

4.8.2 Simple process demo of all-element modelling on gravity field using SRBFs in orthometric height system

Exercise purpose: From the observed terrestrial, marine and airborne gravity disturbances and GNSS-leveling geoidal heights in orthometric height system, make the all-element models on gravity field using spherical radial basis functions (SRBFs) in six steps, in which all the terrain effects are not processed, to quickly master the essentials in observation analysis, computation quality control and all-element modelling on regional gravity field.

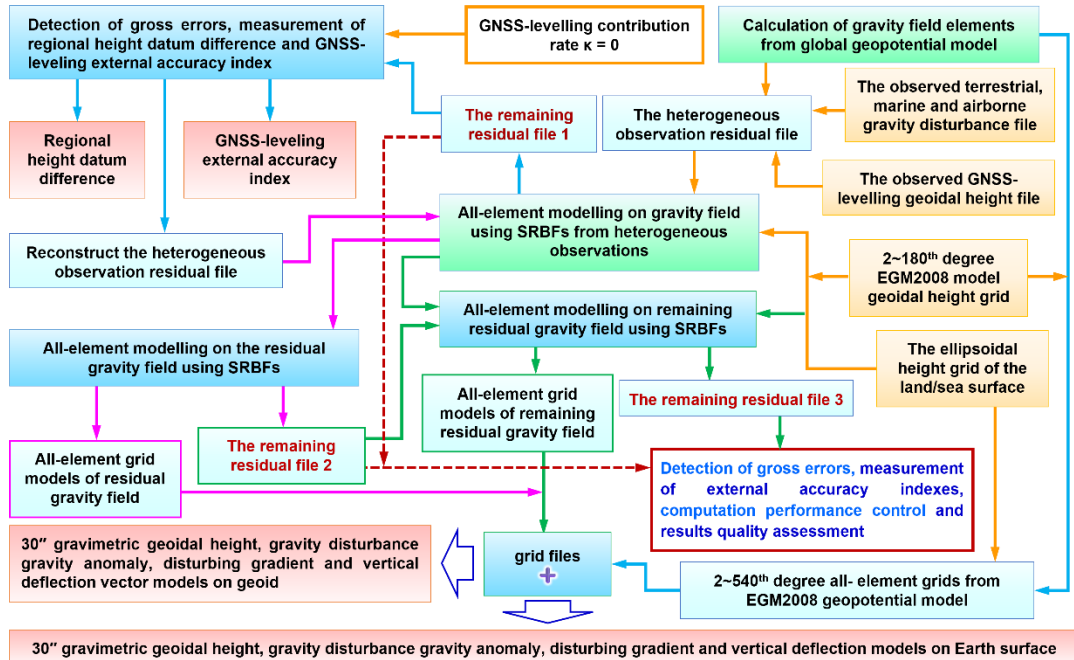
After the terrain effect processing omitted, SRBF approach process of gravity field is very simple because there is no need for additional continuation reduction, gridding and GNSS leveling fusion process.

• The observed gravity disturbance and GNSS-leveling data

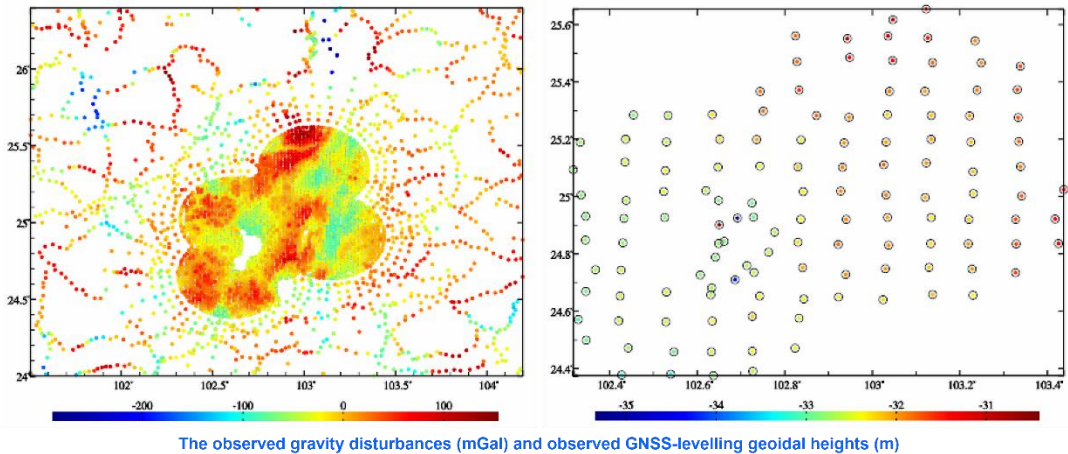
The observed terrestrial, marine and airborne gravity disturbance file obsdistgrav.txt. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed gravity disturbance (mGal), ...

The observed GNSS-leveling geoidal height file obsGNSSlgeoid.txt in orthometric height system. The file record format: ID, longitude (degree decimal), latitude, observed geoidal height (m), ...

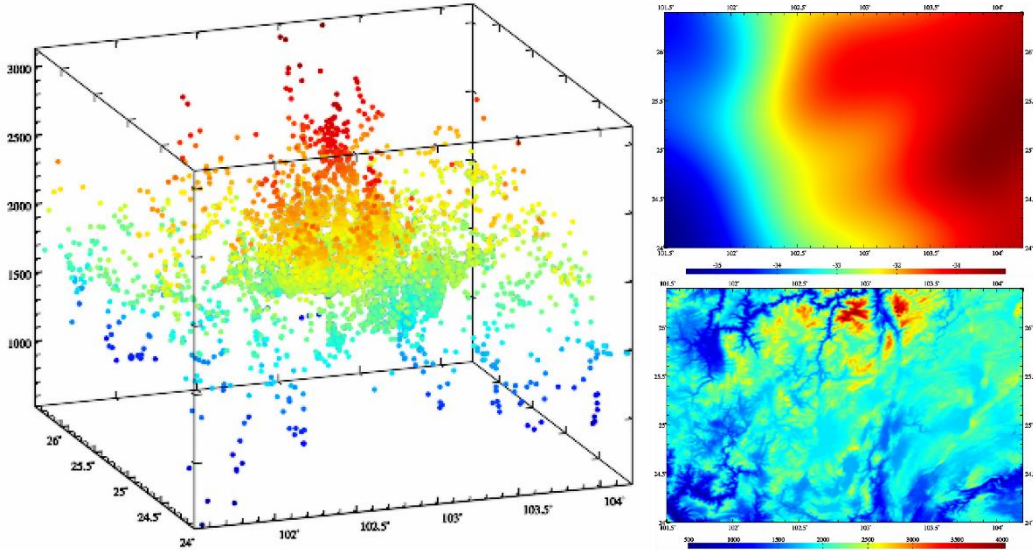
In the example, the observed gravity disturbance and GNSS-leveling geoidal heights are simulated from the EGM2008 model (the 2~1800th degree) in advance.



Simple process demo of All-element modelling on gravity filed using SRBFs in orthometric height system



It should be noted that since the observed geoidal height by GNSS-leveling is essentially the height anomaly on the geoid in orthometric height system, the ellipsoidal height at GNSS-leveling sites must be the geoidal height, which can be employed by the observed GNSS-leveling geoidal height or the model geoidal height from the EGM2008 model (the 2~180th degree).



● **The ellipsoidal height grid of calculation surface:**

The model geoidal height grid file mdlgeoidh30s.dat calculated from the 2~180th degree geopotential model, which is employed for modelling on gravity field on geoid.

The ellipsoidal height grid file surfhgt30s.dat of the land/sea surface equal to the sum of the digital elevation model grid DEM30s.dat and the model geoidal height grid mdlgeoidh30s.dat, which is employed for modelling on ground gravity field.

Here, it is required that the grid range of the calculation surface is larger than the range of the target area to absorb edge effects.

- (1) **Remove reference model value from all the observations and then**

construct the heterogeneous observation residual file.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, and input the file EGM2008.gfc, observed gravity disturbance file obsdistgrav.txt and observed GNSS-leveilling geoidal height file obsGNSSlgeoid.txt, calculate and remove the 2~540th degree model value of these observations to generate the heterogeneous observation file obsresiduals0.txt according to the agreed format.

The agreed format of the heterogeneous observation file record: ID (point no / name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual geoidal height (m).

It should be noted that the ellipsoidal height of GNSS-leveilling site must be the geoidal height and not the observed ellipsoidal height at GNSS-leveilling site.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Calculation of gravity field elements from global geopotential model

Calculation of model value for residual terrain (complete Bouguer) effects

Global geopotential coefficient model Calculator

Calculation and analysis of spectral character of Earth's gravity field

Open global geopotential coefficient model file

Save computation process as

Algorithmic Formulas

Select calculation file format

Discrete calculation points file

Open space calculation points file

Set input point file format

Number of rows of file header 1

Column ordinal number of ellipsoidal height in the record 5

Select elements to be calculated

height anomaly (m)

gravity anomaly (mGal)

gravity disturbance (mGal)

vertical deflection (" , SW)

disturbing gravity gradient (E, radial)

tangential gravity gradient (E, NW)

Laplace operator (E)

Minimum degree 2

Maximum degree 540

Extract elements to be plot

Plot

The window below only shows the geopotential coefficients data with no more than 2000 rows in it.

>> Open space calculation points file C:/PAGrav4_5_win64en/examples/Gravfmdlexercise/SRBFApprgeoidexercise/obsGNSSlgeoid.txt

>> Look at the file information in the window below and set the discrete point file format.

>> Save the results as C:/PAGrav4_5_win64en/examples/Gravfmdlexercise/SRBFApprgeoidexercise/obsGNSSlgeoidh_GM540.txt

** Behind the record of the calculation point file, appends one or more columns of model values of anomalous field elements, and keeps 4 significant figures.

>> The parameter settings have been entered into the system!

** Click the [Start Computation] control button, or the [Start Computation] tool button...

** The calculation process need wait, during which you can open the output file to look at the calculation progress.

>> Complete the calculation of the model value of (residual) gravity field element!

>> Computation end time: 2023-03-21 09:47:17

Save the results as

Import setting parameters

Start Computation

The observed gravity disturbances

The observed GNSS-leveilling geoidal heights

The heterogeneous observation residuals

ID	lon(degree decimal)	lat	ellip(m)	resid	kind	weight
1	102.4424	24.4742	1973.56	-32.7581	0	1
2	102.5467	24.4580	1659.69	-32.9577	0	1
3	102.6324	24.4582	2260.59	-32.5792	0	1
4	102.7259	24.4605	2112.00	-32.3917	0	1
5	102.8208	24.5663	1991.24	-32.4938	0	1
6	102.5286	24.5627	1637.23	32.5636	0	1
7	102.6344	24.5656	2193.72	-32.3922	0	1
8	102.7259	24.5919	2304.57	-32.2197	0	1
9	102.8326	24.5795	1978.11	-32.5408	0	1
10	102.3929	24.4944	2228.19	54.9765	0	1
2	102.3959	24.5989	2170.20	50.0971	0	1
4	102.3927	24.5296	2013.33	28.3822	0	1
5	102.3966	24.5453	2122.50	38.3822	0	1
6	102.3966	24.5453	2122.50	20.6411	0	1
7	102.3966	24.5453	2122.50	15.5784	0	1
8	102.3966	24.6056	1965.58	14.5045	0	1
9	102.3991	24.6178	1997.72	14.9731	0	1
10	102.3935	24.6384	1916.15	7.4068	0	1
4221	102.4424	24.4717	-32.6525	-0.1056	1	1
4222	102.5467	24.4580	-32.5340	-0.4237	1	1
4223	102.6324	24.4582	-32.4433	-0.1359	1	1
4224	102.7259	24.4605	-32.3324	-0.0593	1	1
4225	102.8208	24.5663	-32.5734	-0.0304	1	1
4226	102.5286	24.5627	-32.4239	-0.1397	1	1
4227	102.6344	24.5656	-32.3922	-0.0694	1	1
4228	102.7259	24.5919	-32.2197	-0.0128	1	1
4229	102.8326	24.5795	-32.5408	-0.4474	1	1
4230	102.3455	24.6689	-32.6394	-0.2903	1	1
4231	102.4239	24.6529	-32.4801	0.0740	1	1
4232	102.5297	24.6670	-32.3057	-0.2186	1	1

The ellipsoidal height here at GNSS-leveilling point is the observed or model geoidal height, not the observed ellipsoidal height.

The model geoidal height (m) at the GNSS-leveilling points

When the geoidal height to be set is equal, the program calculates the contribution of the degree n geopotential coefficients to the geoidal height, which can be employed to analyze and evaluate the spectral and space properties of the geopotential coefficient model.

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file.

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$, and input the heterogeneous residual file obsresiduals0.txt and model geoidal height grid file mdlgeoidh30s.dat to estimate the residual gravity field grid rntSRBFgeoidh30s0.xxx on geoid, and get the remaining

residual file rntSRBFgeoidh30s0.chs.

Where, xxx=ksi stands for residual geoidal height (m), xxx=rga stands for residual gravity disturbance (mGal), xxx=gra stands for residual gravity anomaly (mGal), xxx=grr stands for residual disturbing gravity gradient (radial, E) and xx=dft stands for residual vertical deflection (SW, ").

Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbances from the remaining residual file rntSRBFgeoidh30s0.chs, detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresiduals01.txt.

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresiduals01.txt and repeat the step (2) to re-estimate the residual gravity field grid rntSRBFdatum30s.xxx on geoid and get the new remaining residual file rntSRBFdatum30s.chs.

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file. Follow example

Open the discrete heterogeneous residual observations file

number of rows of file header 1
column ordinal number of ellipsoidal height in the record 6
column ordinal number of weight 7

Select SRBF radial multipole kernel
Order m 5
Minimum degree 360
Maximum degree 1800
Burial depth of Bjerhammar sphere 10.0km
Action distance of SBRF center 100km
Reuter network level K 3600

Select the adjustable observations height anomaly (m)
Contribution rate k of adjustable observations 0.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.
The parameter settings have been entered into the system!
Click the [Start Computation] control button, or the [Start Computation] tool button...
Computation start time: 2024-09-28 18:03:50
Complete the computation!
Computation end time: 2024-09-28 18:10:25
The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *rga (mGal), residual height anomaly *ksi (m), residual gravity anomaly *gra (mGal), residual disturbing gravity gradient *grr (E, radial) and residual vertical deflection vector *dft (" SW), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.
The program also outputs SRBF center file *center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval ("). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (").

Type 0 of source observations: mean 0.3186 standard deviation 42.1772 minimum -296.0915 maximum 165.2611
Residual observations: mean 0.7368 standard deviation 16.9838 minimum -105.2839 maximum 114.8811
Type 1 of source observations: mean -0.3510 standard deviation 0.2774 minimum -0.9962 maximum 0.3436
Residual observations: mean -0.0410 standard deviation 0.0287 minimum -0.1943 maximum 0.0132

ID	lon	lat	ellipsoidht	ksi	rga	gra	grr	dft
1	101.50417	24.0	0.3186	42.1772	-296.0915	165.2611	0.7368	16.9838
2	101.51250	24.0	-0.3510	0.2774	-0.9962	0.3436	-0.0410	0.0287
3	101.52083	24.0	1.0238590	24.45440	2226.190	16.4159	0.76765	0.1000
4	101.52917	24.0	1.0238930	24.50590	2170.200	-4.7680	0.00951	0.1000
5	101.53750	24.0	1.0239270	24.52960	2013.330	-10.3076	0.00951	0.1000
6	101.54583	24.0	1.0239610	24.54530	2122.500	-1.0012	0.00951	0.1000
7	101.55417	24.0	1.0239950	24.56360	1971.290	-0.0946	0.00951	0.1000
8	101.56250	24.0	1.0239980	24.58130	1940.310	-12.0941	0.00951	0.1000
9	101.57083	24.0	1.0239920	24.60360	1965.680	12.1550	0.00951	0.1000
10	101.57917	24.0	1.0239510	24.61780	1997.720	20.6312	0.00951	0.1000
11	101.58750	24.0	1.0239360	24.63640	1916.150	3.6948	0.00951	0.1000

Select the remaining residuals (column 5) as the statistical reference.

Algorithm of gravity field approach using SRBFs

- After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.
- The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

- The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.
- The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations spherical radial basis function spatial curve residual gravity disturbance (mGal)

residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S (")

Separate the remaining residuals of the observed GNSS-leveiling and observed gravity disturbances from mtsSRBFgeoidh30s0.chs.

Reconstruct the heterogeneous observation residual file obsresiduals01.txt.

Since the contribution rate of GNSS-leveiling $\kappa = 0$ is set in advance, it is essentially here directly to measure the external accuracy index of the observed GNSS levelling only using the observed gravity disturbances. Before and after gross error removed, the statistical results on the observation residuals are as follows.

		number of points	mean	standard deviation	minimum	maximum
Gravity disturbance (mGal)	Original residuals	4219	0.3186	42.1772	-296.0915	165.2611
	Residuals without error	4215	0.2695	42.0737	-296.0915	165.2611
	Remaining residuals	4215	-0.4584	13.6071	-61.1040	64.8276
GNSS levelling geoidal height (m)	Original residuals	125	-0.3510	0.2774	-0.9982	0.3435
	Residuals without error	124	-0.3482 ^①	0.2768	-0.9982	0.3435
	Remaining residuals	124	-0.0070 ^②	0.0243 ^③	-0.1328	0.0561

The statistical mean ① minus ② of the GNSS-leveiling remaining residuals in the table, that is, $-0.3482^{①} - (-0.0070^{②}) = -0.3412$ m, is the difference between the regional height datum and global height datum (gravimetric geoid). Here provides the SRBF measurement method for regional height datum difference.

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

Open the discrete heterogeneous residual observations file

number of rows of file header: 1
 column ordinal number of ellipsoidal height in the record: 6
 column ordinal number of weight: 7

Select SRBF: radial multipole kernel
 Order m: 3
 Minimum degree: 240
 Maximum degree: 1800
 Burial depth of Bjerranmar sphere: 10.0km
 Action distance of SBRF center: 100km
 Reuter network level K: 3600

Select the adjustable observations: height anomaly (m)
 Contribution rate κ of adjustable observations: 0.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, type, weight.

0.2768m The external accuracy index (SD) of the 2-540th degree model geoid
 0.0243m The external accuracy index (SD) of GNSS-leveling

Residual observations: mean -0.5621 standard deviation 13.7996 minimum -80.4161 maximum 64.8276
 Residual observations: mesh -0.0070 standard deviation 0.0243 minimum -0.1327 maximum 0.0961

Solution of normal equation LU triangular decomposition

ID	lon	lat	ellipsoid height (m)	gravity disturbance (mGal)	height anomaly (m)	gravity anomaly (mGal)	gravity gradient (E)	vertical deflection (")
1	101.50437	24.00417	-35.598	-28.0425	-0.4135	-27.9147	-15.9774	9.2237
2	101.51250	24.00417	-35.519	-36.6655	-0.4692	-36.5312	-31.8163	10.0940
3	101.52083	24.00417	-35.510	-43.9560	-0.5174	-43.8292	-40.0000	7.8452
4	101.52917	24.00417	-35.511	-52.5841	-0.5709	-52.4572	-50.0000	5.5952
5	101.53750	24.00417	-35.491	-62.9602	-0.6293	-62.8334	-60.0000	3.3452
6	101.54583	24.00417	-35.481	-63.3818	-0.6500	-63.2550	-60.0000	1.0952

-0.3482 - (-0.0070) = -0.3412m
 The measured height datum difference

Only using the observed gravity disturbances.

Algorithm of gravity field approach using SRBFs

- After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.
- The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations spherical radial basis function spatial curve residual gravity disturbance (mGal)

residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S (")

In the table, 0.0243^③m is the external accuracy index of the observed GNSS-leveling expressed as standard deviation, that is, 2.43 cm. Here provides the SRBF measurement method for the external accuracy index of GNSS-leveling. The result indicates that the external accuracy of the observed GNSS-leveling is not bad than 2.43 cm (standard deviation).

In general, it is necessary to make 1 to 2 cumulative SRBF approach with *.chs as the input file to obtain the minimum of the standard deviation of the GNSS-leveling remaining residuals as the external accuracy index, and this process is omitted in this example.

After removing the regional height datum difference of -0.3412m from the GNSS-leveling residuals, the new heterogeneous observation residual file obsresiduals1.txt is reconstructed again.

(4) All-element modelling on the residual gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the heterogeneous residual file obsresiduals1.txt and the model geoidal height grid mdlgeoidh30s.dat to estimate the 30" residual gravity field grid rntSRBFgeoidh30s1.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s1.chs.

(4) All-element modelling on the residual gravity field using SRBFs

Open the discrete heterogeneous residual observations file

number of rows of file header: 1

column ordinal number of ellipsoidal height in the record: 6

column ordinal number of weight: 7

Select SRBF: radial multipole kernel

Order m: 3

Minimum degree: 360

Maximum degree: 1800

Bjerrhammar sphere

Action distance of SBRF center: 100km

Reuter network level K: 3600

Select the adjustable observations: height anomaly (m)

Contribution rate κ of adjustable observations: 1.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.

>> The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] tool button...
 >> Computation start time: 2024-09-28 18:36:27
 >> Complete the computation!
 >> Computation end time: 2024-09-28 18:41:35

>> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *rga (mGal), residual height anomaly *ksi (m), residual gravity anomaly *gra (mGal), residual disturbing gravity gradient *gr (E, radial) and residual vertical deflection vector *dft (*, SW), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.

>> The program also outputs SRBF center file "center.txt" into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval (*). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (*).

>> Type 0 of source observations: mean 0.2695 standard deviation 42.0737 minimum -296.0915 maximum 165.2611
 Residual observations: mean 0.0196 standard deviation 12.9866 minimum -80.4161 maximum 64.8276

>> Type 1 of source observations: mean -0.0071 standard deviation 0.2768 minimum -0.6571 maximum 0.6846
 Residual observations: mean -0.0002 standard deviation 0.0276 minimum -0.1059 maximum 0.0768

Solution of normal equation: LU triangular decomposition

ID	lon	lat	ellipheight	gravity disturbance (mGal)	height anomaly (m)	gravity gradient (E, radial)	vertical deflection (E, radial)
1	101.50437	24.00417	-35.528	-40.8696	-0.3641	-4	-4
2	101.53250	24.00417	-35.519	-47.9108	-0.4135	-4	-4
3	101.52083	24.00417	-35.510	-55.2656	-0.4640	-5	-5
4	101.52917	24.00417	-35.511	-64.0909	-0.5229	-6	-6
5	101.53750	24.00417	-35.491	-73.4852	-0.5948	-7	-7
6	101.54583	24.00417	-35.481	-72.3357	-0.5786	-72.1577	-106.5435

Can further detect and remove the observation gross errors from *.chs, and then repeat the step (4).

Algorithm of gravity field approach using SRBFs

- After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.
- The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot →

- The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.
- The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations **spherical radial basis function spatial curve** **residual gravity disturbance (mGal)**

All-element models rntSRBFgeoidh30s1.xxx of the residual gravity field

residual height anomaly (m)
residual disturbing gradient (E)
residual vertical deflection S (*)

[The quality control scheme] You can further detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-leveilling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file rntSRBFgeoidh30s1.chs, and then repeat the step (4). This process is omitted in this example.

(5) All-element modelling on the remaining residual gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the remaining residual file rntSRBFgeoidh30s1.chs and model geoidal height grid file mdlgeoidh30s.dat to estimate the 30" remaining residual gravity field grid rntSRBFgeoidh30s1.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s1.chs.

(5) All-element modelling on the remaining residual gravity field using SRBFs

Open the discrete heterogeneous residual observations file

number of rows of file header: 2
 column ordinal number of ellipsoidal height in the record: 7
 column ordinal number of weight: 8

Select SRBF: Poisson wavelet kernel
 Order m: 5
 Minimum degree: 540
 Maximum degree: 5400
 Burial depth of Bjerrnhamer sphere: 6.0km
 Action distance of SBRF center: 50km
 Reuter network level K: 5400

Select the adjustable observations: height anomaly (m)
 Contribution rate k of adjustable observations: 1.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, observation type, weight.

>> The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] control button, or the [Start Computation] control button.

Computation start time: 2024-09-28 19:56:11
 Computation end time: 2024-09-28 20:03:11

>> Complete the computation!
 >> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *rga (mGal), residual height anomaly *ksi (m), residual gravity anomaly *gra (mGal), residual disturbing gravity gradient *gr (E, radial) and residual vertical deflection vector *dft (*, SW), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.

>> The program also outputs SRBF center file *center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval (*). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (*).

>> Type 0 of source observations: mean 0.0196 standard deviation 12.9866 minimum -80.4161 maximum 64.8276
 Residual observations: mean 0.0200 standard deviation 8.4565 minimum -54.9649 maximum 58.6241

>> Type 1 of source observations: mean -0.0002 standard deviation 0.0276 minimum -0.1059 maximum 0.0768
 Residual observations: mean 0.0008 standard deviation 0.0147 minimum -0.0511 maximum 0.0345

Solution of normal equation LU triangular decomposition

Save the results as Import setting parameters Start Computation

ID	lon	lat	ellipshgt	gravity disturbance(mGal)	height anomaly(m)	gravity anomaly(mGal)	gravity gradient(E)	vertical deflect.
1	101.50437	24.00417	-35.508	-12.7137	-0.0168	-12.7045	-97.1997	-0.6321
2	101.51250	24.00417	-35.519	-6.6258	-0.0077	-6.6234	-54.9695	-1.0260
3	101.52083	24.00417	-35.510	2.3531	0.0053	2.35	1.1958	0.0321
4	101.52917	24.00417	-35.501	11.0246	0.0174	11.03	1.1958	0.0321
5	101.53750	24.00417	-35.491	16.8356	0.0255	16.82	1.1958	0.0321
6	101.54583	24.00417	-35.481	17.2077	0.0259	17.19	1.1958	0.0321

0.0147m ≈ 1.5cm The accuracy index (SD) of geoid modelling.

Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.

The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations spherical radial basis function spatial curve residual gravity disturbance (mGal)

All-element models rntSRBFgeoidh30s2.xxx of the remaining residual gravity field

residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S (*)

		mean	standard deviation	minimum	maximum
Residual gravity disturbance (mGal)	Residuals	0.3523	42.1561	-296.0915	165.2611
	First SRBF	0.0196	12.9866	-80.4161	64.8276
	Second SRBF	0.0200	8.4565	-54.9649	58.6241
Residual GNSS-levelling geoidal height (m)	Residuals	-0.0071	0.2768	-0.6571	0.6846
	First SRBF	-0.0002	0.0276	-0.1059	0.0768
	Second SRBF	0.0008	0.0147④	-0.0511	0.0345

In the table, 0.0147④m = 1.5cm can be considered as the accuracy index of geoid modelling.

[The quality control scheme] You can furtherly detect and remove again the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFgeoidheight30s2.chs, and then repeat from step (4). This process is omitted in this example.

You can also do further cumulative SRBF approach to improve the results. This example omits this process.

(6) Restore the reference gravity field and generate the 30" all-element models of the gravity field on the geoid.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, input the file EGM2008.gfc, and the model geoidal height grid file mdlgeoidh30srst.dat (from mdlgeoidh30s.dat with grid edge removed), to calculate the all-element grid 5Mgeoidh30s540.xxx of the reference gravity field on geoid.

(6) Restore the reference gravity field and generate the 30" all-element models of the gravity field on the geoid.

The screenshot shows the 'Geopotential model' software interface. The 'Calculation of gravity field elements' section is active. The 'Minimum degree' is set to 2 and the 'Maximum degree' is set to 540. Below the input fields, there are several heatmaps showing gravity anomaly, gravity disturbance, and gravity vector components. A table of coefficients is also visible.

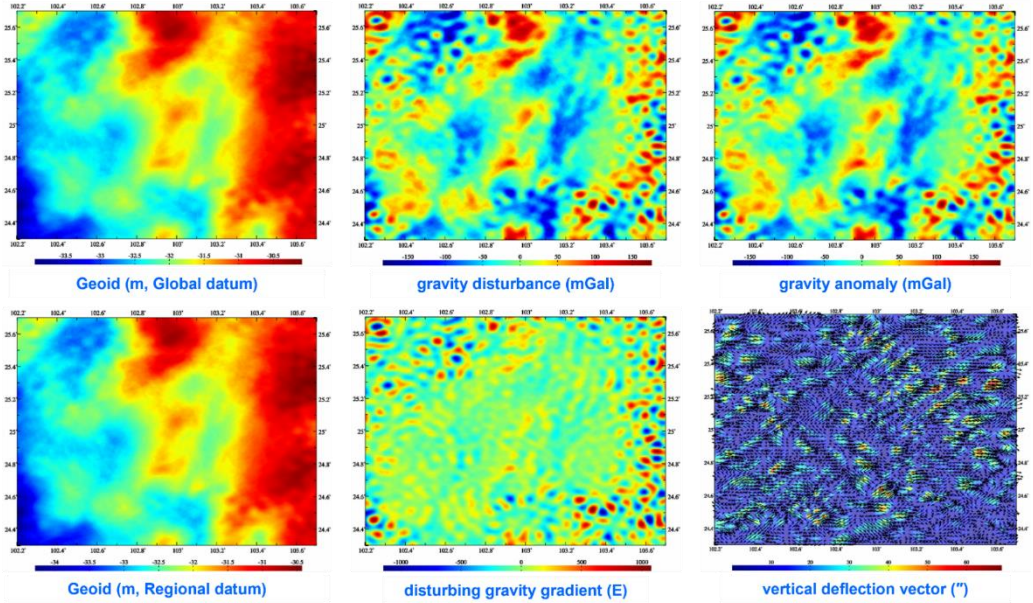
The screenshot shows the 'Program Process' window. It displays the execution of the 'Calculate gravity field elements' process. The start time is 2022-04-01 11:42:23 and the end time is 2022-04-01 11:42:50. The output files are listed in the 'Display of the output files' section.

The diagram shows the relationship between the input files and the output files. It shows 'Residuals geoidh30s1' plus 'Remaining residuals geoidh30s2' plus 'Reference models GMgeoidh30s540' resulting in the final output.

Add the residual gravity field grid geoidh30s1.xxx (from SRBFgeoidheight30s1.xxx with grid edge removed) and remaining residual gravity field grid geoidh30s2.xxx (from SRBFgeoidheight30s2.xxx with grid edge removed) to the reference gravity field grid GMgeoidh30s540.xxx, the 30" all-element gravity field models geoidh30srst.xxx on the geoid are obtained, which include the 30" gravimetric geoidal height grid (geoidh30srst.ksi, m), gravity disturbance grid (geoidh30srst.rga, mGal), gravity anomaly grid (geoidh30srst.gra, mGal), disturbing gravity gradient grid (geoidh30srst.grr, radial, E) and vertical deflection vector grid (geoidh30srst.dft, SW, ").

Add the regional height datum difference -0.3411m to the 30" gravimetric geoidal height grid geoidh30srst.ksi in global height datum, the 30" gravimetric geoidal height grid geoidh30srsgn.ksi in regional height datum can be obtained.

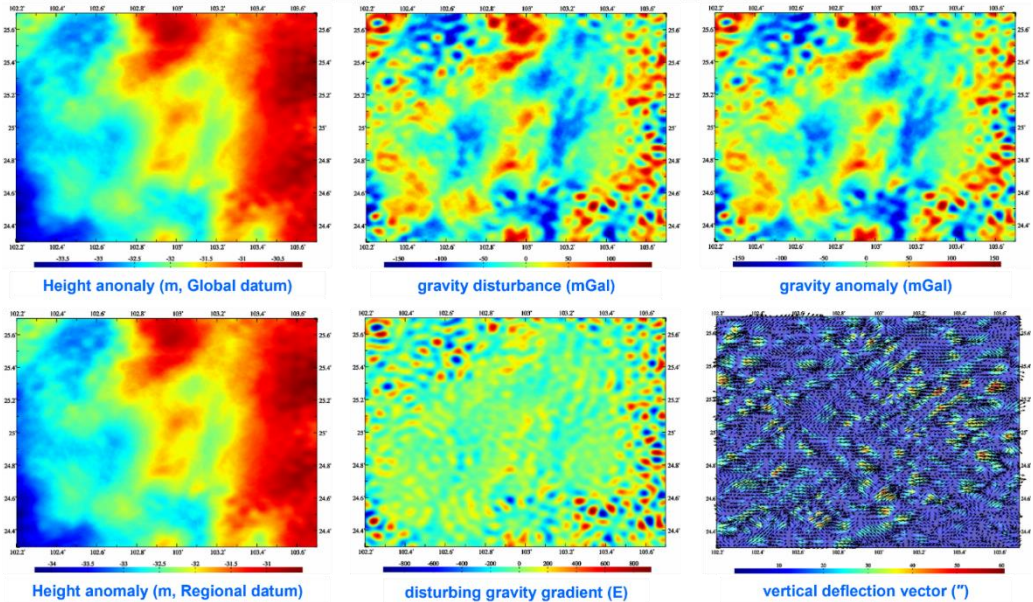
30"×30" full element models of gravity field on geoid



So far, the all-element modelling on gravity field on the geoid have been completed.

- Let the terrain surface as the calculation surface, and directly generate the 30" all-element models of the gravity field on the terrain surface.

30"×30" full element models of gravity field on terrain surface

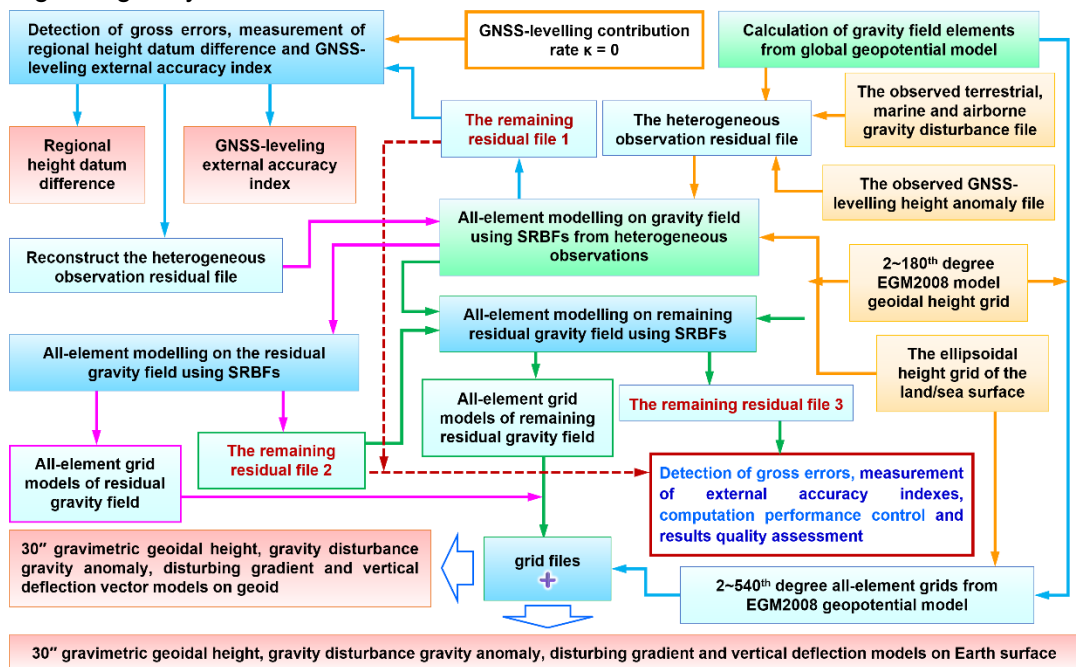


In step (3) to step (6) above, the input data file and all the parameter settings are kept the same, and only the calculation surface is changed to the terrain surface surfhgt30s.dat. Using the same computation process, you can synchronously obtain the 30" all-element models surfhgt30srst.xxx of the gravity field on the terrain surface, which include the 30" gravimetric ground height anomaly grid (surfhgt30srst.ksi, m, in global

height datum), ground gravity disturbance grid (surfhgt30srst.rga, mGal), ground gravity anomaly grid (surfhgt30srst.gra, mGal), ground disturbing gravity gradient grid (surfhgt30srst.grr, radial, E), ground vertical deflection vector grid (surfhgt30srst.dft, SW, ") and ground height anomaly grid (surfhgt30srgn.ksi, m, in regional height datum).

4.8.3 Simple process demo of all-element modelling on gravity field using SRBFs in normal height system

Exercise purpose: From the observed terrestrial, marine and airborne gravity disturbances and GNSS-leveling height anomalies in normal height system, make the all-element models on gravity field using spherical radial basis functions (SRBFs) in six steps, in which all the terrain effects are not processed, to quickly master the essentials in observation analysis, computation performance control and all-element modelling on regional gravity field.



Simple process demo of all-element modelling on gravity field using SRBFs in normal height system

In this section, the observed GNSS-leveling height anomaly in the normal height system is employed to replace the observed GNSS-leveling geoidal height in orthometric height system in the 4.8.2, and the simple process of all-element modelling on gravity field using SRBFs is introduced. In the both cases, there is only a slight difference in the processing of the observed GNSS-leveling data, and the other modelling processes are the same. For the convenience, here gives the complete quick process.

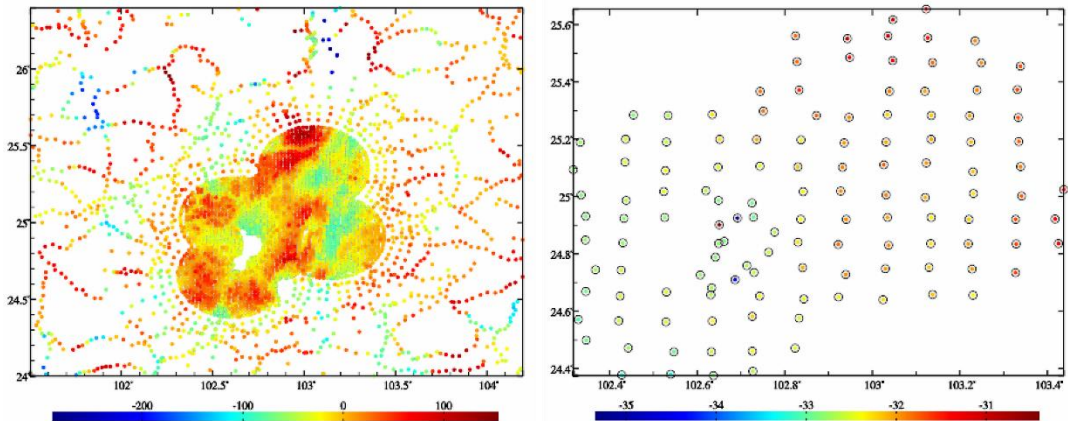
After the terrain effect processing omitted, SRBF approach process of gravity field is very simple because there is no need for additional continuation reduction, gridding and GNSS-leveling fusion process.

- **The observed gravity disturbance and GNSS-levelling data**

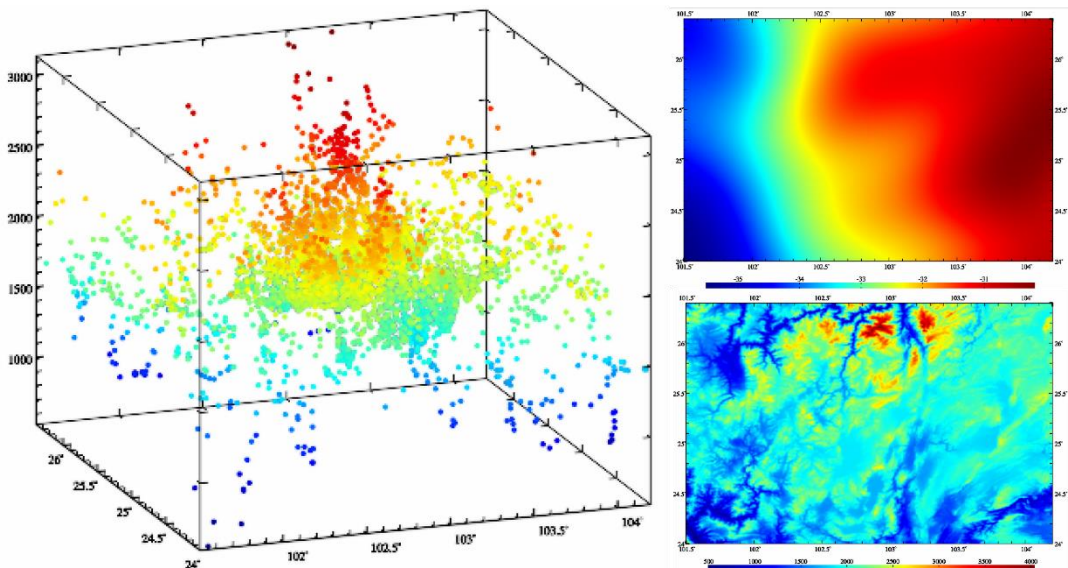
The observed terrestrial, marine and airborne gravity disturbance file obsdistgrav.txt. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed gravity disturbance (mGal), ...

The observed GNSS-levelling height anomaly file obsGNSSlksi.txt in normal height system. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed height anomaly (m), ...

In the example, the observed gravity disturbances and GNSS-levelling anomalies are simulated from the EGM2008 model (the 2~1800th degree) in advance.



The observed gravity disturbances (mGal) and observed GNSS-levelling height anomalies (m)



The distribution of gravity points, 2~180th degree model geoidal height and ellipsoidal height of the terrain surface

- **The ellipsoidal height grid of calculation surface:**

The model geoidal height grid file mdlgeoidh30s.dat calculated from the 2~180th degree geopotential model, which is employed for modelling on gravity field on geoid.

The ellipsoidal height grid file surfhgt30s.dat of the land/sea surface equal to the

sum of the digital elevation model grid DEM30s.dat and model geoidal height grid mdlgeoidh30s.dat, which is employed for modelling on ground gravity field.

Here, it is required that the grid range of the calculation surface is larger than the range of the target area to absorb edge effects.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, and input the file EGM2008.gfc, observed gravity disturbance file obsdistgrav.txt and observed GNSS-leveilling height anomaly file obsGNSSlksi.txt, calculate and remove the 2~540th degree model value of these observations to generate the heterogeneous observation file obsresiduals0.txt according to the agreed format.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Calculation of gravity field elements from global geopotential model

Calculation of model value for residual terrain (complete Bouguer) effects

Global geopotential coefficient model Calculator

Calculation and analysis of spectral character of Earth's gravity field

Save computation process as

Algorithmic Formulas

Open global geopotential coefficient model file

Select calculation file format

Discrete calculation points file

Open space calculation points file

Set input point file format

Number of rows of file header 1

Column ordinal number of ellipsoidal height in the record 5

Select elements to be calculated

height anomaly (m)

gravity anomaly (mGal)

gravity disturbance (mGal)

vertical deflection (τ , SW)

disturbing gravity gradient (E, radial)

tangential gravity gradient (E, NW)

Laplace operator (E)

Minimum degree 2

Maximum degree 540

Extract elements to be plot

Plot

The ellipsoidal height here at GNSS-leveilling point is the observed ellipsoidal height.

ID	lon(degree decimal)	lat	ellip(h)(m)	kst(m)
1	102.4424	24.4757	1973.56	-32.7581
2	102.5467	24.4580	1659.69	-32.9577
3	102.6324	24.4582	2262.99	-32.5792
4	102.7259	24.4605	2112.00	-32.3917
5	102.4208	24.5663	1991.56	-32.6038
6	102.5286	24.5627	1937.23	-32.5836
7	102.6344	24.5656	2193.72	-33.3822
8	102.7258	24.5819	2304.57	-32.2197
9	102.8326	24.5755	1978.11	-32.8408
				-32.0934

ID	lon(degree decimal)	lat	ellip(h)(m)	rent	kind	weight	
1	102.3929	24.4944	2228.19	54.9765	0	1	
2	102.3959	24.5089	2170.20	50.0971	0	1	
3	102.3927	24.5296	2013.33	28.3652	0	1	
4	102.3956	24.5453	2122.50	38.3822	0	1	
5	102.4208	24.5663	1973.28	20.6411	0	1	
6	102.5286	24.5627	1937.23	15.5784	0	1	
7	102.6344	24.5656	2193.72	14.5045	0	1	
8	102.7258	24.5819	2304.57	14.9731	0	1	
9	102.8326	24.5755	1978.11	7.4068	0	1	
4221	1	102.4424	24.4717	1973.56	-0.0882	1	1
4222	2	102.5467	24.4580	1659.69	-0.4184	1	1
4223	3	102.6324	24.4582	2120.99	-0.1378	1	1
4224	4	102.7259	24.4605	2112.20	-0.0659	1	1
4225	5	102.4208	24.5663	1991.56	-0.0029	1	1
4226	6	102.5286	24.5627	1937.23	-0.1219	1	1
4227	7	102.6344	24.5656	2193.72	-0.0607	1	1
4228	8	102.7258	24.5819	2304.57	-0.0100	1	1
4229	9	102.8326	24.5755	1978.11	-0.4484	1	1
4230	10	102.9455	24.6689	1920.60	-0.2580	1	1
4231	11	102.4239	24.6529	1960.26	-0.0416	1	1
4232	12	102.5297	24.6670	2158.55	-0.1896	1	1

The model height anomaly (m) at the GNSS-leveilling points to be set is equal, the program calculates the contribution of the degree n geopotential to be set is equal, the program calculates the contribution of the degree n geopotential can be employed to analyze and evaluate the spectral and space properties of the geopotential coefficient model.

The agreed format of the heterogeneous observation file record: ID (point no/name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual height anomaly (m).

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file.

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$, and input the heterogeneous residual file

obsresiduals0.txt and terrain surface ellipsoidal height grid file surfhgt30s.dat to estimate the residual gravity field grid SRBFsurfhtg30s0.xxx on geoid, and get the remaining residual file SRBFsurfhtg30s0.chs.

Where, xxx=ksi stands for residual height anomaly (m), xxx=rga stands for residual gravity disturbance (mGal), xxx=gra stands for residual gravity anomaly (mGal), xxx=grr stands for residual disturbing gravity gradient (radial, E) and xx=dft stands for residual vertical deflection (SW, ").

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file. Follow example

Open the discrete heterogeneous residual observations file

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.

number of rows of file header: 1
 column ordinal number of ellipsoidal height in the record: 6
 column ordinal number of weight: 7

Select SRBF radial multipole kernel

Order m: 5
 Minimum degree: 360
 Maximum degree: 1800
 Burial depth of Bjerrhammar sphere: 10.0km
 Action distance of SBRF center: 100km
 Reuter network level K: 3600

Select the adjustable observations height anomaly (m)
 Contribution rate κ of adjustable observations: 0.00

Open the ellipsoidal height grid file of calculation surface

Solution of normal equation LU tria

ID	lon	lat	ellipsoidht	gravit	residuals								
1	101.50417	24.0041	2	1	0.3186	42.1772	-296.0915	165.2611	residuals:	0.7368	16.9638	-105.2839	114.8811
2	101.51259	24.0041	3	1	-0.3510	3.2774	0.9962	0.3435	residuals:	-0.0410	0.0287	-0.1943	0.0132
3	101.52083	24.0041	4	2	102.39590	24.49440	2220.1300	54.9765	0.0000	54.9765	0.0000	0.0000	0.0000
4	101.52917	24.0041	5	2	102.39590	24.50930	2170.2000	-4.7688	50.9971	0.0000	0.0000	0.0000	0.0000
5	101.53750	24.0041	6	4	102.39660	24.52340	2013.3300	-10.3976	20.3682	0.0000	0.0000	0.0000	0.0000
6	101.54583	24.0041	7	5	102.39690	24.54530	2122.5000	1.0011	0.0000	0.0000	0.0000	0.0000	0.0000
7			8	6	102.39830	24.56340	1971.2800	-0.0346	0.0000	0.0000	0.0000	0.0000	0.0000
8			9	6	102.39830	24.58130	1840.3100	-10.0941	0.0000	0.0000	0.0000	0.0000	0.0000
9			10	7	102.39520	24.60360	1965.5800	12.1650	0.0000	0.0000	0.0000	0.0000	0.0000
10			11	8	102.39810	24.61700	1997.1200	20.5312	0.0000	0.0000	0.0000	0.0000	0.0000
11			12	9	102.39350	24.63840	1916.1500	3.5245	0.0000	0.0000	0.0000	0.0000	0.0000

Residual observations: mean 0.3186 standard deviation 42.1772 minimum -296.0915 maximum 165.2611
 Type 0 of source observations: mean -0.3452 standard deviation 0.2739 minimum -0.9755 maximum 0.3702
 Type 1 of source observations: mean -0.0405 standard deviation 0.0271 minimum -0.1876 maximum 0.0099

Select the remaining residuals (column 5) as the statistical reference.

Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.

The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations spherical radial basis function spatial curve residual gravity disturbance (mGal)

residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S (")

Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbances from the remaining residual file SRBFsurfhtg30s0.chs, detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresiduals01.txt.

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresiduals01.txt and repeat the step (2) to re-estimate the residual gravity field grid rntSRBFdatum30s.xxx on terrain surface and get the new remaining residual file rntSRBFdatum30s.chs.

Since the contribution rate of GNSS-levelling $\kappa = 0$ is set in advance, it is

essentially here directly to measure the external accuracy index of the observed GNSS levelling only using the observed gravity disturbances.

Separate the remaining residuals of the observed GNSS-levelling and observed gravity disturbance from `rntSRBFgeoidh30s0.chs`.

Before and after gross error removed, the statistical results on the observation residuals are as follows.

		number of points	mean	standard deviation	minimum	maximum
Gravity disturbance (mGal)	Original residuals	4219	0.3186	42.1772	-296.0915	165.2611
	Residuals without error	4215	0.2695	42.0737	-296.0915	165.2611
	Remaining residuals	4215	-0.5677	13.8957	-80.4161	64.8276
GNSS levelling height anomaly (m)	Original residuals	125	-0.3452	0.2739	-0.9755	0.3702
	Residuals without error	123	-0.3404 ^①	0.2735	-0.9755	0.3702
	Remaining residuals	123	-0.0069 ^②	0.0233 ^③	-0.1295	0.0528

The statistical mean ① minus ② of the GNSS-levelling remaining residuals in the table, that is, $-0.3404^{①} - (-0.0069^{②}) = -0.3335\text{m}$, is the difference between the regional height datum and the global height datum (gravimetric geoid). Here provides the SRBF measurement method for regional height datum difference.

In the table, $0.0233^{③}\text{m}$ is the external accuracy index of the observed GNSS-levelling expressed as standard deviation, that is, 2.33cm . Here provides the SRBF

measurement method for the external accuracy index of GNSS-leveling. The result indicates that the external accuracy of GNSS-leveling is not bad than 2.33 cm (SD).

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual, observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, height, type, weight.

>> The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] button.
 >> Computation start time: 2024-09-28 21:17:31
 >> Complete the computation!
 >> Computation end time: 2024-09-28 21:22:53
 >> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *rga (mGal), residual height anomaly *ksi (m), residual gravity anomaly *gra (mGal), residual disturbing gravity gradient *gr (E, radial) and residual vertical deflection vector *dft (*, SW), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.
 >> The program also outputs SRBF center file "center.txt" into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval (*). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (*).
 >> Type 0 of source observations: mean 0.2695 standard deviation 42.0737 minimum -296.0915 maximum 165.2611
 Residual observations: mean -0.5677 standard deviation 13.8957 minimum -80.4161 maximum 44.8276
 >> Type 1 of source observations: mean -0.3404 standard deviation 0.2735 minimum -0.9755 maximum 0.3702
 Residual observations: mean -0.0069 standard deviation 0.0233 minimum -0.1295 maximum 0.0528

Solution of normal equation LU triangular decomposition

ID	lon	lat	ellipsoid gravity disturbance (mGal)	height	type	weight
1	101.50417	24.00417	2427.222	-25.2719	0	1
2	101.53250	24.00417	2480.991	-33.0111	0	1
3	101.52083	24.00417	2435.157	-39.4282	0	1
4	101.52917	24.00417	2239.999	-41.4915	0	1
5	101.53750	24.00417	2032.509	-57.3974	0	1
6	101.54583	24.00417	1906.019	-58.2186	0	1
7	101.55417	24.00417	1780.019	-61.102	0	1
8	101.56250	24.00417	1660.019	-58.0311	0	1
9	101.57083	24.00417	1545.019	-67.6348	0	1
10	101.57917	24.00417	1435.019	-72.2220	0	1
11	101.58750	24.00417	1330.019	-77.2220	0	1
12	101.59583	24.00417	1230.019	-82.2220	0	1
13	101.60417	24.00417	1135.019	-87.2220	0	1
14	101.61250	24.00417	1045.019	-92.2220	0	1
15	101.62083	24.00417	960.019	-97.2220	0	1
16	101.62917	24.00417	880.019	-102.2220	0	1
17	101.63750	24.00417	805.019	-107.2220	0	1
18	101.64583	24.00417	735.019	-112.2220	0	1
19	101.65417	24.00417	670.019	-117.2220	0	1
20	101.66250	24.00417	610.019	-122.2220	0	1

0.2735m The external accuracy index (SD) of the 2-540th degree model height anomaly
0.0233 m The external accuracy index (SD) of GNSS-leveling

-0.3404 (-0.0069) -0.3335m
 The measured height datum difference

Only using the observed gravity disturbances.

Algorithm of gravity field approach using SRBFs

- After the first estimation is completed, it is recommended to refine the output residual observation file *.chs as the input observation file again to employ target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.
- The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

- The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.
- The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations spherical radial basis function spatial curve residual gravity disturbance (mGal)

residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S (")

In general, it is necessary to make 1 to 2 cumulative SRBF approach with *.chs as the input file to obtain the minimum of standard deviation of GNSS-leveling remaining residuals as the external accuracy index, and this process is omitted in this example.

After removing the regional height datum difference of -0.3345m from GNSS-leveling residuals, the new heterogeneous observation residual file obsresiduals1.txt is reconstructed again.

(4) All-element modelling on the regional gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the heterogeneous residual file obsresiduals1.txt and terrain surface ellipsoidal height grid file surfhgt30s.dat to estimate the 30" residual gravity field grid SRBFsurfhgt30s1.xxx on terrain surface, and get the remaining residual file SRBFsurfhgt30s1.chs.

[The quality control scheme] You can furtherly detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-leveling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFsurfhgt30s1.chs, and then repeat the step (4). This process is omitted in this example.

(4) All-element modelling on the residual gravity field using SRBFs

Open the discrete heterogeneous residual observations file

number of rows of file header: 1
 column ordinal number of ellipsoidal height in the record: 6
 column ordinal number of weight: 7

Select SRBF: radial multipole kernel
 Order m: 3
 Minimum degree: 360
 Maximum degree: 1800
 Burial depth of Bjerrnhamar sphere: 10.0km
 Action distance of SBRF center: 100km
 Reuter network level K: 3600

Select the adjustable observations: height anomaly (m)
 Contribution rate κ of adjustable observations: 1.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.

>> The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] tool button...
 >> Computation start time: 2024-09-28 21:43:45
 >> Complete the computation!
 >> Computation end time: 2024-09-28 21:48:27
 >> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *rga (mGal), residual height anomaly *ksi (m), residual gravity anomaly *gra (mGal), residual disturbing gravity gradient *gr (E, radial) and residual vertical deflection vector *dft (*, SW), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.
 >> The program also outputs SRBF center file "center.txt" into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval (*). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (*).

>> Type 0 of source observations: mean 0.2695 standard deviation 42.0737 minimum -296.0915 maximum 165.2611
 Residual observations: mean 0.0620 standard deviation 12.9866 minimum -80.4161 maximum 64.8276
 >> Type 1 of source observations: mean -0.0107 standard deviation 0.2739 minimum -0.6410 maximum 0.7047
 Residual observations: mean -0.0014 standard deviation 0.0291 minimum -0.1886 maximum 0.0595

Solution of normal equation: LU triangular decomposition

Save the results as: Import setting parameters Start Computation

ID	lon	lat	ellipshgt	gravity disturbance (mGal)	height anomaly (m)	gravity gradient (E, radial)	gravity gradient (S, SW)
1	101.504317	24.00417	2427.202	-33.8830	-0.3067	-3	0.0000
2	101.51250	24.00417	2460.991	-41.3359	-0.3579	-4	0.0000
3	101.52083	24.00417	2435.167	-47.3401	-0.3988	-4	0.0000
4	101.52917	24.00417	2229.999	-55.4930	-0.4344	-3	0.0000
5	101.53750	24.00417	2032.509	-65.0026	-0.5171	-6	0.0000
6	101.54583	24.00417	1906.019	-65.4479	-0.5213	-65.2877	-96.5237

Can furtherly detect and remove the observation gross errors from *.chs, and then repeat the step (4).

Algorithm of gravity field approach using SRBFs

After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.

The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot Plot

The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.

The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations spherical radial basis function spatial curve residual gravity disturbance (mGal)

All-element models SRBFsurfhtg30s1.xxx of the residual gravity field

residual height anomaly (m) residual disturbing gradient (E) residual vertical deflection S (*)

(5) All-element modelling on the remaining residual gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the remaining residual file SRBFsurfhtg30s1.chs and terrain surface ellipsoidal height grid file surfhtg30s.dat to estimate the 30" remaining residual field grid SRBFsurfhtg30s2.xxx on the terrain surface, and get the remaining residual file SRBFsurfhtg30s2.chs.

In the table below, $0.0154^{(4)}\text{m} = 1.5\text{cm}$ can be considered as the accuracy index of ground height anomaly (quasigeoid) modelling.

		mean	standard deviation	minimum	maximum
Residual gravity disturbance (mGal)	Residuals	0.2695	42.0737	-296.0915	165.2611
	First SRBF	0.0620	12.9866	-80.4161	64.8276
	Second SRBF	0.1309	8.5135	-50.6030	57.3920
Residual GNSS-levelling height anomaly (m)	Residuals	-0.0071	0.2768	-0.6571	0.6846
	First SRBF	-0.0014	0.0291	-0.1886	0.0595
	Second SRBF	-0.0013	0.0154 ⁽⁴⁾	-0.0708	0.0315

(5) All-element modelling on the remaining residual gravity field using SRBFs

Open the discrete heterogeneous residual observations file

number of rows of file header: 2
 column ordinal number of ellipsoidal height in the record: 7
 column ordinal number of weight: 8

Select SRBF: Poisson wavelet kernel
 Order m: 5
 Minimum degree: 540
 Maximum degree: 5400
 Burial depth of Bjerranmar sphere: 6.0km
 Action distance of SBRF center: 50km
 Reuter network level K: 5400

Select the adjustable observations: height anomaly (m)
 Contribution rate k of adjustable observations: 1.00

Open the ellipsoidal height grid file of calculation surface

of file header, whose format: observation type (0-5), source observation mean, standard deviation, minimum, maximum, residual observation mean, standard deviation, minimum, maximum. The record format: ID, longitude, latitude, ellipsoidal height, residual observation, source observation, observation type, weight.
 ** The parameter settings have been entered into the system!
 ** Click the [Start Computation] control button, or the [Start Computation] control button.
 >> Computation start time: 2024-09-28 19:56:11
 >> Complete the computation!
 >> Computation end time: 2024-09-28 20:03:11
 >> The program outputs the all-element grid files into the current directory. These grid files include the residual gravity disturbance *rga (mGal), residual height anomaly *ksi (m), residual gravity anomaly *gra (mGal), residual disturbing gravity gradient *gr (E, radial) and residual vertical deflection vector *dft (*, SW), where * is the output file name, and whose grid specification are the same as the input ellipsoidal height grid of calculation surface.
 >> The program also outputs SRBF center file *center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid number in meridian circle direction, maximum cell grid number in prime vertical circle direction, latitude interval (*). The record format: ID, longitude (degree decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (*).
 >> Type 0 of source observations: mean 0.0196 standard deviation 12.9866 minimum -80.4161 maximum 64.8276
 Residual observations: mean 0.0200 standard deviation 8.4565 minimum -54.9649 maximum 58.6241
 >> Type 1 of source observations: mean -0.0002 standard deviation 0.0275 minimum -0.1059 maximum 0.0766
 Residual observations: mean 0.0008 standard deviation 0.0147 minimum -0.0511 maximum 0.0345

Solution of normal equation: LU triangular decomposition

Save the results as | Import setting parameters | Start Computation

ID	lon	lat	ellipshgt	gravity disturbance (mGal)	height anomaly (m)	gravity anomaly (mGal)	gravity gradient (E)	vertical deflect.
1	101.504317	24.00417	-35.508	-12.7117	-0.0168	-12.7045	-97.1997	-0.6321
2	101.51250	24.00417	-35.519	-6.6258	-0.0077	-6.6234	-1.2702	-0.0260
3	101.52083	24.00417	-35.510	2.3531	0.0053	2.35	1.1588	0.0000
4	101.52917	24.00417	-35.511	11.0246	0.0174	11.01	1.1588	0.0000
5	101.53750	24.00417	-35.491	16.8356	0.0255	16.82	1.1588	0.0000
6	101.54583	24.00417	-35.481	17.2077	0.0259	17.19	1.1588	0.0000

0.0147m ≈ 1.5 cm The accuracy index (SD) of geoid modeling.

Algorithm of gravity field approach using SRBFs

- After the first estimation is completed, it is recommended to employ the output residual observation file *.chs as the input observation file again to refine target field elements. Generally, the stable solution can be achieved by 1 to 3 times cumulative SRBF approach, and the target field elements are equal to the sum of these SRBF approach solutions.
- The validity principle of once SRBF approach: (1) The residual target field element grid are continuous and differentiable, and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

Extract data to be plot | Plot

- The program is a high performance and adaptable modelling tool on local gravity field. Various observations with heterogeneity, different altitudes, cross-distribution and land-sea coexisting can be directly employed to estimate the all-element models of gravity field without reduction, continuation and gridding.
- The program has strong capacity on the detection of observation gross errors, measurement of external accuracy indexes and control of computation performance.

Spatial distribution of observations | spherical radial basis function spatial curve | residual gravity disturbance (mGal)

All-element models rntSRBFgeoidh30s2.xxx of the remaining residual gravity field

residual height anomaly (m) | residual disturbing gradient (E) | residual vertical deflection S (*)

[The quality control scheme] You can furtherly detect and remove again the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFsurfhtg30s2.chs, and then repeat from step (4). This process is omitted in this example.

You can also do further cumulative SRBF approach to improve the results. This example omits this process.

(6) Restore the reference gravity field and generate the 30" all-element models of the gravity field on the terrain surface.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, input the file EGM2008.gfc, and the terrain surface ellipsoidal height grid file surfhgt30srst.dat (from surfhgt30s.dat with grid edge removed), to calculate the all-element grid GMsurfhtg30s540.xxx of the reference gravity field on the terrain surface.

Add the residual gravity field grid surfhgt30s1.xxx (from SRBFsurfhtg30s0.xxx with grid edge removed) and remaining residual gravity field grid surfhgt30s2.xxx (from SRBFsurfhtg30s1.xxx with grid edge removed) to the reference gravity field grid GMsurfhtg30s540.xxx, the 30" all-element gravity field models surfhgt30srst.xxx on the terrain surface are obtained, which include the 30" gravimetric ground height anomaly grid (surfhtg30srst.ksi, m), ground gravity disturbance grid (surfhtg30srst.rga, mGal),

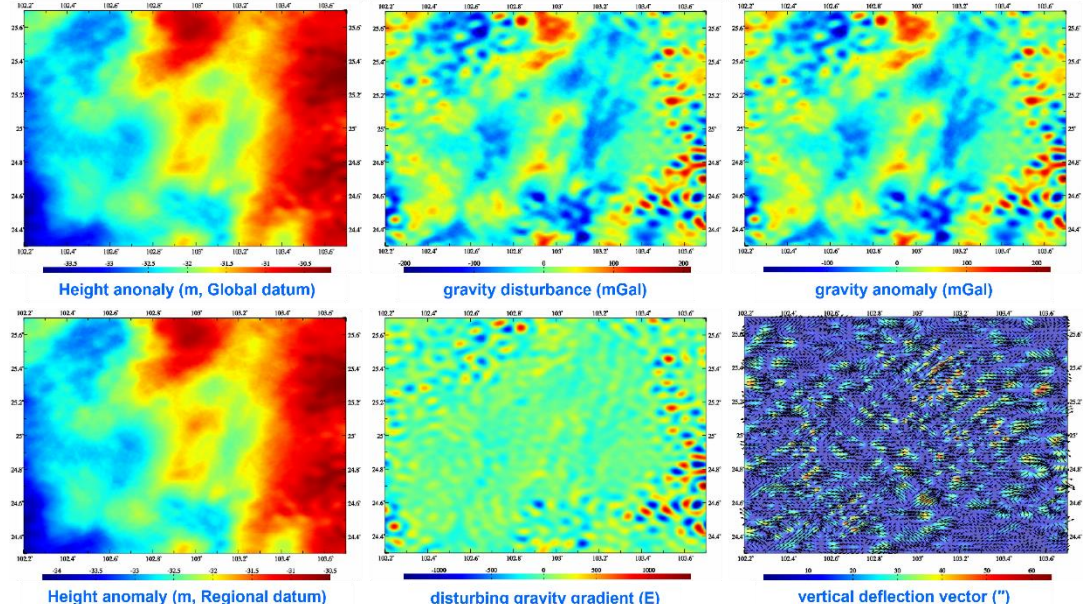
ground gravity anomaly grid (surfhgt30srst.gra, mGal), ground disturbing gravity gradient grid (surfhgt30srst.grr, radial, E) and ground vertical deflection vector grid (surfhgt30srst.dft, SW, ").

(6) Restore the reference gravity field and generate the 30" all-element models of the gravity field on terrain surface.

The screenshot shows the software interface for generating gravity field models. Key components include:

- Full element models window:** Contains input fields for 'Minimum longitude', 'Maximum longitude', 'Minimum latitude', and 'Maximum latitude'. It also has a 'Save the results as' field and an 'Import setting parameters' button.
- Residuals window:** Displays a table of residuals for 'surfhtg30s1'. The table has columns for 'lon', 'lat', 'residual', and 'weight'. The residuals are listed in scientific notation.
- Maps:** Three maps are shown:
 - Model ground disturbing gravity gradient (E):** A map showing the disturbing gravity gradient in E (radial).
 - Remaining residuals surfhgt30s2:** A map showing the remaining residuals after the model is applied.
 - Reference models GMSurfhgt30s40:** A map showing the reference gravity field model.

30"×30" full element models of gravity field on terrain surface



Add the regional height datum difference -0.3411m to the 30" gravimetric height anomaly grid surfhgt30srst.ksi in global height datum, the 30" gravimetric height

anomaly grid surfhgt30srgrn.ksi in regional height datum can be obtained.

So far, the all-element modelling on gravity field on the terrain surface have been completed.

- **Let the geoid as the calculation surface, and directly generate the 30" all-element models of the gravity field on the geoid.**

In step (3) to step (6) above, the input data file and all the parameter settings are kept same, and only the calculation surface is changed to the geoid. Using the same process, you can synchronously obtain the 30" all-element models geoidh30srst.xxx of the gravity field on the geoid, which include the 30" gravimetric geoidal height grid (geoidh30srst.ksi, m, in global height datum), gravity disturbance grid (geoidh30srst.rga, mGal), gravity anomaly grid (surfhgt30srst.gra, mGal), disturbing gravity gradient grid (geoidh30srst.grr, radial, E), vertical deflection vector grid (geoidh30srst.dft, SW, ") and geoidal height grid (geoidh30srgrn.ksi, m) in regional height datum.

30"×30" full element models of gravity field on geoid

