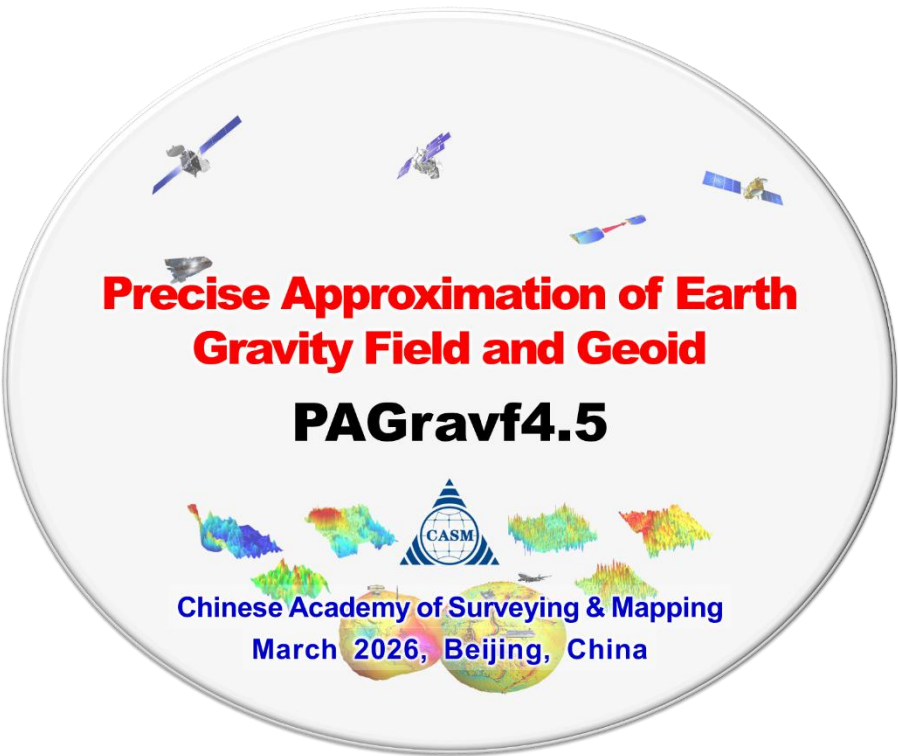
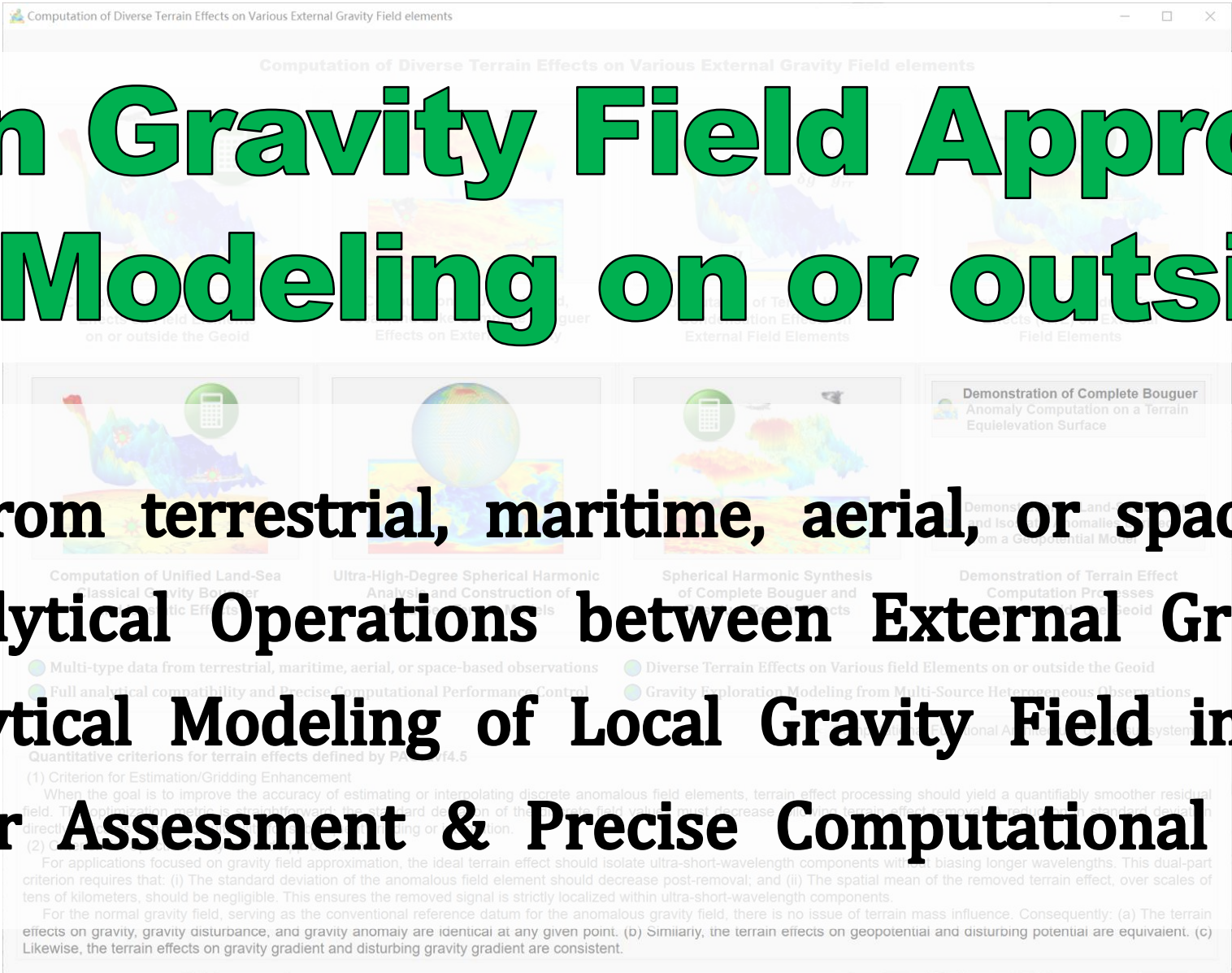


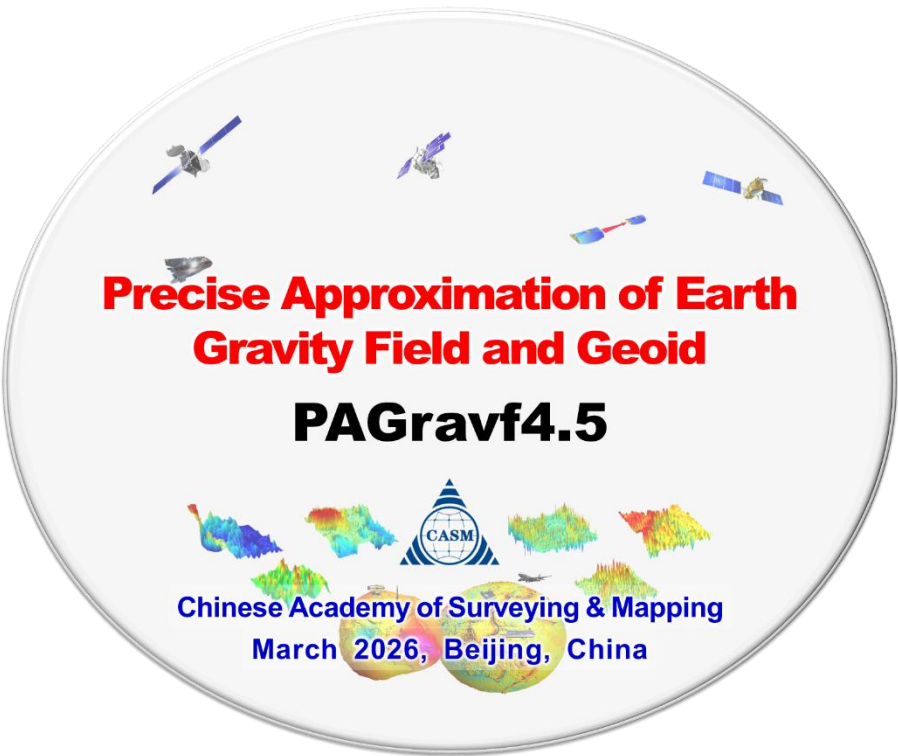


High-Precision Gravity Field Approximation and Full-Element Modeling on or outside the Geoid

- 🌐 Multi-type data from terrestrial, maritime, aerial, or space-based observations
- 🌐 Closed-Loop Analytical Operations between External Gravity Field Elements
- 🌐 All-Element Analytical Modeling of Local Gravity Field in Full External Space
- 🌐 Observation Error Assessment & Precise Computational Performance Control

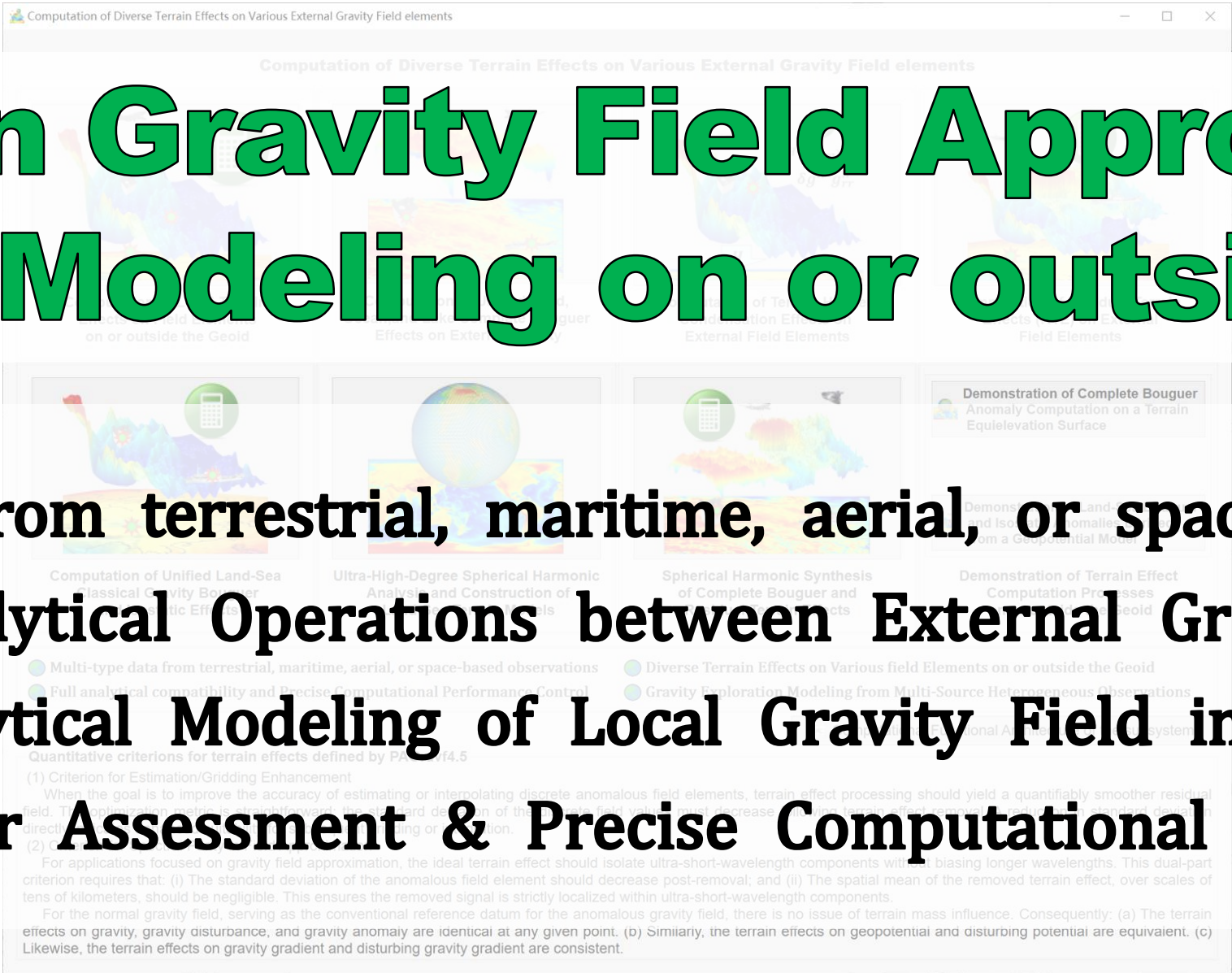


ZHANG Chuanyin, March 2026, Beijing, China

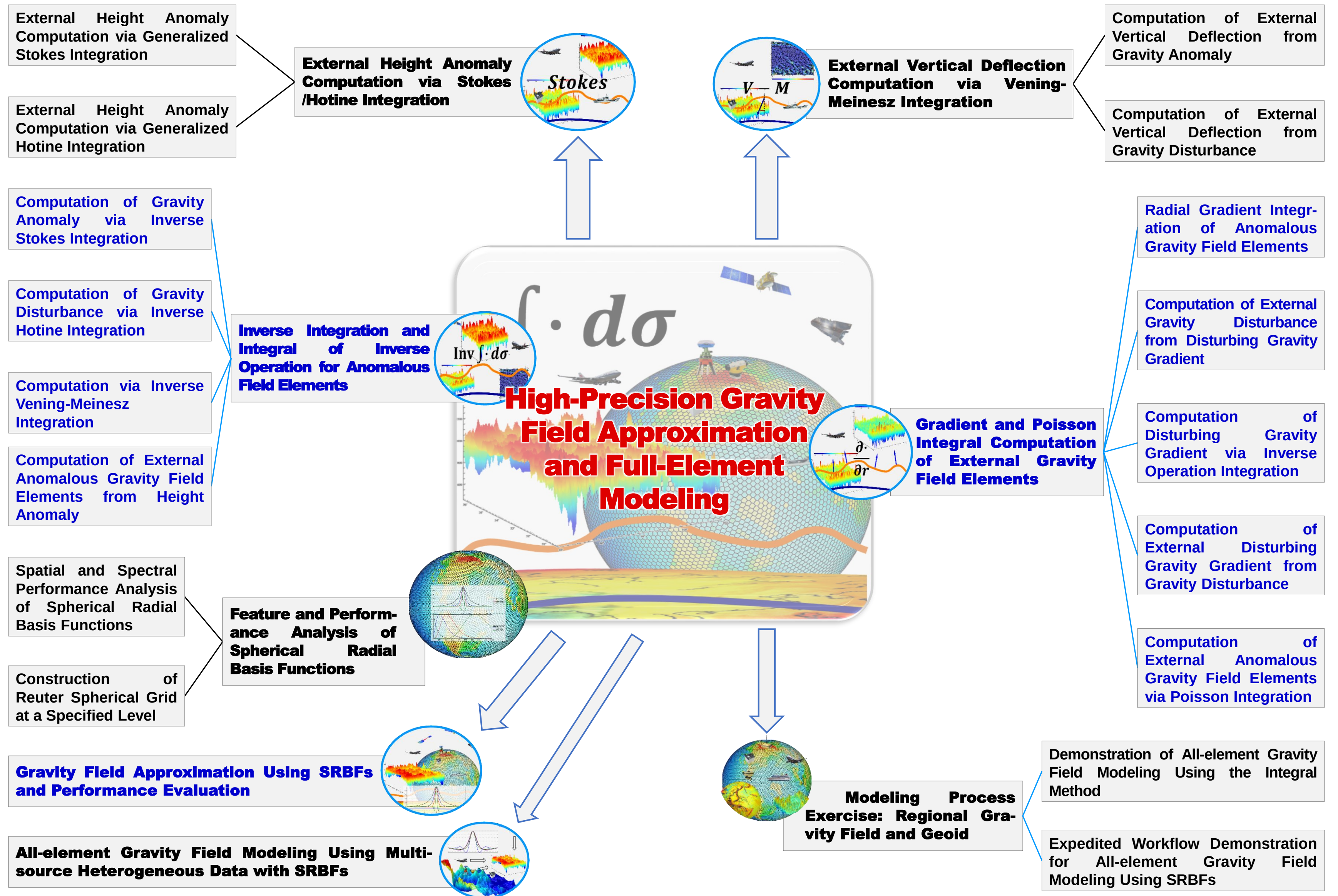


High-Precision Gravity Field Approximation and Full-Element Modeling on or outside the Geoid

- 🌐 Multi-type data from terrestrial, maritime, aerial, or space-based observations
- 🌐 Closed-Loop Analytical Operations between External Gravity Field Elements
- 🌐 All-Element Analytical Modeling of Local Gravity Field in Full External Space
- 🌐 Observation Error Assessment & Precise Computational Performance Control



ZHANG Chuanyin, March 2026, Beijing, China



External Height Anomaly Computation via Generalized Stokes Integration

External Height Anomaly Computation
via Generalized Stokes Integration

External Height Anomaly Computation
via Generalized Hotine Integration

Stokes and Hotine integral formulas

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Gravity Anomaly Grid File

Select computation file format

space computation point file

Open Space Computation Point File

Set input point file format

number of rows of file header 1

Ellipsoidal Height Column Index 4

Integral radius 180 km

Save the results as

Import setting parameters

Start Computation

>> Computation Process ** Operation Prompts

>> [Purpose] To compute height anomalies (m) on the geoid or in its outer space using generalized rigorous numerical integration or Fast Fourier Transform (FFT) algorithms based on Stokes or Hotine theory. Inputs include an ellipsoidal height grid defining an equipotential boundary surface and the corresponding gravity anomaly or gravity disturbance grid (mGal) on that surface.

>> On the geoid, the height anomaly is equivalent to the geoid undulation (geoidal/ ellipsoidal height).

>> To perform integration over a finite radius in local regions, a Remove-Restore scheme based on a reference global geopotential model is employed.

>> [Function] Computes the external residual height anomaly (m) via Stokes integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity anomaly grid (mGal) on that surface.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenStokesHotine/landgeoidhgt.dat.

>> Open Residual Gravity Anomaly Grid File C:/PAGravf4.5_win64en/examples/IntgenStokesHotine/resGMIgeoid541_1800.gra.

>> Open Space Computation Point File C:/PAGravf4.5_win64en/examples/IntgenStokesHotine/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save the results as C:/PAGravf4.5_win64en/examples/IntgenStokesHotine/rststk.txt.

** Appends a column of computed residual height anomalies to the source file, retaining 4 significant figures.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

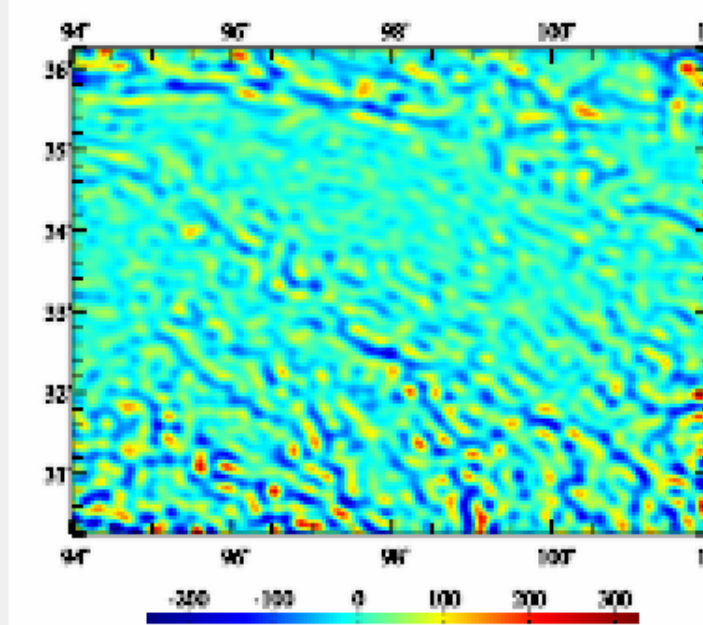
>> Computation start time: 2026-04-12 11:34:16

>> Complete the computation of external height anomaly!

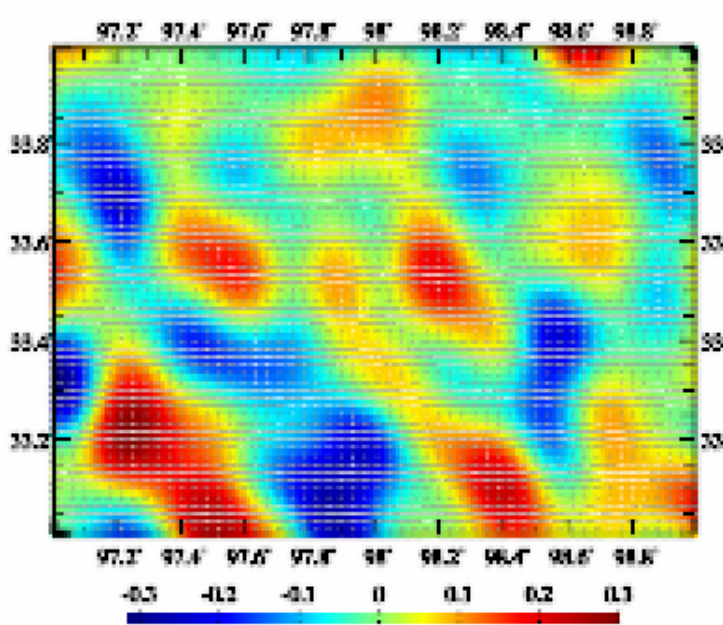
no	lon(degree/decimal)	lat	ellipHeight(m)	
1	97.008333	33.008333	3942.764	-0.0294
2	97.025000	33.008333	3989.787	-0.0340
3	97.041667	33.008333	4034.817	-0.0404
4	97.058333	33.008333	4070.847	-0.0485
5	97.075000	33.008333	4106.877	-0.0582
6	97.091667	33.008333	4119.913	-0.0693
7	97.108333	33.008333	4115.946	-0.0817
8	97.125000	33.008333	4090.977	-0.0952
9	97.141667	33.008333	4070.007	-0.1090
10	97.158333	33.008333	3991.047	-0.1235
11	97.175000	33.008333	3985.070	-0.1362
12	97.191667	33.008333	3956.107	-0.1475
13	97.208333	33.008333	3965.137	-0.1552
14	97.225000	33.008333	3964.173	-0.1592
15	97.241667	33.008333	3983.205	-0.1581
16	97.258333	33.008333	3953.251	-0.1526

Extract height anomaly

Plot



gravity anomaly (mGal)



height anomaly (m)

- Stokes Boundary Value Theory mandates that the boundary surface be an equipotential boundary surface; thus, input gravity anomalies/disturbances must reside on such a surface.
- Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

External Height Anomaly Computation via Generalized Stokes Integration – 2D FFT

External Height Anomaly Computation
via Generalized Stokes Integration

External Height Anomaly Computation
via Generalized Hotine Integration

Stokes and Hotine integral formulas

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Gravity Anomaly Grid File

Select computation file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal
Height Grid File

Select integral algorithm

2D FFT algorithm

>> Computation Process ** Operation Prompts

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 11:34:16

>> Complete the computation of external height anomaly!

>> Computation end time: 2026-04-12 11:35:43

>> [Function] Computes the external residual height anomaly (m) via Stokes integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity anomaly grid (mGal) on that surface.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landgeoidhgt.dat.

>> Open Residual Gravity Anomaly Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/resGMlgeoid541_1800.gra.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landbmsurfhgt.dat.

>> Compute external residual height anomaly by 2D FFT algorithm...

>> Save the results as C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/stokesFFT2.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 11:38:54

>> Complete the computation of external height anomaly!

>> Computation end time: 2026-04-12 11:38:55

Integral radius 180 km

Save the results as

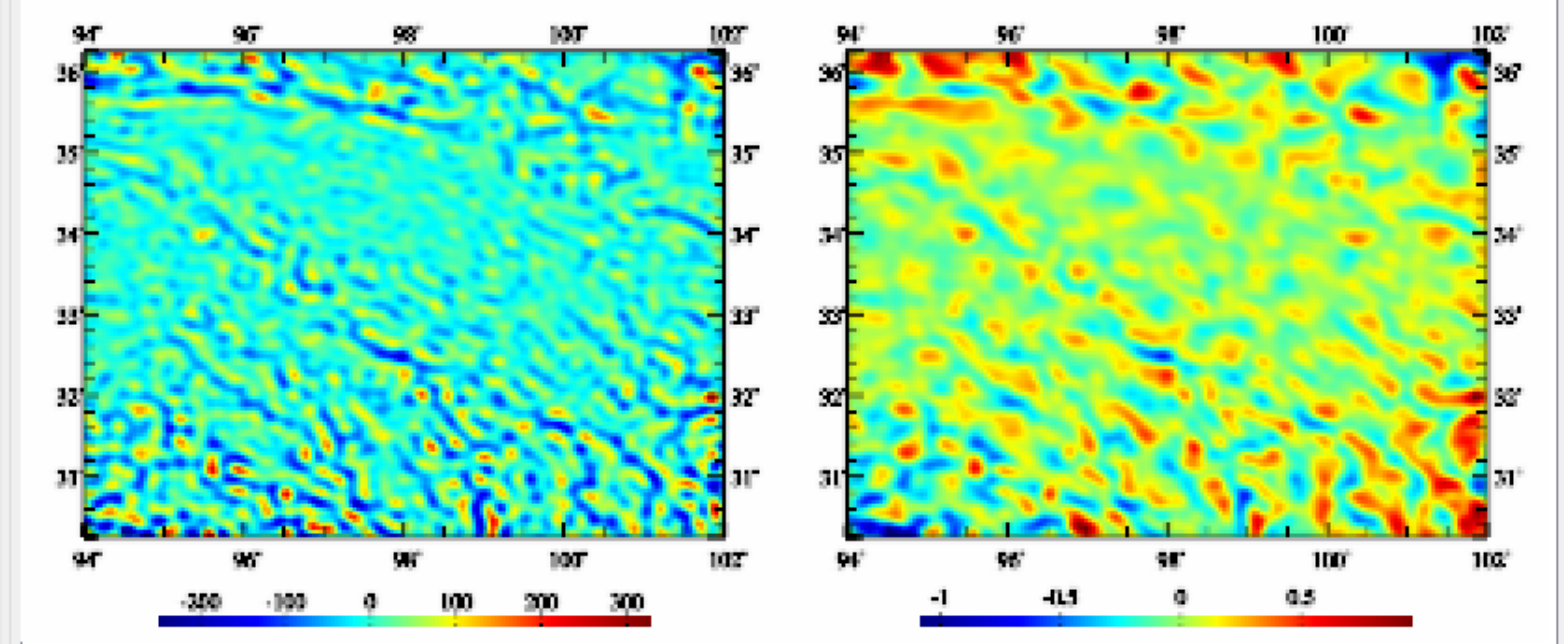
Import setting parameters

Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667	
-0.0801	-0.0775	-0.0825	-0.0952	-0.1146	-0.1390	-0.1667
-0.3914	-0.4036	-0.4126	-0.4191	-0.4241	-0.4292	-0.4362
-0.7545	-0.7992	-0.8366	-0.8651	-0.8842	-0.8944	-0.8971
-0.8988	-0.8960	-0.8837	-0.8596	-0.8213	-0.7679	-0.7000
0.0897	0.1378	0.1713	0.1904	0.1958	0.1884	0.1697
-0.2694	-0.3182	-0.3601	-0.3932	-0.4158	-0.4257	-0.4217
0.1558	0.2243	0.2769	0.3098	0.3208	0.3092	0.2763
-0.2183	-0.2149	-0.1962	-0.1656	-0.1276	-0.0868	-0.0479
-0.0505	-0.0730	-0.0945	-0.1144	-0.1326	-0.1492	-0.1642
-0.2305	-0.2288	-0.2252	-0.2196	-0.2115	-0.2005	-0.1861
0.0563	0.0686	0.0703	0.0607	0.0397	0.0086	-0.0306
-0.2157	-0.1802	-0.1347	-0.0816	-0.0234	0.0371	0.0978
0.3944	0.3889	0.3769	0.3592	0.3374	0.3126	0.2862
0.0878	0.0701	0.0532	0.0379	0.0251	0.0160	0.0116
0.1639	0.1731	0.1734	0.1640	0.1448	0.1166	0.0807

Extract height anomaly

Plot



gravity anomaly (mGal)

height anomaly (m)

- Stokes Boundary Value Theory mandates that the boundary surface be an equipotential boundary surface; thus, input gravity anomalies/disturbances must reside on such a surface.
- Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

External Height Anomaly Computation via Generalized Stokes Integration – 1D FFT

External Height Anomaly Computation
via Generalized Stokes Integration

External Height Anomaly Computation
via Generalized Hotine Integration

Stokes and Hotine integral formulas

Save computation process as

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Gravity Anomaly Grid File

Select computation file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal
Height Grid File

Select integral algorithm

1D FFT algorithm

>> Computation Process ** Operation Prompts

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 11:38:54

>> Complete the computation of external height anomaly!

>> Computation end time: 2026-04-12 11:38:55

>> [Function] Computes the external residual height anomaly (m) via Stokes integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity anomaly grid (mGal) on that surface.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landgeoidhgt.dat.

>> Open Residual Gravity Anomaly Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/resGMIgeoid541_1800.gra.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landbmsurfhgt.dat.

>> Compute external residual height anomaly by 1D FFT algorithm...

>> Save the results as C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/stokesFFT1.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 11:40:19

>> Complete the computation of external height anomaly!

>> Computation end time: 2026-04-12 11:40:40

Integral radius 180 km

Save the results as

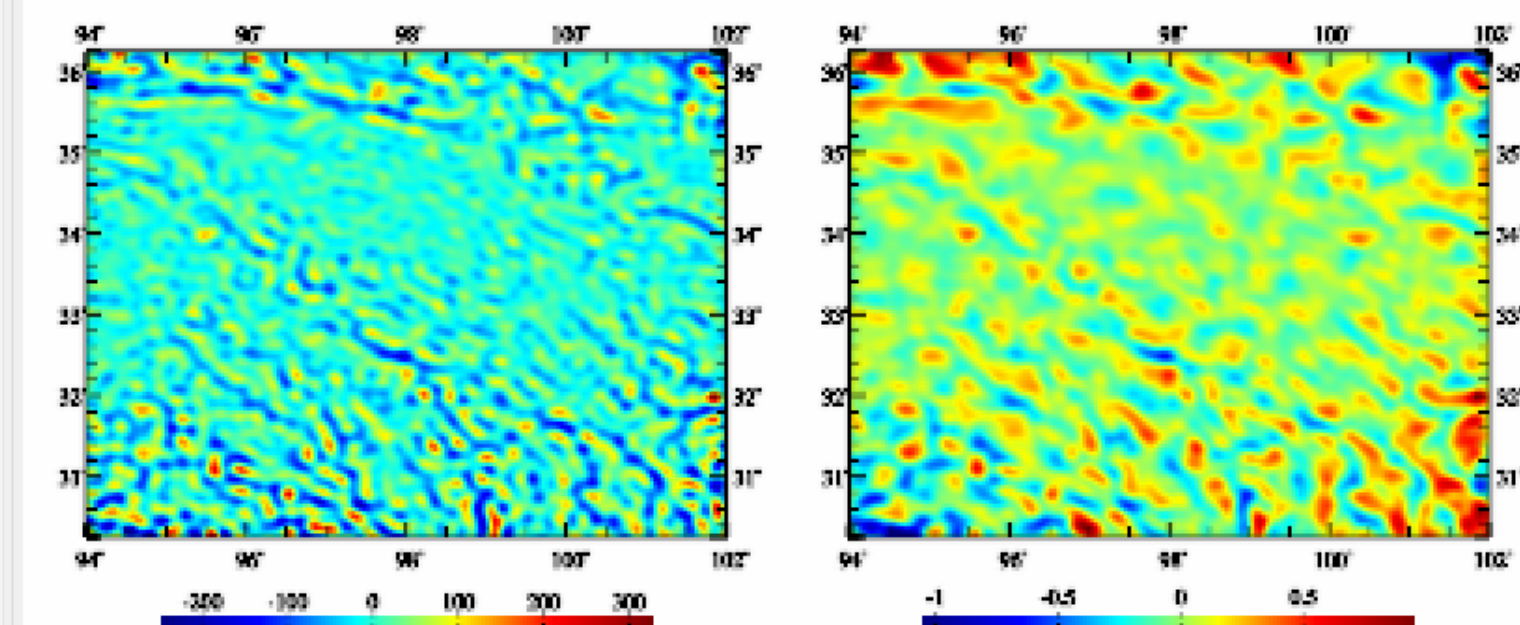
Import setting parameters

Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667	
-0.0952	-0.0880	-0.0882	-0.0965	-0.1123	-0.1340	-0.1600
-0.3732	-0.3838	-0.3920	-0.3989	-0.4053	-0.4126	-0.4224
-0.7210	-0.7623	-0.7987	-0.8291	-0.8527	-0.8694	-0.8802
-0.8921	-0.8822	-0.8643	-0.8363	-0.7968	-0.7448	-0.6806
0.0945	0.1479	0.1862	0.2089	0.2161	0.2087	0.1878
-0.2825	-0.3320	-0.3751	-0.4100	-0.4351	-0.4483	-0.4479
0.1639	0.2432	0.3053	0.3456	0.3612	0.3510	0.3162
-0.2177	-0.2100	-0.1866	-0.1515	-0.1100	-0.0683	-0.0304
-0.0415	-0.0652	-0.0882	-0.1099	-0.1299	-0.1480	-0.1642
-0.2320	-0.2323	-0.2309	-0.2270	-0.2199	-0.2087	-0.1924
0.0941	0.1026	0.0979	0.0796	0.0488	0.0077	-0.0403
-0.2009	-0.1641	-0.1212	-0.0745	-0.0260	0.0228	0.0710
0.4179	0.4290	0.4295	0.4189	0.3977	0.3672	0.3294
0.0566	0.0509	0.0472	0.0443	0.0414	0.0385	0.0360
0.1418	0.1571	0.1655	0.1650	0.1543	0.1328	0.1010

Extract height anomaly

Plot↓



gravity anomaly (mGal)

height anomaly (m)

- Stokes Boundary Value Theory mandates that the boundary surface be an equipotential boundary surface; thus, input gravity anomalies/disturbances must reside on such a surface.
- Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Table 4.1: Statistics of Model Reference Truth (Unit: m)

Item	Mean	Std. Dev.	Minimum	Maximum
Ground Height Anomalies (Deg 541 – 1800)	0.0010	0.1182	-0.6745	0.4760

Table 4.2: Differences between Stokes Computed Results and Reference Truth (m)

Algorithm	Mean	Std. Dev.	Minimum	Maximum
Numerical Integration	-0.0002	0.0324	-0.1159	0.1211
FFT2	0.0018	0.0326	-0.1150	0.1280
FFT1	0.0018	0.0327	-0.1178	0.1251

Algorithm Performance: The ratio of the standard deviation of Differences in Table 4.2 to the standard deviation of Reference Truths in Table 4.1 reflects the approximation capability of the corresponding algorithm.

External Height Anomaly Computation via Generalized Hotine Integration

External Height Anomaly Computation
via Generalized Stokes Integration

External Height Anomaly Computation
via Generalized Hotine Integration

Stokes and Hotine integral formulas

Save computation process as

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Gravity Disturbance Grid File

Select computation file format

space computation point file

Open Space Computation Point File

Set input point file format

number of rows of file header 1

Ellipsoidal Height Column Index 4

>> Computation Process ** Operation Prompts

>> Complete the computation of external height anomaly!

>> Computation end time: 2026-04-12 11:40:40

>> [Function] Computes the external residual height anomaly (m) via Hotine integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity disturbance grid (mGal) on that surface.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landgeoidhgt.dat.

>> Open Residual Gravity Anomaly Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/resGMIgeoid541_1800.rga.

>> Open Space Computation Point File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Open Space Computation Point File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save the results as C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/rsthtn.txt.

** Appends a column of computed residual height anomalies to the source file, retaining 4 significant figures.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 11:42:19

>> Complete the computation of external height anomaly!

>> Computation end time: 2026-04-12 11:44:04

Integral radius 180 km

Save the results as

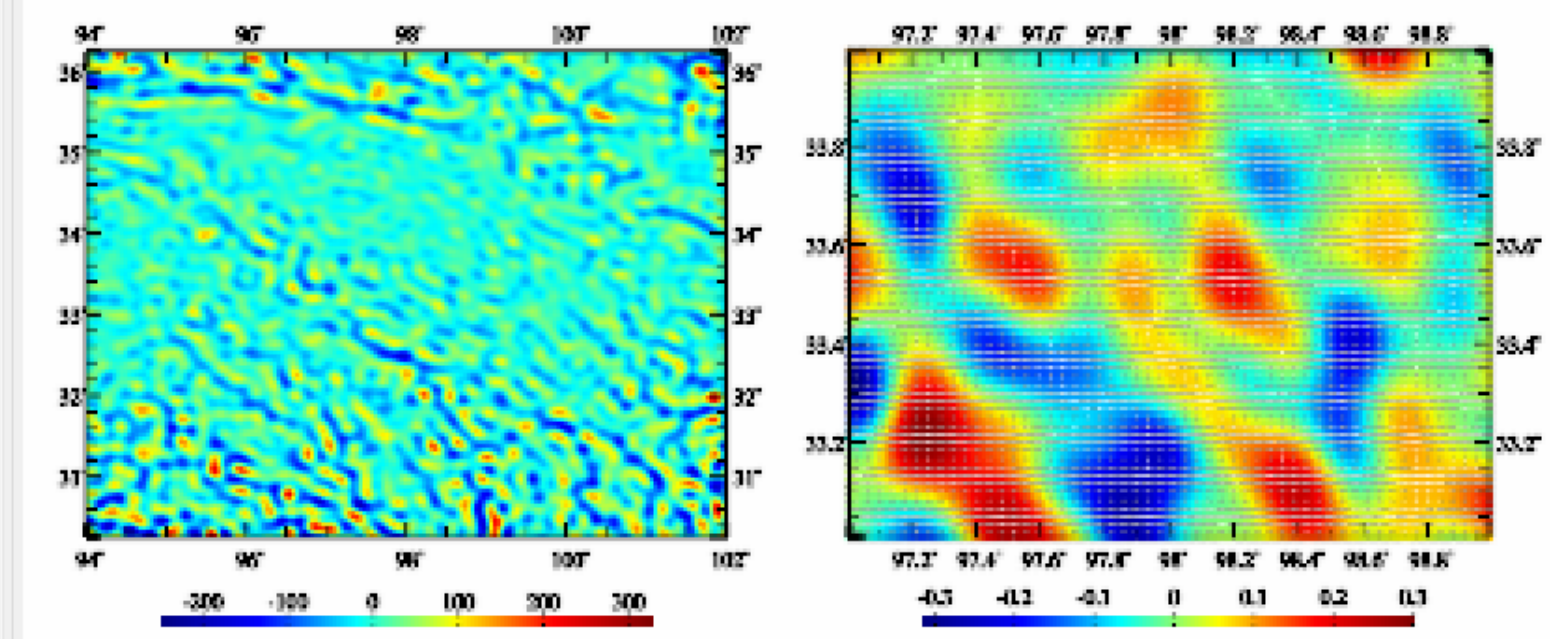
Import setting parameters

Start Computation

no	lon(degree/decimal)	lat	ellipHeight(m)	
1	97.008333	33.008333	3942.764	-0.0297
2	97.025000	33.008333	3989.787	-0.0343
3	97.041667	33.008333	4034.817	-0.0407
4	97.058333	33.008333	4070.847	-0.0488
5	97.075000	33.008333	4106.877	-0.0585
6	97.091667	33.008333	4119.913	-0.0697
7	97.108333	33.008333	4115.946	-0.0821
8	97.125000	33.008333	4090.977	-0.0955
9	97.141667	33.008333	4070.007	-0.1094
10	97.158333	33.008333	3991.047	-0.1239
11	97.175000	33.008333	3985.070	-0.1366
12	97.191667	33.008333	3956.107	-0.1479
13	97.208333	33.008333	3965.137	-0.1556
14	97.225000	33.008333	3964.173	-0.1596
15	97.241667	33.008333	3983.205	-0.1585
16	97.258333	33.008333	3953.251	-0.1530

Extract height anomaly

Plot



gravity disturbance (mGal)

height anomaly (m)

- Stokes Boundary Value Theory mandates that the boundary surface be an equipotential boundary surface; thus, input gravity anomalies/disturbances must reside on such a surface.
- Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

External Height Anomaly Computation via Generalized Hotine Integration – 2D FFT

External Height Anomaly Computation
via Generalized Stokes Integration

External Height Anomaly Computation
via Generalized Hotine Integration

Stokes and Hotine integral formulas

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Gravity Disturbance Grid File

Select computation file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal
Height Grid File

Select integral algorithm

2D FFT algorithm

>> Computation Process ** Operation Prompts

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 11:42:19

>> Complete the computation of external height anomaly!

>> Computation end time: 2026-04-12 11:44:04

>> [Function] Computes the external residual height anomaly (m) via Hotine integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity disturbance grid (mGal) on that surface.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landgeoidhgt.dat.

>> Open Residual Gravity Anomaly Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/resGMIgeoid541_1800.rga.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/landbmsurfhgt.dat.

>> Compute external residual height anomaly by 2D FFT algorithm...

>> Save the results as C:/PAGrav4.5_win64en/examples/IntgenStokesHotine/HotineFFT2.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 11:45:59

>> Complete the computation of external height anomaly!

>> Computation end time: 2026-04-12 11:46:00

Integral radius 180 km

Save the results as

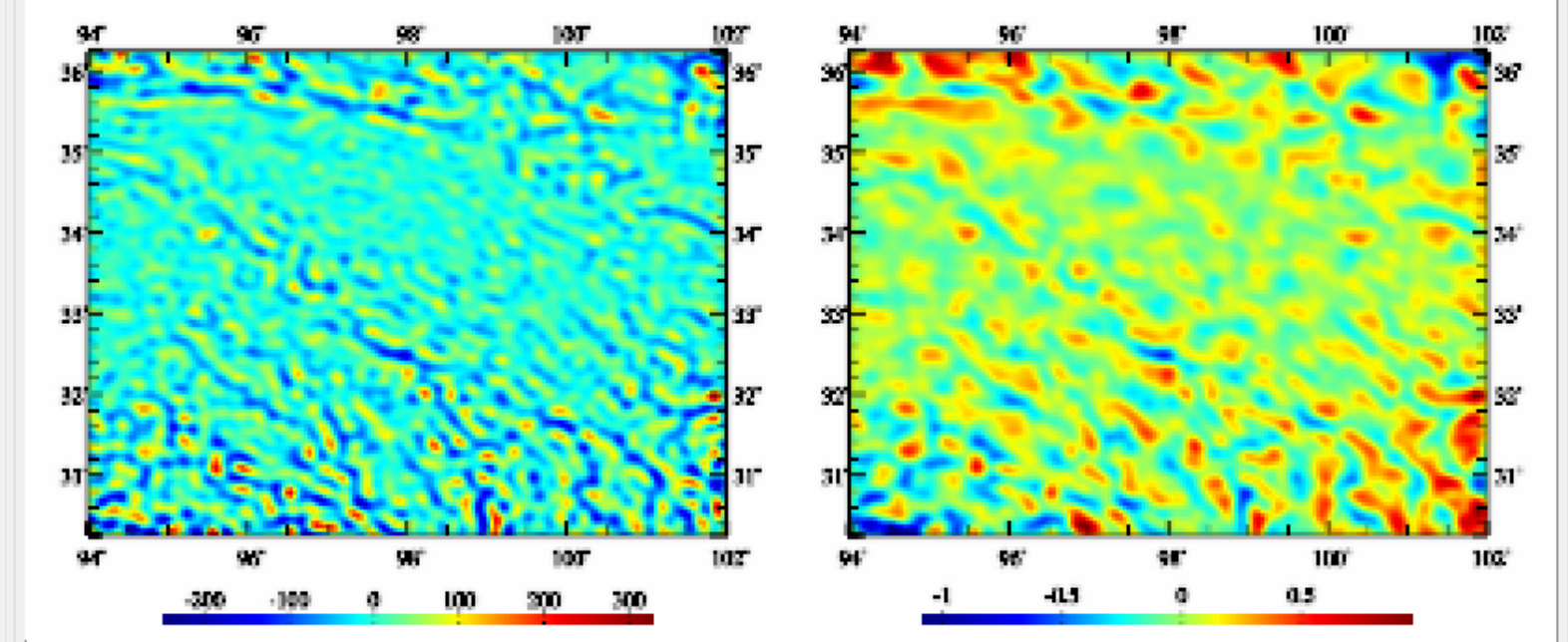
Import setting parameters

Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667	
-0.0804	-0.0778	-0.0828	-0.0956	-0.1150	-0.1395	-0.1672
-0.3925	-0.4048	-0.4138	-0.4203	-0.4253	-0.4304	-0.4375
-0.7564	-0.8011	-0.8386	-0.8672	-0.8863	-0.8965	-0.8993
-0.9010	-0.8982	-0.8859	-0.8617	-0.8233	-0.7698	-0.7018
0.0895	0.1377	0.1713	0.1905	0.1959	0.1886	0.1698
-0.2700	-0.3189	-0.3609	-0.3941	-0.4166	-0.4267	-0.4226
0.1560	0.2246	0.2772	0.3102	0.3212	0.3097	0.2768
-0.2186	-0.2152	-0.1965	-0.1658	-0.1278	-0.0870	-0.0480
-0.0506	-0.0732	-0.0948	-0.1147	-0.1330	-0.1496	-0.1646
-0.2312	-0.2296	-0.2260	-0.2203	-0.2122	-0.2013	-0.1869
0.0560	0.0683	0.0701	0.0604	0.0395	0.0083	-0.0309
-0.2158	-0.1803	-0.1347	-0.0814	-0.0231	0.0376	0.0984
0.3955	0.3901	0.3780	0.3603	0.3384	0.3135	0.2871
0.0882	0.0704	0.0535	0.0382	0.0254	0.0163	0.0120
0.1644	0.1736	0.1739	0.1645	0.1453	0.1170	0.0811

Extract height anomaly

Plot



gravity disturbance (mGal)

height anomaly (m)

- Stokes Boundary Value Theory mandates that the boundary surface be an equipotential boundary surface; thus, input gravity anomalies/disturbances must reside on such a surface.
- Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

External Height Anomaly Computation via Generalized Hotine Integration – 1D FFT

External Height Anomaly Computation
via Generalized Stokes Integration

External Height Anomaly Computation
via Generalized Hotine Integration

Stokes and Hotine integral formulas

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Gravity Disturbance Grid File

Select computation file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal
Height Grid File

Select integral algorithm

1D FFT algorithm

>> Computation Process ** Operation Prompts

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 11:45:59

>> Complete the computation of external height anomaly!

>> Computation end time: 2026-04-12 11:46:00

>> [Function] Computes the external residual height anomaly (m) via Hotine integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity disturbance grid (mGal) on that surface.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenStokesHotine/landgeoidhgt.dat.

>> Open Residual Gravity Anomaly Grid File C:/PAGravf4.5_win64en/examples/IntgenStokesHotine/resGMlgeoid541_1800.rga.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenStokesHotine/landbmsurfhgt.dat.

>> Compute external residual height anomaly by 1D FFT algorithm...

>> Save the results as C:/PAGravf4.5_win64en/examples/IntgenStokesHotine/HotineFFT1.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 11:47:16

>> Complete the computation of external height anomaly!

>> Computation end time: 2026-04-12 11:47:38

Integral radius 180 km

Save the results as

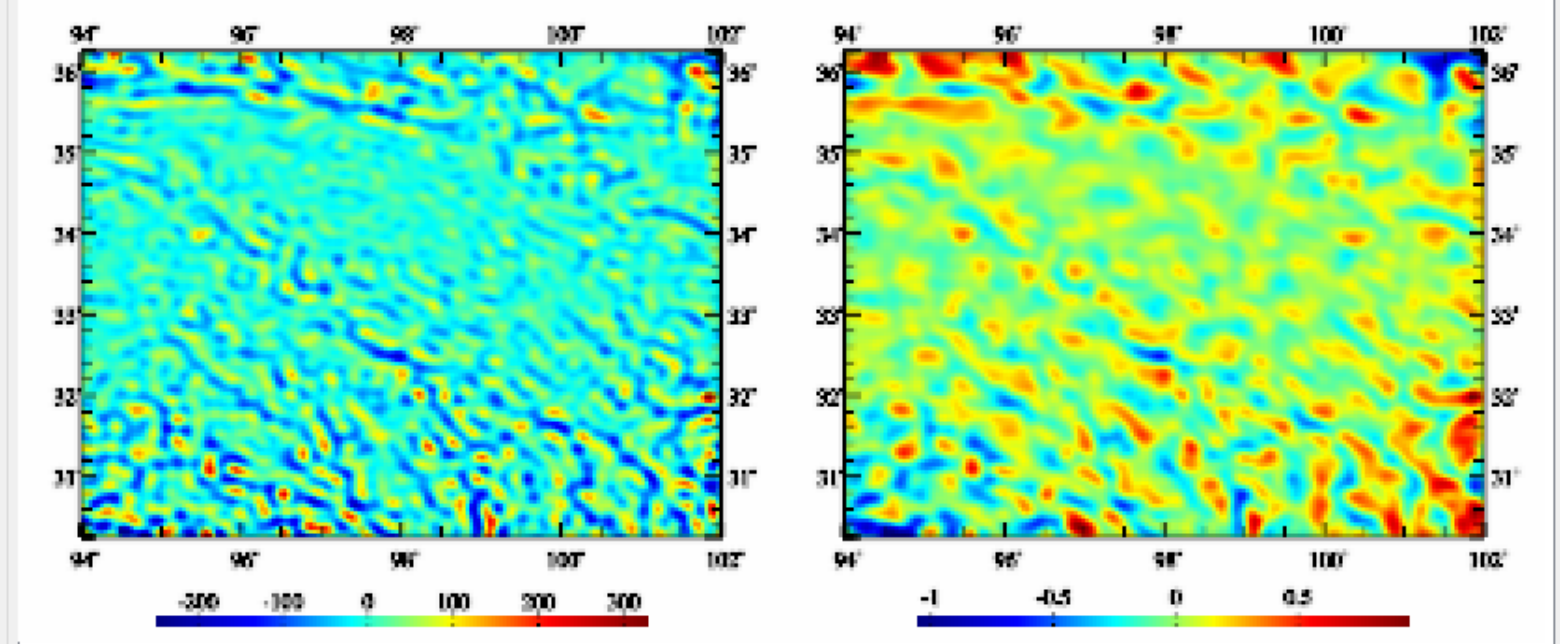
Import setting parameters

Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667	
-0.0955	-0.0883	-0.0885	-0.0969	-0.1127	-0.1345	-0.1605
-0.3743	-0.3849	-0.3932	-0.4000	-0.4065	-0.4139	-0.4236
-0.7228	-0.7642	-0.8007	-0.8311	-0.8547	-0.8715	-0.8824
-0.8943	-0.8844	-0.8664	-0.8384	-0.7988	-0.7467	-0.6824
0.0943	0.1478	0.1862	0.2091	0.2163	0.2088	0.1880
-0.2832	-0.3327	-0.3759	-0.4109	-0.4360	-0.4493	-0.4489
0.1640	0.2435	0.3057	0.3460	0.3617	0.3515	0.3167
-0.2180	-0.2103	-0.1868	-0.1517	-0.1102	-0.0684	-0.0304
-0.0416	-0.0654	-0.0884	-0.1102	-0.1302	-0.1483	-0.1646
-0.2327	-0.2331	-0.2316	-0.2278	-0.2207	-0.2095	-0.1932
0.0938	0.1024	0.0977	0.0794	0.0485	0.0074	-0.0406
-0.2011	-0.1642	-0.1211	-0.0743	-0.0257	0.0232	0.0716
0.4191	0.4302	0.4307	0.4201	0.3988	0.3682	0.3303
0.0570	0.0513	0.0475	0.0446	0.0418	0.0388	0.0363
0.1424	0.1576	0.1660	0.1655	0.1547	0.1332	0.1013

Extract height anomaly

Plot↓



gravity disturbance (mGal)

height anomaly (m)

- Stokes Boundary Value Theory mandates that the boundary surface be an equipotential boundary surface; thus, input gravity anomalies/disturbances must reside on such a surface.
- Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Table 4.1: Statistics of Model Reference Truth (Unit: m)

Item	Mean	Std. Dev.	Minimum	Maximum
Ground Height Anomalies (Deg 541 – 1800)	0.0010	0.1182	-0.6745	0.4760

Table 4.4: Differences between Hotine Computed Results and Reference Truth (m)

Algorithm	Mean	Std. Dev.	Minimum	Maximum
Numerical Integration	-0.0001	0.0256	-0.0915	0.0957
FFT2	0.0019	0.0259	-0.0911	0.1065
FFT1	0.0018	0.0261	-0.0934	0.1036

The performance of the Hotine integration algorithm is slightly better than that of the Stokes integration algorithm.

Computation of External Vertical Deflection from Gravity Anomaly

Computation of External Vertical Deflection from Gravity Anomaly

Computation of External Vertical Deflection from Gravity Disturbance

Vening-Meinesz integral formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Residual Gravity Anomaly Grid File

Select computation file format

space computation point file

Open Space Computation Point File

Set input point file format

number of rows of file header 1

Ellipsoidal Height Column Index 4

Integral radius 180 km

Extract vertical deflection

Plot

>> Computation Process ** Operation Prompts

>> [Function] Computes the external residual vertical deflection (" , south and west components) via the generalized Vening-Meinesz integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity anomaly grid (mGal).

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/landgeoidhgt.dat.

>> Open Residual Gravity Anomaly Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/resGMIgeoid541_1800.gra.

>> Open Space Computation Point File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save the results as C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/rstrga.txt.

** Record format: Appends two columns (southward and westward components) of computed residual vertical deflections to the source file, retaining 4 significant figures.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

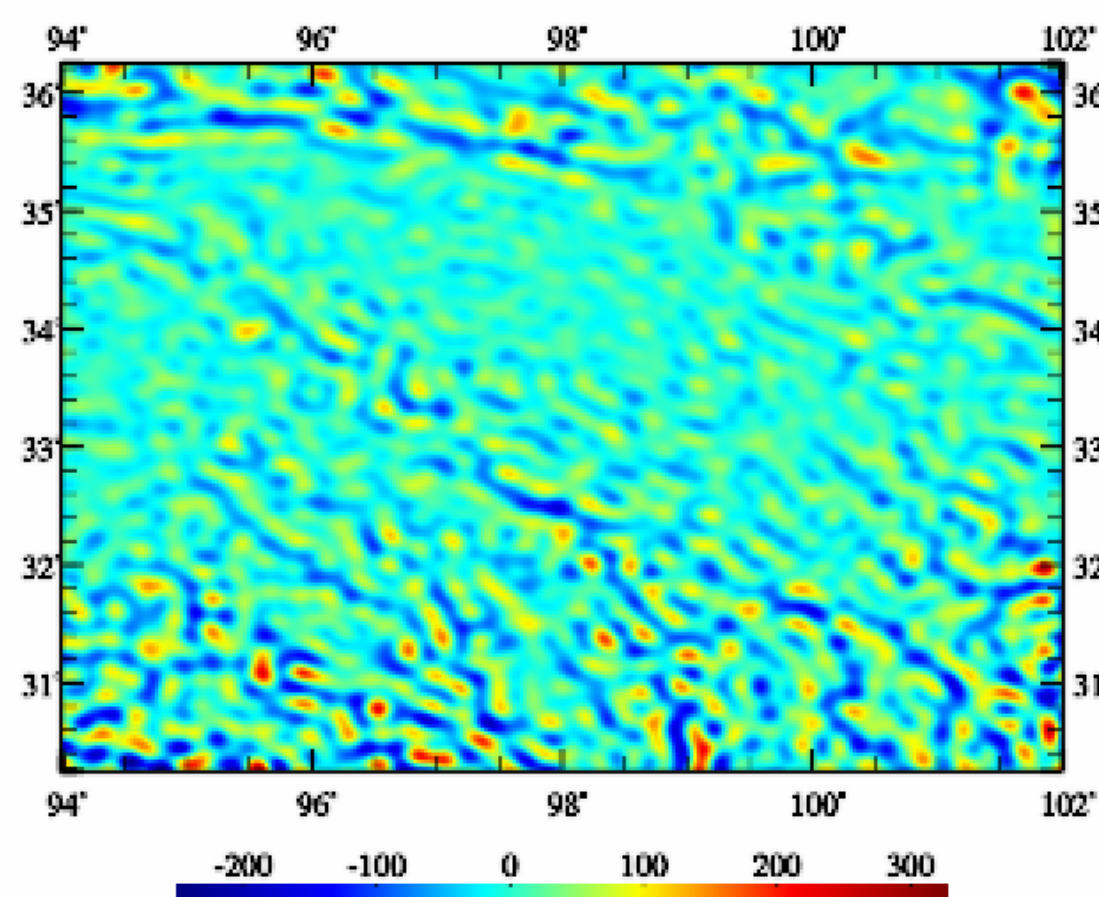
>> Computation start time: 2026-04-12 15:04:16

Save the results as

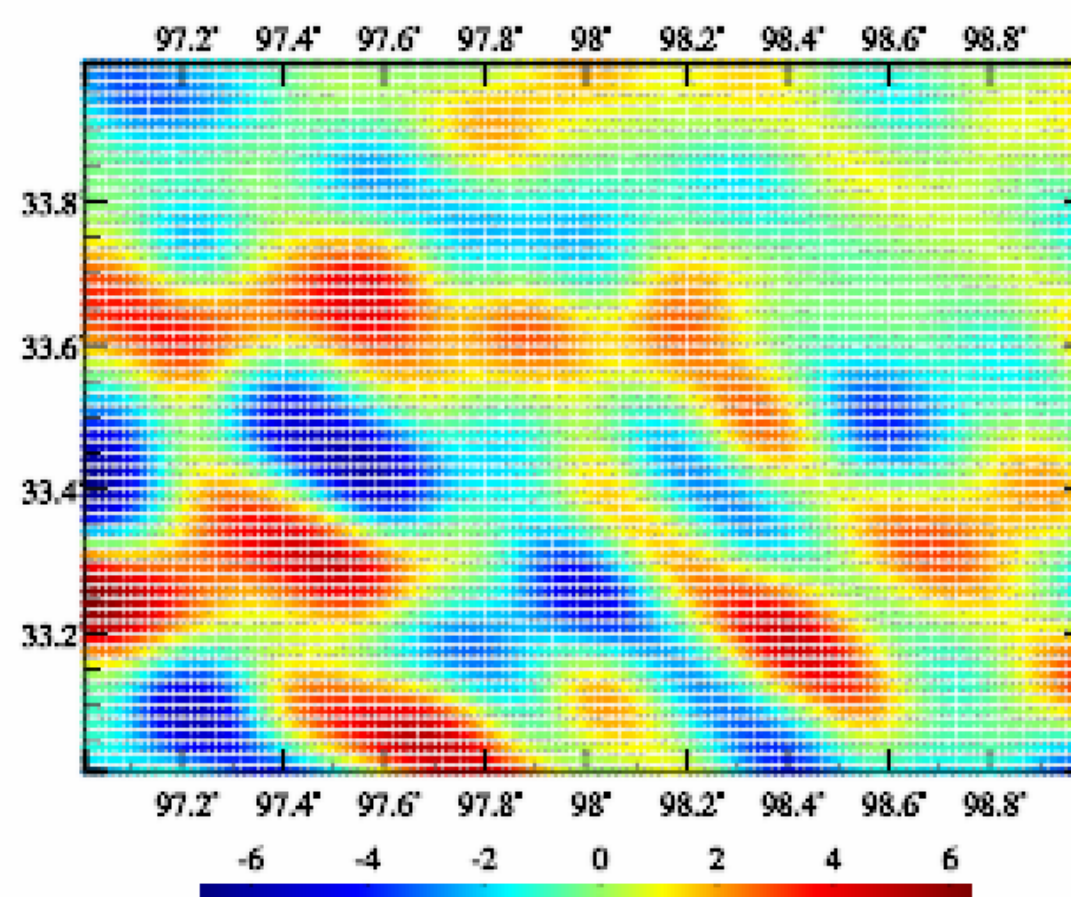
Import setting parameters

Start Computation

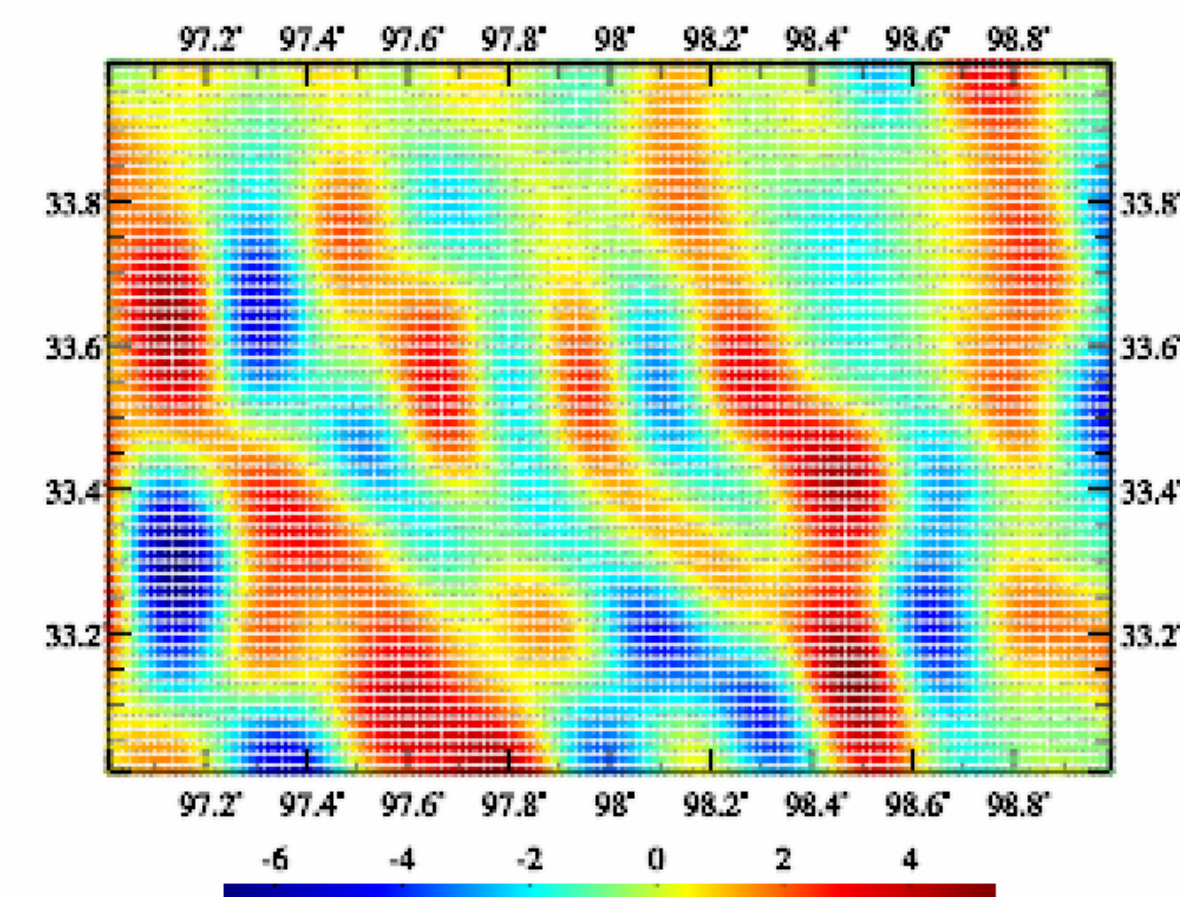
no	lon(degree/decimal)	lat	ellipHeight(m)		
1	97.008333	33.008333	3942.764	-2.4975	0.4726
2	97.025000	33.008333	3989.787	-2.4200	0.6841
3	97.041667	33.008333	4034.817	-2.3012	0.9131
4	97.058333	33.008333	4070.847	-2.1495	1.1375
5	97.075000	33.008333	4106.877	-1.9758	1.3348
6	97.091667	33.008333	4119.913	-1.7943	1.4946
7	97.108333	33.008333	4115.946	-1.6197	1.6015



gravity anomaly (mGal)



vertical deflection (" , S)



vertical deflection (" , W)

Computation of External Vertical Deflection from Gravity Anomaly – 2D FFT

Computation of External Vertical Deflection from Gravity Anomaly

Computation of External Vertical Deflection from Gravity Disturbance

Vening-Meinesz integral formulas

Save computation process as

Open Equipotential Surface Ellipsoidal Height Grid File

Open Residual Gravity Anomaly Grid File

Select computation file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal Height Grid File

Select integral algorithm

2D FFT algorithm

Integral radius 180 km

>> Computation Process ** Operation Prompts

>> [Function] Computes the external residual vertical deflection (" , south and west components) via the generalized Vening-Meinesz integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity anomaly grid (mGal).

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/landgeoidhgt.dat.

>> Open Residual Gravity Anomaly Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/resGMIgeoid541_1800.gra.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/landbmsurfhgt.dat.

>> Compute external residual vertical deflection by 2D FFT algorithm...

>> Save the results as C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/gratovmFFT2.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 15:08:28

>> Complete the computation of external vertical deflection!

>> Computation end time: 2026-04-12 15:08:30

Save the results as

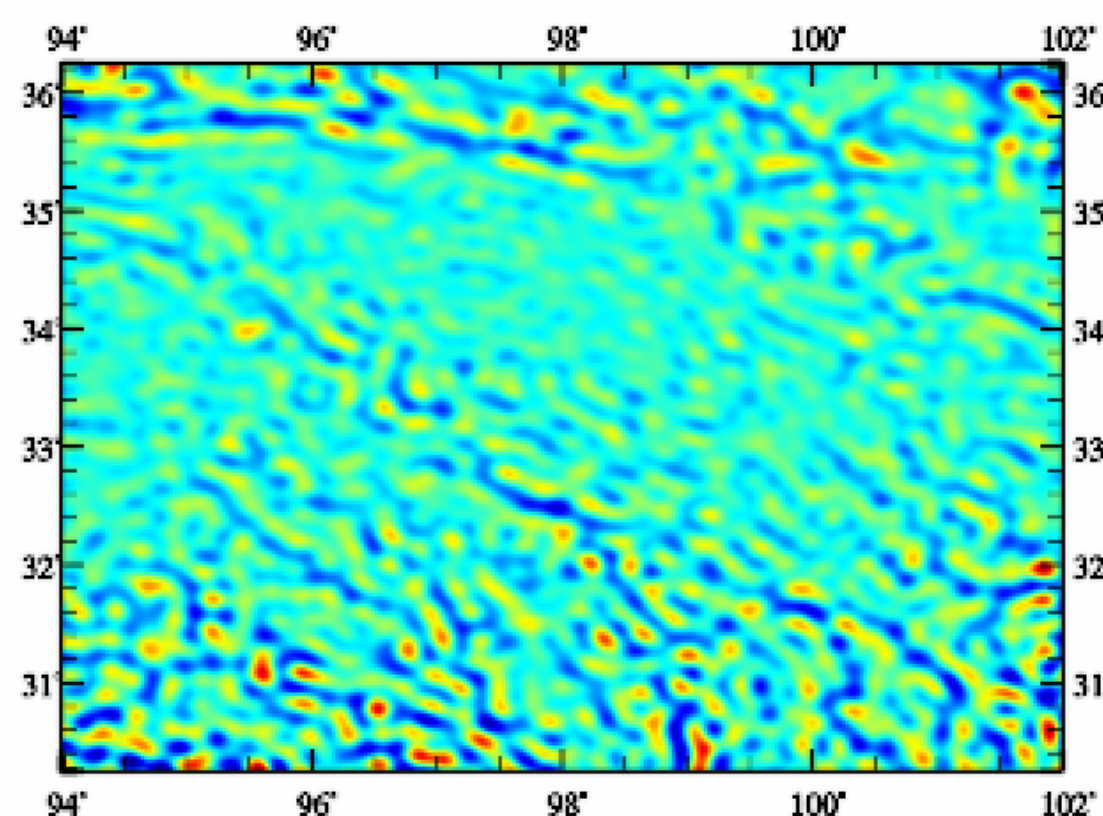
Import setting parameters

Start Computation

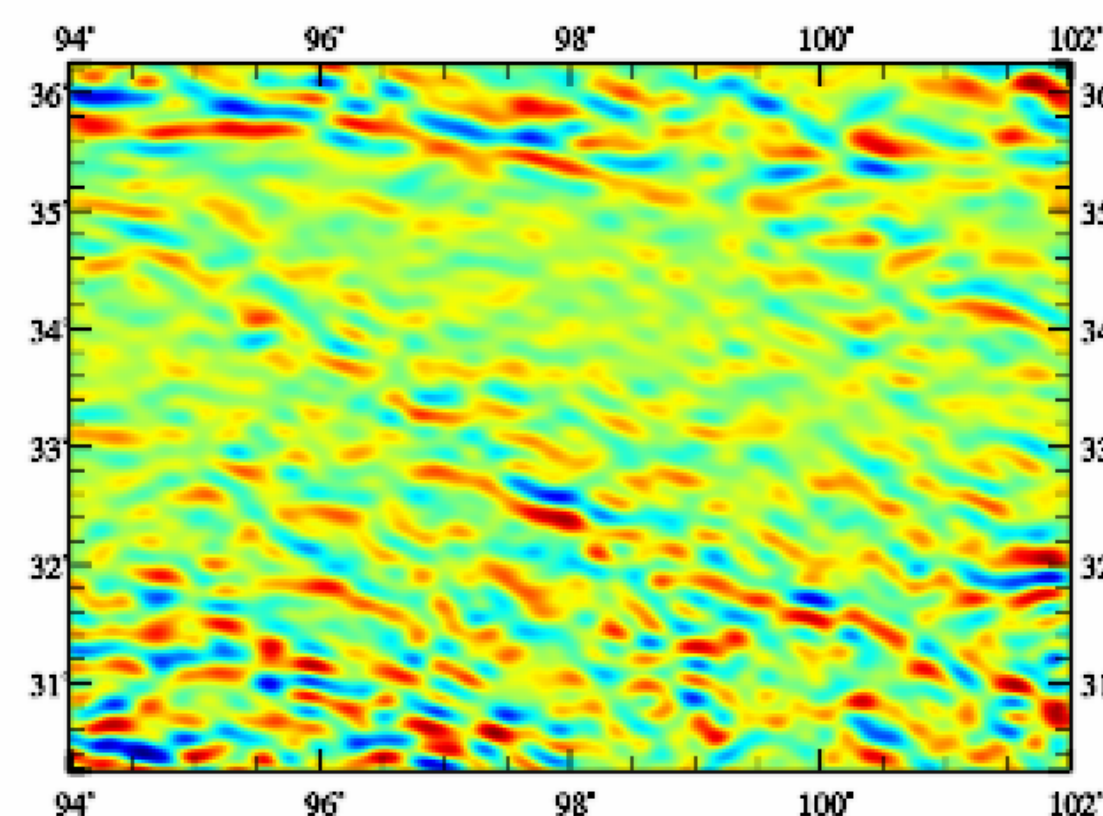
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667			
-2.2955	-2.5663	-2.5312	-2.2038	-1.6530	-0.9588	-0.1916	0.5943	1.3597
4.4045	4.3352	4.1336	3.8281	3.4637	3.0993	2.8020	2.6392	2.6689
8.1418	8.9666	9.5719	9.9219	10.0144	9.8802	9.5792	9.1888	8.7932
9.2908	9.5062	9.5374	9.3162	8.7965	7.9607	6.8206	5.4140	3.7999
-7.1705	-7.9937	-8.5383	-8.7986	-8.7766	-8.4811	-7.9276	-7.1379	-6.1402
3.4382	4.6931	5.8090	6.7512	7.4831	7.9666	8.1626	8.0351	7.5542

Extract vertical deflection

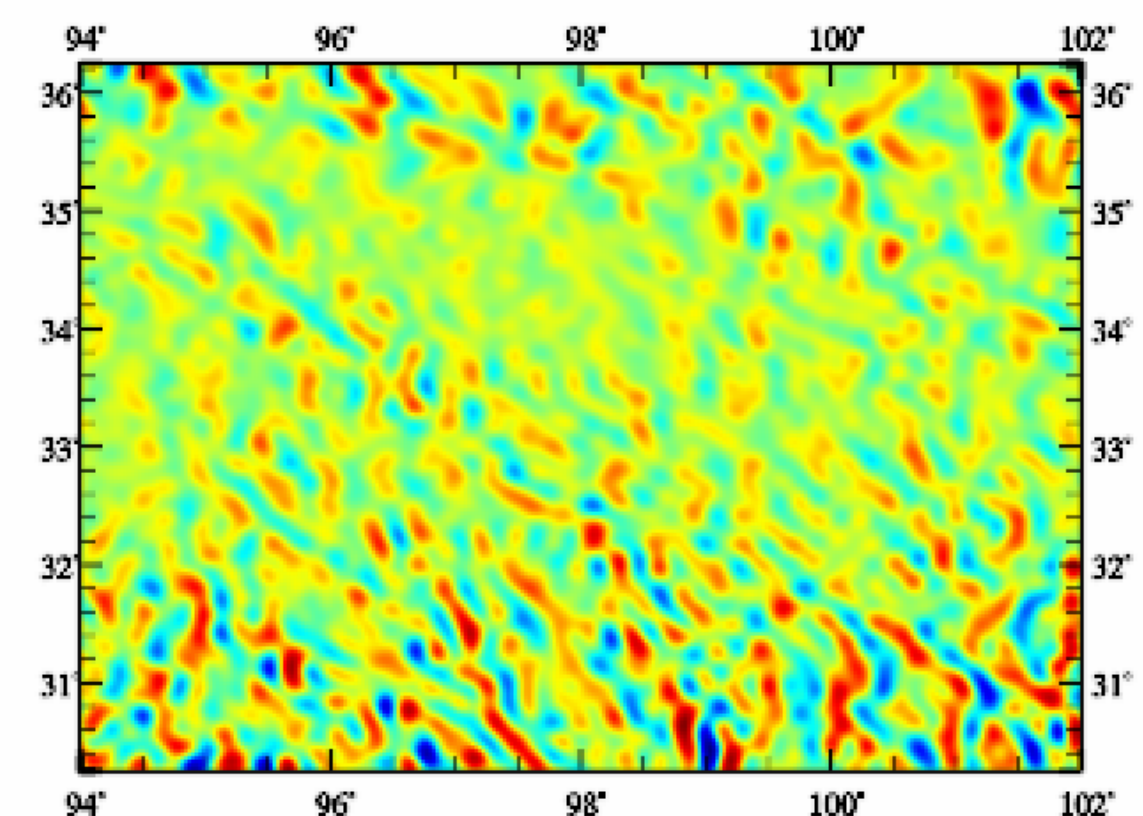
Plot



gravity anomaly (mGal)



vertical deflection (" , S)



vertical deflection (" , W)

Computation of External Vertical Deflection from Gravity Anomaly – 1D FFT

Computation of External Vertical Deflection from Gravity Anomaly

Computation of External Vertical Deflection from Gravity Disturbance

Vening-Meinesz integral formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Residual Gravity Anomaly Grid File

Select computation file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal Height Grid File

Select integral algorithm

1D FFT algorithm

Integral radius 180 km

>> Computation Process ** Operation Prompts

>> [Function] Computes the external residual vertical deflection (" , south and west components) via the generalized Vening-Meinesz integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity anomaly grid (mGal).

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/landgeoidhgt.dat.

>> Open Residual Gravity Anomaly Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/resGMIgeoid541_1800.gra.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/landbmsurfhgt.dat.

>> Compute external residual vertical deflection by 1D FFT algorithm...

>> Save the results as C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/gratovmFFT1.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 15:09:58

>> Complete the computation of external vertical deflection!

>> Computation end time: 2026-04-12 15:10:23

Save the results as

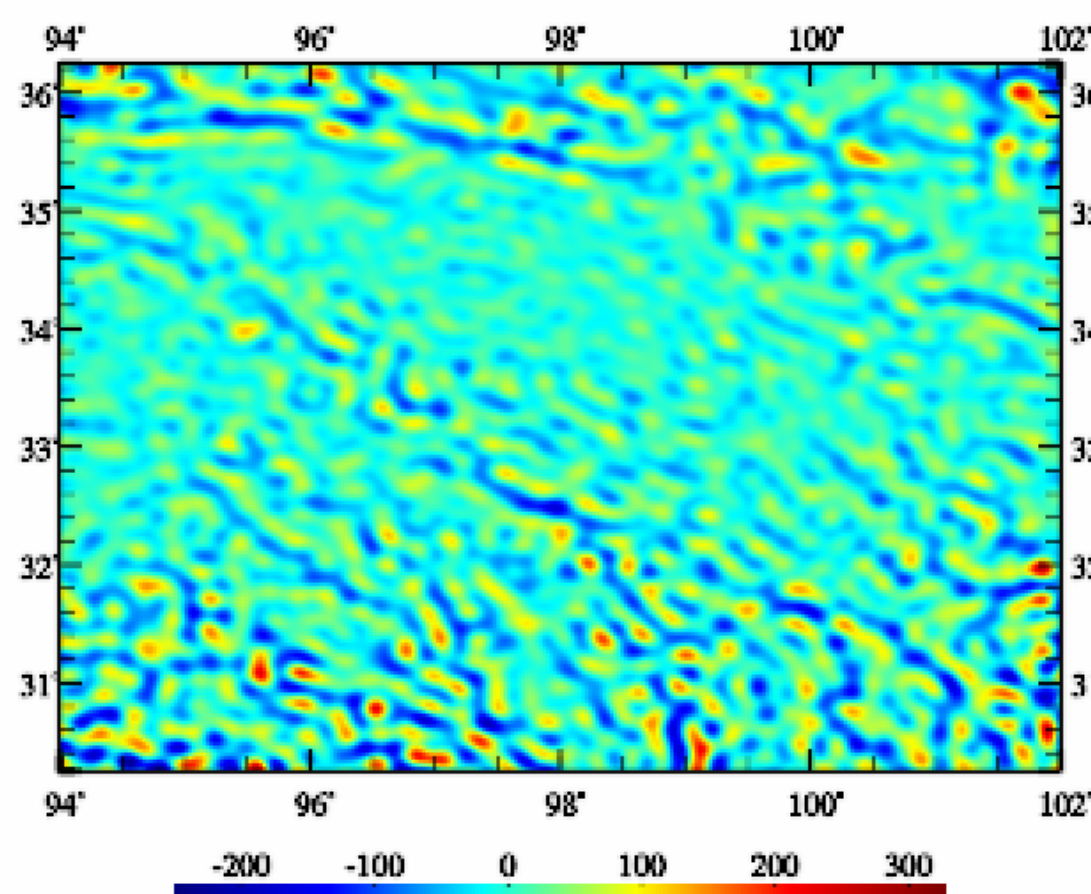
Import setting parameters

Start Computation

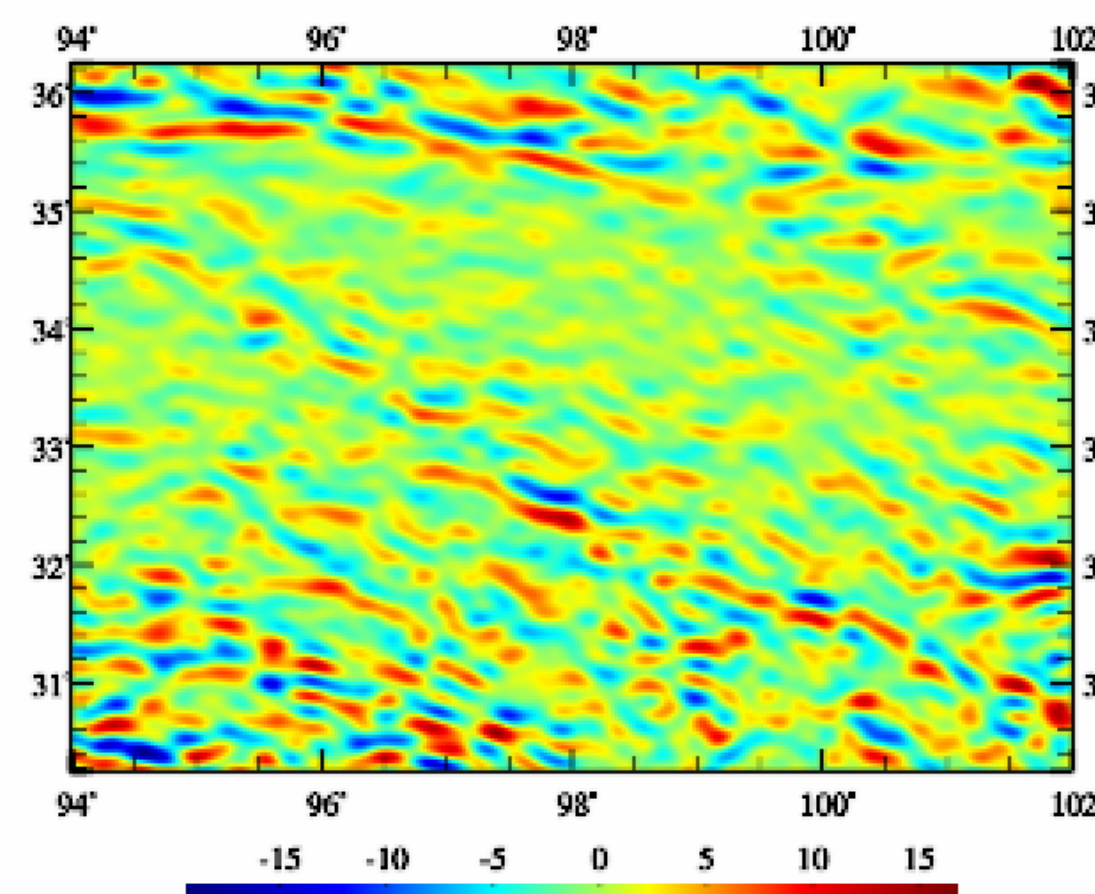
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667			
-2.2866	-2.5693	-2.5365	-2.2060	-1.6513	-0.9553	-0.1892	0.5928	1.3523
4.3578	4.2812	4.0701	3.7522	3.3727	2.9908	2.6747	2.4928	2.5043
7.9132	8.7324	9.3302	9.6709	9.7517	9.6042	9.2891	8.8855	8.4788
9.0235	9.2631	9.3211	9.1282	8.6380	7.8323	6.7225	5.3462	3.7619
-7.0608	-7.8788	-8.4227	-8.6862	-8.6706	-8.3840	-7.8409	-7.0624	-6.0762
3.4219	4.6659	5.7721	6.7067	7.4337	7.9155	8.1136	7.9915	7.5192

Extract vertical deflection

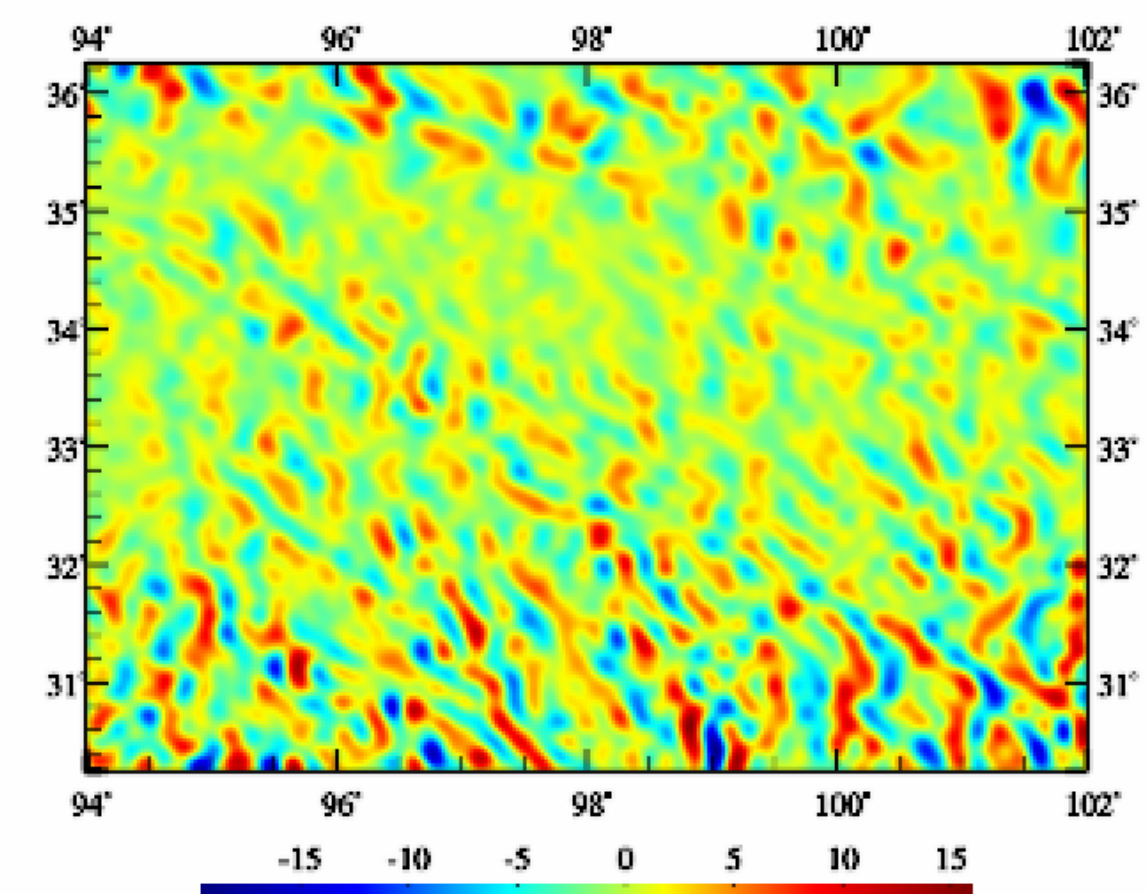
Plot↓



gravity anomaly (mGal)



vertical deflection (" , S)



vertical deflection (" , W)

Table 4.6: Statistics of Model Reference Truth (Unit: ")

Component	Mean	Std. Dev.	Min	Max
Ground Vertical Deflection (South, ξ)	0.0014	2.4951	-12.7789	14.2346
Ground Vertical Deflection (West, η)	0.0097	2.1772	-9.1577	10.4499

Table 4.7: Differences between Computed Results (Anomaly-based) and Reference Truth (")

Algorithm	Component	Mean	Std. Dev.	Min	Max
Numerical integral	South (ξ)	0.0003	0.0380	-0.1061	0.1387
	West (η)	0.0011	0.0289	-0.0830	0.1091
FFT2	South (ξ)	0.0002	0.1107	-0.8974	0.7395
	West (η)	0.0011	0.1003	-0.6887	1.0743
FFT1	South (ξ)	0.0007	0.1090	-0.8189	0.6581
	West (η)	0.0011	0.0984	-0.6078	1.0866

Algorithm Performance: The ratio of the standard deviation of Differences in Table 4.7 to the standard deviation of Reference Truths in Table 4.6 reflects the approximation capability of the corresponding algorithm.

Computation of External Vertical Deflection from Gravity Disturbance

Computation of External Vertical Deflection from Gravity Anomaly

Computation of External Vertical Deflection from Gravity Disturbance

Vening-Meinesz integral formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Residual Gravity Disturbance Grid File

Select computation file format

space computation point file

Open Space Computation Point File

Set input point file format

number of rows of file header 1

Ellipsoidal Height Column Index 4

Integral radius 180 km

>> Computation Process ** Operation Prompts

>> [Function] Computes the external residual vertical deflection (" , south and west components) via the generalized Vening-Meinesz integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity disturbance grid (mGal)

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/landgeoidhgt.dat.

>> Open Residual Gravity Disturbance Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/resGMIgeoid541_1800.rga.

>> Open Space Computation Point File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save the results as C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/rstgra.txt.

** Record format: Appends two columns (southward and westward components) of computed residual vertical deflections to the source file, retaining 4 significant figures.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 15:11:46

Save the results as

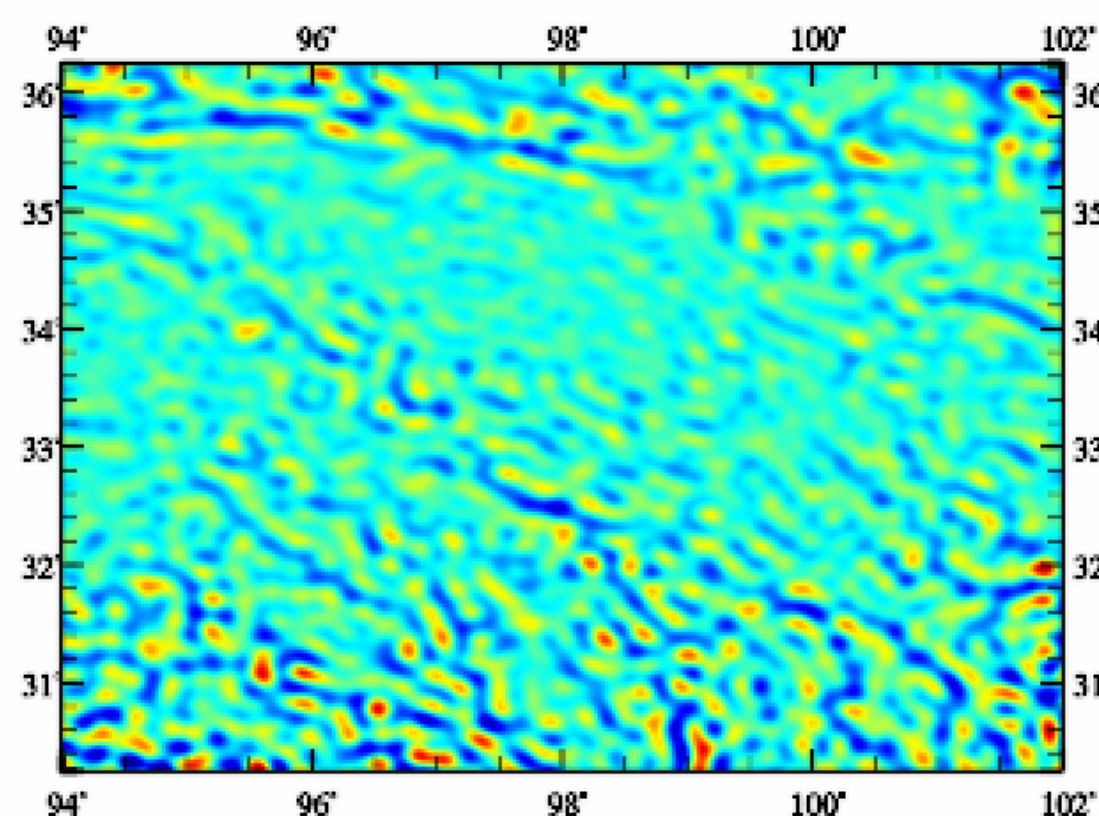
Import setting parameters

Start Computation

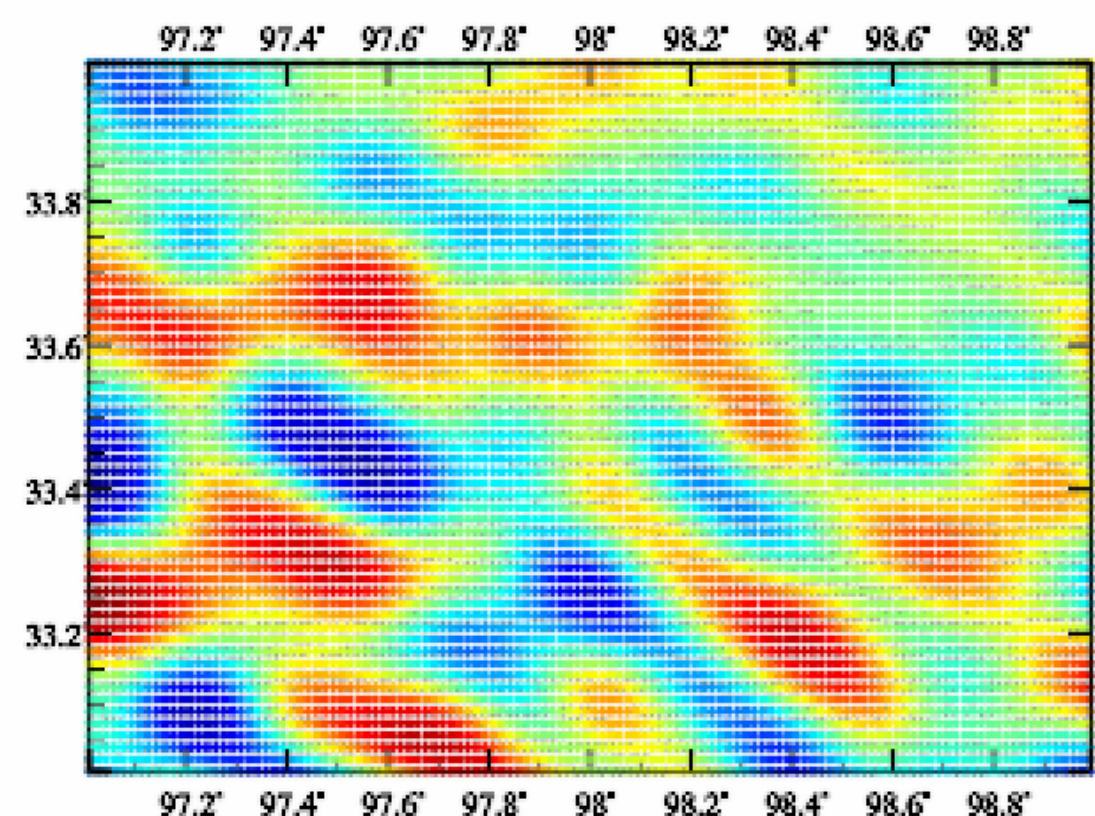
no	lon(degree/decimal)	lat	ellipHeight(m)		
1	97.008333	33.008333	3942.764	-2.4923	0.4718
2	97.025000	33.008333	3989.787	-2.4149	0.6833
3	97.041667	33.008333	4034.817	-2.2964	0.9122
4	97.058333	33.008333	4070.847	-2.1450	1.1367
5	97.075000	33.008333	4106.877	-1.9717	1.3340
6	97.091667	33.008333	4119.913	-1.7905	1.4938
7	97.108333	33.008333	4115.946	-1.6162	1.6008

Extract vertical deflection

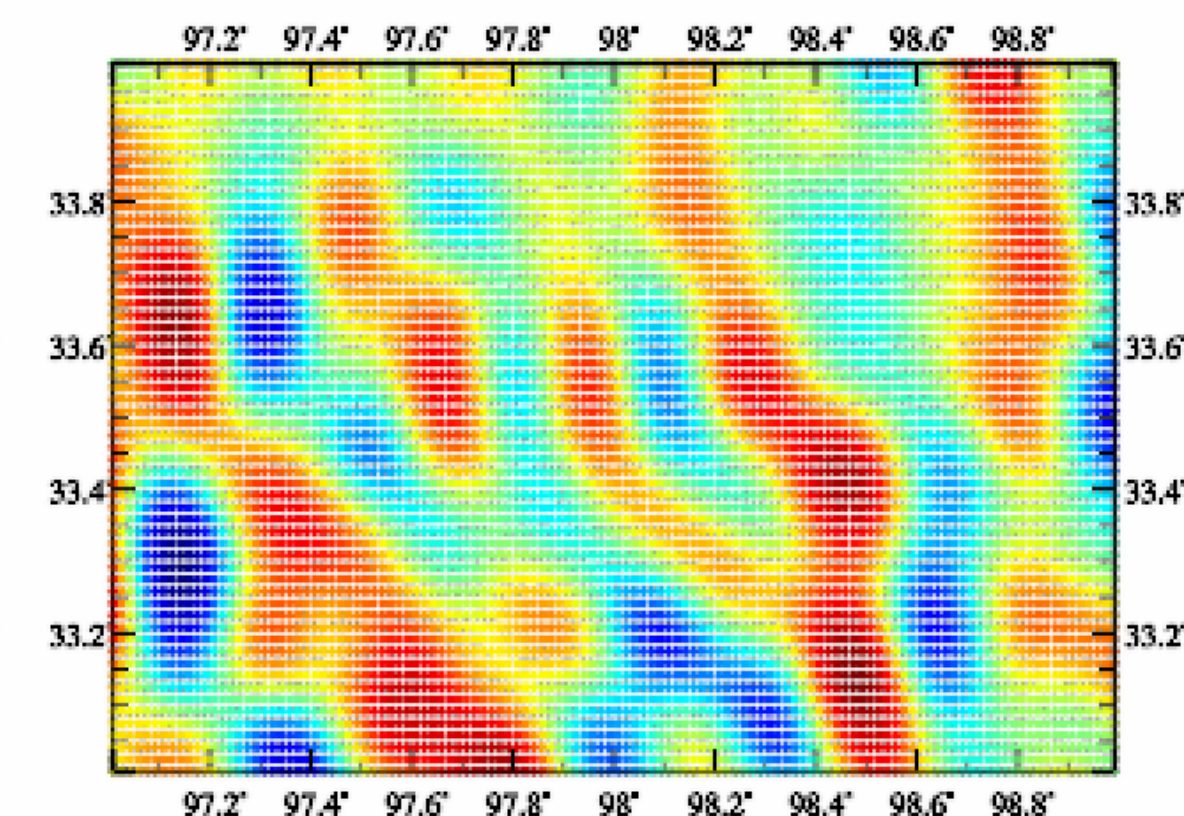
Plot



gravity disturbance (mGal)



vertical deflection (" , S)



vertical deflection (" , W)

Computation of External Vertical Deflection from Gravity Disturbance – 2D FFT

Computation of External Vertical Deflection from Gravity Anomaly

Computation of External Vertical Deflection from Gravity Disturbance

Vening-Meinesz integral formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Residual Gravity Disturbance Grid File

Select computation file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal Height Grid File

Select integral algorithm

2D FFT algorithm

Integral radius 180 km

>> Computation Process ** Operation Prompts

>> [Function] Computes the external residual vertical deflection (" , south and west components) via the generalized Vening-Meinesz integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity disturbance grid (mGal)

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/landgeoidhgt.dat.

>> Open Residual Gravity Disturbance Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/resGMIgeoid541_1800.rga.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/landbmsurfhgt.dat.

>> Compute external residual vertical deflection by 2D FFT algorithm...

>> Save the results as C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/rgatovmFFT2.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 15:16:01

>> Complete the computation of external vertical deflection!

>> Computation end time: 2026-04-12 15:16:03

Save the results as

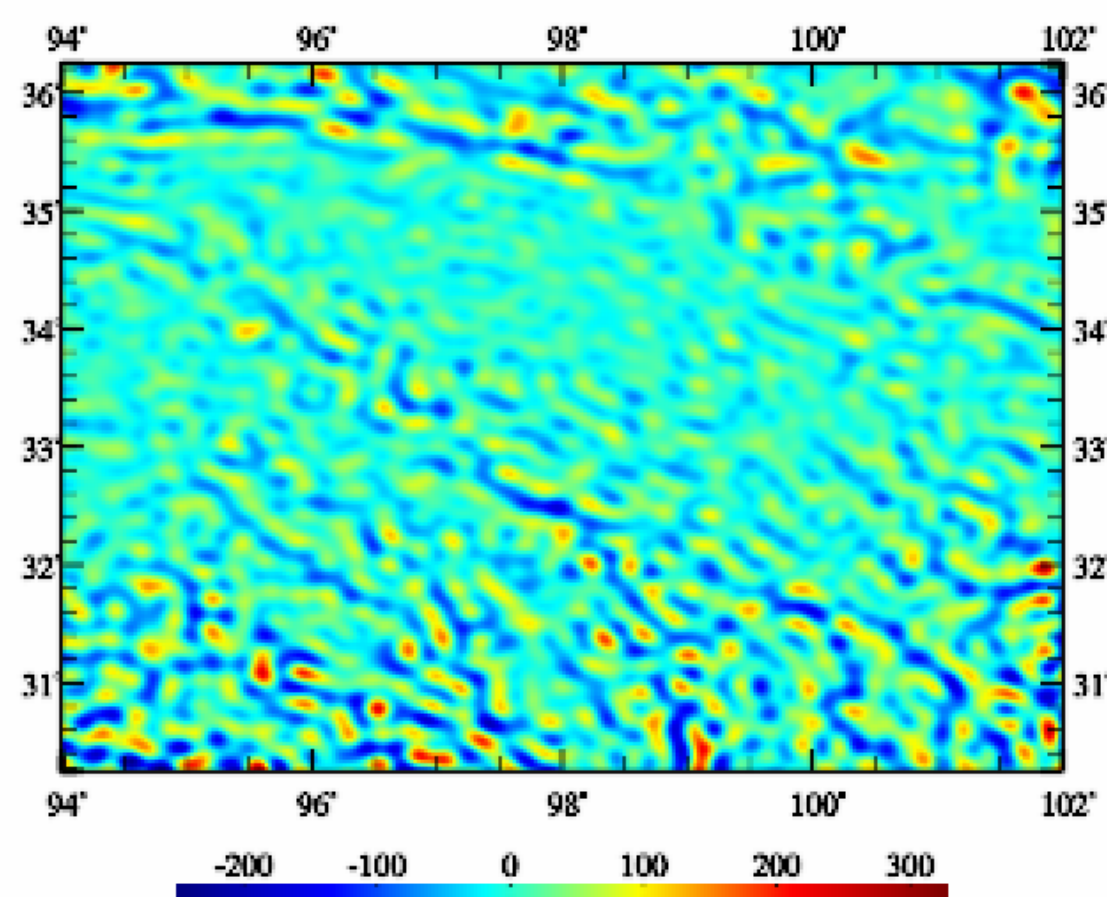
Import setting parameters

Start Computation

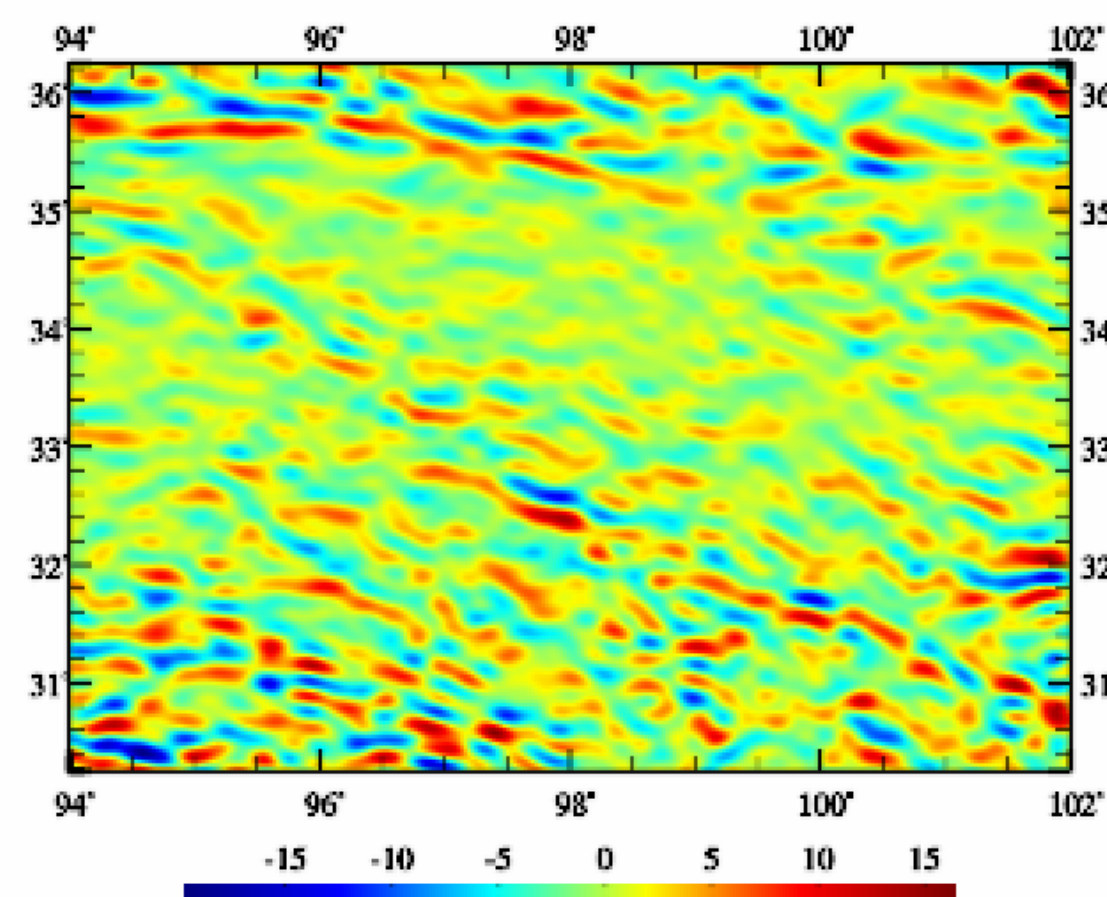
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667			
-2.2983	-2.5692	-2.5347	-2.2082	-1.6585	-0.9656	-0.1997	0.5849	1.3491
4.3900	4.3211	4.1201	3.8155	3.4521	3.0887	2.7923	2.6303	2.6604
8.1264	8.9502	9.5548	9.9047	9.9975	9.8642	9.5643	9.1752	8.7810
9.2803	9.4953	9.5263	9.3052	8.7862	7.9515	6.8129	5.4083	3.7964
-7.1600	-7.9825	-8.5270	-8.7875	-8.7662	-8.4717	-7.9195	-7.1314	-6.1356
3.4291	4.6825	5.7972	6.7385	7.4698	7.9529	8.1491	8.0220	7.5421

Extract vertical deflection

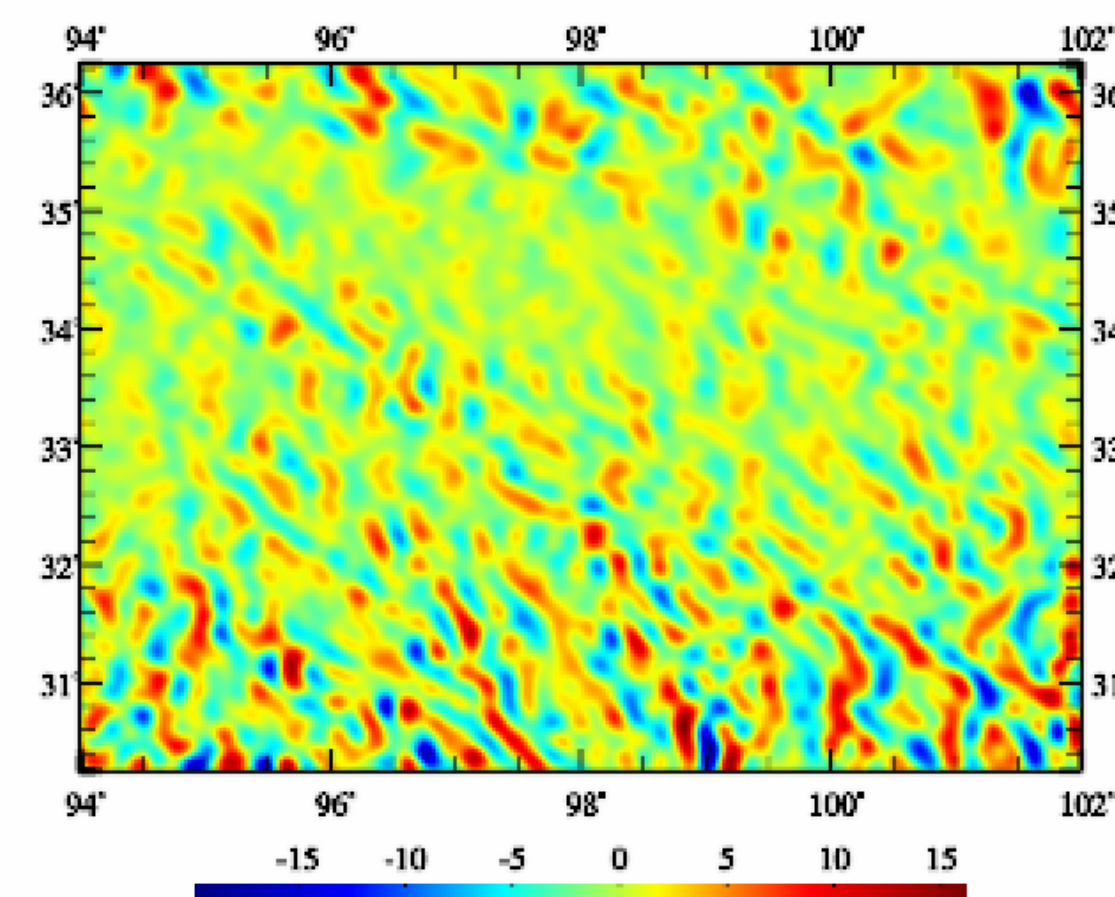
Plot



gravity disturbance (mGal)



vertical deflection (" , S)



vertical deflection (" , W)

Computation of External Vertical Deflection from Gravity Disturbance – 1D FFT

Computation of External Vertical Deflection from Gravity Anomaly

Computation of External Vertical Deflection from Gravity Disturbance

Vening-Meinesz integral formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Residual Gravity Disturbance Grid File

Select computation file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal Height Grid File

Select integral algorithm

1D FFT algorithm

Integral radius 180 km

>> Computation Process ** Operation Prompts

>> [Function] Computes the external residual vertical deflection (" , south and west components) via the generalized Vening-Meinesz integration, utilizing an ellipsoidal height grid of an equipotential boundary surface and the corresponding residual gravity disturbance grid (mGal)

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/landgeoidhgt.dat.

>> Open Residual Gravity Disturbance Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/resGMIgeoid541_1800.rga.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/landbmsurfhgt.dat.

>> Compute external residual vertical deflection by 1D FFT algorithm...

>> Save the results as C:/PAGravf4.5_win64en/examples/IntgenVeningMeinesz/rgatovmFFT1.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-12 15:17:57

>> Complete the computation of external vertical deflection!

>> Computation end time: 2026-04-12 15:18:51

Save the results as

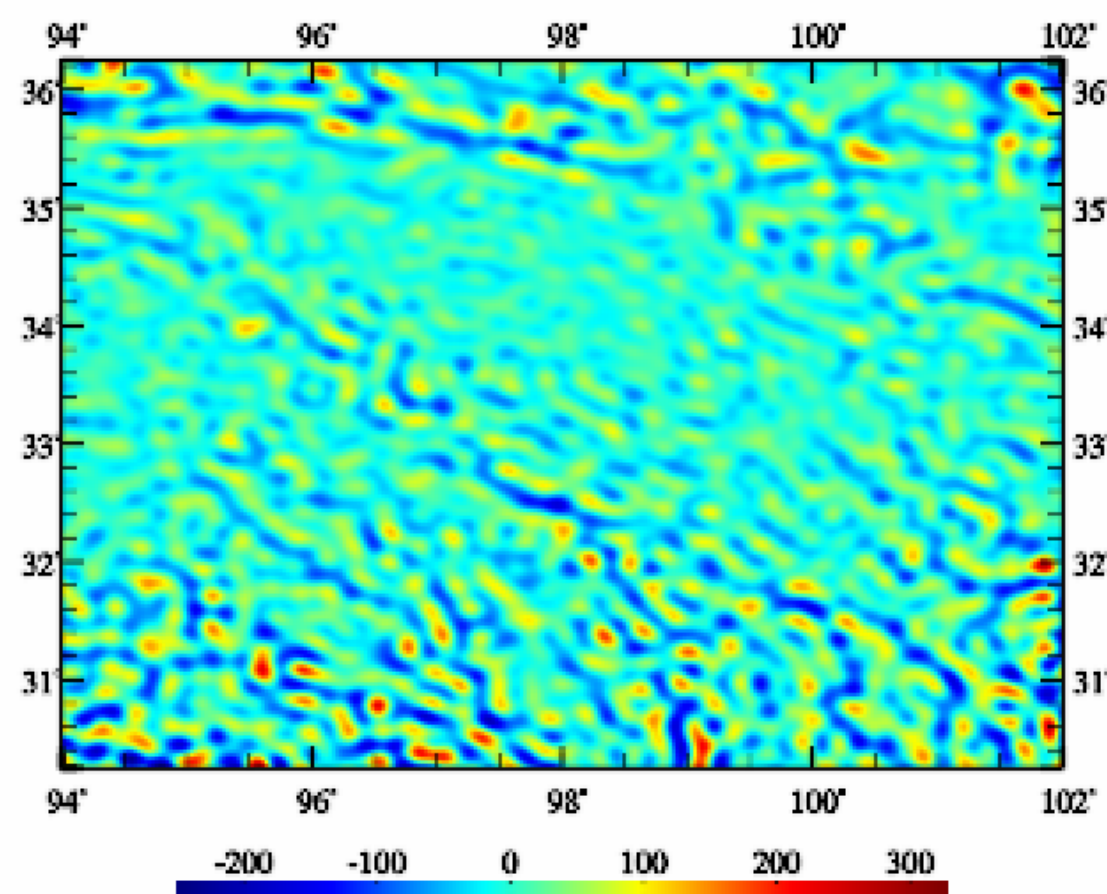
Import setting parameters

Start Computation

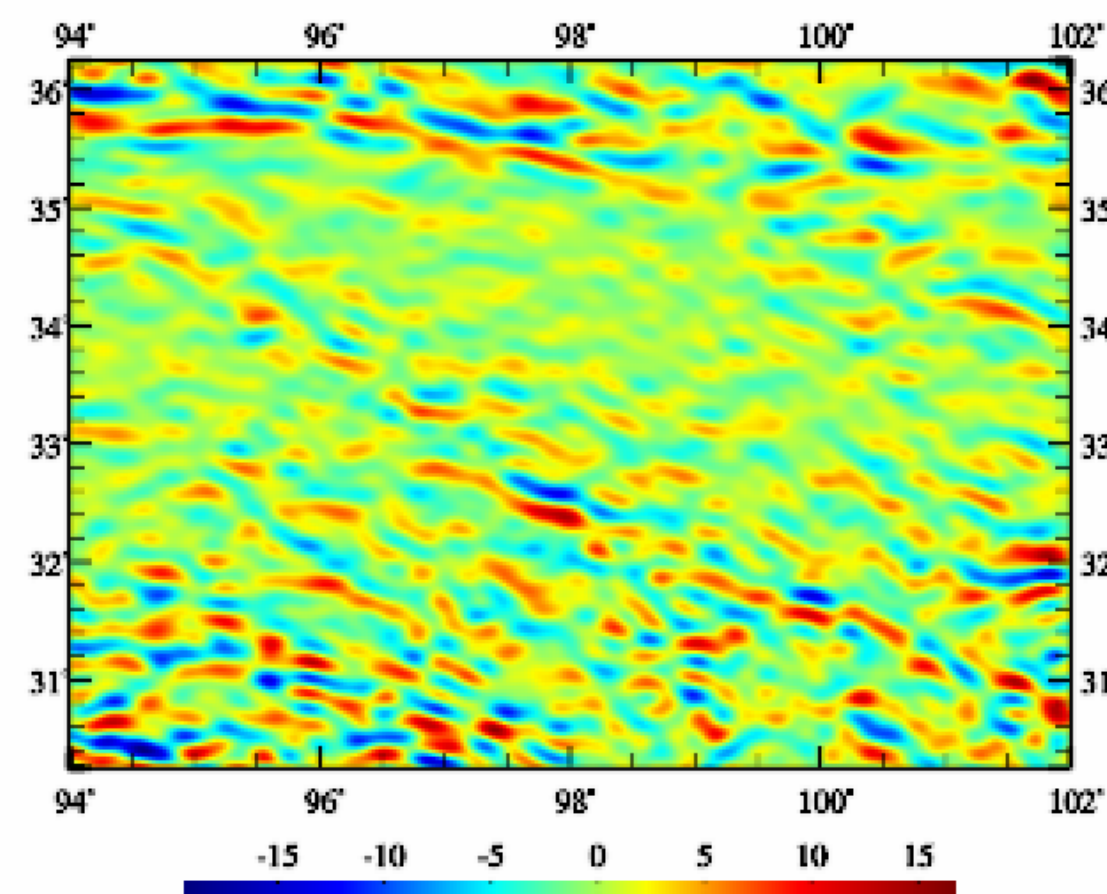
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667			
-2.2902	-2.5728	-2.5404	-2.2106	-1.6569	-0.9620	-0.1970	0.5837	1.3422
4.3438	4.2677	4.0572	3.7402	3.3616	2.9808	2.6656	2.4845	2.4963
7.8987	8.7168	9.3140	9.6544	9.7356	9.5887	9.2746	8.8722	8.4667
9.0128	9.2522	9.3100	9.1173	8.6277	7.8231	6.7148	5.3404	3.7583
-7.0502	-7.8673	-8.4108	-8.6744	-8.6593	-8.3737	-7.8319	-7.0551	-6.0709
3.4117	4.6540	5.7589	6.6925	7.4188	7.9003	8.0984	7.9768	7.5056

Extract vertical deflection

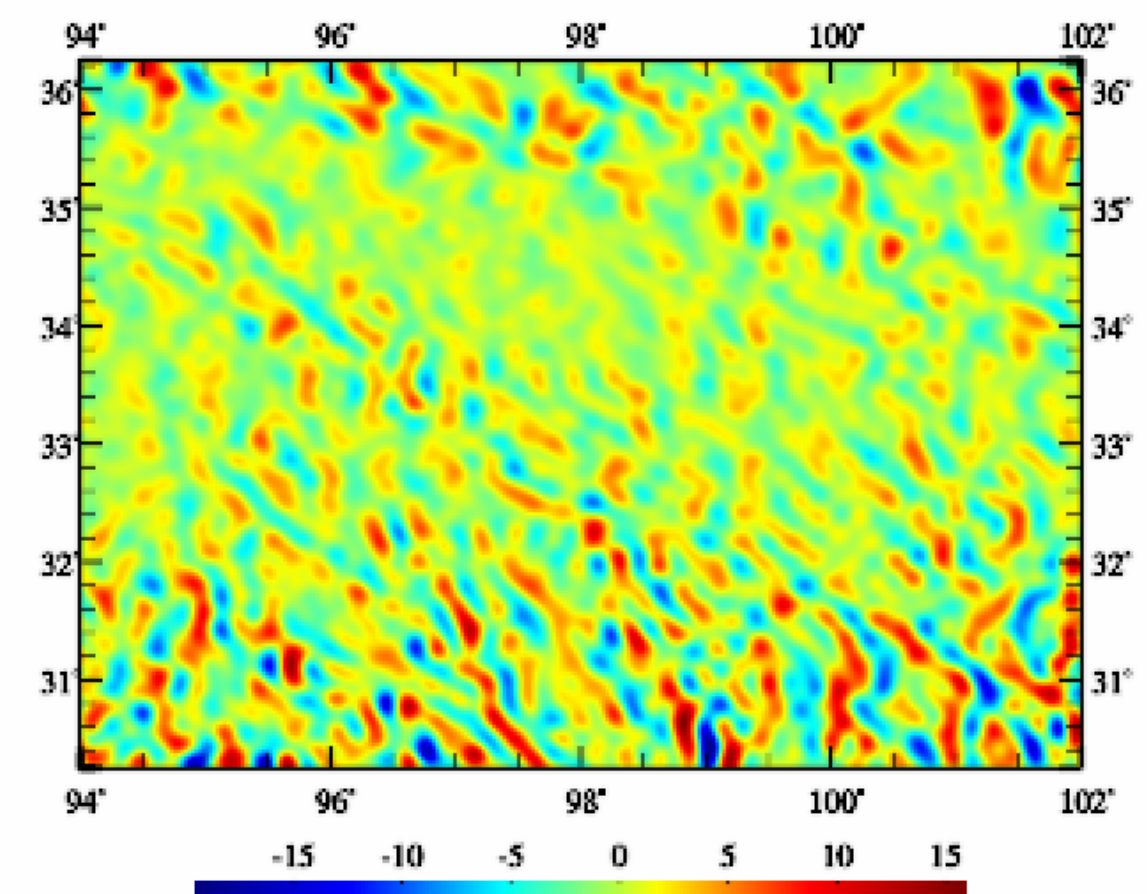
Plot



gravity disturbance (mGal)



vertical deflection (" , S)



vertical deflection (" , W)

Table 4.6: Statistics of Model Reference Truth (Unit: ")

Component	Mean	Std. Dev.	Min	Max
Ground Vertical Deflection (South, ξ)	0.0014	2.4951	-12.7789	14.2346
Ground Vertical Deflection (West, η)	0.0097	2.1772	-9.1577	10.4499

Table 4.8: Differences between Computed Results (Disturbance-based) and Reference Truth (")

Algorithm	Component	Mean	Std. Dev.	Min	Max
Numerical integral	South (ξ)	-0.0003	0.0274	-0.0872	0.0994
	West (η)	0.0005	0.0200	-0.0646	0.0836
FFT2	South (ξ)	0.0002	0.1072	-0.8692	0.7094
	West (η)	0.0010	0.0959	-0.6204	1.0981
FFT1	South (ξ)	0.0007	0.1058	-0.7910	0.6481
	West (η)	0.0010	0.0959	-0.6204	1.0981

The performance of the integration using gravity disturbance is slightly better than that using gravity anomaly.

Computation of Gravity Anomaly via Inverse Stokes Integration

Save as Import parameters Start Computation Save process Follow example

Computation of Gravity Anomaly via Inverse Stokes Integration

Computation of Gravity Disturbance via Inverse Hotine Integration

Computation via Inverse Vening-Meinesz Integration

Computation of External Anomalous Field Elements from Height Anomaly

Integral Formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Height Anomaly Grid File

Select computation point file format

discrete computation point file

Open Discrete Computation Point File

Set input point file format

number of rows of file header 1

>> Computation Process ** Operation Prompts

using either the Inverse Integration method or the Integral of Inverse Operation method.

>> Select the computation function from the 4 control buttons at the top of the interface...

>> [Function] Computes the residual gravity anomaly (mGal) on an equipotential boundary surface via Inverse Stokes Integration, utilizing the surface's ellipsoidal height grid and the corresponding residual height anomaly (m) grid.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/landgeoidhgt.dat.

>> Open Height Anomaly Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/resGMlgeoid541_1800.ksi.

>> Open Discrete Computation Point File C:/PAGravf4.5_win64en/examples/Integralgrainverse/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save the results as C:/PAGravf4.5_win64en/examples/Integralgrainverse/invstk.txt.

>> Appends two columns to the source file: Interpolated Ellipsoidal Height of the computation point, and Computed Residual Gravity Anomaly value.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-13 10:37:01

>> Complete the computation!

>> Computation end time: 2026-04-13 10:38:15

Integral radius 150 km

Save the results as

Import setting parameters

Start Computation

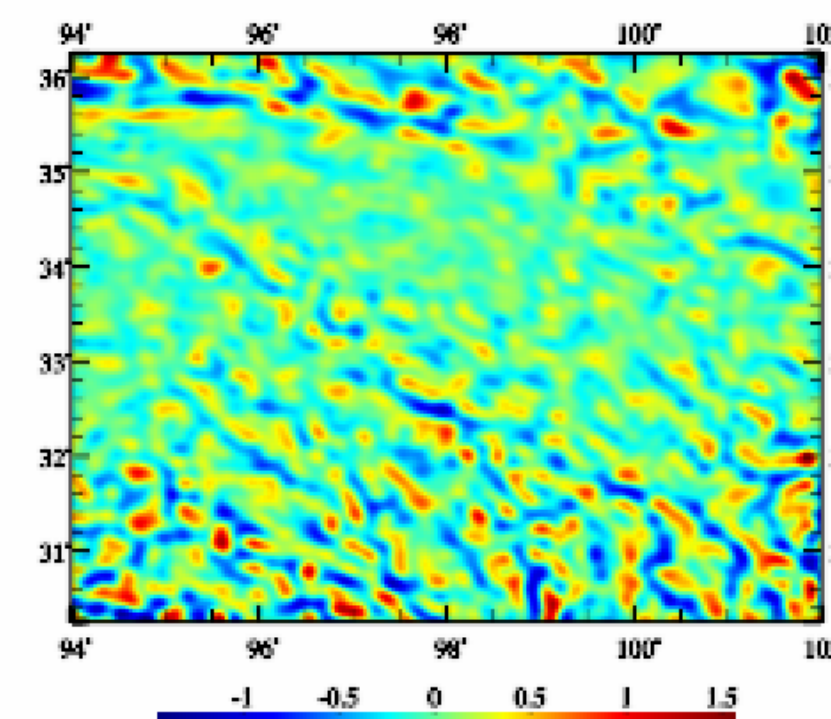
no lon(degree/decimal) lat ellipHeight (m)

1	97.008333	33.008333	3942.764	-37.2501	67.6375
2	97.025000	33.008333	3989.787	-37.2203	67.3423
3	97.041667	33.008333	4034.817	-37.1899	62.7938
4	97.058333	33.008333	4070.847	-37.1590	53.7191
5	97.075000	33.008333	4106.877	-37.1276	40.5471
6	97.091667	33.008333	4119.913	-37.0959	23.9785
7	97.108333	33.008333	4115.946	-37.0640	4.3967
8	97.125000	33.008333	4090.977	-37.0318	-16.7515
9	97.141667	33.008333	4070.007	-36.9990	-38.8489
10	97.158333	33.008333	3991.047	-36.9665	-60.2521
11	97.175000	33.008333	3985.070	-36.9327	-80.1620
12	97.191667	33.008333	3956.107	-36.8988	-96.9897
13	97.208333	33.008333	3965.137	-36.8642	-110.2047
14	97.225000	33.008333	3964.173	-36.8295	-118.0853
15	97.241667	33.008333	3983.205	-36.7943	-120.4004
16	97.258333	33.008333	3953.251	-36.7595	-116.0276
17	97.275000	33.008333	4016.279	-36.7238	-104.8662
18	97.291667	33.008333	4054.318	-36.6883	-87.1099
19	97.308333	33.008333	4090.360	-36.6528	-63.3095

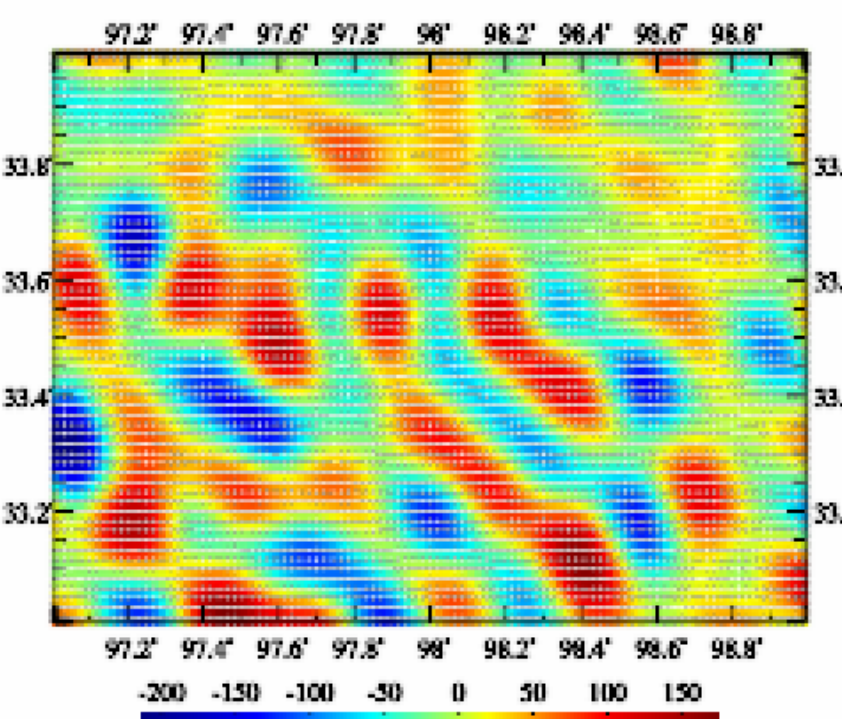
Ignore the ellipsoidal height

Extract result element

Plot



height anomaly (m)



gravity anomaly (mGal)

Integral of Inverse Operation: A rigorous solution to the Stokes-type Boundary Value Problem. It strictly requires the integrand (height anomaly or vertical deflection) to reside on an equipotential boundary surface.

Inverse Integration: Employs a hybrid algorithm combining Poisson integration with the differentiation of anomalous field elements. This approach is more flexible and does not mandate the boundary surface to be an equipotential boundary surface.

Computation of Gravity Anomaly via Inverse Stokes Integration – 2D FFT

Computation of Gravity Anomaly via Inverse Stokes Integration

Computation of Gravity Disturbance via Inverse Hotine Integration

Computation via Inverse Vening-Meinesz Integration

Computation of External Anomalous Field Elements from Height Anomaly

Integral Formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Height Anomaly Grid File

Select computation point file format

surface ellipsoidal height grid file

Select integral algorithm

2D FFT algorithm

>> Computation Process ** Operation Prompts

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-13 11:02:42

>> Complete the computation!

>> Computation end time: 2026-04-13 11:11:09

>> The program computation process have saved to the file C:/PAGravf4.5_win64en/examples/Integralgrainverse/process.txt!

>> [Function] Computes the residual gravity anomaly (mGal) on an equipotential boundary surface via Inverse Stokes Integration, utilizing the surface's ellipsoidal height grid and the corresponding residual height anomaly (m) grid.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/landgeoidhgt.dat.

>> Open Height Anomaly Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/resGMIgeoid541_1800.ksi.

>> Compute by 2D FFT algorithm...

>> Save the results as C:/PAGravf4.5_win64en/examples/Integralgrainverse/invstokesFFT2.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-13 11:15:05

>> Complete the computation!

>> Computation end time: 2026-04-13 11:15:07

Integral radius 150 km

Save the results as

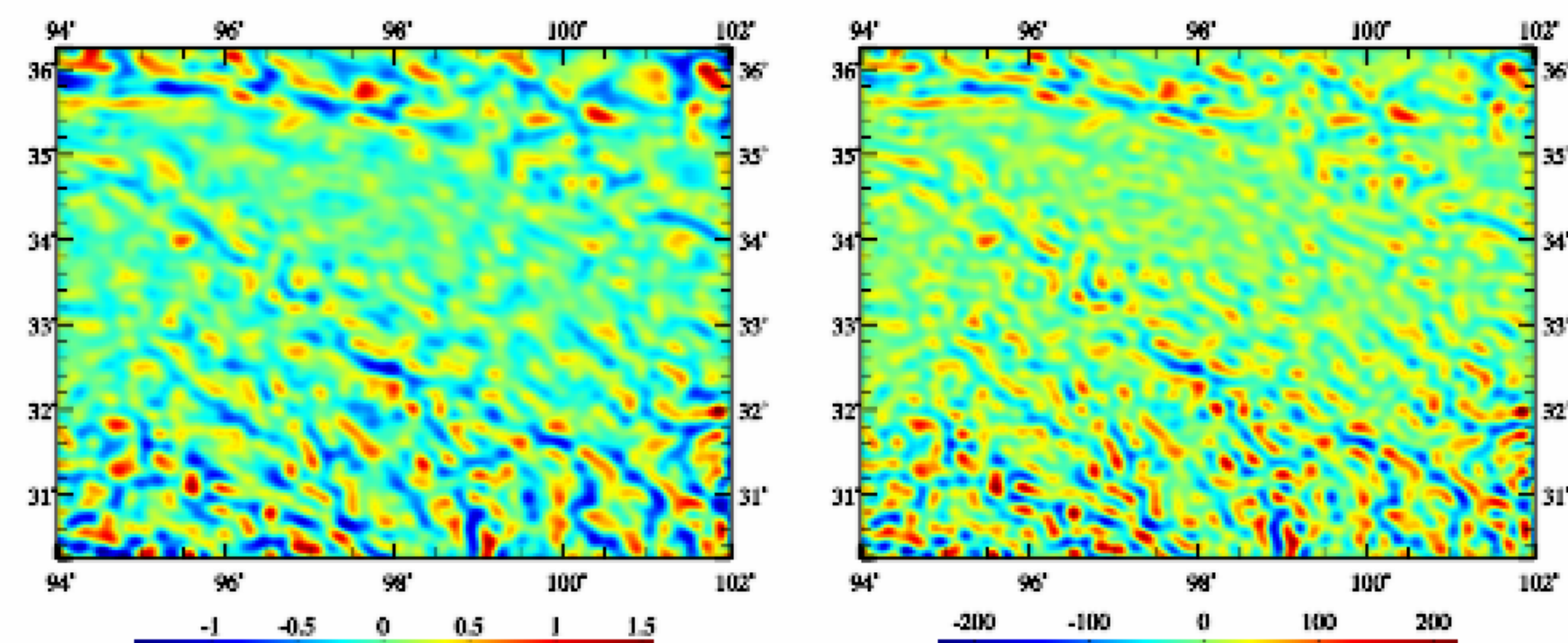
Import setting parameters

Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
-39.6477	-31.7555	-23.0010	-13.3460	-3.1905	7.3758
68.7137	66.2175	63.6459	61.8294	61.1919	61.9060
85.4454	88.5755	92.8940	98.3290	104.2779	109.8428
-20.5549	-42.5148	-60.3258	-73.0308	-80.1262	-81.5883
1.5651	3.2947	3.7253	3.2002	2.3803	1.7674
50.4526	51.4144	47.9648	39.6026	26.4345	9.2700
41.7132	78.7630	112.9903	140.3739	157.4710	161.9463
-116.6430	-108.3396	-89.6918	-63.5529	-33.3099	-2.4602
34.2004	25.2184	18.4197	13.7955	11.4404	10.9439
58.3004	57.9660	54.0121	46.0891	34.5079	20.1147
12.7681	30.0333	44.9480	55.6834	61.1593	60.9161
-35.4209	-39.1119	-43.6884	-50.2486	-59.2394	-70.9157
-141.6149	-133.2776	-123.9171	-114.2731	-104.7800	-95.4982
36.9525	44.8229	47.4242	44.3201	35.6144	22.1620
-16.0591	5.5549	28.0217	49.7083	68.9606	84.7837
53.8984	38.1050	20.1477	0.2555	-21.0772	-43.0778
-54.1878	-31.6831	-10.0942	8.7349	23.4213	33.2218
10.4447	14.6742	19.5786	23.8960	26.8234	27.6294

Extract result element

Plot



height anomaly (m)

gravity anomaly (mGal)

Integral of Inverse Operation: A rigorous solution to the Stokes-type Boundary Value Problem. It strictly requires the integrand (height anomaly or vertical deflection) to reside on an equipotential boundary surface.

Inverse Integration: Employs a hybrid algorithm combining Poisson integration with the differentiation of anomalous field elements. This approach is more flexible and does not mandate the boundary surface to be an equipotential boundary surface.

Computation of Gravity Anomaly via Inverse Stokes Integration – 1D FFT

Computation of Gravity Anomaly via Inverse Stokes Integration

Computation of Gravity Disturbance via Inverse Hotine Integration

Computation via Inverse Vening-Meinesz Integration

Computation of External Anomalous Field Elements from Height Anomaly

Integral Formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Height Anomaly Grid File

Select computation point file format

surface ellipsoidal height grid file

Select integral algorithm

1D FFT algorithm

>> Computation Process ** Operation Prompts

```
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2026-04-13 10:40:32
>> Complete the computation!
>> Computation end time: 2026-04-13 10:40:33
>> [Function] Computes the residual gravity anomaly (mGal) on an equipotential boundary surface via Inverse Stokes Integration, utilizing the surface's ellipsoidal height grid and the corresponding residual height anomaly (m) grid.
>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/landgeoidhgt.dat.
>> Open Height Anomaly Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/resGMlgeoid541_1800.ksi.
>> Compute by 1D FFT algorithm...
>> Save the results as C:/PAGravf4.5_win64en/examples/Integralgrainverse/invstokesFFT1.dat.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2026-04-13 10:41:16
>> Complete the computation!
>> Computation end time: 2026-04-13 10:41:48
```

Save computation process as

Integral radius 150 km

Save the results as

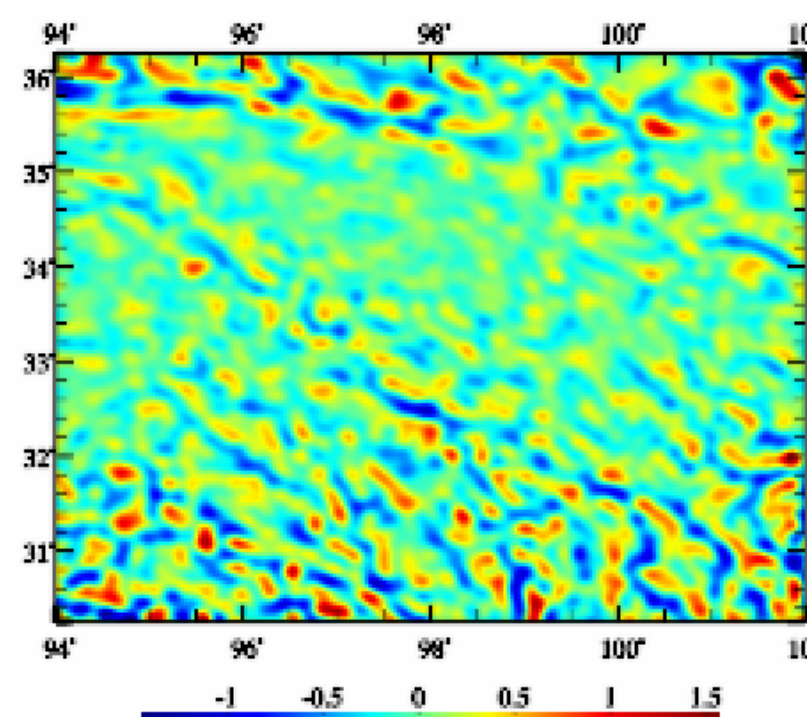
Import setting parameters

Start Computation

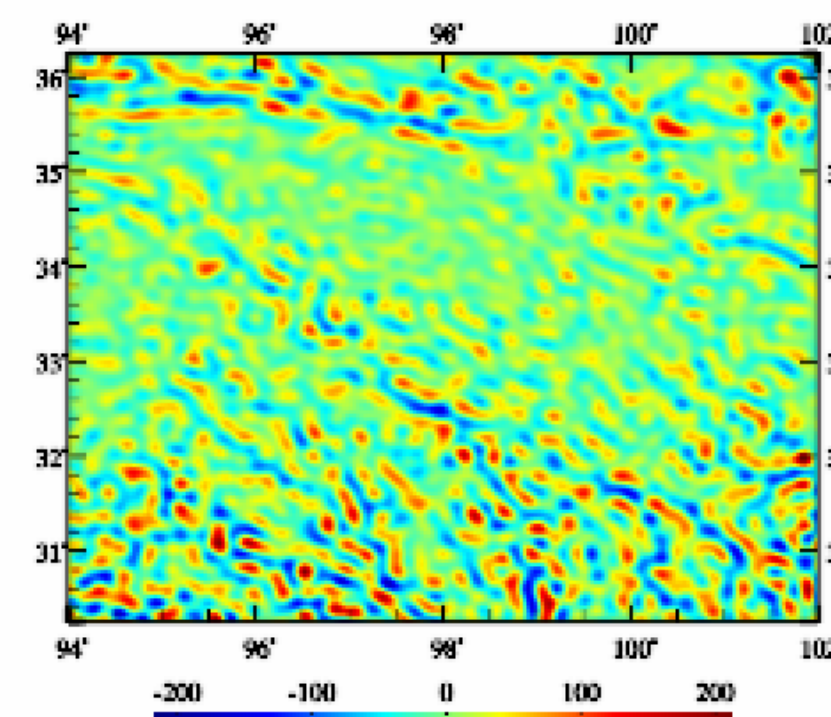
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
-38.8436	-31.6669	-23.1151	-13.5783	-3.5313	6.8983
67.0584	64.6259	62.0594	60.1520	59.3231	59.7621
83.3528	86.6034	90.8779	96.0863	101.6546	106.7426
-18.5990	-39.3488	-56.1960	-68.2546	-75.0628	-76.6044
-0.4045	1.1473	1.5172	1.0340	0.3231	-0.1369
49.1110	50.2744	47.2085	39.4367	27.0601	10.8463
39.0631	73.8112	105.9681	131.7520	147.9208	152.2701
-109.3804	-101.8005	-84.4823	-60.0815	-31.7596	-2.7911
33.1716	24.7000	18.2457	13.8216	11.5360	11.0218
56.3368	56.0106	52.2388	44.6968	33.6809	19.9907
11.0672	27.2974	41.4079	51.6809	57.0925	57.1928
-32.8821	-36.8786	-41.7506	-48.4961	-57.5067	-68.9966
-137.5939	-129.7212	-120.8182	-111.5682	-102.3731	-93.2874
35.8661	43.3681	45.8191	42.8191	34.4810	21.6239
-16.0694	4.4015	25.7657	46.4817	64.9832	80.3144
52.9143	37.6282	20.2416	0.9994	-19.6029	-40.8092
-52.5338	-31.1322	-10.5583	7.4401	21.5472	31.0469
9.8553	13.7442	18.3160	22.3948	25.2230	26.1047

Extract result element

Plot



height anomaly (m)



gravity anomaly (mGal)

- Integral of Inverse Operation: A rigorous solution to the Stokes-type Boundary Value Problem. It strictly requires the integrand (height anomaly or vertical deflection) to reside on an equipotential boundary surface.
- Inverse Integration: Employs a hybrid algorithm combining Poisson integration with the differentiation of anomalous field elements. This approach is more flexible and does not mandate the boundary surface to be an equipotential boundary surface.

Table 4.10: Statistics of Model Reference Truth (Unit: mGal)

Item	Mean	Std. Dev.	Min	Max
Residual Gravity Anomalies (Deg 541 – 1800)	0.4334	34.0852	-170.9556	177.8780

Table 4.11: Differences between Inverse Operation Integration Results and Reference Truth (Unit: mGal)

Algorithm	mean	standard deviation	minimum	maximum
Numerical Integral	-0.0415	2.9285	-15.2734	14.6970
FFT2	-0.0433	2.8208	-11.9201	13.3607
FFT1	-0.0444	3.0509	-15.7950	15.5242

Algorithm Performance: The ratio of the standard deviation of Differences in Table 4.11 to the standard deviation of Reference Truths in Table 4.10 reflects the approximation capability of the corresponding algorithm.

Computation of Gravity Anomaly via Inverse Hotine Integration

Computation of Gravity Anomaly via Inverse Stokes Integration

Computation of Gravity Disturbance via Inverse Hotine Integration

Computation via Inverse Vening-Meinesz Integration

Computation of External Anomalous Field Elements from Height Anomaly

Integral Formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Height Anomaly Grid File

Select computation point file format

discrete computation point file

Open Discrete Computation Point File

Set input point file format

number of rows of file header 1

>> Computation Process ** Operation Prompts

>> Computation start time: 2026-04-13 10:41:16

>> Complete the computation!

>> Computation end time: 2026-04-13 10:41:48

>> [Function] Computes the residual gravity disturbance (mGal) on the surface via the Integral of Inverse Hotine Operation, utilizing the ellipsoidal height grid of the equipotential boundary surface and the corresponding residual height anomaly (m) grid.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Integralgrainverse/landgeoidhgt.dat.

>> Open Height Anomaly Grid File C:/PAGrav4.5_win64en/examples/Integralgrainverse/resGMIgeoid541_1800.ksi.

>> Open Discrete Computation Point File C:/PAGrav4.5_win64en/examples/Integralgrainverse/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save the results as C:/PAGrav4.5_win64en/examples/Integralgrainverse/invhtn.txt.

>> Appends columns for interpolated Ellipsoidal Height and computed Residual Gravity Disturbance.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-13 10:43:55

>> Complete the computation!

>> Computation end time: 2026-04-13 10:45:10

Integral radius 150 km

Save the results as

Import setting parameters

Start Computation

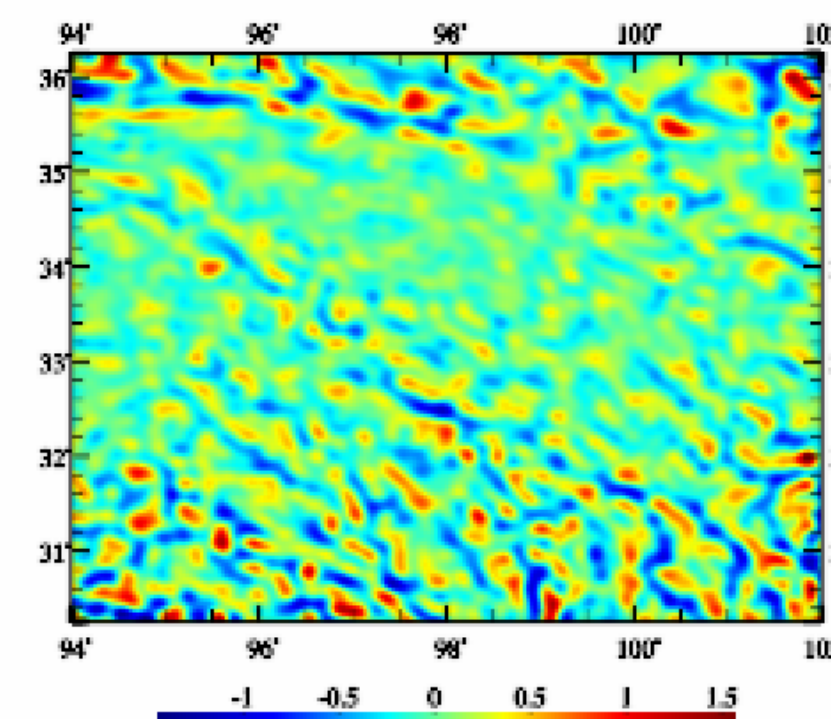
no lon(degree/decimal) lat ellipHeight (m)

1	97.008333	33.008333	3942.764	-37.2501	67.6349
2	97.025000	33.008333	3989.787	-37.2203	67.3400
3	97.041667	33.008333	4034.817	-37.1899	62.7924
4	97.058333	33.008333	4070.847	-37.1590	53.7191
5	97.075000	33.008333	4106.877	-37.1276	40.5490
6	97.091667	33.008333	4119.913	-37.0959	23.9828
7	97.108333	33.008333	4115.946	-37.0640	4.4036
8	97.125000	33.008333	4090.977	-37.0318	-16.7417
9	97.141667	33.008333	4070.007	-36.9990	-38.8361
10	97.158333	33.008333	3991.047	-36.9665	-60.2365
11	97.175000	33.008333	3985.070	-36.9327	-80.1437
12	97.191667	33.008333	3956.107	-36.8988	-96.9693
13	97.208333	33.008333	3965.137	-36.8642	-110.1827
14	97.225000	33.008333	3964.173	-36.8295	-118.0627
15	97.241667	33.008333	3983.205	-36.7943	-120.3779
16	97.258333	33.008333	3953.251	-36.7595	-116.0064
17	97.275000	33.008333	4016.279	-36.7238	-104.8475
18	97.291667	33.008333	4054.318	-36.6883	-87.0946
19	97.308333	33.008333	4090.360	-36.6528	-63.2988

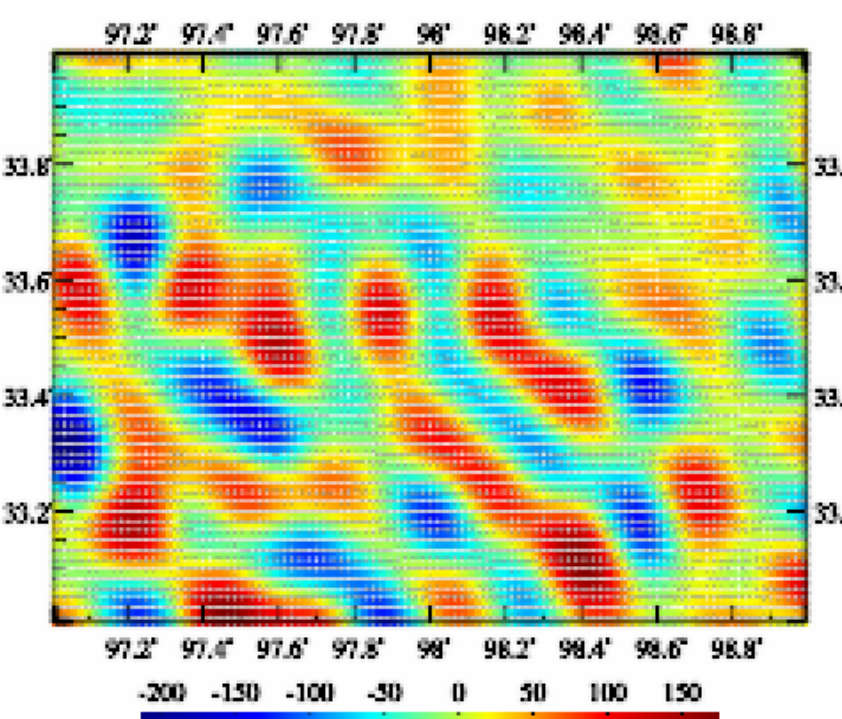
Ignore the ellipsoidal height

Extract result element

Plot



height anomaly (m)



gravity disturbance (mGal)

Integral of Inverse Operation: A rigorous solution to the Stokes-type Boundary Value Problem. It strictly requires the integrand (height anomaly or vertical deflection) to reside on an equipotential boundary surface.

Inverse Integration: Employs a hybrid algorithm combining Poisson integration with the differentiation of anomalous field elements. This approach is more flexible and does not mandate the boundary surface to be an equipotential boundary surface.

Computation of Gravity Disturbance via Inverse Hotine Integration – 2D FFT

Computation of Gravity Anomaly via Inverse Stokes Integration

Computation of Gravity Disturbance via Inverse Hotine Integration

Computation via Inverse Vening-Meinesz Integration

Computation of External Anomalous Field Elements from Height Anomaly

Integral Formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Height Anomaly Grid File

Select computation point file format

surface ellipsoidal height grid file

Select integral algorithm

2D FFT algorithm

>> Computation Process ** Operation Prompts

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-13 10:43:55

>> Complete the computation!

>> Computation end time: 2026-04-13 10:45:10

>> [Function] Computes the residual gravity disturbance (mGal) on the surface via the Integral of Inverse Hotine Operation, utilizing the ellipsoidal height grid of the equipotential boundary surface and the corresponding residual height anomaly (m) grid.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/landgeoidhgt.dat.

>> Open Height Anomaly Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/resGMlgeoid541_1800.ksi.

>> Compute by 2D FFT algorithm...

>> Save the results as C:/PAGravf4.5_win64en/examples/Integralgrainverse/invhotineFFT2.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-13 10:47:17

>> Complete the computation!

>> Computation end time: 2026-04-13 10:47:19

Integral radius 150 km

Save the results as

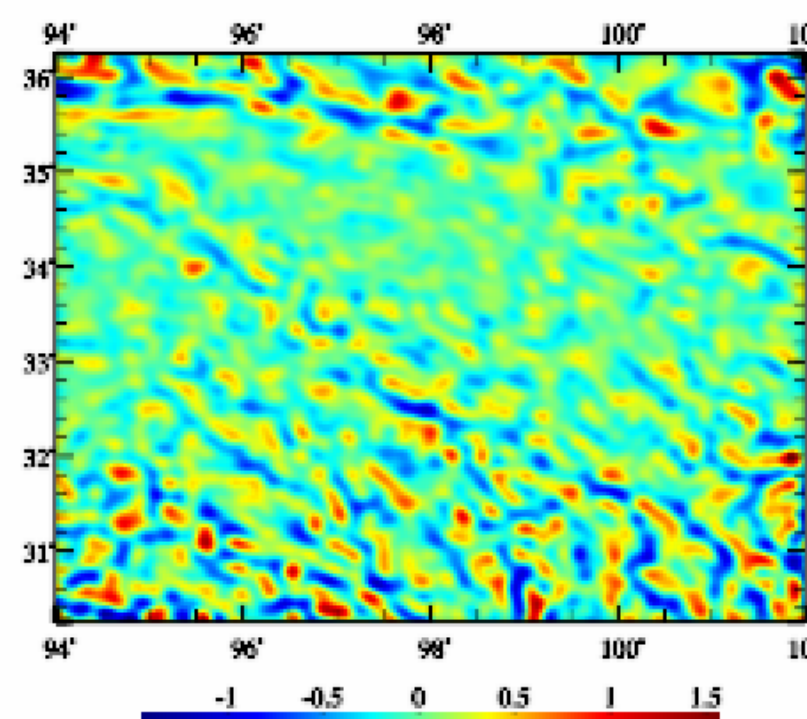
Import setting parameters

Start Computation

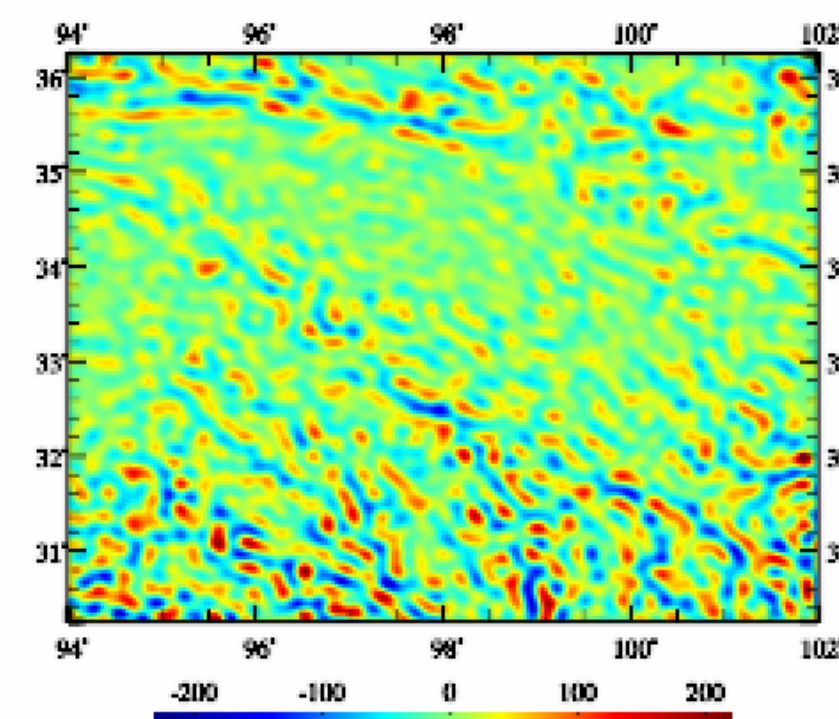
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
-39.6723	-31.7828	-23.0297	-13.3751	-3.2193	7.3476
68.6958	66.2006	63.6293	61.8125	61.1742	61.8872
85.4591	88.5938	92.9155	98.3523	104.3015	109.8659
-20.4677	-42.4183	-60.2231	-72.9255	-80.0225	-81.4903
1.5178	3.2397	3.6650	3.1371	2.3165	1.7048
50.4502	51.4207	47.9804	39.6280	26.4697	9.3145
41.6604	78.6860	112.8925	140.2611	157.3504	161.8263
-116.5733	-108.2735	-89.6355	-63.5110	-33.2851	-2.4532
34.1834	25.2076	18.4141	13.7940	11.4418	10.9471
58.2936	57.9596	54.0072	46.0869	34.5094	20.1203
12.7326	29.9892	44.8982	55.6314	61.1093	60.8720
-35.3719	-39.0639	-43.6423	-50.2046	-59.1971	-70.8742
-141.5860	-133.2540	-123.8984	-114.2587	-104.7690	-95.4896
36.9439	44.8132	47.4147	44.3125	35.6102	22.1623
-16.0796	5.5231	27.9794	49.6573	68.9034	84.7234
53.9134	38.1295	20.1815	0.2984	-21.0256	-43.0180
-54.1560	-31.6638	-10.0865	8.7326	23.4111	33.2061
10.4285	14.6561	19.5591	23.8760	26.8041	27.6124

Extract result element

Plot



height anomaly (m)



gravity disturbance (mGal)

Integral of Inverse Operation: A rigorous solution to the Stokes-type Boundary Value Problem. It strictly requires the integrand (height anomaly or vertical deflection) to reside on an equipotential boundary surface.

Inverse Integration: Employs a hybrid algorithm combining Poisson integration with the differentiation of anomalous field elements. This approach is more flexible and does not mandate the boundary surface to be an equipotential boundary surface.

Computation of Gravity Disturbance via Inverse Hotine Integration – 1D FFT

Computation of Gravity Anomaly via Inverse Stokes Integration

Computation of Gravity Disturbance via Inverse Hotine Integration

Computation via Inverse Vening-Meinesz Integration

Computation of External Anomalous Field Elements from Height Anomaly

Integral Formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Height Anomaly Grid File

Select computation point file format

surface ellipsoidal height grid file

Select integral algorithm

1D FFT algorithm

>> Computation Process ** Operation Prompts

```
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2026-04-13 10:47:17
>> Complete the computation!
>> Computation end time: 2026-04-13 10:47:19
>> [Function] Computes the residual gravity disturbance (mGal) on the surface via the Integral of Inverse Hotine Operation, utilizing the ellipsoidal height grid of the equipotential boundary surface and the corresponding residual height anomaly (m) grid.
>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/landgeoidhgt.dat.
>> Open Height Anomaly Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/resGMlgeoid541_1800.ksi.
>> Compute by 1D FFT algorithm...
>> Save the results as C:/PAGravf4.5_win64en/examples/Integralgrainverse/invhotineFFT1.dat.
>> The parameter settings have been entered into the system!
** Click the [Start Computation] control button, or the [Start Computation] tool button...
>> Computation start time: 2026-04-13 10:48:47
>> Complete the computation!
>> Computation end time: 2026-04-13 10:49:21
```

Save computation process as

Integral radius 150 km

Save the results as

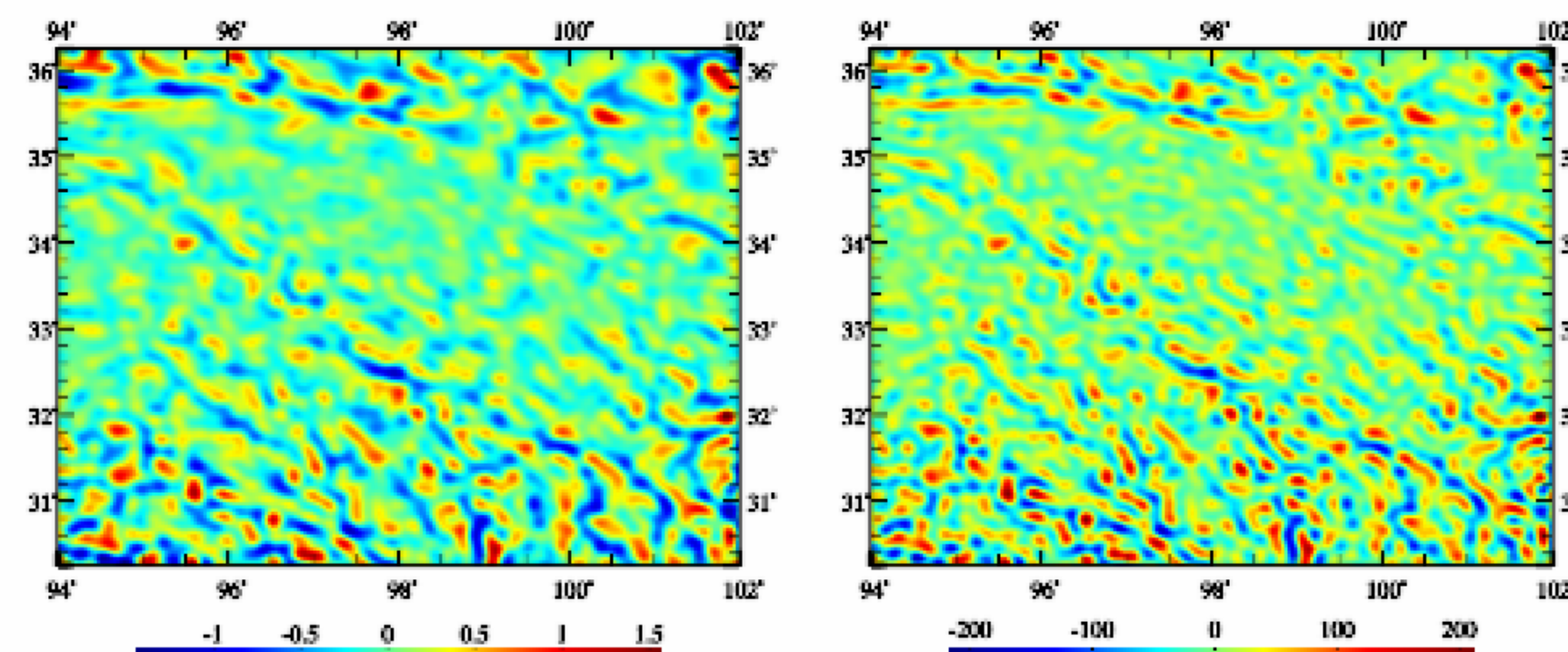
Import setting parameters

Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
-38.8682	-31.6941	-23.1438	-13.6073	-3.5601	6.8701
67.0406	64.6090	62.0428	60.1351	59.3054	59.7434
83.3665	86.6217	90.8994	96.1095	101.6782	106.7658
-18.5118	-39.2524	-56.0933	-68.1493	-74.9591	-76.5065
-0.4518	1.0922	1.4569	0.9709	0.2593	-0.1994
49.1086	50.2807	47.2241	39.4621	27.0953	10.8908
39.0104	73.7341	105.8704	131.6391	147.8002	152.1501
-109.3107	-101.7344	-84.4260	-60.0396	-31.7348	-2.7841
33.1546	24.6893	18.2401	13.8202	11.5374	11.0251
56.3300	56.0042	52.2339	44.6946	33.6823	19.9963
11.0317	27.2532	41.3580	51.6289	57.0424	57.1487
-32.8331	-36.8306	-41.7045	-48.4521	-57.4644	-68.9551
-137.5650	-129.6975	-120.7996	-111.5539	-102.3621	-93.2787
35.8576	43.3584	45.8096	42.8115	34.4767	21.6241
-16.0898	4.3696	25.7234	46.4307	64.9261	80.2540
52.9293	37.6527	20.2755	1.0423	-19.5513	-40.7495
-52.5020	-31.1128	-10.5506	7.4378	21.5370	31.0312
9.8390	13.7261	18.2965	22.3748	25.2038	26.0876

Extract result element

Plot



height anomaly (m)

gravity disturbance (mGal)

Integral of Inverse Operation: A rigorous solution to the Stokes-type Boundary Value Problem. It strictly requires the integrand (height anomaly or vertical deflection) to reside on an equipotential boundary surface.

Inverse Integration: Employs a hybrid algorithm combining Poisson integration with the differentiation of anomalous field elements. This approach is more flexible and does not mandate the boundary surface to be an equipotential boundary surface.

Table 4.12: Statistics of Model Reference Truth (Unit: mGal)

Item	Mean	Std. Dev.	Min	Max
Residual Gravity Disturbances (Deg 541 – 1800)	0.4334	34.0852	-170.9556	177.8780

Table 4.13: Differences between Inverse Operation Integration Results and Reference Truth (Unit: mGal)

Algorithm	Mean	Std. Dev.	Min	Max
Numerical Integral	-0.0415	2.9285	-15.2734	14.6970
FFT2	-0.0433	2.8208	-11.9201	13.3607
FFT1	-0.0444	3.0509	-15.7950	15.5242

Algorithm Performance: The ratio of the standard deviation of Differences in Table 4.13 to the standard deviation of Reference Truths in Table 4.12 reflects the approximation capability of the corresponding algorithm.

The Inverse Hotine numerical integration yields a standard deviation of ~2.93 mGal, comparable to the Inverse Stokes method but with a slightly reduced maximum error range (± 9.7 mGal vs. ± 15.3 mGal). This suggests marginally better numerical stability when solving for gravity disturbances directly.

Computation via Inverse Vening-Meinesz Integration

Save as Import parameters Start Computation Save process Follow example

Computation of Gravity Anomaly via Inverse Stokes Integration

Computation of Gravity Disturbance via Inverse Hotine Integration

Computation via Inverse Vening-Meinesz Integration

Computation of External Anomalous Field Elements from Height Anomaly

Integral Formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Vertical Deflection Vector Grid File

Select computation point file format

discrete computation point file

Open Discrete Computation Point File

Set input point file format

number of rows of file header 1

>> Computation Process ** Operation Prompts

>> Computation end time: 2026-04-13 10:49:21

>> [Function] Computes the residual height anomaly (m), gravity anomaly (mGal), and gravity disturbance (mGal) on an equipotential boundary surface. This is achieved via Inverse Vening-Meinesz Integration, utilizing the surface's ellipsoidal height grid and the corresponding residual vertical deflection vector grid (" , south and west components).

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/landgeoidhgt.dat.

>> Open Vertical Deflection Vector Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/resGMIgeoid541_1800.dft.

>> Open Discrete Computation Point File C:/PAGravf4.5_win64en/examples/Integralgrainverse/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save the results as C:/PAGravf4.5_win64en/examples/Integralgrainverse/invVM.txt.

>> Appends four columns to the source file: Interpolated Ellipsoidal Height, Residual Height Anomaly, Gravity Anomaly, and Gravity Disturbance.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-13 10:51:09

>> Complete the computation!

>> Computation end time: 2026-04-13 10:52:39

Integral radius 150 km

Save the results as

Import setting parameters

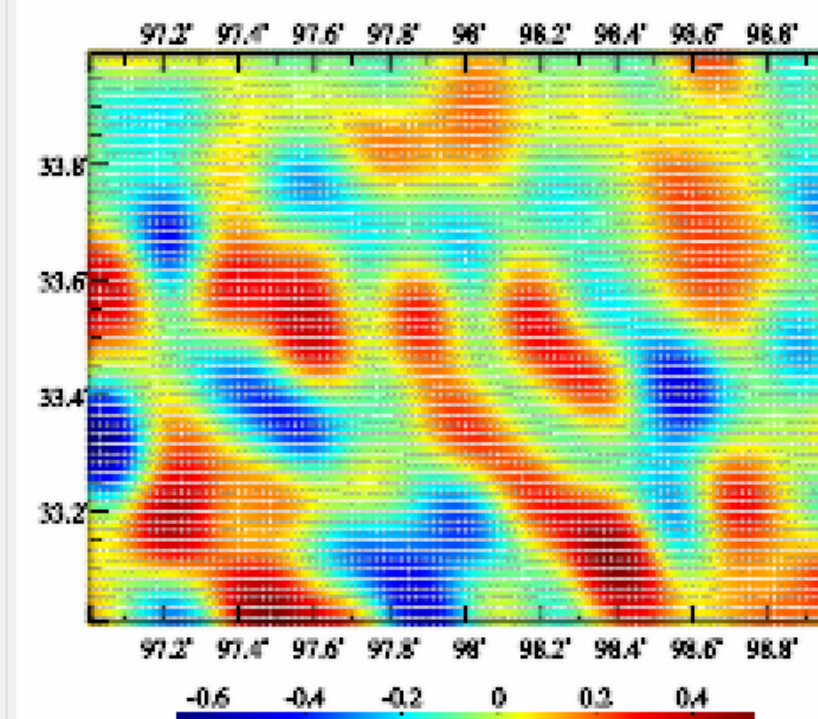
Start Computation

no	lon(degree/decimal)	lat	ellipHeight (m)			
1	97.008333	33.008333	3942.764	-37.2501	0.0986	2
2	97.025000	33.008333	3989.787	-37.2203	0.0846	2
3	97.041667	33.008333	4034.817	-37.1899	0.0639	2
4	97.058333	33.008333	4070.847	-37.1590	0.0366	1
5	97.075000	33.008333	4106.877	-37.1276	0.0033	1
6	97.091667	33.008333	4119.913	-37.0959	-0.0351	-
7	97.108333	33.008333	4115.946	-37.0640	-0.0772	-
8	97.125000	33.008333	4090.977	-37.0318	-0.1213	-1
9	97.141667	33.008333	4070.007	-36.9990	-0.1655	-1
10	97.158333	33.008333	3991.047	-36.9665	-0.2077	-2
11	97.175000	33.008333	3985.070	-36.9327	-0.2458	-3
12	97.191667	33.008333	3956.107	-36.8988	-0.2773	-4
13	97.208333	33.008333	3965.137	-36.8642	-0.2999	-4
14	97.225000	33.008333	3964.173	-36.8295	-0.3115	-5
15	97.241667	33.008333	3983.205	-36.7943	-0.3100	-5
16	97.258333	33.008333	3953.251	-36.7595	-0.2941	-5
17	97.275000	33.008333	4016.279	-36.7238	-0.2629	-4
18	97.291667	33.008333	4054.318	-36.6883	-0.2165	-3

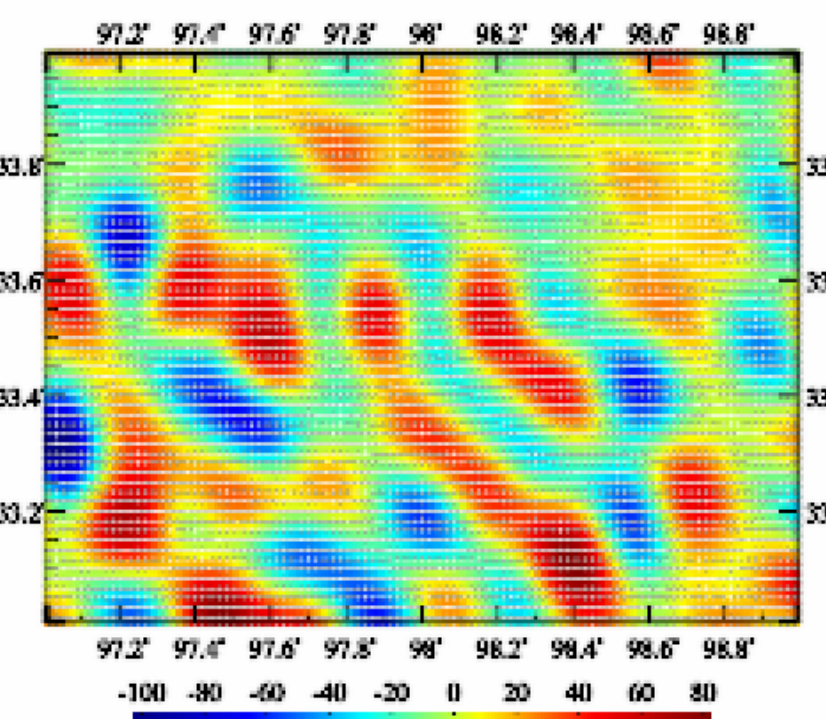
Ignore the ellipsoidal height

Extract result element

Plot



height anomaly (m)



gravity disturbance (mGal)

Integral of Inverse Operation: A rigorous solution to the Stokes-type Boundary Value Problem. It strictly requires the integrand (height anomaly or vertical deflection) to reside on an equipotential boundary surface.

Inverse Integration: Employs a hybrid algorithm combining Poisson integration with the differentiation of anomalous field elements. This approach is more flexible and does not mandate the boundary surface to be an equipotential boundary surface.

Computation via Inverse Vening-Meinesz Integration – 2D FFT

Computation of Gravity Anomaly via Inverse Stokes Integration

Computation of Gravity Disturbance via Inverse Hotine Integration

Computation via Inverse Vening-Meinesz Integration

Computation of External Anomalous Field Elements from Height Anomaly

Integral Formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Vertical Deflection Vector Grid File

Select computation point file format

surface ellipsoidal height grid file

Select integral algorithm

2D FFT algorithm

>> Computation Process ** Operation Prompts

>> Computation end time: 2026-04-13 10:52:39

>> [Function] Computes the residual height anomaly (m), gravity anomaly (mGal), and gravity disturbance (mGal) on an equipotential boundary surface. This is achieved via Inverse Vening-Meinesz Integration, utilizing the surface's ellipsoidal height grid and the corresponding residual vertical deflection vector grid (" , south and west components).

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/landgeoidhgt.dat.

>> Open Vertical Deflection Vector Grid File C:/PAGravf4.5_win64en/examples/Integralgrainverse/resGMIgeoid541_1800.dft.

>> Compute by 2D FFT algorithm...

>> Save the results as C:/PAGravf4.5_win64en/examples/Integralgrainverse/invVMFFT2.txt.

>> Save Height Anomaly Grid as C:/PAGravf4.5_win64en/examples/Integralgrainverse/invVMFFT2.ksi.

>> Save Gravity Disturbance Grid as C:/PAGravf4.5_win64en/examples/Integralgrainverse/invVMFFT2.rga.

>> Save Gravity Anomaly Grid as C:/PAGravf4.5_win64en/examples/Integralgrainverse/invVMFFT2.gra.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-13 10:56:58

>> Complete the computation!

>> Computation end time: 2026-04-13 10:57:02

Integral radius 150 km

Save the results as

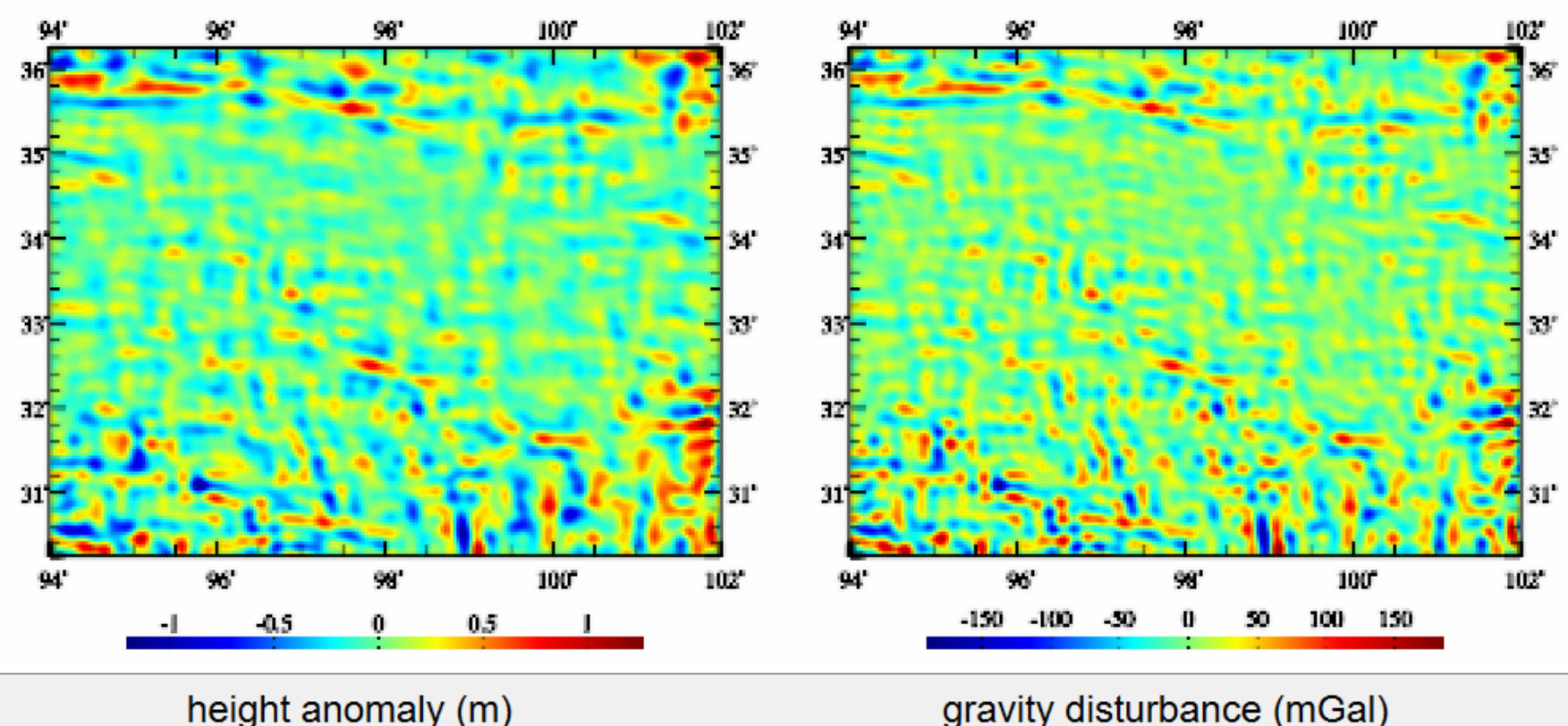
Import setting parameters

Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
0.1280	0.1327	0.0986	0.0433	-0.0224	-0.0910
-0.4254	-0.3999	-0.3598	-0.3066	-0.2436	-0.1756
-0.1179	-0.1453	-0.1565	-0.1483	-0.1207	-0.0768
-0.0151	-0.0785	-0.1354	-0.1797	-0.2064	-0.2132
0.3349	0.3741	0.4003	0.4122	0.4089	0.3904
-0.3038	-0.3751	-0.4408	-0.5001	-0.5515	-0.5929
0.0786	0.2103	0.3196	0.3958	0.4316	0.4235
-0.2695	-0.2007	-0.1085	-0.0047	0.0984	0.1893
0.0264	-0.0020	-0.0203	-0.0307	-0.0365	-0.0410
-0.2177	-0.2401	-0.2634	-0.2873	-0.3108	-0.3316
0.0467	0.0913	0.1121	0.1052	0.0701	0.0100
-0.1209	0.0013	0.1284	0.2511	0.3614	0.4538
0.6658	0.6550	0.6320	0.5938	0.5387	0.4673
-0.1721	-0.1628	-0.1465	-0.1274	-0.1089	-0.0926
0.1315	0.1496	0.1536	0.1402	0.1083	0.0588
-0.2766	-0.2228	-0.1617	-0.0990	-0.0397	0.0128
0.2671	0.2814	0.2876	0.2833	0.2677	0.2411
0.0669	0.0819	0.0903	0.0883	0.0736	0.0461

Extract result element

Plot



Integral of Inverse Operation: A rigorous solution to the Stokes-type Boundary Value Problem. It strictly requires the integrand (height anomaly or vertical deflection) to reside on an equipotential boundary surface.

Inverse Integration: Employs a hybrid algorithm combining Poisson integration with the differentiation of anomalous field elements. This approach is more flexible and does not mandate the boundary surface to be an equipotential boundary surface.

Computation via Inverse Vening-Meinesz Integration – 1D FFT

Computation of Gravity Anomaly via Inverse Stokes Integration

Computation of Gravity Disturbance via Inverse Hotine Integration

Computation via Inverse Vening-Meinesz Integration

Computation of External Anomalous Field Elements from Height Anomaly

Integral Formulas

Open Equipotential Surface Ellipsoidal Height Grid File

Open Vertical Deflection Vector Grid File

Select computation point file format

surface ellipsoidal height grid file

Select integral algorithm

1D FFT algorithm

>> Computation Process ** Operation Prompts

>> Computation end time: 2026-04-13 10:57:02

>> [Function] Computes the residual height anomaly (m), gravity anomaly (mGal), and gravity disturbance (mGal) on an equipotential boundary surface. This is achieved via Inverse Vening-Meinesz Integration, utilizing the surface's ellipsoidal height grid and the corresponding residual vertical deflection vector grid (" , south and west components).

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Integralgrainverse/landgeoidhgt.dat.

>> Open Vertical Deflection Vector Grid File C:/PAGrav4.5_win64en/examples/Integralgrainverse/resGMIgeoid541_1800.dft.

>> Compute by 1D FFT algorithm...

>> Save the results as C:/PAGrav4.5_win64en/examples/Integralgrainverse/invVMFFT1.txt.

>> Save Height Anomaly Grid as C:/PAGrav4.5_win64en/examples/Integralgrainverse/invVMFFT1.ksi.

>> Save Gravity Disturbance Grid as C:/PAGrav4.5_win64en/examples/Integralgrainverse/invVMFFT1.rga.

>> Save Gravity Anomaly Grid as C:/PAGrav4.5_win64en/examples/Integralgrainverse/invVMFFT1.gra.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-13 10:58:06

>> Complete the computation!

>> Computation end time: 2026-04-13 11:00:12

Integral radius 150 km

Save the results as

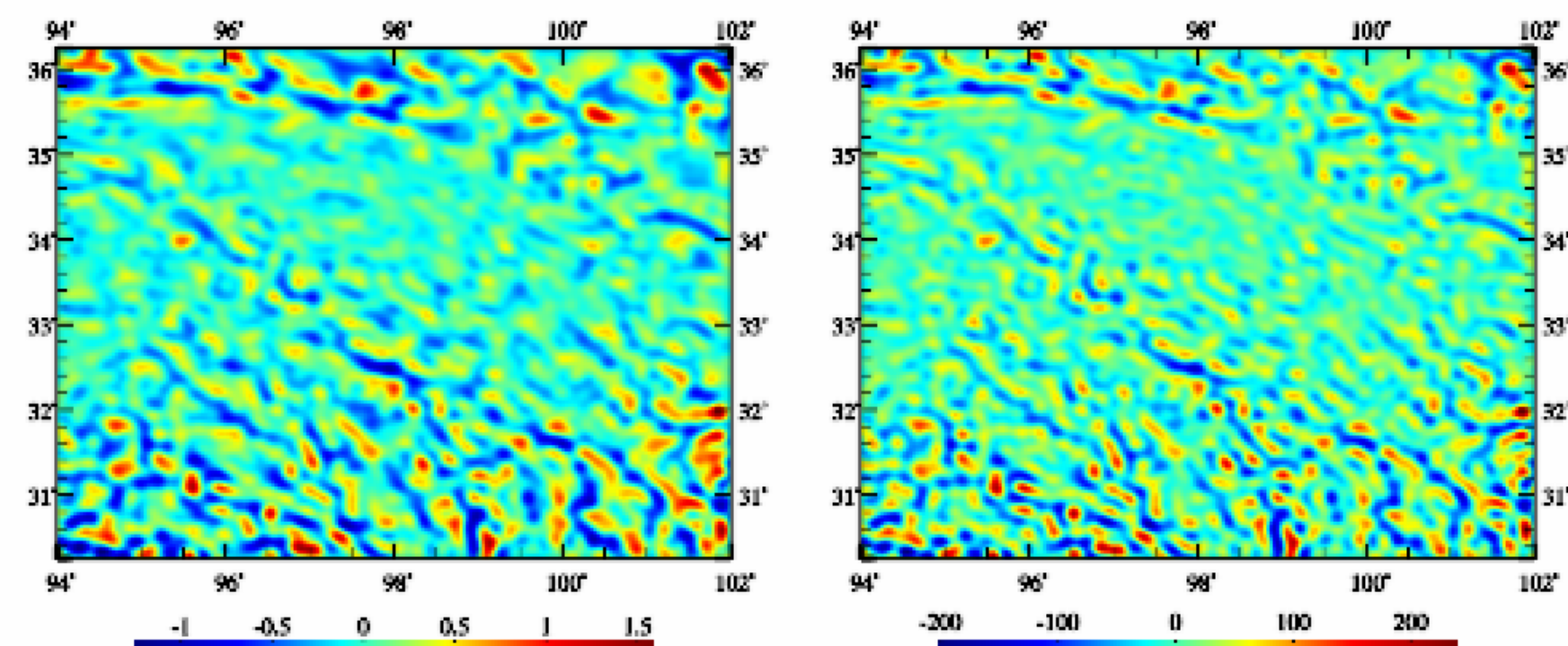
Import setting parameters

Start Computation

94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
0.0580	0.0564	0.0585	0.0612	0.0638	0.0665
0.0288	0.0192	0.0142	0.0145	0.0196	0.0275
-0.2325	-0.2663	-0.2868	-0.2930	-0.2868	-0.2721
-0.6373	-0.6958	-0.7310	-0.7377	-0.7127	-0.6565
0.4076	0.4529	0.4809	0.4930	0.4911	0.4768
-0.0231	-0.0915	-0.1622	-0.2336	-0.3027	-0.3653
0.4099	0.5883	0.7397	0.8481	0.9025	0.8942
-0.5901	-0.5632	-0.4873	-0.3747	-0.2404	-0.1003
0.1089	0.0615	0.0211	-0.0111	-0.0350	-0.0515
0.0047	0.0079	0.0040	-0.0074	-0.0250	-0.0461
0.2976	0.3571	0.3940	0.4020	0.3780	0.3228
-0.3702	-0.3424	-0.3057	-0.2670	-0.2321	-0.2049
-0.0947	-0.0690	-0.0468	-0.0309	-0.0231	-0.0235
-0.0131	0.0018	0.0096	0.0082	-0.0031	-0.0230
0.1404	0.2157	0.2854	0.3430	0.3836	0.4036
-0.1584	-0.2263	-0.2908	-0.3517	-0.4083	-0.4590
-0.2058	-0.1199	-0.0404	0.0281	0.0820	0.1198
0.1510	0.1570	0.1570	0.1483	0.1292	0.0996

Extract result element

Plot



height anomaly (m)

gravity disturbance (mGal)

Integral of Inverse Operation: A rigorous solution to the Stokes-type Boundary Value Problem. It strictly requires the integrand (height anomaly or vertical deflection) to reside on an equipotential boundary surface.

Inverse Integration: Employs a hybrid algorithm combining Poisson integration with the differentiation of anomalous field elements. This approach is more flexible and does not mandate the boundary surface to be an equipotential boundary surface.

Table 4.14: Statistics of Model Reference Truths

Field Element(Deg 541 – 1800)	Unit	Mean	Std. Dev.	Min	Max
Residual Geoidal Heights	m	0.0045	0.2172	-1.1490	0.9110
Residual Gravity Disturbances	mGal	0.4348	34.1479	-171.3088	178.1561
Residual Gravity Anomalies	mGal	0.4334	34.0852	-170.9556	177.8780

Algorithm Performance: The ratio of the standard deviation of Differences in Table 4.15 to the standard deviation of Reference Truths in Table 4.14 reflects the approximation capability of the corresponding algorithm.

Geoidal Height Recovery: All three algorithms demonstrate comparable performance in recovering geoidal heights, with standard deviations around 3.3 cm and maximum errors within ± 13 cm. This confirms the high stability of the Inverse Vening-Meinesz formula.

Internal Consistency: The nearly identical error statistics for gravity anomalies and gravity disturbances validate the internal logical consistency of the algorithm implementation.

Table 4.15: Differences between Inverse Operation Integration Results and Reference Truth

Algorithm	Field Element	Unit	Mean	Std. Dev.	Min	Max
Numerical integral	Residual Geoidal Heights	m	-0.0002	0.0331	-0.1066	0.1266
	Residual Gravity Disturbances	mGal	-0.0281	3.5304	-19.8306	15.5799
	Residual Gravity Anomalies	mGal	-0.0280	3.5293	-19.8233	15.5701
FFT2	Residual Geoidal Heights	m	0.0001	0.0337	-0.1057	0.1271
	Residual Gravity Disturbances	mGal	-0.0131	2.4732	-12.3026	9.8919
	Residual Gravity Anomalies	mGal	-0.0131	2.4729	-12.2973	9.8831
FFT1	Residual Geoidal Heights	m	-0.0001	0.0332	-0.1068	0.1263
	Residual Gravity Disturbances	mGal	-0.0283	3.5527	-19.9907	15.7164
	Residual Gravity Anomalies	mGal	-0.0283	3.5516	-19.9830	15.7072

Computation of External Anomalous Gravity Field Elements from Height Anomaly

Computation of Gravity Anomaly via Inverse Stokes Integration

Computation of Gravity Disturbance via Inverse Hotine Integration

Computation via Inverse Vening-Meinesz Integration

Computation of External Anomalous Field Elements from Height Anomaly

Integral Formulas

Open Boundary Surface Ellipsoidal Height Grid File

Open Height Anomaly Grid File

Select computation point file format

discrete computation point file

Open Space Computation Point File

Set input point file format

number of rows of file header 1

>> Computation Process ** Operation Prompts

>> Computation end time: 2026-04-13 11:00:12

>> [Function] Computes the residual gravity anomaly (mGal), residual gravity disturbance (mGal), and residual vertical deflection vector (" south and west components) on or outside the geoid. Inputs include an ellipsoidal height grid of a boundary surface and the corresponding residual height anomaly (m) grid.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Integralgrainverse/landgeoidhgt.dat.

>> Open Height Anomaly Grid File C:/PAGrav4.5_win64en/examples/Integralgrainverse/resGMIgeoid541_1800.ksi.

>> Open Space Computation Point File C:/PAGrav4.5_win64en/examples/Integralgrainverse/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save the results as C:/PAGrav4.5_win64en/examples/Integralgrainverse/invksiouter.txt.

>> Appends four attribute columns to the source file: Residual Gravity Anomaly, Residual Gravity Disturbance, Residual Vertical Deflection (Southward), and Residual Vertical Deflection (Westward).

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-13 11:02:42

>> Complete the computation!

>> Computation end time: 2026-04-13 11:11:09

Integral radius 150 km

Save the results as

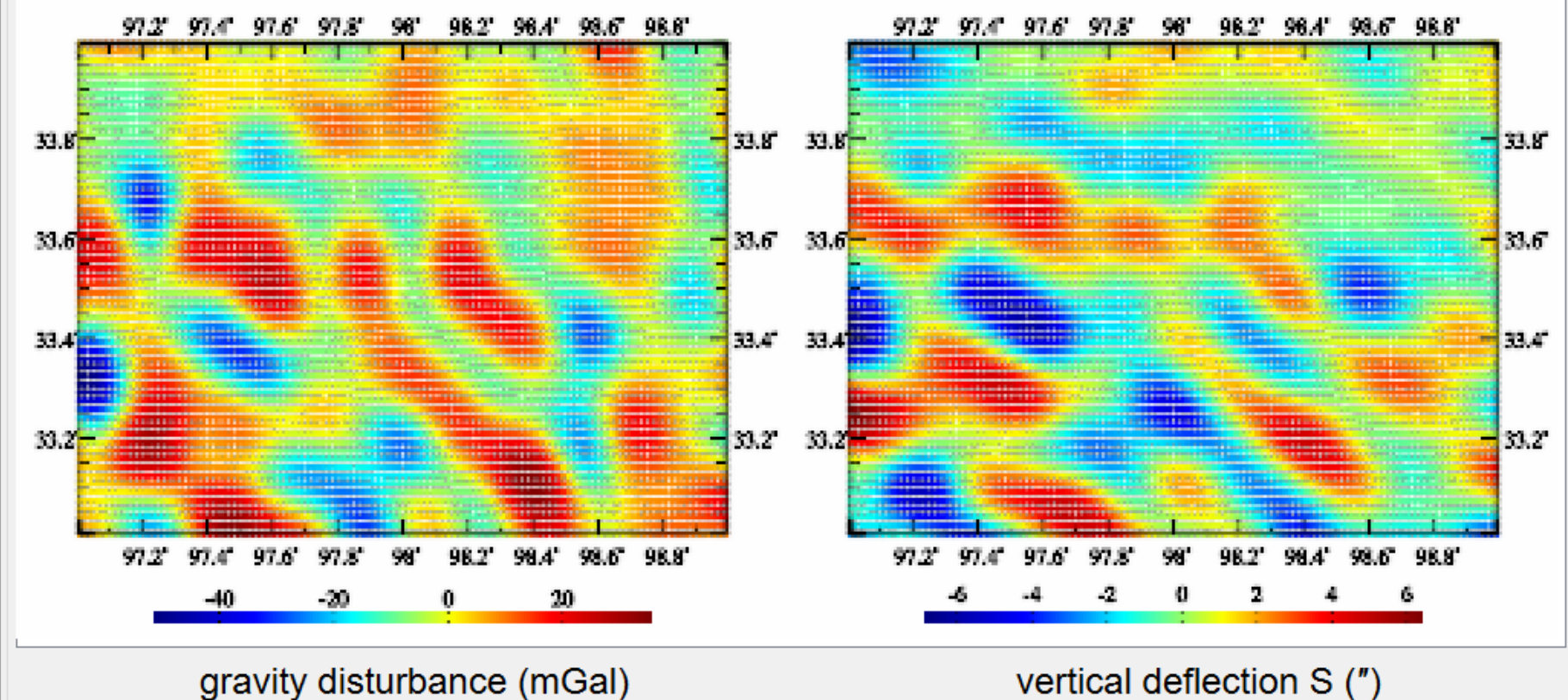
Import setting parameters

Start Computation

imal)	lat	ellipHeight (m)				
333	33.008333	3942.764	6.2612	6.2592	-2.5007	0.4847
000	33.008333	3989.787	6.0388	6.0365	-2.4248	0.6987
667	33.008333	4034.817	5.2179	5.2151	-2.3076	0.9304
333	33.008333	4070.847	3.8190	3.8156	-2.1573	1.1571
000	33.008333	4106.877	1.8631	1.8588	-1.9846	1.3566
667	33.008333	4119.913	-0.5421	-0.5472	-1.8034	1.5177
333	33.008333	4115.946	-3.3350	-3.3411	-1.6284	1.6250
000	33.008333	4090.977	-6.4301	-6.4372	-1.4752	1.6670
667	33.008333	4070.007	-9.6925	-9.7007	-1.3639	1.6181
333	33.008333	3991.047	-13.0955	-13.1048	-1.3008	1.4939
000	33.008333	3985.070	-16.1623	-16.1724	-1.3230	1.2209
667	33.008333	3956.107	-18.9050	-18.9158	-1.4222	0.8360
333	33.008333	3965.137	-20.8767	-20.8879	-1.6097	0.3179
000	33.008333	3964.173	-22.0755	-22.0867	-1.8711	-0.3092
667	33.008333	3983.205	-22.1895	-22.2002	-2.1941	-1.0304
333	33.008333	3953.251	-21.4492	-21.4591	-2.5624	-1.8234
000	33.008333	4016.279	-19.0783	-19.0868	-2.9355	-2.6206
667	33.008333	4054.318	-15.7055	-15.7122	-3.2973	-3.3943

Extract result element

Plot



Integral of Inverse Operation: A rigorous solution to the Stokes-type Boundary Value Problem. It strictly requires the integrand (height anomaly or vertical deflection) to reside on an equipotential boundary surface.

Inverse Integration: Employs a hybrid algorithm combining Poisson integration with the differentiation of anomalous field elements. This approach is more flexible and does not mandate the boundary surface to be an equipotential boundary surface.

Table 4.16: Statistics of Model Reference Truths

Field Element	Unit	Mean	Std. Dev.	Min	Max
Ground Residual Gravity anomalies	mGal	-0.0349	15.7184	-93.7784	66.5507
Ground Residual Gravity disturbances	mGal	-0.0346	15.7527	-93.9854	66.6638
Ground Residual Vertical Deflection (S)	"	0.0014	2.4951	-12.7789	14.2346
Ground Residual Vertical Veflection (W)	"	0.0097	2.1772	-9.1577	10.4499

Table 4.17: Differences between Computed Results and Reference Truth

Field Element	Unit	Mean	Std. Dev.	Min	Max
Ground Residual Gravity anomalies	mGal	-0.0104	2.0577	-8.2097	10.7064
Ground Residual Gravity disturbances	mGal	-0.0100	2.0929	-8.3217	10.8982
Ground Residual Vertical Deflection (S)	"	0.0004	0.0075	-0.0357	0.0388
Ground Residual Vertical Veflection (W)	"	0.0003	0.0076	-0.0362	0.0358

Algorithm Performance: The ratio of the standard deviation of Differences in Table 4.17 to the standard deviation of Reference Truths in Table 4.16 reflects the approximation capability of the corresponding algorithm.

Exceptional Inversion Accuracy: The inversion from height anomalies to ground gravity field elements demonstrates exceptional precision. The standard deviations for gravity anomalies and disturbances are merely $\sim 2.06 - 2.09$ mGal, representing a significant signal-to-noise improvement over the original field variability (~ 15.7 mGal).

High-Precision Vertical Deflection Recovery: The recovery of vertical deflections is remarkably accurate, with standard deviations of only $0.0075''$ (7.5 mas) and maximum errors confined within $\pm 0.04''$. This underscores the superior numerical stability and high-frequency preservation capabilities of the combined Poisson integration and differentiation algorithm when transforming geoidal heights to vertical deflections.

Algorithm Robustness: Given that this method does not require an equipotential boundary, these results further validate the algorithm's robustness and reliability for practical applications involving quasi-geoids or non-strict reference surfaces.

Radial Gradient Integration of Anomalous Gravity Field Elements

Save as Import parameters Start computation Save process Follow example

Radial Gradient Integration
of Anomalous Gravity
Field Elements

Computation of External Gravity
Disturbance from
Disturbing Gravity Gradient

Computation of Disturbing
Gravity Gradient via
Inverse Operation Integration

Computation of External Disturbing
Gravity Gradient
from Gravity Disturbance

Computation of External Anomalous
Gravity Field Elements
via Poisson Integration

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Field Element Grid File

Select computation point file format

discrete computation point file

Open Discrete Computation Point File

Set input point file format

number of rows of file header

1

>> Computation Process ** Operation Prompts

>> [Purpose] To perform radial gradient integration, integral inverse operations, inverse operation integrations, and Poisson integrations on anomalous gravity field elements using rigorous numerical integration methods.

>> Select the computation function from the 5 control buttons at the top of the interface...

>> [Function] Computes the radial gradient (/km) of a specific field element on a surface via numerical integration, utilizing the ellipsoidal height grid of the equipotential boundary surface and the corresponding anomalous gravity field element grid.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.

>> Open Residual Field Element Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/resGMIgeoid541_1800.ksi.

>> Open Discrete Computation Point File C:/PAGrav4.5_win64en/examples/Intgendistgradient/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save Radial Gradient as C:/PAGrav4.5_win64en/examples/Intgendistgradient/rgradient.txt.

>> Appends two columns to the source file: Interpolated Ellipsoidal Height, and Computed Radial Gradient value (/km).

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 07:02:30

>> Complete the computation!

>> Computation end time: 2026-04-14 07:03:19

Save computation process as

Integral radius 120 km

Save the results as

Import setting parameters

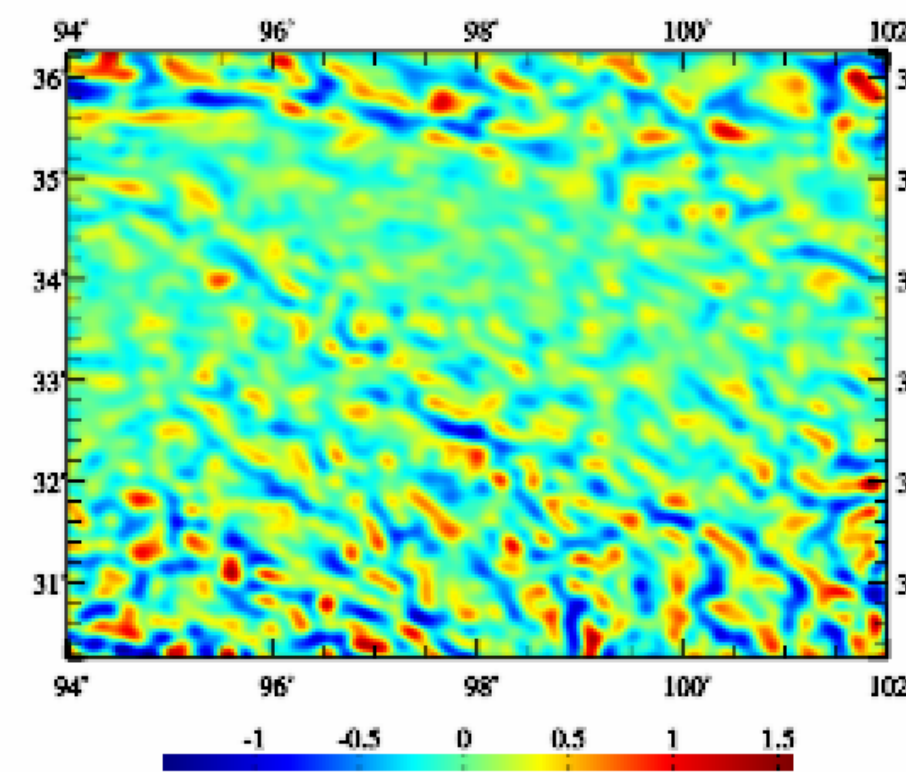
Start Computation

no lon(degree/decimal) lat ellipHeight (m)

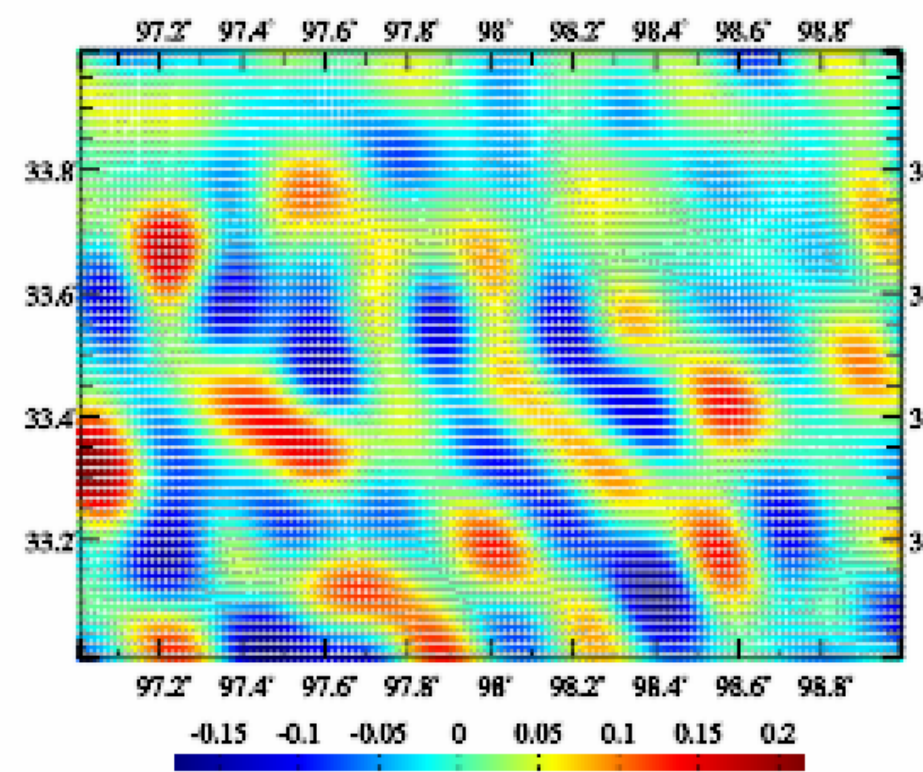
1	97.008333	33.008333	3942.764	-37.2501	-0.0690
2	97.025000	33.008333	3989.787	-37.2203	-0.0687
3	97.041667	33.008333	4034.817	-37.1899	-0.0641
4	97.058333	33.008333	4070.847	-37.1590	-0.0548
5	97.075000	33.008333	4106.877	-37.1276	-0.0414
6	97.091667	33.008333	4119.913	-37.0959	-0.0246
7	97.108333	33.008333	4115.946	-37.0640	-0.0046
8	97.125000	33.008333	4090.977	-37.0318	0.0169
9	97.141667	33.008333	4070.007	-36.9990	0.0394
10	97.158333	33.008333	3991.047	-36.9665	0.0612
11	97.175000	33.008333	3985.070	-36.9327	0.0814
12	97.191667	33.008333	3956.107	-36.8988	0.0986
13	97.208333	33.008333	3965.137	-36.8642	0.1120
14	97.225000	33.008333	3964.173	-36.8295	0.1200
15	97.241667	33.008333	3983.205	-36.7943	0.1224
16	97.258333	33.008333	3953.251	-36.7595	0.1180
17	97.275000	33.008333	4016.279	-36.7238	0.1066
18	97.291667	33.008333	4054.318	-36.6883	0.0886
19	97.308333	33.008333	4090.360	-36.6528	0.0644
20	97.325000	33.008333	4192.388	-36.6171	0.0351
21	97.341667	33.008333	4287.429	-36.5816	0.0020

Extract radial gradient of element

Plot↓



anomalous field element on surface



radial gradient (/km) of element

Radial Gradient Integration algorithm is derived from the solution to the Stokes Boundary Value Problem, this requires the integrand field elements to reside on an equipotential boundary surface.

Equipotential Surface Construction: Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Radial Gradient Integration of Anomalous Gravity Field Elements

Radial Gradient Integration
of Anomalous Gravity
Field Elements

Computation of External Gravity
Disturbance from
Disturbing Gravity Gradient

Computation of Disturbing
Gravity Gradient via
Inverse Operation Integration

Computation of External Disturbing
Gravity Gradient
from Gravity Disturbance

Computation of External Anomalous
Gravity Field Elements
via Poisson Integration

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Field Element Grid File

Select computation point file format

surface ellipsoidal height grid file

>> Computation Process ** Operation Prompts

>> Appends two columns to the source file: Interpolated Ellipsoidal Height, and Computed Radial Gradient value (/km).

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 07:02:30

>> Complete the computation!

>> Computation end time: 2026-04-14 07:03:19

>> [Function] Computes the radial gradient (/km) of a specific field element on a surface via numerical integration, utilizing the ellipsoidal height grid of the equipotential boundary surface and the corresponding anomalous gravity field element grid.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.

>> Open Residual Field Element Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/resGMIgeoid541_1800.ksi.

>> Save Radial Gradient as C:/PAGrav4.5_win64en/examples/Intgendistgradient/rgradient.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 07:05:14

>> Complete the computation!

>> Computation end time: 2026-04-14 07:21:28

Integral radius 120 km

Save the results as

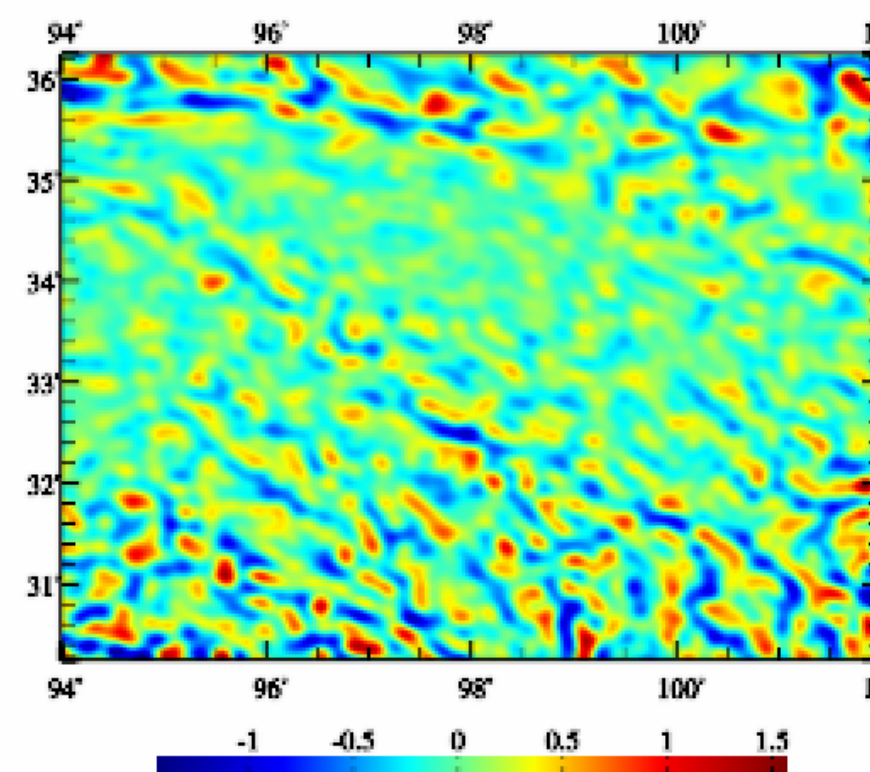
Import setting parameters

Start Computation

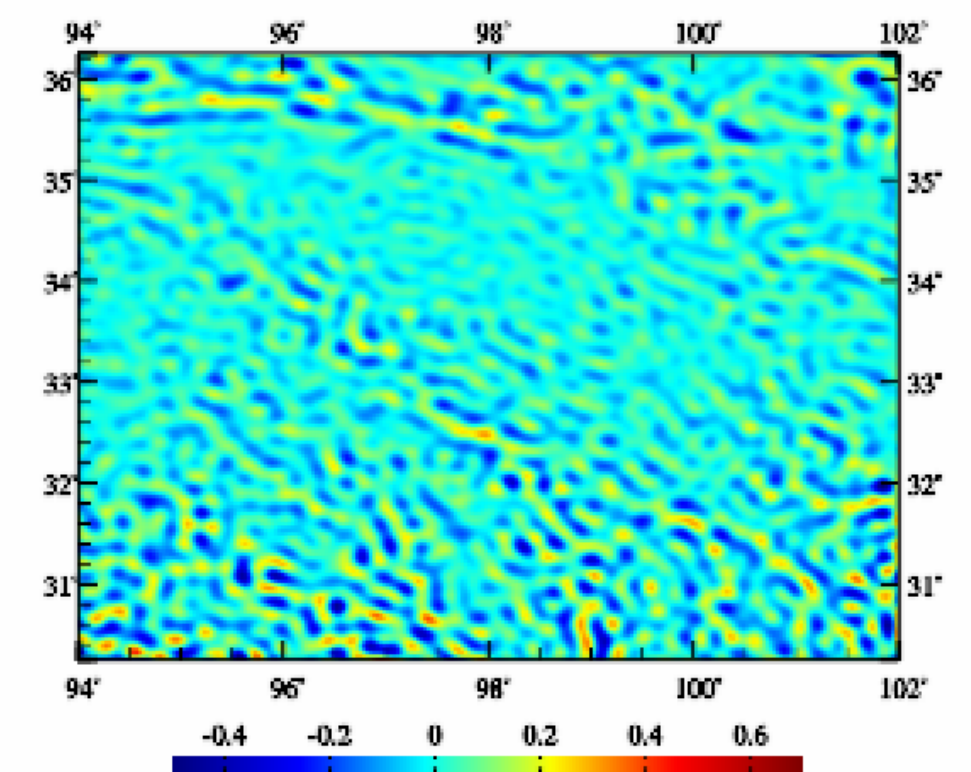
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
0.1994	0.1616	0.1299	0.0966	0.0618	0.0187
-0.2230	-0.2129	-0.2012	-0.1909	-0.1838	-0.1814
-0.3206	-0.3410	-0.3625	-0.3845	-0.4051	-0.4217
-0.0254	0.0381	0.0910	0.1313	0.1579	0.1710
0.0889	0.0928	0.0974	0.1016	0.1035	0.1017
-0.1844	-0.2024	-0.2053	-0.1910	-0.1595	-0.1125
-0.0629	-0.1566	-0.2467	-0.3222	-0.3733	-0.3932
0.2883	0.2679	0.2207	0.1538	0.0761	-0.0035
-0.1036	-0.0806	-0.0632	-0.0514	-0.0454	-0.0445
-0.1906	-0.1900	-0.1781	-0.1537	-0.1175	-0.0717
0.0379	-0.0101	-0.0551	-0.0921	-0.1179	-0.1305
0.0444	0.0620	0.0847	0.1145	0.1522	0.1975
0.4743	0.4530	0.4273	0.3990	0.3692	0.3379
-0.1393	-0.1662	-0.1758	-0.1670	-0.1400	-0.0975
0.0919	0.0327	-0.0318	-0.0970	-0.1582	-0.2123
-0.2250	-0.1789	-0.1243	-0.0623	0.0052	0.0756
0.1622	0.0982	0.0360	-0.0190	-0.0626	-0.0924
-0.0143	-0.0254	-0.0398	-0.0541	-0.0659	-0.0729
0.0383	0.0427	0.0461	0.0490	0.0522	0.0552
-0.1294	-0.1505	-0.1593	-0.1532	-0.1318	-0.0968

Extract radial gradient of element

Plot↓



anomalous field element on surface



radial gradient (/km) of element

Radial Gradient Integration algorithm is derived from the solution to the Stokes Boundary Value Problem, this requires the integrand field elements to reside on an equipotential boundary surface.

Equipotential Surface Construction: Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Computation of External Gravity Disturbance from Disturbing Gravity Gradient

Save as Import parameters Start Computation Save process Follow example

Radial Gradient Integration
of Anomalous Gravity
Field Elements

Computation of External Gravity
Disturbance from
Disturbing Gravity Gradient

Computation of Disturbing
Gravity Gradient via
Inverse Operation Integration

Computation of External Disturbing
Gravity Gradient
from Gravity Disturbance

Computation of External Anomalous
Gravity Field Elements
via Poisson Integration

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Gravity Gradient Grid File

Select computation point file format

discrete computation point file

Open Space Computation Point File

Set input point file format

number of rows of file header 1

>> Computation Process ** Operation Prompts

>> [Purpose] To perform radial gradient integration, integral inverse operations, inverse operation integrations, and Poisson integrations on anomalous gravity field elements using rigorous numerical integration methods.

>> Select the computation function from the 5 control buttons at the top of the interface...

>> [Function] Computes the residual gravity disturbance (mGal) on or outside the geoid via numerical integration, utilizing the ellipsoidal height grid of the equipotential boundary surface and the corresponding residual disturbing gravity gradient grid (E, radial component).

** This function performs the inversion (downward/upward continuation) from gravity gradients to gravity disturbances.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.

>> Open Residual Gravity Gradient Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/resGMIgeoid541_1800.grr.

>> Open Space Computation Point File C:/PAGrav4.5_win64en/examples/Intgendistgradient/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save Gravity Disturbance as C:/PAGrav4.5_win64en/examples/Intgendistgradient/grrtorgadbm.txt.

>> Appends one column containing the residual gravity disturbance value.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 07:26:58

>> Complete the computation!

>> Computation end time: 2026-04-14 07:27:47

Integral radius 120 km

Save the results as

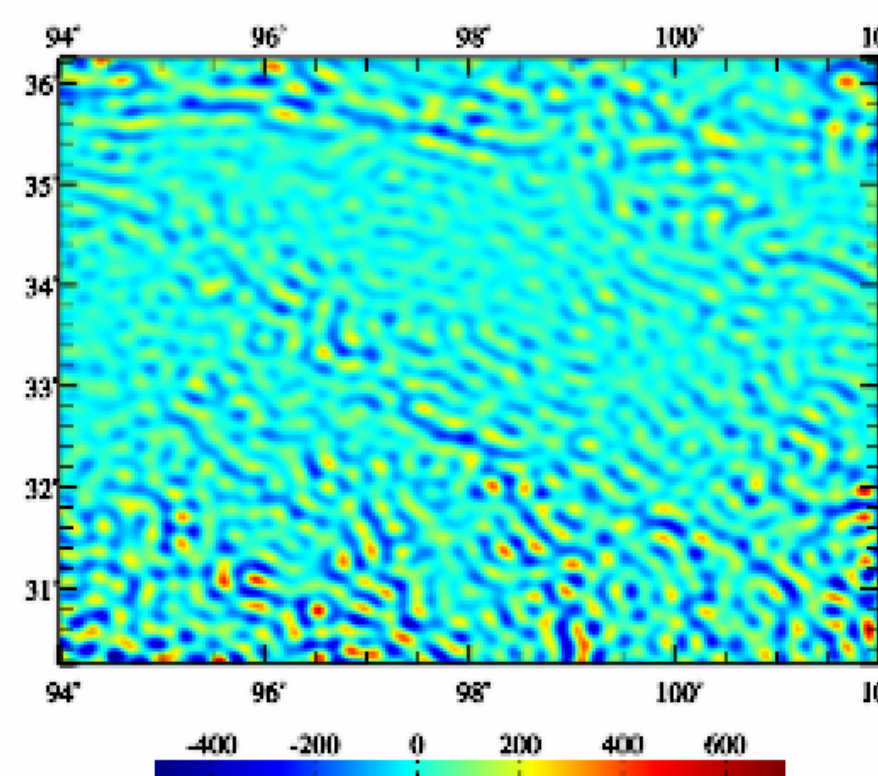
Import setting parameters

Start Computation

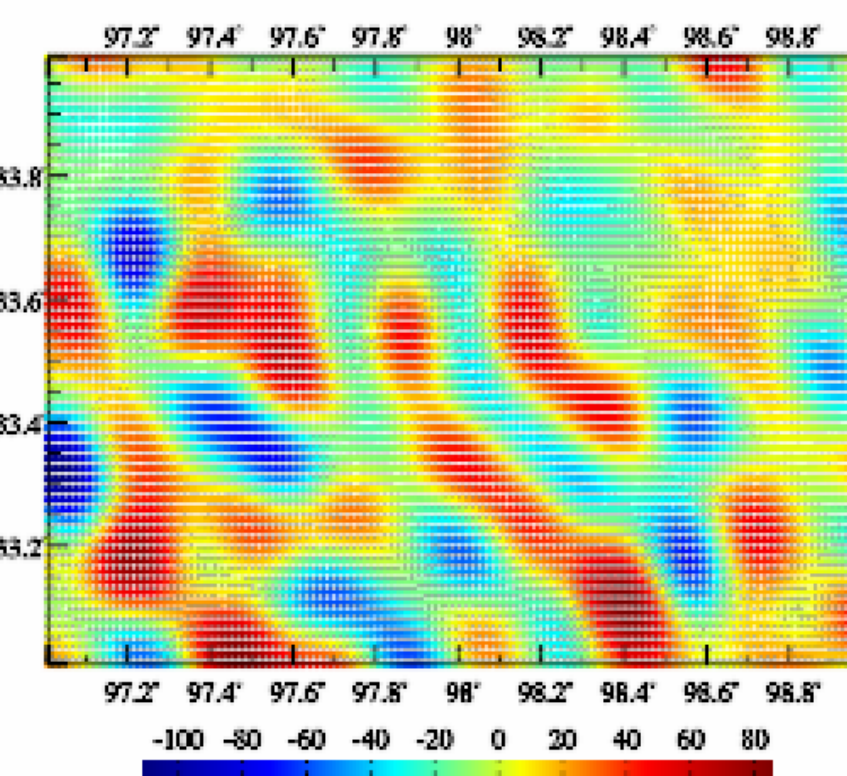
no	lon(degree/decimal)	lat	ellipHeight(m)	
1	97.008333	33.008333	3942.764	22.5501
2	97.025000	33.008333	3989.787	22.5614
3	97.041667	33.008333	4034.817	20.8934
4	97.058333	33.008333	4070.847	17.5000
5	97.075000	33.008333	4106.877	12.4499
6	97.091667	33.008333	4119.913	5.9167
7	97.108333	33.008333	4115.946	-1.8356
8	97.125000	33.008333	4090.977	-10.4680
9	97.141667	33.008333	4070.007	-19.5753
10	97.158333	33.008333	3991.047	-28.6981
11	97.175000	33.008333	3985.070	-37.3345
12	97.191667	33.008333	3956.107	-44.9514
13	97.208333	33.008333	3965.137	-51.0062
14	97.225000	33.008333	3964.173	-54.9764
15	97.241667	33.008333	3983.205	-56.3949
16	97.258333	33.008333	3953.251	-54.8958
17	97.275000	33.008333	4016.279	-50.2604
18	97.291667	33.008333	4054.318	-42.4609
19	97.308333	33.008333	4090.360	-31.6924
20	97.325000	33.008333	4192.388	-18.3837
21	97.341667	33.008333	4287.429	-3.1899

Extract external gravity disturbance

Plot



disturbing gravity gradient (E, radial)



external gravity disturbance (mGal)

Radial Gradient Integration algorithm is derived from the solution to the Stokes Boundary Value Problem, this requires the integrand field elements to reside on an equipotential boundary surface.

Equipotential Surface Construction: Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Computation of External Gravity Disturbance from Disturbing Gravity Gradient

Radial Gradient Integration
of Anomalous Gravity
Field Elements

Computation of External Gravity
Disturbance from
Disturbing Gravity Gradient

Computation of Disturbing
Gravity Gradient via
Inverse Operation Integration

Computation of External Disturbing
Gravity Gradient
from Gravity Disturbance

Computation of External Anomalous
Gravity Field Elements
via Poisson Integration

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Gravity Gradient Grid File

Select computation point file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal
Height Grid File

>> Computation Process ** Operation Prompts

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 07:26:58

>> Complete the computation!

>> Computation end time: 2026-04-14 07:27:47

>> [Function] Computes the residual gravity disturbance (mGal) on or outside the geoid via numerical integration, utilizing the ellipsoidal height grid of the equipotential boundary surface and the corresponding residual disturbing gravity gradient grid (E, radial component).

** This function performs the inversion (downward/upward continuation) from gravity gradients to gravity disturbances.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.

>> Open Residual Gravity Gradient Grid File C:/PAGravf4.5_win64en/examples/Intgendistgradient/resGMIgeoid541_1800.grr.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/Intgendistgradient/landbmsurfhgt.dat.

>> Save Gravity Disturbance as C:/PAGravf4.5_win64en/examples/Intgendistgradient/grrtorgadbm.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 07:29:24

>> Complete the computation!

>> Computation end time: 2026-04-14 07:45:40

Integral radius 120 km

Save the results as

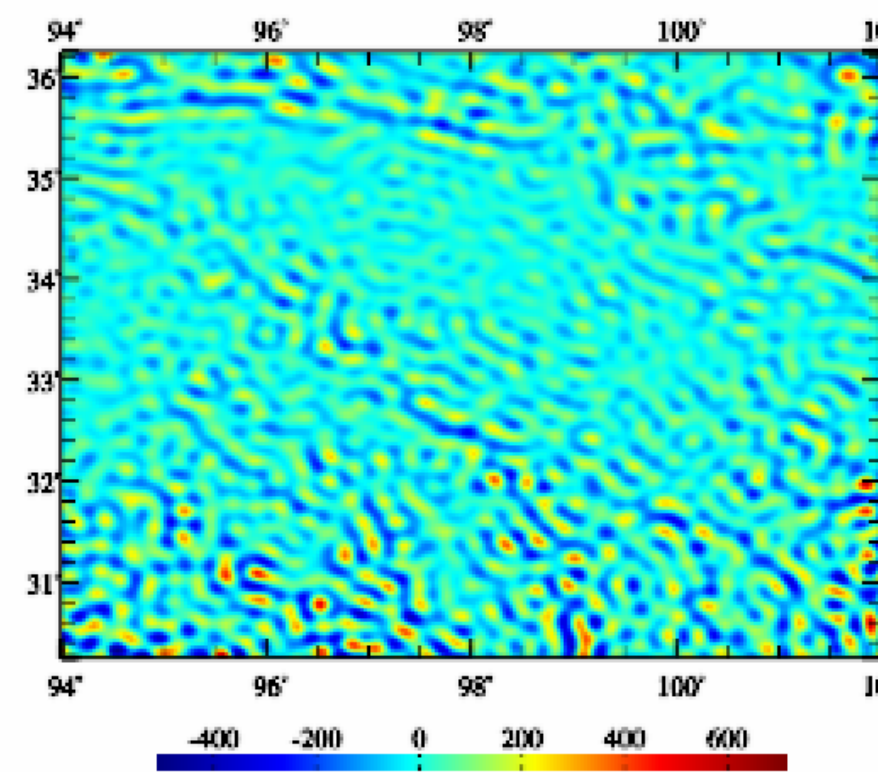
Import setting parameters

Start Computation

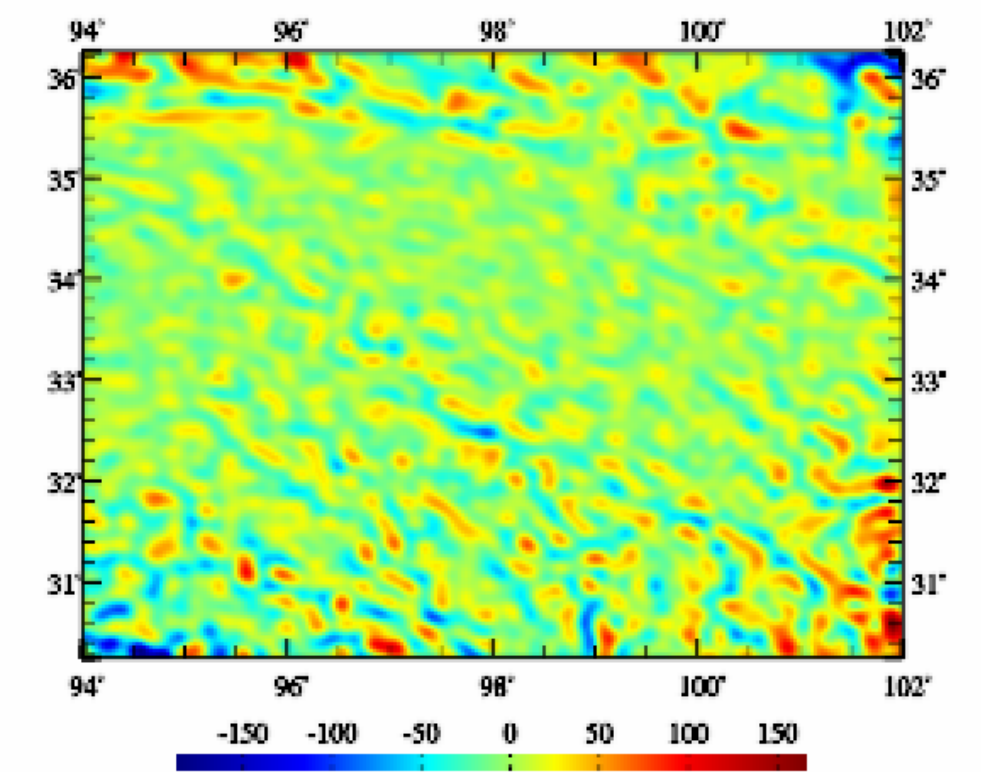
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
-5.8779	-6.8887	-9.0490	-12.2946	-16.1783	-20.4347
-48.7033	-48.3274	-47.1508	-45.5078	-43.9008	-42.2746
-101.6110	-110.1029	-118.1538	-124.3369	-128.4160	-132.2481
-149.3842	-152.4005	-152.8367	-147.3838	-136.6681	-123.3897
41.2586	48.8025	53.1999	55.8075	56.6694	54.4517
-41.5562	-52.3111	-62.2337	-69.9137	-76.8626	-81.6524
23.1595	36.1268	46.9121	53.8192	56.4041	56.5276
-51.6454	-47.2762	-38.7213	-27.7525	-15.5018	-4.4056
-7.9294	-12.8406	-16.6274	-19.1768	-20.5113	-20.6005
-2.4648	-2.0945	-2.1912	-2.7243	-3.4289	-4.0043
34.6174	37.4087	37.7245	35.7470	31.0625	23.9802
-52.2752	-47.5792	-41.7309	-35.3883	-28.6450	-22.6639
34.0288	39.2193	43.6125	46.9344	48.6715	48.6397
1.7688	-2.1718	-5.7483	-8.9916	-11.8712	-14.2903
11.1982	16.5337	20.6686	23.5081	24.6648	23.9631
-58.2206	-65.1046	-69.5639	-71.7567	-72.8949	-72.0902
-12.2046	-8.7767	-6.9390	-6.4350	-6.9716	-8.1888
-8.5910	-8.2107	-8.6697	-10.0133	-12.1953	-14.9998
-13.0860	-10.5907	-9.2138	-9.2451	-10.7382	-13.5382
-19.1313	-16.2120	-14.5117	-14.5602	-16.4463	-20.2273

Extract external gravity disturbance

Plot



disturbing gravity gradient (E, radial)



external gravity disturbance (mGal)

Radial Gradient Integration algorithm is derived from the solution to the Stokes Boundary Value Problem, this requires the integrand field elements to reside on an equipotential boundary surface.

Equipotential Surface Construction: Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Table 4.18: Statistics of Model Reference Truth

Field Element	Unit	Mean	Std. Dev.	Min	Max
Ground Residual Gravity Disturbances	mGal	-0.0346	15.7527	-93.9854	66.6638

Table 4.19: Differences between Computed Results and Reference Truth

Field Element	Unit	Mean	Std. Dev.	Min	Max
Ground Residual Gravity Disturbances	mGal	0.0071	4.2456	-18.5325	16.5266

Algorithm Performance: The ratio of the standard deviation of Differences in Table 4.19 to the standard deviation of Reference Truths in Table 4.18 reflects the approximation capability of the corresponding algorithm.

Robustness: The inversion from gravity gradients to disturbances yields a standard deviation of approximately 4.25 mGal. Given the ill-posed nature of differentiation and the high-frequency content of gradient data, this accuracy demonstrates the algorithm's effectiveness in suppressing noise amplification while successfully recovering medium-to-long wavelength gravity signals.

Computation of Disturbing Gravity Gradient via Inverse Operation Integration

Radial Gradient Integration of Anomalous Gravity Field Elements

Computation of External Gravity Disturbance from Disturbing Gravity Gradient

Computation of Disturbing Gravity Gradient via Inverse Operation Integration

Computation of External Disturbing Gravity Gradient from Gravity Disturbance

Computation of External Anomalous Gravity Field Elements via Poisson Integration

Open Equipotential Surface Ellipsoidal Height Grid File

Open Residual Gravity Disturbance Grid File

Select computation point file format
discrete computation point file

Open Discrete Computation Point File

Set input point file format
number of rows of file header 1

>> Computation Process ** Operation Prompts

>> Complete the computation!
 >> Computation end time: 2026-04-14 07:45:40
 >> [Function] Computes the residual disturbing gravity gradient (E, radial component) on a surface via inverse operation integration, utilizing the ellipsoidal height grid of the equipotential boundary surface and the corresponding residual gravity disturbance grid (mGal).
 ** This function effectively implements the differentiation of gravity field elements through an integral formulation.
 >> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.
 >> Open Residual Gravity Disturbance Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/resGMlgeoid541_1800.rga.
 >> Open Discrete Computation Point File C:/PAGrav4.5_win64en/examples/Intgendistgradient/calcpnt.txt.
 ** Look at the file information in the window below, set the input file format parameters...
 >> Save Disturbing Gravity Gradient as C:/PAGrav4.5_win64en/examples/Intgendistgradient/rgatogrddwm.txt.
 >> Appends two columns to the source file: Interpolated Ellipsoidal Height, and Computed residual disturbing gravity gradient value.
 >> The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] tool button...
 >> Computation start time: 2026-04-14 07:47:59
 >> Complete the computation!
 >> Computation end time: 2026-04-14 07:48:44

Save computation process as

Integral radius 120 km

Save the results as

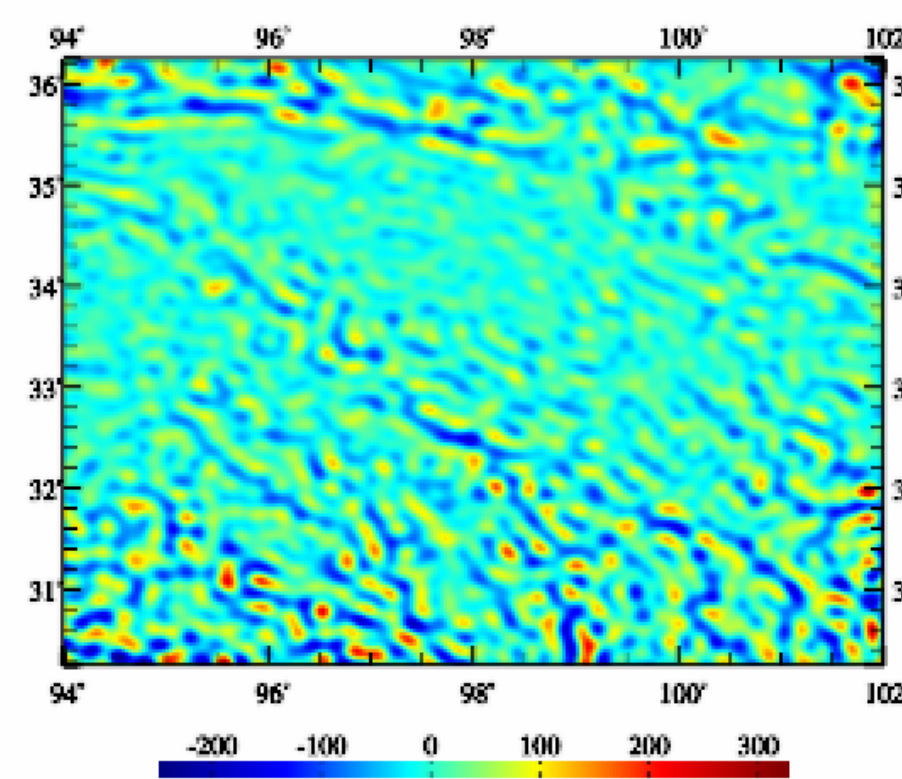
Import setting parameters

Start Computation

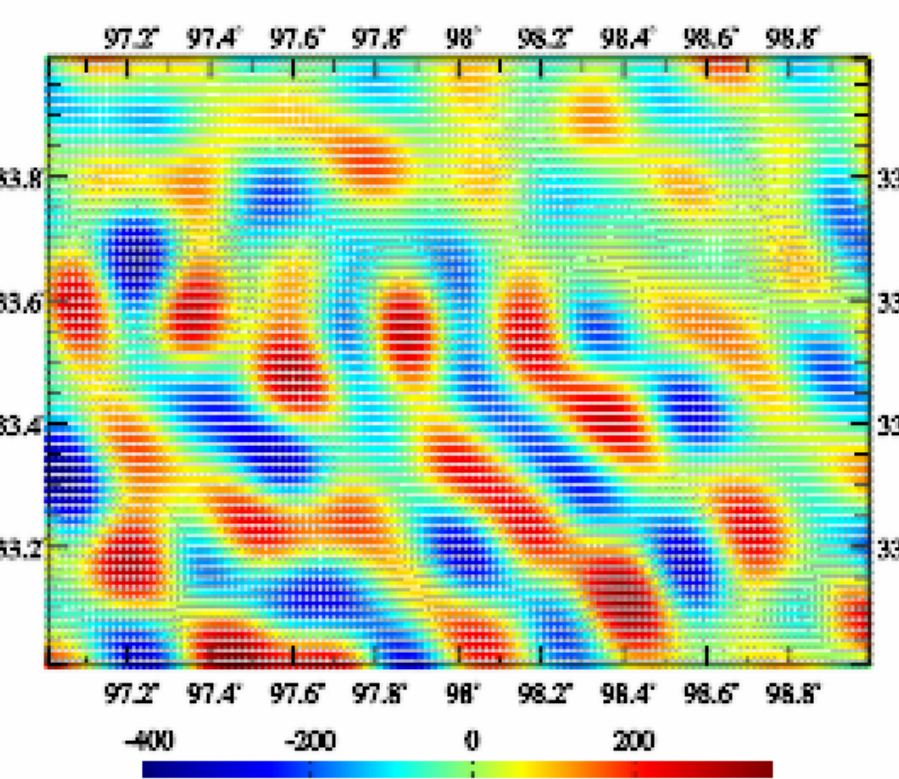
no	lon(degree/decimal)	lat	ellipHeight (m)		
1	97.008333	33.008333	3942.764	-37.2501	196.3239
2	97.025000	33.008333	3989.787	-37.2203	195.2762
3	97.041667	33.008333	4034.817	-37.1899	182.6273
4	97.058333	33.008333	4070.847	-37.1590	158.4840
5	97.075000	33.008333	4106.877	-37.1276	123.7975
6	97.091667	33.008333	4119.913	-37.0959	80.2016
7	97.108333	33.008333	4115.946	-37.0640	29.8584
8	97.125000	33.008333	4090.977	-37.0318	-24.6377
9	97.141667	33.008333	4070.007	-36.9990	-80.4870
10	97.158333	33.008333	3991.047	-36.9665	-134.7652
11	97.175000	33.008333	3985.070	-36.9327	-184.4728
12	97.191667	33.008333	3956.107	-36.8988	-226.7272
13	97.208333	33.008333	3965.137	-36.8642	-258.8156
14	97.225000	33.008333	3964.173	-36.8295	-278.2994
15	97.241667	33.008333	3983.205	-36.7943	-283.2352
16	97.258333	33.008333	3953.251	-36.7595	-272.3134
17	97.275000	33.008333	4016.279	-36.7238	-245.0733
18	97.291667	33.008333	4054.318	-36.6883	-202.0572
19	97.308333	33.008333	4090.360	-36.6528	-144.8927
20	97.325000	33.008333	4192.388	-36.6171	-76.4053
21	97.341667	33.008333	4287.429	-36.5816	-0.3795

Extract disturbing gravity gradient

Plot↓



gravity disturbance (mGal) on surface



disturbing gravity gradient (E, radial)

Radial Gradient Integration algorithm is derived from the solution to the Stokes Boundary Value Problem, this requires the integrand field elements to reside on an equipotential boundary surface.

Equipotential Surface Construction: Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Computation of Disturbing Gravity Gradient via Inverse Operation Integration

Radial Gradient Integration
of Anomalous Gravity
Field Elements

Computation of External Gravity
Disturbance from
Disturbing Gravity Gradient

Computation of Disturbing
Gravity Gradient via
Inverse Operation Integration

Computation of External Disturbing
Gravity Gradient
from Gravity Disturbance

Computation of External Anomalous
Gravity Field Elements
via Poisson Integration

Open Equipotential Surface Ellipsoidal
Height Grid File

Open Residual Gravity Disturbance Grid File

Select computation point file format

surface ellipsoidal height grid file

>> Computation Process ** Operation Prompts

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 07:47:59

>> Complete the computation!

>> Computation end time: 2026-04-14 07:48:44

>> [Function] Computes the residual disturbing gravity gradient (E, radial component) on a surface via inverse operation integration, utilizing the ellipsoidal height grid of the equipotential boundary surface and the corresponding residual gravity disturbance grid (mGal).

** This function effectively implements the differentiation of gravity field elements through an integral formulation.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.

>> Open Residual Gravity Disturbance Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/resGMlgeoid541_1800.rga.

>> Save Disturbing Gravity Gradient as C:/PAGrav4.5_win64en/examples/Intgendistgradient/rgatogrddwm.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 07:51:25

>> Complete the computation!

>> Computation end time: 2026-04-14 08:06:35

Integral radius 120 km

Save the results as

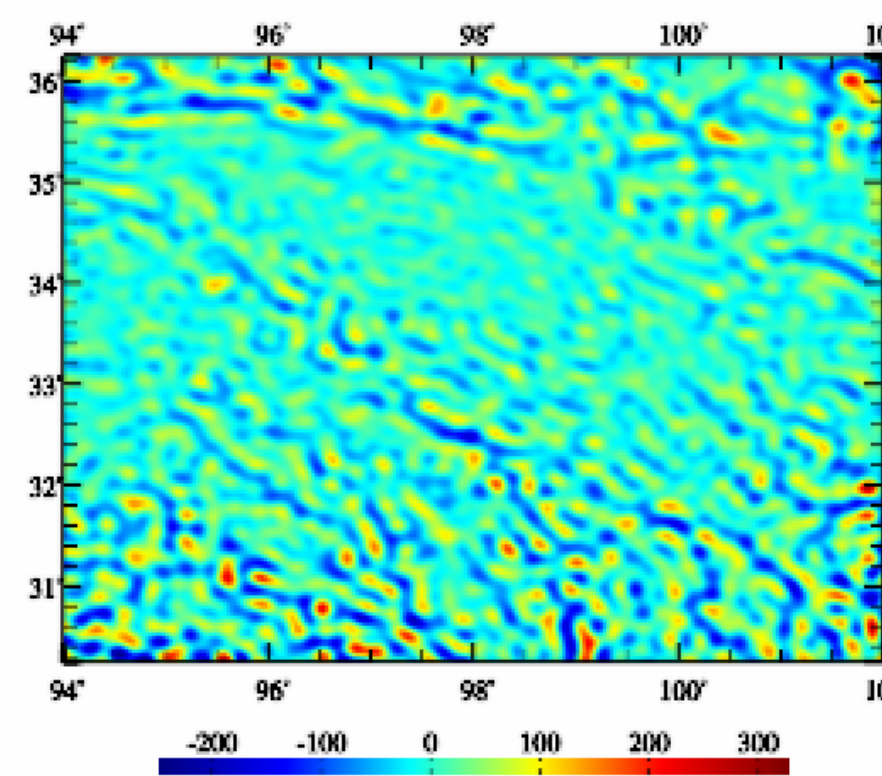
Import setting parameters

Start Computation

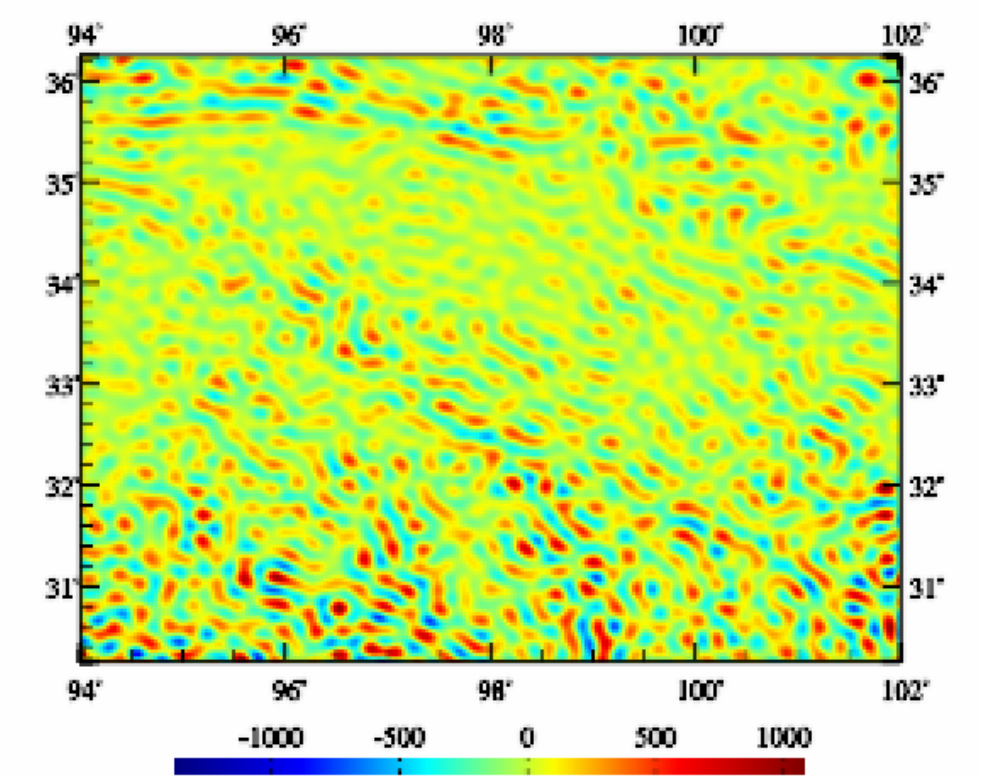
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
-505.4299	-429.7192	-375.1353	-318.9850	-259.0175	-178.4956
320.7604	295.5932	267.9471	244.0138	228.7793	225.3513
561.1932	609.1060	661.1394	715.1226	766.7232	809.7038
-133.4266	-279.6643	-397.6367	-481.6297	-529.7104	-543.6878
-224.3589	-233.7219	-247.5993	-262.5230	-274.9934	-281.5644
313.2648	368.5782	390.7834	373.2899	313.7112	214.9406
78.3233	312.9719	544.6997	746.3038	893.2953	967.2313
-509.5290	-464.7210	-359.7168	-211.5690	-40.9560	130.8348
202.3081	140.9889	91.4380	53.8376	27.1802	10.1833
284.4640	298.8265	286.6105	243.8790	170.3773	70.0537
-281.5145	-176.5681	-77.0418	5.5935	63.1846	91.8935
-127.7336	-134.1163	-153.4278	-191.2082	-250.1097	-329.2804
-831.1787	-784.8146	-733.7349	-681.9185	-630.9929	-580.0215
398.4436	460.7426	481.5642	456.7132	386.8177	277.5634
-314.8760	-189.4597	-51.4814	87.5984	217.8968	332.0093
437.3080	358.6982	261.0526	145.4304	15.2953	-123.2869
-230.4060	-91.7501	37.8178	145.7641	222.5161	262.6057
-103.3651	-67.1048	-20.6186	27.7371	70.0580	100.1148
-59.1371	-57.8878	-55.5052	-55.1573	-58.8136	-66.5979
246.8788	292.7210	312.0650	298.5954	250.2514	170.0304

Extract disturbing gravity gradient

Plot↓



gravity disturbance (mGal) on surface



disturbing gravity gradient (E, radial)

Radial Gradient Integration algorithm is derived from the solution to the Stokes Boundary Value Problem, this requires the integrand field elements to reside on an equipotential boundary surface.

Equipotential Surface Construction: Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Table 4.20: Statistics of Model Reference Truth

Field Element	Unit	Mean	Std. Dev.	Min	Max
Residual Disturbing Gravity Gradients (Radial) on Geoid	E	0.4763	68.2499	-288.1750	387.7286

Table 4.21: Differences between Computed Results and Reference Truth

Field Element	Unit	Mean	Std. Dev.	Min	Max
Residual Disturbing Gravity Gradients (Radial) on Geoid	E	-0.0562	6.6405	-37.1465	28.3557

Algorithm Performance: The ratio of the standard deviation of Differences in Table 4.21 to the standard deviation of Reference Truths in Table 4.20 reflects the approximation capability of the corresponding algorithm.

Stability of Differentiation: Computing gradients from gravity disturbances is a differentiation process, typically sensitive to high-frequency noise. In this case, while the true signal exhibits a large variability (Std. Dev. = 68.25 E), the error standard deviation is well-controlled at ~6.64 E (approximately 10% of the signal strength).

Conclusion: The inverse operation integration algorithm offers superior numerical stability for differentiation tasks. It effectively preserves high-frequency signal characteristics while avoiding the severe oscillations often associated with traditional finite difference methods, making it suitable for high-precision gravity gradient field modeling.

Computation of External Disturbing Gravity Gradient from Gravity Disturbance

Save as Import parameters Start Computation Save process Follow example

Radial Gradient Integration
of Anomalous Gravity
Field Elements

Computation of External Gravity
Disturbance from
Disturbing Gravity Gradient

Computation of Disturbing
Gravity Gradient via
Inverse Operation Integration

Computation of External Disturbing
Gravity Gradient
from Gravity Disturbance

Computation of External Anomalous
Gravity Field Elements
via Poisson Integration

Open Boundary Surface Ellipsoidal
Height Grid File

Open Residual Gravity Disturbance Grid File

Select computation point file format

discrete computation point file

Open Space Computation Point File

Set input point file format

number of rows of file header 1

>> Computation Process ** Operation Prompts

>> Complete the computation!

>> Computation end time: 2026-04-14 08:06:35

>> [Function] Computes the residual disturbing gravity gradient (E, radial component) on or outside the geoid. Inputs include an ellipsoidal height grid of a boundary surface and the corresponding residual gravity disturbance grid (mGal).

** The inverse integration of gravity disturbances employs a hybrid algorithm combining Poisson integration with differentiation. This approach does not require the boundary surface to be an equipotential surface.

>> Open Boundary Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.

>> Open Residual Gravity Disturbance Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/resGMIgeoid541_1800.rga.

>> Open Space Computation Point File C:/PAGrav4.5_win64en/examples/Intgendistgradient/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save Poisson Integration Results as C:/PAGrav4.5_win64en/examples/Intgendistgradient/rgatogrrdbm.txt.

>> Appends one column containing the residual anomalous field element value.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 08:09:04

>> Complete the computation!

>> Computation end time: 2026-04-14 08:09:49

Integral radius 120 km

Save the results as

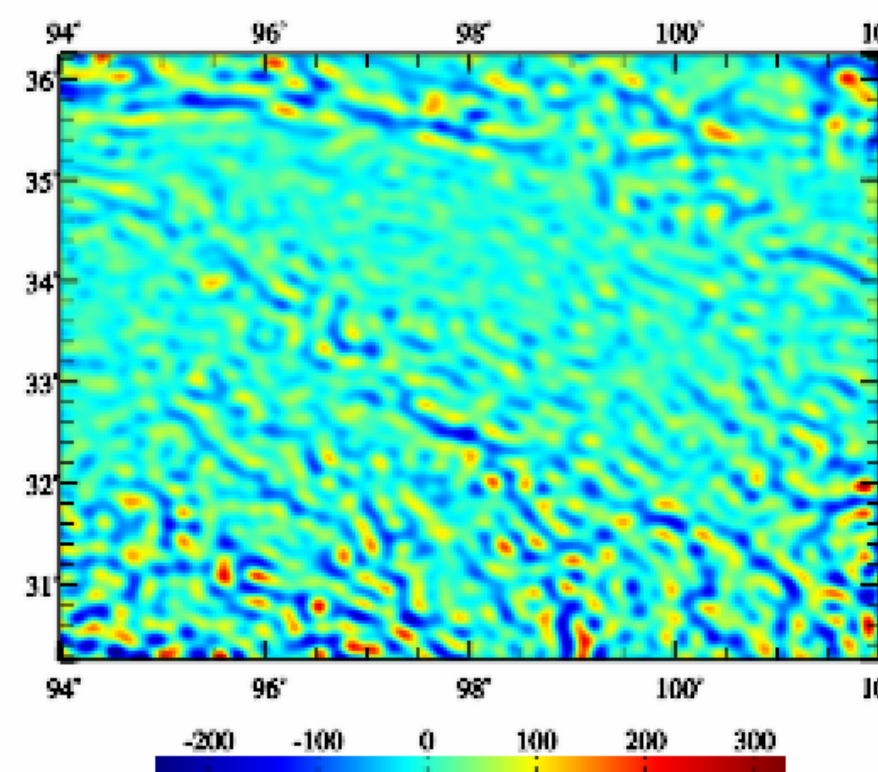
Import setting parameters

Start Computation

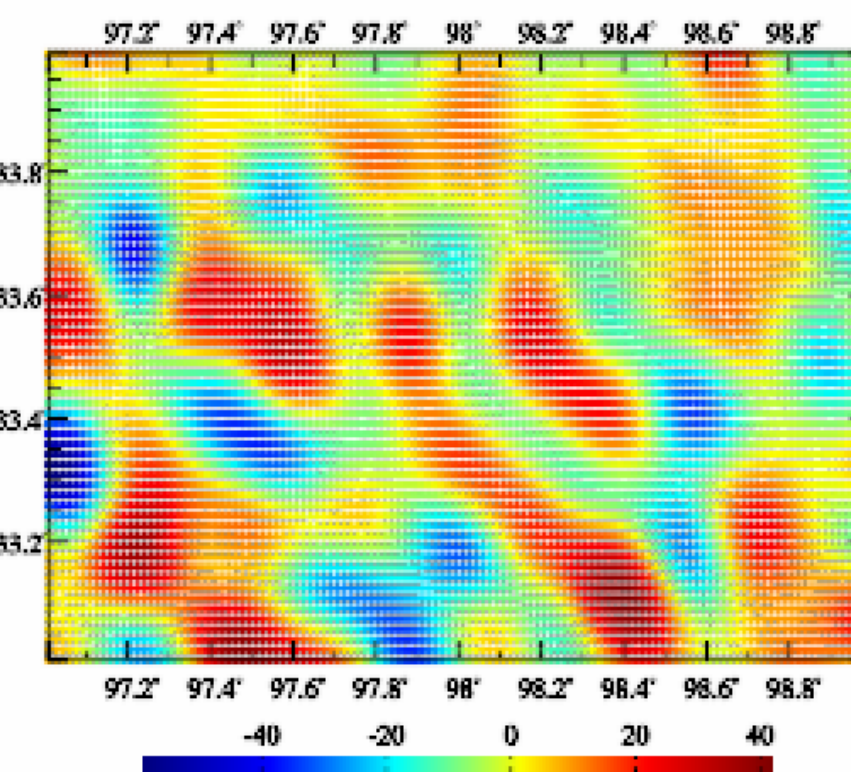
no	lon (degree/decimal)	lat	ellipHeight (m)	
1	97.008333	33.008333	3942.764	7.3569
2	97.025000	33.008333	3989.787	7.1106
3	97.041667	33.008333	4034.817	6.1598
4	97.058333	33.008333	4070.847	4.5194
5	97.075000	33.008333	4106.877	2.2173
6	97.091667	33.008333	4119.913	-0.6277
7	97.108333	33.008333	4115.946	-3.9317
8	97.125000	33.008333	4090.977	-7.5796
9	97.141667	33.008333	4070.007	-11.4084
10	97.158333	33.008333	3991.047	-15.3450
11	97.175000	33.008333	3985.070	-18.9119
12	97.191667	33.008333	3956.107	-22.0661
13	97.208333	33.008333	3965.137	-24.3539
14	97.225000	33.008333	3964.173	-25.7245
15	97.241667	33.008333	3983.205	-25.8555
16	97.258333	33.008333	3953.251	-24.9234
17	97.275000	33.008333	4016.279	-22.2078
18	97.291667	33.008333	4054.318	-18.2787
19	97.308333	33.008333	4090.360	-13.1378
20	97.325000	33.008333	4192.388	-6.8831
21	97.341667	33.008333	4287.429	-0.0506

Extract residual disturbing gradient

Plot



residual gravity disturbance



residual disturbing gradient

Radial Gradient Integration algorithm is derived from the solution to the Stokes Boundary Value Problem, this requires the integrand field elements to reside on an equipotential boundary surface.

Equipotential Surface Construction: Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Computation of External Disturbing Gravity Gradient from Gravity Disturbance

Radial Gradient Integration of Anomalous Gravity Field Elements

Computation of External Gravity Disturbance from Disturbing Gravity Gradient

Computation of Disturbing Gravity Gradient via Inverse Operation Integration

Computation of External Disturbing Gravity Gradient from Gravity Disturbance

Computation of External Anomalous Gravity Field Elements via Poisson Integration

Open Boundary Surface Ellipsoidal Height Grid File

Open Residual Gravity Disturbance Grid File

Select computation point file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal Height Grid File

>> Computation Process ** Operation Prompts

>> Computation start time: 2026-04-14 08:09:04

>> Complete the computation!

>> Computation end time: 2026-04-14 08:09:49

>> [Function] Computes the residual disturbing gravity gradient (E, radial component) on or outside the geoid. Inputs include an ellipsoidal height grid of a boundary surface and the corresponding residual gravity disturbance grid (mGal).

** The inverse integration of gravity disturbances employs a hybrid algorithm combining Poisson integration with differentiation. This approach does not require the boundary surface to be an equipotential surface.

>> Open Boundary Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.

>> Open Residual Gravity Disturbance Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/resGMIgeoid541_1800.rga.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/landbmsurfhgt.dat.

>> Save Poisson Integration Results as C:/PAGrav4.5_win64en/examples/Intgendistgradient/rgatogrdbm.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 08:15:41

>> Complete the computation!

>> Computation end time: 2026-04-14 08:31:32

Integral radius 120 km

Save the results as

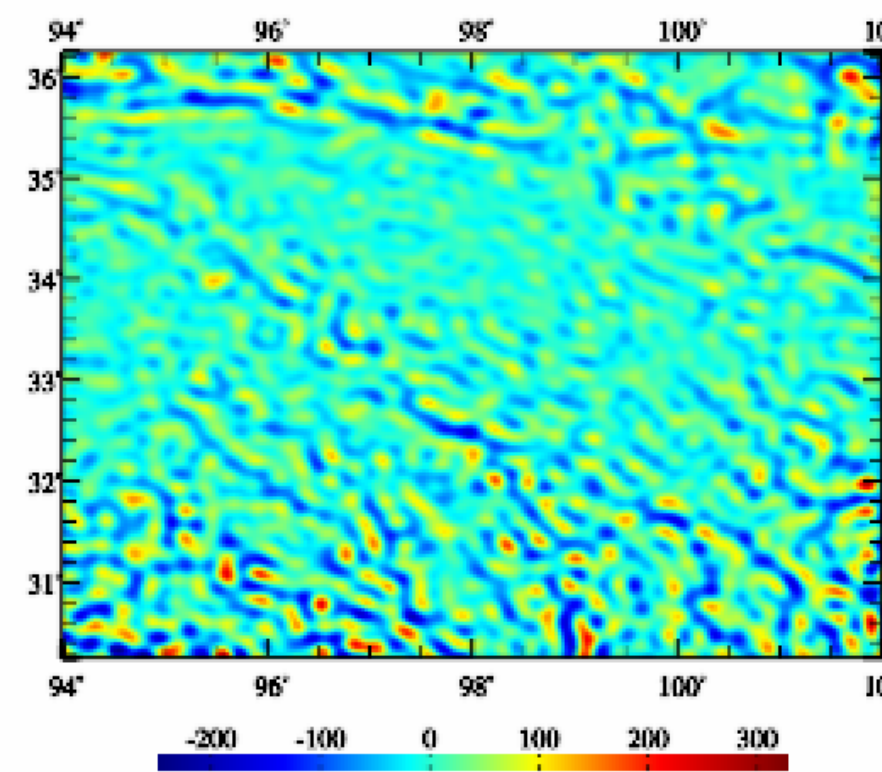
Import setting parameters

Start Computation

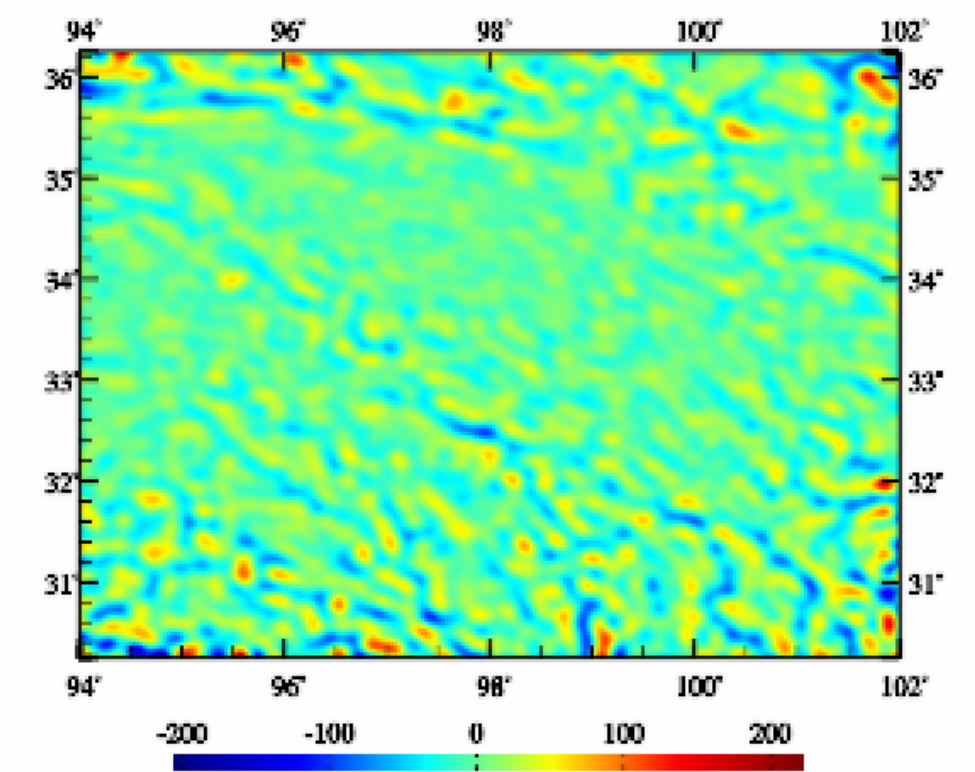
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
25.5482	32.2428	34.7751	34.4845	32.7565	30.5713
23.1387	24.0504	26.1586	29.2803	32.9778	37.0922
-40.9652	-52.4488	-60.2859	-64.0934	-64.2180	-61.8296
-168.9207	-188.2162	-201.5886	-205.7324	-200.1540	-186.4230
97.2243	108.0579	113.3779	114.2648	111.2597	104.2769
-22.5602	-36.0182	-50.5114	-65.5670	-81.4032	-96.6037
99.7997	144.4861	182.6666	209.9054	223.2932	222.6037
-129.9665	-120.0988	-98.2952	-68.4071	-34.2581	-0.6680
20.9084	8.9722	-0.7863	-8.2100	-13.4662	-16.8121
7.8213	9.7997	9.7165	7.2302	2.6829	-3.2340
68.8623	82.9914	91.4576	93.0122	86.6506	72.5596
-114.4181	-108.9383	-101.6061	-94.1016	-87.4459	-82.9561
-32.1668	-20.9009	-10.4904	-2.0746	3.9615	7.4281
22.0577	23.2345	21.5692	16.6097	8.4376	-2.2785
20.2172	40.4930	58.6649	73.1995	82.7652	86.6726
-46.1260	-58.3513	-69.5337	-80.0170	-90.5309	-100.4354
-36.7005	-17.3713	-0.7322	12.0406	20.1988	23.5788
2.8070	9.4586	15.4368	19.5678	20.9498	19.1877
-15.8866	-11.3266	-7.7563	-6.1785	-7.1564	-10.7033
4.9674	13.4445	17.6264	16.0554	8.2735	-5.6347

Extract residual disturbing gradient

Plot↓



residual gravity disturbance



residual disturbing gradient

Radial Gradient Integration algorithm is derived from the solution to the Stokes Boundary Value Problem, this requires the integrand field elements to reside on an equipotential boundary surface.

Equipotential Surface Construction: Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Table 4.22: Statistics of Model Reference Truth

Field Element	Unit	Mean	Std. Dev.	Min	Max
Ground Residual Disturbing Gravity Gradients (Radial)	E	-0.2872	26.9448	-148.0282	136.7864

Table 4.23: Differences between Computed Results and Reference Truth

Field Element	Unit	Mean	Std. Dev.	Min	Max
Ground Residual Disturbing Gravity Gradients (Radial)	E	-0.0361	2.7361	-14.2089	13.6802

Algorithm Performance: The ratio of the standard deviation of Differences in Table 4.23 to the standard deviation of Reference Truths in Table 4.22 reflects the approximation capability of the corresponding algorithm.

Comparative Analysis: The inversion integration of gravity disturbance demonstrates comparable performance and accuracy. Both approaches effectively achieve high-precision differential inversion from gravity disturbances to gradients, with error standard deviations controlled to approximately 10% of the signal strength, validating the consistency of the algorithms across different mathematical formulations.

Computation of External Anomalous Gravity Field Elements via Poisson Integration

Save as Import parameters Start Computation Save process Follow example

Radial Gradient Integration
of Anomalous Gravity
Field Elements

Computation of External Gravity
Disturbance from
Disturbing Gravity Gradient

Computation of Disturbing
Gravity Gradient via
Inverse Operation Integration

Computation of External Disturbing
Gravity Gradient
from Gravity Disturbance

Computation of External Anomalous
Gravity Field Elements
via Poisson Integration

Open Boundary Surface Ellipsoidal
Height Grid File

Open Residual Field Element Grid File

Select computation point file format

discrete computation point file

Open Space Computation Point File

Set input point file format

number of rows of file header 1

>> Computation Process ** Operation Prompts

>> Complete the computation!

>> Computation end time: 2026-04-14 08:31:32

>> [Function] Computes residual anomalous gravity field elements on or outside the geoid via Poisson integration. Inputs include an ellipsoidal height grid of a boundary surface and the corresponding residual anomalous field element grid on that surface.

** It does not require the boundary surface to be an equipotential surface, making it ideal for analytical continuation (both upward and downward) across arbitrary geometric boundaries.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGravf4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.

>> Open Residual Field Element Grid File C:/PAGravf4.5_win64en/examples/Intgendistgradient/resGMIgeoid541_1800.ksi.

>> Open Space Computation Point File C:/PAGravf4.5_win64en/examples/Intgendistgradient/calcpnt.txt.

** Look at the file information in the window below, set the input file format parameters...

>> Save the results as C:/PAGravf4.5_win64en/examples/Intgendistgradient/possonksi.txt.

>> Appends one column containing the residual disturbing gravity gradient value.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 08:35:13

>> Complete the computation!

>> Computation end time: 2026-04-14 08:36:45

Save computation process as

Integral radius 120 km

Save the results as

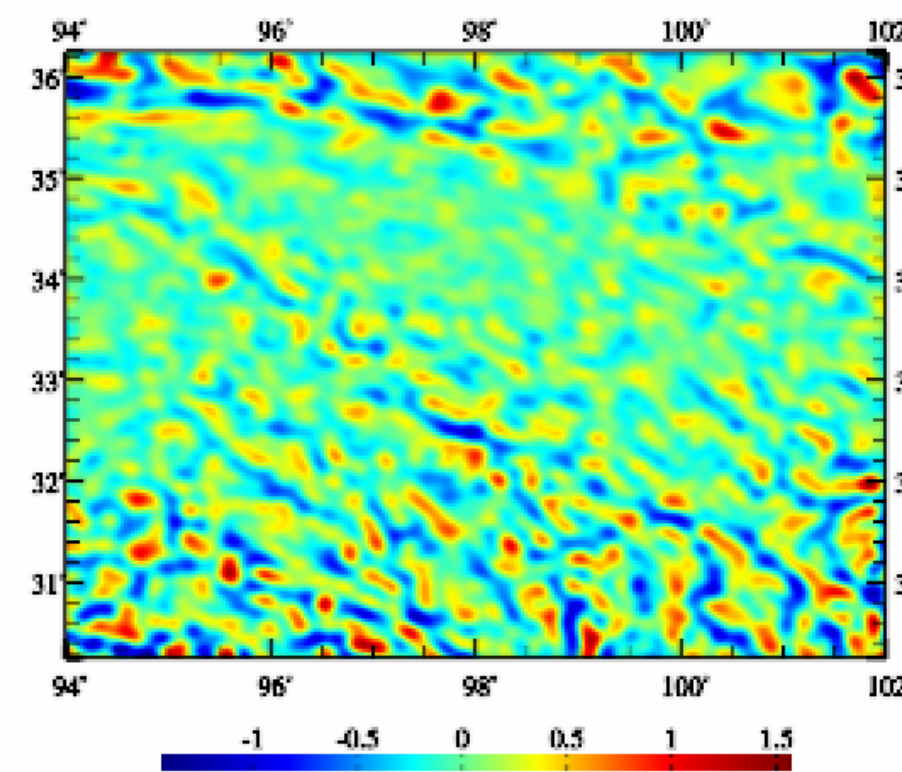
Import setting parameters

Start Computation

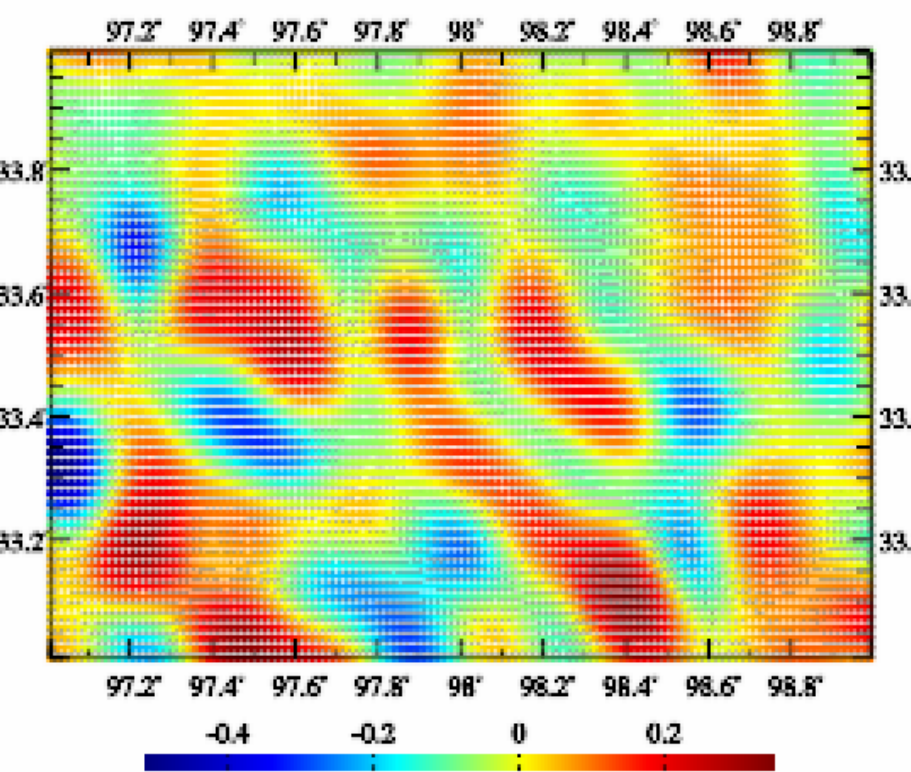
no	lon (degree/decimal)	lat	ellipHeight (m)	
1	97.008333	33.008333	3942.764	0.0636
2	97.025000	33.008333	3989.787	0.0614
3	97.041667	33.008333	4034.817	0.0531
4	97.058333	33.008333	4070.847	0.0391
5	97.075000	33.008333	4106.877	0.0195
6	97.091667	33.008333	4119.913	-0.0046
7	97.108333	33.008333	4115.946	-0.0326
8	97.125000	33.008333	4090.977	-0.0637
9	97.141667	33.008333	4070.007	-0.0964
10	97.158333	33.008333	3991.047	-0.1305
11	97.175000	33.008333	3985.070	-0.1613
12	97.191667	33.008333	3956.107	-0.1889
13	97.208333	33.008333	3965.137	-0.2087
14	97.225000	33.008333	3964.173	-0.2208
15	97.241667	33.008333	3983.205	-0.2220
16	97.258333	33.008333	3953.251	-0.2146
17	97.275000	33.008333	4016.279	-0.1909
18	97.291667	33.008333	4054.318	-0.1572
19	97.308333	33.008333	4090.360	-0.1132
20	97.325000	33.008333	4192.388	-0.0596
21	97.341667	33.008333	4287.429	-0.0017

Extract external field element

Plot



field element on surface



external field element

Radial Gradient Integration algorithm is derived from the solution to the Stokes Boundary Value Problem, this requires the integrand field elements to reside on an equipotential boundary surface.

Equipotential Surface Construction: Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Computation of External Anomalous Gravity Field Elements via Poisson Integration

Radial Gradient Integration of Anomalous Gravity Field Elements

Computation of External Gravity Disturbance from Disturbing Gravity Gradient

Computation of Disturbing Gravity Gradient via Inverse Operation Integration

Computation of External Disturbing Gravity Gradient from Gravity Disturbance

Computation of External Anomalous Gravity Field Elements via Poisson Integration

Open Boundary Surface Ellipsoidal Height Grid File

Open Residual Field Element Grid File

Select computation point file format

surface ellipsoidal height grid file

Open Computation Surface Ellipsoidal Height Grid File

>> Computation Process ** Operation Prompts

>> Complete the computation!

>> Computation end time: 2026-04-14 08:36:45

>> [Function] Computes residual anomalous gravity field elements on or outside the geoid via Poisson integration. Inputs include an ellipsoidal height grid of a boundary surface and the corresponding residual anomalous field element grid on that surface.

** It does not require the boundary surface to be an equipotential surface, making it ideal for analytical continuation (both upward and downward) across arbitrary geometric boundaries.

>> Open Equipotential Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/landgeoidhgt.dat.

>> Open Residual Field Element Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/resGMIgeoid541_1800.ksi.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/possessionksi.dat.

>> Open Computation Surface Ellipsoidal Height Grid File C:/PAGrav4.5_win64en/examples/Intgendistgradient/landbmsurfhgt.dat.

>> Save the results as C:/PAGrav4.5_win64en/examples/Intgendistgradient/possessionksi.dat.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-04-14 08:41:27

>> Complete the computation!

>> Computation end time: 2026-04-14 09:13:01

Integral radius 120 km

Save the results as

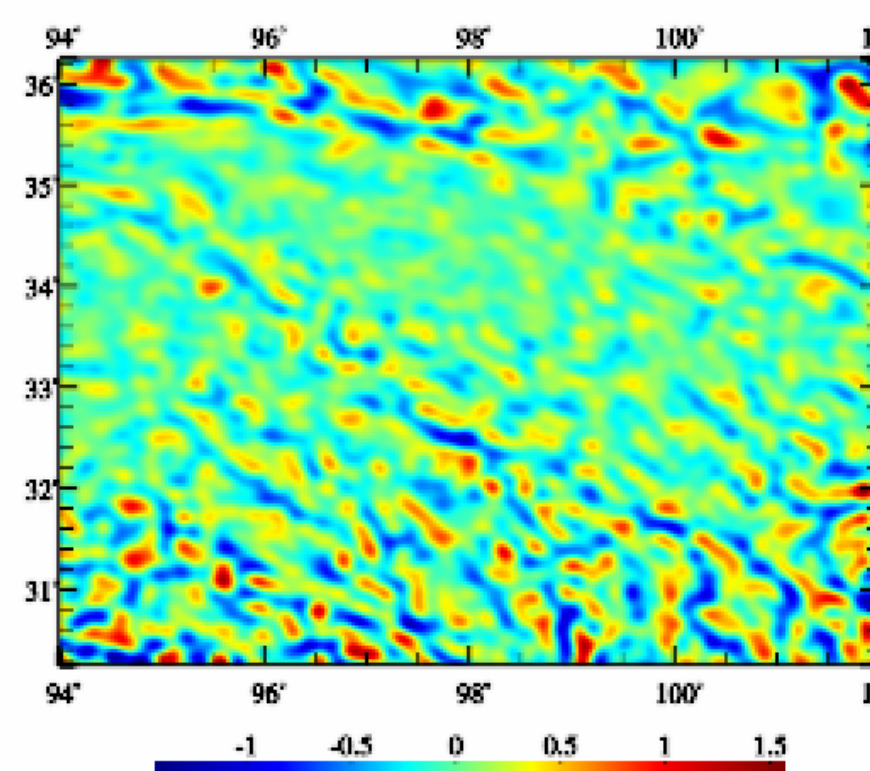
Import setting parameters

Start Computation

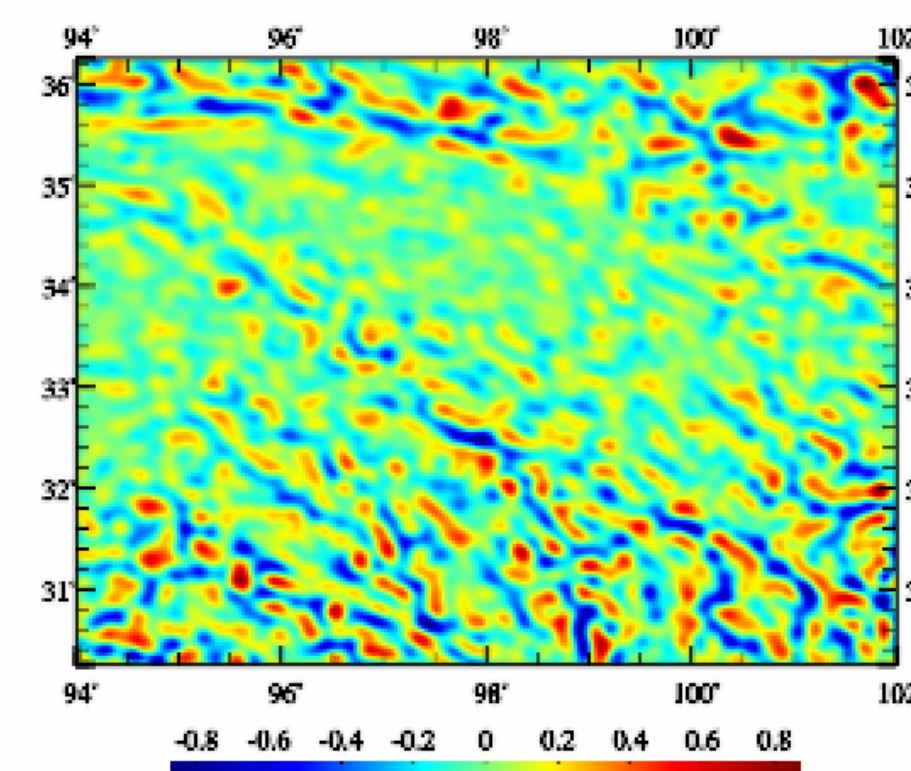
94.000000	102.000000	30.250000	36.250000	0.01666667	0.01666667
0.1157	0.1319	0.1369	0.1356	0.1311	0.1250
0.1244	0.1314	0.1401	0.1473	0.1512	0.1590
-0.0049	-0.0420	-0.0602	-0.0680	-0.0658	-0.0428
-0.3743	-0.4508	-0.5145	-0.5484	-0.5487	-0.5269
0.2770	0.3131	0.3341	0.3437	0.3429	0.3307
-0.0089	-0.0507	-0.0940	-0.1428	-0.1913	-0.2405
0.2850	0.3812	0.4591	0.5059	0.5179	0.5101
-0.4162	-0.3848	-0.3189	-0.2331	-0.1353	-0.0466
0.0699	0.0457	0.0258	0.0114	0.0022	-0.0027
0.0521	0.0519	0.0495	0.0406	0.0294	0.0177
0.2419	0.2725	0.2878	0.2879	0.2697	0.2346
-0.3044	-0.3000	-0.2908	-0.2811	-0.2715	-0.2698
-0.1457	-0.1294	-0.1080	-0.0928	-0.0774	-0.0647
-0.0071	-0.0030	-0.0050	-0.0138	-0.0288	-0.0475
0.1051	0.1566	0.1991	0.2315	0.2518	0.2596
-0.0938	-0.1315	-0.1692	-0.2057	-0.2443	-0.2838
-0.1469	-0.0852	-0.0329	0.0080	0.0377	0.0565
0.0687	0.0805	0.0893	0.0931	0.0897	0.0787
-0.0526	-0.0446	-0.0380	-0.0353	-0.0376	-0.0444
0.0321	0.0542	0.0632	0.0560	0.0352	-0.0037

Extract external field element

Plot↓



field element on surface



external field element

Radial Gradient Integration algorithm is derived from the solution to the Stokes Boundary Value Problem, this requires the integrand field elements to reside on an equipotential boundary surface.

Equipotential Surface Construction: Typically constructed from a global geopotential model (degree ≤ 360). For equipotential surfaces within an altitude of 10 km, a normal (or orthometric) equiheight surface may serve as a valid approximation.

Table 4.24: Statistics of Model Reference Truths

Field Element	Unit	Mean	Std. Dev.	Min	Max
Ground Residual Height Anomalies	m	0.0010	0.1182	-0.6745	0.4760
Ground Residual Gravity Disturbances	mGal	-0.0346	15.7527	-93.9854	66.6638

Table 4.25: Differences between Computed Results and Reference Truth

Field Element	Unit	Mean	Std. Dev.	Min	Max
Ground Residual Height Anomalies	m	0.0004	0.0074	-0.0326	0.0334
Ground Residual Gravity Disturbances	mGal	0.0476	1.1919	-5.0173	6.5138

Algorithm Performance: The ratio of the standard deviation of Differences in Table 4.25 to the standard deviation of Reference Truths in Table 4.24 reflects the approximation capability of the corresponding algorithm.

Superior Short-Wavelength Preservation: The algorithm effectively suppresses the attenuation of short-wavelength signals, demonstrating high fidelity for both upward and downward analytical continuation.

Iterative Strategy: Supports iterative execution to perform multi-step Poisson integrations. In general, a single iteration suffices to meet accuracy requirements for most applications. In scenarios involving extreme topography or large vertical separations, at most three iterations are typically required to achieve convergence.

Spatial and Spectral Performance Analysis of Spherical Radial Basis Functions

Spatial and Spectral Performance Analysis of Spherical Radial Basis Functions

Construction of Reuter Spherical Grid at a Specified Level

Select anomalous field element gravity disturbance

Algorithms of the spherical radial basis function and Reuter grid

Save computation process as

[SRBFs] Point-mass kernel, Poisson kernel, m-order radial multipole and m-order Poisson wavelet kernel functions.

degrees of the SRBF Legendre expansion, the burial depth of the reference surface (Bjerhammar sphere), and the order (m) of multipolar SRBFs – users can analyze and characterize the spatial localization and spectral behavior of SRBFs for diverse gravity field elements.

>> Save the results as C:/PAGrav4.5_win64en/examples/SRBFperformance/rgaSRBF.txt.

>> The program generates both spatial-domain and spectral-domain curve files. Contains the kernel values for the four SRBF types corresponding to the selected field element. Record Format: Spherical Angular Distance (Independent Variable), Point Mass Kernel Value, Poisson Kernel Value, m-order Radial Multipole Kernel Value, m-order Poisson Wavelet Kernel Value.

** Automatically saved to the current directory with the user-defined prefix (*). Contains the degree variances for the four SRBF types. Record Format: SRBF Legendre Degree (Independent Variable), Point Mass Degree Variance, Poisson Degree Variance, m-order Radial Multipole Degree Variance, m-order Poisson Wavelet Degree Variance.

Set SRBF parameters

Minimum Degree 30
Maximum Degree 1440
Bjerhammar Sphere Burial Depth 5km
Multipole/Wavelet Order 5
Action Distance of SBRF Center 1.5°
Spatial Sampling Interval 0.20'

Calculation and save

Display of the input-output file↓

1.446667	-0.004675	-0.006049	-0.008050	-0.017435
1.450000	-0.004839	-0.006275	-0.008335	-0.017861
1.453333	-0.004972	-0.006457	-0.008560	-0.018159
1.456667	-0.005075	-0.006593	-0.008723	-0.018330
1.460000	-0.005147	-0.006683	-0.008825	-0.018372
1.463333	-0.005188	-0.006726	-0.008864	-0.018285
1.466667	-0.005197	-0.006723	-0.008840	-0.018072
1.470000	-0.005174	-0.006674	-0.008755	-0.017735
1.473333	-0.005120	-0.006579	-0.008609	-0.017276
1.476667	-0.005037	-0.006440	-0.008404	-0.016700
1.480000	-0.004923	-0.006257	-0.008142	-0.016012
1.483333	-0.004781	-0.006032	-0.007824	-0.015217
1.486667	-0.004611	-0.005768	-0.007454	-0.014321
1.490000	-0.004416	-0.005466	-0.007034	-0.013332
1.493333	-0.004195	-0.005128	-0.006567	-0.012256
1.496667	-0.003952	-0.004758	-0.006058	-0.011102
1.500000	-0.003689	-0.004357	-0.005509	-0.009878

Curve type Spatial curves of SRBFs

Line thickness 3

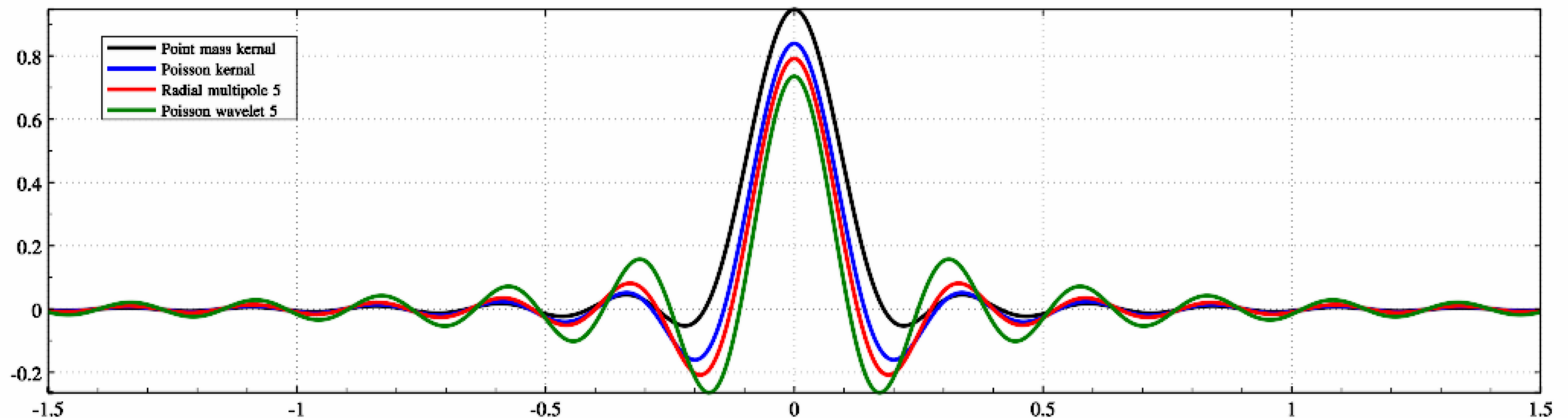
Start end row number 1 3600

Extract plot data

Chart plot↓

Spatial and spectral curves of spherical radial basis functions (SRBF)

Save current plot as



Spatial and Spectral Performance Analysis of Spherical Radial Basis Functions

Spatial and Spectral Performance Analysis of Spherical Radial Basis Functions

Construction of Reuter Spherical Grid at a Specified Level

Select anomalous field element gravity disturbance

Algorithms of the spherical radial basis function and Reuter grid

Save computation process as

[SRBFs] Point-mass kernel, Poisson kernel, m-order radial multipole and m-order Poisson wavelet kernel functions.

degrees of the SRBF Legendre expansion, the burial depth of the reference surface (Bjerhammar sphere), and the order (m) of multipolar SRBFs – users can analyze and characterize the spatial localization and spectral behavior of SRBFs for diverse gravity field elements.

>> Save the results as C:/PAGrav4.5_win64en/examples/SRBFperformance/rgaSRBF.txt.

>> The program generates both spatial-domain and spectral-domain curve files. Contains the kernel values for the four SRBF types corresponding to the selected field element. Record Format: Spherical Angular Distance (Independent Variable), Point Mass Kernel Value, Poisson Kernel Value, m-order Radial Multipole Kernel Value, m-order Poisson Wavelet Kernel Value.

** Automatically saved to the current directory with the user-defined prefix (*). Contains the degree variances for the four SRBF types. Record Format: SRBF Legendre Degree (Independent Variable), Point Mass Degree Variance, Poisson Degree Variance, m-order Radial Multipole Degree Variance, m-order Poisson Wavelet Degree Variance.

Set SRBF parameters

Minimum Degree 30
Maximum Degree 1440
Bjerhammar Sphere Burial Depth 5km
Multipole/Wavelet Order 5
Action Distance of SBRF Center 1.5°
Spatial Sampling Interval 0.20'

Calculation and save

Display of the input-output file↓

1424	0.991387	0.980618	0.958923	0.876948
1425	0.991223	0.981833	0.961462	0.884220
1426	0.991058	0.983047	0.964004	0.891546
1427	0.990892	0.984261	0.966550	0.898927
1428	0.990725	0.985475	0.969100	0.906362
1429	0.990557	0.986688	0.971653	0.913853
1430	0.990388	0.987900	0.974211	0.921399
1431	0.990219	0.989112	0.976773	0.929001
1432	0.990048	0.990324	0.979338	0.936660
1433	0.989876	0.991536	0.981907	0.944375
1434	0.989704	0.992746	0.984480	0.952147
1435	0.989531	0.993957	0.987057	0.959977
1436	0.989356	0.995167	0.989638	0.967864
1437	0.989181	0.996376	0.992223	0.975810
1438	0.989005	0.997585	0.994811	0.983814
1439	0.988828	0.998794	0.997404	0.991877
1440	0.988650	1.000002	1.000000	1.000000

Curve type Spectral curves of SRBFs

Line thickness 3

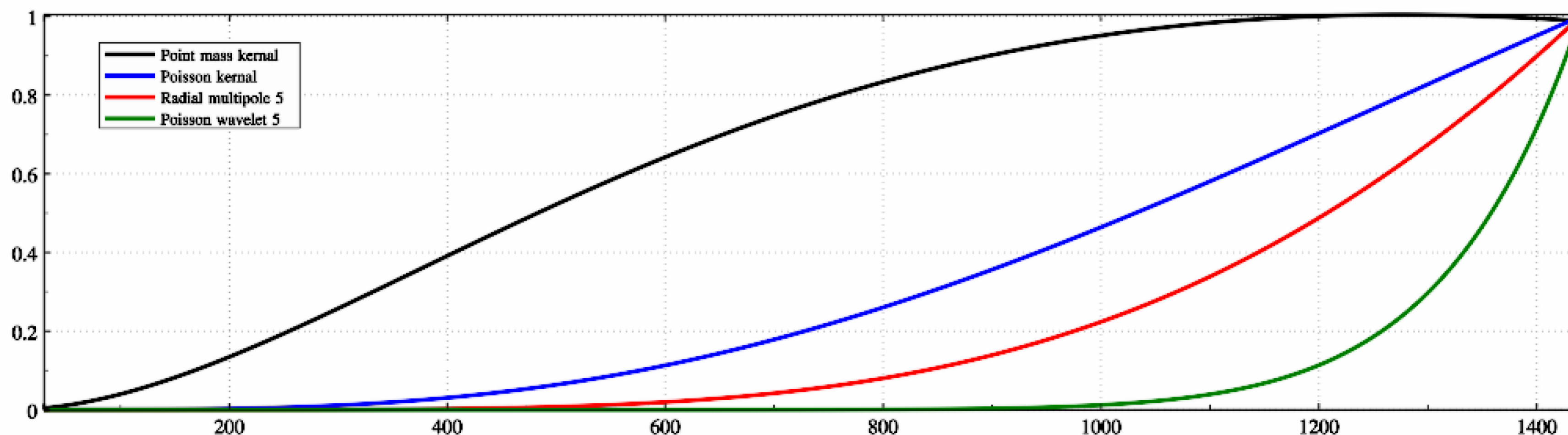
Start end row number 1 3600

Extract plot data

Chart plot↓

Spatial and spectral curves of spherical radial basis functions (SRBF)

Save current plot as



Spatial and Spectral Performance Analysis of Spherical Radial Basis Functions

Spatial and Spectral Performance Analysis of Spherical Radial Basis Functions

Construction of Reuter Spherical Grid at a Specified Level

Select anomalous field element : anomaly / disturbing potential

Algorithms of the spherical radial basis function and Reuter grid

Save computation process as

[SRBFs] Point-mass kernel, Poisson kernel, m-order radial multipole and m-order Poisson wavelet kernel functions.

** Automatically saved to the current directory with the user-defined prefix (*). Contains the degree variances for the four SRBF types. Record Format: SRBF Legendre Degree (Independent Variable), Point Mass Degree Variance, Poisson Degree Variance, m-order Radial Multipole Degree Variance, m-order Poisson Wavelet Degree Variance.

>> Save the results as C:/PAGravf4.5_win64en/examples/SRBFperformance/ksiSRBF.txt.

>> The program generates both spatial-domain and spectral-domain curve files. Contains the kernel values for the four SRBF types corresponding to the selected field element. Record Format: Spherical Angular Distance (Independent Variable), Point Mass Kernel Value, Poisson Kernel Value, m-order Radial Multipole Kernel Value, m-order Poisson Wavelet Kernel Value.

** Automatically saved to the current directory with the user-defined prefix (*). Contains the degree variances for the four SRBF types. Record Format: SRBF Legendre Degree (Independent Variable), Point Mass Degree Variance, Poisson Degree Variance, m-order Radial Multipole Degree Variance, m-order Poisson Wavelet Degree Variance.

Set SRBF parameters

Minimum Degree 30
Maximum Degree 1440
Bjerhammar Sphere Burial Depth 5km
Multipole/Wavelet Order 5
Action Distance of SBRF Center 1.5°
Spatial Sampling Interval 0.20'

Calculation and save

Display of the input-output file↓

1.446667	0.010961	-0.004686	-0.006052	-0.015008
1.450000	0.010781	-0.004850	-0.006278	-0.015414
1.453333	0.010616	-0.004984	-0.006460	-0.015710
1.456667	0.010464	-0.005087	-0.006597	-0.015894
1.460000	0.010327	-0.005158	-0.006687	-0.015967
1.463333	0.010204	-0.005199	-0.006730	-0.015928
1.466667	0.010097	-0.005207	-0.006727	-0.015778
1.470000	0.010004	-0.005185	-0.006678	-0.015519
1.473333	0.009925	-0.005131	-0.006583	-0.015153
1.476667	0.009860	-0.005047	-0.006443	-0.014685
1.480000	0.009810	-0.004933	-0.006260	-0.014117
1.483333	0.009773	-0.004791	-0.006035	-0.013455
1.486667	0.009749	-0.004622	-0.005771	-0.012703
1.490000	0.009738	-0.004426	-0.005468	-0.011868
1.493333	0.009738	-0.004205	-0.005130	-0.010955
1.496667	0.009749	-0.003962	-0.004759	-0.009972
1.500000	0.009770	-0.003698	-0.004359	-0.008927

Curve type Spatial curves of SRBFs

Line thickness 3

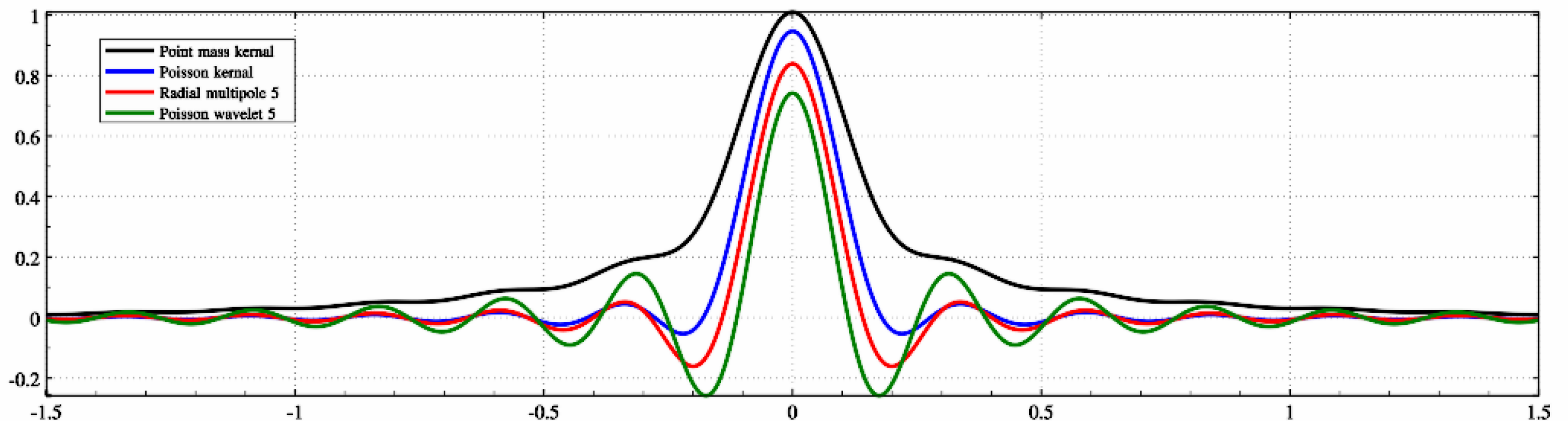
Start end row number 1 3600

Extract plot data

Chart plot↓

Spatial and spectral curves of spherical radial basis functions (SRBF)

Save current plot as



Spatial and Spectral Performance Analysis of Spherical Radial Basis Functions

Spatial and Spectral Performance Analysis of Spherical Radial Basis Functions

Construction of Reuter Spherical Grid at a Specified Level

Select anomalous field element : anomaly / disturbing potential

Algorithms of the spherical radial basis function and Reuter grid

Save computation process as

[SRBFs] Point-mass kernel, Poisson kernel, m-order radial multipole and m-order Poisson wavelet kernel functions.

** Automatically saved to the current directory with the user-defined prefix (*). Contains the degree variances for the four SRBF types. Record Format: SRBF Legendre Degree (Independent Variable), Point Mass Degree Variance, Poisson Degree Variance, m-order Radial Multipole Degree Variance, m-order Poisson Wavelet Degree Variance.

>> Save the results as C:/PAGravf4.5_win64en/examples/SRBFperformance/ksiSRBF.txt.

>> The program generates both spatial-domain and spectral-domain curve files. Contains the kernel values for the four SRBF types corresponding to the selected field element. Record Format: Spherical Angular Distance (Independent Variable), Point Mass Kernel Value, Poisson Kernel Value, m-order Radial Multipole Kernel Value, m-order Poisson Wavelet Kernel Value.

** Automatically saved to the current directory with the user-defined prefix (*). Contains the degree variances for the four SRBF types. Record Format: SRBF Legendre Degree (Independent Variable), Point Mass Degree Variance, Poisson Degree Variance, m-order Radial Multipole Degree Variance, m-order Poisson Wavelet Degree Variance.

Set SRBF parameters

Minimum Degree 30

Maximum Degree 1440

Bjerhammar Sphere Burial Depth 5km

Multipole/Wavelet Order 5

Action Distance of SBRF Center 1.5°

Spatial Sampling Interval 0.20'

Calculation and save

Display of the input-output file↓

1424	0.126130	0.991339	0.980579	0.896752
1425	0.125932	0.991175	0.981797	0.902920
1426	0.125735	0.991011	0.983013	0.909126
1427	0.125538	0.990846	0.984230	0.915368
1428	0.125341	0.990679	0.985446	0.921648
1429	0.125145	0.990512	0.986661	0.927966
1430	0.124949	0.990344	0.987876	0.934322
1431	0.124753	0.990174	0.989091	0.940715
1432	0.124557	0.990004	0.990305	0.947147
1433	0.124362	0.989833	0.991518	0.953617
1434	0.124167	0.989661	0.992732	0.960126
1435	0.123973	0.989488	0.993944	0.966674
1436	0.123778	0.989315	0.995157	0.973260
1437	0.123584	0.989140	0.996369	0.979886
1438	0.123391	0.988964	0.997580	0.986551
1439	0.123197	0.988787	0.998791	0.993256
1440	0.123004	0.988610	1.000002	1.000000

Curve type Spectral curves of SRBFs

Line thickness 3

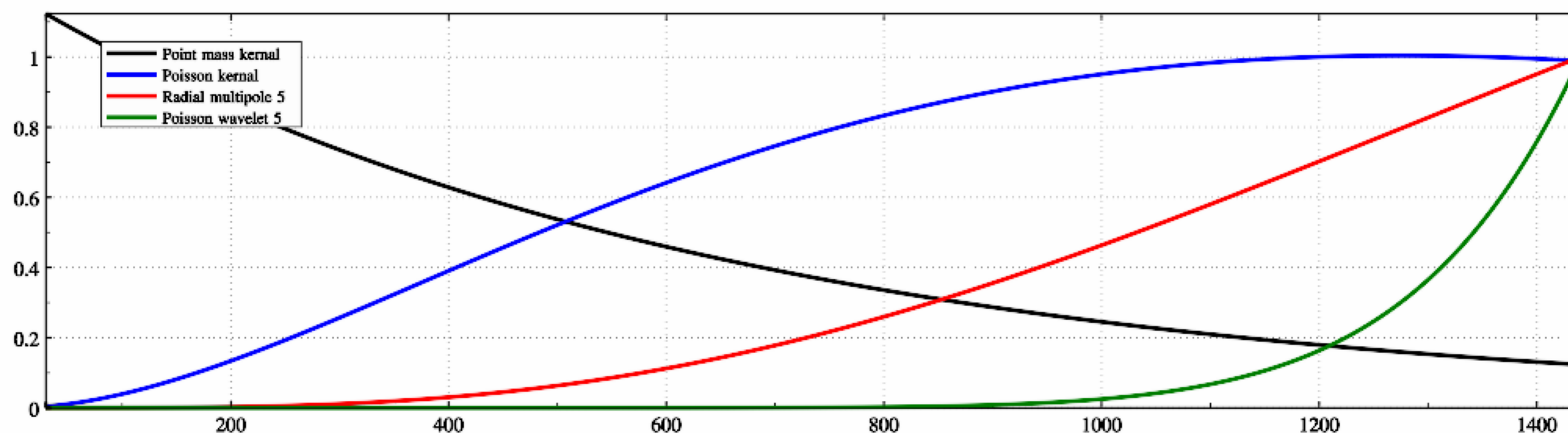
Start end row number 1 3600

Extract plot data

Chart plot↓

Spatial and spectral curves of spherical radial basis functions (SRBF)

Save current plot as



Construction of Reuter Spherical Grid at a Specified Level

Construction of Reuter Spherical Grid at a Specified Level

Minimum Maximum geocentric latitude

-90.0°

90.0°

Minimum Maximum longitude

0.0°

360°

Set Reuter level Q 90

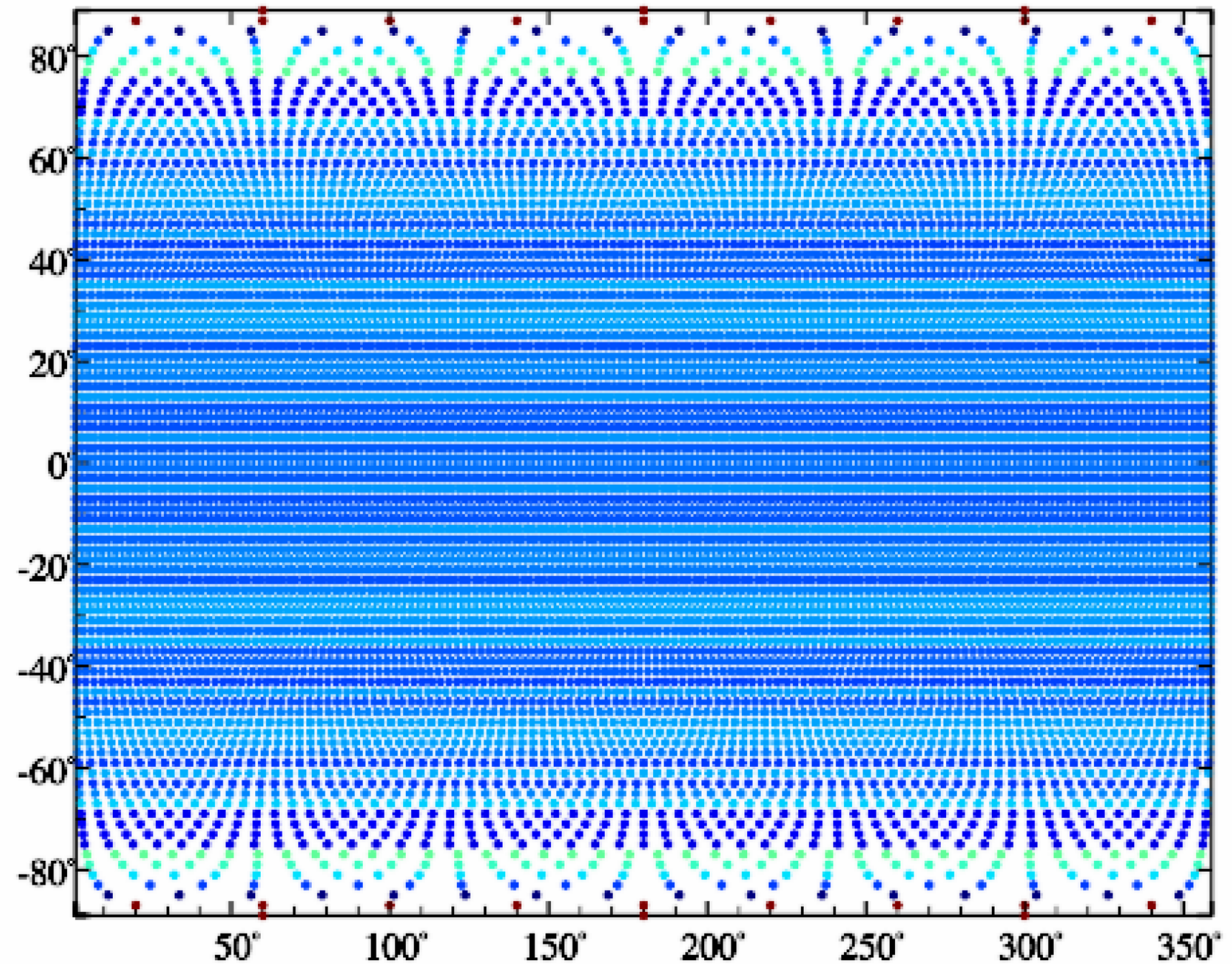
Plot mode longitude, latitude and area of the center of cell grid

Calculation and save

Extract plot data

Reuter grid plot →

Save current plot as



Reuter spatial grid results file

10291	191.250000	85.000000	-1.95	-0.085481	-0.017003	0.996195
10292	213.750000	85.000000	-1.95	-0.072467	-0.048421	0.996195
10293	236.250000	85.000000	-1.95	-0.048421	-0.072467	0.996195
10294	258.750000	85.000000	-1.95	-0.017003	-0.085481	0.996195
10295	281.250000	85.000000	-1.95	0.017003	-0.085481	0.996195
10296	303.750000	85.000000	-1.95	0.048421	-0.072467	0.996195
10297	326.250000	85.000000	-1.95	0.072467	-0.048421	0.996195
10298	348.750000	85.000000	-1.95	0.085481	-0.017003	0.996195
10299	20.000000	87.000000	4.67	0.049180	0.017900	0.998630
10300	60.000000	87.000000	4.67	0.026168	0.045324	0.998630
10301	100.000000	87.000000	4.67	-0.009088	0.051541	0.998630
10302	140.000000	87.000000	4.67	-0.040092	0.033641	0.998630

Construction of Reuter Spherical Grid at a Specified Level

Construction of Reuter Spherical Grid at a Specified Level

?

×

Minimum Maximum geocentric latitude

-90.0°

90.0°

Minimum Maximum longitude

0.0°

360°

Set Reuter level Q

90

Plot mode

Cartesian space coordinates of the center of cell grid



Calculation and save



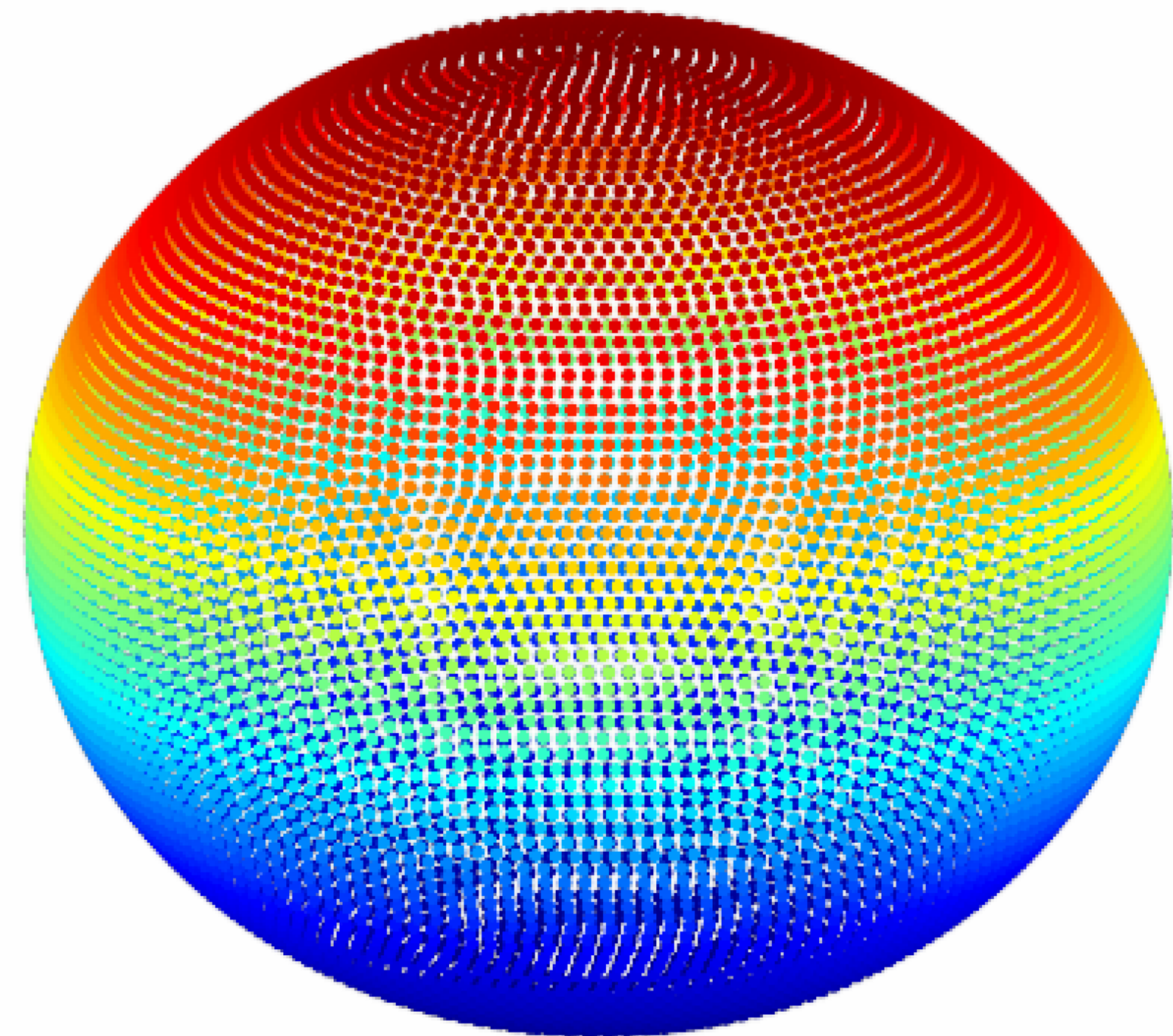
Extract plot data



Reuter grid plot →



Save current plot as



Reuter spatial grid results file

10291	191.250000	85.000000	-1.95	-0.085481	-0.017003	0.996195
10292	213.750000	85.000000	-1.95	-0.072467	-0.048421	0.996195
10293	236.250000	85.000000	-1.95	-0.048421	-0.072467	0.996195
10294	258.750000	85.000000	-1.95	-0.017003	-0.085481	0.996195
10295	281.250000	85.000000	-1.95	0.017003	-0.085481	0.996195
10296	303.750000	85.000000	-1.95	0.048421	-0.072467	0.996195
10297	326.250000	85.000000	-1.95	0.072467	-0.048421	0.996195
10298	348.750000	85.000000	-1.95	0.085481	-0.017003	0.996195
10299	20.000000	87.000000	4.67	0.049180	0.017900	0.998630
10300	60.000000	87.000000	4.67	0.026168	0.045324	0.998630
10301	100.000000	87.000000	4.67	-0.009088	0.051541	0.998630
10302	140.000000	87.000000	4.67	-0.040092	0.033641	0.998630

Construction of Reuter Spherical Grid at a Specified Level

Construction of Reuter Spherical Grid at a Specified Level

Minimum Maximum geocentric latitude

28.0°

34.0°

Minimum Maximum longitude

96.0°

104°

Set Reuter level Q 1800

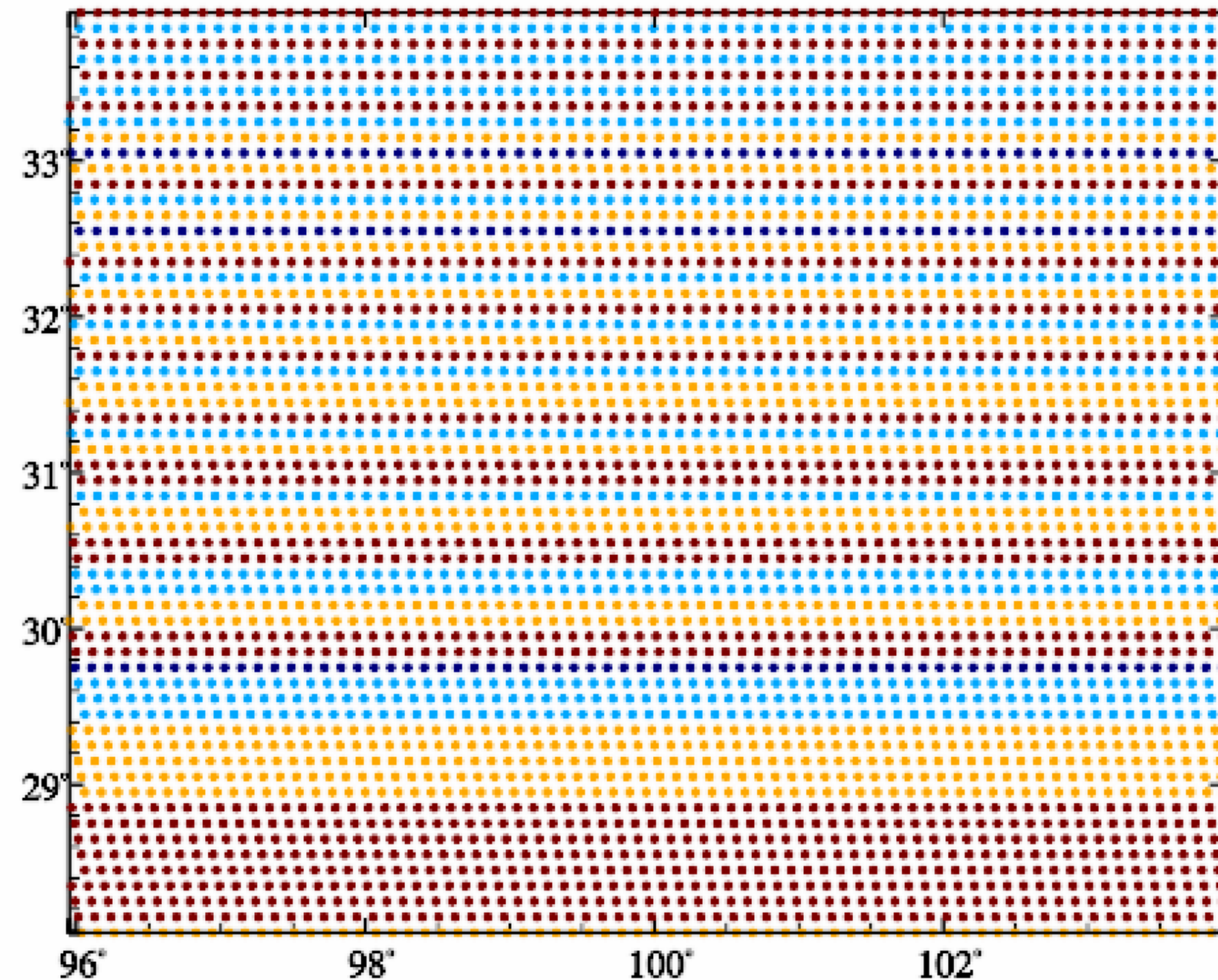
Plot mode longitude, latitude and area of the center of cell grid

Calculation and save

Extract plot data

Reuter grid plot →

Save current plot as



Reuter spatial grid results file

4094	102.297388	33.950000	0.01	-0.176677	0.810492	0.558469
4095	102.417950	33.950000	0.01	-0.178382	0.810118	0.558469
4096	102.538513	33.950000	0.01	-0.180086	0.809741	0.558469
4097	102.659076	33.950000	0.01	-0.181790	0.809361	0.558469
4098	102.779638	33.950000	0.01	-0.183493	0.808976	0.558469
4099	102.900201	33.950000	0.01	-0.185194	0.808588	0.558469
4100	103.020764	33.950000	0.01	-0.186895	0.808197	0.558469
4101	103.141326	33.950000	0.01	-0.188596	0.807802	0.558469
4102	103.261889	33.950000	0.01	-0.190295	0.807403	0.558469
4103	103.382451	33.950000	0.01	-0.191994	0.807001	0.558469
4104	103.503014	33.950000	0.01	-0.193691	0.806595	0.558469
4105	103.623577	33.950000	0.01	-0.195388	0.806186	0.558469

Gravity Field Approximation Using SRBFs: Gravity disturbance → Height anomaly

Open Single-Type Residual Observation File

Select type of observations

gravity disturbance (mGal)

Set observations file format

Number of rows of file header

1

Ellipsoidal Height Column Index

4

Observation Weight Column Index

0

Gravity Disturbance Column Index

7

Select SRBF

radial multipole kernel

Set SRBF parameters

Order m

1

Minimum degree

360

Maximum degree

1800

Burial depth of Bjerhammar sphere

10.0km

Action distance of SRBF center

100km

Reuter network level Q

1800

Open Computation Surface ellipsoidal Height grid File

Solution of normal equation

LU triangular decomposition

External Accuracy Evaluation

☐ Synchronous computation of target field elements at discrete points

By setting computation points identical to the observation points of the target field element, the program generates SRBF-based estimates using the input observation data. The residuals between the observed and estimated values are then employed for gross error detection and the assessment of external accuracy indices.

Extract data to be plot

Plot→

SRBF approach algorithms

>> [Purpose] Select a specific type of residual observation and a kind of Spherical Radial Basis Function (SRBF) kernel to evaluate the spatial, spectral, and analytical performance of the SRBF approximation algorithm. This is achieved by estimating specific residual gravity field elements – such as gravity disturbances, height anomalies, gravity anomalies, disturbing potential gradients, or vertical deflections – on or outside the geoid.

** Selecting different type of observations and constructing different figure of SRBF, we can calculate different type of target field elements, and then fully verify and analyze the spatial and spectral properties of gravity field approach algorithms using SRBFs.

>> The program itself can be employed for analytical continuation, gridding, type conversion and all-element modelling on gravity field from various single type of observed field elements.

>> Open Single-Type Residual Observation File C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/resdbm541_1800.txt.

** Look at the file information in the window below and set the discrete point file format...

>> Open Computation Surface ellipsoidal Height grid File C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/surfhgt.dat.

>> Save the results as C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/rgatoksi.dat.

>> The program outputs Single-Type Observation Residual File (*.chs) into the current directory. The file header format: Original Obs (Mean, StdDev, Min, Max); Residuals (Mean, StdDev, Min, Max). Record format: Point ID, Lon, Lat, Height, Weight, Original Obs [, West Component if Vert. Defl.], Residual Value [, Residual West Component]. where * is the output file name.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-03-26 10:25:38

>> Complete the computation!

>> Computation end time: 2026-03-26 10:26:00

>> The program also outputs various field elements' SRBF spatial curve file *spc.rbf, various field elements' SRBF spectral curve files *dgr.rbf and SRBF center file* center.txt into the current directory.

>> Mean -0.4113 standard deviation 21.8940 minimum -141.1997 maximum 112.4878 of the source observations.

** mean -0.0216 standard deviation 1.9088 minimum -54.0885 maximum 53.0770 of the result residuals.

Type of target element

height anomaly (m)

Save the results as

Import setting parameters

Start Computation

94.00000000	100.00000000	30.00000000	35.00000000	0.05000000	0.05000000	0.0000			
-0.4934	-0.4466	-0.3004	-0.1309	-0.0358	0.0254	0.0649	0.0679	0.0223	-0.0732
0.1289	0.2125	0.1641	-0.0056	-0.2094	-0.2817	-0.1712	0.0125	0.1665	0.2722
0.0273	-0.2539	-0.4608	-0.4318	-0.2215	0.0453	0.2281	0.2436	0.1685	0.0364
0.2761	0.2306	0.1674	0.1678	0.1902	0.2424	0.2110	0.0717	-0.1042	-0.2823
-0.5242	-0.5674	-0.6053	-0.6055	-0.5592	-0.4309	-0.2461	-0.0668	0.0980	0.2084

observed residual gravity disturbance (mGal)

spherical radial basis function spatial curve

target residual height anomaly (m)

- After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input residual observations file again to refine target field elements by cumulative SRBF approximation scheme. Typically, stable and high-precision solutions are achieved after accumulating just 1 to 2 SRBF approximation steps.
- Validity Principles for Single SRBF Approximation: (1) The target field element grid should remain spatially continuous and differentiable, and the standard deviation of residual observations in file *.chs is as small as possible. (2) As cumulative steps proceed, the statistical mean of the residual observations must converge toward zero without exhibiting significant sign reversals.
- Observations with zero weights can be extracted from the *.chs file (indicating they were excluded from the SRBF coefficient estimation) to determine and evaluate their external accuracy.

Gravity Field Approximation Using SRBFs: cumulative SRBF approach

Open Single-Type Residual Observation File

Select type of observations

gravity disturbance (mGal)

Set observations file format

Number of rows of file header

1

Ellipsoidal Height Column Index

4

Observation Weight Column Index

5

Gravity Disturbance Column Index

7

Select SRBF

radial multipole kernel

Set SRBF parameters

Order m

1

Minimum degree

540

Maximum degree

1800

Burial depth of Bjerhammar sphere

6.0km

Action distance of SRBF center

75km

Reuter network level Q

3600

Open Computation Surface ellipsoidal Height grid File

Solution of normal equation

LU triangular decomposition

External Accuracy Evaluation

☐ Synchronous computation of target field elements at discrete points

By setting computation points identical to the observation points of the target field element, the program generates SRBF-based estimates using the input observation data. The residuals between the observed and estimated values are then employed for gross error detection and the assessment of external accuracy indices.

Extract data to be plot

Plot→

SRBF approach algorithms

Click the [Start Computation] control button, or the [Start Computation] tool button...

Computation start time: 2026-03-26 10:25:38

Complete the computation!

Computation end time: 2026-03-26 10:26:00

The program also outputs various field elements' SRBF spatial curve file *spc.rbf, various field elements' SRBF spectral curve files *dgr.rbf and SRBF center file* center.txt into the current directory.

Mean -0.4113 standard deviation 21.8940 minimum -141.1997 maximum 112.4878 of the source observations.

mean -0.0216 standard deviation 1.9088 minimum -54.0885 maximum 53.0770 of the result residuals.

Open Single-Type Residual Observation File C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/rgatoksi.chs.

Look at the file information in the window below and set the discrete point file format...

Open Computation Surface ellipsoidal Height grid File C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/surfhgt.dat.

Save the results as C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/rgatoksi1.dat.

The program outputs Single-Type Observation Residual File (*.chs) into the current directory. The file header format: Original Obs (Mean, StdDev, Min, Max); Residuals (Mean, StdDev, Min, Max). Record format: Point ID, Lon, Lat, Height, Weight, Original Obs [, West Component if Vert. Defl.], Residual Value [, Residual West Component]. where * is the output file name.

The parameter settings have been entered into the system!

Click the [Start Computation] control button, or the [Start Computation] tool button...

Computation start time: 2026-03-26 10:29:20

Complete the computation!

Computation end time: 2026-03-26 10:30:44

The program also outputs various field elements' SRBF spatial curve file *spc.rbf, various field elements' SRBF spectral curve files *dgr.rbf and SRBF center file* center.txt into the current directory.

Mean -0.0216 standard deviation 1.9088 minimum -54.0885 maximum 53.0770 of the source observations.

mean -0.0044 standard deviation 0.6536 minimum -28.2780 maximum 11.2439 of the result residuals.

Type of target element

height anomaly (m)

Save the results as

Import setting parameters

Start Computation

94.00000000	100.00000000	30.00000000	35.00000000	0.05000000	0.05000000	0.0000			
0.0062	0.0156	0.0198	0.0140	0.0131	0.0070	0.0129	0.0219	0.0341	0.0222
-0.0377	-0.0595	-0.0418	-0.0222	-0.0199	-0.0378	-0.0153	-0.0140	-0.0054	-0.0043
-0.0170	-0.0175	-0.0034	0.0075	0.0119	0.0092	0.0314	0.0430	0.0474	0.0319
0.0164	0.0141	0.0022	-0.0088	0.0031	0.0067	0.0020	-0.0066	-0.0183	-0.0052
0.0253	0.0220	0.0056	0.0005	0.0232	0.0116	0.0053	-0.0034	-0.0063	0.0051

observed residual gravity disturbance (mGal)

spherical radial basis function spatial curve

target residual height anomaly (m)

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input residual observations file again to refine target field elements by cumulative SRBF approximation scheme. Typically, stable and high-precision solutions are achieved after accumulating just 1 to 2 SRBF approximation steps.

Validity Principles for Single SRBF Approximation: (1)The target field element grid should remain spatially continuous and differentiable, and the standard deviation of residual observations in file *.chs is as small as possible. (2) As cumulative steps proceed, the statistical mean of the residual observations must converge toward zero without exhibiting significant sign reversals.

Observations with zero weights can be extracted from the *.chs file (indicating they were excluded from the SRBF coefficient estimation) to determine and evaluate their external accuracy.

Gravity Field Approximation Using SRBFs: Vertical deflection → Height anomaly

Open Single-Type Residual Observation File

Select type of observations
vertical deflection vector (")

Set observations file format

Number of rows of file header 1
Ellipsoidal Height Column Index 4
Observation Weight Column Index 0
Vertical Deflection Westward Column Index 8
Vertical Deflection Southward Column Index 9

Select SRBF radial multipole kernel

Set SRBF parameters

Order m 1
Minimum degree 360
Maximum degree 1800
Burial depth of Bjerhammar sphere 10.0km
Action distance of SRBF center 100km
Reuter network level Q 1800

Open Computation Surface ellipsoidal Height grid File

Solution of normal equation
LU triangular decomposition

External Accuracy Evaluation

☐ Synchronous computation of target field elements at discrete points

By setting computation points identical to the observation points of the target field element, the program generates SRBF-based estimates using the input observation data. The residuals between the observed and estimated values are then employed for gross error detection and the assessment of external accuracy indices.

Extract data to be plot

Plot→



SRBF approach algorithms

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-03-26 10:29:20

>> Complete the computation!

>> Computation end time: 2026-03-26 10:30:44

>> The program also outputs various field elements' SRBF spatial curve file *spc.rbf, various field elements' SRBF spectral curve files *dgr.rbf and SRBF center file* center.txt into the current directory.

>> Mean -0.0216 standard deviation 1.9088 minimum -54.0885 maximum 53.0770 of the source observations.

** mean -0.0044 standard deviation 0.6536 minimum -28.2780 maximum 11.2439 of the result residuals.

>> Open Single-Type Residual Observation File C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/resdbm541_1800.txt.

** Look at the file information in the window below and set the discrete point file format...

>> Open Computation Surface ellipsoidal Height grid File C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/surfhgt.dat.

>> Save the results as C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/dfttoksi.dat.

>> The program outputs Single-Type Observation Residual File (*.chs) into the current directory. The file header format: Original Obs (Mean, StdDev, Min, Max); Residuals (Mean, StdDev, Min, Max). Record format: Point ID, Lon, Lat, Height, Weight, Original Obs [, West Component if Vert. Defl.], Residual Value [, Residual West Component]. where * is the output file name.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-03-26 10:38:03

>> Complete the computation!

>> Computation end time: 2026-03-26 10:38:38

>> The program also outputs various field elements' SRBF spatial curve file *spc.rbf, various field elements' SRBF spectral curve files *dgr.rbf and SRBF center file* center.txt into the current directory.

>> Mean -0.0129 standard deviation 3.2354 minimum -19.8241 maximum 23.1114 of the source observations.

** mean 0.0039 standard deviation 0.3118 minimum -9.2421 maximum 10.5513 of the result residuals.

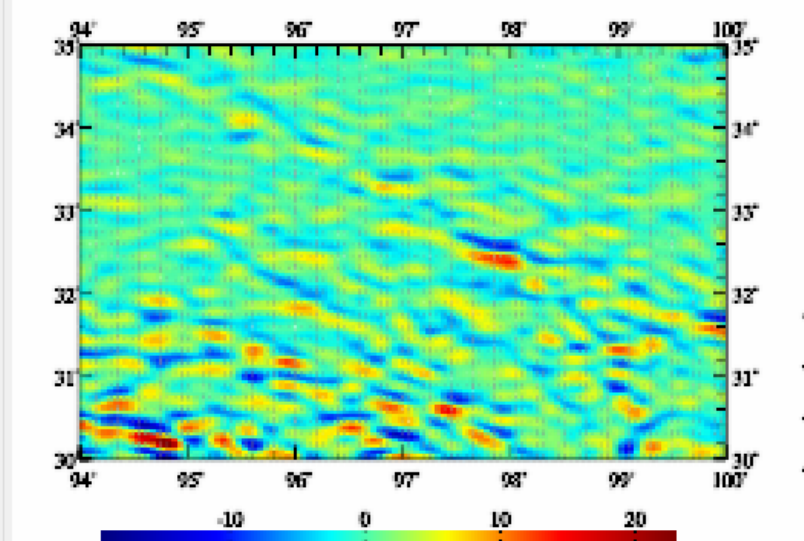
Type of target element height anomaly (m)

Save the results as

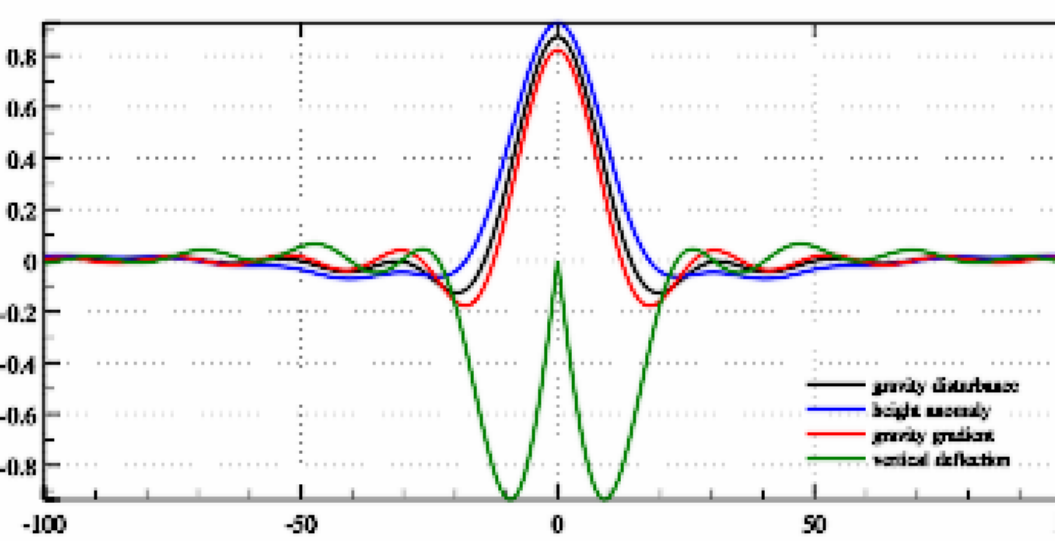
Import setting parameters

Start Computation

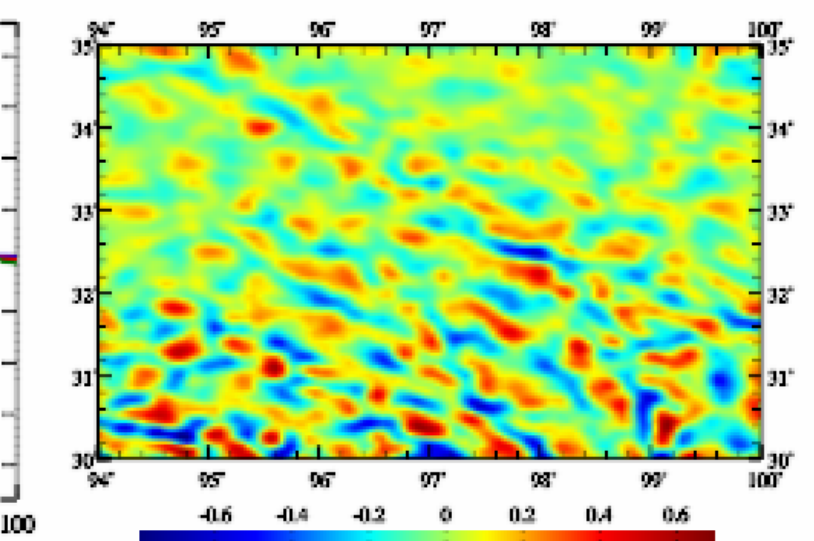
94.00000000	100.00000000	30.00000000	35.00000000	0.05000000	0.05000000	0.0000			
-0.4808	-0.4329	-0.2845	-0.1120	-0.0144	0.0492	0.0874	0.1029	0.0741	-0.0079
0.1176	0.2158	0.1885	0.0295	-0.1768	-0.2662	-0.1820	-0.0265	0.1247	0.2425
0.0318	-0.2592	-0.4819	-0.4693	-0.2501	0.0327	0.2356	0.2747	0.2107	0.0684
0.2274	0.1964	0.1420	0.1525	0.1764	0.2116	0.1653	0.0179	-0.1573	-0.3123
-0.4060	-0.4720	-0.5360	-0.5683	-0.5500	-0.4631	-0.3231	-0.1673	-0.0224	0.0857



observed residual vertical deflection (" , S)



spherical radial basis function spatial curve



target residual height anomaly (m)

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input residual observations file again to refine target field elements by cumulative SRBF approximation scheme. Typically, stable and high-precision solutions are achieved after accumulating just 1 to 2 SRBF approximation steps.

Validity Principles for Single SRBF Approximation: (1)The target field element grid should remain spatially continuous and differentiable, and the standard deviation of residual observations in file *.chs is as small as possible. (2) As cumulative steps proceed, the statistical mean of the residual observations must converge toward zero without exhibiting significant sign reversals.

Observations with zero weights can be extracted from the *.chs file (indicating they were excluded from the SRBF coefficient estimation) to determine and evaluate their external accuracy.

Gravity Field Approximation Using SRBFs: Disturbing gradient → Height anomaly

Open Single-Type Residual Observation File

Select type of observations

disturbing gradient (E, radial)

Set observations file format

Number of rows of file header

1

Ellipsoidal Height Column Index

4

Observation Weight Column Index

0

Disturbing Gradient Column Index

10

Select SRBF

radial multipole kernel

Set SRBF parameters

Order m

1

Minimum degree

360

Maximum degree

1800

Burial depth of Bjerhammar sphere

10.0km

Action distance of SRBF center

100km

Reuter network level Q

1800

Open Computation Surface ellipsoidal Height grid File

Solution of normal equation

LU triangular decomposition

External Accuracy Evaluation

☐ Synchronous computation of target field elements at discrete points

By setting computation points identical to the observation points of the target field element, the program generates SRBF-based estimates using the input observation data. The residuals between the observed and estimated values are then employed for gross error detection and the assessment of external accuracy indices.

Extract data to be plot

Plot→

SRBF approach algorithms

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-03-26 10:38:03

>> Complete the computation!

>> Computation end time: 2026-03-26 10:38:38

>> The program also outputs various field elements' SRBF spatial curve file *spc.rbf, various field elements' SRBF spectral curve files *dgr.rbf and SRBF center file* center.txt into the current directory.

>> Mean -0.0129 standard deviation 3.2354 minimum -19.8241 maximum 23.1114 of the source observations.

** mean 0.0039 standard deviation 0.3118 minimum -9.2421 maximum 10.5513 of the result residuals.

>> Open Single-Type Residual Observation File C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/resdbm541_1800.txt.

** Look at the file information in the window below and set the discrete point file format...

>> Open Computation Surface ellipsoidal Height grid File C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/surfhgt.dat.

>> Save the results as C:/PAGrav4.5_win64en/examples/SRBFestimateVerify/grtoksi.dat.

>> The program outputs Single-Type Observation Residual File (*.chs) into the current directory. The file header format: Original Obs (Mean, StdDev, Min, Max); Residuals (Mean, StdDev, Min, Max). Record format: Point ID, Lon, Lat, Height, Weight, Original Obs [, West Component if Vert. Defl.], Residual Value [, Residual West Component]. where * is the output file name.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

>> Computation start time: 2026-03-26 10:40:47

>> Complete the computation!

>> Computation end time: 2026-03-26 10:41:09

>> The program also outputs various field elements' SRBF spatial curve file *spc.rbf, various field elements' SRBF spectral curve files *dgr.rbf and SRBF center file* center.txt into the current directory.

>> Mean -0.8635 standard deviation 38.2935 minimum -262.7565 maximum 232.6519 of the source observations.

** mean -0.0373 standard deviation 3.7041 minimum -90.4115 maximum 78.2329 of the result residuals.

Type of target element

height anomaly (m)

Save the results as

Import setting parameters

Start Computation

94.00000000	100.00000000	30.00000000	35.00000000	0.05000000	0.05000000	0.0000			
-0.5445	-0.5102	-0.3560	-0.1739	-0.0708	-0.0012	0.0419	0.0482	-0.0085	-0.1114
0.1839	0.2645	0.1883	0.0030	-0.2013	-0.2539	-0.1254	0.0529	0.1988	0.2703
0.1064	-0.1544	-0.3614	-0.3513	-0.1629	0.0673	0.2065	0.1871	0.0870	-0.0395
0.2908	0.2241	0.1481	0.1425	0.1681	0.2248	0.2043	0.0655	-0.1207	-0.3326
-0.7192	-0.7325	-0.7358	-0.6986	-0.6140	-0.4451	-0.2224	-0.0131	0.1616	0.2708

observed residual disturbing gradient (E, R)

spherical radial basis function spatial curve

target residual height anomaly (m)

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input residual observations file again to refine target field elements by cumulative SRBF approximation scheme. Typically, stable and high-precision solutions are achieved after accumulating just 1 to 2 SRBF approximation steps.

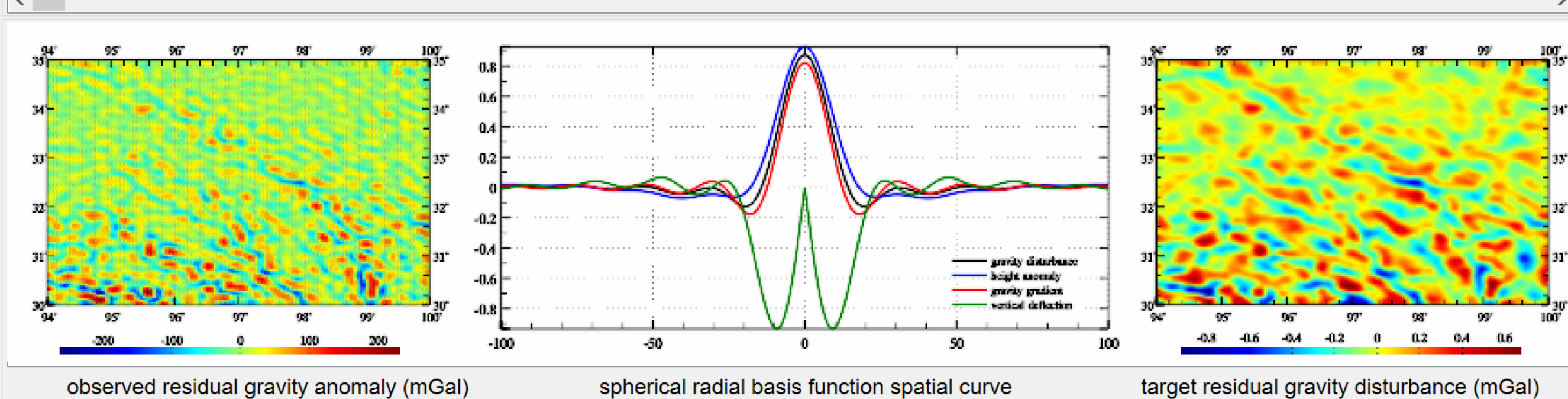
Validity Principles for Single SRBF Approximation: (1)The target field element grid should remain spatially continuous and differentiable, and the standard deviation of residual observations in file *.chs is as small as possible. (2) As cumulative steps proceed, the statistical mean of the residual observations must converge toward zero without exhibiting significant sign reversals.

Observations with zero weights can be extracted from the *.chs file (indicating they were excluded from the SRBF coefficient estimation) to determine and evaluate their external accuracy.

Observation file Save as Import parameters Start Computation Follow example Save process

 SRBF approach algorithms

94.00000000	100.00000000	30.00000000	35.00000000	0.05000000	0.05000000	0.0000			
-0.4274	-0.3868	-0.2398	-0.0702	0.0199	0.0651	0.0962	0.1104	0.0885	0.010
0.1013	0.1975	0.1669	0.0044	-0.2117	-0.3170	-0.2383	-0.0673	0.0919	0.209
-0.0204	-0.3159	-0.5152	-0.4741	-0.2542	0.0301	0.2403	0.2867	0.2114	0.069
0.2554	0.2373	0.1896	0.1880	0.2189	0.2546	0.2057	0.0555	-0.1233	-0.269
-0.3326	-0.3908	-0.4684	-0.5152	-0.5078	-0.4270	-0.2902	-0.1481	-0.0078	0.095

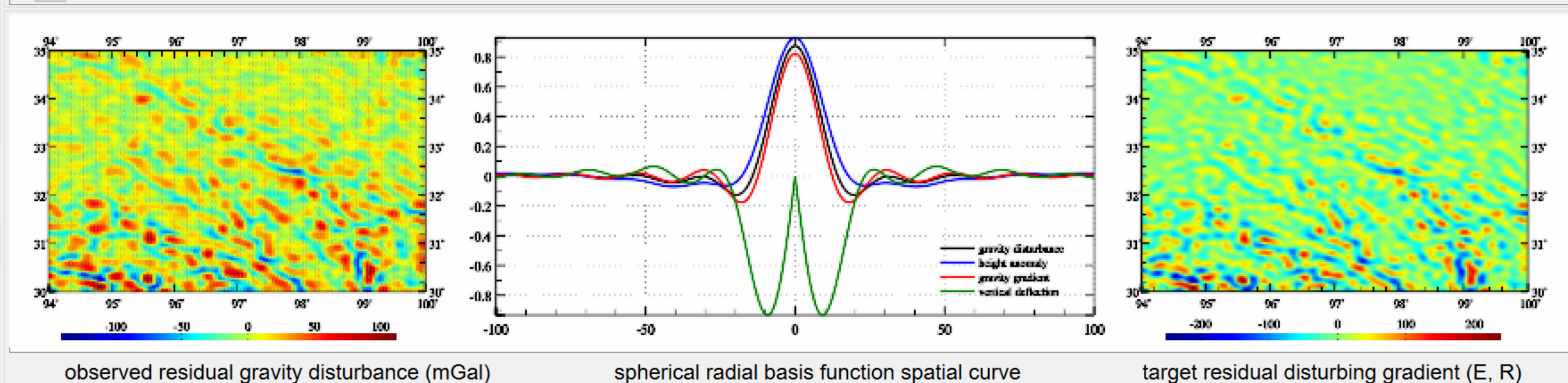


- After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input residual observations file again to refine target field elements by cumulative SRBF approximation scheme. Typically, stable and high-precision solutions are achieved after accumulating just 1 to 2 SRBF approximation steps.
- Validity Principles for Single SRBF Approximation: (1)The target field element grid should remain spatially continuous and differentiable, and the standard deviation of residual observations in file *.chs is as small as possible. (2) As cumulative steps proceed, the statistical mean of the residual observations must converge toward zero without exhibiting significant sign reversals.
- Observations with zero weights can be extracted from the *.chs file (indicating they were excluded from the SRBF coefficient estimation) to determine and evaluate their external accuracy.

Observation file Save as Import parameters Start Computation Follow example Save process

 SRBF approach algorithms

94.00000000	100.00000000	30.00000000	35.00000000	0.05000000	0.05000000	0.0000			
-137.4913	-128.5689	-75.7644	-21.5254	-1.7301	0.0090	7.5791	20.3746	19.7973	-10.7891
-9.5433	43.9876	47.6359	-6.5605	-77.1569	-94.1416	-42.8307	15.0738	31.2255	9.8689
47.3224	-44.9296	-109.9247	-94.9134	-16.2553	76.4291	124.0372	104.8406	50.7895	-8.2329
45.1466	28.4229	2.7440	1.0298	24.3953	52.6813	41.2375	-13.0719	-74.3834	-113.3740
-26.6862	-32.8644	-62.8344	-90.7690	-98.7158	-80.0600	-48.4845	-19.0314	9.7789	27.5468



After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input residual observations file again to refine target field elements by cumulative SRBF approximation scheme. Typically, stable and high-precision solutions are achieved after accumulating just 1 to 2 SRBF approximation steps.

Validity Principles for Single SRBF Approximation: (1)The target field element grid should remain spatially continuous and differentiable, and the standard deviation of residual observations in file *.chs is as small as possible. (2) As cumulative steps proceed, the statistical mean of the residual observations must converge toward zero without exhibiting significant sign reversals.

Observations with zero weights can be extracted from the *.chs file (indicating they were excluded from the SRBF coefficient estimation) to determine and evaluate their external accuracy.

Table 4.26: Statistics of Reference Truth Model (Degrees 541 – 1800)

Field Element	Unit	Mean	Std. Dev.	Min	Max
Residual Geoidal Heights	m	-0.0020	0.1590	-0.8621	0.6546
Residual Gravity Disturbances	mGal	-0.4113	21.8940	-141.1997	112.4878

Table 4.27: Differences between SRBF Approximation and Reference Model

Observation Residuals	Target Residual Element	1st SRBF Approx.		2nd SRBF Approx.	
		Mean	Std. Dev.	Mean	Std. Dev.
Gravity Disturbance	Ground Height Anomaly (m)	0.0201	0.0185	0.0189	0.0162
Vertical Deflection		0.0058	0.0147	0.0058	0.0072
Disturbing Gradient		-0.0055	0.0202	-0.0056	0.0185
Height Anomaly	Ground Gravity Disturbance (mGal)	0.1731	1.4421	0.2075	1.4382
Vertical Deflection		0.3550	1.6905	0.3566	1.6845
Disturbing Gradient		0.0840	1.5780	0.0906	1.5298

Algorithm Performance: The approximation capability of the algorithm is quantified by the ratio of the standard deviation of the approximation errors (Table 4.27) to the standard deviation of the reference truth (Table 4.26).

Comparative analysis against traditional gravity field integration algorithms reveals that even a single-step SRBF approximation surpasses integration methods in accuracy.

All-element Gravity Field Modeling Using Multi-source Heterogeneous Data with SRBFs

Open Multi-source Heterogeneous Residual Observations File

number of rows of file header 1
 Ellipsoidal Height Column Index 6
 Observation Weight Column Index 7
 System Weight Column Index 8

Select SRBF radial multipole kernel

Order m 3
 Minimum degree 360
 Maximum degree 1800
 Burial depth of Bjerhammar sphere 10.0km
 Action distance of SBRF center 100km
 Reuter network level Q 1800

Open Computation Surface ellipsoidal Height grid File

>> Save the results as C:/PAGrav4.5_win64en/examples/SRBFheterogeneous/rgagr_ksi.txt.

** Record Format: Point ID, Lon, Lat, Height (m), Res. Grav. Dist. (mGal), Res. Height Anom. (m), Res. Grav. Anom. (mGal), Res. Grad. Radial (E), Res. V.D. South ("), Res. V.D. West (").

>> The program also outputs Heterogeneous Observation Residual File *.chs into the current directory. Header Format: Field Element Type (0 – 5), System Weight, Number of Observations (Group), Original Obs (Mean, StdDev, Min, Max); Residuals (Mean, StdDev, Min, Max).

** Record Format: Point ID/Station Name, Longitude, Latitude, Ellipsoidal height, Residual Value, Original Observation, Field Element Type, Observation Weight, Observation System Weight.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] control button.

>> Computation start time: 2026-03-26 15:50:21

>> Complete the computation!

>> Computation end time: 2026-03-26 15:51:29

>> The program outputs Result All Residual Gravity Field Element Grid Files (Prefix * specs match input grid): Residual Gravity Disturbance (*.rga), Residual Height Anomaly (*.ksi), Residual Gravity Anomaly (*.gra), Residual Disturbing Gravity Gradient (*.grr), and Residual Vertical Deflection Vector (*.dft)

>> The program also outputs SRBF center file *center.txt into the current directory.

>> Observation Type 0 System Weight 1.000 Number of observations 12000

** Source observations: mean -0.4107 standard deviation 21.8478 minimum -140.9351 maximum 112.3153

** Residual observations: mean -0.0149 standard deviation 2.0457 minimum -53.9731 maximum 52.9464

>> Observation Type 3 System Weight 1.000 Number of observations 12000

** Source observations: mean -0.8635 standard deviation 38.2935 minimum -262.7565 maximum 232.6519

** Residual observations: mean -0.0490 standard deviation 4.1126 minimum -90.4115 maximum 78.2329

Observation: gravity disturbance +
disturbing gravity gradient

Solution of normal equation LU triangular decomposition

Save the results as

Import setting parameters

Start Computation

ID	lon	lat	ellipshgt	gravity disturbance(mGal)	height anomaly(m)	gravity anomaly(mGal)	gravity gradient(E)	vertical deflection(S,")
1	94.02500	30.02500	3984.353	-77.1825	-0.4406	-77.0471	-150.3665	-2.8223 -0.6840
2	94.07500	30.02500	4226.989	-74.7742	-0.4196	-74.6452	-145.9857	-3.3652 1.4543
3	94.12500	30.02500	4461.719	-47.0745	-0.2759	-46.8897	-86.5023	-3.6640 0.0702
4	94.17500	30.02500	4422.914	-15.5930	-0.1036	-15.3612	22.0351	-3.8390 0.9292
5	94.22500	30.02500	4335.893	-2.4854	-0.0121	-2.4817	3.0334	-3.0188 0.5979
6	94.27500	30.02500	4463.689	0.5039	0.0290	0.4850	0.7881	-1.9157 1.1147

Output all-element estimations of residual gravity field on the computation surface



Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input residual observations file again to refine target field elements by cumulative SRBF approximation scheme. Typically, stable and high-precision solutions are achieved after accumulating just 1 to 2 SRBF approximation steps.

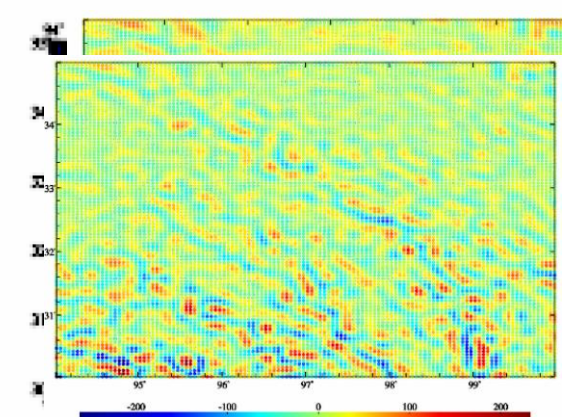
Validity Principles for Single SRBF Approximation: (1)The target field element grid should remain spatially continuous and differentiable, and the standard deviation of residual observations in file *.chs is as small as possible. (2) As cumulative steps proceed, the statistical mean of the residual observations must converge toward zero without exhibiting significant sign reversals.

Extract data to be plot

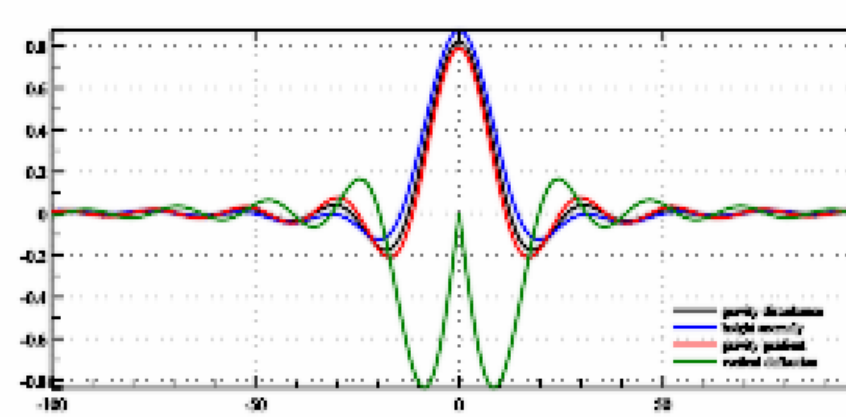
Plot →

Observations with zero observation weights or zero system weights can be extracted from the *.chs file (indicating they were excluded from the SRBF coefficient estimation) to determine and evaluate their external accuracy.

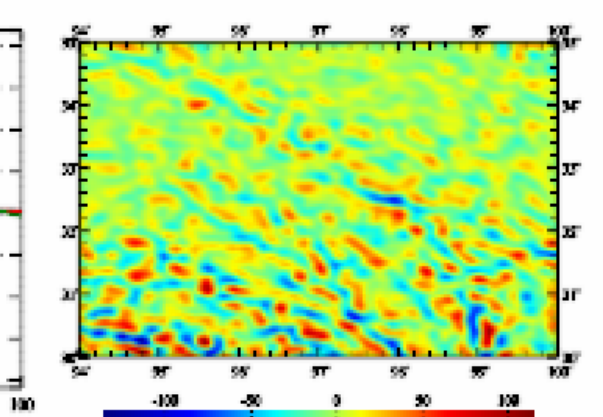
Spatial distribution of observations



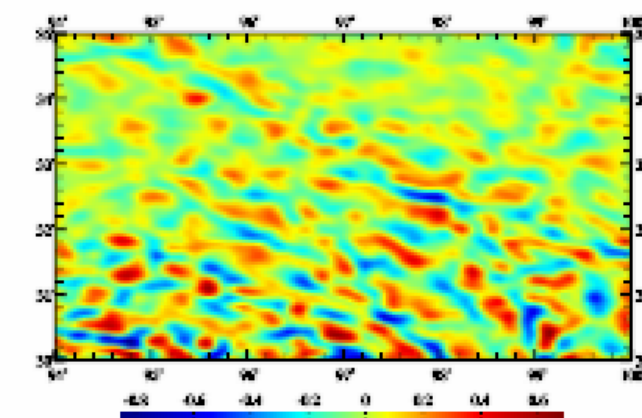
spherical radial basis function spatial curve



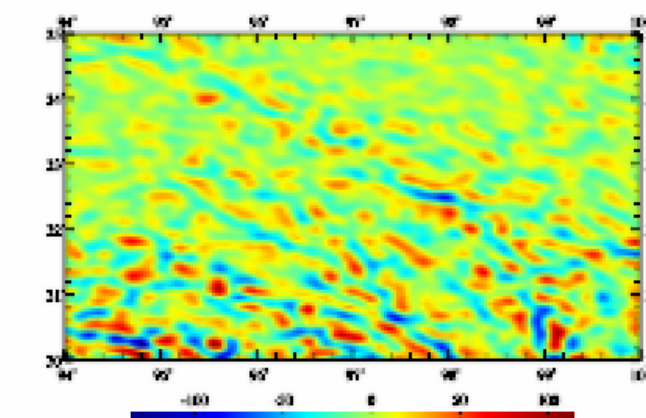
residual gravity disturbance (mGal)



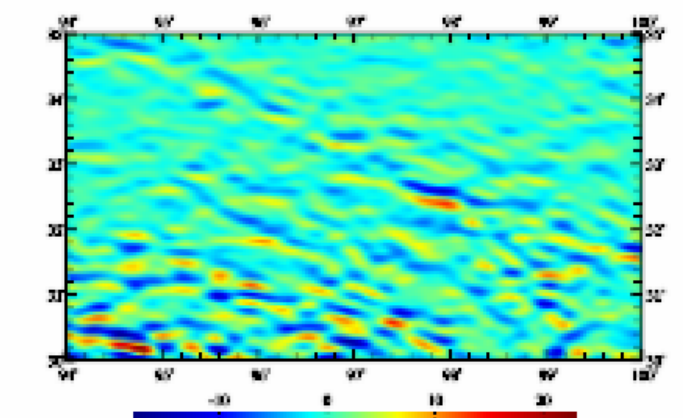
residual height anomaly (m)



residual disturbing gradient (E)



residual vertical deflection S (")



All-element Gravity Field Modeling with SRBFs- cumulative SRBF approach

Open Multi-source Heterogeneous Residual Observations File

number of rows of file header 2
Ellipsoidal Height Column Index 7
Observation Weight Column Index 8
System Weight Column Index 9

Select SRBF Poisson wavelet kernel

Order m 3
Minimum degree 540
Maximum degree 1800
Burial depth of Bjerhammar sphere 6.0km
Action distance of SBRF center 60km
Reuter network level Q 3600

Open Computation Surface ellipsoidal Height grid File

>> Save the results as C:/PAGrav4.5_win64en/examples/SRBFheterogeneous/rgagr_ksi1.txt.

** Record Format: Point D, Lon, Lat, Height (m), Res. Grav. Dist. (mGal), Res. Height Anom. (m), Res. Grav. Anom. (mGal), Res. Grad. Radial (E), Res. V.D. South ("), Res. V.D. West (").

>> The program also outputs Heterogeneous Observation Residual File *.chs into the current directory. Header Format: Field Element Type (0 – 5), System Weight, Number of Observations (Group), Original Obs (Mean, StdDev, Min, Max); Residuals (Mean, StdDev, Min, Max).

** Record Format: Point D/Station Name, Longitude, Latitude, Ellipsoidal height, Residual Value, Original Observation, Field Element Type, Observation Weight, Observation System Weight.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] button.

>> Computation start time: 2026-03-26 15:54:44

>> Complete the computation!

>> Computation end time: 2026-03-26 15:57:03

>> The program outputs Result All Residual Gravity Field Element Grid Files (Prefix * specs match input grid): Residual Gravity Disturbance (*.rga), Residual Height Anomaly (*.ksi), Residual Gravity Anomaly (*.gra), Residual Disturbing Gravity Gradient (*.grg), and Residual Vertical Deflection Vector (*.dft)

>> The program also outputs SRBF center file *center.txt into the current directory.

>> Observation Type 0 System Weight 1.000 Number of observations 12000

** Source observations: mean -0.0149 standard deviation 2.0457 minimum -53.9731 maximum 52.9464

** Residual observations: mean -0.0046 standard deviation 0.8894 minimum -34.4709 maximum 16.1693

>> Observation Type 3 System Weight 1.000 Number of observations 12000

** Source observations: mean -0.0490 standard deviation 4.1126 minimum -90.4115 maximum 78.2329

** Residual observations: mean -0.0091 standard deviation 1.8594 minimum -66.0576 maximum 18.2884

Input the remaining residual file *.chs output by previous SRBF approach

Solution of normal equation LU triangular decomposition

Save the results as

Import setting parameters

Start Computation

ID	lon	lat	ellipshgt	gravity disturbance(mGal)	height anomaly(m)	gravity anomaly(mGal)	gravity gradient(E)	vertical deflection(S,")
1	94.02500	30.02500	3984.353	-2.4836	-0.0035	-2.4825	-8.9872	0.5849
2	94.07500	30.02500	4226.989	2.6691	0.0184	2.6634	3.6244	1.2741
3	94.12500	30.02500	4461.719	4.2047	0.0241	4.1973	7.7089	1.3746
4	94.17500	30.02500	4422.914	-0.1919	0.0064	-0.1939	-3.7468	0.6048
5	94.22500	30.02500	4335.893	-1.8799	-0.0038	-1.8787	-6.0238	0.6010
6	94.27500	30.02500	4463.689	-0.8672	-0.0040	-0.8659	-2.7805	-0.0567

Output all-element estimations of residual gravity field on the computation surface



Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input residual observations file again to refine target field elements by cumulative SRBF approximation scheme. Typically, stable and high-precision solutions are achieved after accumulating just 1 to 2 SRBF approximation steps.

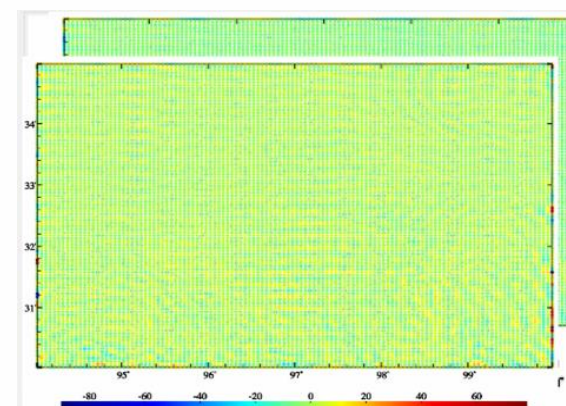
Validity Principles for Single SRBF Approximation: (1)The target field element grid should remain spatially continuous and differentiable, and the standard deviation of residual observations in file *.chs is as small as possible. (2) As cumulative steps proceed, the statistical mean of the residual observations must converge toward zero without exhibiting significant sign reversals.

Extract data to be plot

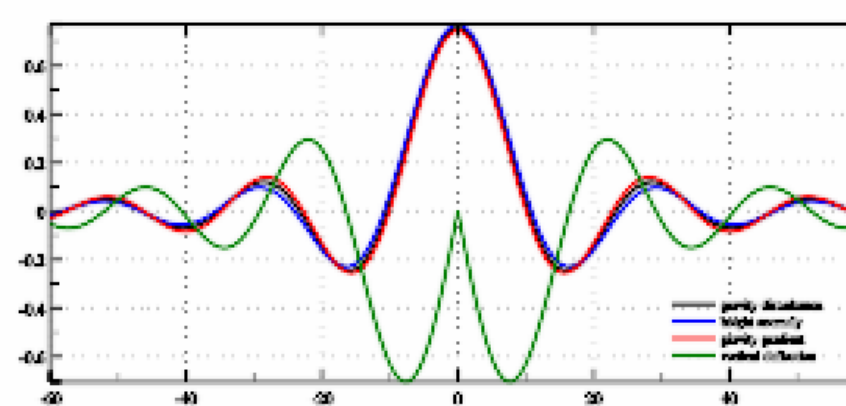
Plot →

Observations with zero observation weights or zero system weights can be extracted from the *.chs file (indicating they were excluded from the SRBF coefficient estimation) to determine and evaluate their external accuracy.

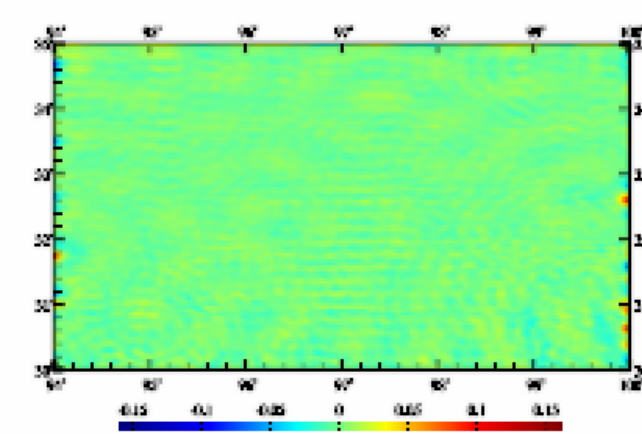
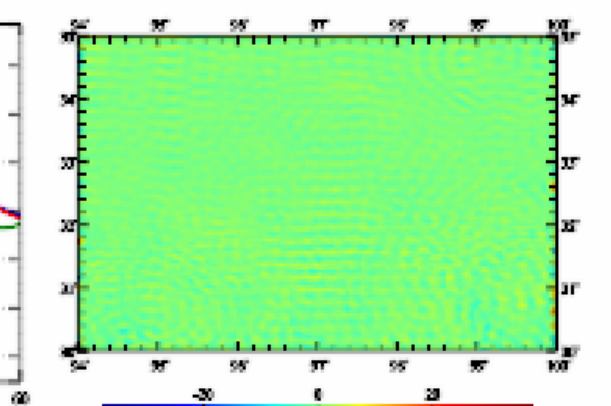
Spatial distribution of observations



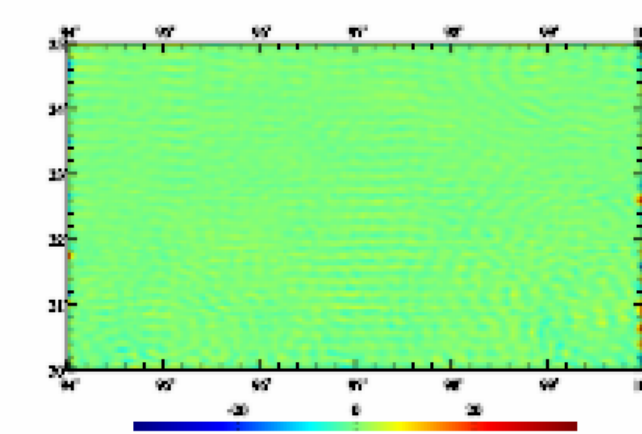
spherical radial basis function spatial curve



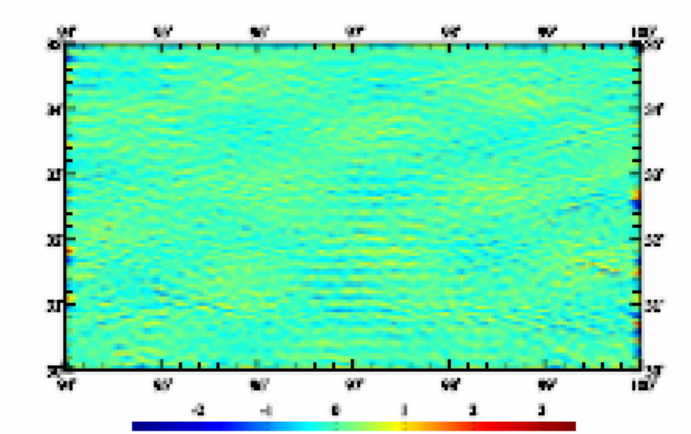
residual gravity disturbance (mGal)



residual height anomaly (m)



residual disturbing gradient (E)



residual vertical deflection S (")

All-element Gravity Field Modeling Using Multi-source Heterogeneous Data with SRBFs

Open Multi-source Heterogeneous Residual Observations File

number of rows of file header 1

Ellipsoidal Height Column Index 6

Observation Weight Column Index 7

System Weight Column Index 8

Select SRBF radial multipole kernel

Order m 3

Minimum degree 360

Maximum degree 1800

Burial depth of Bjerhammar sphere 10.0km

Action distance of SBRF center 100km

Reuter network level Q 1800

Open Computation Surface ellipsoidal Height grid File

>> The program also outputs Heterogeneous Observation Residual File *.chs into the current directory. Header Format: Field Element Type (0 – 5), System Weight, Number of Observations (Group), Original Obs (Mean, StdDev, Min, Max); Residuals (Mean, StdDev, Min, Max).
 ** Record Format: Point ID/Station Name, Longitude, Latitude, Ellipsoidal height, Residual Value, Original Observation, Field Element Type, Observation Weight, Observation System Weight.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button

>> Computation start time: 2026-03-26 16:00:10

>> Complete the computation!

>> Computation end time: 2026-03-26 16:01:39

>> The program outputs Result All Residual Gravity Field Element Grid Files (Prefix * specs match input grid): Residual Gravity Disturbance (*.rga), Residual Height Anomaly (*.ksi), Residual Gravity Anomaly (*.gra), Residual Disturbing Gravity Gradient (*.grr), and Residual Vertical Deflection Vector (*.dft)

>> The program also outputs SRBF center file *center.txt into the current directory.

>> Observation Type 0 System Weight 1.000 Number of observations 12000

** Source observations: mean -0.4107 standard deviation 21.8478 minimum -140.9351 maximum 112.3153

** Residual observations: mean -0.0748 standard deviation 2.1496 minimum -53.9731 maximum 52.9464

>> Observation Type 4 System Weight 1.000 Number of observations 12000

** Source observations: mean -0.0159 standard deviation 3.2930 minimum -19.5319 maximum 23.1114

** Residual observations: mean -0.0118 standard deviation 0.3797 minimum -9.0410 maximum 10.5513

>> Observation Type 5 System Weight 1.000 Number of observations 12000

** Source observations: mean -0.0098 standard deviation 3.1766 minimum -19.8241 maximum 17.6561

** Residual observations: mean -0.0021 standard deviation 0.2857 minimum -5.4896 maximum 6.1347

Observation: gravity disturbance
+ vertical deflection

Solution of normal equation LU triangular decomposition

Save the results as

Import setting parameters

Start Computation

ID	lon	lat	ellipshgt	gravity disturbance(mGal)	height anomaly(m)	gravity anomaly(mGal)	gravity gradient(E)	vertical deflection(S)
1	94.02500	30.02500	3984.353	-74.2232	-0.4258	-74.0924	-144.2746	-1.4992
2	94.07500	30.02500	4226.989	-70.1105	-0.3965	-69.9887	-136.1047	-2.3884
3	94.12500	30.02500	4461.719	-42.8068	-0.2539	-42.7289	-77.9105	-3.0800
4	94.17500	30.02500	4422.914	-12.7597	-0.0862	-12.7332	-17.1010	-3.3899
5	94.22500	30.02500	4335.893	-0.6512	0.0023	-0.6518	-0.2113	-2.8733
6	94.27500	30.02500	4463.689	2.7049	0.0452	2.6410	0.3265	-1.7001

Output all-element estimations of residual gravity field on the computation surface



Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input residual observations file again to refine target field elements by cumulative SRBF approximation scheme. Typically, stable and high-precision solutions are achieved after accumulating just 1 to 2 SRBF approximation steps.

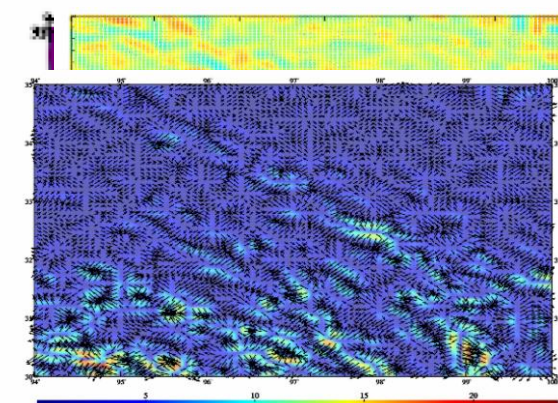
Validity Principles for Single SRBF Approximation: (1)The target field element grid should remain spatially continuous and differentiable, and the standard deviation of residual observations in file *.chs is as small as possible. (2) As cumulative steps proceed, the statistical mean of the residual observations must converge toward zero without exhibiting significant sign reversals.

Extract data to be plot

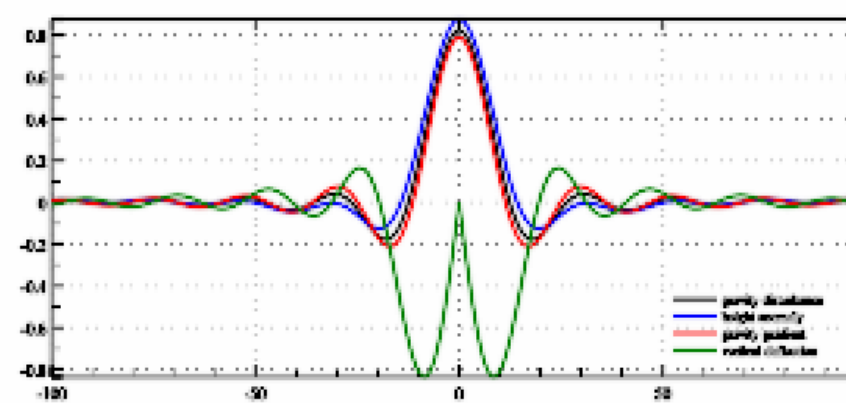
Plot →

Observations with zero observation weights or zero system weights can be extracted from the *.chs file (indicating they were excluded from the SRBF coefficient estimation) to determine and evaluate their external accuracy.

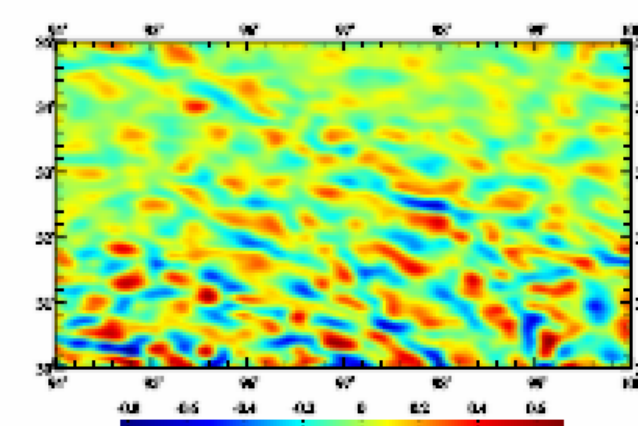
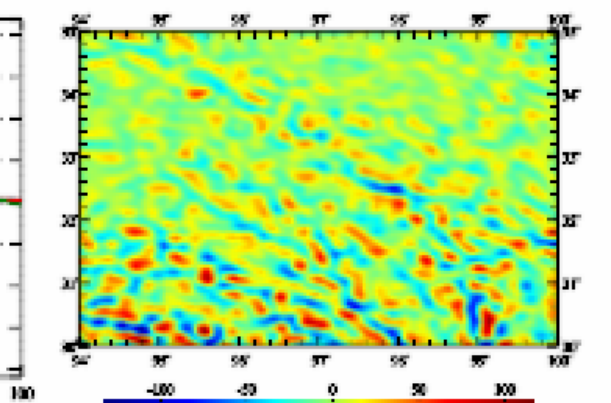
Spatial distribution of observations



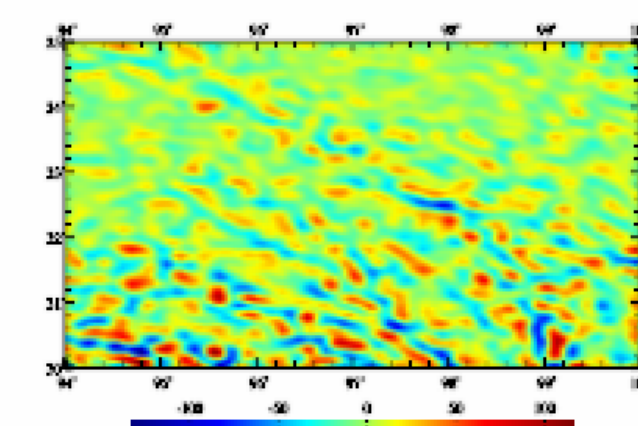
spherical radial basis function spatial curve



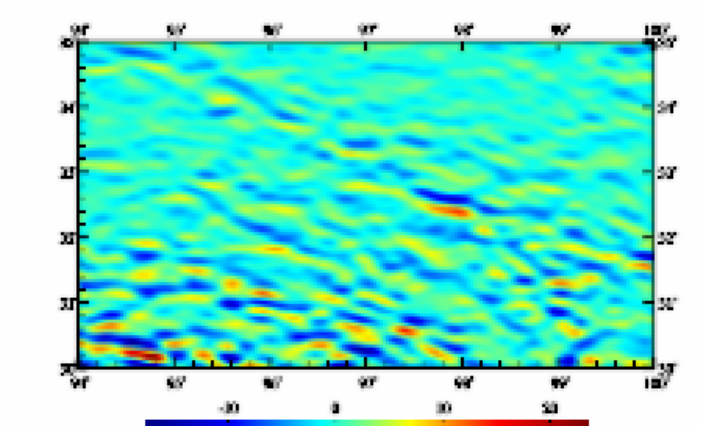
residual gravity disturbance (mGal)



residual height anomaly (m)



residual disturbing gradient (E)



residual vertical deflection S (")

All-element Gravity Field Modeling Using Multi-source Heterogeneous Data with SRBFs

Open Multi-source Heterogeneous Residual Observations File

number of rows of file header 1
Ellipsoidal Height Column Index 6
Observation Weight Column Index 7
System Weight Column Index 8

Select SRBF radial multipole kernel

Order m 3
Minimum degree 360
Maximum degree 1800
Burial depth of Bjerhammar sphere 10.0km
Action distance of SRBF center 100km
Reuter network level Q 1800

Open Computation Surface ellipsoidal Height grid File

>> Computation start time: 2026-03-26 16:05:24

>> Complete the computation!

>> Computation end time: 2026-03-26 16:07:28

>> The program outputs Result All Residual Gravity Field Element Grid Files (Prefix "specs match input grid"): Residual Gravity Disturbance (*.rga), Residual Height Anomaly (*.ksi), Residual Gravity Anomaly (*.gra), Residual Disturbing Gravity Gradient (*.grr), and Residual Vertical Deflection Vector (*.dft)

>> The program also outputs SRBF center file *center.txt into the current directory.

>> Observation Type 0 System Weight 1.000 Number of observations 12000
** Source observations: mean -0.4107 standard deviation 21.8478 minimum -140.9351 maximum 112.3153
** Residual observations: mean -0.0605 standard deviation 2.0780 minimum -53.9731 maximum 52.9464
>> Observation Type 4 System Weight 1.000 Number of observations 12000
** Source observations: mean -0.0159 standard deviation 3.2930 minimum -19.5319 maximum 23.1114
** Residual observations: mean -0.0131 standard deviation 0.4016 minimum -9.0135 maximum 10.5513
>> Observation Type 5 System Weight 1.000 Number of observations 12000
** Source observations: mean -0.0098 standard deviation 3.1766 minimum -19.8241 maximum 17.6561
** Residual observations: mean -0.0006 standard deviation 0.2998 minimum -5.4896 maximum 6.1347
>> Observation Type 3 System Weight 1.000 Number of observations 12000
** Source observations: mean -0.8635 standard deviation 38.2935 minimum -262.7565 maximum 232.6519
** Residual observations: mean -0.1550 standard deviation 4.8631 minimum -90.4115 maximum 78.2329
>> Observation Type 1 System Weight 1.000 Number of observations 12000
** Source observations: mean -0.0020 standard deviation 0.1590 minimum -0.8621 maximum 0.6546
** Residual observations: mean 0.0003 standard deviation 0.0153 minimum -0.3763 maximum 0.4258

Solution of normal equation LU triangular decomposition

Save the results as

Import setting parameters

Start Computation

ID	lon	lat	ellipshgt	gravity disturbance(mGal)	height anomaly(m)	gravity anomaly(mGal)	gravity gradient(E)	vertical deflection(S)
1	94.02500	30.02500	3984.353	-75.3080	-0.4302	-75.1758	-146.8345	-1.8472
2	94.07500	30.02500	4226.989	-71.5472	-0.4018	-71.4237	-139.5876	-2.6240
3	94.12500	30.02500	4461.719	-43.7214	-0.2566	-43.5425	-80.1711	-3.1808
4	94.17500	30.02500	4422.914	-12.6373	-0.0841	-12.6114	-16.8881	-5.4288
5	94.22500	30.02500	4335.893	0.0479	0.0075	0.0456	-1.0237	0.7430
6	94.27500	30.02500	4463.689	3.2054	0.0506	3.1999	-0.8232	-1.8272

Output all-element estimations of residual gravity field on the computation surface



Algorithm of gravity field approach using SRBFs

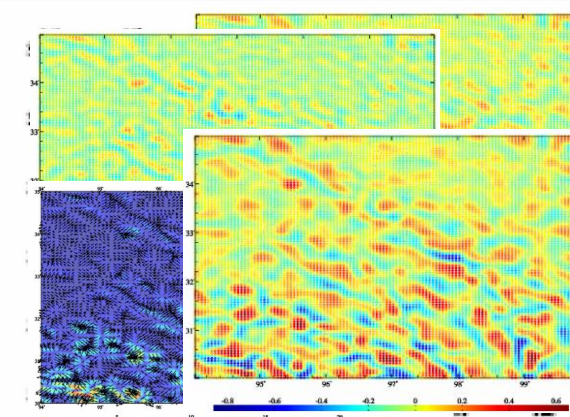
- After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input residual observations file again to refine target field elements by cumulative SRBF approximation scheme. Typically, stable and high-precision solutions are achieved after accumulating just 1 to 2 SRBF approximation steps.
- Validity Principles for Single SRBF Approximation: (1)The target field element grid should remain spatially continuous and differentiable, and the standard deviation of residual observations in file *.chs is as small as possible. (2) As cumulative steps proceed, the statistical mean of the residual observations must converge toward zero without exhibiting significant sign reversals.

Extract data to be plot

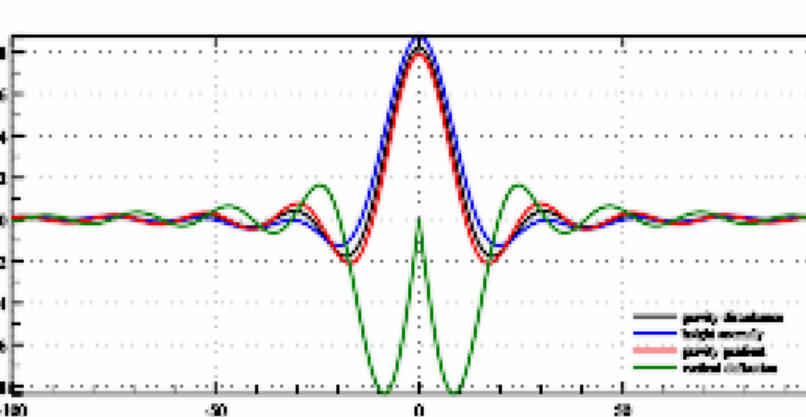
Plot →

- Observations with zero observation weights or zero system weights can be extracted from the *.chs file (indicating they were excluded from the SRBF coefficient estimation) to determine and evaluate their external accuracy.

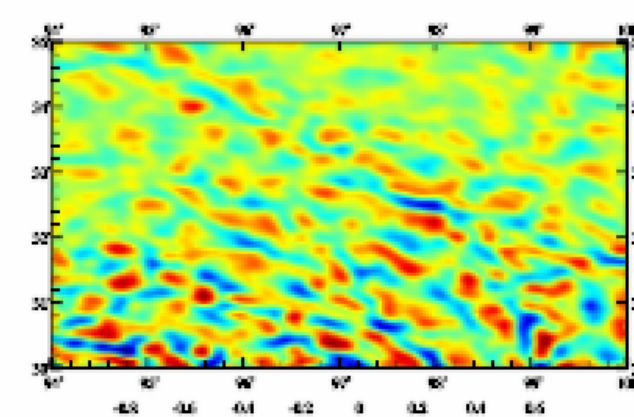
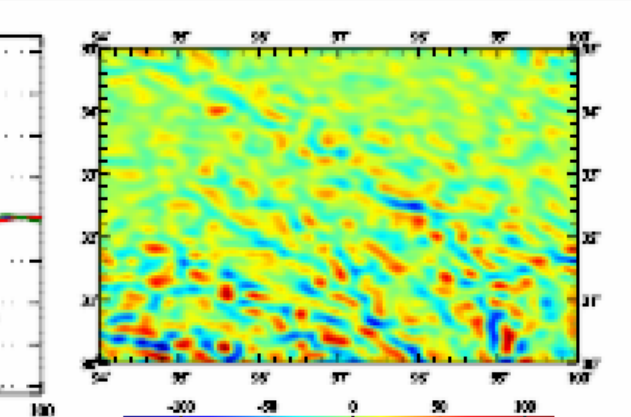
Spatial distribution of observations



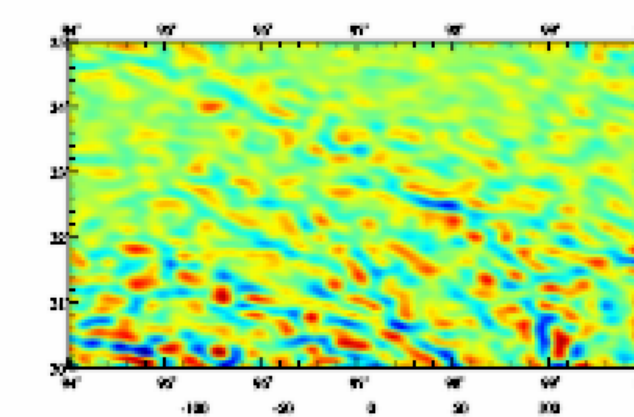
spherical radial basis function spatial curve



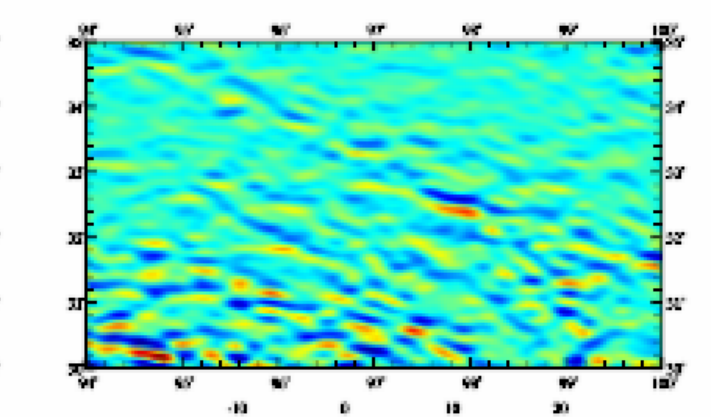
residual gravity disturbance (mGal)



residual height anomaly (m)



residual disturbing gradient (E)



residual vertical deflection S (")

Table 4.28: Statistics of Reference Truth Model (Degrees 541 – 1800)

Field Element	Unit	Mean	Std. Dev.	Min	Max
Residual Geoidal Heights	m	-0.0020	0.1590	-0.8621	0.6546

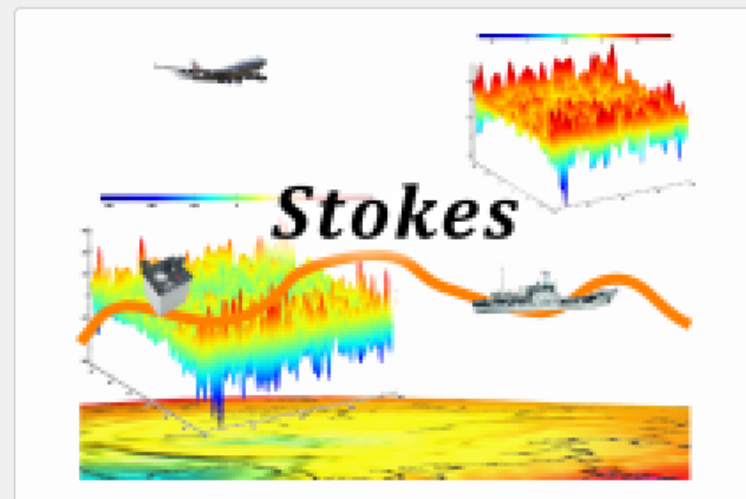
Table 4.29: Differences between SRBF Approximation and Reference Model

Residual Observations	Target Residual Element	1st SRBF Approx.		2nd SRBF Approx.	
		Mean	Std. Dev.	Mean	Std. Dev.
Gravity Disturbances	Ground Height Anomaly (m)	0.0023	0.0095	0.0005	0.0093
Geoidal Heights					
Gravity Disturbances	Ground Height Anomaly (m)	0.0036	0.0190	0.0031	0.0158
Disturbing Gradients					

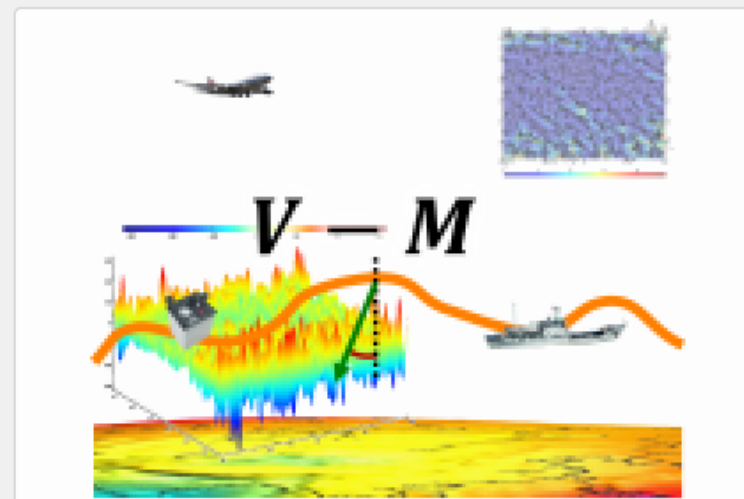
Algorithm Performance: The approximation capability of the algorithm is quantified by the ratio of the standard deviation of the approximation errors (Table 4.29) to the standard deviation of the reference truth (Table 4.28).

Multi-source fusion combined with a second approximation iteration further reduces standard deviations, enhancing overall accuracy.

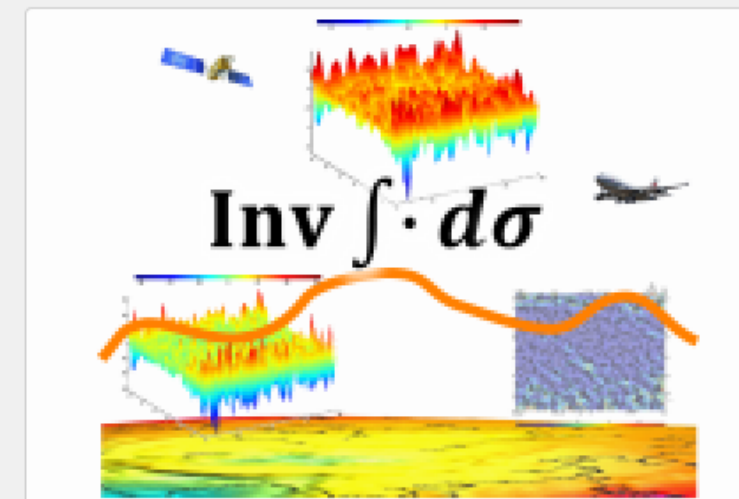
High-Precision Gravity Field Approximation and Full-Element Modeling



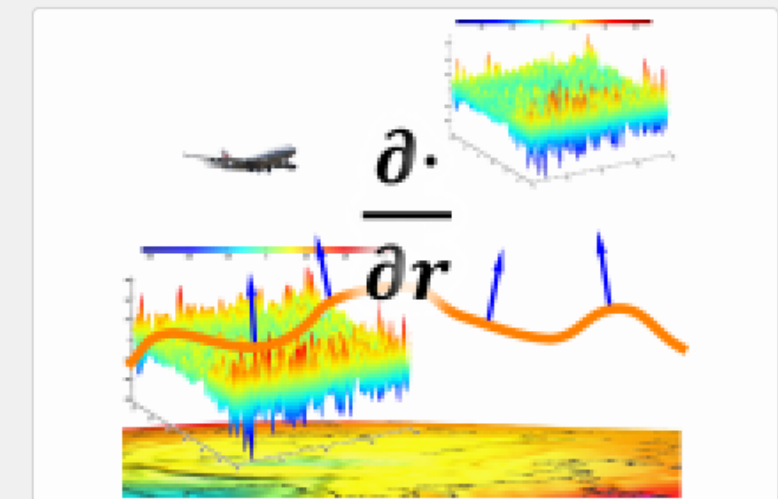
External Height Anomaly
Computation via Stokes/
Hotine Integration



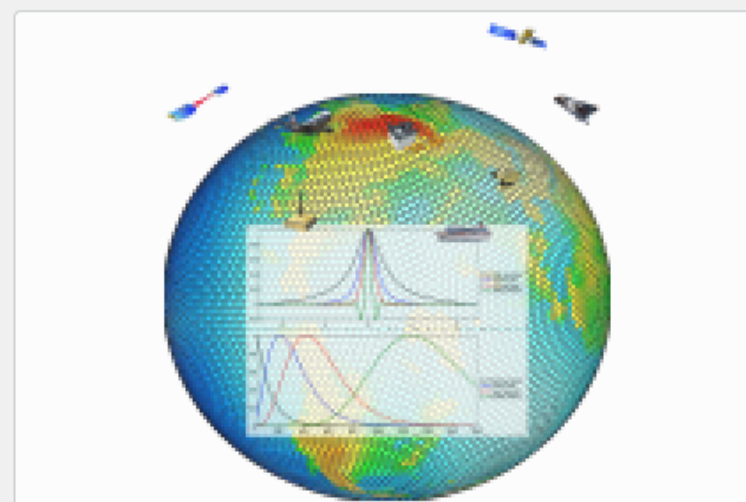
External Vertical Deflection
Computation via Vening-
Meinesz Integration



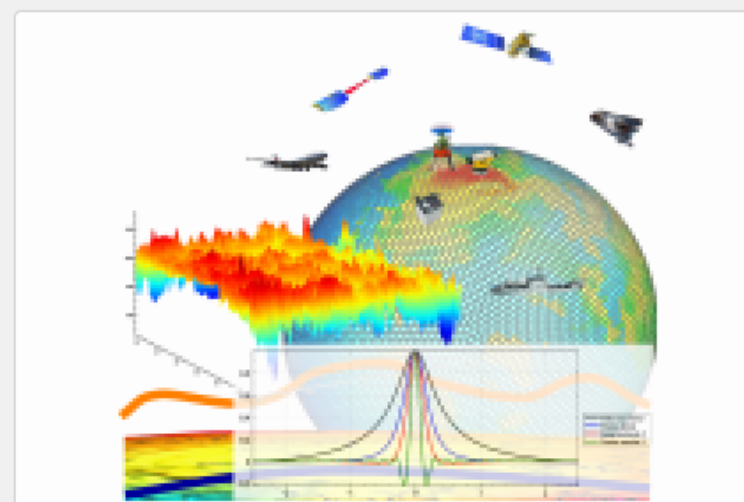
Inverse Integration and Integral
of Inverse Operation for
Anomalous Field Elements



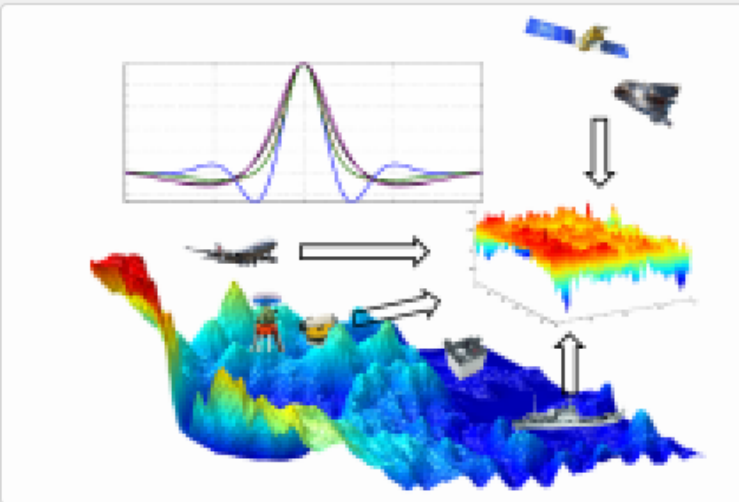
Gradient and Poisson Integral
Computation of External
Gravity Field Elements



Feature and Performance
Analysis of Spherical
Radial Basis Functions



Gravity Field Approximation
Using SRBFs and
Performance Evaluation



All-element Gravity Field Modeling
Using Multi-source Hetero-
geneous Data with SRBFs

Demonstration of All-element
Gravity Field Modeling
Using the Integral Method

Expedited Workflow Demonstr-
ation for All-element Gravity
Field Modeling Using SRBFs

Modeling Process Exercise:
Regional Gravity
Field and Geoid

- Multi-type data from terrestrial, maritime, aerial, or space-based observations
- Closed-Loop Analytical Operations between External Gravity Field Elements
- All-Element Analytical Modeling of Local Gravity Field in Full External Space
- Observation Error Assessment & Precise Computational Performance Control

Computational Functional Architecture of the subsystem

To achieve analytical full-element modeling of the gravity field on or outside the geoid, PAGrav4.5 establishes a rigorous gravity field approximation framework. It integrates spatial-domain numerical integration algorithms, based on Boundary Value Theory, with spectral-domain Spherical Radial Basis Function (SRBF) approximation techniques. This subsystem seamlessly processes multi-source, heterogeneous data of varying altitudes, cross-distributions, and multiple types, encompassing terrestrial, maritime, aerial, and space-based observations.

This software suite utilizes a representative region characterized by complex gravity field signatures, where the range of residual gravity disturbances exceeds 300 mGal after removing a 540-degree reference model. Users can verify and evaluate the performance of diverse gravity field approximation algorithms, facilitating a comprehensive understanding of their characteristics and optimal application strategies.