4.8.2 Simple process demo of all-element modelling on gravity field using SRBFs in orthometric height system

Exercise purpose: From the observed terrestrial, marine and airborne gravity disturbances and GNSS-leveling geoidal heights in orthometric height system, make the all-element models on gravity field using spherical radial basis functions (SRBFs) in six steps, in which all the terrain effects are not processed, to quickly master the essentials in observation analysis, computation quality control and all-element modelling on regional gravity field.

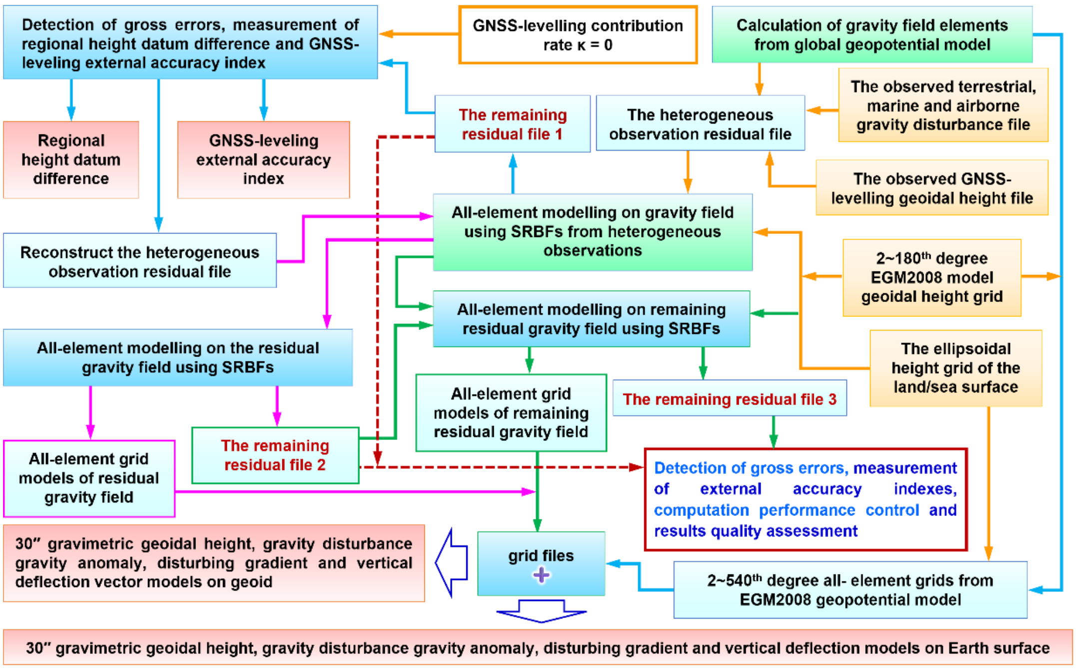
After the terrain effect processing omitted, SRBF approach process of gravity field is very simple because there is no need for additional continuation reduction, gridding and GNSS leveling fusion process.

● **The observed gravity disturbance and GNSS-levelling data**

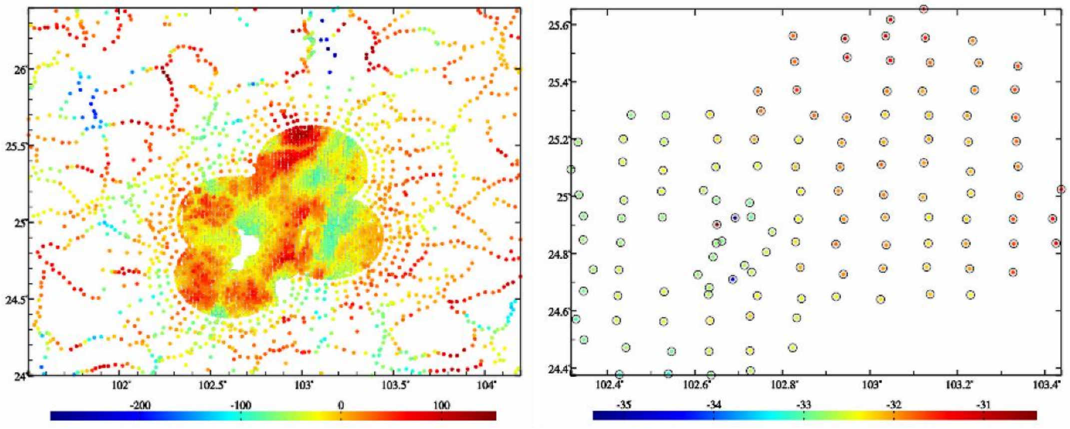
The observed terrestrial, marine and airborne gravity disturbance file obsdistgrav.txt. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed gravity disturbance (mGal), …

The observed GNSS-leveling geoidal height file obsGNSSlgeoid.txt in orthometric height system. The file record format: ID, longitude (degree decimal), latitude, observed geoidal height (m), …

In the example, the observed gravity disturbance and GNSS-leveling geoidal heights are simulated from the EGM2008 model (the 2~1800th degree) in advance.

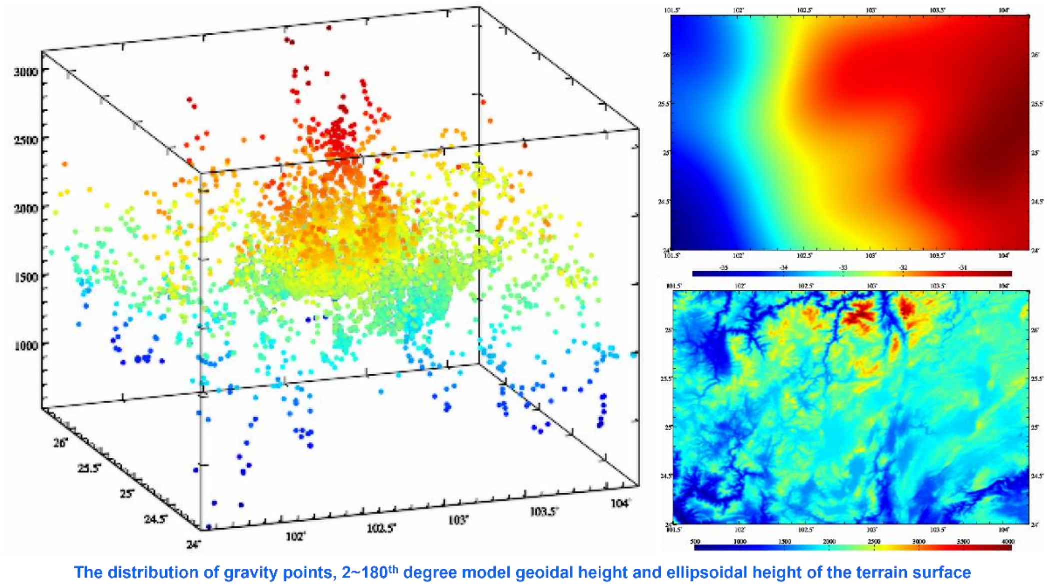


Simple process demo of All-element modelling on gravity filed using SRBFs in orthometric height system



The observed gravity disturbances (mGal) and observed GNSS-levelling geoidal heights (m)

It should be noted that since the observed geoidal height by GNSS-leveling is essentially the height anomaly on the geoid in orthometric height system, the ellipsoidal height at GNSS-leveling sites must be the geoidal height, which can be employed by the observed GNSS-leveling geoidal height or the model geoidal height from the EGM2008 model (the 2~180th degree).



The distribution of gravity points, 2~180th degree model geoidal height and ellipsoidal height of the terrain surface

● The ellipsoidal height grid of calculation surface:

The model geoidal height grid file mdlgeoidh30s.dat calculated from the 2~180th degree geopotential model, which is employed for modelling on gravity field on geoid.

The ellipsoidal height grid file surfhgt30s.dat of the land/sea surface equal to the sum of the digital elevation model grid DEM30s.dat and the model geoidal height grid mdlgeoidh30s.dat, which is employed for modelling on ground gravity field.

Here, it is required that the grid range of the calculation surface is larger than the range of the target area to absorb edge effects.

(1) Remove reference model value from all the observations and then

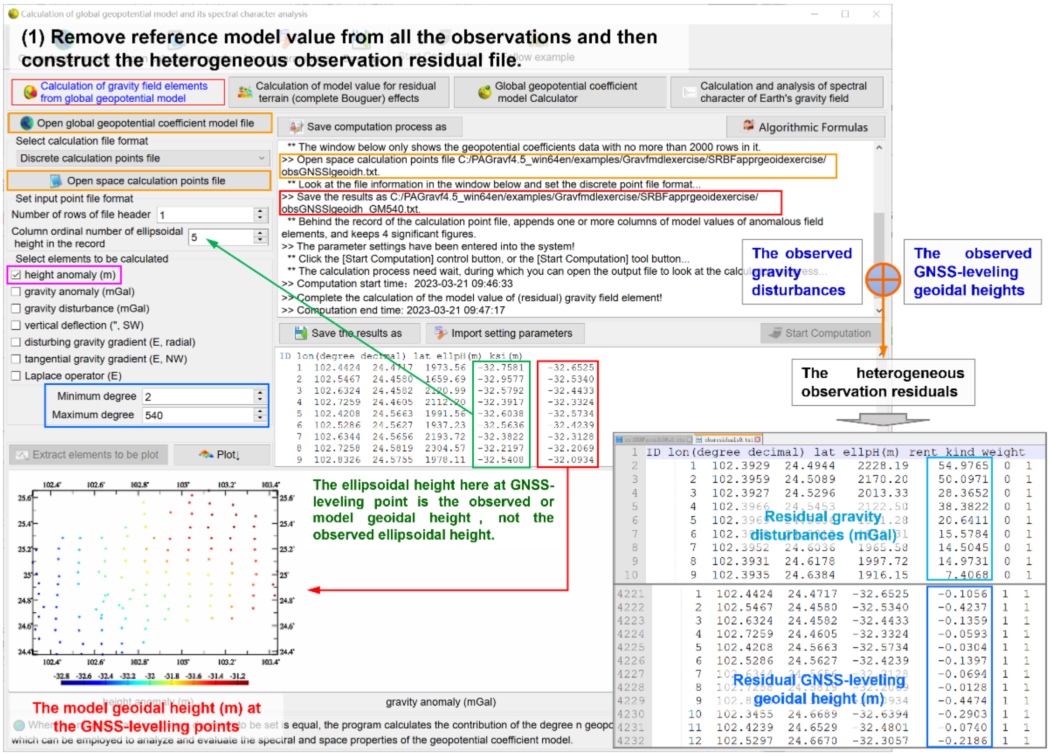
construct the heterogeneous observation residual file.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, and input the file EGM2008.gfc, observed gravity disturbance file obsdistgrav.txt and observed GNSS-levelling geoidal height file obsGNSSlgeoid.txt, calculate and remove the 2~540th degree model value of these observations to generate the heterogeneous observation file obsresiduals0.txt according to the agreed format.

The agreed format of the heterogeneous observation file record: ID (point no / name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual geoidal height (m).

It should be noted that the ellipsoidal height of GNSS-leveling site must be the geoidal height and not the observed ellipsoidal height at GNSS-levelling site.



(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file.

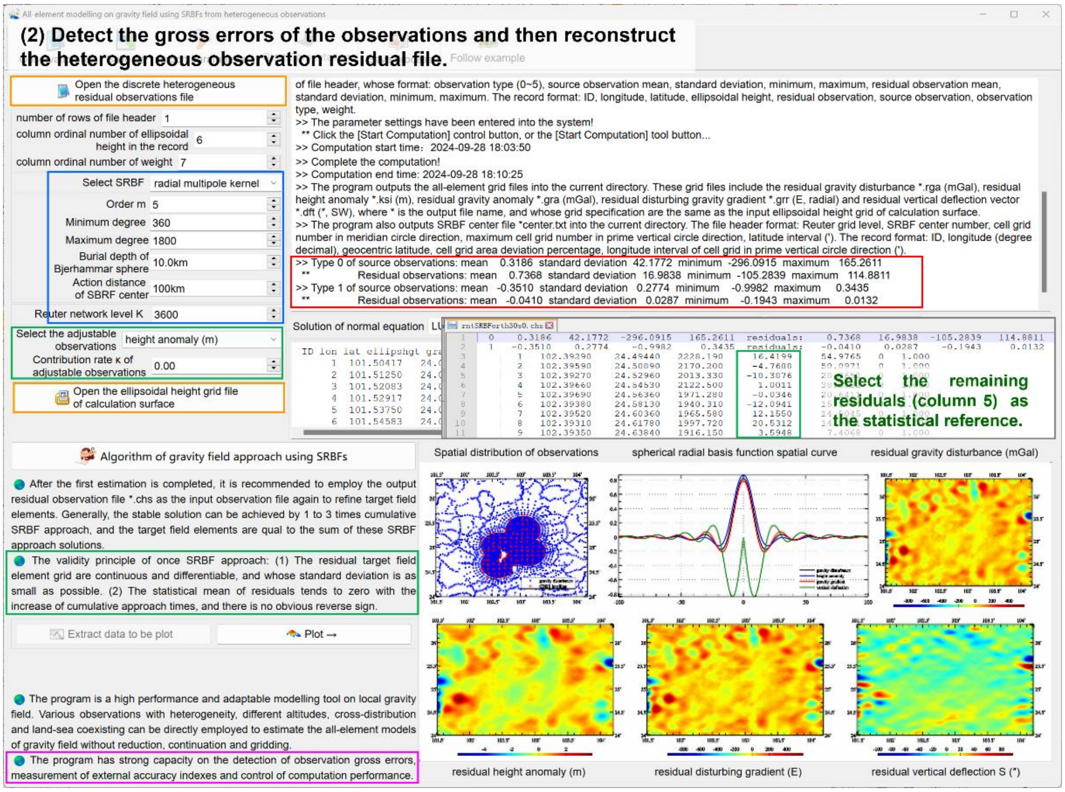
Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$, and input the heterogeneous residual file obsresiduals0.txt and model geoidal height grid file mdlgeoidh30s.dat to estimate the residual gravity field grid rntSRBFgeoidh30s0.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s0.chs.

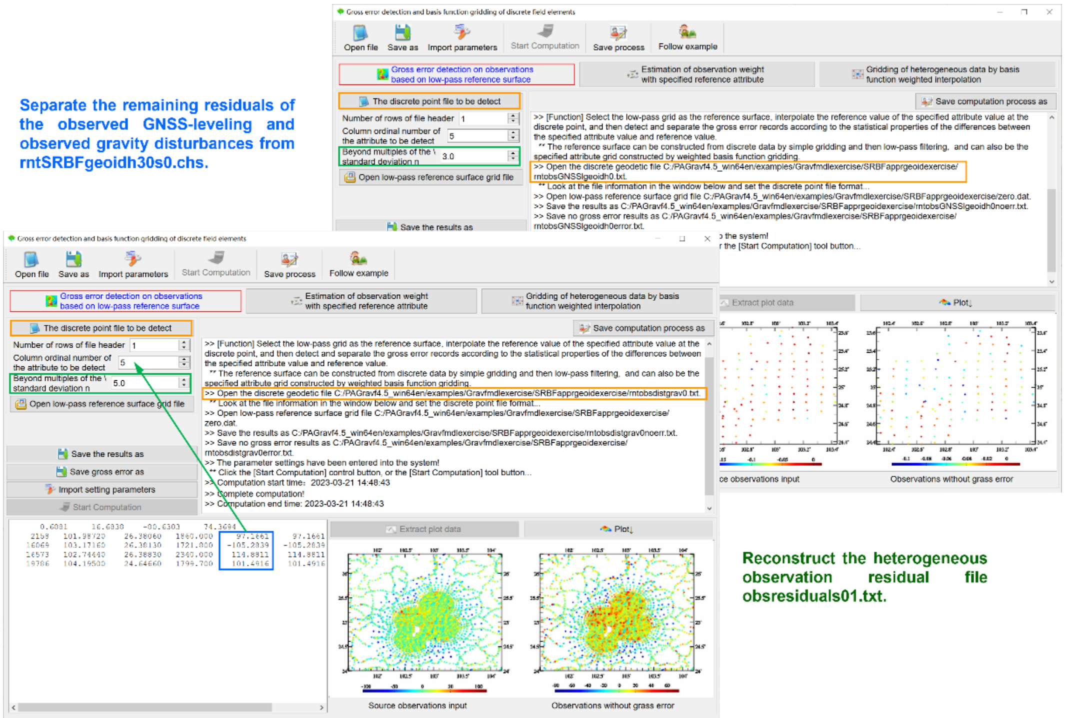
Where, xxx=ksi stands for residual geoidal height (m), xxx=rga stands for residual gravity disturbance (mGal), xxx=gra stands for residual gravity anomaly (mGal), xxx=grr stands for residual disturbing gravity gradient (radial, E) and xx=dft stands for residual vertical deflection (SW, ″).

Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbances from the remaining residual file rntSRBFgeoidh30s0.chs, detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresiduals01.txt.

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresiduals01.txt and repeat the step (2) to re-estimate the residual gravity field grid rntSRBFdatum30s.xxx on geoid and get the new remaining residual file rntSRBFdatum30s.chs.

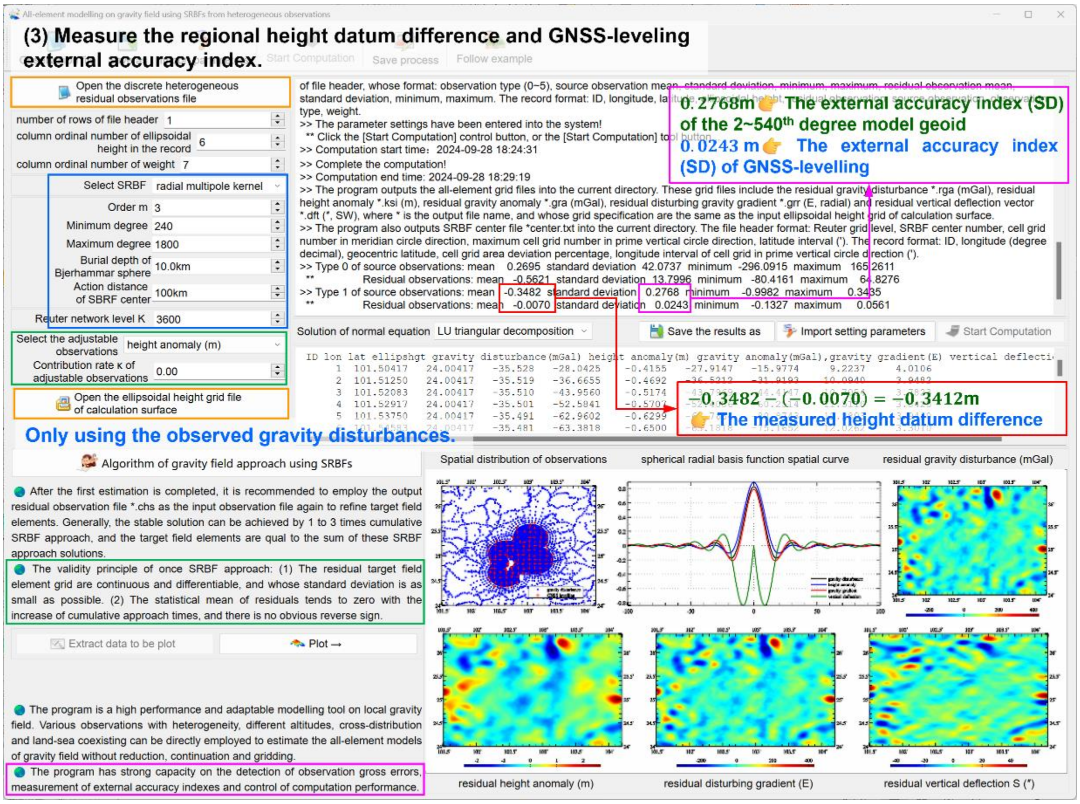




Since the contribution rate of GNSS-levelling $\kappa = 0$ is set in advance, it is essentially here directly to measure the external accuracy index of the observed GNSS levelling only using the observed gravity disturbances. Before and after gross error removed, the statistical results on the observation residuals are as follows.

		number of points	mean	standard deviation	minimum	maximum
Gravity disturbance (mGal)	Original residuals	4219	0.3186	42.1772	-296.0915	165.2611
	Residuals without error	4215	0.2695	42.0737	-296.0915	165.2611
	Remaining residuals	4215	-0.4584	13.6071	-61.1040	64.8276
GNSS levelling geoidal height (m)	Original residuals	125	-0.3510	0.2774	-0.9982	0.3435
	Residuals without error	124	-0.3482①	0.2768	-0.9982	0.3435
	Remaining residuals	124	-0.0070②	0.0243③	-0.1328	0.0561

The statistical mean ① minus ② of the GNSS-levelling remaining residuals in the table, that is, -0.3482① - (-0.0070②) = -0.3412 m, is the difference between the regional height datum and global height datum (gravimetric geoid). Here provides the SRBF measurement method for regional height datum difference.



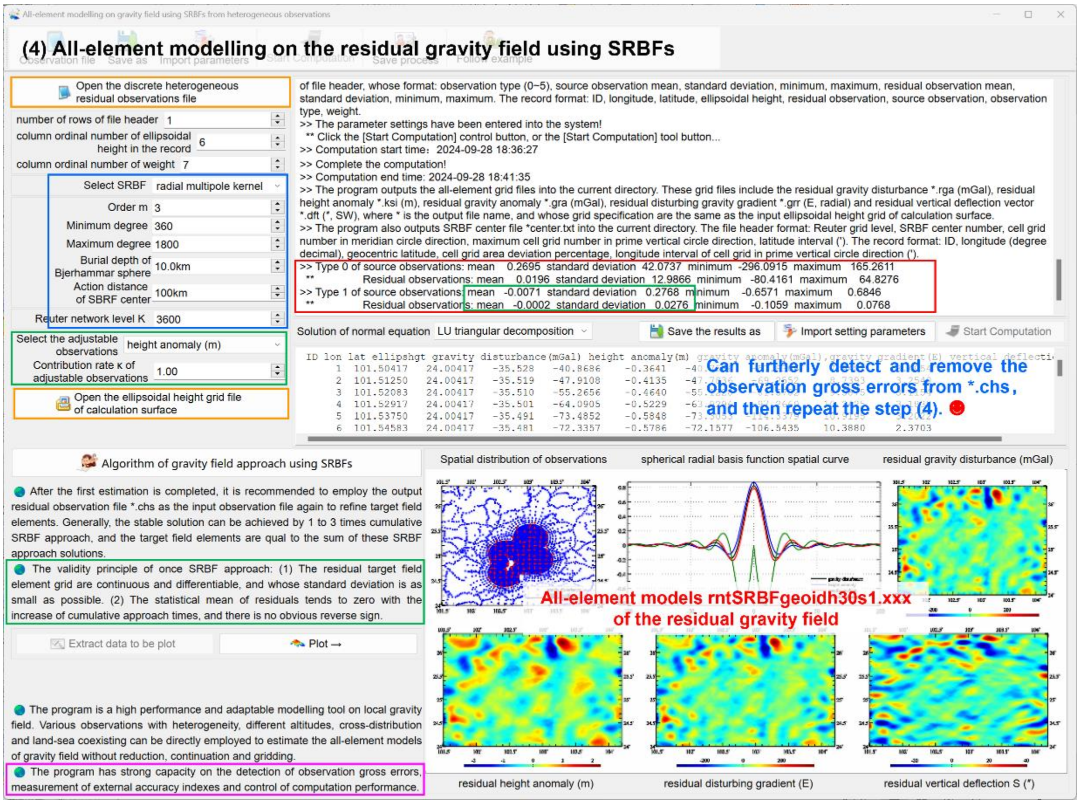
In the table, 0.0243③m is the external accuracy index of the observed GNSS-levelling expressed as standard deviation, that is, 2.43 cm. Here provides the SRBF measurement method for the external accuracy index of GNSS-leveling. The result indicates that the external accuracy of the observed GNSS-leveling is not bad than 2.43 cm (standard deviation).

In general, it is necessary to make 1 to 2 cumulative SRBF approach with *.chs as the input file to obtain the minimum of the standard deviation of the GNSS-levelling remaining residuals as the external accuracy index, and this process is omitted in this example.

After removing the regional height datum difference of -0.3412m from the GNSS-levelling residuals, the new heterogeneous observation residual file obsresiduals1.txt is reconstructed again.

(4) All-element modelling on the residual gravity field using SRBFs

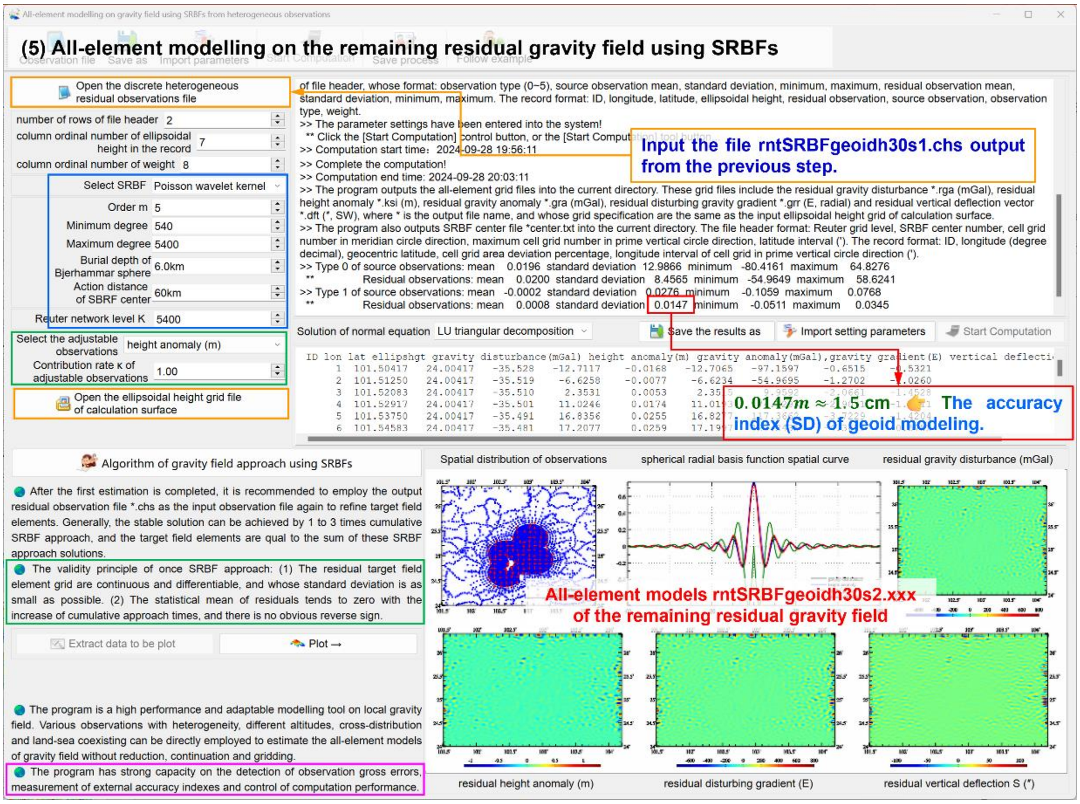
Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the heterogeneous residual file obsresiduals1.txt and the model geoidal height grid file mdlgeoidh30s.dat to estimate the 30″ residual gravity field grid rntSRBFgeoidh30s1.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s1.chs.



[The quality control scheme] You can furtherly detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file rntSRBFgeoidh30s1.chs, and then repeat the step (4). This process is omitted in this example.

(5) All-element modelling on the remaining residual gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the remaining residual file rntSRBFgeoidh30s1.chs and model geoidal height grid file mdlgeoidh30s.dat to estimate the 30″ remaining residual gravity field grid rntSRBFgeoidh30s1.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s1.chs.



		mean	standard deviation	minimum	maximum
Residual gravity disturbance (mGal)	Residuals	0.3523	42.1561	-296.0915	165.2611
	First SRBF	0.0196	12.9866	-80.4161	64.8276
	Second SRBF	0.0200	8.4565	-54.9649	58.6241
Residual GNSS-levelling geoidal height (m)	Residuals	-0.0071	0.2768	-0.6571	0.6846
	First SRBF	-0.0002	0.0276	-0.1059	0.0768
	Second SRBF	0.0008	0.0147④	-0.0511	0.0345

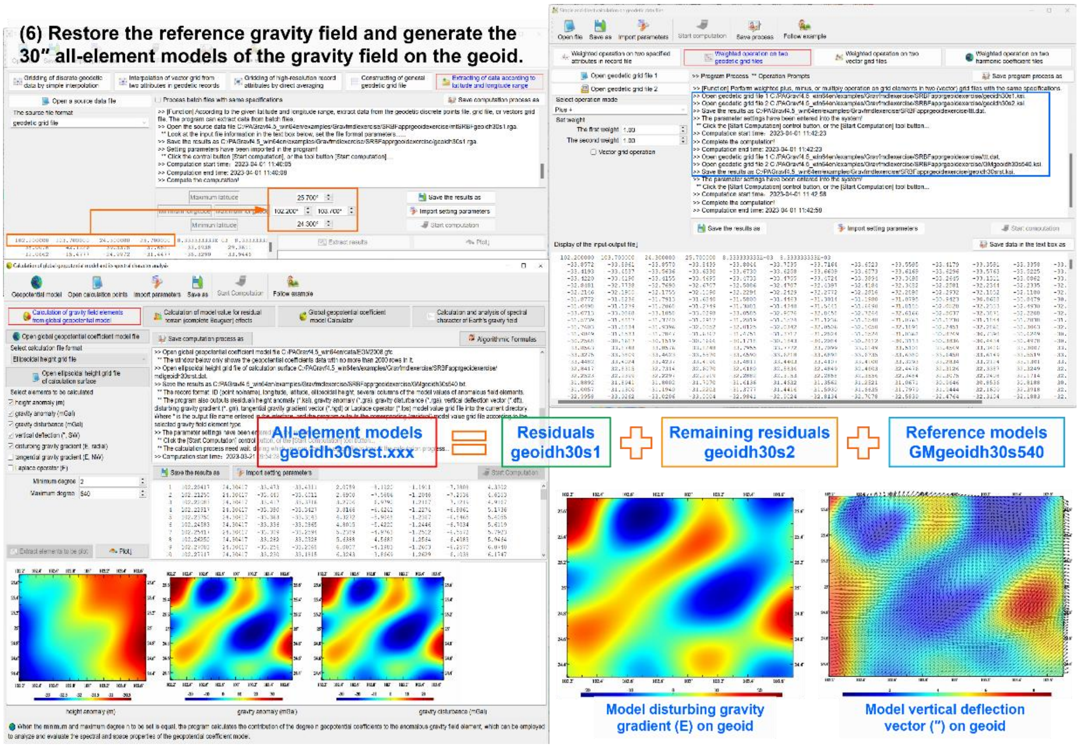
In the table, 0.0147④m = 1.5cm can be considered as the accuracy index of geoid modelling.

[The quality control scheme] You can furtherly detect and remove again the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFgeoidheight30s2.chs, and then repeat from step (4). This process is omitted in this example.

You can also do further cumulative SRBF approach to improve the results. This example omits this process.

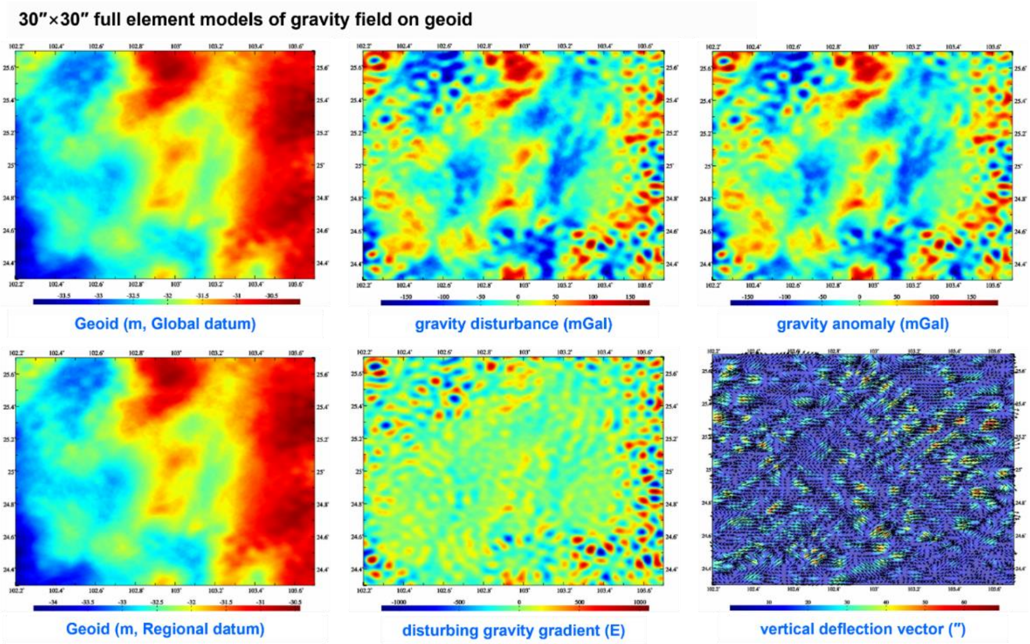
(6) Restore the reference gravity field and generate the 30″ all-element models of the gravity field on the geoid.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, input the file EGM2008.gfc, and the model geoidal height grid file mdlgeoidh30srst.dat (from mdlgeoidh30s.dat with grid edge removed), to calculate the all-element grid 5Mgeoidh30s540.xxx of the reference gravity field on geoid.



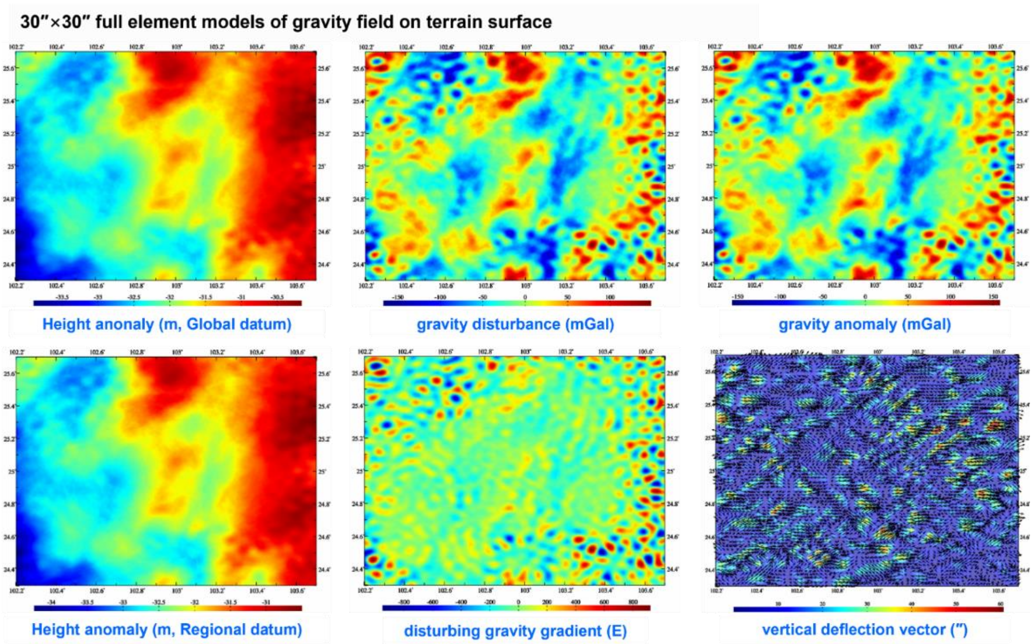
Add the residual gravity field grid geoidh30s1.xxx (from SRBFgeoidheight30s1.xxx with grid edge removed) and remaining residual gravity field grid geoidh30s2.xxx (from SRBFgeoidheight30s2.xxx with grid edge removed) to the reference gravity field grid GMgeoidh30s540.xxx, the 30″ all-element gravity field models geoidh30srst.xxx on the geoid are obtained, which include the 30″ gravimetric geoidal height grid (geoidh30srst.ksi, m), gravity disturbance grid (geoidh30srst.rga, mGal), gravity anomaly grid (geoidh30srst.gra, mGal), disturbing gravity gradient grid (geoidh30srst.grr, radial, E) and vertical deflection vector grid (geoidh30srst.dft, SW, ″).

Add the regional height datum difference -0.3411m to the 30″ gravimetric geoidal height grid geoidh30srst.ksi in global height datum, the 30″ gravimetric geoidal height grid geoidh30srgn.ksi in regional height datum can be obtained.



So far, the all-element modelling on gravity field on the geoid have been completed.

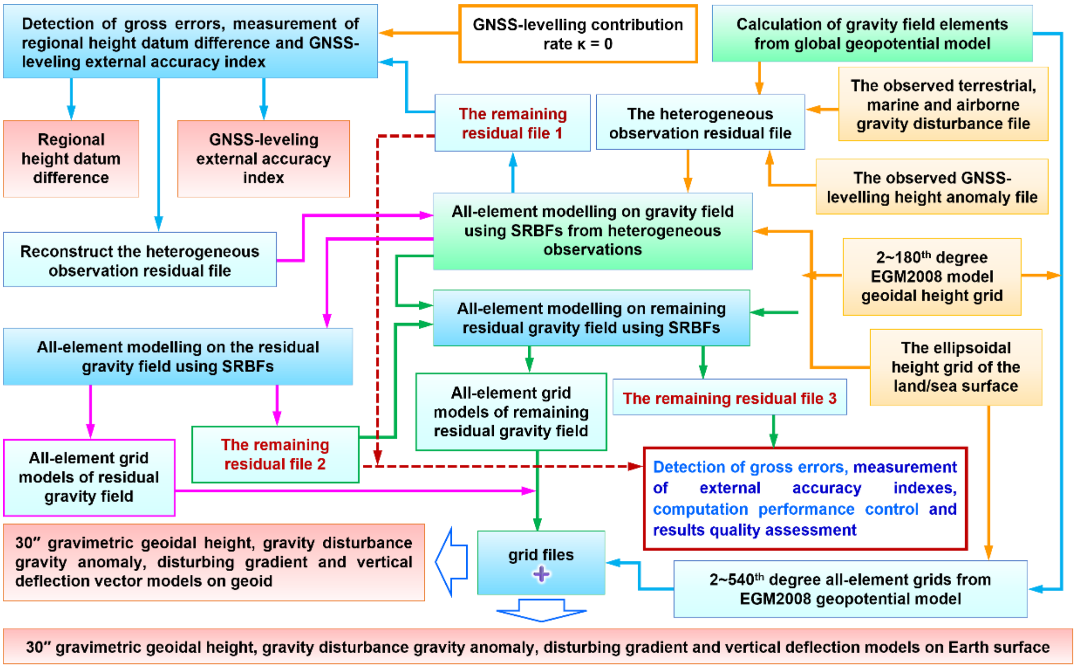
○ **Let the terrain surface as the calculation surface, and directly generate the 30″ all-element models of the gravity field on the terrain surface.**



In step (3) to step (6) above, the input data file and all the parameter settings are kept the same, and only the calculation surface is changed to the terrain surface surfhgt30s.dat. Using the same computation process, you can synchronously obtain the 30″ all-element models surfhgt30srst.xxx of the gravity field on the terrain surface, which include the 30″ gravimetric ground height anomaly grid (surfhgt30srst.ksi, m, in global height datum), ground gravity disturbance grid (surfhgt30srst.rga, mGal), ground gravity anomaly grid (surfhgt30srst.gra, mGal), ground disturbing gravity gradient grid (surfhgt30srst.grr, radial, E), ground vertical deflection vector grid (surfhgt30srst.dft, SW, ″) and ground height anomaly grid (surfhgt30srgn.ksi, m, in regional height datum).

4.8.3 Simple process demo of all-element modelling on gravity field using SRBFs in normal height system

Exercise purpose: From the observed terrestrial, marine and airborne gravity disturbances and GNSS-leveling height anomalies in normal height system, make the all-element models on gravity field using spherical radial basis functions (SRBFs) in six steps, in which all the terrain effects are not processed, to quickly master the essentials in observation analysis, computation performance control and all-element modelling on regional gravity field.



Simple process demo of all-element modelling on gravity filed using SRBFs in normal height system

In this section, the observed GNSS-levelling height anomaly in the normal height system is employed to replace the observed GNSS-levelling geoidal height in orthometric height system in the 4.8.2, and the simple process of all-element modelling on gravity filed using SRBFs is introduced. In the both cases, there is only a slight difference in the processing of the observed GNSS-levelling data, and the other modelling processes are the same. For the convenience, here gives the complete quick process.

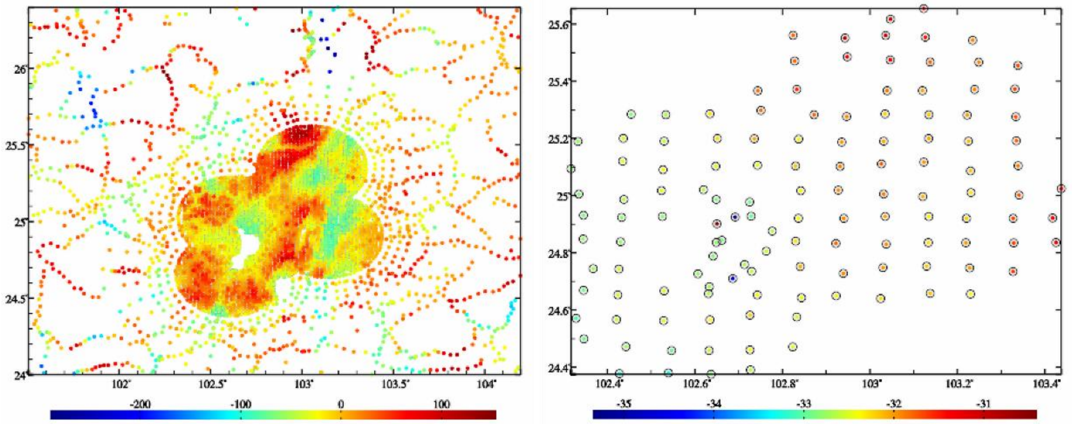
After the terrain effect processing omitted, SRBF approach process of gravity field is very simple because there is no need for additional continuation reduction, gridding and GNSS-leveling fusion process.

● **The observed gravity disturbance and GNSS-levelling data**

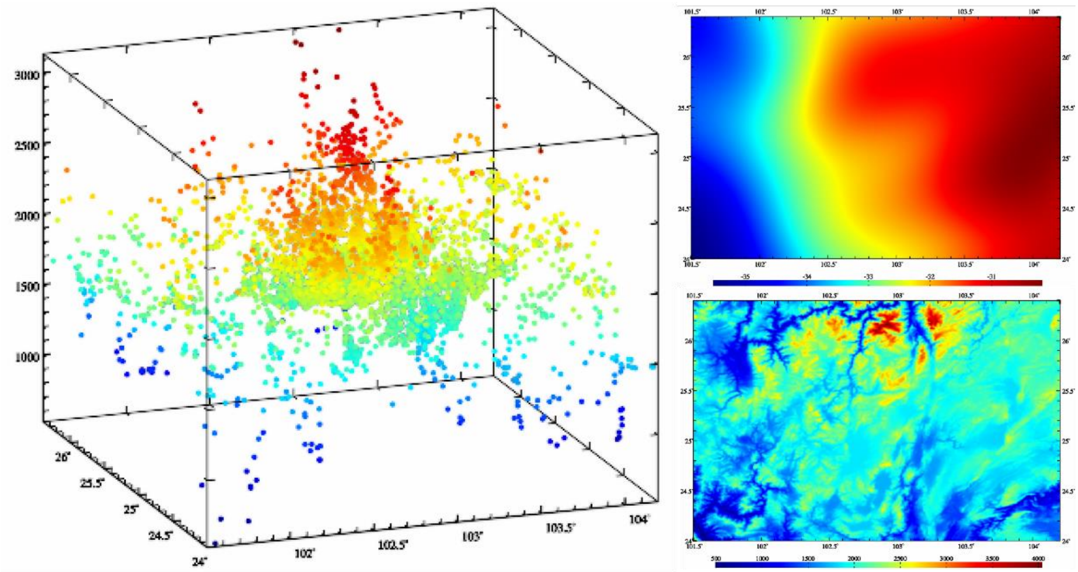
The observed terrestrial, marine and airborne gravity disturbance file obsdistgrav.txt. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed gravity disturbance (mGal), …

The observed GNSS-leveling height anomaly file obsGNSSlksi.txt in normal height system. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed height anomaly (m), …

In the example, the observed gravity disturbances and GNSS-leveling anomalies are simulated from the EGM2008 model (the 2~1800th degree) in advance.



The observed gravity disturbances (mGal) and observed GNSS-levelling height anomalies (m)



The distribution of gravity points, 2~180th degree model geoidal height and ellipsoidal height of the terrain surface

● **The ellipsoidal height grid of calculation surface:**

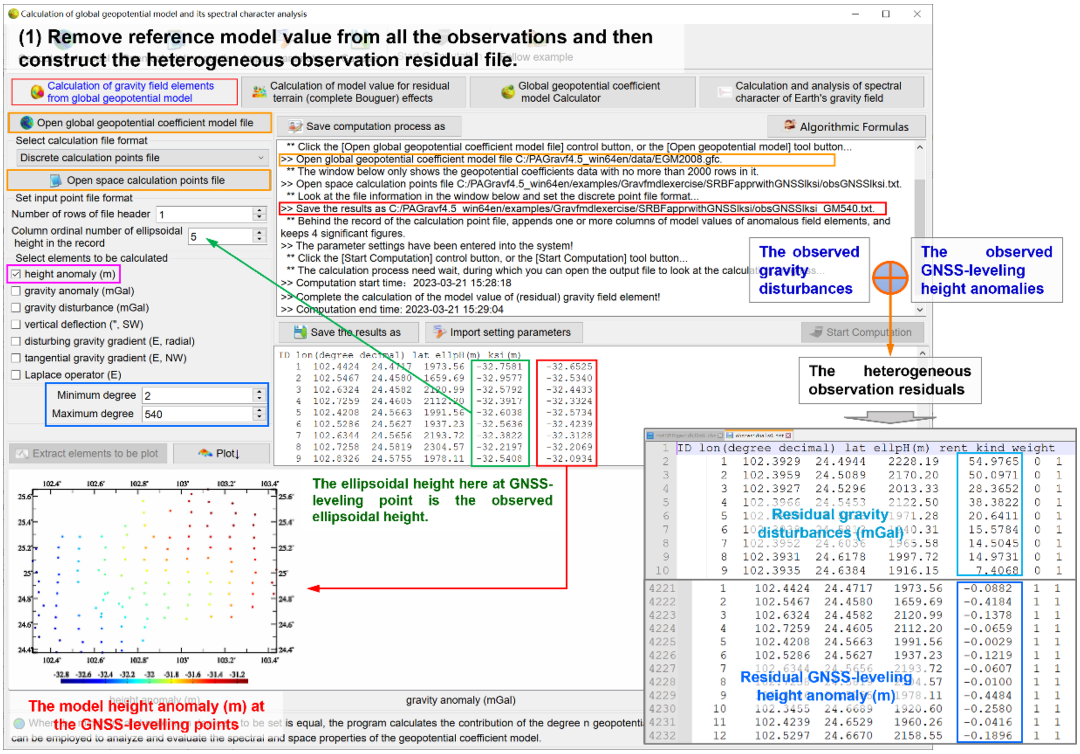
The model geoidal height grid file mdlgeoidh30s.dat calculated from the 2~180th degree geopotential model, which is employed for modelling on gravity field on geoid.

The ellipsoidal height grid file surfhgt30s.dat of the land/sea surface equal to the sum of the digital elevation model grid DEM30s.dat and model geoidal height grid mdlgeoidh30s.dat, which is employed for modelling on ground gravity field.

Here, it is required that the grid range of the calculation surface is larger than the range of the target area to absorb edge effects.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, and input the file EGM2008.gfc, observed gravity disturbance file obsdistgrav.txt and observed GNSS-levelling height anomaly file obsGNSSlksi.txt, calculate and remove the 2~540th degree model value of these observations to generate the heterogeneous observation file obsresiduals0.txt according to the agreed format.



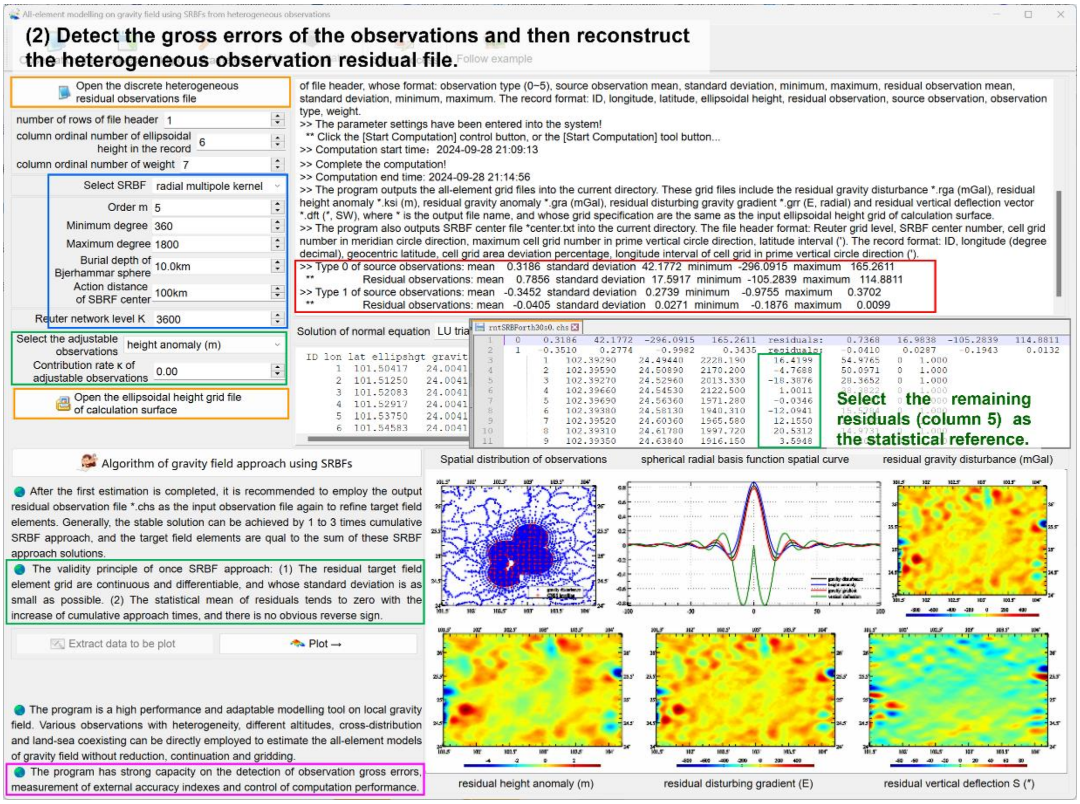
The agreed format of the heterogeneous observation file record: ID (point no/name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual height anomaly (m).

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file.

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$, and input the heterogeneous residual file obsresiduals0.txt and terrain surface ellipsoidal height grid file surfhgt30s.dat to estimate the residual gravity field grid SRBFsurfhgt30s0.xxx on geoid, and get the remaining residual file SRBFsurfhgt30s0.chs.

Where, xxx=ksi stands for residual height anomaly (m), xxx=rga stands for residual gravity disturbance (mGal), xxx=gra stands for residual gravity anomaly (mGal), xxx=grr stands for residual disturbing gravity gradient (radial, E) and xx=dft stands for residual vertical deflection (SW, ″).

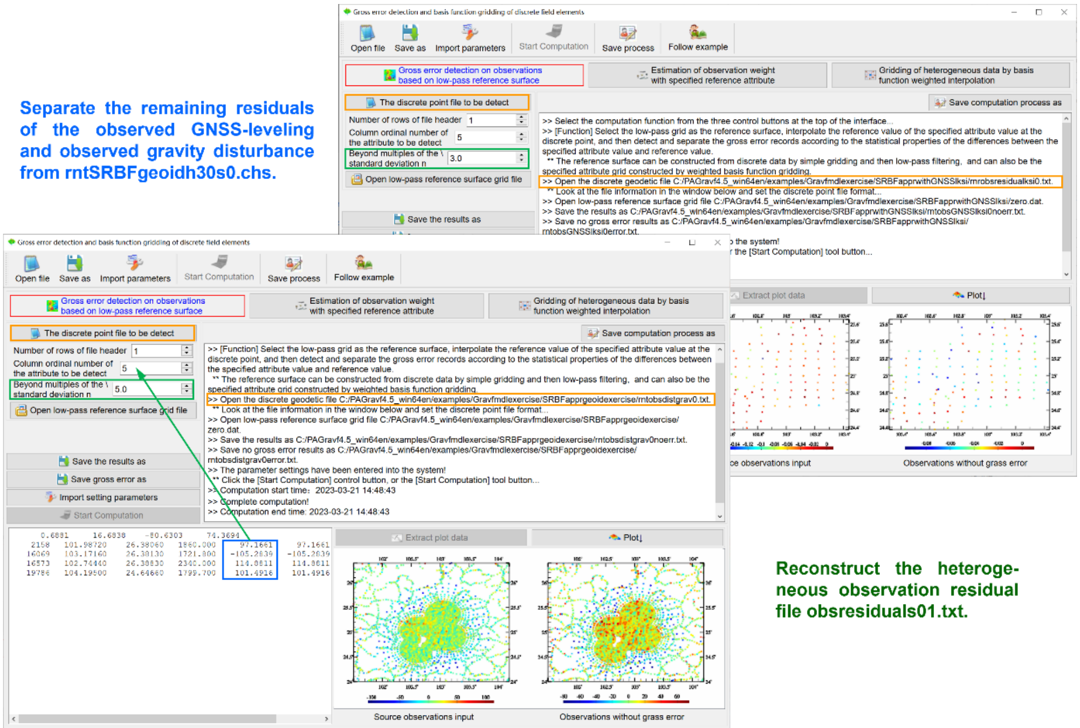


Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbances from the remaining residual file SRBFsurfhgt30s0.chs, detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresiduals01.txt.

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresiduals01.txt and repeat the step (2) to re-estimate the residual gravity field grid rntSRBFdatum30s.xxx on terrain surface and get the new remaining residual file rntSRBFdatum30s.chs.

Since the contribution rate of GNSS-levelling $\kappa = 0$ is set in advance, it is essentially here directly to measure the external accuracy index of the observed GNSS levelling only using the observed gravity disturbances.

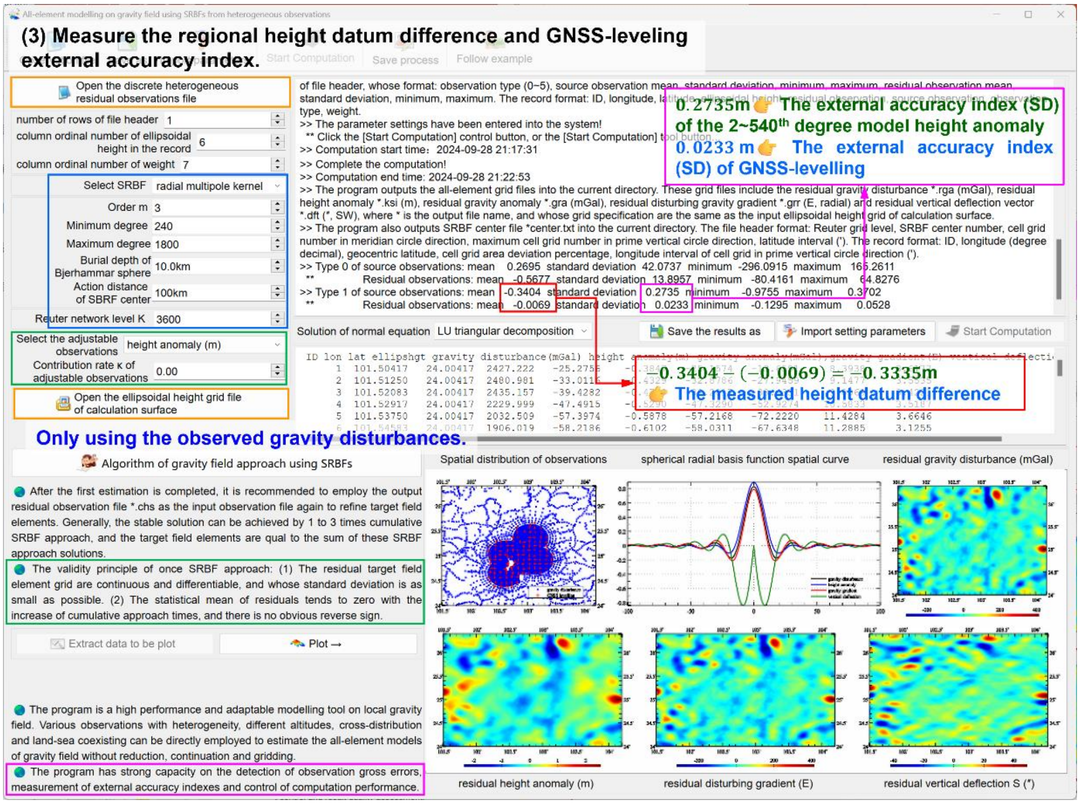


Before and after gross error removed, the statistical results on the observation residuals are as follows.

		number of points	mean	standard deviation	minimum	maximum
Gravity disturbance (mGal)	Original residuals	4219	0.3186	42.1772	-296.0915	165.2611
	Residuals without error	4215	0.2695	42.0737	-296.0915	165.2611
	Remaining residuals	4215	-0.5677	13.8957	-80.4161	64.8276
GNSS levelling height anomaly (m)	Original residuals	125	-0.3452	0.2739	-0.9755	0.3702
	Residuals without error	123	-0.3404①	0.2735	-0.9755	0.3702
	Remaining residuals	123	-0.0069②	0.0233③	-0.1295	0.0528

The statistical mean ① minus ② of the GNSS-levelling remaining residuals in the table, that is, -0.3404① - (-0.0069②) = -0.3335m, is the difference between the regional height datum and the global height datum (gravimetric geoid). Here provides the SRBF measurement method for regional height datum difference.

In the table, 0.0233③m is the external accuracy index of the observed GNSS-levelling expressed as standard deviation, that is, 2.33cm. Here provides the SRBF measurement method for the external accuracy index of GNSS- leveling. The result indicates that the external accuracy of GNSS-leveling is not bad than 2.33 cm (SD).



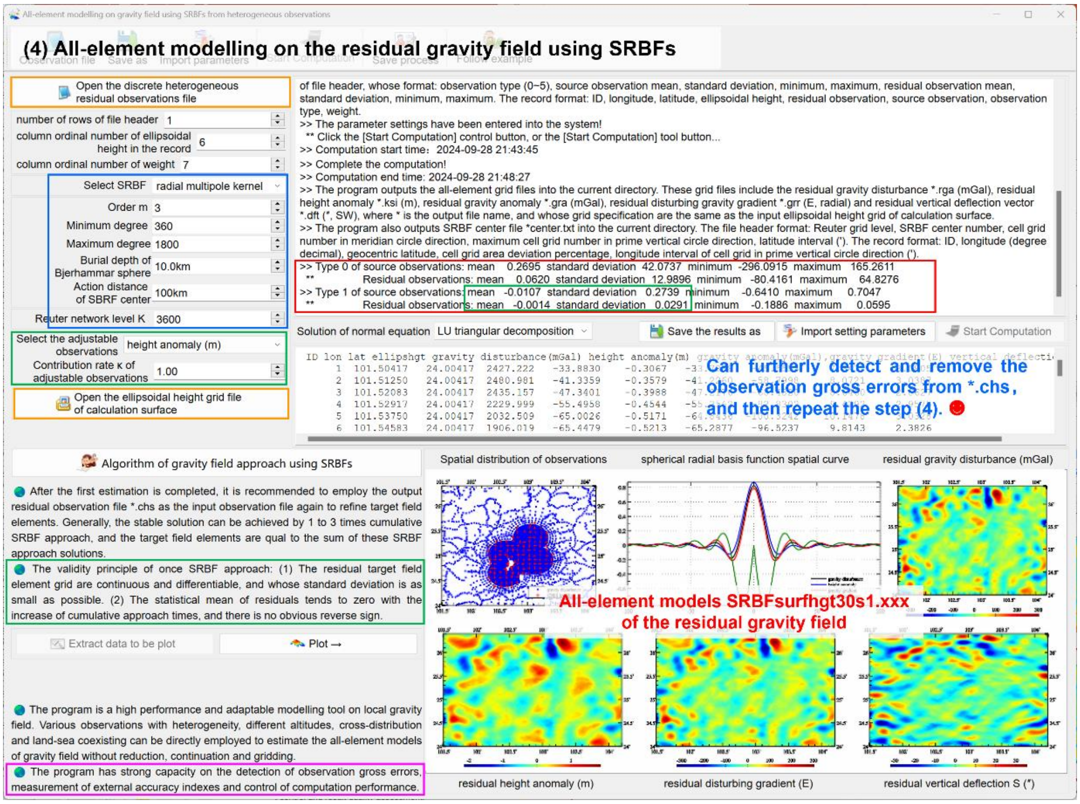
In general, it is necessary to make 1 to 2 cumulative SRBF approach with *.chs as the input file to obtain the minimum of standard deviation of GNSS-levelling remaining residuals as the external accuracy index, and this process is omitted in this example.

After removing the regional height datum difference of -0.3345m from GNSS-levelling residuals, the new heterogeneous observation residual file obsresiduals1.txt is reconstructed again.

(4) All-element modelling on the residual gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the heterogeneous residual file obsresiduals1.txt and terrain surface ellipsoidal height grid file surfhgt30s.dat to estimate the 30″ residual gravity field grid SRBFsurfhgt30s1.xxx on terrain surface, and get the remaining residual file SRBFsurfhgt30s1.chs.

[The quality control scheme] You can furtherly detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFsurfhgt30s1.chs, and then repeat the step (4). This process is omitted in this example.

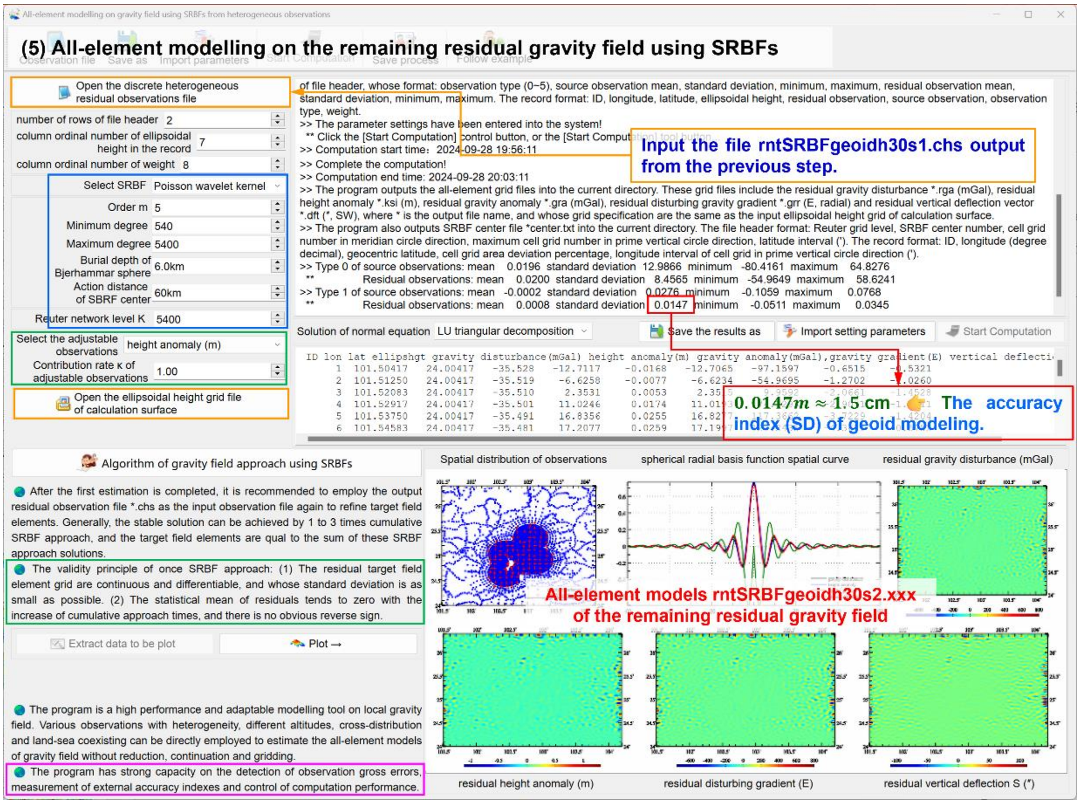


(5) All-element modelling on the remaining residual gravity field using SRBFs

Call the program [All-element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the remaining residual file SRBFsurfhgt30s1.chs and terrain surface ellipsoidal height grid file surfhgt30s.dat to estimate the 30″ remaining residual field grid SRBFsurfhgt30s2.xxx on the terrain surface, and get the remaining residual file SRBFsurfhgt30s2.chs.

In the table below, 0.0154④m = 1.5cm can be considered as the accuracy index of ground height anomaly (quasigeoid) modelling.

		mean	standard deviation	minimum	maximum
Residual gravity disturbance (mGal)	Residuals	0.2695	42.0737	-296.0915	165.2611
	First SRBF	0.0620	12.9866	-80.4161	64.8276
	Second SRBF	0.1309	8.5135	-50.6030	57.3920
Residual GNSS-levelling height anomaly (m)	Residuals	-0.0071	0.2768	-0.6571	0.6846
	First SRBF	-0.0014	0.0291	-0.1886	0.0595
	Second SRBF	-0.0013	0.0154④	-0.0708	0.0315



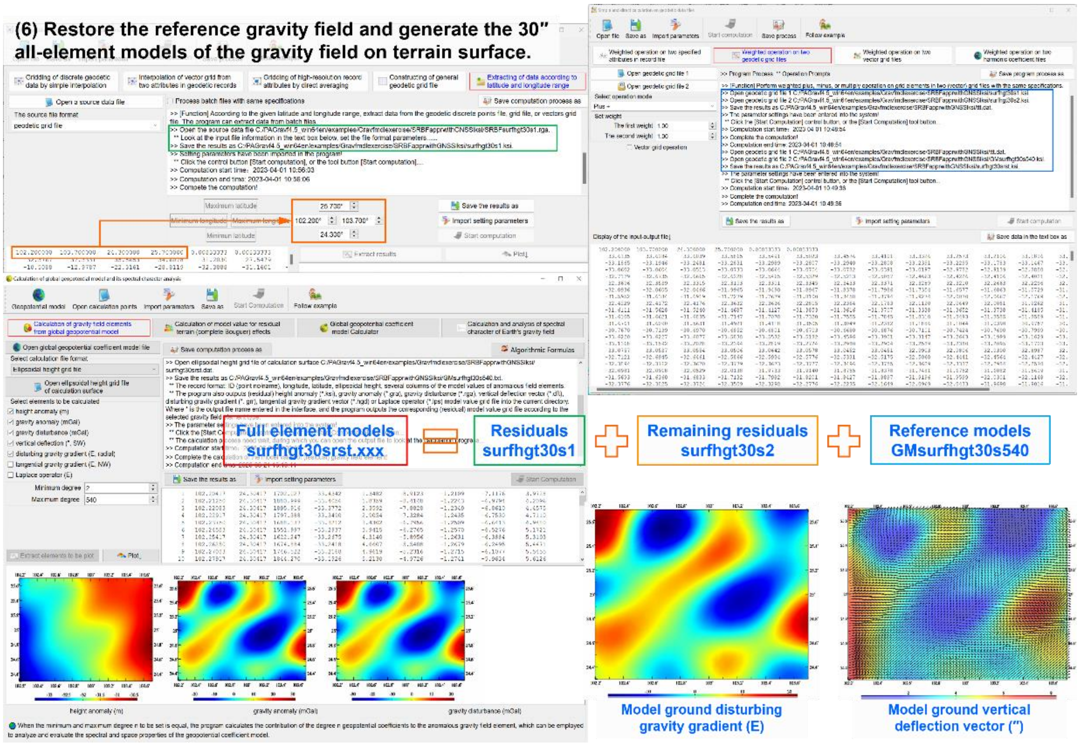
[The quality control scheme] You can furtherly detect and remove again the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFsurfhgt30s2.chs, and then repeat from step (4). This process is omitted in this example.

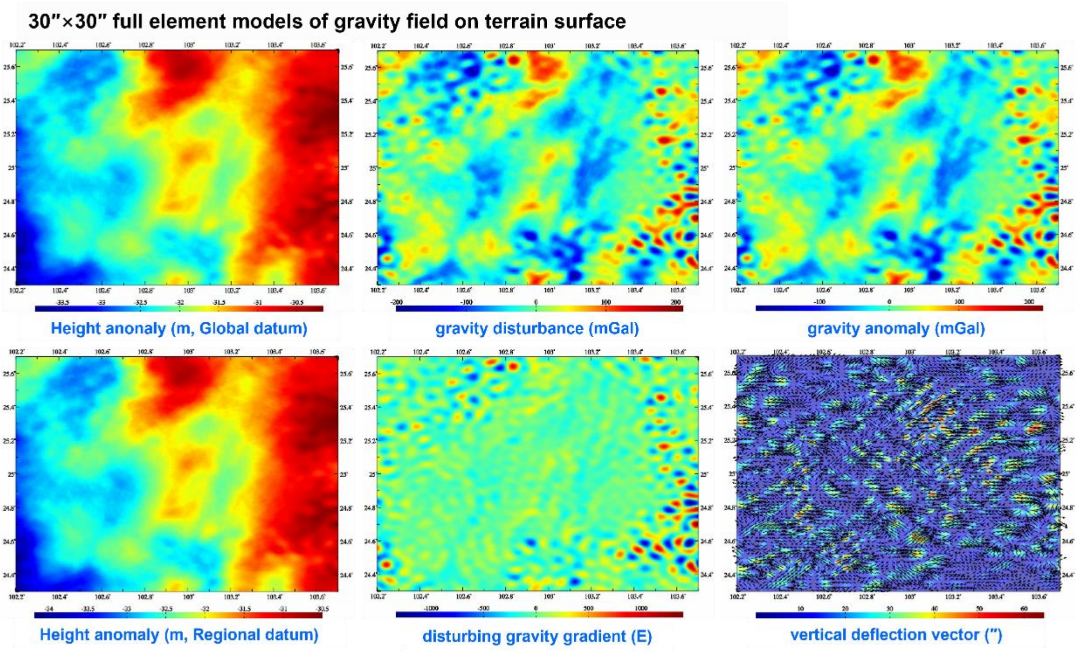
You can also do further cumulative SRBF approach to improve the results. This example omits this process.

(6) Restore the reference gravity field and generate the 30″ all-element models of the gravity field on the terrain surface.

Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, input the file EGM2008.gfc, and the terrain surface ellipsoidal height grid file surfhgt30srst.dat (from surfhgt30s.dat with grid edge removed), to calculate the all-element grid GMsurfhgt30s540.xxx of the reference gravity field on the terrain surface.

Add the residual gravity field grid surfhgt30s1.xxx (from SRBFsurfhgt30s0.xxx with grid edge removed) and remaining residual gravity field grid surfhgt30s2.xxx (from SRBFsurfhgt30s1.xxx with grid edge removed) to the reference gravity field grid GMsurfhgt30s540.xxx, the 30″ all-element gravity field models surfhgt30srst.xxx on the terrain surface are obtained, which include the 30″ gravimetric ground height anomaly grid (surfhgt30srst.ksi, m), ground gravity disturbance grid (surfhgt30srst.rga, mGal), ground gravity anomaly grid (surfhgt30srst.gra, mGal), ground disturbing gravity gradient grid (surfhgt30srst.grr, radial, E) and ground vertical deflection vector grid (surfhgt30srst.dft, SW, ″).





Add the regional height datum difference -0.3411m to the 30″ gravimetric height anomaly grid surfhgt30srst.ksi in global height datum, the 30″ gravimetric height anomaly grid surfhgt30srgn.ksi in regional height datum can be obtained.

So far, the all-element modelling on gravity field on the terrain surface have been completed.

○ **Let the geoid as the calculation surface, and directly generate the 30″ all-element models of the gravity field on the geoid.**

In step (3) to step (6) above, the input data file and all the parameter settings are kept same, and only the calculation surface is changed to the geoid. Using the same process, you can synchronously obtain the 30″ all-element models geoidh30srst.xxx of the gravity field on the geoid, which include the 30″ gravimetric geoidal height grid (geoidh30srst.ksi, m, in global height datum), gravity disturbance grid (geoidh30srst.rga, mGal), gravity anomaly grid (surfhgt30srst.gra, mGal), disturbing gravity gradient grid (geoidh30srst.grr, radial, E), vertical deflection vector grid (geoidh30srst.dft, SW, ″) and geoidal height grid (geoidh30srgn.ksi, m) in regional height datum.