravity field approach and modelling.

Construction of terrain equiheight surface passing through specified point

Grid range Save as Import parameters Start Computation Save process Follow example

Calculation of the model geopotential of the normal or orthometric equiheight surface Refinement of the normal equiheight surface passing through specified point Summation of the model values and residual values Save computation process as

Open the equiheight surface range grid file Open the geopotential coefficient model file

Input position of the specified point

longitude 110.24560000° latitude 27.46720000° normal or orthometric height 1346.0240 m

Maximum calculation degree for the geopotential model 360

Height system normal height system

>> [Purpose] Firstly, calculate the model geopotential of the normal or orthometric equiheight surface from the reference gravity field model, and then further refine the geopotential from the anomalous field element grid using the remove-restore scheme.
 ** Select the function module the 3 control buttons at the top of the interface...
 >> [Function] From the geopotential coefficient model, calculate the model geopotential, model ellipsoidal height (m) and model gravity (mGal) grid of the normal or orthometric equiheight surface passing through the specified point (B, L).
 >> The equiheight surface range grid file is only employed to give the latitude and longitude range and resolution of the surface grid.
 >> Open the equiheight surface range grid file C:/PAGrav4.5_win64en/examples/AppEqhgtpotential/areagrid.dat.
 >> Open the geopotential coefficient model file C:/PAGrav4.5_win64en/data/EGM2008.gfc.
 ** The window below only shows the geopotential coefficients data with no more than 2000 rows in it.
 >> Save the model gravity grid as C:/PAGrav4.5_win64en/examples/AppEqhgtpotential/eqhgtmdlgrv.dat.
 >> Save the model geopotential grid as C:/PAGrav4.5_win64en/examples/AppEqhgtpotential/eqhgmptential.dat.
 >> Save the model ellipsoidal height as C:/PAGrav4.5_win64en/examples/AppEqhgtpotential/eqhgtmdlhtg.dat.
 ** The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] tool button...
 The program needs to iteratively calculate the model geopotential grid of the equiheight surface, please wait...
 >> Computation start time: 2024-09-26 10:29:19
 >> Complete to calculate the model equiheight surface!
 >> Computation end time: 2024-09-26 10:37:47

106.000000	112.000000	27.000000	32.000000	0.04166667	0.04166667
-47.0779	-48.5710	-50.0626	-51.5117	-52.8773	-54.1205
-55.6160	-54.8211	-53.9773	-53.1293	-52.3217	-51.5968
-53.7911	-54.8783	-56.0160	-57.1638	-58.2815	-59.3310
-61.6286	-61.0857	-60.4887	-59.8692	-59.2583	-58.6846
-57.8114	-58.0957	-58.3711	-58.6137	-58.8012	-58.9143
-54.9842	-54.2986	-53.6328	-53.0050	-52.4305	-51.9211
-50.1814	-50.1167	-50.0220	-49.8912	-49.7219	-49.5160
-48.2901	-48.6244	-49.0894	-49.6836	-50.3996	-51.2246
-59.9952	-60.3345	-60.5051	-60.5073	-60.3479	-60.0403
-54.5244	-54.3209	-54.2460	-54.2972	-54.4654	-54.7357
-43.9476	-45.5158	-47.0858	-48.6168	-50.0682	-51.4013
-53.8577	-53.1327	-52.3463	-51.5418	-50.7621	-50.0484
-51.4914	-52.4531	-53.4667	-54.4949	-55.5002	-56.4472
-58.5753	-58.0995	-57.5792	-57.0436	-56.5213	-56.0383
-55.7158	-55.9891	-56.2396	-56.4438	-56.5801	-56.6301
-51.6479	-50.8404	-50.0538	-49.3070	-48.6162	-47.9938
-45.3853	-45.2636	-45.1176	-44.9411	-44.7319	-44.4920
-43.3261	-43.7170	-44.2496	-44.9224	-45.7282	-46.6537
-56.7661	-57.2295	-57.5125	-57.6128	-57.5353	-57.2914

Save the model gravity grid as Save the model geopotential grid as Save model ellipsoidal height as Import setting parameters Start Computation

Extract results Plot

model geopotential (m²/s²) model ellipsoidal height (m)

The model ellipsoidal height = the cell grid value in the model ellipsoidal height grid file + 9th number of the file header. The model gravity = the cell grid value in the model gravity grid file + 10th number of the file header. The ellipsoidal height = the model ellipsoidal height + correction of the height. The gravity = the model gravity + correction of the gravity.

[Output files] The model geopotential grid file, model ellipsoidal height grid file and model gravity grid file of model equiheight surface.

The model ellipsoidal height = the cell-grid value in the model ellipsoidal height grid file + 9th number of the file header.

The model gravity = the cell-grid value in the model gravity grid file + 10th number of the file header.

5.3.2 Refinement of the normal equiheight surface passing through specified point

[Function] From the ellipsoidal height of some an equipotential boundary surface and residual gravity disturbance on the surface as well as the calculated model gravity and model ellipsoidal height grid, refine the ellipsoidal height grid of the normal or orthometric equiheight surface passing through the specified point (B, L) by the Hotine

integral.

[Input files] The ellipsoidal height grid file of the equipotential boundary surface and residual gravity disturbance grid file on the surface, the model ellipsoidal height grid file and model gravity grid file on the model equipotential surface.

The residual geopotential correction and residual ellipsoidal height correction of the equiheight surface are calculated from the residual gravity disturbance after the reference model values removed.

[Parameter settings] Input geodetic coordinates of the specified point and the residual integral radius and select the type of height system.

[Output files] The geopotential correction grid file and ellipsoidal height correction grid file of the equiheight surface.

Construction of terrain equiheight surface passing through specified point

Grid range Save as Import parameters Start Computation Save process Follow example

Calculation of the model geopotential of the normal or orthometric equiheight surface Refinement of the normal equiheight surface passing through specified point Summation of the model values and residual values Save computation process as

Open the ellipsoidal height grid file of equipotential boundary surface

Open residual gravity disturbance grid file on equipotential boundary surface

Input position of the specified point

longitude 110.24560000°

latitude 27.46720000°

normal or orthometric height 1346.0240 m

Residual integral radius 150 km

Height system normal height system

** The program needs to iteratively calculate the model geopotential grid of the equiheight surface, please wait...

>> Computation start time: 2024-09-26 10:29:19

>> Complete to calculate the model equiheight surface!

>> [Function] From the ellipsoidal height of some an equipotential boundary surface and residual gravity disturbance on the surface as well as the calculated model gravity and model ellipsoidal height grid, refine the ellipsoidal height grid of the normal or orthometric equiheight surface passing through the specified point (B, L) by the Hotine integral.

>> Open the ellipsoidal height grid file of equipotential boundary surface C:/PAGrav4.5_win64en/examples/AppEquiheightpotential/dwmhgt150s.dat.

>> Open residual gravity disturbance grid file on equipotential boundary surface C:/PAGrav4.5_win64en/examples/AppEquiheightpotential/dwmchrgs.dat.

>> Save geopotential correction grid as C:/PAGrav4.5_win64en/examples/AppEquiheightpotential/eqhpotadj.dat.

>> Save ellipsoidal height corrections as C:/PAGrav4.5_win64en/examples/AppEquiheightpotential/eqhgtadj.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-26 10:41:14

>> Complete to refine the equiheight surface!

>> Computation end time: 2024-09-26 10:42:34

Save the geopotential corrections as Save ellipsoidal height corrections as Import setting parameters Start Computation

106.000000	112.000000	27.000000	32.000000	0.04166667	0.04166667
-47.0779	-48.5710	-50.0626	-51.5117	-52.8773	-54.1205
-55.6160	-54.8211	-53.9773	-53.1293	-52.3217	-51.5968
-53.7911	-54.8783	-56.0160	-57.1638	-58.2815	-59.3310
-61.6286	-61.0857	-60.4887	-59.8692	-59.2583	-58.6946
-57.8114	-58.0957	-59.3711	-58.6137	-58.8012	-58.9143
-54.9842	-54.2986	-53.6328	-53.0050	-52.4305	-51.9211
-50.1814	-50.1167	-50.0220	-49.8912	-49.7219	-49.5160
-48.2901	-48.6244	-49.0894	-49.6836	-50.3996	-51.2246
-59.9952	-60.3345	-60.5051	-60.5073	-60.3479	-60.0403
-54.5244	-54.3209	-54.2460	-54.2972	-54.4654	-54.7357
-43.9476	-45.5158	-47.0858	-48.6168	-50.0682	-51.4013
-53.8577	-53.1327	-52.3463	-51.5418	-50.7621	-50.0484
-51.4914	-52.4531	-53.4667	-54.4949	-55.5002	-56.4472
-58.5753	-58.0995	-57.5792	-57.0436	-56.5213	-56.0383
-55.7158	-55.9991	-56.2396	-56.4438	-56.5801	-56.6301
-51.6479	-50.8404	-50.0538	-49.3070	-48.6162	-47.9938
-45.3853	-45.2636	-45.1176	-44.9411	-44.7319	-44.4920
-43.3261	-43.7170	-44.2496	-44.9224	-45.7282	-46.6537
-56.7661	-57.2295	-57.5125	-57.6128	-57.5353	-57.2914

The model ellipsoidal height = the cell grid value in the model ellipsoidal height grid file + 9th number of the file header. The model gravity = the cell grid value in the model gravity grid file + 10th number of the file header. The ellipsoidal height = the model ellipsoidal height + correction of the height. The gravity = the model gravity + correction of the gravity.

Extract results Plot

residual geopotential (m²/s²) residual ellipsoidal height (m)

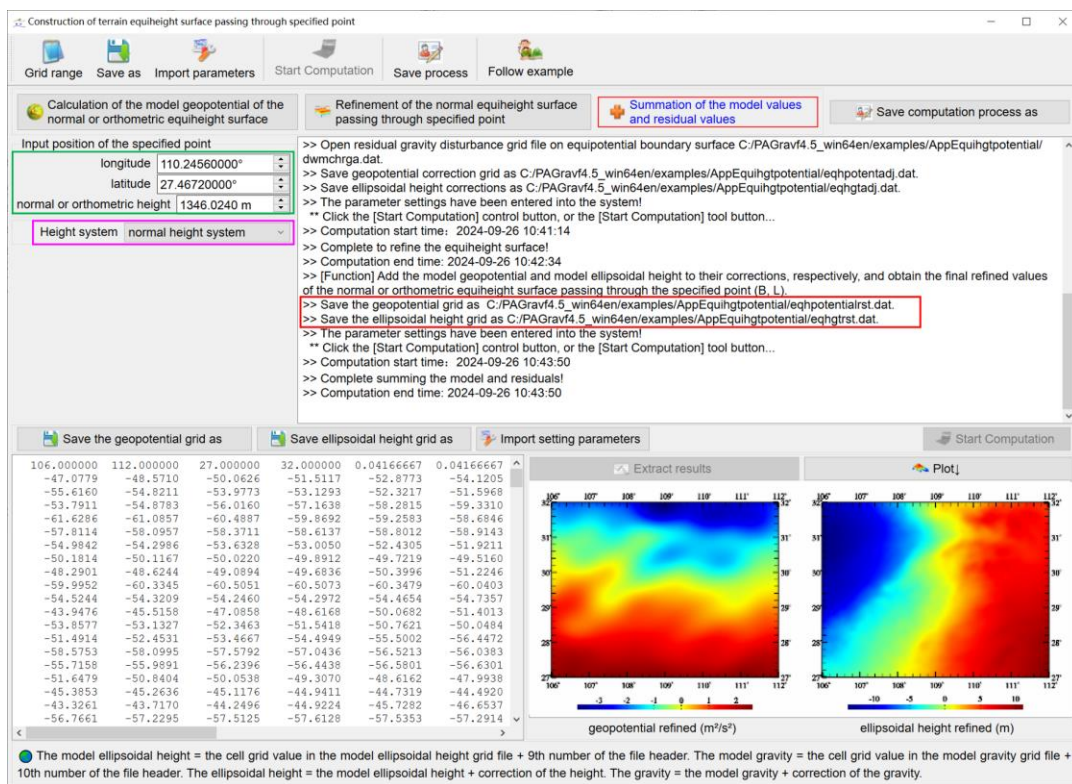
5.3.3 Summation of the model values and residual values

[Function] Add the model geopotential and model ellipsoidal height to their corrections, respectively, and obtain the final refined values of the normal or orthometric equiheight surface passing through the specified point (B, L).

[Output files] The geopotential grid file and ellipsoidal height grid file of the normal or orthometric equiheight surface passing through the specified point (B, L).

The ellipsoidal height = the model ellipsoidal height + correction of the height.

The gravity = the model gravity + correction of the gravity.



5.4 Assessment of gravimetric geoid using GNSS-levelling data

[Function] According to the spectral domain error characteristics of GNSS-levelling height anomaly and gravity field, statistically analyze the GNSS-levelling residual height anomalies (m), and then estimate the error (cm) of gravimetric height anomalies, inner coincidence error of hybrid height anomalies, error of hybrid height anomaly differences and error of GNSS-levelling height anomaly differences.

Replacing the ground height anomaly with the geoidal height, the program can evaluate the gravimetric geoid of the orthometric system.

[Input file] The discrete GNSS-levelling residual height anomaly file.

[Parameter settings] Set the input file format parameters, enter the GNSS-levelling network parameters, mean distance between GNSS benchmarks, the constant and proportional error of ellipsoidal height differences of the GNSS baselines, set the number of groups in the GNSS benchmarks combined in pairs, the maximum distance and distance interval for the distance-error curve.

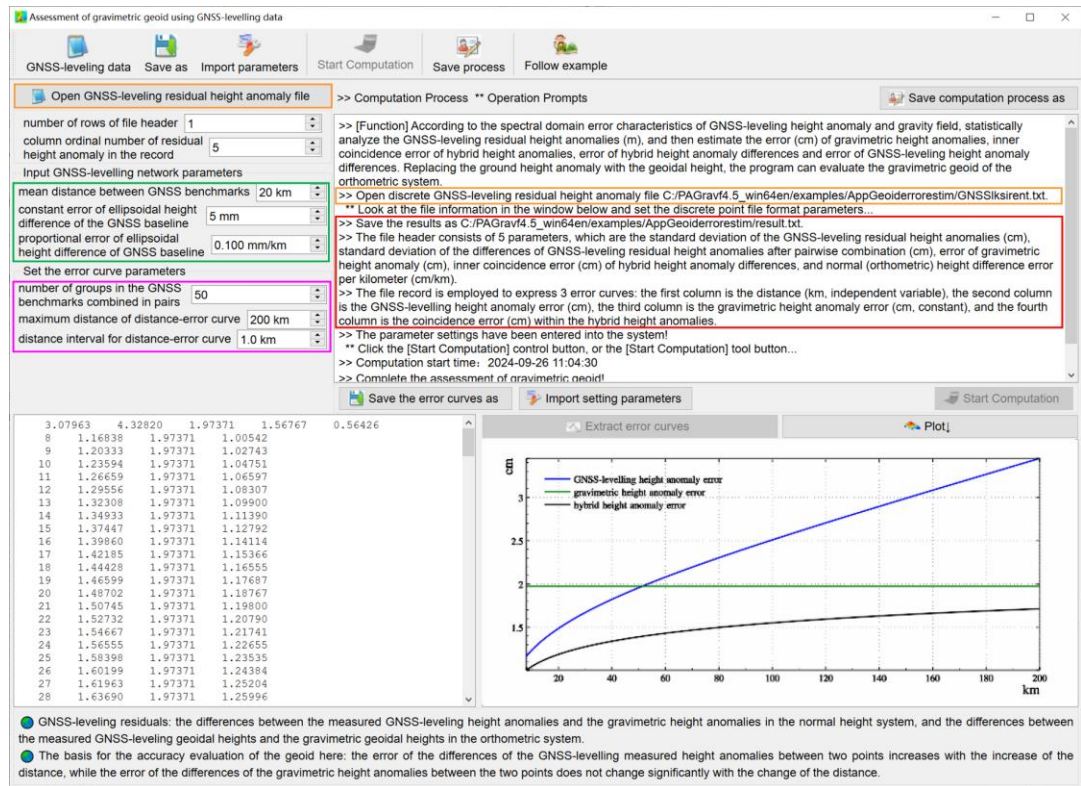
The program combines each of the n GNSS benchmarks to form $n(n-1)/2$ sides, and after sorting them according to the side length, the GNSS-levelling residual height anomaly differences are grouped and then counted.

[Output file] The error curve file of gravimetric geoid.

The file header consists of 5 parameters, which are the standard deviation of the GNSS-levelling residual height anomalies (cm), standard deviation of the differences of

GNSS-leveling residual height anomalies after pairwise combination (cm), error of gravimetric height anomaly (cm), inner coincidence error (cm) of hybrid height anomaly differences, and normal (orthometric) height difference error per kilometer (cm/km).

The file record is employed to express 3 error curves: the first column is the distance (km, independent variable), the second column is the GNSS-levelling height anomaly error (cm), the third column is the gravimetric height anomaly error (cm, constant), and the fourth column is the coincidence error (cm) within the hybrid height anomalies.



GNSS-leveling residuals: the differences between the measured GNSS-leveling height anomalies and the gravimetric height anomalies in the normal height system, and the differences between the measured GNSS-leveling geoidal heights and the gravimetric geoidal heights in the orthometric system.

The basis for the accuracy evaluation of the geoid here: the error of the differences of the GNSS-levelling measured height anomalies between two points increases with the increase of the distance, while the error of the differences of the gravimetric height anomalies between the two points does not change significantly with the change of the distance.

5.5 GNSS-levelling data fusion and regional height datum optimization

[Purpose] From the GNSS-leveling residual geoidal heights or residual height anomalies, estimate the geopotential difference of regional height datum and its zero-point parameters, calculate the correction of hybrid geoid, and then optimize the regional

height datum (GNSS-leveilling network).

5.5.1 Calculation of geopotential of the zero-height surface for regional height datum

[Function] From the GNSS-leveilling measured geoidal heights (height anomalies, m) and residual geoidal heights (height anomalies, m), estimate the zero-height surface geopotential W_r of the regional height datum. Then with the given geopotential W_o of the global geoid, obtain the zero-height surface geopotential difference $W_r - W_o$ of the regional height datum relative to the global geoid.

[Input file] The discrete GNSS-leveilling residual file.

GNSS-leveilling residuals: the differences between the measured GNSS-leveilling height anomalies and the gravimetric height anomalies in the normal height system, or the differences between the measured GNSS-leveilling geoidal heights and the gravimetric geoidal heights in the orthometric system.

[Parameter settings] Set the input file format parameters, enter the geopotential W_o of global zero-height surface.

GNSS-leveilling data fusion and regional height datum optimization

GNSS-leveilling residuals Save as Import parameters Start Computation Save process Follow example

Calculation of geopotential of the zero-height surface for regional height datum

GNSS-leveilling data fusion with constraints of the Poisson integral

Leveling network quasi-stable adjustment with remaining GNSS-leveilling residuals

Open discrete GNSS-leveilling residuals file

Set input file format

number of rows of file header 1

Column ordinal number of the GNSS-leveilling observation 5

column ordinal number of GNSS-leveilling residual 6

column ordinal number of weight 7

Input geopotential of global zero-height surface (m²/s²) 62636853.400

Operation Prompts

>> Open discrete GNSS-leveilling residual file C:/PAGrav4.5...win64en/examples/AppGNSSLivingdatum/GNSSImtksi.txt

>> Look at the file information in the window below and set the discrete point file format parameters...

>> Save the remaining GNSS-leveilling residuals as C:/PAGrav4.5...win64en/examples/AppGNSSLivingdatum/mtksich.txt

** The file header has 6 attributes, namely W_r , U_o , W_o , $W_r - U_o$, $W_r - W_o$ and $U_o - W_o$, where W_r is the zero-height surface geopotential of the regional height datum, U_o is the normal geopotential of the level ellipsoidal surface which is equal to the geopotential of the gravimetric geoid, and W_o is the geopotential of the global geoid considered as the global orthometric (normal) zero-height surface.

** Behind the record of the GNSS-leveilling residual file, appends a column of correction after reduction to gravimetric geoid, and keeps 4 significant figures.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-26 11:14:15

Geopotential of regional zero-height surface $W_r = 62636850.846\text{m}^2/\text{s}^2$

Geopotential of global gravimetric geoid $U_o = 62636858.709\text{m}^2/\text{s}^2$

Geopotential of global zero-height surface $W_o = 62636853.400\text{m}^2/\text{s}^2$

Geopotential difference of regional zero-height surface relative to global geoid $W_r - U_o = -7.863\text{m}^2/\text{s}^2$

Geopotential difference of regional zero-height surface relative to global zero-height surface $W_r - W_o = -2.554\text{m}^2/\text{s}^2$

Geopotential difference of global gravimetric geoid relative to global zero-height $U_o - W_o = 5.309\text{m}^2/\text{s}^2$

>> Complete to calculate the geopotential differences of height datum!

Save the remaining GNSS-leveilling residuals as

Import setting parameters

Start Computation

no lon(degree/decimal) lat ellipheight(m) geoidheight(m) residual weight

no	lon(degree/decimal)	lat	ellipheight(m)	geoidheight(m)	residual	weight
1	102.546777	24.458002	1659.0410	-33.6150	-0.8046	0.94
2	102.632412	24.458211	2120.2558	-33.3212	-0.7142	1.23
3	102.725921	24.460578	2111.3872	-33.2058	-0.7612	1.68
4	102.420803	24.566357	1990.6386	-33.5334	-0.7157	1.95
5	102.528697	24.562786	1936.4260	-33.3720	-0.7491	2.93
6	102.832641	24.575505	1977.4949	-33.1581	-0.8223	3.04
7	102.345532	24.668953	1919.7825	-33.7565	-0.7782	3.53
8	102.423972	24.652933	1959.3369	-33.4781	-0.7548	2.02
9	102.529771	24.667079	2157.7877	-33.2933	-0.7317	1.46
10	102.631063	24.670755	1906.3415	-33.3155	-0.8185	3.53
11	102.742718	24.652871	1935.7882	-33.1128	-0.7767	3.39
12	102.843573	24.642787	1880.7707	-33.1133	-0.8319	0.81
13	103.137778	24.658224	1838.4387	-32.7463	-0.7730	0.53
14	102.426305	24.743284	1929.0476	-33.4579	-0.7771	1.48
15	102.729945	24.734509	1856.2213	-33.2087	-0.8356	6.12
16	102.840819	24.752018	2117.8582	-32.8948	-0.7459	1.56
17	102.939253	24.728089	2050.9590	-32.8500	-0.7907	0.81
18	103.029713	24.748496	2034.1964	-32.8194	-0.8217	0.88
19	103.129600	24.753135	1575.0654	-32.8486	-0.8477	1.41

Extract remaining residuals

Plot

The data fusion surface for the normal height system is the ground, and the ground ellipsoidal height grid file should be input. The data fusion surface for the orthometric system is the geoid, and the geoidal height grid file should be input.

The remaining GNSS-leveilling residual file after GNSS-leveilling data fusion can be employed to evaluate the quality of the measured GNSS-leveilling data.

[Output file] The remaining GNSS-leveilling residual file.

The file header has 6 attributes, namely W_r , U_o , W_o , $W_r - U_o$, $W_r - W_o$ and $U_o - W_o$, where W_r is the zero-height surface geopotential of the regional height datum, U_o is the normal geopotential of the level ellipsoidal surface which is equal to the geopotential of the gravimetric geoid, and W_o is the geopotential of the global geoid considered as the

global orthometric (normal) zero-height surface.

Behind the record of the GNSS-levelling residual file, appends a column of correction after reduction to gravimetric geoid, and keeps 4 significant figures.

The normal zero-height surface and the orthometric zero-height surface always coincide everywhere and are the same, and there is no need to distinguish them.

PAGravf4.5 recommends that the geoidal geopotential W_0 or U_0 should replace the empirical appoint W_0 in the IERS numerical standard. The latter is calculated from the global geopotential model and mean sea surface height model according to the Gaussian geoid definition.

5.5.2 GNSS-leveling data fusion with constraints of the Poisson integral

[Function] From the GNSS-leveling residuals and the ellipsoidal height grid of the data fusion surface, estimate the correction of gravimetric geoidal height/ground height anomalies with constraints of the Poisson integral, to realize the analytical fusion of GNSS-leveling data and gravimetric geoidal heights.

[Input files] The discrete (remaining) GNSS-leveling residual file, the data fusion range grid file.

The data fusion surface for the normal height system is the ground, and the ground ellipsoidal height grid file should be input. The data fusion surface for the orthometric system is the geoid, and the geoidal height grid file should be input.

[Parameter settings] Set the number of rows of the GNSS-leveling file header, the column ordinal number of the ellipsoidal height, GNSS-levelling residual and weight, enter the iterative calculation times, residual integral radius, Laplace operator parameter and edge effect suppressing parameter.

Column ordinal number of the ellipsoidal height. The geodetic height is consistent with the geodetic height of the fusion calculation surface, that is, the orthometric system is the geoidal height, and the normal height system is the ellipsoidal height at the GNSS benchmark.

The integral radius. The smaller the integral radius, the faster the calculation.

Set Laplace operator parameter. The larger the smoothing parameter, the stronger the filtering.

The edge effect suppression parameter n . The program suppresses edge and far-zone effects with the residual of the cell-grid at the area margins equal to zero as the observation equation.

The local gravity field approach method does not have the capacity to deal with the problem of systematic deviation. The program automatically removes the statistical mean of GNSS-leveling residuals.

[Output file] The remaining residual geoidal height/height anomaly grid file.

The interface displays the statistical properties of the remaining GNSS-leveling residuals during the iterative calculation process.

After the first iterative calculation, the appropriate number of iterations should be

selected according to the residual statistical properties of the iterative process, and the calculation should be performed once again!

The remaining GNSS-leveling residual file after GNSS-levelling data fusion can be employed to evaluate the quality of the measured GNSS-levelling data.

The screenshot shows the software interface for GNSS-levelling data fusion. The main workspace displays the following information:

- Computation Process:**
 - >> Open discrete GNSS-leveling residual file C:/PAGrav4.5_win64en/examples/AppGNSSlvhgt datum/mtksich.txt.
 - ** Look at the file information in the window below and set the discrete point file format parameters...
 - >> Open the ellipsoidal height grid file of the data fusion surface C:/PAGrav4.5_win64en/examples/AppGNSSlvhgt datum/GeoidEGM150s.dat.
 - >> Save the remaining GNSS-leveling residuals as C:/PAGrav4.5_win64en/examples/AppGNSSlvhgt datum/mtksich01.txt.
 - >> Save the remaining residual grid results as C:/PAGrav4.5_win64en/examples/AppGNSSlvhgt datum/residualgeoid.dat.
 - >> The parameter settings have been entered into the system!
 - ** Click the [Start Computation] control button, or the [Start Computation] tool button...
 - ** The iterative calculation process needs to wait... During the period, you can open the file C:/PAGrav4.5_win64en/examples/AppGNSSlvhgt datum/mtksich01.txt and look at the iterative calculation progress!
 - >> Computation start time: 2024-09-26 11:25:11
 - >> GNSS-levelling residuals: mean, standard deviation, minimum, maximum
 - >> Source GNSS-levelling residuals: -0.7932 0.0369 -0.8550 -0.7142
 - >> the 1th iterative remaining residuals: 0.0024 0.0254 -0.0401 0.0663
 - >> the 2th iterative remaining residuals: 0.0024 0.0224 -0.0419 0.0655
 - >> the 3th iterative remaining residuals: 0.0022 0.0204 -0.0412 0.0607
 - >> the 4th iterative remaining residuals: 0.0020 0.0188 -0.0400 0.0557
 - >> the 5th iterative remaining residuals: 0.0018 0.0175 -0.0387 0.0511
 - >> Complete GNSS-leveling data fusion!
- Save the remaining GNSS-leveling residuals as:**

101.400000	104.400000	23.500000	26.500000	0.04166667	0.04166667	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
-0.0003	-0.0004	-0.0004	-0.0005	-0.0005	-0.0006	-0.0006
-0.0009	-0.0009	-0.0009	-0.0009	-0.0009	-0.0009	-0.0007
0.0002	0.0003	0.0004	0.0005	0.0005	0.0006	0.0006
0.0004	0.0004	0.0003	0.0002	0.0002	0.0001	0.0001
0.0000	-0.0000	-0.0000	-0.0000	-0.0001	-0.0001	-0.0001
-0.0009	-0.0010	-0.0012	-0.0013	-0.0015	-0.0016	-0.0018
-0.0026	-0.0026	-0.0026	-0.0025	-0.0024	-0.0022	-0.0020
0.0005	0.0008	0.0011	0.0013	0.0015	0.0016	0.0017
0.0012	0.0010	0.0009	0.0007	0.0006	0.0004	0.0003
0.0000	-0.0000	-0.0000	-0.0001	-0.0001	-0.0002	-0.0003
-0.0017	-0.0020	-0.0022	-0.0025	-0.0028	-0.0031	-0.0034
-0.0050	-0.0049	-0.0048	-0.0047	-0.0045	-0.0042	-0.0038
0.0011	0.0016	0.0020	0.0024	0.0028	0.0031	0.0033
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
- Plot:** Two plots are shown: 'remaining residuals (m)' and 'residual height anomaly (m)'. The 'remaining residuals (m)' plot shows a scatter of points with a color scale from -0.02 to 0.02. The 'residual height anomaly (m)' plot shows a heatmap of the residual height anomaly with a color scale from -0.05 to 0.05.

Legend:

- The data fusion surface for the normal height system is the ground, and the ground ellipsoidal height grid file should be input. The data fusion surface for the orthometric system is the geoid, and the geoidal height grid file should be input.
- The remaining GNSS-leveling residual file after GNSS-levelling data fusion can be employed to evaluate the quality of the measured GNSS-levelling data.

5.5.3 Leveling network quasi-stable adjustment with remaining GNSS-levelling residuals

[Function] Taking all GNSS-levelling points as quasi-stable benchmarks, from the remaining GNSS-levelling residuals and leveling routes file in GNSS-levelling network, the indirect least squares adjustment method with quasi-stable benchmark constraints is employed to estimate the normal (orthometric) height corrections of leveling benchmarks and the height anomaly corrections of GNSS benchmarks.

[Input files] The discrete (remaining) GNSS-leveling residual file, the leveling routine file in GNSS-levelling network.

The leveling routine file adopts the agreed format, please refer to the main interface [Format convention for geodetic data file].

[Parameter settings] Set the number of rows of the GNSS-leveling file header, the column ordinal number of the ellipsoidal height, GNSS-levelling residual and weight.

[Output files] The normal (orthometric) height correction file of leveling benchmarks and the height anomaly correction file of GNSS benchmarks.

5.6 GNSS replaces leveling to calculate the orthometric or normal height

[Function] From the modelling results of the regional gravity field and geoid, calculate the orthometric (normal) height of the GNSS positioning point.

Please select height system firstly...

When the normal height system selected, input the ground height anomaly grid file, ground ellipsoidal height grid file, and ground gravity disturbance grid file.

When the orthometric height system selected, input the geoidal height grid file.

The ground ellipsoidal height grid is employed to stand for the location of the ground height anomaly.

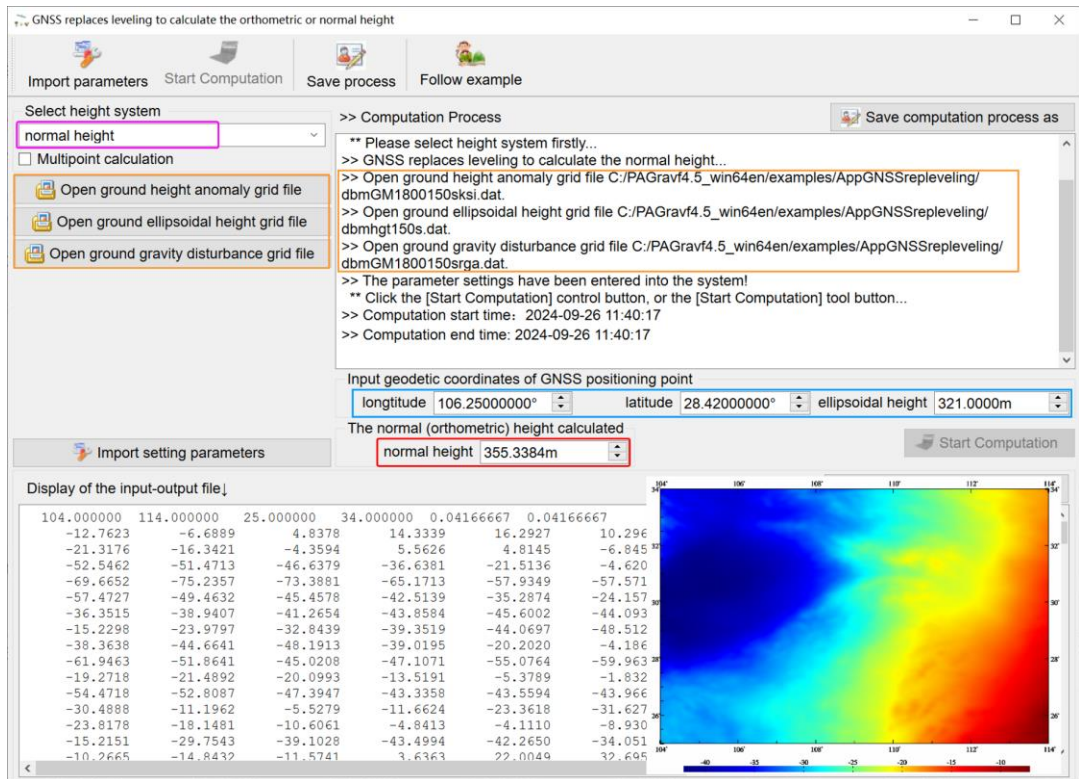
The ground gravity disturbance grid is employed to compute the correction of height anomaly at GNSS point with height change.

The geodetic coordinates of the calculation point can be input repeatedly, and the orthometric (normal) height at GNSS point can be calculated and displayed in time.

GNSS positioning point should be within the latitude and longitude range of the geoid model.

The ground gravity disturbances can be synchronously calculated while the gravimetric ground height anomalies are refined.

When the discrete GNSS position point file input, the program can calculate the orthometric (normal) heights for batch GNSS positioning points.



GNSS replaces leveling to calculate the orthometric or normal height

Import parameters Start Computation Save process Follow example

Select height system
orthometric height
☐ Multipoint calculation
Open the geoidal height grid file

>> Computation Process
dbmGM1800150srga.dat.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2024-09-26 11:40:17
>> Computation end time: 2024-09-26 11:40:17
>> GNSS replaces leveling to calculate the orthometric height...
>> Open the geoidal height grid file C:/PAGrav4.5_win64en/examples/AppGNSSreleveling/dwmGM1800150sksi.dat.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2024-09-26 11:41:13
>> Computation end time: 2024-09-26 11:41:13

Input geodetic coordinates of GNSS positioning point
longitude 106.25000000° latitude 28.42000000° ellipsoidal height 321.0000m

The normal (orthometric) height calculated
orthometric height 355.3301m

Import setting parameters Start Computation

Display of the input-output file

104.000000	114.000000	25.000000	34.000000	0.04166667	0.04166667															
-30.3888	-30.3353	-30.2474	-30.1782	-30.1695	-30.2333	-30.3529	-30.4928	-30.6097	-30.6											
-30.5079	-30.4773	-30.4178	-30.3818	-30.4241	-30.5528	-30.7173	-30.8459	-30.8988	-30.8											
-30.9666	-30.9265	-30.8488	-30.7279	-30.5689	-30.3983	-30.2589	-30.1897	-30.2039	-30.2											
-30.7087	-30.7124	-30.6640	-30.5747	-30.4772	-30.3990	-30.3378	-30.2657	-30.1592	-30.0											
-29.4489	-29.3199	-29.2045	-29.0906	-28.9585	-28.8137	-28.6941	-28.6379	-28.6437	-28.6											
-28.0546	-27.9933	-27.9349	-27.8688	-27.7835	-27.6670	-27.5190	-27.3588	-27.2144	-27.1											
-26.6019	-26.6005	-26.6046	-26.6013	-26.5906	-26.5754	-26.5548	-26.5268	-26.4927	-26.4											
-25.5602	-25.4324	-25.2871	-25.0836	-24.8338	-24.5998	-24.4445	-24.3811	-24.3680	-24.3											
-24.0863	-23.9551	-23.8277	-23.7292	-23.6460	-23.5405	-23.3871	-23.1946	-22.9939	-22.8											
-21.7590	-21.6183	-21.4670	-21.2995	-21.1346	-21.0048	-20.9343	-20.9193	-20.9265	-20.9											
-20.2280	-20.0810	-19.9070	-19.7213	-19.5329	-19.3250	-19.0671	-18.7527	-18.4272	-18.1											
-17.5519	-17.2803	-17.0668	-16.9249	-16.8241	-16.7266	-16.6149	-16.4921	-16.3651	-16.2											
-15.2983	-15.0905	-14.8649	-14.6429	-14.4439	-14.2719	-14.1124	-13.9403	-13.7362	-13.5											
-12.8449	-12.8349	-12.7969	-12.7235	-12.6087	-12.4527	-12.2738	-12.1088	-11.9898	-11.9											
-10.8914	-10.7385	-10.5461	-10.2937	-10.0234	-9.8083	-9.6879	-9.6324	-9.5699	-9.4											

GNSS replaces leveling to calculate the orthometric or normal height

Import parameters Start Computation Save process Follow example

Select height system
normal (orthometric) height
☐ Multipoint calculation
Open ground height anomaly grid file
Open ground ellipsoidal height grid file
Open ground gravity disturbance grid file
Open the geoidal height grid file

>> Computation Process
GNSS replaces leveling to calculate both the normal height and orthometric height...
>> Open ground height anomaly grid file C:/PAGrav4.5_win64en/examples/AppGNSSreleveling/dbmGM1800150sksi.dat.
>> Open ground ellipsoidal height grid file C:/PAGrav4.5_win64en/examples/AppGNSSreleveling/dbmhgt150s.dat.
>> Open ground gravity disturbance grid file C:/PAGrav4.5_win64en/examples/AppGNSSreleveling/dbmGM1800150srga.dat.
>> Open the geoidal height grid file C:/PAGrav4.5_win64en/examples/AppGNSSreleveling/dwmGM1800150sksi.dat.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2024-09-26 11:42:09
>> Computation end time: 2024-09-26 11:42:09

Input geodetic coordinates of GNSS positioning point
longitude 106.25000000° latitude 28.42000000° ellipsoidal height 321.0000m

The normal (orthometric) height calculated
normal height 355.3384m orthometric height 355.3301m

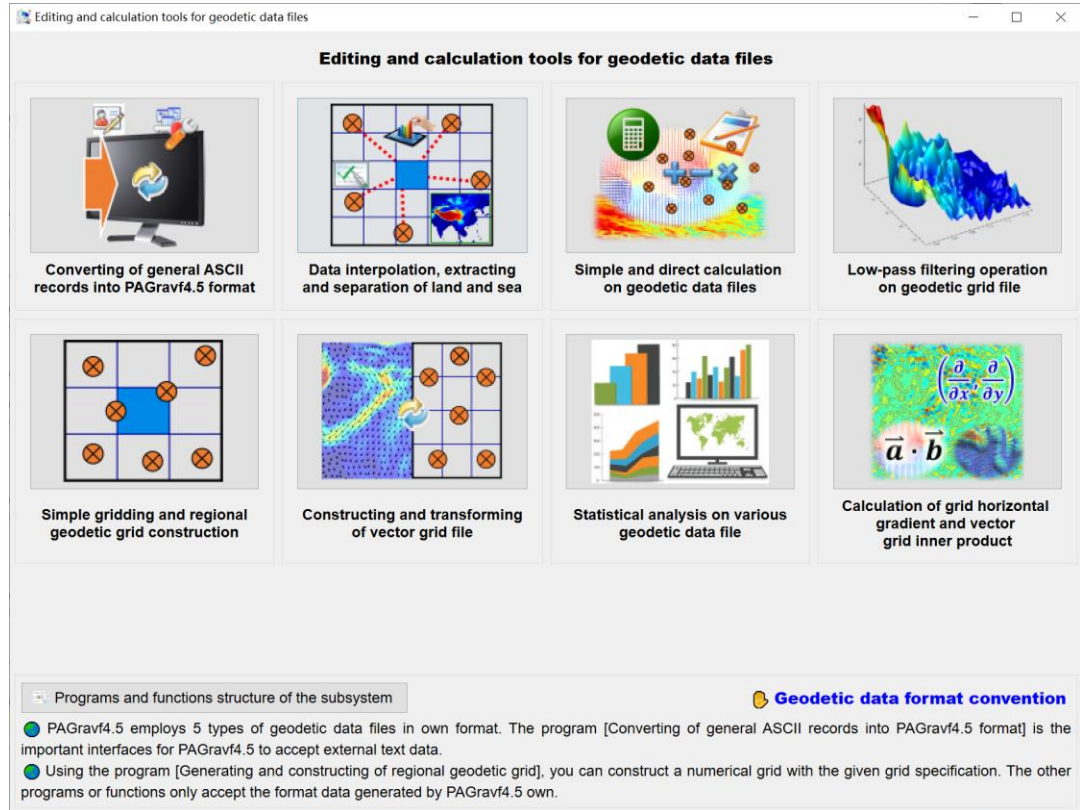
Import setting parameters Start Computation

Display of the input-output file

104.000000	114.000000	25.000000	34.000000	0.04166667	0.04166667															
-30.3888	-30.3353	-30.2474	-30.1782	-30.1695	-30.2333	-30.4928	-30.6097	-30.6												
-30.5079	-30.4773	-30.4178	-30.3818	-30.4241	-30.5528	-30.8459	-30.8988	-30.8												
-30.9666	-30.9265	-30.8488	-30.7279	-30.5689	-30.3983	-30.2589	-30.1897	-30.2039	-30.2											
-30.7087	-30.7124	-30.6640	-30.5747	-30.4772	-30.3990	-30.3378	-30.2657	-30.1592	-30.0											
-29.4489	-29.3199	-29.2045	-29.0906	-28.9585	-28.8137	-28.6941	-28.6379	-28.6437	-28.6											
-28.0546	-27.9933	-27.9349	-27.8688	-27.7835	-27.6670	-27.5190	-27.3588	-27.2144	-27.1											
-26.6019	-26.6005	-26.6046	-26.6013	-26.5906	-26.5754	-26.5548	-26.5268	-26.4927	-26.4											
-25.5602	-25.4324	-25.2871	-25.0836	-24.8338	-24.5998	-24.4445	-24.3811	-24.3680	-24.3											
-24.0863	-23.9551	-23.8277	-23.7292	-23.6460	-23.5405	-23.3871	-23.1946	-22.9939	-22.8											
-21.7590	-21.6183	-21.4670	-21.2995	-21.1346	-21.0048	-20.9343	-20.9193	-20.9265	-20.9											
-20.2280	-20.0810	-19.9070	-19.7213	-19.5329	-19.3250	-19.0671	-18.7527	-18.4272	-18.1											
-17.5519	-17.2803	-17.0668	-16.9249	-16.8241	-16.7266	-16.6149	-16.4921	-16.3651	-16.2											
-15.2983	-15.0905	-14.8649	-14.6429	-14.4439	-14.2719	-14.1124	-13.9403	-13.7362	-13.5											
-12.8449	-12.8349	-12.7969	-12.7235	-12.6087	-12.4527	-12.2738	-12.1088	-11.9898	-11.9											
-10.8914	-10.7385	-10.5461	-10.2937	-10.0234	-9.8083	-9.6879	-9.6324	-9.5699	-9.4											

6 Editing, calculation, and visualization tools for geodetic data files

The geodetic data file editing and calculation assembly are mainly employed for data file format conversion, interpolation and gridding, data extraction, separation and merging, vector grid and numerical grid data processing, basic operations on multiple sets of data and other data preprocessing etc.



PAGravf4.5 employs 5 types of geodetic data files in own format. The program [Converting of general ASCII records into PAGravf4.5 format] is the important interfaces for PAGravf4.5 to accept external text data. Using the program [Generating and constructing of regional geodetic grid], you can construct a numerical grid with the given grid specification. The other programs or functions only accept the format data generated by PAGravf4.5 own.

6.1 Converting of general ASCII records into PAGravf4.5 format

[Function] Convert the general ASCII data record file from different sources and non-standard format into the discrete geodetic record file in PAGravf4.5 format.

Please click the [Open the source ASCII record file] control button to open the general text record file to be standardized...

[Input file] The general ASCII data record file.

After entering the number of rows of the input file header, click the control button [Exact and edit data] to open the dialog [Exact and edit data from the source text file].

If the target file does not need the record table header, please clear the text responding to the input text box.

The program needs some time to organize the target record attributes, please wait...



The program is the important interface for PAGravf4.5 to accept the external text data.

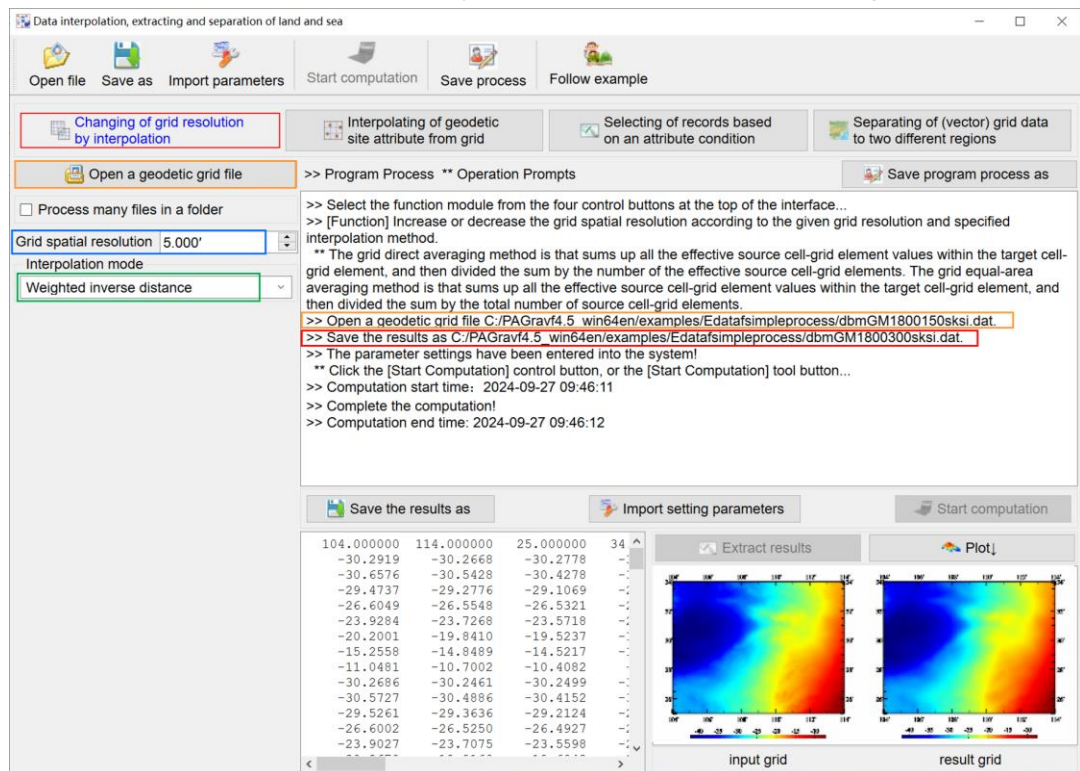
6.2.1 Changing of grid resolution by interpolation

[Input file] The geodetic numerical grid file.

[Parameter settings] Enter the spatial resolution for target grid and select the interpolation mode.

[Output file] The target geodetic numerical grid file.

The grid direct averaging method is that sums up all the effective source cell-grid element values within the target cell-grid element, and then divided the sum by the number of the effective source cell-grid elements. The grid equal-area averaging method is that sums up all the effective source cell-grid element values within the target cell-grid element, and then divided the sum by the total number of source cell-grid elements.



The grid direct averaging method or the grid equal-area averaging method can be employed to decrease grid resolution. When the resolution of the target grid is lower than that of the source grid, the program automatically adopts the inverse distance weighted interpolation method.

6.2.2 Interpolating of geodetic site attribute from grid

[Function] From a numerical grid, interpolate the attribute values of the geodetic sites using the specified interpolation method.

[Input files] The discrete geodetic point file to be interpolated. The geodetic numerical grid file for interpolation.

[Parameter settings] Enter number of rows of the discrete geodetic point file header and select the interpolation mode.

[Output file] The interpolated discrete geodetic point file.

The file format is the same as the input discrete geodetic point file. Behind the input

file record, add one column of the interpolated value as the output file record.

Data interpolation, extracting and separation of land and sea

Open file Save as Import parameters Start computation Save process Follow example

Changing of grid resolution by interpolation Interpolating of geodetic site attribute from grid Selecting of records based on an attribute condition Separating of (vector) grid data to two different regions

Open a discrete points file

Set format parameters of the file
Number of rows of the file header 1

Open the grid file for interpolation

Interpolation mode
Weighted inverse distance

Program Process ** Operation Prompts

```
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2024-09-27 09:46:11
>> Complete the computation!
>> Computation end time: 2024-09-27 09:46:12
>> [Function] From a numerical grid, interpolate the attribute values of the geodetic sites using the specified interpolation method.
>> Open a discrete points file C:\PA\Grav4.5_win64en/examples/Edatfaisimpleprocess/pntdata.txt.
** Look at the input file information in the text box above, set the file format parameters...
>> Open the grid file for interpolation C:\PA\Grav4.5_win64en/examples/Edatfaisimpleprocess/pntgrid.dat.
>> Save the results as C:\PA\Grav4.5_win64en/examples/Edatfaisimpleprocess/rstpnt.txt.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2024-09-27 09:56:47
>> Complete the computation!
>> Computation end time: 2024-09-27 09:56:47
```

Save the results as Import setting parameters Start computation

number	lon(deg/decimal)	lat	ellipHeight(m)
1	102.442457	24.471769	1972.7703
2	102.546777	24.458002	1659.0410
3	102.632412	24.458211	2120.2558
4	102.725921	24.460578	2111.3872
5	102.420803	24.566357	1990.6386
6	102.528697	24.562786	1936.4260
7	102.634437	24.565660	2192.9271
8	102.725888	24.581970	2303.7797
9	102.832641	24.575505	1977.4949
10	102.345532	24.668953	1919.7825
11	102.423972	24.652933	1959.3369
12	102.529771	24.667079	2157.7877
13	102.631063	24.657055	1906.3415

Extract results Plot

input grid results interpolated

Data interpolation, extracting and separation of land and sea

Open file Save as Import parameters Start computation Save process Follow example

Changing of grid resolution by interpolation Interpolating of geodetic site attribute from grid Selecting of records based on an attribute condition Separating of (vector) grid data to two different regions

Open a discrete points file

Set format parameters of the file
Number of rows of the file header 1
Column ordinal number of the condition attribute 5

Minimum 0.00
Maximum 9000.00

Program Process ** Operation Prompts

```
>> Save the results as C:\PA\Grav4.5_win64en/examples/Edatfaisimpleprocess/rstpnt.txt.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2024-09-27 09:56:47
>> Complete the computation!
>> Computation end time: 2024-09-27 09:56:47
>> [Function] Select the geodetic records from a geodetic record file according to the maximum and minimum range of the specified attribute.
>> Open a discrete points file C:\PA\Grav4.5_win64en/examples/Edatfaisimpleprocess/chksinterp.txt.
** Look at the input file information in the text box above, set the file format parameters...
>> Save the results as C:\PA\Grav4.5_win64en/examples/Edatfaisimpleprocess/chksinterp.txt.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2024-09-27 09:57:45
>> Complete the computation!
>> Computation end time: 2024-09-27 09:57:57
```

Save the results as Import setting parameters Start computation

no	lon(degree/decimal)	lat	ellipHeight(m)
11581	106.520833	27.020833	-30
11582	106.562500	27.020833	-30
11583	106.604167	27.020833	-29
11584	106.645833	27.020833	-29
11585	106.687500	27.020833	-29
11586	106.729167	27.020833	-29
11587	106.770833	27.020833	-29
11588	106.812500	27.020833	-29
11589	106.854167	27.020833	-29
11590	106.895833	27.020833	-29
11591	106.937500	27.020833	-29
11598	107.229167	27.020833	-28
11599	107.270833	27.020833	-28

Extract results Plot

input points result points

6.2.3 Selecting of records based on an attribute condition

[Function] Select the geodetic records from a geodetic record file according to the maximum and minimum range of the specified attribute.

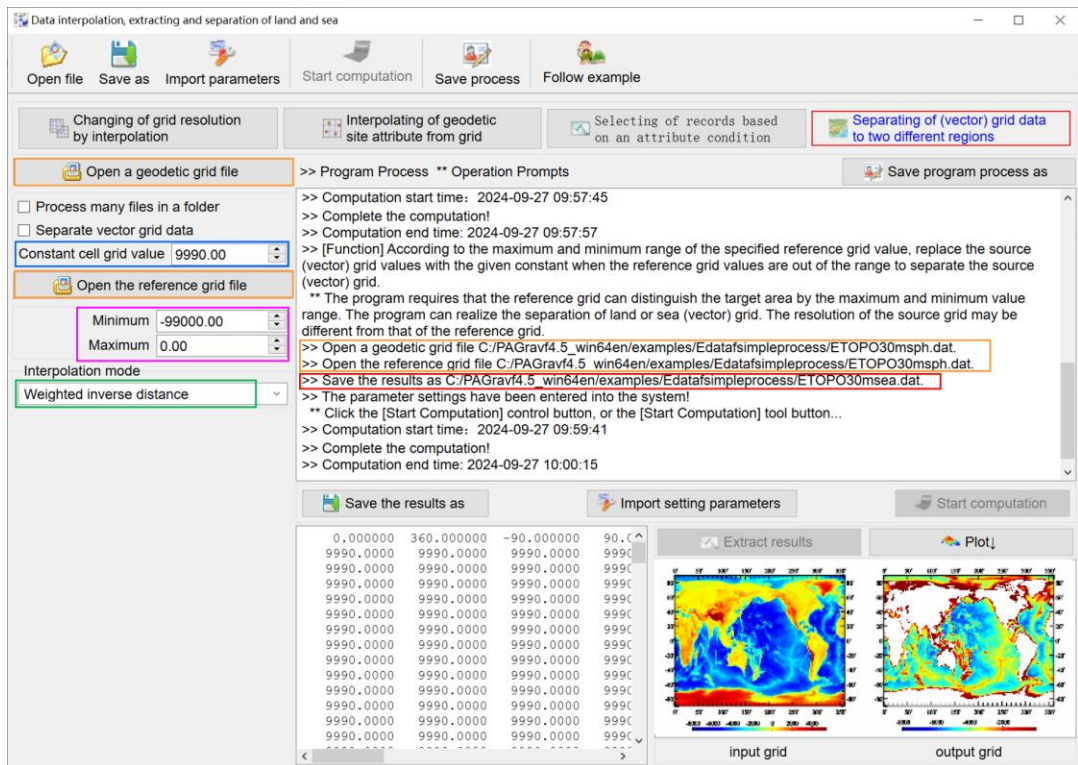
[Parameter settings] Enter number of rows of the input file header, column ordinal number of the condition attribute in the file record, and minimum and maximum of the attribute.

6.2.4 Separating of (vector) grid data to two different regions

[Function] According to the maximum and minimum range of the specified reference grid value, replace the source (vector) grid values with the given constant when the reference grid values are out of the range, to separate the source (vector) grid.

The program requires that the reference grid can distinguish the target area by the maximum and minimum value range.

[Input files] The source geodetic (vector) grid file. The reference grid file whose grid range and resolution should not be smaller than that of the source grid file.



The program can realize the separation of land or sea (vector) grid. The resolution of the source grid may be different from that of the reference grid.

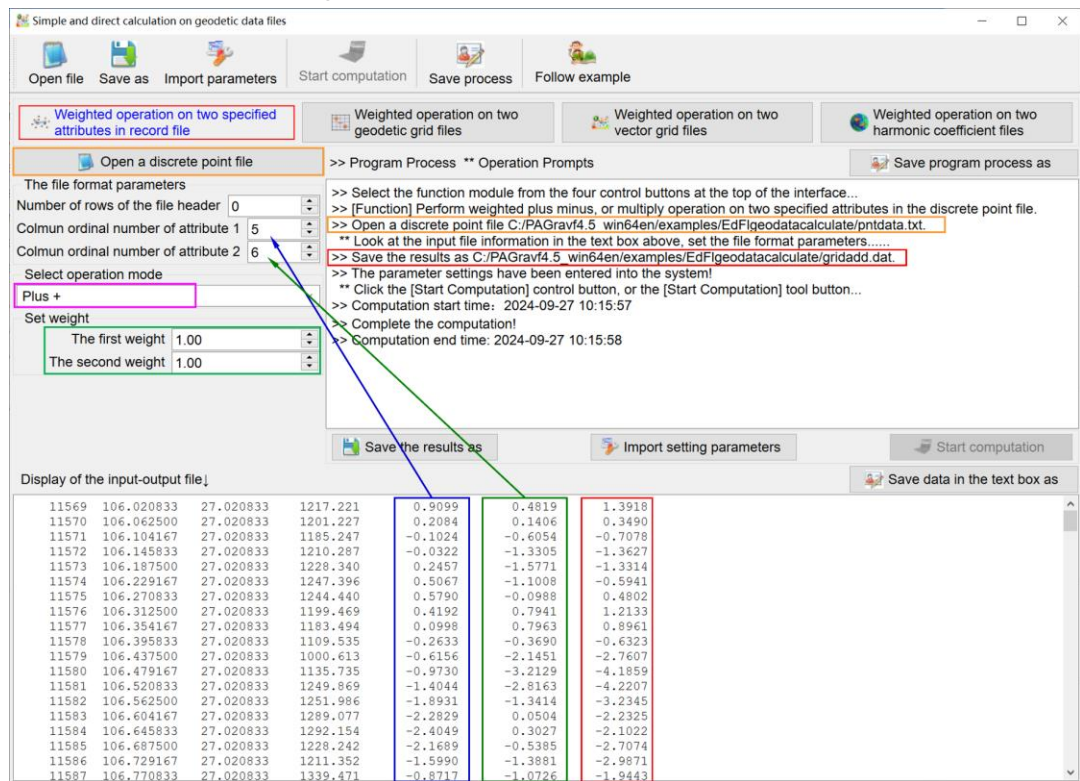
6.3 Simple and direct calculation on geodetic data files

6.3.1 Weighted operation on two specified attributes in record file

[Function] Perform weighted plus, minus or multiply operation on two specified attributes in the discrete point file.

[Input file] The discrete geodetic point file.

[Parameter settings] Enter number of rows of the discrete geodetic point file header, column ordinal number of the attribute 1 and its weight, and column ordinal number of the attribute 2 and its weight. Select operation mode.



[Output file] The operated discrete geodetic point file.

The file format is the same as the input discrete geodetic point file. Behind the input file record, add 1 column of the calculation result as the output file record.

6.3.2 Weighted operation on two geodetic grid files

[Function] Perform weighted plus, minus or multiply operation on grid elements in two (vector) grid files with the same specification.

6.3.3 Product operation on two vector grid files

[Function] Perform outer product or inner product operations on vector grid elements in two vector grid files with the same specification.

6.3.4 Weighted operation on two harmonic coefficient files

[Function] Perform weighted operation on two normalized spherical harmonic coefficient model files.

The file header occupies a row and consists of two attributes for the scale parameters of the spherical harmonic coefficients model, namely the geocentric gravitational constant GM ($\times 10^{14} \text{m}^2/\text{s}^2$) and equatorial radius a (m) of the Earth.

6.4 Low-pass filtering operation on geodetic grid file

[Function] Using the low-pass filters such as the moving average, Gaussian, exponential or Butterworth, perform low-pass filtering on the geodetic grid file. Before and after filtering, the grid specification (Latitude and longitude range and spatial resolution) remain unchanged.

[Input file] The geodetic grid file.

[Parameter settings] Select low-pass filter, and set filter parameter n .

For the moving average filtering, the greater the filtering parameter n , the greater the filtering strength. For 'Exponential' or 'Butterworth' filters, the smaller the n , the greater the filtering strength.

[Output file] The filtered geodetic grid file.

Low-pass filtering operation on geodetic grid file

Open grid file Import parameters Save as Start Computation Save process Follow example

Open the geodetic grid file >> Computation Process ** Operation Prompts Save computation process as

Select low-pass filter
moving average filter

Set filter parameter n 2

Save the results as

Import setting parameters

Start Computation

>> [Function] Using the low-pass filters such as the moving average, Gaussian, exponential or Butterworth, perform low-pass filtering on the geodetic grid file. Before and after filtering, the grid specification (Latitude and longitude range and spatial resolution) remain unchanged.

** For the moving average filtering, the greater the filtering parameter n , the greater the filtering strength. For 'Exponential' or 'Butterworth' filters, the smaller the n , the greater the filtering strength.

>> Open the geodetic grid file C:/PAGrav4.5_win64en/examples/EdGrdlowpassfilter/dbmGM1800150sgr.dat.

>> Save the results as C:/PAGrav4.5_win64en/examples/EdGrdlowpassfilter/result.dat.

** The program output the low-pass filtered file with the same grid specification as the input file.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2024-09-27 10:32:30

>> Complete the Low-pass filtering operation!

>> Computation end time: 2024-09-27 10:32:30

104.000000	114.000000	25.000000	34.000000	0.041667
-5.962	4.177	11.466	23.289	28.210
-10.021	-2.122	7.745	14.268	12.628
-6.945	-2.312	6.767	18.694	31.652
-36.281	-36.125	-29.339	-19.418	-11.661
-32.602	-28.244	-23.450	-17.043	-7.400
2.814	7.501	6.571	1.403	-4.846
11.234	8.115	3.029	-3.837	-11.794
2.271	-6.546	-13.002	-10.816	0.961
-18.356	-9.850	-3.663	-4.016	-9.983
13.916	11.283	9.246	10.708	15.153
-20.631	-18.825	-17.034	-15.637	-13.492
-24.640	-23.201	-25.161	-31.713	-38.808
-6.841	-6.836	-4.822	-2.367	-1.876
-3.447	-14.404	-21.274	-23.791	-21.795
-37.146	-36.745	-32.950	-24.857	-14.622
-35.635	-30.386	-17.813	-6.002	-0.671
-8.413	0.194	6.596	17.027	21.671

Extract filter result Plot

input grid low-pass filter grid

6.5 Simple gridding and regional geodetic grid construction

6.5.1 Gridding of discrete geodetic data by simple interpolation

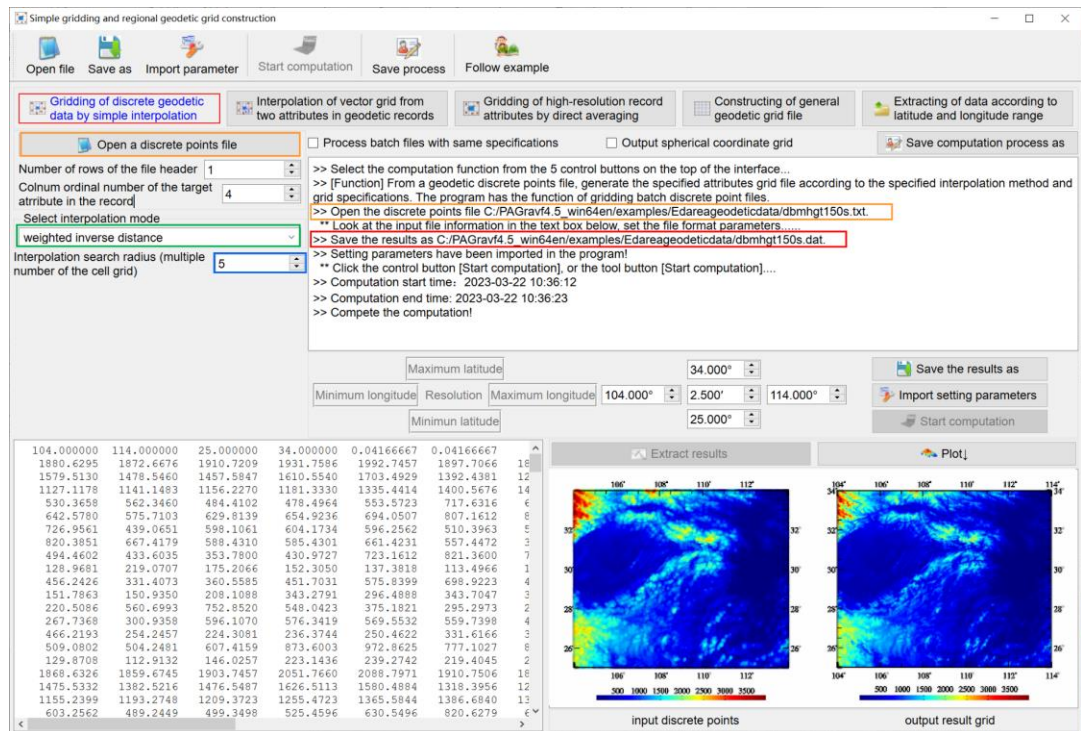
[Function] From a geodetic discrete point file, generate the specified attribute grid file according to the specified interpolation method and grid specification. The program has the function of gridding batch discrete point files.

[Input files] The discrete geodetic point file to be interpolated. The geodetic numerical grid file for interpolation.

[Parameter settings] Enter number of rows of the discrete point file header, column ordinal number of the target attribute in the file record, interpolation search radius

(multiple of the grid element) and grid specification parameters. Select the interpolation mode.

[Output file] The interpolated geodetic grid file.



6.5.2 Interpolation of vector grid from two attributes in geodetic records

[Function] From a geodetic discrete point file, generate the vector grid file according to the two specified component attributes, specified interpolation method and given grid specification.

6.5.3 Gridding of high-resolution record attributes by direct averaging

[Function] Using the direct averaging method, grid the high-resolution discrete observations.

6.5.4 Constructing of general geodetic grid file

[Function] According to the given latitude and longitude range and spatial resolution, generate the constant value, random number, 2D array index value or Gaussian surface grid file.

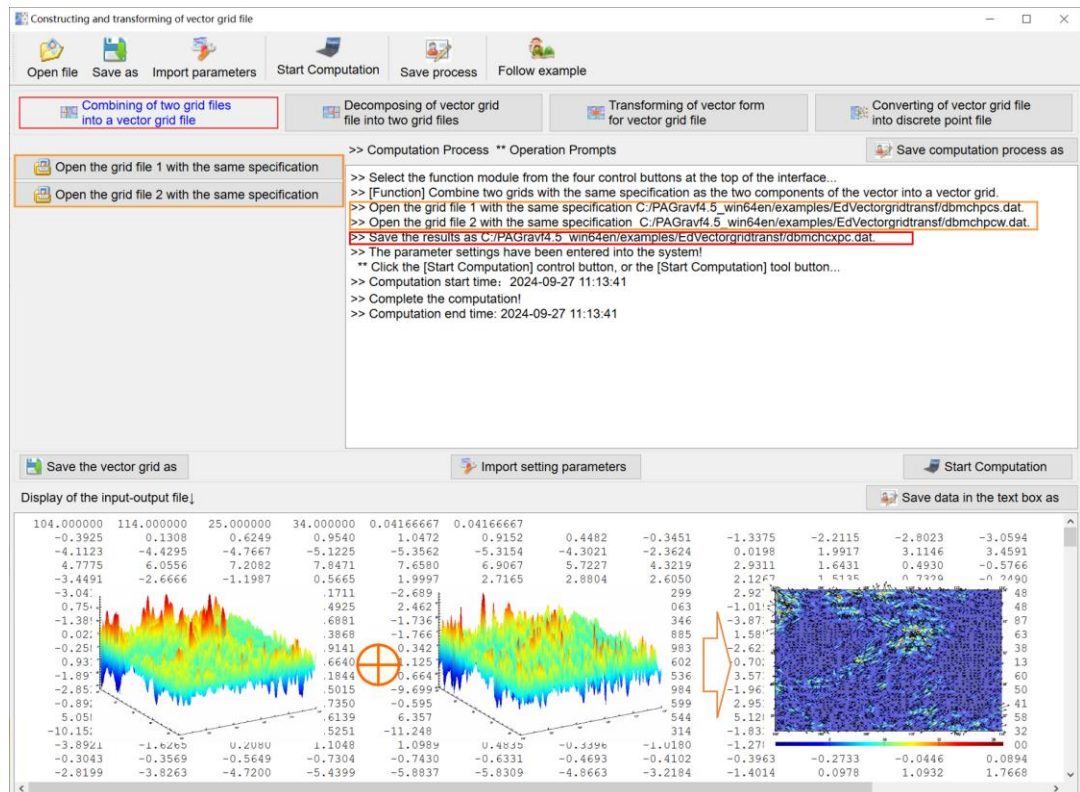
6.5.5 Extracting of data according to latitude and longitude range

[Function] According to the given latitude and longitude range, extract data from the geodetic discrete point file, grid file or vector grid file. The program can extract data from batch files.

6.6 Constructing and transforming of vector grid file

6.6.1 Combining of two grid files into a vector grid file

[Function] Combine two grids with the same specification as the two components of the vector into a vector grid.



6.6.2 Decomposing of vector grid file into two grid files

[Function] Decompose a vector grid file into two component grid files.

6.6.3 Transforming of vector form for vector grid file

[Function] Transform the vectors in a vector grid file between plane coordinates (in-phase/cross-phase amplitude) and polar coordinates (amplitude/phase).

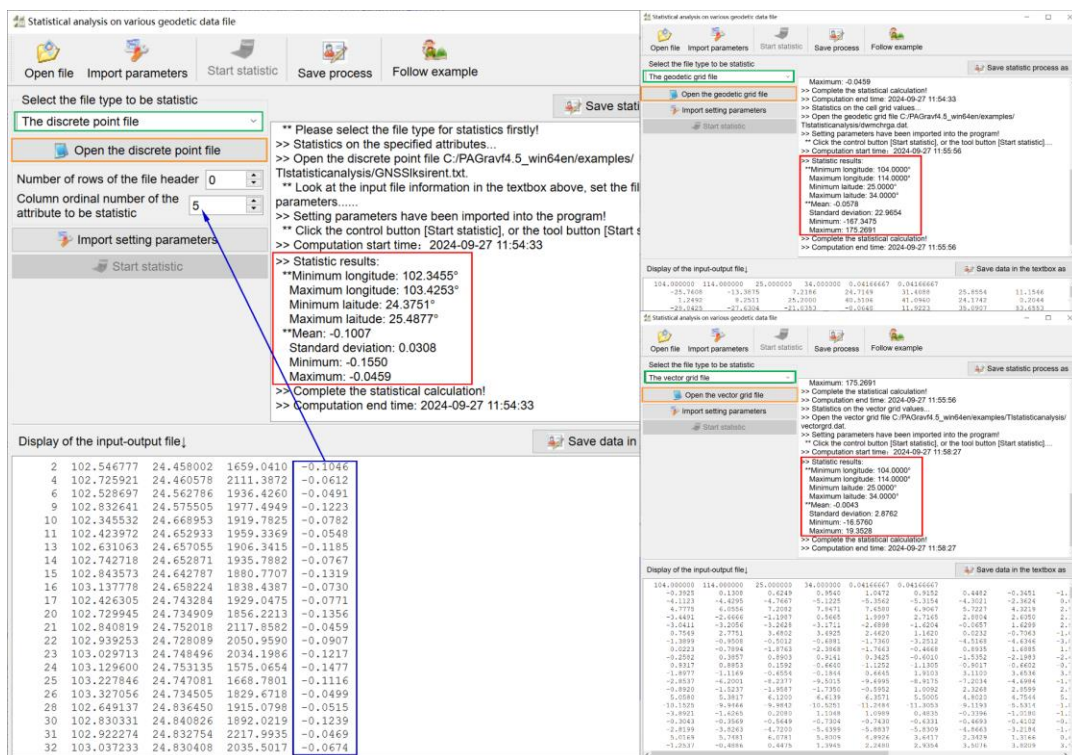
6.6.4 Converting of vector grid file into discrete point file

[Function] Convert the (vector) grid file into the discrete point file.

The latitude and longitude of discrete point are the latitude and longitude of the center point of the cell grid.

6.7 Statistical analysis on various geodetic data file

[Purpose] Extract the latitude and longitude range, mean, standard deviation, minimum, maximum and other statistical information from the specified attributes in the discrete point file, geodetic grid file or vector grid file.



6.8 Calculation of grid horizontal gradient and vector grid inner product

[Purpose] Calculate the first and second order horizontal gradient of the geodetic grid, or perform an inner product operation on the two vector grids.

6.8.1 Horizontal gradient calculation on geodetic grid

[Function] Using the least square method, estimate the first-order horizontal gradient vector (/km) or the second-order horizontal gradient vector (/km²) of the parameters of the geodetic grid. The horizontal gradient vector can be output in the form of polar coordinates or EN horizontal coordinates.

[Input files] The geodetic parameter grid file, and the ellipsoidal height grid file of the surface where the parameter located.

6.8.2 Inner product operation on two vector grids

[Function] Perform an inner product operation on two vector grids with the same grid specification.

6.9 Visualization plot tools for various geodetic data files

6.9.1 Visualization of multi-attribute curves from 2D geodetic data

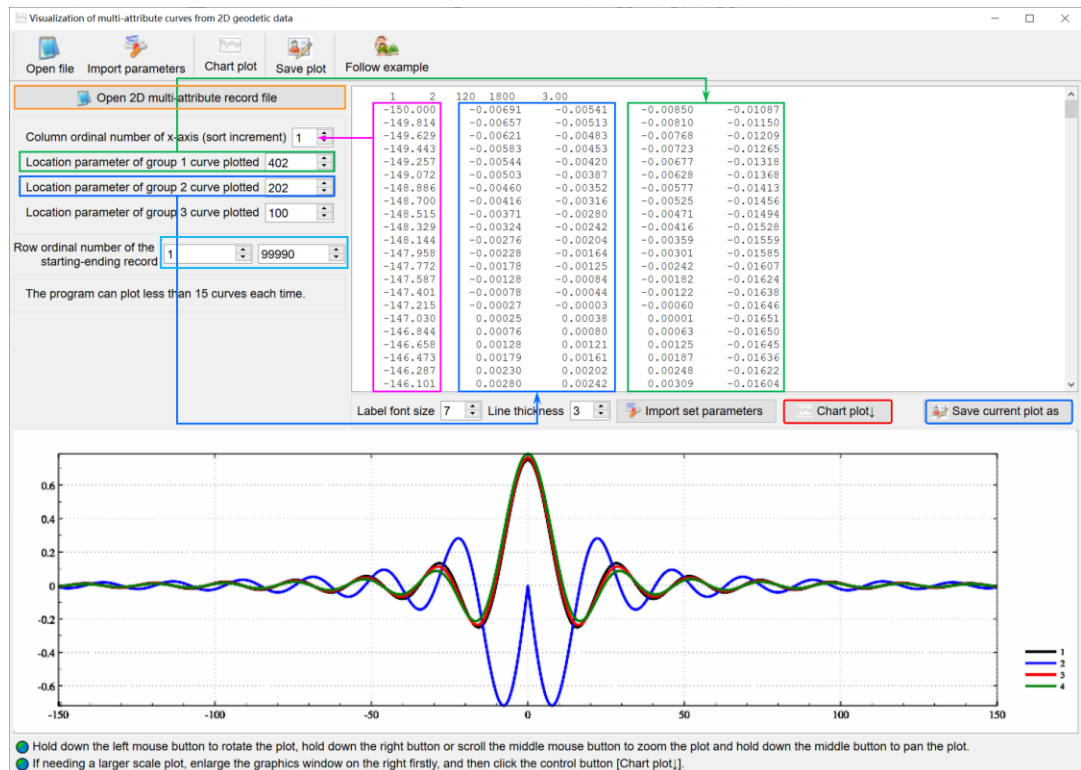
[Function] Plot multi- attribute curves stored in a 2D geodetic data file.

The program requires the file header to occupy a row, and the x-axis column values are strictly sorted in an incremental manner.

The program can plot less than 15 curves each time.

Hold down the left mouse button to rotate the plot, hold down the right button or scroll the middle mouse button to zoom the plot and hold down the middle button to pan the plot.

If needing a larger scale plot, enlarge the graphics window on the right firstly, and then click the control button [Chart plot].



6.9.2 Visualization for specified attribute in discrete point record file

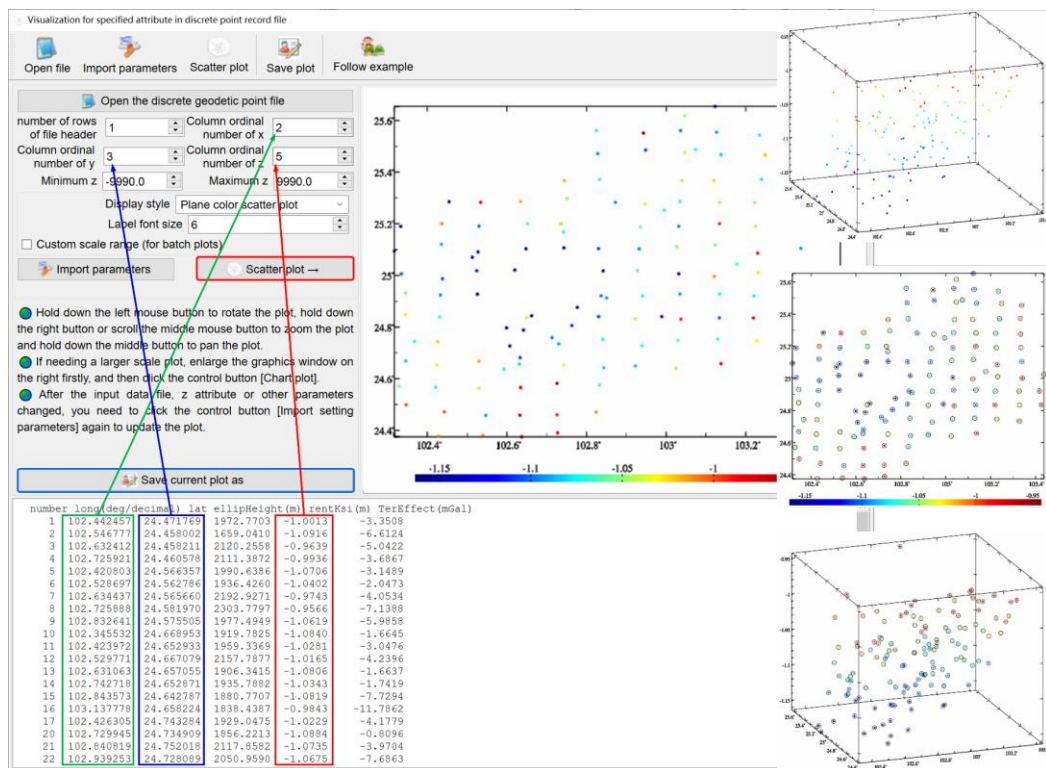
[Function] Displays the point locations and their specified attributes in a geodetic discrete point file.

If needing a larger scale plot, enlarge the graphics window on the right firstly, and then click the control button [Scatter plot].

After the input data file, z attribute or other parameters changed, you need to click the control button [Import setting parameters] again to update the plot.

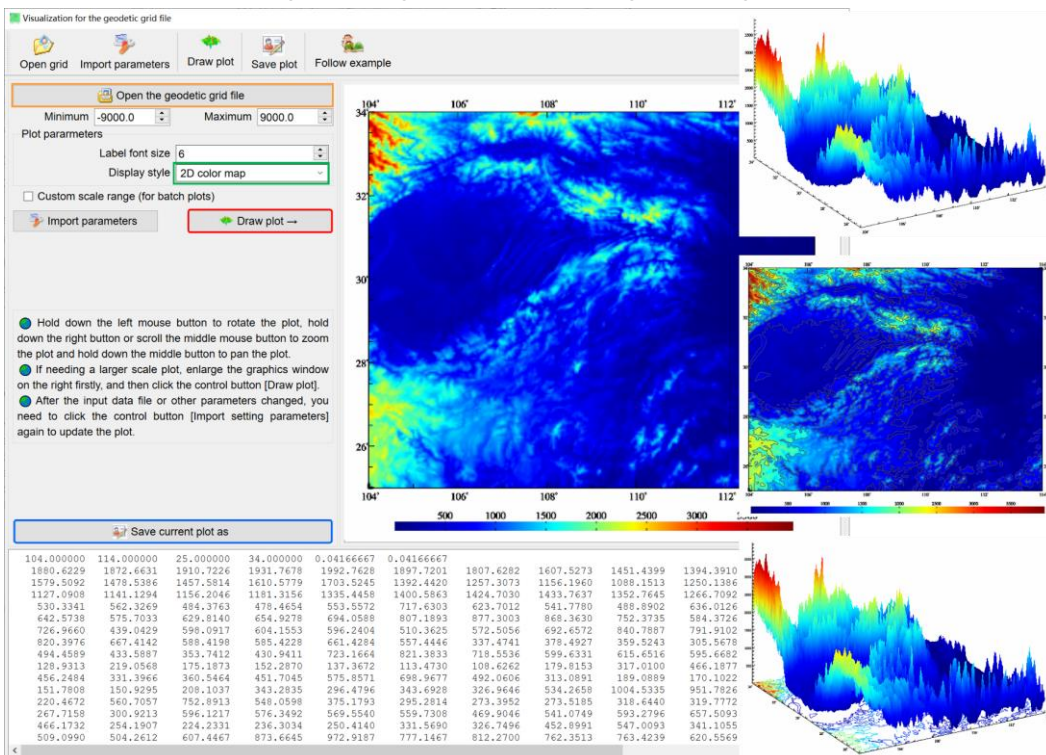
You can unify the scales by fixing the scale range for batch plots.

Adjust the size of the graphics window on the right and the plot requirements to an appropriate state before drawing batch plots. During plot period, the parameters and the size of the graphics window are kept unchanged, and no mouse operation is performed on the plot.



6.9.3 Visualization for the geodetic grid file

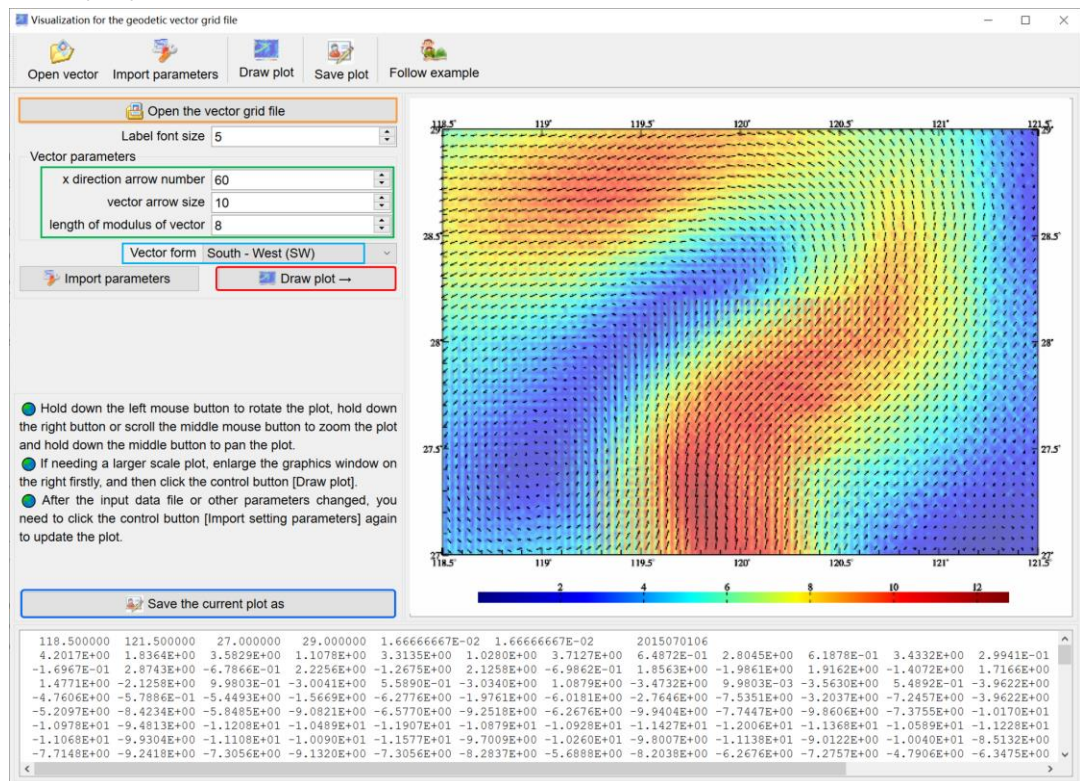
[Function] Plot the geodetic grid data from the geodetic grid file.



You can unify the scales by fixing the scale range for batch plots. Adjust the size of the graphics window on the right and the plot requirements to an appropriate state before drawing batch plots. During plot period, the parameters and the size of the graphics window are kept unchanged, and no mouse operation is performed on the plot.

6.9.4 Visualization for the geodetic vector grid file

The X-axis and Y-axis of the plot coordinate system respectively point east and north (EN), which is the same with horizontal displacement vector.



Vector form: East-North (EN, e.g., horizontal displacement vector), South-West (SW, e.g., vertical deflection vector), North - West (NW, e.g., Tangential gravity gradient vector).

7 Featured algorithms and formulas in PAGravf4.5

More than 300 algorithm formulas here have been derived, cross-validated, programmed and verified repeatedly. In which, some of the algorithm formulas belong to PAGravf4.5 own. The performance and reliability of all these algorithmic formulas can be verified by calling the relative programs or functions in PAGravf4.5.

7.1 Calculation formulas of normal gravity field at any point in Earth space

(1) The normal geopotential U at the Earth space point (r, θ, λ) in the spherical coordinate system can be expressed as spherical harmonic series:

$$U(r, \theta) = \frac{GM}{r} \left[1 - \sum_{n=1}^{\infty} \left(\frac{a}{r} \right)^{2n} J_{2n} P_{2n}(\cos \theta) \right] + \frac{1}{2} \omega^2 r^2 \sin^2 \theta \quad (1.1)$$

$$J_{2n} = (-1)^{n+1} \frac{3e^{2n}}{(2n+1)(2n+3)} \left(1 - n + \frac{5nJ_2}{e^2} \right) \quad (1.2)$$

where r is the distance from the calculation point to the center of the level ellipsoid, λ is the longitude of the calculation point, $\theta = \pi/2 - \varphi$ is the geocentric colatitude, φ is the geocentric latitude, a is the semimajor axis of the ellipsoid, J_2 is the dynamic shape factor of the Earth, GM is the geocentric gravitational constant, ω is the mean rotation rate of the Earth, e is the first eccentricity of the level ellipsoid, and $P_{2n}(\cos \theta)$ is the Legendre function.

(2) Taking the partial derivative of the normal gravitational potential $U(r, \theta)$ formula (1.1) in the spherical coordinate system, the normal gravity vector at the Earth space point can be obtained:

$$\gamma(r, \theta) = \gamma_r e_r + \gamma_\theta e_\theta \quad (1.3)$$

$$\gamma_r = -\frac{GM}{r^2} \left[1 - \sum_{n=1}^{\infty} (2n+1) \left(\frac{a}{r} \right)^{2n} J_{2n} P_{2n}(\cos \theta) \right] + \omega^2 r \sin^2 \theta \quad (1.4)$$

$$\gamma_\theta = \frac{\partial U}{r \partial \theta} = -\frac{GM}{r^2} \left[\sum_{n=1}^{\infty} \left(\frac{a}{r} \right)^{2n} J_{2n} \frac{\partial}{\partial \theta} P_{2n}(\cos \theta) \right] + \omega^2 r \sin \theta \cos \theta \quad (1.5)$$

Since $e_r \perp e_\theta$, the normal gravity scalar value can be obtained:

$$\gamma = \sqrt{\gamma_r^2 + \gamma_\theta^2} \quad (1.6)$$

and the north declination angle of the normal gravity line direction relative to the Earth center of mass can also be obtained:

$$\vartheta_\gamma = \tan^{-1} \frac{\gamma_\theta}{\gamma_r} \quad (1.7)$$

(3) Furtherly taking the partial derivative of the normal gravity vector $\gamma(r, \theta)$ of the formula (1.3), the diagonal elements of the normal gravity gradient tensor at the earth space point in the spherical coordinate system can be obtained:

$$U_{rr} = -2 \frac{GM}{r^3} \left[1 - \sum_{n=1}^{\infty} (n+1)(2n+1) \left(\frac{a}{r} \right)^{2n} J_{2n} P_{2n}(\cos \theta) \right] + \omega^2 \sin^2 \theta \quad (1.8)$$

$$U_{\theta\theta} = \frac{\partial^2 U}{r^2 \partial \theta^2} = \frac{\partial \gamma_\theta}{r \partial \theta} = -\frac{GM}{r^3} \left[\sum_{n=1}^{\infty} \left(\frac{a}{r} \right)^{2n} J_{2n} \frac{\partial^2}{\partial \theta^2} P_{2n}(\cos \theta) \right] + \omega^2 \cos 2\theta \quad (1.9)$$

Since $e_r \perp e_\theta$, the normal gravity gradient scalar value can be obtained:

$$U_{nn} = \sqrt{U_{rr}^2 + U_{\theta\theta}^2} \quad (1.10)$$

and the north declination angle of the normal gravity gradient direction relative to the mass center of the Earth can also be obtained:

$$\vartheta_E = \tan^{-1} \frac{U_{\theta\theta}}{U_{rr}} \quad (1.11)$$

(4) Low-degree Legendre function $P_n(t)$ and its first and second derivative algorithms with respect to θ

$$\text{Let } t = \cos \theta, \quad u = \sin \theta, \quad (1.12)$$

$$\text{then } P_n(t) = \frac{2n-1}{n} t P_{n-1}(t) - \frac{n-1}{n} P_{n-2}(t) \quad (1.13)$$

$$P_1 = t, \quad P_2 = \frac{1}{2}(3t^2 - 1) \quad (1.14)$$

$$\frac{\partial}{\partial \theta} P_n(t) = \frac{2n-1}{n} t \frac{\partial}{\partial \theta} P_{n-1}(t) - \frac{2n-1}{n} u P_{n-1}(t) - \frac{n-1}{n} \frac{\partial}{\partial \theta} P_{n-2}(t) \quad (1.15)$$

$$\frac{\partial}{\partial \theta} P_1(t) = -u, \quad \frac{\partial}{\partial \theta} P_2(t) = -3ut \quad (1.16)$$

$$\frac{\partial^2}{\partial \theta^2} P_n(t) = \frac{2n-1}{n} \left(t \frac{\partial^2}{\partial \theta^2} P_{n-1} - 2u \frac{\partial}{\partial \theta} P_{n-1} - t P_{n-1} \right) - \frac{n-1}{n} \frac{\partial^2}{\partial \theta^2} P_{n-2} \quad (1.17)$$

$$\frac{\partial^2}{\partial \theta^2} P_1(t) = -t, \quad \frac{\partial^2}{\partial \theta^2} P_2(t) = 3(1 - 2t^2) \quad (1.18)$$

7.2 Calculation formulas of Earth gravity field from geopotential coefficient model

The disturbing potential T or height anomaly ζ at the space point (r, θ, λ) outside the Earth can be expressed as the following spherical harmonic series:

$$T(r, \theta, \lambda) = \zeta \gamma = \frac{GM}{r} \sum_{n=2}^{\infty} \left(\frac{a}{r} \right)^n \sum_{m=0}^n (\delta \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm} \quad (2.1)$$

where $\bar{C}_{nm}, \bar{S}_{nm}$ are called as the fully normalized Stokes coefficients, also known as the geopotential coefficients, $\bar{P}_{nm} = \bar{P}_{nm}(t)$ is the fully normalized associative Legendre function, n is called as the degree of the geopotential coefficient, m is called as order of geopotential coefficients. And

$$\delta \bar{C}_{2n,0} = \bar{C}_{2n,0} + \frac{J_{2n}}{\sqrt{4n+1}} \quad (2.2)$$

$$\delta \bar{C}_{2n,m} = \bar{C}_{2n,m} (m > 0) \quad \delta \bar{C}_{2n+1,m} = \bar{C}_{2n+1,m} \quad (2.3)$$

The spherical harmonic series of gravity anomaly Δg , gravity disturbance δg , vertical deflection (ξ, η) , disturbing gravity gradient T_{rr} and tangential gravity gradient (T_{NN}, T_{WW}) at the space point (r, θ, λ) outside the Earth can be respectively expressed as:

$$\Delta g = \frac{GM}{r^2} \sum_{n=2}^{\infty} (n-1) \left(\frac{a}{r} \right)^n \sum_{m=0}^n (\delta \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm} \quad (2.4)$$

$$\delta g = -T_r = \frac{GM}{r^2} \sum_{n=2}^{\infty} (n+1) \left(\frac{a}{r} \right)^n \sum_{m=0}^n (\delta \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm} \quad (2.5)$$

$$\xi = \frac{T_\theta}{\gamma r} = \frac{GM}{\gamma r^2} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (\delta \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \frac{\partial}{\partial \theta} \bar{P}_{nm} \quad (2.6)$$

$$\eta = -\frac{T_\lambda}{\gamma r \sin \theta} = \frac{GM}{\gamma r^2 \sin \theta} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=1}^n m (\delta \bar{C}_{nm} \sin m\lambda - \bar{S}_{nm} \cos m\lambda) \bar{P}_{nm} \quad (2.7)$$

$$T_{rr} = \frac{GM}{r^3} \sum_{n=2}^{\infty} (n+1)(n+2) \left(\frac{a}{r}\right)^n \sum_{m=0}^n (\delta \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm} \quad (2.8)$$

$$T_{NN} = \frac{1}{r} T_r + \frac{1}{r^2} T_{\theta\theta} \\ = -\frac{\delta g}{r} + \frac{GM}{r^3} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (\delta \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \frac{\partial^2}{\partial \theta^2} \bar{P}_{nm} \quad (2.9)$$

$$T_{WW} = \frac{1}{r} T_r + \frac{1}{r^2} T_\theta \cot \theta + \frac{1}{r^2 \sin^2 \theta} T_{\lambda\lambda} = -\frac{\delta g}{r} + \frac{\xi \gamma}{r} \cot \theta \\ - \frac{GM}{r^3 \sin^2 \theta} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n m^2 (\delta \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm} \quad (2.10)$$

$$\text{here, } T_r = \frac{\partial}{\partial r} T(r, \theta, \lambda), \quad T_{rr} = \frac{\partial^2}{\partial r^2} T(r, \theta, \lambda) \quad (2.11)$$

$$T_\theta = \frac{\partial}{\partial \theta} T(r, \theta, \lambda), \quad T_{\theta\theta} = \frac{\partial^2}{\partial \theta^2} T(r, \theta, \lambda) \quad (2.12)$$

$$T_\lambda = \frac{\partial}{\partial \lambda} T(r, \theta, \lambda), \quad T_{\lambda\lambda} = \frac{\partial^2}{\partial \lambda^2} T(r, \theta, \lambda) \quad (2.13)$$

$$T_{rr} + T_{NN} + T_{WW} \equiv 0, \quad T_{rr}^n + T_{NN}^n + T_{NN}^n \equiv 0, \quad T_* = \sum_{n=2}^{\infty} T_*^n \quad (2.14)$$

where T_*^n represents the degree n harmonic component of T_* . The N axis points North and the W axis points West.

Equation (2.14) is the Laplace equation, which can be employed to check the spatial and spectral domain performance of the geopotential model.

7.3 Algorithms of normalized associative Legendre function and its derivative

(1) Standard forward column recursion algorithm for $\bar{P}_{nm}(t)$ ($n < 1900$)

$$\begin{cases} \bar{P}_{nm}(t) = a_{nm} t \bar{P}_{n-1,m}(t) - b_{nm} \bar{P}_{n-2,m}(t) & \forall n > 1, m < n \\ \bar{P}_{nn}(t) = u \sqrt{\frac{2n+1}{2n}} \bar{P}_{n-1,n-1} & n > 1 \end{cases} \quad (3.1)$$

$$a_{nm} = \sqrt{\frac{(2n-1)(2n+1)}{(n+m)(n-m)}}, \quad b_{nm} = \sqrt{\frac{(2n+1)(n+m-1)(n-m-1)}{(2n-3)(n+m)(n-m)}} \\ \bar{P}_{00}(t) = 1, \quad \bar{P}_{10}(t) = \sqrt{3}t, \quad \bar{P}_{11}(t) = \sqrt{3}u \quad (3.2)$$

(2) Improved Belikov recursion algorithm for $\bar{P}_{nm}(t)$ ($n < 64800$)

When $n = 0, 1$, $\bar{P}_{nm}(t)$ is calculated according to (3.2). And when $n \geq 2$, we have:

$$\bar{P}_{n0}(t) = a_n t \bar{P}_{n-1,0}(t) - b_n \frac{u}{2} \bar{P}_{n-1,1}(t), \quad m = 0 \quad (3.3)$$

$$\bar{P}_{nm}(t) = c_{nm} t \bar{P}_{n-1,m}(t) - d_{nm} u \bar{P}_{n-1,m+1}(t) + e_{nm} u \bar{P}_{n-1,m-1}(t), \quad m > 0 \quad (3.4)$$

$$a_n = \sqrt{\frac{2n+1}{2n-1}}, \quad b_n = \sqrt{\frac{2(n-1)(2n+1)}{n(2n-1)}} \quad (3.5)$$

$$c_{nm} = \frac{1}{n} \sqrt{\frac{(n+m)(n-m)(2n+1)}{2n-1}}, \quad d_{nm} = \frac{1}{2n} \sqrt{\frac{(n-m)(n-m-1)(2n+1)}{2n-1}} \quad (3.6)$$

here when $m > 0$,

$$e_{nm} = \frac{1}{2n} \sqrt{\frac{2}{2-\delta_0^{m-1}}} \sqrt{\frac{(n+m)(n+m-1)(2n+1)}{2n-1}} \quad (3.7)$$

(3) Cross degree-order recursive algorithm for $\bar{P}_{nm}(t)$ ($n < 20000$)

When $n = 0, 1$, $\bar{P}_{nm}(t)$ is calculated according to (3.2). And when $n \geq 2$, we have:

$$\bar{P}_{nm}(t) = \alpha_{nm} \bar{P}_{n-2,m}(t) + \beta_{nm} \bar{P}_{n-2,m-2}(t) - \gamma_{nm} \bar{P}_{n,m-2}(t) \quad (3.8)$$

$$\alpha_{nm} = \sqrt{\frac{(2n+1)(n-m)(n-m-1)}{(2n-3)(n+m)(n+m-1)}}$$

$$\beta_{nm} = \sqrt{1 + \delta_0^{m-2}} \sqrt{\frac{(2n+1)(n+m-2)(n+m-3)}{(2n-3)(n+m)(n+m-1)}} \quad (3.9)$$

$$\gamma_{nm} = \sqrt{1 + \delta_0^{m-2}} \sqrt{\frac{(n-m+1)(n-m+2)}{(n+m)(n+m-1)}}$$

(4) Non-singular recursive algorithm for $\frac{\partial}{\partial \theta} \bar{P}_{nm}(\cos \theta)$

Considering that the first derivative of $\bar{P}_{nm}(\cos \theta)$ with respect to θ is

$$\frac{\partial}{\partial \theta} \bar{P}_{nm}(\cos \theta) = -\sin \theta \frac{\partial}{\partial t} \bar{P}_{nm}(t) \quad (3.10)$$

we have

$$\begin{cases} \frac{\partial}{\partial \theta} \bar{P}_{n0} = -\sqrt{\frac{n(n+1)}{2}} \bar{P}_{n1}, & \frac{\partial}{\partial \theta} \bar{P}_{n1} = \sqrt{\frac{n(n+1)}{2}} \bar{P}_{n0} - \frac{\sqrt{(n-1)(n+2)}}{2} \bar{P}_{n2} \\ \frac{\partial}{\partial \theta} \bar{P}_{nm} = \frac{\sqrt{(n+m)(n-m+1)}}{2} \bar{P}_{n,m-1} - \frac{\sqrt{(n-m)(n+m+1)}}{2} \bar{P}_{n,m+1}, & m > 2 \end{cases} \quad (3.11)$$

$$\frac{\partial}{\partial \theta} \bar{P}_{00}(t) = 0, \quad \frac{\partial}{\partial \theta} \bar{P}_{10}(t) = -\sqrt{3}u, \quad \frac{\partial}{\partial \theta} \bar{P}_{11}(t) = \sqrt{3}t \quad (3.12)$$

(5) Non-singular recursive algorithm for $\frac{\partial^2}{\partial \theta^2} \bar{P}_{nm}(\cos \theta)$

$$\begin{cases} \frac{\partial^2}{\partial \theta^2} \bar{P}_{n0} = -\frac{n(n+1)}{2} \bar{P}_{n0} + \sqrt{\frac{n(n-1)(n+1)(n+2)}{8}} \bar{P}_{n2} \\ \frac{\partial^2}{\partial \theta^2} \bar{P}_{n1} = -\frac{2n(n+1)+(n-1)(n+2)}{4} \bar{P}_{n1} + \frac{\sqrt{(n-2)(n-1)(n+2)(n+3)}}{4} \bar{P}_{n3} \end{cases} \quad (3.13)$$

$$\begin{aligned} \frac{\partial^2}{\partial \theta^2} \bar{P}_{nm} &= \frac{\sqrt{(n-m+1)(n-m+2)(n+m-1)(n+m)}}{4} \bar{P}_{n,m-2} \\ &\quad - \frac{(n+m)(n-m+1)+(n-m)(n+m+1)}{4} \bar{P}_{nm} \\ &\quad + \frac{\sqrt{(n-m-1)(n-m)(n+m+1)(n+m+2)}}{4} \bar{P}_{n,m+2}, \quad m > 2 \end{aligned} \quad (3.14)$$

$$\frac{\partial^2}{\partial \theta^2} \bar{P}_{00}(t) = 0, \quad \frac{\partial^2}{\partial \theta^2} \bar{P}_{10}(t) = -\sqrt{3}t, \quad \frac{\partial^2}{\partial \theta^2} \bar{P}_{11}(t) = -\sqrt{3}u \quad (3.15)$$

7.4 Boundary value correction for ellipsoid and spherical boundary surface

(1) Ellipsoid correction of gravity. The correction of the gravity g on an ellipsoid surface outside the Earth from the vertical direction to the normal gravity direction, also

known as the vertical deflection correction of gravity.

$$\varepsilon_p = \gamma \sin\theta \cos\theta \left[3J_2 \left(\frac{a}{r} \right)^2 + \frac{\omega^3 r^3}{GM} \right] \xi \quad (4.1)$$

(2) The correction of the gravity g from the normal gravity direction to the geocentric direction

$$\varepsilon_h = \gamma e^2 \sin\theta \cos\theta \xi \quad (4.2)$$

(3) The correction of the normal gravity γ from the normal gravity direction to the geocentric direction

$$\varepsilon_\gamma = 3\gamma \left[J_2 \frac{a^2}{r^3} (3\cos^2\theta - 1) - \frac{\omega^3 r^3}{GM} \sin^2\theta \right] T \quad (4.3)$$

When the boundary surface is an ellipsoidal surface, only one ellipsoid correction is required in equation (4.1). When the boundary surface is spherical surface, it is necessary to perform three items boundary value corrections using equations (4.1) ~ (4.3).

7.5 Terrain effect algorithms on various gravity field elements outside geoid

The theory of the Earth's gravitational field points out that any type of anomalous gravity field element outside the Earth can be expressed as a linear combination of the disturbing potential, gravity disturbance or their partial derivatives on some an equipotential surface. For example, the vertical deflection can be expressed by the local horizontal partial derivative of the disturbing potential, and the disturbing gravity gradient can be expressed by the vertical derivative of the gravity disturbance. Therefore, if we can get the terrain effects on disturbing potential and gravity disturbance, we also can get the terrain effects on other types of gravity field elements since there is no terrain effect in normal gravity field..

7.5.1 Expression of land terrain mass gravitational field - land complete Bouguer effect

The land terrain mass gravitational field is also called as as the complete Bouguer effect of land terrain, which is defined as the effect of the terrain mass above the geoid on the Earth's gravity field.

(1) Land complete Bouguer effect on disturbing potential

Ignoring the mass effect of the Earth's atmosphere, the disturbing potential T at the calculation point outside the Earth can be expressed as the sum of the terrain mass gravitational potential T^t (land complete Bouguer effect) and the disturbing potential T^{NT} after the terrain removed:

$$T = T^{NT} + T^t = T^{NT} + T^B + T^R \quad (5.1)$$

where T^t is the gravitational potential generated by the total terrain mass at the calculation point, which is called as the terrain effect on the disturbing potential or land complete Bouguer effect. T^R is the effect of the local terrain mass on the gravitational

potential at the calculation point, called as the local terrain effect on the disturbing potential. T^B is the gravitational potential at the calculation point generated by the mass of the spherical shell with a thickness equal to the terrain height, which is called as the spherical shell Bouguer effect on the disturbing potential.

From the harmonic properties of the disturbing potential T outside the Earth, it can be known that the land complete Bouguer effect T^t , local terrain effect T^R and spherical shell Bouguer effect T^B on the disturbing potential outside the Earth are all harmonic.

Under the spherical approximation, the complete Bouguer effect on disturbing potential in the near-Earth space outside the Earth ($r \geq R + h$, R is the mean radius of the Earth) can be expressed as:

$$T^t = T^B + T^R = 4\pi G \rho_0 \frac{R^2 h}{r} \left(1 + \frac{h}{R} + \frac{h^2}{3R^2}\right) + T^R \quad (5.2)$$

where G is the gravitational constant, h is the terrain height directly below the calculation point, r is the geocentric distance of the calculation point, ρ_0 is the geometric mean density of the terrain from the ground to the geoid, and $\rho_0 = 2.67 \times 10^3 \text{ kg/m}^3$.

(2) Land complete Bouguer effect on gravity disturbance

Substituting (5.1) into the definition of gravity disturbance, we get

$$\delta g = -\frac{\partial T^{NT}}{\partial r} - \frac{\partial T^t}{\partial r} = \delta g^{NT} + \delta g^t = \delta g^{NT} + \delta g^B + \delta g^R \quad (5.3)$$

where δg^t is called as the complete Bouguer effect on gravity disturbance, δg^B is the spherical shell Bouguer effect, and δg^R is called as the local terrain effect on gravity disturbance.

Under spherical approximation, the complete Bouguer effect on gravity disturbance:

$$\delta g^t = \delta g^B + \delta g^R = 4\pi G \tilde{\rho} \frac{R^2 h}{r^2} \left(1 + \frac{h}{R} + \frac{h^2}{3R^2}\right) + \delta g^R \quad (5.4)$$

Equations (5.2) and (5.4) are truncated to the quadratic term of h/R , which are suitable on ground and near-Earth space (such as aviation altitude), but not suitable on satellite altitude.

7.5.2 Integral formula of local terrain effect outside the Earth

(1) The local terrain effect on disturbing potential

According to the definition, only considering the surface density ρ , the local terrain effect on disturbing potential can be expressed as:

$$T^R = \gamma \zeta^R = G \rho \iint_s \int_{R+h}^{R+h'} L^{-1}(r, \psi, r') dr' ds \quad (5.5)$$

where $ds = r'^2 \cos \varphi' d\varphi' d\lambda'$ is the integral areal element on the ground, and $L = \sqrt{r^2 + r'^2 - 2rr' \cos \psi}$ is the space distance from the move point (namely the integral volume element $dV = dr' ds$) to the calculation point.

$$\int L^{-1}(r, \psi, r') dr' = \ln(r' - rt + L) + C \quad (5.6)$$

where $t = \cos \psi$, C is the integral constant.

When the calculation point is the same as the move point, the integral of local terrain

effect on disturbing potential is singular:

$$T^R|_0 = \frac{1}{6}G\rho_0 A_0 \sqrt{A_0/\pi} (h_{xx} + h_{yy}) \quad (5.7)$$

where ρ_0 is the terrain density at the calculation point, A_0 is the area of the integral areal element at the calculation point, and h_{xx}, h_{yy} are the second-order horizontal partial derivatives of the terrain at the calculation point in the north direction x and the east direction y .

(2) The local terrain effect on gravity disturbance

According to the definition, the local terrain effect on gravity disturbance can be expressed as:

$$\delta g^R = -T_r^R = -\frac{\partial T^R}{\partial r} = -G\rho \iint_S \int_{R+h}^{R+h'} \frac{\partial L^{-1}(r, \psi, r')}{\partial r} dr' ds \quad (5.8)$$

$$\text{where, } \int \frac{\partial L^{-1}(r, \psi, r')}{\partial r} dr' = -\int \frac{r-r't}{L^3} dr' = -\frac{r'}{rL} + C \quad (5.9)$$

When the calculation point is the same as the move point, the integral of local terrain effect on gravity disturbance is singular:

$$\delta g^R|_0 = \frac{1}{2}G\rho_0 \sqrt{\pi A_0} (h_x^2 + h_y^2) \quad (5.10)$$

where (h_x, h_y) is the terrain slope vector at the calculation point

(3) The local terrain effect on vertical deflection

Considering $\frac{\partial \psi}{\partial \varphi} = -\cos \alpha$, $\frac{\partial \psi}{\partial \lambda} = -\cos \varphi \sin \alpha$, we have:

$$\begin{aligned} \xi^R &= \frac{T_\theta^R}{\gamma r} = -\frac{\partial T^R}{\gamma r \partial \varphi} = -\frac{\partial T^R}{\gamma r \partial \psi} \frac{\partial \psi}{\partial \varphi} = \frac{\partial T^R}{\gamma r \partial \psi} \cos \alpha \\ &= \frac{G\rho}{\gamma r} \iint_S \int_{R+h}^{R+h'} \frac{\partial L^{-1}(r, \psi, r')}{\partial \psi} dr' \cos \alpha ds \end{aligned} \quad (5.11)$$

$$\begin{aligned} \eta^R &= -\frac{T_\lambda^R}{\gamma r \sin \theta} = -\frac{\partial T^R}{\gamma r \cos \varphi \partial \lambda} = -\frac{\partial T^R}{\gamma r \cos \varphi \partial \psi} \frac{\partial \psi}{\partial \lambda} = \frac{\partial T^R}{\gamma r \cos \varphi \partial \psi} \cos \varphi \sin \alpha \\ &= \frac{G\rho}{\gamma r} \iint_S \int_{R+h}^{R+h'} \frac{\partial L^{-1}(r, \psi, r')}{\partial \psi} dr' \sin \alpha ds \end{aligned} \quad (5.12)$$

$$\text{where } \int \frac{\partial L^{-1}(r, \psi, r')}{\partial \psi} dr' = \frac{r-r't}{L \sin \psi} + C, \quad (5.14)$$

α is the geodetic azimuth of ψ , which can be obtained from the spherical trigonometric formula:

$$\sin \psi \cos \alpha = \cos \varphi \sin \varphi' - \sin \varphi \cos \varphi' \cos(\lambda' - \lambda) \quad (5.14)$$

$$\sin \psi \sin \alpha = \cos \varphi' \sin(\lambda' - \lambda) \quad (5.15)$$

(4) The local terrain effect on disturbing gravity gradient

$$T_{rr}^R = \frac{\partial^2}{\partial r^2} T^R = G\rho \iint_S \int_{R+h}^{R+h'} \frac{\partial^2 L^{-1}(r, \psi, r')}{\partial r^2} dr' ds \quad (5.16)$$

$$\text{where } \int \frac{\partial^2 L^{-1}(r, \psi, r')}{\partial r^2} dr' = \int \left[-\frac{1}{L^3} + \frac{3(r-r't)^2}{L^5} \right] dr' = \frac{r'}{r^2 L} + \frac{r'(r-r't)}{r L^3} + C \quad (5.17)$$

(5) The local terrain effect on tangential gravity gradient

$$T_{NN}^R = \frac{1}{r} T_r^R + \frac{1}{r^2} T_{\theta\theta}^R = -\frac{1}{r} \delta g^R - \frac{1}{r^2} T_{\varphi\varphi}^R \quad (5.18)$$

$$T_{ww}^R = \frac{1}{r}T_r + \frac{1}{r^2}T_\theta ctg\theta + \frac{1}{r^2\sin^2\theta}T_{\lambda\lambda} = -\frac{1}{r}\delta g^R + \frac{\gamma}{r}\xi^R ctg\theta + \frac{1}{r^2\cos^2\varphi}T_{\lambda\lambda}^R \quad (5.19)$$

$$T_{\varphi\varphi}^R = \frac{\partial^2 T^R}{\partial \psi^2} \frac{\partial^2 \psi}{\partial \varphi^2}, \quad T_{\lambda\lambda}^R = \frac{\partial^2 T^R}{\partial \psi^2} \frac{\partial^2 \psi}{\partial \lambda^2} \quad (5.20)$$

Taking the partial derivative with respect to φ on both sides of equation (5.14), considering $\frac{\partial \psi}{\partial \varphi} = -\cos\alpha$, $\frac{\partial \psi}{\partial \lambda} = -\cos\varphi\sin\alpha$, we have:

$$\sin\psi \frac{\partial^2 \psi}{\partial \varphi^2} = -\sin\varphi\sin\varphi' - \cos\varphi\cos\varphi'\cos(\lambda' - \lambda) + \cos\psi\cos^2\alpha \quad (5.21)$$

In the same way, taking the partial derivatives of both sides of (5.15) with respect to λ , we have: $-\cos\psi\cos\varphi\sin^2\alpha + \sin\psi \frac{\partial^2 \psi}{\partial \lambda^2} = -\cos\varphi'\sin(\lambda' - \lambda)$, so that we can get:

$$\sin\psi \frac{\partial^2 \psi}{\partial \lambda^2} = -\cos\varphi'\sin(\lambda' - \lambda) + \cos\psi\cos\varphi\sin^2\alpha \quad (5.22)$$

Calculate the second-order partial derivative with respect to the spherical angular distance ψ on both sides of the integral of the local terrain effect on disturbing potential, we have:

$$\frac{\partial^2 T^R}{\partial \psi^2} = G\rho \iint_s \int_{R+h}^{R+h'} \frac{\partial^2}{\partial \psi^2} \frac{1}{L} dr' ds = G\rho \iint_s \int_{R+h}^{R+h'} \frac{\partial^2}{\partial \psi^2} \frac{1}{\sqrt{r^2+r'^2-2rr'\cos\psi}} dr' ds \quad (5.23)$$

$$\int \frac{\partial^2}{\partial \psi^2} \frac{1}{L} dr' = \frac{r'(6r^2+4r'^2+6r^2\cos 2\psi - rr'\cos 3\psi) - rt(4r^2+11r'^2)}{4L^3\sin^2\psi} \quad (5.24)$$

7.5.3 Fast FFT algorithms of the integral of local terrain effects on various field elements

(1) Fast algorithm of the integral of local terrain effect on disturbing potential

Using the local spherical polar coordinate system, let the z-axis be the radial direction from the Earth center of mass, that is, the zenith direction, the $z = 0$ is the terrain surface, and \tilde{h} is the height of the calculation point relative to the terrain surface. In this case, $dz = dr'$, $d\tilde{h} = dr$, then the local terrain effect on disturbing potential (5.5) is equivalent to:

$$\begin{aligned} T^R &= G\rho \iint_s \int_0^{\Delta h} \frac{dz}{L} ds = G\rho \iint_s \int_0^{\Delta h} \frac{dz}{\sqrt{(\tilde{h}-z)^2 + l^2}} ds \\ &= G\rho \iint_s \left[\ln \frac{\sqrt{(\tilde{h}-\Delta h)^2 + l^2} - \tilde{h} + \Delta h}{\sqrt{(\tilde{h}-\Delta h)^2 + l^2} + \tilde{h} - \Delta h} - \ln \frac{\sqrt{\tilde{h}^2 + l^2} - H}{\sqrt{\tilde{h}^2 + l^2} + H} \right] ds \end{aligned} \quad (5.25)$$

where Δh is the height difference of the integral move areal element ds on the surface relative to the terrain surface directly below the calculation point, $l = 2r_0\sin(\psi/2)$ is the spherical distance from the move areal element to the calculation point, and r_0 is the geocentric distance of the surface directly below the calculation point, and ψ is the spherical angular distance from the move point to the calculation point.

Expand the integrand in Eq. (5.25) to order 3 near $z = 0$, we have:

$$T^R = G\rho \iint_s \left[\frac{1}{\mathcal{L}}\Delta h + \frac{\tilde{h}}{2\mathcal{L}^3}\Delta h^2 + \frac{2\tilde{h}^2 - l^2}{6\mathcal{L}^5}\Delta h^3 \right] ds \quad (5.26)$$

where $\mathcal{L} = \sqrt{\tilde{h}^2 + l^2}$ is the space distance from the move areal element ds to the

calculation point in the case of $z = 0$. Here $\mathcal{L} \neq L$, and L is the space distance from the move volume element $dzds$ to the calculation point.

Considering $\Delta h^2 = h'^2 - 2h'h + h^2$, $\Delta h^3 = h'^3 - 3h'^2h + 3h'h^2 - h^3$, each item on the right side of the formula (5.26) can be quickly calculated using the FFT algorithm. where, h is the terrain height directly below the calculation point, and h' is the terrain height of the move areal element.

(2) Fast algorithm of the integral of local terrain effect on gravity disturbance

In the same way, the local terrain effect on gravity disturbance (5.8) is equivalent to:

$$\delta g^R = \frac{G\rho}{r} \iint_s \left[\frac{(r_0+z)}{\sqrt{(\tilde{h}-z)^2+l^2}} \right]_0^{\Delta h} ds = \frac{G\rho}{r} \iint_s \left[\frac{r_0+\Delta h}{\sqrt{(\tilde{h}-\Delta h)^2+l^2}} - \frac{r_0}{L} \right] ds \quad (5.27)$$

Expand the integrand in Eq. (5.27) to order 4 near $z = 0$, we have:

$$\delta g^R = \frac{G\rho}{r} \iint_s \left[\frac{r\tilde{h}+L^2}{L^3} \Delta h + \frac{2\tilde{h}L^2+r_0(2\tilde{h}^2-l^2)}{2L^5} \Delta h^2 + \frac{2\tilde{h}^3r+\tilde{h}^2l^2-3r_0\tilde{h}l^2-l^4}{2L^7} \Delta h^3 + \frac{8r\tilde{h}^4-4\tilde{h}^3l^2-12\tilde{h}l^4-24r_0\tilde{h}^2l^2+3l^4r_0}{8L^9} \Delta h^4 \right] ds \quad (5.28)$$

where $\Delta h^4 = h'^4 - 4h'^3h + 6h^2h'^2 - 4h'h^3 + h^4$.

Each item on the right side of the formula (5.28) can be quickly calculated using the FFT algorithm. In PAGravf4.5, the numerical integral of the local terrain effect on gravity disturbance is calculated using formula (5.27).

(3) Fast algorithm of the integral of local terrain effect on vertical deflection

Expand the integrand in the integral formula of the local terrain effect on vertical deflection to order 3 near $z = 0$:

$$\int_{R+h}^{R+h'} \frac{\partial L^{-1}(r,\psi,r')}{\partial \psi} dr' = -\frac{r^2 \sin \psi}{L^3} \Delta h - \frac{3\tilde{h}r^2 \sin \psi}{2L^5} \Delta h^2 - \left[\frac{r^2 \sin \psi}{3L^5} + \frac{5r^2 \sin \psi (2\tilde{h}^2-l^2)}{6L^7} \right] \Delta h^3 \quad (5.29)$$

Substitute (5.29) into (5.11) and (5.12) formulas, and the FFT algorithm can be employed to quickly calculate the local terrain effect on vertical deflection.

(4) Fast algorithm of the integral of local terrain effect on disturbing gravity gradient

The local terrain effect on disturbing gravity gradient (5.16) is equivalent to:

$$T_{rr}^R = G\rho \iint_s \left[\frac{\tilde{h}-\Delta h}{((\tilde{h}-\Delta h)^2+l^2)^{3/2}} - \frac{\tilde{h}}{L^3} \right] ds \quad (5.30)$$

Expand the integrand in Eq. (5.30) to order 3 near $z = 0$, we have:

$$T_{rr}^R = G\rho \iint_s \left[\frac{2\tilde{h}^2-l^2}{L^5} \Delta h - \frac{3\tilde{h}(2\tilde{h}^2-3l^2)}{2L^7} \Delta h^2 + \frac{4\tilde{h}^4+6r^4-12\tilde{h}^2l^2-(6r^4+3r^2l^2)t}{L^9} \Delta h^3 \right] ds \quad (5.31)$$

(5) Fast algorithm of the integral of local terrain effect on tangential gravity gradient

The integrand function in Equation (5.23) is equivalent to:

$$\begin{aligned}
\int_{R+h}^{R+h'} \frac{\partial^2}{\partial \psi^2} \frac{1}{L} dr' &= \int_0^{\Delta h} \frac{\partial^2}{\partial \psi^2} \frac{1}{\sqrt{(\tilde{h}-z)^2 + 4r_0^2 \sin^2(\psi/2)}} dz \\
&= \frac{1}{8\sin^2 \frac{\psi}{2}} \left[\frac{\tilde{h}(2\mathcal{L}^2 + r_0^2 \sin^2 \psi)}{\mathcal{L}^3} - \frac{(\tilde{h}-\Delta h)(2\mathcal{L}^2 + r_0^2 \sin^2 \psi - 4\tilde{h}\Delta h + 2\Delta h^2)}{(\mathcal{L}^2 - 2\tilde{h}\Delta h + \Delta h^2)^{3/2}} \right]
\end{aligned} \quad (5.32)$$

Expand it to order 3 near $z=0$, we have:

$$\begin{aligned}
\int_{R+h}^{R+h'} \frac{\partial^2}{\partial \psi^2} \frac{1}{L} dr' &= -\frac{2(\tilde{h}^2 + 2r_0^2)\cos\psi + r_0^2(-5 + \cos 2\psi)}{2\mathcal{L}^5} r_0^2 \Delta h \\
&+ \frac{6(\tilde{h}^2 + 2r_0^2)\cos\psi + 3r_0^2(-7 + 3\cos 2\psi)}{4\mathcal{L}^7} \tilde{h} r_0^2 \Delta h^2 \\
&+ \frac{(8\tilde{h}^4 + 12\tilde{h}^2 r_0^2 - 19r_0^4)\cos\psi - r_0^2(36\tilde{h}^2 - 18r_0^2 - (24\tilde{h}^2 - 2r_0^2)\cos 2\psi + 3r_0^2 \cos 3\psi)}{4\mathcal{L}^9} r_0^2 \Delta h^3
\end{aligned} \quad (5.33)$$

If the calculation point is also on the terrain surface, there are $\tilde{h} = 0$, $\mathcal{L} = l$, formulas (5.25) ~ (5.33) can be greatly simplified.

7.6 Seawater Bouguer effect and land-sea residual terrain effect

7.6.1 Marine terrain gravitational field - seawater complete Bouguer effect

The marine terrain gravitational field is usually represented by the seawater complete Bouguer effect. The seawater complete Bouguer effect is defined as the effect on the Earth's gravity field (various gravity field elements) because of the density of seawater compensated to the density of land terrain.

The seawater complete Bouguer effect on disturbing potential can be directly expressed by the integral formula as:

$$T^o = G\beta \iint_s \int_{R+d}^R L^{-1}(r, \psi, r') dr' ds \quad (6.1)$$

where $d < 0$ is the seafloor depth, β is the seawater compensation density, equal to the difference between the terrain density and the seawater density, $\beta = \rho_0 - \rho_w = 1.64 \times 10^3 \text{ kg/m}^3$, and L is the space distance of the move volume element of the water body ($dV' = dr' ds$) to the calculation point.

Using the local spherical polar coordinate system, let the z -axis be the radial direction from the Earth center of mass, that is, the zenith direction, $z = 0$ represents the sea level, then equation (6.1) is equivalent to:

$$\begin{aligned}
T^o &= G\beta \iint_s \int_d^0 \frac{dz}{L} ds = G\beta \iint_s \int_d^0 \frac{dz}{\sqrt{(\tilde{h}-z)^2 + l^2}} ds \\
&= G\beta \iint_s \left[\ln \frac{\sqrt{\tilde{h}^2 + l^2} - \tilde{h}}{\sqrt{\tilde{h}^2 + l^2} + \tilde{h}} - \ln \frac{\sqrt{(\tilde{h}-d)^2 + l^2} - \tilde{h} + d}{\sqrt{(\tilde{h}-d)^2 + l^2} + \tilde{h} - d} \right] ds
\end{aligned} \quad (6.2)$$

where, $ds = r'^2 d\sigma = r'^2 \cos\varphi' d\varphi' d\lambda'$ is the areal element on the sea surface, $\mathcal{L} = \sqrt{\tilde{h}^2 + l^2}$ is the space distance between the move area element ds on the sea surface and the calculation point ($\mathcal{L} \neq L$), and \tilde{h} is the altitude of the calculation point. $l = 2r_0 \sin \frac{\psi}{2}$ is the distance between the calculation point and the move point on the sea surface, r_0 is the mean geocentric distance of the sea surface, the mean radius R of the

Earth.

In the same way, the seawater complete Bouguer effect on gravity disturbance is:

$$\delta g^o = -\frac{\partial T^o}{\partial r} = -G\beta \iint_s \int_{R+d}^R \frac{\partial L^{-1}(r,\psi,r')}{\partial r} dr' ds \quad (6.3)$$

Equation (6.3) is equivalent to

$$\delta g^o = \frac{G}{r} \iint_s \beta \int_d^0 \frac{(r_0+z)dz}{\sqrt{(\tilde{h}-z)^2+l^2}} ds = \frac{G\beta}{r} \iint_s \left[\frac{r_0}{\tilde{L}} - \frac{r_0+d}{\sqrt{(\tilde{h}-d)^2+l^2}} \right] ds \quad (6.4)$$

Considering $\frac{\partial \psi}{\partial \varphi} = -\cos\alpha$, $\frac{\partial \psi}{\partial \lambda} = -\cos\varphi \sin\alpha$, the seawater complete Bouguer effect on vertical deflection is equal to:

$$\xi^o = \frac{T_\theta^o}{\gamma r} = \frac{G\beta}{\gamma r} \iint_s \int_{R+d}^R \frac{\partial L^{-1}(r,\psi,r')}{\partial \psi} dr' \cos\alpha ds \quad (6.5)$$

$$\eta^o = -\frac{T_\lambda^o}{\gamma r \sin\theta} = \frac{G\beta}{\gamma r} \iint_s \int_{R+d}^R \frac{\partial L^{-1}(r,\psi,r')}{\partial \psi} dr' \sin\alpha ds \quad (6.6)$$

The seawater complete Bouguer effect on disturbing gravity gradient is:

$$T_{rr}^o = \frac{\partial^2}{\partial r^2} T^o = G\beta \iint_s \int_{R+d}^R \frac{\partial^2 L^{-1}(r,\psi,r')}{\partial r^2} dr' ds \quad (6.7)$$

Equation (6.7) is equivalent to

$$T_{rr}^o = G\beta \iint_s \left[\frac{\tilde{h}-d}{(\tilde{h}-d)^2+l^2} - \frac{h}{\tilde{L}^3} \right] ds \quad (6.8)$$

Similarly, by expanding the integrand in the above integral formula near the sea level $z = 0$, the fast FFT algorithm formula can be derived.

Expand the integrand in Eq. (6.2) to order 3 near $z = 0$, we have:

$$T^o = G\beta \int_d^0 \frac{1}{\tilde{L}} dz ds = G\beta \iint_s \left(\frac{1}{\tilde{L}} d + \frac{\tilde{h}}{2\tilde{L}^3} d^2 + \frac{2\tilde{h}^2-l^2}{6\tilde{L}^5} d^3 \right) ds \quad (6.9)$$

Expand the integrand in Eq. (6.3) to order 4 near $z = 0$, we have:

$$\delta g^o = \frac{G}{r} \iint_s \beta \left[\frac{r\tilde{h}+L^2}{\tilde{L}^3} d + \frac{2\tilde{h}L^2+r_0(\tilde{h}^2+L^2)}{2\tilde{L}^5} d^2 + \frac{2\tilde{h}^3 r+\tilde{h}^2 l^2-3r_0\tilde{h}l^2-l^4}{2\tilde{L}^7} d^3 + \frac{8r\tilde{h}^4-4\tilde{h}^3 l^2-12\tilde{h}l^4-24r_0\tilde{h}^2 l^2+3l^4 r_0}{8\tilde{L}^9} d^4 \right] ds \quad (6.10)$$

Expand the integrand in the integral formula of the seawater complete Bouguer effect on vertical deflection to order 3 near $z = 0$:

$$\begin{aligned} & \int_{R+d}^R \frac{\partial L^{-1}(r,\psi,r')}{\partial \psi} dr' \\ &= -\frac{r^2 \sin\psi}{\tilde{L}^3} d - \frac{3\tilde{h}r^2 \sin\psi}{2\tilde{L}^5} d^2 - \left[\frac{r^2 \sin\psi}{3\tilde{L}^5} + \frac{5r^2 \sin\psi(2\tilde{h}^2-l^2)}{6\tilde{L}^7} \right] d^3 \end{aligned} \quad (6.11)$$

From equation (6.8), we can get the seawater complete Bouguer effect on disturbing gravity gradient:

$$T_{rr}^o = -\frac{\partial \delta g^o}{\partial r} = G\beta \iint_s \left[\frac{2\tilde{h}^2-l^2}{\tilde{L}^5} d + \frac{3\tilde{h}(2\tilde{h}^2-3l^2)}{2\tilde{L}^7} d^2 + \frac{4\tilde{h}^4+6r^4-12\tilde{h}^2 l^2-(6r^4+3r^2 l^2)\cos\psi}{\tilde{L}^9} d^3 \right] ds \quad (6.12)$$

The items on the right side of equations (6.9) to (6.12) can be quickly calculated by

the FFT algorithm. If the calculation point is also on the sea surface, with $h = 0$, $\mathcal{L} = l$, formulas (6.2) ~ (6.12) can be greatly simplified.

The seawater complete Bouguer effects on various gravity field elements are relatively large, and a larger integral radius should be employed for the integral calculation, such as not less than 250km.

7.6.2 Integral algorithms of residual terrain effects on various field elements outside the geoid

The land-sea residual terrain effect is defined as the short-wave and ultra-short-wave components of the land-sea complete Bouguer effect. The residual terrain effects on various types of field elements can be calculated by firstly constructing the land-sea residual terrain model and then using the integral method.

The residual terrain model (RTM) can be obtained by subtracting the low-resolution land-sea terrain model from the high-resolution land-sea terrain model with the same grid specification.

The integral formula of residual terrain effect is similar in form to the integral formula of local terrain effect/seawater complete Bouguer effect, the difference lies in the adopted the density of the move element and the radial integral domain.

(1) Numerical Integral of residual terrain effects on various field elements outside the geoid

The residual terrain effects on disturbing potential can be directly expressed as:

$$T^{rtm} = G \iint_S \int_R^{R+\delta'} \beta' L^{-1}(r, \psi, r') dr' ds \quad (6.13)$$

where, δ' , β' are the residual terrain height and density at the move areal element $ds = r'^2 \cos\varphi' d\varphi' d\lambda'$, respectively. When ds is located in the land area, δ' is the residual terrain height δh , and β' is the terrain density ρ_0 ($= 2.67 \times 10^3 \text{kg/m}^3$), while when ds is located in the ocean area, δ' is the residual seafloor depth δd , β' is seawater compensation density $\rho_0 - \rho_w$ (where seawater density $\rho_w = 1.03 \times 10^3 \text{kg/m}^3$).

It is not difficult to find that whether the areal element ds is in the land area or in the sea area, the residual terrain δ' could be positive or negative.

In the same way, the residual terrain effect on gravity disturbing is equal to:

$$\delta g^{rtm} = -\frac{\partial T^{rtm}}{\partial r} = -G \iint_S \beta' \int_R^{R+\delta'} \frac{\partial L^{-1}(r, \psi, r')}{\partial r} dr' ds \quad (6.14)$$

The residual terrain effect on vertical deflection is equal to:

$$\begin{aligned} \xi^{rtm} &= \frac{G}{\gamma r} \iint_S \beta' \int_R^{R+\delta'} \frac{\partial L^{-1}(r, \psi, r')}{\partial \psi} dr' \cos\alpha ds \\ \eta^{rtm} &= \frac{G}{\gamma r} \iint_S \beta' \int_R^{R+\delta'} \frac{\partial L^{-1}(r, \psi, r')}{\partial \psi} dr' \sin\alpha ds \end{aligned} \quad (6.15)$$

The residual terrain effect on disturbing gravity gradient is equal to:

$$T_{rr}^{rtm} = \frac{\partial^2}{\partial r^2} T^{rtm} = G \iint_S \beta' \int_R^{R+\delta'} \frac{\partial^2 L^{-1}(r, \psi, r')}{\partial r^2} dr' ds \quad (6.16)$$

Similarly, using a local spherical coordinate system, let the z-axis be the radial

direction (zenith direction), and $z = 0$ is the terrain surface/sea surface. Let $\mathcal{L} = \sqrt{\tilde{h}^2 + l^2}$ be the three-dimensional space distance between the move areal element and the calculation point, then formulas (6.13) ~ (6.16) can be rewritten as:

$$\begin{aligned} T^{rtm} &= G \iint_S \beta' \int_0^{\delta'} \frac{dz}{\sqrt{(\tilde{h}-z)^2 + l^2}} ds \\ &= G \iint_S \beta' \left[\ln \frac{\sqrt{(\tilde{h}-\delta')^2 + l^2} - \tilde{h} + \delta'}{\mathcal{L} + \tilde{h} - \delta'} - \ln \frac{\mathcal{L} - \tilde{h}}{\mathcal{L} + \tilde{h}} \right] ds \end{aligned} \quad (6.17)$$

$$\delta g^{rtm} = \frac{G}{r} \iint_S \beta' \iint_0^{\Delta h} \frac{\partial}{\partial \tilde{h}} \frac{dz}{\sqrt{(\tilde{h}-z)^2 + l^2}} ds = \frac{G}{r} \iint_S \beta' \left[\frac{1}{\sqrt{(\tilde{h}-\Delta h)^2 + l^2}} - \frac{1}{\mathcal{L}} \right] ds \quad (6.18)$$

$$\int_R^{R+\delta'} \frac{\partial L^{-1}(r, \psi, r')}{\partial \psi} dr' = \frac{1}{2} ctg \frac{\psi}{2} \left[\frac{\tilde{h} - \delta'}{\sqrt{(\tilde{h}-\delta')^2 + l^2}} - \frac{\tilde{h}}{\mathcal{L}} \right] \quad (6.19)$$

$$T_{rr}^{rtm} = G \iint_S \beta' \left[\frac{\tilde{h} - \delta'}{[(\tilde{h}-\delta')^2 + l^2]^{3/2}} - \frac{\tilde{h}}{\mathcal{L}^3} \right] ds \quad (6.20)$$

(2) Fast FFT algorithms of the integral of residual terrain effect on various field elements

The integrand in the above integral formula is expanded near $z=0$, where $z=0$ is the terrain surface/sea surface.

Expand the integrand in Eq. (6.17) to order 3 near $z = 0$, we have:

$$T^{rtm} = -G \iint_S \beta' \left(\frac{1}{\mathcal{L}} \delta' + \frac{\tilde{h}}{2\mathcal{L}^3} \delta'^2 + \frac{2\tilde{h}^2 - l^2}{6\mathcal{L}^5} \delta'^3 \right) ds \quad (6.21)$$

Expand the integrand in Eq. (6.18) to order 4 near $z = 0$, we have:

$$\delta g^{rtm} = \frac{G}{r} \iint_S \beta' \left[\frac{\tilde{h}}{\mathcal{L}^3} \delta' + \frac{2\tilde{h}^2 - l^2}{2\mathcal{L}^5} \delta'^2 + \frac{\tilde{h}(2\tilde{h}^2 - 3l^2)}{2\mathcal{L}^7} \delta'^3 + \frac{8\tilde{h}^4 - 24\tilde{h}^2 l^2 + 3l^4}{8\mathcal{L}^9} \delta'^4 \right] ds \quad (6.22)$$

Expand the integrand in the integral formula (6.19) of the residual terrain effect on vertical deflection to order 3 near $z = 0$:

$$\begin{aligned} \int_R^{R+\delta'} \frac{\partial L^{-1}(r, \psi, r')}{\partial \psi} dr' &= -\frac{r^2 \sin \psi}{\mathcal{L}^3} \delta' - \frac{3\tilde{h} r^2 \sin \psi}{2\mathcal{L}^5} \delta'^2 - \left[\frac{r^2 \sin \psi}{3\mathcal{L}^5} + \frac{5r^2 \sin \psi (2\tilde{h}^2 - l^2)}{6\mathcal{L}^7} \right] \delta'^3 \end{aligned} \quad (6.23)$$

Expand the integrand in Eq. (6.20) to order 4 near $z=0$, we have:

$$T_{rr}^{rtm} = G \iint_S \beta' \left[\frac{2\tilde{h}^2 - l^2}{\mathcal{L}^5} \delta' + \frac{3\tilde{h}(2\tilde{h}^2 - 3l^2)}{2\mathcal{L}^7} \delta'^2 + \frac{8\tilde{h}^4 - 24\tilde{h}^2 l^2 + 3l^4}{2\mathcal{L}^9} \delta'^3 \right] ds \quad (6.24)$$

7.6.3 Spherical harmonic analysis and synthesis of land-sea terrain masses

(1) The terrain areal density $q(\varphi, \lambda)$ of any point $P(R, \varphi, \lambda)$ on the land-sea surface can be expressed as:

$$q(\varphi, \lambda) = \beta h = R \sum_{n=1}^{\infty} \sum_{m=0}^n [A_{nm} \cos m\lambda + B_{nm} \sin m\lambda] \bar{P}_{nm}(\sin \varphi) \quad (6.25)$$

where R is the mean radius of the Earth (PAGrav4.5 replaces R with the semimajor axis

a of the Earth to facilitate the combination with the geopotential model), and A_{nm}, B_{nm} are the degree n order m normalized terrain mass spherical harmonic coefficients.

In formula (6.25), when P is on the land terrain surface, h is the terrain height ($h > 0$), β is the terrain density, $\beta = \rho_0 = 2.67 \times 10^3 \text{kg/m}^3$, while when P is on the sea surface, h is the seafloor depth ($h < 0$), β is the compensation density of seawater, equal to the difference between terrain density ρ_0 and seawater density ρ_w , $\beta = \rho_0 - \rho_w = 1.64 \times 10^3 \text{kg/m}^3$.

(2) The land-sea complete Bouguer effect on the gravitational potential at the point (r, θ, λ) outside geoid in spherical coordinates can be expressed by the spherical harmonic series of global land-sea terrain masses as:

$$V^{tbg}(r, \theta, \lambda) = \frac{3GM}{r\rho_e} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (A_{nm} \cos m\lambda + B_{nm} \sin m\lambda) \bar{P}_{nm}(\cos \theta) \quad (6.26)$$

where $\rho_e = 5.517 \times 10^3 \text{kg/m}^3$ is the mean density of the Earth.

(3) The land-sea residual terrain effect on the gravitational potential at the point (r, θ, λ) outside geoid in spherical coordinates can be expressed by the spherical harmonic series of global land-sea terrain masses as:

$$V^{rtm}(r, \theta, \lambda) = \frac{3GM}{r\rho_e} \sum_{n=N}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (A_{nm} \cos m\lambda + B_{nm} \sin m\lambda) \bar{P}_{nm}(\cos \theta) \quad (6.27)$$

where N is the minimum degree of the residual terrain model.

(4) The relationship between the normalized terrain masses spherical harmonic coefficient and the normalized terrain Stokes coefficient:

$$\bar{C}_{nm}^t = \frac{3}{\rho_e} \frac{1}{2n+1} A_{nm}, \quad \bar{S}_{nm}^t = \frac{3}{\rho_e} \frac{1}{2n+1} B_{nm} \quad (6.28)$$

The terrain masses between the surface and the geoid, and the seawater compensation masses between the sea surface and the seafloor together generate the terrain gravitational field, that is, the complete Bouguer effect. The terrain Stokes coefficients $\bar{C}_{nm}^t, \bar{S}_{nm}^t$ are the normalized spherical harmonic expansion coefficients of the terrain gravitational field., that is, the complete Bouguer effects on the Stokes coefficients of global geopotential.

7.7 Local terrain compensation and terrain Helmert condensation

7.7.1 Terrain Helmert condensation effects on various gravity field elements outside geoid

The Helmert condensation of terrestrial terrain involves a concept called as terrain masses compensation, or terrain compensation for short. Terrain compensation effect on any type of gravity field element outside the geoid is defined as the amount of mass compensation for this type of gravity field element to offset the change in the Earth's gravitational field caused by the removal of terrain mass.

The Helmert condensation process of the terrain masses can be decomposed into two steps: the first step is to deduct the gravitational field generated by the terrain masses, that is, to subtract the effect of the terrain, and the second step is to

compensate for the change of the gravitational field caused by the deduction of the terrain masses, that is, to add the terrain compensate effect.

For any type of gravity field element α outside the geoid, the change of the field element caused by terrain Helmert condensation is called as the terrain Helmert condensation effect on this type of field element, which can be uniformly expressed as:

$$\alpha^h = \alpha^t - \alpha^c \quad (7.1)$$

where, α^h is the terrain Helmert condensation effect on the gravity field element α , α^t is the complete Bouguer effect on α , and α^c is the terrain compensation effect on α .

The residual terrain effect is equal to the difference between the high-resolution and low-resolution complete Bouguer effect, and similarly, the terrain Helmert condensation effect is equal to the difference between the complete Bouguer effect and the terrain compensation effect.

The space outside the geoid after terrain Helmert condensation is called as the Helmert space, and the corresponding gravitational field is called as the Helmert gravitational field, which is harmonious with one difference from the actual Earth's gravitational field due to terrain Helmert condensation.

7.7.2 Algorithm formulas of terrain compensation and Helmert condensation effect

The following presents the spherical approximation algorithms for terrain compensation effects on various gravity field elements in the near-Earth harmonic space outside the geoid.

(1) The terrain compensation effect on disturbing potential

$$T^c = T^B + T^{cR} = T^B + G \iint_S \frac{\mu' - \mu}{L} ds \quad (7.2)$$

where, T^{cR} is called as the local terrain compensation effect on disturbing potential, ds is the move areal element on the unit sphere, μ is called as the terrain compensation density, and under the spherical approximation:

$$\mu = \rho_0 h \left(1 + \frac{h}{R} + \frac{h^2}{3R^2} \right) \quad (7.3)$$

where, h is the terrain height directly below the calculation point and ρ_0 is the terrain density.

The geocentric distance is replaced by the mean geocentric distance of the calculation surface and the terrain surface, respectively, then the local terrain compensation effect integral of the second term on the right side of (7.2) can be directly calculated by the FFT algorithm.

When the calculation point is the same as the move point, the integral of compensation effect on disturbing potential is singular:

$$T^{cR}|_0 = \frac{R^2}{6F^2} G A_0 \sqrt{A_0/\pi} (\mu_{xx} + \mu_{yy}) \quad (7.4)$$

where μ_{xx}, μ_{yy} are the second-order partial derivatives of the terrain compensation

density at the calculation point in the north direction x and the east direction y .

(2) The terrain compensation effect on gravity disturbance

$$\delta g^c = \delta g^B + \delta g^{cR} = \delta g^B + G \iint_S (\mu' - \mu) \frac{r-r't}{L^3} ds \quad (7.5)$$

where, δg^{cR} is called as the local terrain compensation effect on gravity disturbance.

When the calculation point is the same as the move point, the integral of compensation effect on gravity disturbance is singular:

$$\delta g^{cR}|_0 = \frac{R^2}{12\tilde{r}^3} GA_0 \sqrt{A_0/\pi} (\mu_{xx} + \mu_{yy}) \quad (7.6)$$

(3) Calculation of the terrain Helmert condensation effect

Combining formulas (7.2) and (7.5), the terrain Helmert condensation effects on various gravity field elements outside geoid under the spherical approximation can be expressed as:

$$\alpha^h = \alpha^t - \alpha^c = (\alpha^B + \alpha^R) - (\alpha^B + \alpha^{cR}) = \alpha^R - \alpha^{cR} \quad (7.7)$$

In formula (7.7), the spherical shell Bouguer effect α^B cancels each other, so the terrain Helmert condensation effect is also equal to the difference between the local terrain effect α^R and the local terrain compensation effect α^{cR} .

Substituting formulas (5.5) and (7.4), and formulas (5.7) and (7.5) into formula (7.7), respectively, the terrain Helmert condensation effects on the disturbing potential and gravity disturbance can be obtained, and then terrain Helmert condensation effects on various other types of gravity field elements can be obtained.

(4) Fast algorithm of local terrain compensation effect on gravity disturbance

Using the local spherical polar coordinate system, let the z -axis be the radial direction (the zenith direction), in this case, $dr = d\tilde{h}$, we have

$$\begin{aligned} \delta g^{cR} &= -G \iint_S (\mu' - \mu) \frac{\partial}{\partial \tilde{h}} \frac{1}{L} ds = G \iint_S (\mu' - \mu) \frac{\tilde{h}}{L^3} - \frac{\mu' - \mu}{L^3} (h' - h) ds \\ &= G \iint_S (\mu' - \mu) \frac{\tilde{h}}{L^3} ds - G \iint_S \frac{\mu' h'}{L^3} \frac{t}{L} ds \\ &\quad + G \iint_S \frac{\mu' h}{L^3} ds + G \iint_S \frac{\mu h'}{L^3} ds - G \iint_S \frac{\mu h}{L^3} ds \end{aligned} \quad (7.8)$$

Each item on the right side of the formula (7.8) can be quickly calculated using the FFT algorithm.

(5) Fast algorithm of local terrain compensation effect on vertical deflection

Considering $\frac{\partial L^{-1}(r, \psi, r')}{\partial \psi} = \frac{rr' \sin \psi}{L^3}$, $\frac{\partial \psi}{\partial \varphi} = -\cos \alpha$, $\frac{\partial \psi}{\partial \lambda} = -\cos \varphi \sin \alpha$, we have

$$\begin{aligned} \xi^{cR} &= -\frac{\partial T^{cR}}{\gamma r \partial \varphi} = -\frac{\partial T^{cR}}{\gamma r \partial \psi} \frac{\partial \psi}{\partial \varphi} = \frac{\partial T^{cR}}{\gamma r \partial \psi} \cos \alpha = \frac{G}{\gamma r} \iint_S (\mu' - \mu) \frac{\partial L^{-1}(r, \psi, r')}{\partial \psi} \cos \alpha ds \\ &= \frac{G}{\gamma} \int_S (\mu' - \mu) \frac{r' \sin \psi}{L^3} \cos \alpha ds \end{aligned} \quad (7.9)$$

$$\begin{aligned} \eta^{cR} &= -\frac{\partial T^{cR}}{\gamma r \cos \varphi \partial \psi} \frac{\partial \psi}{\partial \lambda} = \frac{\partial T^{cR}}{\gamma r \partial \psi} \sin \alpha = \frac{G}{\gamma r} \iint_S \frac{\partial L^{-1}(r, \psi, r')}{\partial \psi} (\mu' - \mu) \sin \alpha ds \\ &= \frac{G}{\gamma} \iint_S (\mu' - \mu) \frac{r' \sin \psi}{L^3} \sin \alpha ds \end{aligned} \quad (7.10)$$

(6) Fast algorithm of local terrain compensation effect on disturbing gravity gradient

$$\begin{aligned} T_{rr}^{cR} &= \frac{\partial^2}{\partial r^2} T^{cR} = G \iint_S (\mu' - \mu) \frac{\partial^2}{\partial r^2} \left(\frac{1}{L} \right) ds \\ &= G \iint_S (\mu' - \mu) \left(3 \frac{r-r' \cos \psi}{L^5} - \frac{1}{L^3} \right) ds \end{aligned} \quad (7.11)$$

7.8 Land-sea unified classic Bouguer and equilibrium effects

7.8.1 The classical reduction method for land Bouguer gravity anomaly

In Stokes theory, the Bouguer gravity anomaly is defined on the geoid, which is equal to the gravity anomaly on the geoid minus the effect of all terrain masses outside the geoid on the gravity at the ground point. The classical algorithm for the Bouguer gravity anomaly on the geoid is:

$$\Delta g_B = \Delta g - g^R - 2\pi G \rho h \quad (8.1)$$

where Δg is the gravity anomaly on the geoid, $-g^R$ is the classic plane terrain correction as well as g^R is equal to the plane approximation of the local terrain effect in PAGravf4.5, and $-2\pi G \rho h$ is called as the layer correction as well as $2\pi G \rho h$ is equal to the plane approximation of spherical shell Bouguer effect in PAGravf4.5.

In terrestrial mountainous area, the layer correction $-2\pi G \rho h$ is much less than zero, so the Bouguer gravity anomaly is generally less than zero.

Since the gravity observed point is generally not on the geoid, it is necessary to make downward continuation of the observed gravity from the observed point to the geoid to obtain the gravity anomaly Δg on the geoid, and then according to the algorithm formula (8.1) to calculate the classical Bouguer gravity anomaly.

In the classical gravity reduction process, the observed gravity is made downward continuation to the geoid using the space correction $-0.3086h + O(h^2)$ (mGal), which only considers the normal gravity gradient. While the actual situation is that even in hilly area with the terrain altitude of several hundred meters, the contribution of disturbing gravity gradient may reach or exceed the mGal level.

Considering that the height of the observed point can be easily and accurately measured at present, and the normal gravity at the observed point can be strictly calculated, PAGravf4.5 will not continue to employ the concept of space correction.

PAGravf4.5 firstly calculates the gravity anomaly at the observed point from the observed gravity and height, in which, the normal gravity calculated using the analytical formula (see Section 2.3.1). Then, the rigorous method is employed to obtain the analytical continuation value of the gravity anomaly from the observed point to the geoid.

From an ultra-high degree geopotential model, the difference between the model gravity anomaly at the observed point and on the geoid can be directly the analytical continuation value within the altitude of 1000m. Which is equivalent to removing the model gravity anomaly at observed point firstly and then restoring model gravity anomaly on the geoid. In mountainous area, the analytical continuation value can be furtherly improved by using the radial gradient continuation of residual gravity anomaly

(see Section 2.5).

From the observed gravity and height, PAgavf4.5 calculate the classic Bouguer gravity anomaly on geoid according to the general formula in following:

$$\Delta g_B = \Delta g^s - g^R - 2\pi G\rho h - \Delta g^c \quad (8.2)$$

where Δg^s is the gravity anomaly at the observed point (see section 2.3 for the calculation method), and Δg^c is the analytical continuation value.

Since the object affected by terrain is gravity itself, it has nothing to do with the normal gravity. Therefore, the algorithm formula for the Bouguer gravity disturbance on the geoid is:

$$\delta g_B = \delta g^s - g^R - 2\pi G\rho h - \delta g^c \quad (8.3)$$

where δg^s is the gravity disturbance at the observed point (see section 2.3 for the calculation method), and δg^c is the analytical continuation value, which is almost equal to Δg^c .

In formula (8.2) or (8.3), the observed points can be on the ground or in near-Earth space outside the ground.

It should be emphasized that no matter whether the observed point is on the ground or in the near-Earth space (such as aviation altitude), the classical Bouguer gravity anomaly and classical Bouguer gravity disturbance can only be defined on the geoid, and the terrain effect can only be the effect of terrain masses on the ground gravity. g^R in the formula (8.2) or (8.3) can only be the local terrain effect on the ground gravity, even if the observed point is at the altitude of the air.

7.8.2 Calculation of land-sea unified Bouguer gravity anomaly

The existence of terrestrial terrain makes the space outside the geoid exist mass, which need to be removed, resulting in the land complete Bouguer effect. In the marine area, the density of seawater below the sea level (geoid) is less than the terrain density, and the mass loss of seawater layer need to be compensated, resulting in the seawater complete Bouguer effect.

Referring to Section 7.5.4 for the calculation method of the seawater complete Bouguer effect on gravity (gravity anomaly or gravity disturbance), the exact integral formula is:

$$g_b^w = \frac{G\beta}{r} \iint_S \left[\frac{r_0}{\mathcal{L}} - \frac{r_0+d}{\sqrt{(\tilde{h}-d)^2+l^2}} \right] ds \quad (8.4)$$

there $d < 0$ is the seafloor depth, β is the seawater compensation density (the difference between the terrain density ρ_0 and the seawater density ρ_w), and $\rho_w = 1.64 \times 10^3 \text{ kg/m}^3$, \tilde{h} is the height of the calculation point relative to the sea surface, r_0 is the geocentric distance of the sea surface directly below the calculation point, ds is the move areal element on sea surface, \mathcal{L} is the space distance from the move areal element to the calculation point, and l is the spherical distance between the projection

point of the calculation point on the sea surface and the move areal element ds .

Since the local terrain effect g^R in (8.1) and the seawater complete Bouguer effect g_b^w in (8.4) are both integral values of a certain range of areas, the offshore ocean gravity is affected by land terrain, and the coastal land gravity is affected by seawater, so both are not zero. It is necessary the land-sea unified Bouguer effect algorithm in the coastal zone.

The height of the sea level is equal to zero, so if the integral range of the local terrain effect includes the sea area, the contribution of the sea area to the local terrain effect is equal to zero. Similarly, the seafloor depth on land is equal to zero, so if the integral range of the seawater complete Bouguer effect includes land area, the contribution of land area to the seawater Bouguer effect is also zero. It is easy to find that the local terrain effect and seawater complete Bouguer effect are completely separated and seamlessly spliced in the integral domain. Therefore, the two integral formulas can be directly added to obtain the calculation formula for the land-sea unified Bouguer gravity anomaly and Bouguer gravity disturbance:

$$\Delta g_B = \Delta g^s - g^R - 2\pi G\rho h - g_b^w - \Delta g^c \quad (8.5)$$

$$\delta g_B = \delta g^s - g^R - 2\pi G\rho h - g_b^w - \delta g^c \quad (8.6)$$

$$\text{Let } g^B = g^R + 2\pi G\rho h + g_b^w \quad (8.7)$$

In PAGravf 4.5, g^B is called as the classical Bouguer effect (see Section 3.5). It is not difficult to see that the classical Bouguer effect g^B for gravity anomaly or gravity disturbance are unified and need not be distinguished.

7.8.3 Airy-Heiskanen terrain equilibrium effect on land

The Bouguer gravity anomaly usually has a large negative value in mountainous areas, and people therefore associate the ‘excess’ material with irregular undulating mountains on the crust, which may be compensated by the corresponding loss material in the magma layer below.

Let the depth from the sea level (geoid) to the magma level be the compensation depth D . The Airy-Heiskanen model believes that the lower crust is a magma layer with a density of $\rho_1=3.27\times10^3\text{kg/m}^3$, and a mountain floats above the magma layer with a density of the crustal density $\rho_0=2.67\times10^3\text{kg/m}^3$. The part of the mountain body above the sea level is the visible terrain. The higher the mountain body is, the deeper the part that sinks into the magma (called as the mountain root), and the mountain body and the mountain root are approximately symmetrical to the magma surface. A density difference $\Delta\rho_1 = \rho_1 - \rho_0=0.6\times10^3\text{kg/m}^3$ is formed between the mountain body and the mountain root, which is the local density deficit in the magma layer.

Suppose that the surplus material of the terrain is filled into the depleted part below it and compensated. The compensation density is exactly equal to the depletion density $\Delta\rho_1=0.6\times10^3\text{kg/m}^3$, and the compensation density make the gravity increase. The gravity value change caused by the compensation is the terrain equilibrium effect.

Let the terrain height be h and the mountain root depth be b , it can be known from the floating static equilibrium condition

$$b\Delta\rho_1 = \rho_0 h \Rightarrow b = \frac{\rho_0}{\Delta\rho_1} h = 4.45h \quad (8.8)$$

Let the z -axis be the direction of the plumb line, then the terrain equilibrium effect is equal to

$$g_I = -G\Delta\rho_1 \iint_{\sigma} \int_D^{D+b} \frac{z-z'}{L^3} dz d\sigma \quad (8.9)$$

7.8.4 Calculation of land-sea unified equilibrium gravity anomaly

The ocean has a layer of low-density seawater $\rho_w=1.03\times 10^3\text{kg/m}^3$ and a layer of oceanic crust with a density equal to $\rho_0=2.67\times 10^3\text{kg/m}^3$. The self-weight of the two layers of material is less than the buoyancy of the magma, so it need supplement the material to achieve static balance, which leads to the magma material upwelling to the ocean area, the formation of mountain anti-root.

The compensation $\beta = \rho_0 - \rho_w = 1.64\times 10^3\text{kg/m}^3$ for the density deficit of the seawater layer, which produces the seawater complete Bouguer effect, has been expressed by formula (8.4). After the seawater density compensated, the static equilibrium condition of the ocean mountain anti-root becomes:

$$b'\Delta\rho_1 = \beta d \Rightarrow b' = \frac{\beta}{\Delta\rho_1} d = 2.73d \quad (8.10)$$

where d is the seafloor depth.

The mass loss of the land mountain root requires mass compensation, so the land equilibrium effect and the plane Bouguer effect are roughly inverse. On the contrary, the ocean mountain anti-root is excess mass that need be removed, the ocean equilibrium effect and the seawater Bouguer effect are also roughly inverse. The ocean equilibrium effect is equal to

$$g_I^o = -G\Delta\rho_1 \iint_{\sigma} \int_{D-b'}^D \frac{z-z'}{L^3} dz d\sigma \quad (8.11)$$

Since the terrain equilibrium effect in (8.9) and the ocean equilibrium effect in (8.11) are both integral values of a certain range of areas, the terrain equilibrium effect is not zero in the offshore ocean area, and the ocean equilibrium effect is also not zero in the coastal land area. Thence it is necessary the land-sea unified equilibrium effect algorithm in the coastal zone.

The height of the sea level is equal to zero, so if the integral range of the terrain equilibrium effect includes the sea area, the contribution of the sea area to the terrain equilibrium effect is equal to zero. Similarly, the seafloor depth on land is equal to zero, so if the integral range of the ocean equilibrium effect includes land area, the contribution of land area to the ocean equilibrium effect is also zero. It is easy to find that the terrain equilibrium effect and the ocean equilibrium effect are completely separated and seamlessly spliced in the integral domain. Therefore, the two integral formulas can be directly added to obtain the calculation formula for the land-sea unified equilibrium

gravity anomaly and equilibrium gravity disturbance:

$$\Delta g_B = \Delta g^s - g^B - g_I - g_I^o - \Delta g^c \quad (8.12)$$

$$\delta g_B = \delta g^s - g^R - g_I - g_I^o - \delta g^c \quad (8.13)$$

$$\text{Let } g^I = g_I + g_I^o \quad (8.14)$$

In PAGravf 4.5, g^I is called as the classical equilibrium effect (see Section 3.5). Similarly, the classical equilibrium effect g^I for gravity anomaly or gravity disturbance is unified and need not be distinguished.

7.8.5 Sign analysis of the land and sea Bouguer / equilibrium effect

The land layer effect is to remove the effect of terrain mass outside geoid on surface gravity, and the seawater Bouguer effect is the effect on surface gravity after seawater density compensated to terrain density. Therefore, the sign of the land layer effect is inverse to that of the seawater Bouguer effect.

The land equilibrium effect is the effect of filling the depleted part of the mountain root with excess terrain material on surface gravity, and the sign of land equilibrium effect is inverse to that of the layer effect. The ocean equilibrium effect is the effect on surface gravity after the process mass of the ocean mountain anti-root removed, and the sign of ocean equilibrium effect is inverse to that of the seawater Bouguer effect.

In PAGravf4.5, the layer effect (equal to the negative layer correction) is greater than zero, so the seawater Bouguer effect and the land equilibrium effect are less than zero, and the ocean equilibrium effect is greater than zero.

If + means greater than zero, - means less than zero, we have: the layer effect (+), sea water Bouguer effect (-), land equilibrium effect (-), and ocean equilibrium effect (+).

If described by the concept of classical terrain correction, we have: the layer correction (-), sea water Bouguer correction (+), land equilibrium correction (+), and ocean equilibrium correction (-).

7.9 Integral algorithm formula of anomalous gravity field

7.9.1 Stokes and Hotine integral formulas outside geoid

(1) It is known that the gravity anomaly Δg on an equipotential surface outside the geoid, the disturbing potential $T(r, \theta, \lambda)$ or the height anomaly $\zeta(r, \theta, \lambda)$ at the calculation point (r, θ, λ) outside the geoid can be calculated by Stokes integral Formula:

$$T(r, \theta, \lambda) = \gamma \zeta(r, \theta, \lambda) = \frac{1}{4\pi} \iint_S \Delta g' S(r, \psi, r') ds \quad (9.1)$$

where r' is the geocentric distance of the move areal element ds (the move point) on the equipotential surface where the gravity anomaly $\Delta g'$ is located, $S(r, \psi, r')$ is called as the generalized Stokes kernel function, and:

$$S(r, \psi, r') = \frac{2}{L} + \frac{1}{r} - \frac{3L}{r^2} - \frac{5r' \cos \psi}{r^2} - \frac{3r'}{r^2} \cos \psi \ln \frac{r - r' \cos \psi + L}{2r} \quad (9.2)$$

where L is the space distance from the move point to the calculation point.

When the calculation point is the same as the move point, the integral is singular:

$$\zeta|_0 = \frac{A_0}{\gamma} \Delta g_0 \quad (9.3)$$

(2) It is known that the gravity disturbance δg on an equipotential surface outside the geoid, the disturbing potential $T(r, \theta, \lambda)$ or the height anomaly $\zeta(r, \theta, \lambda)$ at the calculation point (r, θ, λ) outside the geoid can be calculated by Hotine integral Formula:

$$T(r, \theta, \lambda) = \gamma \zeta(r, \theta, \lambda) = \frac{1}{4\pi} \iint_S \delta g' H(r, \psi, r') ds \quad (9.4)$$

where $H(r, \psi, r')$ is called as the generalized Hotine kernel function, and:

$$H(r, \psi, r') = \frac{2}{L} - \frac{1}{r} - \frac{3r' \cos \psi}{r^2} - \frac{1}{r'} \ln \frac{r - r' \cos \psi + L}{r(1 - \cos \psi)} \quad (9.5)$$

When the calculation point is the same as the move point, the integral is singular:

$$\zeta|_0 = \frac{A_0}{\gamma} \delta g_0 \quad (9.6)$$

If r and r' are taken as constants, the generalized Stokes/Hotine integral can be calculated by the fast FFT algorithm.

7.9.2 Vening-Meinesz integral formulas outside geoid

Taking the horizontal derivatives on both sides of the generalized Stokes formula, we have:

$$\xi = \frac{-1}{4\pi r \gamma} \iint_S \Delta g' \frac{\partial S(r, \psi, r')}{\partial \psi} \frac{\partial \psi}{\partial \varphi} ds, \quad \eta = \frac{-1}{4\pi r \cos \varphi \gamma} \iint_S \Delta g' \frac{\partial S(r, \psi, r')}{\partial \psi} \frac{\partial \psi}{\partial \lambda} ds \quad (9.7)$$

$$\text{From } \cos \psi = \sin \varphi \sin \varphi' + \cos \varphi \cos \varphi' \cos(\lambda' - \lambda), \quad (9.8)$$

differentiating both sides, we get:

$$-\sin \psi \frac{\partial \psi}{\partial \varphi} = \cos \varphi \sin \varphi' - \sin \varphi \cos \varphi' \cos(\lambda' - \lambda) \quad (9.9)$$

$$-\sin \psi \frac{\partial \psi}{\partial \lambda} = \cos \varphi \cos \varphi' \sin(\lambda' - \lambda) \quad (9.10)$$

From the spherical trigonometry formula, we can get:

$$\sin \psi \cos \alpha = \cos \varphi \sin \varphi' - \sin \varphi \cos \varphi' \cos(\lambda' - \lambda) \quad (9.11)$$

$$\sin \psi \sin \alpha = \cos \varphi' \sin(\lambda' - \lambda) \quad (9.12)$$

Combining formulas (9.9) ~ (9.12), we can get:

$$\frac{\partial \psi}{\partial \varphi} = -\cos \alpha, \quad \frac{\partial \psi}{\partial \lambda} = -\cos \varphi \sin \alpha \quad (9.13)$$

Substitute formula (9.13) into (9.7), we have:

$$\xi = \frac{1}{4\pi r \gamma} \iint_S g' \frac{\partial S(r, \psi, r')}{\partial \psi} \cos \alpha ds, \quad \eta = \frac{1}{4\pi r \gamma} \iint_S \Delta g' \frac{\partial S(r, \psi, r')}{\partial \psi} \sin \alpha ds \quad (9.14)$$

Considering $L = \sqrt{r^2 + r'^2 - 2rr' \cos \psi}$, we have:

$$\frac{\partial}{\partial \psi} L = \frac{rr'}{L} \sin \psi, \quad \frac{\partial}{\partial \psi} \frac{1}{L} = -\frac{1}{L^2} \frac{\partial}{\partial \psi} L = -\frac{rr'}{L^3} \sin \psi \quad (9.15)$$

$$\frac{\partial}{\partial \psi} \ln \frac{r - r' \cos \psi + L}{2r} = \frac{1}{r - r' \cos \psi + L} \left(\frac{rr'}{L} \sin \psi + r' \sin \psi \right) = \frac{r' \sin \psi}{r + L - r' \cos \psi} \frac{L + r}{L} \quad (9.16)$$

$$\begin{aligned}
\frac{\partial}{\partial \psi} S(r, \psi, r') &= \frac{\partial}{\partial \psi} \left(\frac{2}{L} + \frac{1}{r} - \frac{3L}{r^2} - \frac{5r' \cos \psi}{r^2} - \frac{3r' \cos \psi}{r^2} \ln \frac{r-r' \cos \psi + L}{2r} \right) \\
&= \frac{\partial}{\partial \psi} \frac{2}{L} - \frac{3}{r^2} \frac{\partial}{\partial \psi} L + \frac{5r' \sin \psi}{r^2} + \frac{3r' \sin \psi}{r^2} \ln \frac{r+L-r' \cos \psi}{2r} - \frac{3r' \cos \psi}{r^2} \frac{\partial}{\partial \psi} \ln \frac{r+L-r' \cos \psi}{2r} \\
&= \left(-\frac{2rr'}{L^3} - \frac{3r'}{rL} + \frac{5r'}{r^2} + \frac{3r'}{r^2} \ln \frac{r-r' \cos \psi + L}{2r} - \frac{3r' \cos \psi}{r^2} \frac{r'}{r-r' \cos \psi + L} \frac{L+r}{L} \right) \sin \psi \\
&= \left[-\frac{2r}{L^3} - \frac{3}{rL} + \frac{5}{r^2} + \frac{3}{r^2} \ln \frac{r-r' \cos \psi + L}{2r} - \frac{3r'(L+r) \cos \psi}{r^2 L (r-r' \cos \psi + L)} \right] r' \sin \psi \quad (9.17)
\end{aligned}$$

In the same way, by calculating the horizontal derivatives on both sides of the generalized Hotine formula, we can get:

$$\xi = \frac{1}{4\pi r \gamma} \iint_S \delta g' \frac{\partial H(r, \psi, r')}{\partial \psi} \cos \alpha ds, \quad \eta = \frac{1}{4\pi r \gamma} \iint_S \delta g' \frac{\partial H(r, \psi, r')}{\partial \psi} \sin \alpha ds \quad (9.18)$$

Because of

$$\begin{aligned}
\frac{\partial}{\partial \psi} \ln \frac{r-r' \cos \psi + L}{r(1-\cos \psi)} &= \frac{r(1-\cos \psi)}{r-r' \cos \psi + L} \frac{\left(\frac{rr'}{L} \sin \psi + r' \sin \psi \right) r(1-\cos \psi) + (r-r' \cos \psi + L) r \sin \psi}{r^2 (1-\cos \psi)^2} \\
&= \frac{\sin \psi}{r-r' \cos \psi + L} \frac{\frac{L+r}{L} r' (1-\cos \psi) + (r-r' \cos \psi + L)}{1-\cos \psi} = \left[\frac{r'(L+r)}{(r-r' \cos \psi + L)L} + \frac{1}{1-\cos \psi} \right] \sin \psi, \quad (9.19)
\end{aligned}$$

we have:

$$\begin{aligned}
\frac{\partial}{\partial \psi} H(r, \psi, r') &= \frac{\partial}{\partial \psi} \left(\frac{2}{L} - \frac{1}{r} - \frac{3r' \cos \psi}{r^2} - \frac{1}{r'} \ln \frac{r-r' \cos \psi + L}{r(1-\cos \psi)} \right) \\
&= \frac{\partial}{\partial \psi} \frac{2}{L} + \frac{3r' \sin \psi}{r^2} - \frac{1}{r'} \frac{\partial}{\partial \psi} \ln \frac{r-r' \cos \psi + L}{r(1-\cos \psi)} \\
&= \left[-\frac{2rr'}{L^3} + \frac{3r'}{r^2} - \frac{L-r}{(r-r' \cos \psi + L)L} + \frac{1}{r'(1-\cos \psi)} \right] \sin \psi \quad (9.20)
\end{aligned}$$

Formulas (9.14) and (9.18) are also called as generalized Vening-Meinesz integral formulas, and formulas (9.17) and (9.20) are generalized Vening-Meinesz kernel functions.

Using the formula (9.14), the vertical deflection at any point outside the geoid be calculated from the gravity anomaly on a certain equipotential surface. And using the formula (9.18), the vertical deflection at any point outside the geoid be calculated from the gravity disturbance on a certain equipotential surface.

If r and r' are taken as constants, the generalized Vening-Meinesz integral formulas (9.14) and (9.18) can be calculated by the fast FFT algorithm.

7.9.3 Integral formula of inverse operation of anomalous gravity field element

(1) Calculation of the gravity disturbance by integral on the height anomaly

According to the definition of gravity disturbance, take the vertical derivative of the Poisson integral formula of disturbing potential T

$$\delta g = \frac{\partial T}{\partial n} \approx -\frac{\gamma \partial \zeta}{\partial r} = -\frac{\gamma}{2\pi} \iint_S \frac{\zeta - \zeta_p}{l^3} ds \quad (9.21)$$

where n is the vertical line direction (reverse to the radial direction r), and l is the distance between the calculation point and the move point on the sphere:

$$l = 2r \sin \frac{\psi}{2} \quad (9.22)$$

Formula (9.21) is also known as the inverse Hotine integral formula under spherical approximation.

When the calculation point is the same as the move point, the integral is singular:

$$\delta g|_0 = \frac{\gamma\sqrt{A_0/\pi}}{4}(\zeta_{xx} + \zeta_{yy}) \quad (9.23)$$

where ζ_{xx} and ζ_{yy} are the second-order horizontal partial derivatives of the height anomaly at the calculation point, and $\gamma\zeta_{xx}$ and $\gamma\zeta_{yy}$ are the northward direction of the horizontal gravity gradient and the eastward direction of the horizontal gravity gradient, respectively.

Using formula (9.21), the gravity disturbance on the equipotential surface can be calculated from the height anomaly on the surface.

Since the gravity disturbance δg is the derivative of the disturbing potential T along the vertical direction n , formula (9.21) requires that the boundary surface where the height anomaly is located should be an equipotential surface.

(2) Calculation of the gravity anomaly by integral on the height anomaly

Substitute the basic gravimetric equation into formula (9.21) to get:

$$\Delta g = -\frac{\gamma}{2\pi} \iint_s \frac{\zeta - \zeta_p}{l^3} ds - \frac{\zeta\gamma}{2r} \quad (9.24)$$

Formula (9.24) is also known as the inverse Stokes integral formula under spherical approximation.

Using formula (9.24), the gravity anomaly on the equipotential surface can be calculated from the height anomaly on the surface.

(3) Calculation of the height anomaly by integral on the vertical deflection

$$\zeta = \frac{r}{4\pi} \iint_\sigma \operatorname{ctg} g \frac{\psi}{2} (\xi \cos \alpha + \eta \sin \alpha) d\sigma \quad (9.25)$$

When the calculation point is the same as the move point, the integral is singular:

$$\zeta|_0 = \frac{A_0}{4\pi} (\xi_y + \eta_x) \quad (9.26)$$

where ξ_y and η_x are the partial derivatives of ξ and η in the east and north directions, respectively.

Using formula (9.26), the height anomaly on the equipotential surface can be calculated from the vertical deflection on the surface.

(4) Calculation of the gravity anomaly by integral on the vertical deflection

$$\Delta g = -\frac{\gamma}{4\pi} \iint_\sigma \left(3 \operatorname{csc} \psi - \operatorname{csc} \psi \operatorname{csc} \frac{\psi}{2} - \operatorname{tg} \frac{\psi}{2} \right) (\xi \cos \alpha + \eta \sin \alpha) d\sigma \quad (9.27)$$

When the calculation point is the same as the move point, the integral is singular:

$$\Delta g|_0 = -\frac{\gamma\sqrt{A_0/\pi}}{4} (\xi_y + \eta_x) \quad (9.28)$$

Using formula (9.27), the gravity anomaly on the equipotential surface can be calculated from the vertical deflection on the surface.

(5) Calculation of the gravity disturbance by integrating on the vertical deflection

From the basic gravimetric equation, and the formulas (9.25) and (9.27), the integral formula for the gravity disturbance from the vertical deflection can be obtained:

$$\delta g = \frac{-\gamma}{4\pi} \iint_{\sigma} \left(3csc\psi - csc\psi csc\frac{\psi}{2} - tg\frac{\psi}{2} - 2ctg\frac{\psi}{2} \right) (\xi \cos\alpha + \eta \sin\alpha) d\sigma \quad (9.29)$$

When the calculation point is the same as the move point, the integral is singular:

$$\delta g|_0 = -\frac{\gamma}{2\pi} \left(\sqrt{\pi A_0} + \frac{A_0}{r} \right) (\xi_y + \eta_x) \quad (9.30)$$

Using formula (9.29), the gravity disturbance on the equipotential surface can be calculated from the vertical deflection on the surface.

Formulas (9.25), (9.27) and (9.29) are also known as the inverse Vening-Meinesz integral formula under spherical approximation.

If r is taken as constant, formulas (9.21), (9.24), (9.25), (9.27) and (9.29) can all be calculated by the fast FFT algorithm.

7.9.4 Positive and negative operation formula of anomalous field element integral

(1) Poisson integral formula of anomalous field element

Any type of anomalous gravity field element μ can be expressed by the linear combination of the disturbing potential or its partial derivatives. Therefore, the radial gradient and Poisson integral formula of field element are similar.

Given the anomalous gravity field element on a certain boundary surface, the Poisson integral relation satisfied by the same type of field element at any point (r, θ, λ) outside the geoid:

$$\mu(r, \theta, \lambda) = \frac{1}{4\pi r} \iint_S \mu' \frac{r^2 - r'^2}{L^3} ds \quad (9.31)$$

When the calculation point is the same as the move point, $\psi \rightarrow 0$, $r' \rightarrow rt$, $L \rightarrow r\psi$ and $r - r't \rightarrow r\psi^2$, the integral is singular. Considering

$$ds = r'^2 \sin\psi d\psi d\alpha = \pi r^2 \psi_0^2 \quad (9.32)$$

$$\text{we have } \frac{1}{4\pi r} \iint_S \frac{r^2 - r'^2}{L^3} ds = \frac{1}{2r} \int_0^{\psi_0} r^2 \frac{\psi^2}{r^3 \psi^3} r^2 \psi d\psi = \frac{1}{2} \psi_0 = \frac{1}{2r} \sqrt{ds/\pi}, \quad (9.33)$$

$$\text{hence } \mu|_0 = \frac{\mu'}{2r} \sqrt{ds/\pi}. \quad (9.34)$$

(2) Radial gradient integral formula for anomalous field elements

Given the anomalous gravity field element on a certain equipotential surface, the radial gradient of the field element in the Stokes boundary value theory can be calculated by the following integral formula:

$$\frac{\partial \mu}{\partial r} = \frac{1}{2\pi} \iint_S \frac{\mu - \mu'}{l^3} ds \quad (9.35)$$

If r and r' are taken as constants, the integral formulas (9.31) and (9.35) can be calculated by the fast FFT algorithm.

(3) Integral positive and negative operation formula for disturbing gravity gradient

Given the disturbing gravity gradient T_{rr} on some an equipotential surface outside the geoid, the gravity disturbance $\delta g = -T_r$ at any calculation point (r, θ, λ) outside the geoid satisfies the following integral formula:

$$\delta g(r, \theta, \lambda) = \frac{1}{4\pi} \iint_S T_{rr} H(r, \psi, r') ds \quad (9.36)$$

where $H(r, \psi, r')$ is the generalized Hotine kernel function.

Given the gravity disturbance δg on a certain equipotential surface, the disturbing gravity gradient at any point on the equipotential surface can be calculated by the following integral formula:

$$T_{rr} = \frac{1}{2\pi} \iint \frac{\delta g - \delta g'}{l^3} ds \quad (9.37)$$

(4) Calculation of the disturbing gravity gradient by integral on the gravity disturbance

Given the gravity disturbance δg on a certain boundary surface, the disturbing gravity gradient T_{rr} at the any point (r, θ, λ) outside the geoid can also be calculated.

Using the Poisson integral formular (9.31) for the gravity disturbance δg , we have:

$$\delta g(r, \theta, \lambda) = \frac{1}{4\pi r} \iint_S \delta g' \frac{r^2 - r'^2}{L^3} ds \quad (9.38)$$

Considering $T_{rr} = \frac{\partial}{\partial r} \left(\frac{\partial}{\partial r} T \right) = -\frac{\partial}{\partial r} (\delta g)$, taking the partial derivatives of both sides of (9.38) with respect to r , we get:

$$T_{rr} = -\frac{1}{4\pi r} \iint_S \delta g' \frac{\partial}{\partial r} \frac{r^2 - r'^2}{L^3} ds = \frac{1}{4\pi r} \iint_S \delta g' \frac{r^3 - 5rr'^2 + (r^2 + 3r'^2)r'^2 \cos \psi}{L^5} ds \quad (9.39)$$

The formula (9.39) for calculating the disturbing gravity gradient outside the geoid from the gravity disturbance on the boundary surface is derived from the Poisson integral formula for solving the first boundary value problem. Therefore, it is not required that the boundary surface should be a gravity equipotential surface.

7.10 Theory and algorithm of gravity field approach using spherical radial basis functions

The disturbing potential $T(x)$ at the point x outside the Earth can be expressed as a linear combination of normalized surface harmonic basis functions:

$$T(x) = \frac{GM}{r} \sum_{n=2}^N \left(\frac{a}{r} \right)^n \sum_{m=-n}^n \bar{F}_{nm} \bar{Y}_{nm}(e) \quad (10.1)$$

where $x = r \cdot e = r(\sin \theta \cos \lambda, \sin \theta \sin \lambda, \cos \theta)$, r, λ, θ are the geocentric distance, longitude and colatitude of the point x outside the Earth, respectively, \bar{F}_{nm} are the fully normalized Stokes coefficients, also known as the geopotential coefficients, GM, a are the geocentric gravitational constant and equatorial radius of the Earth, respectively, called as the scale parameters, and \bar{Y}_{nm} is the normalized surface harmonic function:

$$\begin{aligned} \bar{Y}_{nm}(e) &= \bar{P}_{nm}(\cos \theta) \cos m \lambda, \quad \bar{F}_{nm} = \delta \bar{C}_{nm}, \quad m \geq 0 \\ \bar{Y}_{nm}(e) &= \bar{P}_{n|m|}(\cos \theta) \sin |m| \lambda, \quad \bar{F}_{nm} = \bar{S}_{n|m|}, \quad m < 0 \end{aligned} \quad (10.2)$$

where $\bar{P}_{nm}(\cos \theta)$ is the fully normalized associative Legendre function, n is called as the

degree of the geopotential coefficient, and m is called as order of geopotential coefficients.

The equatorial radius a of the Earth in formula (10.1) represents the geopotential coefficients defined on the sphere whose radius is equal to the equatorial radius a of the Earth. If it is replaced by the radius \mathcal{R} of the Bjerhammar sphere, the geopotential coefficients will be defined on the Bjerhammar spherical surface. In this case, the geopotential coefficient \bar{F}_{nm} is also converted into \bar{E}_{nm} due to the change of the scale parameter, and $a^n \bar{F}_{nm} = \mathcal{R}^n \bar{E}_{nm}$, the formula (10.1) becomes:

$$T(x) = \frac{GM}{r} \sum_{n=2}^N \left(\frac{\mathcal{R}}{r}\right)^n \sum_{m=-n}^n \bar{E}_{nm} \bar{Y}_{nm}(e) \quad (10.3)$$

7.10.1 Spherical radial basis function representation of external disturbing potential

The disturbing potential $T(x)$ at any point x outside the Earth can also be expressed as a linear combination of K spherical radial basis functions (SRBFs):

$$T(x) = \frac{GM}{r} \sum_{k=1}^K d_k \Phi_k(x, x_k) \quad (10.4)$$

where $x_k = \mathcal{R} \cdot e_k$ is the SRBF node defined on the Bjerhammar sphere, also known as the SRBF center or SRBF pole, d_k is the SRBF coefficient, K is the number of the SRBF nodes, equal to the number of SRBF coefficients, $\Phi_k(x, x_k)$ is the spherical radial basis function of the disturbing potential can be abbreviated as $\Phi_k(x) = \Phi_k(x, x_k)$.

The spherical radial basis function $\Phi_k(x, x_k)$ can be furtherly expanded into the Legendre series:

$$\Phi_k(x, x_k) = \sum_{n=2}^N \phi_n P_n(\psi_k) = \sum_{n=2}^N \frac{2n+1}{4\pi} B_n \left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \quad (10.5)$$

where ϕ_n is the degree n Legendre coefficient of SRBF, which characterizes the shape of SRBF and basically determines the spatial and spectral figures of SRBF, also known as the SRBF shape factor. When the spectral domain degree n need be not emphasized, B_n is also called as the Legendre coefficient of SRBF. $\mu = \mathcal{R}/r$ is also called as the bandwidth parameter because it is related to the spectral domain bandwidth of the radial basis function $\Phi_k(x)$.

Substitute the formula (10.5) into (10.4) to get:

$$\begin{aligned} T(x) &= \frac{GM}{4\pi r} \sum_{n=2}^N (2n+1) B_n \left(\frac{\mathcal{R}}{r}\right)^n \sum_{k=1}^K d_k P_n(\psi_k) \\ &= \frac{GM}{4\pi r} \sum_{k=1}^K d_k \sum_{n=2}^N (2n+1) B_n \left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \end{aligned} \quad (10.6)$$

Considering the addition theorem of spherical harmonics:

$$P_n(\psi_k) = P_n(e, e_k) = \frac{4\pi}{2n+1} \sum_{m=-n}^n \bar{Y}_{nm}(e) \bar{Y}_{nm}(e_k), \quad (10.7)$$

then the formula (10.5) can be written as

$$T(x) = \frac{GM}{r} \sum_{n=2}^N B_n \left(\frac{\mathcal{R}}{r}\right)^n \sum_{m=-n}^n \sum_{k=1}^K d_k \bar{Y}_{nm}(e) \bar{Y}_{nm}(e_k) \quad (10.8)$$

Comparing formulas (10.1), (10.3) and (10.8), we have:

$$\bar{F}_{nm} = \left(\frac{\mathcal{R}}{a}\right)^n \bar{E}_{nm} = B_n \left(\frac{\mathcal{R}}{a}\right)^n \sum_{k=1}^K d_k \bar{Y}_{nm}(e_k) \quad (10.9)$$

Using formula (10.9), the geopotential coefficient \bar{F}_{nm} can be calculated from the spherical radial basis function coefficient d_k . After that, the geopotential coefficient can be employed to calculate various anomalous gravity field elements outside the Earth.

The position, distribution and amount of the SRBF nodes (centers) $\{x_k\}$ on the Bjerhammar sphere are the key indicators for gravity field approach using spherical radial basis function, which determine the spatial degrees of freedom (spatial resolution) and spatial feature of regional gravity field, like the degree of the global geopotential coefficient model.

7.10.2 Spherical radial basis functions suitable for gravity field approach

The radial basis function employed for the gravity field approach must satisfy the Laplace equation. Some spherical radial basis kernel function such as the point mass kernel function, Poisson kernel function, radial multipole kernel function and Poisson wavelet kernel function are all harmonic.

Let x be the calculation point outside the Earth and x_k be the SRBF center on the Bjerhammar sphere $\Omega_{\mathcal{R}}$.

(1) The point mass kernel function

The point mass kernel function is an inverse multiquadric function (IMQ) proposed by Hardy (1971), which is the kernel function of the gravitational potential integral formula $V = G \iiint \frac{dm}{L}$, and its analytical expression is:

$$\Phi_{IMQ}(x, x_k) = \frac{1}{L} = \frac{1}{|x - x_k|} \quad (10.10)$$

where L is the space distance between x and x_k . Since $\Delta(1/L) = 0$, the point mass kernel function $\Phi_{IMQ}(x, x_k)$ satisfies the Laplace equation.

(2) The Poisson kernel function

The Poisson kernel function is derived from the Poisson integral formula of the anomalous gravity field element, and its analytical expression is:

$$\Phi_P(x, x_k) = -2r \frac{\partial}{\partial r} \left(\frac{1}{L} \right) - \frac{1}{L} = \frac{r^2 - r_k^2}{L^3} \quad (10.11)$$

(3) The radial multipole kernel function

The analytical expression of the radial multipole kernel function is:

$$\Phi_{RM}^m(x, x_k) = \frac{1}{m!} \left(\frac{\partial}{\partial r_k} \right)^m \frac{1}{L} \quad (10.12)$$

where m can be called as the order of the radial multipole kernel function, and the zero-order radial multipole kernel function is the point mass kernel function $\Phi_{IMQ}(x, x_k) = \Phi_{RM}^0(x, x_k)$.

(4) The Poisson wavelet kernel function

The analytical expression of the Poisson wavelet kernel function is:

$$\Phi_{PW}^m(x, x_k) = 2(\chi_{m+1} - \chi_m), \quad \chi_m = \left(r_k \frac{\partial}{\partial r_k} \right)^m \frac{1}{L} \quad (10.13)$$

The zero-order Poisson wavelet kernel function is the Poisson kernel function $\Phi_P(x, x_k) = \Phi_{PW}^0(x, x_k)$.

(5) Calculation of spherical radial basis functions

The spherical radial basis function analytical expressions (10.10) ~ (10.13) are usually expressed in the Legendre series (10.5), and then calculated according to the Legendre expansion to highlight the spectral domain figures of the anomalous gravity field element.

PAGrav4.5 normalizes the Legendre expansion of the spherical radial basis function $\Phi_k(x, x_k)$, and then calculates the spherical radial basis function (SRBF) using the normalized Legendre expansion. When dealing with different types of observed field elements, the SRBF of various field elements are uniformly divided by the normalization coefficient of disturbing potential SRBF to maintain the analytical relationship between different types of field elements.

Let the spherical angular distance $\psi_k = 0$ from x_k to x , then $\cos\psi_k = 1$, $P_n(\cos\psi_k) = P_n(1) = 1$, substitute it into formula (10.5), we have the general expression of the normalization coefficient of disturbing potential SRBF:

$$\Phi^0 = \sum_{n=2}^N \frac{2n+1}{4\pi} B_n \mu^n \quad (10.14)$$

The Legendre expansion of the normalized spherical radial basis function of disturbing potential (height anomaly) is:

$$\Phi_k(x, x_k) = \frac{1}{\Phi^0} \sum_{n=2}^N \phi_n P_n(\psi_k) = \frac{1}{\Phi^0} \sum_{n=2}^N \frac{2n+1}{4\pi} B_n \mu^n P_n(\psi_k) \quad (10.15)$$

The above four forms of disturbing potential SRBF and their corresponding Legendre coefficients are shown in Table 2, where the constant factor 4π in the Legendre coefficient B_n has been removed in advance.

SRBF	$\Phi_k(x, x_k)$	ϕ_n	B_n
Point mass kernel	$\frac{1}{L} = \frac{1}{ x-x_k }$	μ^n	$\frac{1}{2n+1}$
Poisson kernel function	$\frac{r^2 - r_k^2}{L^3}$	$(2n+1)\mu^n$	1
radial multipole kernel	$\frac{1}{m!} \left(\frac{\partial}{\partial r_k} \right)^m \frac{1}{L}$	$C_n^m \mu^{n-m} (n \geq m)$	$\frac{C_n^m}{2n+1} \mu^{-m}$
Poisson wavelet kernel	$\frac{2(\chi_{m+1} - \chi_m)}{\chi_m} = \left(r_k \frac{\partial}{\partial r_k} \right)^m \frac{1}{L}$	$(-n \ln \mu)^m (2n+1) \mu^n$	$(-n \ln \mu)^m$

7.10.3 Spherical radial basis function representation for various gravity field elements

According to the definition of the anomalous gravity field element, the spherical radial basis function parameterized form for various anomalous gravity field elements can be derived from the disturbing potential SRBF series (the rightmost expression) of (10.6).

$$\zeta(x) = \frac{T}{\gamma} = \frac{GM}{4\pi r \gamma} \sum_{k=1}^K d_k \sum_n (2n+1) B_n \left(\frac{R}{r} \right)^n P_n(\psi_k) \quad (10.16)$$

$$\delta g(x) = -\frac{\partial T}{\partial r} = \frac{GM}{4\pi r^2} \sum_{k=1}^K d_k \sum_n (2n+1)(n+1) B_n \left(\frac{R}{r} \right)^{n-1} P_n(\psi_k) \quad (10.17)$$

$$\Delta g(x) = -\frac{\partial T}{\partial r} - \frac{2T}{r} = \frac{GM}{4\pi r^2} \sum_{k=1}^K d_k \sum_n (2n+1)(n-1) B_n \left(\frac{R}{r} \right)^{n-1} P_n(\psi_k) \quad (10.18)$$

$$\xi(x) = \frac{GM}{4\pi r^2 \gamma} \sum_{k=1}^K d_k \cos \alpha_k \sum_n (2n+1) B_n \left(\frac{R}{r} \right)^n \frac{\partial P_n(\psi_k)}{\partial \psi_k} \quad (10.19)$$

$$\eta(x) = \frac{GM}{4\pi r^2 \gamma} \sum_{k=1}^K d_k \sin \alpha_k \sum_n (2n+1) B_n \left(\frac{R}{r} \right)^n \frac{\partial P_n(\psi_k)}{\partial \psi_k} \quad (10.20)$$

$$T_{rr}(x) = \frac{GM}{4\pi r^3} \sum_{k=1}^K d_k \sum_n (2n+1)(n+1)(n+2) B_n \left(\frac{R}{r} \right)^{n-1} P_n(\psi_k) \quad (10.21)$$

where α_k is the geodetic azimuth of ψ_k .

For the regional gravity field approach, the reference geopotential model (such as the EGM2008 model) is usually employed to remove the reference model value from the observed anomalous field element, and the residual gravity field is refined by the observed residual field element.

In this case, the minimum and maximum degree range in formulas (10.16) ~ (10.21) (the spectral bandwidth of the gravity field) is closely related to the selected reference geopotential model and the feature of regional gravity field in the target area, which can only be determined after verifying and analysis from actual observations.

7.10.4 Spherical Reuter grid construction and SRBF nodes design

PAGrav4.5 adopts the global and regional consistent spherical Reuter grid, constructs the spherical radial basis function SRBF centers according to the given Reuter grid level, and then using the adaptive algorithm, make the spatial distribution of SRBF nodes be consistent with the spatial distribution of observations everywhere.

(1) Unit spherical Reuter grid construction algorithm

Given the Reuter grid level Q (even number), the geocentric latitude interval $d\varphi$ of the unit spherical Reuter grid in the spherical coordinate system and the geocentric latitude φ_i of the center of the cell-grid i are respectively

$$d\varphi = \frac{\pi}{Q}, \quad \varphi_i = -\frac{\pi}{2} + \left(i - \frac{1}{2}\right) d\varphi, \quad 1 \leq i < Q \quad (10.22)$$

The cell-grid number J_i in the prime vertical circle direction at latitude φ_i , the longitude interval $d\lambda_i$ and the side length dl_i are respectively

$$J_i = \left\lceil \frac{2\pi \cos \varphi_i}{d\varphi} \right\rceil, \quad d\lambda_i = \frac{2\pi}{J_i}, \quad dl_i = d\lambda_i \cos \varphi_i \quad (10.23)$$

It is not difficult to find that $dl_i \approx d\varphi$. Let

$$\varepsilon_i = \frac{ds_i - ds}{ds} = \frac{dl_i - d\varphi}{d\varphi} = \frac{d\lambda_i}{d\varphi} \cos \varphi_i - 1 \quad (10.24)$$

where ds is the cell-grid area near the equator, ds_i is the cell-grid area at the prime vertical circle φ_i , and ε_i is the relative deviation of the parallel circle cell-grid area relative to the cell-grid area near the equator.

ε_i is generally small, about a few ten-thousandth, and the value is related to the Reuter grid level Q . Near the equator, we have $ds = d\varphi \cdot d\varphi$, $\varepsilon_{Q/2} = 0$.

Given the latitude and longitude range of the local area, you can directly determine the minimum and maximum value of i according to the formula (10.22), and then calculate the maximum J_i at each prime vertical circle according to the formula (10.23), to determine the regional Reuter grid whose level is Q without calculating the global grid.

(2) Regional SRBF nodes design with adaptive observation distribution

PAGravf4.5 presents a simple Reuter grid fitting algorithm to design the SRBF centers that adapts to the spatial distribution of observations. Firstly, construct a regional Reuter grid from the given level Q , and then count the number J of effective observations in each cell Reuter grid. When J is less than a given number (as the input parameter), eliminate the SRBF center. After traversing all cell Reuter grids, generate the SRBF center set that adapts to the spatial distribution of observations.

Obviously, when the observations data are a regular grid, the SRBF centers are also regularly distributed, and when the observations are irregularly distributed, the SRBF centers are also irregularly distributed. The denser the distribution of observations, the denser the distribution of SRBF centers. That is, the spatial distribution of SRBF centers is consistent with the spatial distribution of observations everywhere.

7.10.5 SRBF coefficients estimation and gravity field approach

After the constant $GM/(4\pi)$ removed, it does not change the analytical relationship between the anomalous gravity field elements. Therefore, formulas (10.16) ~ (10.21) are rewritten as:

$$\zeta(x) = \frac{1}{r\gamma} \sum_{k=1}^K d_k \sum_n (2n+1) B_n \mu^n P_n(\psi_k) \quad (10.25)$$

$$\delta g(x) = \frac{1}{r^2} \sum_{k=1}^K d_k \sum_n (2n+1)(n+1) B_n \mu^{n-1} P_n(\psi_k) \quad (10.26)$$

$$\Delta g(x) = \frac{1}{r^2} \sum_{k=1}^K d_k \sum_n (2n+1)(n-1) B_n \mu^{n-1} P_n(\psi_k) \quad (10.27)$$

$$\xi(x) = \frac{1}{r^2\gamma} \sum_{k=1}^K d_k \cos\alpha_k \sum_n (2n+1) B_n \mu^n \frac{\partial P_n(\psi_k)}{\partial \psi_k} \quad (10.28)$$

$$\eta(x) = \frac{1}{r^2\gamma} \sum_{k=1}^K d_k \sin\alpha_k \sum_n (2n+1) B_n \mu^n \frac{\partial P_n(\psi_k)}{\partial \psi_k} \quad (10.29)$$

$$T_{rr}(x) = \frac{1}{r^3} \sum_{k=1}^K d_k \sum_n (2n+1)(n+1)(n+2) B_n \mu^{n-1} P_n(\psi_k) \quad (10.30)$$

Substituting the Legendre coefficient B_n in Table 2 into the above equations, we can obtain the basic observation equations for regional gravity field approach with the (residual) anomalous gravity field element $F(x_i)$ as the observations and the SRBF coefficients $\{d_k\}$ as the unknowns.

$$L = \{F(x_i)\}^T = A\{d_k\}^T + e \quad (i = 1, \dots, M, k = 1, \dots, K) \quad (10.31)$$

where A is the $M \times K$ design matrix, e is the $M \times 1$ observation error vector, M is the number of observations, K is the number of RBF centers, that is, the number of unknowns $\{d_k\}$, and x_i is the position of the observations.

PAGravf4.5 proposes an algorithm that can improve the performance of parameter estimation by suppressing edge effect. When the RBF center v is located at the margin of the calculation area, its corresponding SRBF coefficient is set to zero, that is, $d_v = 0$ as the observation equation to suppress the edge effect. The normal equation with the additional suppression of edge effect constructed by PAGravf4.5 is:

$$[A^T P A + \epsilon E] \{d_k\}^T = A^T P L \quad (10.32)$$

where \mathcal{E} is a diagonal matrix, whose element is equal to 1 only when the SRBF center corresponding to its subscript is in the margin of the area, and the others are zero. ϵ is equal to the diagonal standard deviation of the matrix $A^T P A$.

In PAGravf4.5, The action distance dr of all SRBF centers is required to be equal to maintain the spatial consistency of the approach performance of gravity field. Where dr corresponds to the domain of the SRBF argument, so any observation is a linear combination of the spherical radial basis functions of the SRBF centers only within the radius dr .

PAGravf4.5 improves the ill-conditioned or singularity of $A^T P A$ by adding some observation equations that can suppresses edge effect to improve the stability and reliability of parameter estimation, to instead of the regularization of the normal equation without geophysical meaning.

You can choose the LU triangular decomposition (square root method), Cholesky decomposition or unknowns smallest norm method to solve the normal equation (10.32).

7.10.6 Regional gravity field modelling from various heterogeneous observations by SRBF

It has always been a hot and difficult issue in physical geodesy to combine various types of observations to model the regional gravity field. Like the surface harmonic coefficient expansion of anomalous gravity field elements, various types of observations can be represented by spherical radial basis function expansion, such as equations (10.25) ~ (10.30). Estimating the spherical radial basis function coefficients with equations (10.25) ~ (10.30) as observation equations, we can model gravity field from various types of observations.

(1) The crucial issues of gravity field modelling using spherical radial basis function from various types of observations

The regional gravity field modelling from various heterogeneous observations by SRBF need deal with three crucial issues, namely ① The SRBF representations from various types of observations should strictly keep the analytical relationship between different types of observations. ② How to determine the contribution of different types of observations to the SRBF coefficients $\{d_k\}$. ③ Investigate the spectral center & bandwidth of target field element, observations and SRBF, and then study the relationship between the three.

For the first issue, only the SRBF Legendre expansion of height anomaly is normalized, and the SRBF Legendre expansion of other types of observations are divided all by the SRBF normalization coefficient of height anomaly. In this way, the analytical relationships can be strictly maintained between different types of observations.

The way to deal with the second issue is to construct observation equations and normal equations from different types of observations firstly, introduce some parameters related to the error or spatial distribution of observations, and then combine these

normal equations to form a new normal equation.

The third issue is related to the observation situations, the nature of gravity field and the modelling algorithm. The spectral center and bandwidth of the observations, target field elements and SRBFs need be comprehensively analyzed in different parameter combinations case, and then according to the principle of fully resolving the spectrum of the target field elements, optimize the relevant scheme and parameters.

(2) Observation contribution adjustment, edge effect suppression and Parameter estimation

PAGravf4.5 recommends a universal multi-source heterogeneous observation deep fusion method by the normalization of the normal equations and adjust the contribution of the given type of observations at the same time. The normal equation is:

$$\sum_i^{i \neq j} \left(\frac{1}{\varepsilon_i} A_i^T P_i A_i \right) + \frac{\kappa^2}{\varepsilon_j} A_j^T P_j A_j + \varepsilon \{d_k\}^T = \sum_i^{i \neq j} \left(\frac{1}{\varepsilon_i} A_i^T P_i L_i \right) + \frac{\kappa^2}{\varepsilon_j} A_j^T P_j L_j \quad (10.33)$$

where ε_j is the combination parameter for the given type of adjustable observations, ε_i is the combination parameter of the i ($i \neq j$) observations, and κ is the contribution rate of the adjustable observations j .

PAGravf4.5 multiplies the normal equation coefficient matrix $A_j^T P_j A_j$ and constant matrix $A_j^T P_j L_j$ of the adjustable observation j by κ^2 respectively, to increase ($\kappa > 1$) or decrease ($\kappa < 1$) the contribution of the adjustable observation.

For example, the GNSS-levelling residual height anomaly in the observations can be set as the adjustable observations with the contribution rate $\kappa > 1$ to improve the analytical fusion of GNSS-levelling and other observations. For another example, let the nearshore altimetric elements adjustable with the contribution rate $\kappa < 1$, we can suppress the influence of the shallow water altimetric errors and improve the separation of sea surface topography.

The above normalization method of the normal equations can effectively control the deep fusion of different types of observations using covariance structure to approach gravity field from heterogeneous observation field elements. This method completely separates the contribution of the observation system model (covariance structure) and influences of observation quality (errors or gross errors), so that the fusions are away from the observation errors (gross error), observation types and spatial distribution differences of measurement points. Which is conducive to the fusion of multiple types of observation field elements with extreme differences in spatial distribution (such as very few astronomical vertical deflection or GNSS levelling data included), and is conducive to the exact detection of observation gross errors.

In this case, the normal equation does not also need to be iteratively calculated, which conducive to improve the analytical nature of the SRBF approach algorithm.

(3) The cumulative SRBF approach method to achieve the best approach of the gravity field

The target field elements are equal to the convolution of the observations and the

filter SRBFs. When the target field elements and the observations are of different types, it is difficult for one SRBF to match the spectral center and bandwidth of the observations and target field elements at the same time, which would make the spectral leakage of the target field elements. In addition, the SRBF type, the minimum and maximum degree of Legendre expansion and the SRBF center distribution also all affect the approach performance of gravity field. Therefore, only the optimal estimation of the SRBF coefficient with the burial depth as the parameter is not enough to ensure the best approach of gravity field.

PAGrav4.5 proposes a cumulative SRBF approach scheme according to the linear additivity of gravity field to solve the key troubles mentioned. Using the multiple cumulative SRBF approach, it is not necessary to determine the optimal burial depth.

When each SRBF approach of gravity field employs a SRBF with different spectral figure, the cumulative SRBF approach can fully resolve the spectral domain signal of the target field elements by combining multiple SRBF spectral centers and bandwidths, and then optimally restore the target field elements in space domain.

The character of cumulative SRBF approach scheme of gravity field: the essence of each SRBF approach is to employ the previous approach results as the reference field, and then refine the residual target field elements by remove -restore scheme.

The validity principle of once SRBF approach: (1) The residual target field element grid is continuous and differentiable (view the drawing), and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

The typical technical features of SRBF approach program in PAGrav4.5

- ① The analytical function relationships between gravity field elements are strict, and the SRBF approach performance has nothing to do with the observation errors.
- ② Various heterogeneous observations in the different altitudes, cross-distribution, and land-sea coexisting cases can be directly employed to model the all-element gravity field models on or outside the geoid without reduction, continuation and gridding.
- ③ Can integrate very few astronomical vertical deflection or GNSS-levelling data, and effectively absorb the edge effect.
- ④ Has the strong capacity in the detection of observation gross errors, measurement of external accuracy indexes and control of computational performance.

7.11 Height system and its analytic relationship with gravity field

7.11.1 Relationship between the height systems and gravity field

Let the geopotential at the ground point A be W_A , the latitude and longitude of point Q is the same as that of point A, and its normal geopotential U_Q is equal to W_A , then Q is located on the telluroid at A, and QA is equal to the height anomaly ζ_A at point A where the arrow downward means $\zeta_A < 0$, as shown in Figure. In the high-altitude areas, the ground height anomaly $\zeta < 0$, and the telluroid is above the ground.

Without loss of generality, let the geopotential W_R of the zero-height surface in regional height datum, the geoidal geopotential $W_G (= U_0)$, and the global geopotential W_0 (which can be understood as the geopotential of the global height datum) are exactly equal, namely:

$$W_0 = W_G = W_R \quad (11.1)$$

In this case, the geoid is a closed surface whose geopotential is equal to the normal potential of the ellipsoidal surface, and the potential difference of the height datum is equal to zero, that is, $\delta W_R = W_0 - W_R = 0$. The difference between the geopotential W_R of the zero-height surface and the geopotential W_A of the ground point A is called as the geopotential number of the point A, namely $c_A = W_R - W_A = W_0 - W_A$.

The orthometric height is defined in the gravity field space, and it is the ratio of the geopotential number c_A of the point A to the mean gravity \bar{g}_A between the point A and the point O, and the point O is on the zero-height surface namely on the height datum surface.

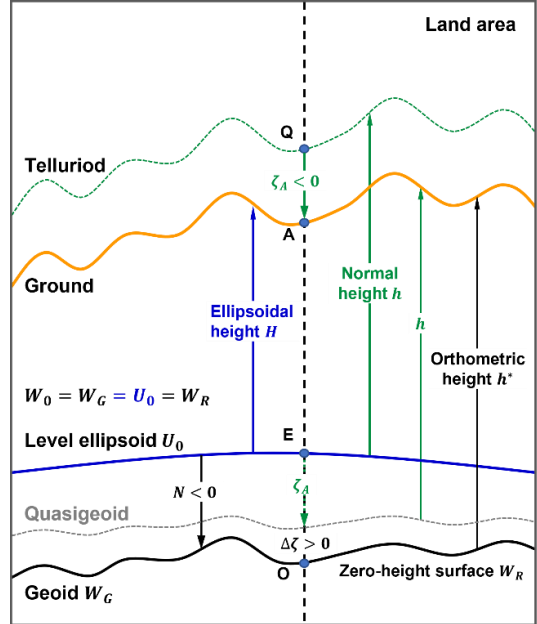
$$h_A^* = \frac{c_A}{\bar{g}_A} = \frac{W_R - W_A}{\bar{g}_A} = \frac{W_0 - W_A}{\bar{g}_A} = \frac{1}{\bar{g}_A} \int_O^A g dn \quad (11.2)$$

where dn is the line element between the ground point A and the O point on the height datum surface, and g is the gravity at the line element.

The normal height is defined in the normal gravity field space, which is the ratio of the normal potential number ($= U_0 - U_Q$) of the telluroid Q at the ground point A to the mean normal gravity $\bar{\gamma}_Q$ between the Q point on the telluroid and the Q point on the normal ellipsoid surface:

$$h_A = \frac{U_0 - U_Q}{\bar{\gamma}_Q} = \frac{1}{\bar{\gamma}_Q} \int_E^Q \gamma dN \quad (11.3)$$

where dN is the line element between the point E and the Q, and γ is the normal gravity at the line element.



The Molodensky condition assumes that the normal geopotential of the point Q located on the telluroid at point A is equal to the geopotential number of point A, that is, $U_0 - U_Q = c_A = W_0 - W_A$, and substituting it into the formula (11.3), then the normal height of Molodensky is:

$$h_A = \frac{U_0 - U_Q}{\bar{\gamma}_Q} = \frac{W_0 - W_A}{\bar{\gamma}_Q} = \frac{c_A}{\bar{\gamma}_Q} = \frac{1}{\bar{\gamma}_Q} \int_0^A g dn \quad (11.4)$$

Equation (11.4) is the normal height definition formula adopted by the current Chinese height system.

If the orthometric height of point A is equal to zero $h_A^* = 0$, the geopotential of point A is equal to zero $c_A = 0$ from the orthometric height definition (11.2). Substituted into the normal height definition (11.4), the normal height is also equal to zero $h_A = 0$. In this case, the geopotential of point A is equal to the geopotential of geoid $W_A = W_G = W_0$, that is, point A is on the geoid. Thus it can be seen that the zero orthometric height surface, zero normal height surface and zero geopotential number surface coincide with the geoid. Therefore, whether it is the orthometric height system, normal height system, or geopotential number system, the height datum surface is unique, and it is the geoid.

7.11.2 The rigorous analytical relationship between orthometric and normal height systems

The solution of the Stokes boundary value problem is the disturbing potential of the geoid and in whole Earth space outside the geoid, that is, the Stokes boundary value problem simultaneously determines the geoidal height and the height anomaly outside the geoid, see the formular (9.1).

It is easy to find that the ground height anomaly is also the solution to the Stokes boundary value problem. In particular, the Stokes boundary value problem solution constrains the rigorous analytical relationship between the ground height anomaly ζ and the geoidal height N :

$$\zeta = N + \Delta\zeta = N + \int_0^{h^*} \frac{\partial \zeta}{\partial h} dh = N - \int_0^{h^*} \frac{\delta g}{\gamma} dh \quad (11.5)$$

where dh is the line element between the ground and the geoid, and δg and γ are the gravity disturbance (analytic gravity disturbance) and normal gravity at the line element dh , respectively.

$\Delta\zeta$ in formula (11.5) is the difference between the height anomaly and the geoidal height, that is, the difference between the orthometric height and the normal height. PAgavf4.5 program 5.1 can be employed for this calculation.

According to the basic conditions for the solution of the Stokes boundary value problem, the gravity between the ground and the geoid should be equivalent to the gravity analytically continued to this point from the gravity outside the ground, which is also called as analytical gravity g^* . Rather than the actual gravity g being affected by the terrain and surrounded by the mass, the analytical gravity g^* has a strict analytical relationship with the solution of the Stokes boundary value problem. The solution condition of the Stokes boundary value problem also requires that the actual gravity and

the analytical gravity are equal everywhere outside the ground.

The solution to the Stokes boundary value problem points out that the height anomaly ζ_o on the geoid is the geoidal height N . So it is easy to find that the zero normal height surface and the zero orthometric height surface coincide everywhere, that is the geoid, whose geopotential are equal.

In high-altitude areas, the determination accuracy of $\Delta\zeta = \zeta - N$ in formula (11.5) can be effectively improved from the observed gravity or by using regional gravity field data to refine the analytical gravity disturbance on the integral line element.

The mean gravity and integral line element gravity in the definition of orthometric height formula (11.2) replaced with analytical gravity, the strict orthometric height definition that satisfies the solution requirements of the Stokes boundary value problem is obtained:

$$h_A^* = \frac{1}{\bar{g}_A^*} \int_0^A g^* dn \quad (11.6)$$

The zero orthometric height surface coincides with the zero normal height surface, both of which are the geoid. Obviously, only using the analytical gravity g^* , we can ensure that the orthometric height, normal height, height anomaly, geoid and their interrelationships are analytically compatible in Stokes boundary value theory. PAGravf4.5 calls the rigorous orthometric height as the analytical orthometric height.

7.11.3 The problem of quasi-geoid as height datum surface

The geoid can be uniquely determined or continuously refined according to its geopotential value, and it can be employed as the orthometric height starting surface, which meets the requirements of the uniqueness of the geodetic datum. However, it is not theoretically rigorous to regard the quasi-geoid as the normal height starting surface.

(1) The zero normal height surface is the equipotential surface whose geopotential is equal to the geopotential at the height datum zero-point. It is the geoid, not the so-called quasi-geoid.

(2) Two points with the same latitude and longitude but different heights have different height anomalies. Therefore, if the normal height is considered to start from the quasi-geoid, there must be two different starting points in the vertical direction.

(3) In most cases, the measurement points will not be just on the specific ground elevation digital model surface employed in the quasi-geoid modelling. It is necessary to add a gradient (or gravity disturbance) correction for the height anomaly at the measurement point from the quasi-geoid model. see the section 5.1 for the calculation procedure.

PAGravf4.5 downplays the concept of quasi-geoid and does not regard so-called quasi-geoid as the starting surface of normal height. The height anomalies in PAGravf4.5 are strictly in one-to-one correspondence with their spatial locations.

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Names table of the sample directories and executable files

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1	PAGravf4.5 system parameter settings	Systemparameterset
2	Calculation of normal Earth gravity field, ellipsoid constant and W_g analysis	PrNormalgravfdcalc
3	Calculation of global geopotential model and its spectral character analysis	PrModelgravfdcalc
4	Calculation of observed anomalous field elements and error analysis of geoid	ProbsAnomousgrav
5	Correction of boundary value problem for field element on non-equipotential surface	PrBoundaryvalueAdj
6	Analytical continuation of anomalous field	PrGradcontinuation

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7	Gross error detection and basis function gridding of discrete gravity field elements	PrGerrweighgride
8	Computation of local terrain effect on various field elements outside the geoid	TerLocalterraininfl
9	Computation of land, ocean, and lake complete Bouguer effect on gravity outside geoid	TerCompleteBougure
10	Computation of terrain Helmert condensation effect on various field elements outside geoid	TerHelmertcondensat
11	Computation of residual terrain effect on various field elements outside geoid	Renterraineffect
12	Computation of land-sea unified classical gravity Bouguer / equilibrium effect	TerSurfacegravinfl
13	Ultrahigh degree spherical harmonic analysis on land-sea terrain and construction of model	TerGloharmanalysis
14	Spherical harmonic synthesis of complete Bouguer or residual terrain effects	TerHarmrntinfluence
15	Computation process demo of various terrain effects outside geoid	Terraininflexercise
16	External height anomaly computation using Stokes/Hotine integral	IntGenStokesHotine
17	External vertical deflection computation using Vening-Meinesz integral	IntGenVeningMeinesz
18	Inverse integral and integral of inverse operation from anomalous gravity field element	Integralgrainverse
19	Gradient and Poisson integral of external gravity field element	Intgendistgradient
20	Feature and performance analysis of spherical radial basis functions	SRBFperformance
21	Gravity field approach using SRBFs in spectral domain and performance test	SRBFestimateVerify
22	All-element modelling on gravity field using SRBFs from heterogeneous observations	SRBFheterogeneous
23	Modelling process exercise of regional gravity field and geoid	Gravfmdlexercise
24	Height difference correction of height anomaly	AppHgtsysdifferent

	and calculation of height system difference	
25	Construction and refinement of the equipotential surface passing through the specified point	AppEquipotentialhgt
26	Construction of the normal equiheight surface passing through the specified point	AppEquihgtpotential
27	Assessment of gravimetric geoid using GNSS-levelling data	AppGeoiderrorestim
28	GNSS-levelling data fusion and regional height datum optimization	AppGNSSlvlhgtdatum
29	GNSS replaces leveling to calculate the orthometric or normal height	AppGNSSreleveling
30	Converting of general ASCII records into PAGrav4.5 format	EdPntrecordstandard
31	Data interpolation, extracting and separation of land and sea	Edatafsimpleprocess
32	Simple and direct calculation on geodetic data files	EdFlgeodatacalculate
33	Low-pass filtering operation on geodetic grid file	EdGrdlowpassfilter
34	Simple gridding and regional geodetic grid construction	Edareageodeticdata
35	Constructing and transforming of vector grid file	EdVectorgridtransf
36	Statistical analysis on various geodetic data file	Tlstatisticanalysis
37	Calculation of grid horizontal gradient and vector grid inner product	AppGerrweighgrdate
38	Visualization of multi-attributes curves from 2D geodetic data	multicurvesplot
39	Visualization for specified attribute in discrete point record file	Viewpntdata
40	Visualization for the geodetic grid file	Viewgridata
41	Visualization for the geodetic vector grid file	Viewvectgrd