Solid tidal effects on various geodetic variations outside solid Earth

The Earth's tidal generating potential (TGP) from celestial body outside the Earth directly cause the variation of geopotential on the ground and outside the Earth, and induce the deformation of the solid Earth, resulting in the redistribution of mass inside the Earth and generation of the associated geopotential. The former is the direct influence of the TGP, and the latter is the indirect influence of TGP. The sum of the two is the solid Earth tidal effect on the geopotential on the ground and outside the Earth, referred to as the body tidal effect.

8.1.1 The unified expression of body tidal effects on geodetic variations outside solid Earth

(1) The Earth's tidal generating potentials from celestial bodies

The tidal generating potential (TGP, or tidal potential) of celestial body outside the Earth can be expressed by the variations of geopotential coefficients $(\Delta\bar{C}_{nm},\Delta\bar{S}_{nm})$ in the Earth-fixed coordinate system as

$$\Delta\bar{C}_{nm}-i\Delta\bar{S}_{nm}=\frac{1}{2n+1}\sum_{j=2}^{10}\frac{GM_j}{GM}\left(\frac{a}{r_j}\right)^{n+1}\bar{P}_{nm}(cos\theta_j)e^{im\lambda_j} \tag{1.1}$$

Where, $j=2\sim10$ represents the moon, sun, Mercury, Venus, Mars, Jupiter, Saturn, Uranus and Neptune, respectively. $\Delta\bar{C}_{nm}-i\Delta\bar{S}_{nm}$ are the variations of the normalize geopotential coefficients with degree-n order-m (with $\Delta\bar{S}_{n0}=0$), $\bar{P}_{nm}$ are the normalized associated Legendre functions, $n=2,3,4,5,6$ for the moon, $n=2,3$ for the sun and $n=2$ for the other celestial bodies. $GM_j,r_j,\varphi_j,\lambda_j$ are respectively the gravitational parameter, distance from geocenter, geocentric latitude, and longitude (from Greenwich) of the celestial body j. GM,a are the geocentric gravitational constant and equatorial radius of the Earth, respectively.

At any epoch time t, the spherical coordinates (r_j,θ_j,λ_j) of the celestial body j in the Earth-fixed coordinate system can be calculated by the SOFA routines using the JPL Planetary Ephemeris DE440.

(2) The representation of solid Earth tidal effects on all-element geodetic variations outside solid Earth

According to the theory of spherical harmonic expansion of gravitational potential, degree-n tidal potential $\Delta V_n(r,\theta,\lambda)$ and degree-n tidal force $\Delta g_n(r,\theta,\lambda)$ at any point (r,φ,λ) on the ground or outside the solid Earth can be expressed by degree-n geopotential coefficient variations $(\Delta\bar{C}_{nm},\Delta\bar{S}_{nm})$ as follows:

$$\begin{aligned}\Delta V_n(r,\theta,\lambda)&=\frac{GM}{r}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda+\Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta)\\ \Delta g_n(r,\theta,\lambda)&=\frac{GM}{r^2}(n+1)\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda+\Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta)\end{aligned} \tag{1.2}$$

For the spherically symmetric non-rotating elastic Earth such as PREM Earth, the potential Love number k_n and displacement Love number (h_n,l_n) are real numbers. When

$n \geq 4,\ k_n = h_n = l_n = 0.$

According to the solid Earth tide theory and spherical harmonic synthesis algorithm, the body tidal effect on geopotential $\Delta V(r,\varphi,\lambda)$ is equal to the sum of the tidal potential $\Delta V_n(r,\theta,\lambda)$ and associated geopotential $\Phi_n^a(r,\theta,\lambda) = k_n \Delta V_n(r,\theta,\lambda)$ of each degree.

$$\Delta V(r,\theta,\lambda) = \sum_{n=2}^{6}[\Delta V_n(r,\theta,\lambda) + \Phi_n^a(r,\theta,\lambda)] = \sum_{n=2}^{6}(1+k_n)\Delta V_n(r,\theta,\lambda)$$

$$= \frac{GM}{r}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.3)$$

Similarly, the body tidal effect expression on height anomaly on the ground or outside the solid Earth can be obtained as follows:

$$\Delta\zeta(r,\theta,\lambda) = \frac{GM}{\gamma r}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)$$

$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.4)$$

The body tidal effect expression on gravity (disturbance) outside the solid Earth as:

$$\Delta g^{\delta}(r,\theta,\lambda) = \frac{GM}{r^2}\sum_{n=2}^{6}(n+1)\left(\frac{a}{r}\right)^n (1+k_n)$$

$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.5)$$

The body tidal effect expression on vertical deflection outside the solid Earth as:

South: $\Delta\xi(r,\theta,\lambda) = \frac{GM}{\gamma r^2}\sin\theta\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)$

$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{nm}(cos\theta) \quad (1.6)$$

West: $\Delta\eta(r,\theta,\lambda) = \frac{GM}{\gamma r^2 \sin\theta}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)$

$$\sum_{m=1}^{n} m(\Delta\bar{C}_{nm}sinm\lambda - \Delta\bar{S}_{nm}cosm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.7)$$

The body tidal effect expression on radial gravity gradient as:

$$\Delta T_{rr}(r,\theta,\lambda) = \frac{GM}{r^3}\sum_{n=2}^{6}(n+1)(n+2)\left(\frac{a}{r}\right)^n (1+k_n)$$

$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.8)$$

The body tidal effect expression on horizontal gravity gradient as:

North: $\Delta T_{NN}(r,\theta,\lambda) = -\frac{GM}{r^3}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)$

$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm}cosm\lambda + \Delta\bar{S}_{nm}sinm\lambda)\frac{\partial^2}{\partial\theta^2}\bar{P}_{nm}(cos\theta) \quad (1.9)$$

West: $\Delta T_{WW}(r,\theta,\lambda) = \frac{GM}{r^3 cos^2\varphi}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n (1+k_n)$

$$\sum_{m=1}^{n} m^2\,(\Delta\bar{C}_{nm}sinm\lambda + \Delta\bar{S}_{nm}cosm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.10)$$

According to whether the contribution of solid Earth tidal deformation by the displacement Love number is directly included, the body tidal effects on geodetic variations are divided into two categories. One class of the body tidal effect on geodetic variations does not directly include the contribution of displacement Love number (h_n, l_n), such as the geopotential, gravity (disturbance), vertical deviation and gravity gradient outside Earth. Another class of geodetic variation whose site is fixed with the solid Earth, on which the body tidal effect includes the contribution of displacement Love number (h_n, l_n), such as the ground gravity, site displacement and ground tilt.

The body tidal effect expression on ground displacement whose site is fixed with the solid Earth◉

East: $\Delta e(r,\theta,\lambda) = -\frac{GM}{\gamma r \sin\theta}\sum_{n=2}^{3}\left(\frac{a}{r}\right)^n l_n$

$$\sum_{m=1}^{n} m(\Delta\bar{C}_{nm} sinm\lambda - \Delta\bar{S}_{nm} cosm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.11)$$

North: $\Delta n(r,\theta,\lambda) = -\frac{GM}{\gamma r}\sin\theta\sum_{n=2}^{3}\left(\frac{a}{r}\right)^n l_n$

$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm} cosm\lambda + \Delta\bar{S}_{nm} sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{nm}(cos\theta) \quad (1.12)$$

Radial: $\Delta r(r,\theta,\lambda) = \frac{GM}{\gamma r}\sum_{n=2}^{3}\left(\frac{a}{r}\right)^n h_n$

$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm} cosm\lambda + \Delta\bar{S}_{nm} sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.13)$$

The body tidal effect expression on ground gravity whose site is fixed with the solid Earth◉

$$\Delta g^s(r,\varphi,\lambda) = \frac{GM}{r^2}\sum_{n=2}^{6}(n+1)\left(\frac{a}{r}\right)^n\left(1+\frac{2}{n}h_n-\frac{n+1}{n}k_n\right)$$

$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm} cosm\lambda + \Delta\bar{S}_{nm} sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.14)$$

The body tidal effect expression on ground tilt whose site is fixed with the solid Earth◉

South: $\Delta\xi^s(r,\theta,\lambda) = \frac{GM}{\gamma r^2}\sin\theta\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n(1+k_n-h_n)$

$$\sum_{m=0}^{n}(\Delta\bar{C}_{nm} cosm\lambda + \Delta\bar{S}_{nm} sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{nm}(cos\theta) \quad (1.15)$$

West: $\Delta\eta^s(r,\theta,\lambda) = \frac{GM}{\gamma r^2 \sin\theta}\sum_{n=2}^{6}\left(\frac{a}{r}\right)^n(1+k_n-h_n)$

$$\sum_{m=1}^{n} m(\Delta\bar{C}_{nm} sinm\lambda - \Delta\bar{S}_{nm} cosm\lambda)\bar{P}_{nm}(cos\theta) \quad (1.16)$$

In the above expressions, the body tidal effects on the geodetic variations marked ◉ are valid only when their sites are fixed with the solid Earth, and that on the remaining geodetic variations are suitable on the ground or outside the solid Earth.

In order to uniformly represent the effects of ocean tide, atmosphere tide, external celestial bodies and non-tidal loads on various geodetic variations on the ground and outside the solid Earth, ETideLoad4.5 looks the sum of the direct and indirect influences of the tidal generating geopoential as the solid tidal effect, rather than only the indirect influence of the tidal generating geopoential as the body tidal effect in some literatures.

(3) Earth's tidal potentials and tidal forces from celestial bodies and their time-varying analysis

Using the solar system ephemeris, degree-n Earth's tidal potential (tidal force) from any celestial object in the solar system can be calculated by (1.1) and (1.2). Then, according to the accuracy requirements of geodesy, the celestial objects and their tidal potential expansion degree-n can be determined, and the periodicity, magnitude and time-varying characteristics of the different degree of tidal potentials can be calculated and investigated.

The Earth's tidal potential and tidal force from celestial body are related to the position of the calculation point in the Earth-fixed coordinate system. Here, the ground point P (105°N, 20°E, H100m) selected, the 2nd to 6th degree Earth's tidal potential ΔV_n (in unit 10^{-5}m²/s²) and tidal force Δg_n (in unit nGal = 10^{-11}m/s²) from 10 external celestial bodies in the solar system are calculated using the formula (1.2). The difference between the maximum and minimum values of the tidal potential (force) of each degree from 10 celestial bodies is calculated as Tab 1.1. 0.0000 in the table indicates that the calculation results are close to zero, and the blank indicates that the value is too small to calculate.

It can be seen from Tab 1.1 that when the tidal potential (force) are calculated, if the cutoff threshold is 10^{-8}m²/s² (11^{-14}m/s²), the moon needs to be expanded to 6 degrees, the sun needs to be expanded to 3 degrees, Venus, Jupiter, Mars, Mercury and Saturn only need to be calculated for the 2nd degree, and Uranus, Neptune and Pluto do not need to be calculated.

Tab 1.1 The difference between the maximum and minimum values of the tidal potential (force) of celestial bodes

	ΔV_2	$\dot{g}_2$	ΔV_3	$\dot{g}_3$	ΔV_4	$\dot{g}_4$
Moon	247660.1100	116532.1527	6176.8512	2906.4098	174.7919	124.7522
Sun	92514.4904	43531.0825	5.6041	2.6369	0.0004	0.0003
Venus	10.8438	5.1023	0.0014	0.0007	0.0000	0.0000
Jupiter	1.4120	0.6644	0.0000	0.0000		
Mars	0.4041	0.1901	0.0000	0.0000		
Mercury	0.0815	0.0383	0.0000	0.0000		
Saturn	0.0383	0.0164	0.0000	0.0000		
Uranus	0.000566	0.000266				
Neptune	0.000194	0.000091				

Pluto	0.00000002	0.00000001				
	ΔV_5	$\dot{g}_5$	ΔV_6	$\dot{g}_6$	ΔV_7	$\dot{g}_7$
Moon	3.0696	2.7402	0.0531	0.0567	0.0000	0.0000

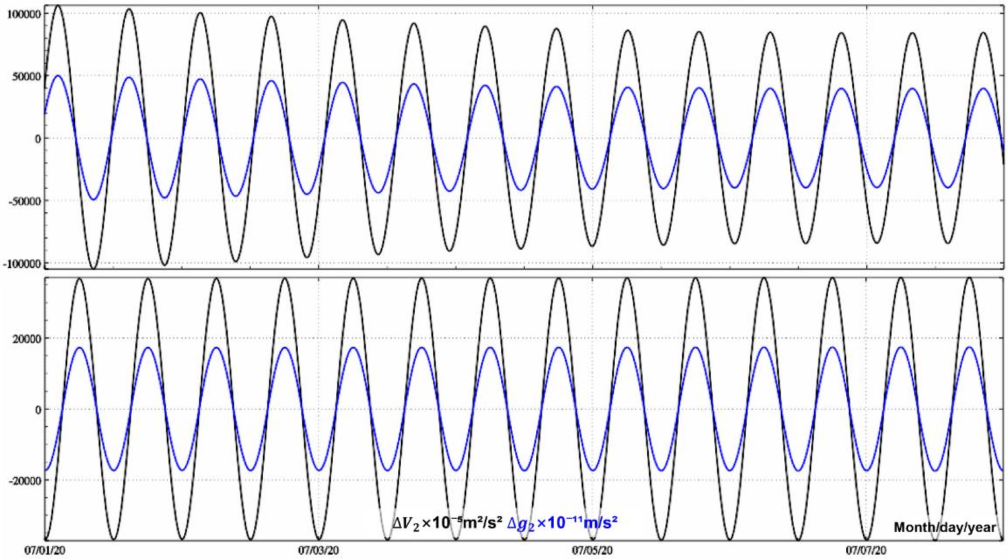

Fig 1.1 Degree-2 Earth's tidal potential (force) time series from Moon and Sun

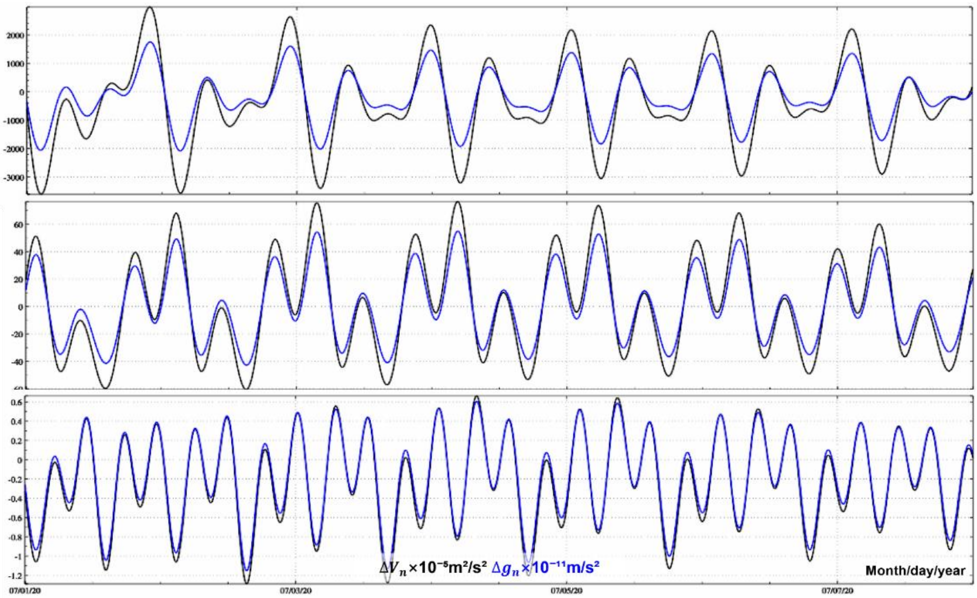

Fig 1.2 Degree 3, 4 and 5 Earth's tidal potential (force) time series from Moon (7 days)

Fig 1.1 shows degree-2 Earth's tidal potential (force) time series from Moon and Sun, and Fig 1.2 shows degree 3, 4 and 5 Earth's tidal potential (force) time series from Moon. The time span is from 0 : 00 on July 1, 2020 to 24 : 00 on July 7, 2022 (7 days), with a time interval of 10 minutes.

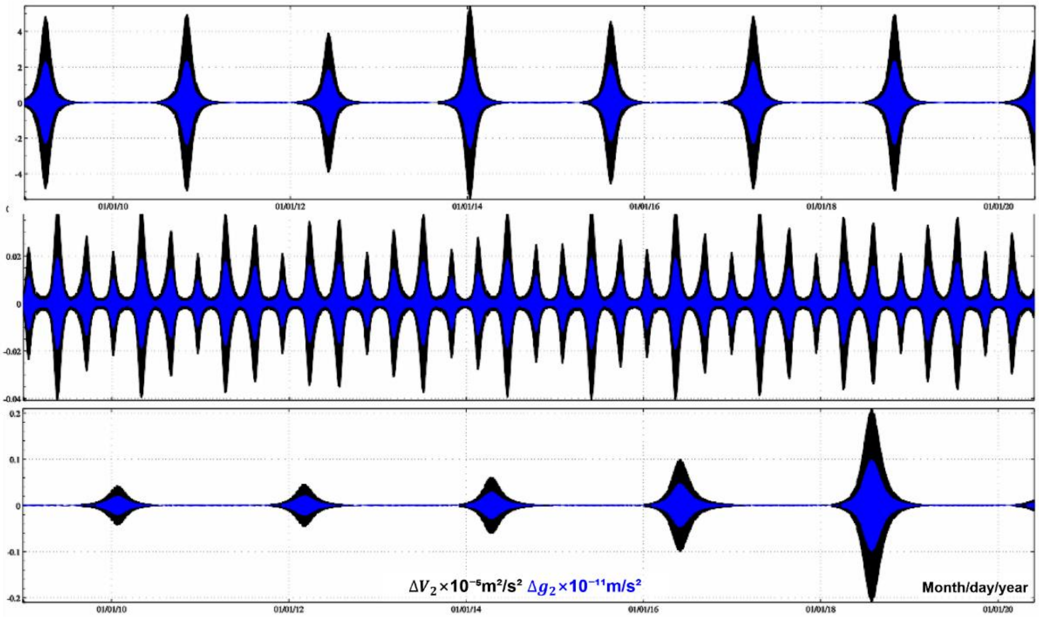

Fig 1.3 Degree-2 Earth’s tidal potential (force) time series from Venus, Jupiter and Mars (12 years)

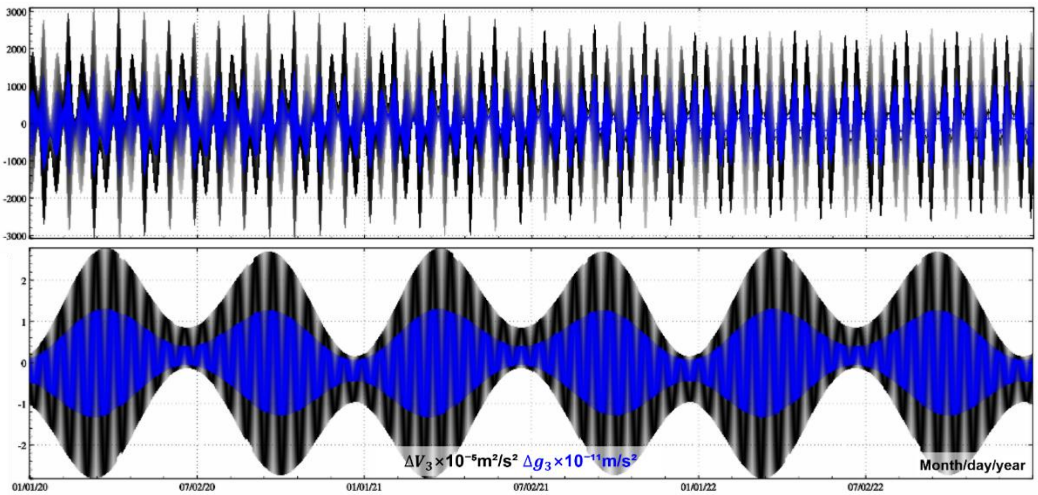

Fig 1.4 Degree 3 Earth’s tidal potential (force) time series from Moon and Sun

Fig 1.3 shows degree-2 Earth’s tidal potential (force) time series from Venus, Jupiter and Mars from January 1, 2010 to December 31, 2022 (12 years), with a time interval of 2 hours. Fig 1.4 shows degree-3 Earth tidal potential (force) time series from Moon and Sun from January 1, 2020 to December 31, 2022 (2 years).

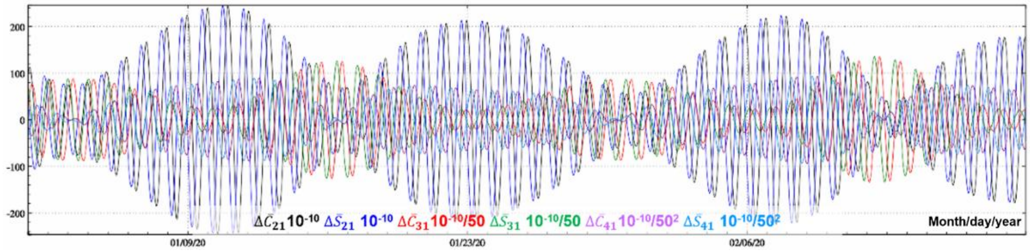

Fig 1.5 The tesseral TGP geopotential coefficient time series (diurnal variation)

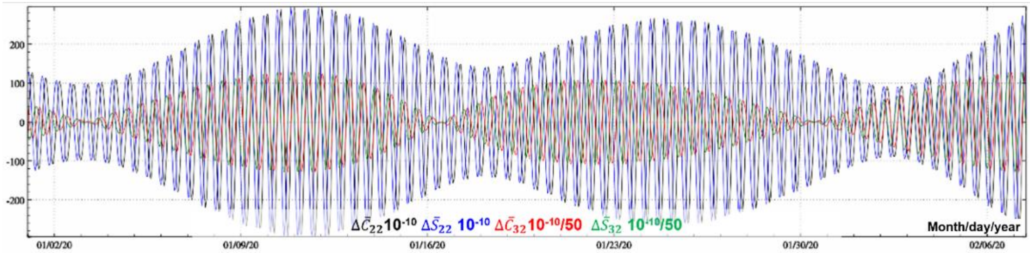

Fig 1.6 The sector TGP geopotential coefficient time series (semi-diurnal variation)

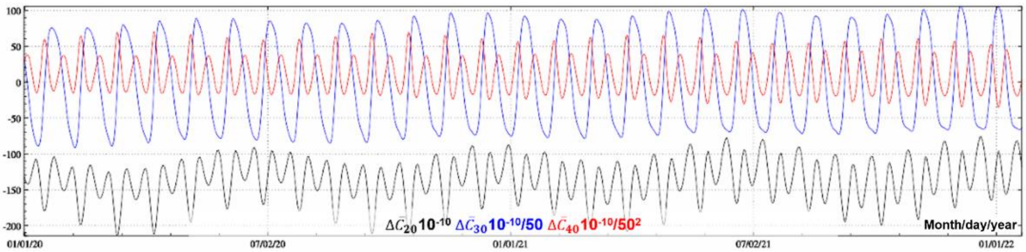

Fig 1.7 The zonal TGP geopotential coefficient time series (long-period variation)

Fig 1.5 ~ Fig 1.7 shows the geopotential coefficient time series of the direct influence of TGP calculated from the formula (1.1). From these three figures, it is not difficult to see that the TGP tesseral harmonic geopotential coefficient (m = 1) time series mainly shows the diurnal variation, the TGP sector harmonic geopotential coefficient (m = 2) time series mainly shows semi-diurnal variation, and the TGP zonal harmonic geopotential coefficient (m = 0) time series mainly shows long-period variation.

8.1.2 The body tidal Love number for the rotating microellipsoidal anelastic Earth

The body tidal effect on geodetic variations are characterized by the linear combination of the body tidal Love numbers, that is, the body tidal factor $\delta = \mathcal{L}(k, h, l)$. There are three kinds of body tidal Love numbers, namely, the potential Love number k, radial (displacement) Love number h and horizontal (displacement) Love number l.

(1) The body tidal Love number for non-spherical rotating Earth

The non-spherical ellipticity of the elastic Earth and the rotation of the Earth make the expression of the Love number complicated. The tidal potential excites the deformation of the solid Earth, and the displacement deformation solution of the ground site excited by the degree-n order-m tidal potential is:

$$\boldsymbol{u} = \frac{W_{mn}}{g_0}\Big[\boldsymbol{e}_r\left(h^0 Y_{nm} + h^+ Y_{n+2,m} + h^- Y_{n-2,m}\right) + \boldsymbol{e}_\theta$$

$$\left(l^0 \frac{\partial Y_{nm}}{\partial\theta} + \omega^+ \frac{m}{sin\theta} Y_{n+1,m} + \omega^- \frac{m}{sin\theta} Y_{n-1,m} + l^+ \frac{\partial Y_{n+2,m}}{\partial\theta} + l^- \frac{\partial Y_{n-2,m}}{\partial\theta}\right) + i\boldsymbol{e}_\lambda$$

$$\left(l^0 \frac{m}{sin\theta}\frac{\partial Y_{nm}}{\partial\theta} + \omega^+ \frac{m}{sin\theta}\frac{\partial Y_{n+1,m}}{\partial\theta} + \omega^- \frac{m}{sin\theta}\frac{\partial Y_{n-1,m}}{\partial\theta} + l^+ \frac{mY_{n+2,m}}{sin\theta} + l^- \frac{mY_{n-2,m}}{sin\theta}\right)\Big] \quad (1.17)$$

Where, $W_{mn}Y_{nm} = W_{mn}(a)Y_{nm}(\theta,\lambda)$ is the degree-n order-m tidal potential, $g_0 = g_0(a)$ is the ground mean gravity, $W_{mn}Y_{nm}/g_0$ is the degree-n order-m equilibrium tidal height. h^0, l^0 are the spherically symmetric parts of displacement Love number, h^+, l^+, h^-, l^- are the spherical coupling parts of corresponding Love numbers, and ω^+, ω^- are the annular coupling parts of corresponding Love numbers.

Formula (1.17) indicates that for the spherically symmetric non-rotating elastic Earth, only two parameters (h and l) are needed to represent the tidal displacement field. But for the rotating microellipsoidal Earth, 8 parameters are needed. Because of the ellipticity and rotation, degree-n spherical displacement field (2 parameters) is coupled into the degree n+1, n-1 annular displacement field (2 parameters) and the degree n+2, n-2 spherical displacement field (4 parameters), forming the latitude dependent parts of the displacement Love numbers.

The tidal potential excites the deformation of the Earth's surface and internal medium, resulting in the redistribution of the Earth's internal density and the generation of associated geopotential. The associated geopotential at (r,θ,λ) can be expressed as:

$$\Phi^a(r,\theta,\lambda) = W_{mn}\left[k^0\left(\frac{a}{r}\right)^{n+1} Y_{nm} + k^+\left(\frac{a}{r}\right)^{n+3} Y_{n+2,m} + k^-\left(\frac{a}{r}\right)^{n-1} Y_{n-2,m}\right] \quad (1.18)$$

Similar to the displacement deformation solution, k^0 represents the spherically symmetric part of potential Love number, and k^+ and k^- represent the spherically coupling parts of corresponding Love number. Since the annular displacement does not involve volume expansion, it will not lead to the variation of geopotential. Therefore, there will be no annular coupling term in the potential Love number k.

The body tidal Love numbers for the microelliptical elastic Earth are still real numbers, which are independent of frequency. Whose value are shown in Tab 1.2.

Tab 1.2 The body tidal Love numbers for the microelliptical elastic Earth

n	m	periods of tidal constituents	k_{nm}	h_{nm}	l_{nm}
2	0	long period	0.29525	0.6078	0.0847

2	1	diurnal	0.29470	0.6078	0.0847
2	2	semi-diurnal	0.29801	0.6078	0.0847
3	0	long period	0.093	0.2920	0.0150
3	1	diurnal	0.093	0.2920	0.0150
3	2	semi-diurnal	0.093	0.2920	0.0150
3	3	1/3-diurnal	0.094	0.2920	0.0150
			0.041	0.175	0.010
			0.025	0.129	
			0.017	0.197	

(2) The latitude dependence of displacement Love numbers for the rotating microellipsoidal Earth

The Earth's ellipticity and rotation destroy the symmetry of the tidal response. The centrifugal force from the microellipsoidal Earth's rotation leads to the latitude dependence of displacement Love number (h_{nm}, l_{nm}). The latitude dependence formulas of degree-2 displacement Love numbers are:

$$\begin{cases} h_{2m}(\varphi) = h_{2m} + h^{\varphi}\frac{3sin^2\varphi-1}{2} \\ l_{2m}(\varphi) = l_{2m} + l^{\varphi}\frac{3sin^2\varphi-1}{2} \end{cases} \tag{1.19}$$

Where, $\varphi = \pi/2 - \theta$ is the geocentric latitude of the ground site, (h_{2m}, l_{2m}) are the displacement Love numbers for the microelliptic non-rotating elastic Earth with these value shown in Table 1.2, and $[h_{2m}(\varphi), l_{2m}(\varphi)]$ are the displacement Love numbers considering the latitude dependence of the site, $(h^{\varphi}, l^{\varphi})$ are the latitude dependence coefficients of displacement Love numbers. In the IERS conventions (2010), $h^{\varphi} = -0.0006, l^{\varphi} = 0.0002$.

The latitude dependence of degree-3 displacement Love numbers are very weak and do not need to be corrected.

(3) The frequency dependence of body tidal Love numbers from the mantle anelasticity

The mantle anelasticity leads to the delay of the deformation response to the tidal potential, which makes the Love numbers (including the spherical symmetry parts and the latitude dependence factors) of each tidal cluster (semi-diurnal $m = 2$, diurnal $m = 1$ and long-period $m = 0$ tidal cluster) become complex number with a relatively small imaginary part whose absolute value is less than 1 % of the real part. On the other hand, the mantle anelasticity also amplifies the tidal deformation, resulting in the frequency dependence of Love number increasing with the increase of the tidal period, so that the real and imaginary parts of Love numbers change slightly at the same time.

Long-period tidal waves ($m = 0$, also known as zonal harmonic tidal waves, with a period of 8 days to 18.6 years) have a low frequency and a large time span. The anelasticity of the mantle leads to a strong frequency dependence of Love numbers for long-period tidal waves. For semi-diurnal tidal waves $(m = 2)$, the radial Love number h increases by 1.4%. For the diurnal tidal waves $(m = 1)$, h increases by 1.4% and the potential Love number k increases by 1.7%. For long-period tidal waves ($m = 0$, such as M_f), the anelasticity of the mantle leads to a 2.5% increase in h and a 3% increase in k.

In order to represent degree-n order-m geopotential coefficient variations caused by the associated geopotential for the anelastic Earth, three forms of potential Love numbers $\left(k_{nm}^{(0)}, k_{nm}^{(\pm)}\right)$, radial Love numbers $\left(h_{nm}^{(0)}, h_{nm}^{(\pm)}\right)$ and horizontal Love numbers $\left(l_{nm}^{(0)}, l_{nm}^{(\pm)}\right)$ are needed to characterize the indirect influence of the degree-n order-m $(n \geq 2, m \leq n)$ tidal potential. Considering the mass conservation of the Earth, when $n = 2$, $k_{2m}^{(-)} = 0, h_{2m}^{(-)} = 0, l_{2m}^{(-)} = 0$, there are only two kinds of potential Love numbers $\left(k_{2m}^{(0)}, k_{2m}^{(+)}\right)$, two kinds of radial Love numbers $\left(h_{2m}^{(0)}, h_{2m}^{(+)}\right)$ and two kinds of horizontal Love numbers $\left(l_{2m}^{(0)}, l_{2m}^{(+)}\right)$.

The anelasticity of the mantle causes the Earth's response to tidal potential to be delayed, so that the Love numbers change with frequency, $\left(k_{2m}^{(0)}, k_{2m}^{(+)}\right)$, $\left(h_{2m}^{(0)}, h_{2m}^{(+)}\right)$ and $\left(l_{2m}^{(0)}, l_{2m}^{(+)}\right)$ have a small imaginary part. The frequency dependence of degree-3 Love numbers are very weak, and their delay effects to tidal potential can be ignored.

For the convenience of compulation, degree-2 long-period $(m = 0)$, diurnal $(m = 1)$ and semi-diurnal $(m = 2)$ tidal clusters for the anelastic Earth in Table 1.3 are denoted as $k_{2m} = Re(k_{2m}) + i\,Im(k_{2m})$ as the nominal potential Love numbers. The displacement Love numbers $h_{2m} = 0.6078$, $l_{2m} = 0.0847$ for the microelliptical elastic Earth in Table 1.2 are taken as the nominal displacement Love numbers.

Tab 1.3 Frequency dependence of potential Love number k for anelastic Earth

nm		periods of tidal constituents	anelastic Earth		anelastic Earth		
n	m		k_{nm}	$k_{2m}^{(+)}$	$Re(k_{nm})$	$Im(k_{nm})$	$k_{2m}^{(+)}$
2	0	long period	0.29525	-0.00087	0.30190	-0.00000	-0.00089
2	1	diurnal	0.29470	-0.00079	0.29830	-0.00144	-0.00080
2	2	semi-diurnal	0.29801	-0.00057	0.30102	-0.00130	-0.00057
3	0	long period	0.093	···			
3	1	diurnal	0.093	···			
3	2	semi-diurnal	0.093	···			
3	3	1/3-diurnal	0.094	···			

8.1.3 Frequency dependence of degree-2 body tidal Love numbers and their corrections

(1) The Earth's near-diurnal wobble and resonance parametersof Love numbers

The excitation of the Earth's near-diurnal free wobble leads to obvious resonance amplification in the observation of diurnal tides (such as P_1、K_1、ψ_1 and ϕ_1 tidal waves) and corresponding free nutation, which are close to the eigen-frequency. The change of centrifugal force from the Earth's wobble excites the deformation of solid Earth, which leads to the coupling change of the Earth's moment of inertia. The contribution to the diurnal tidal Love numbers are proportional to the wobble response of the Earth and its nucleus.

For diurnal tides, the frequency dependent values of any load or body tide Love number L (such as $k_{21}^{(0)}$ or $k_{21}^{(+)}$) may be represented as a function of the tidal excitation frequency σ by a resonance formula

$$L(\sigma) = L_0 + \sum_{\alpha=1}^{3} \frac{L_\alpha}{\sigma-\sigma_\alpha} \tag{1.20}$$

Where, $\sigma_\alpha(\alpha = 1,2,3)$ are the respective resonance frequencies associated with the Chandler wobble (CW), the retrograde FCN and the prograde free core nutation (also known as the free inner core nutation), and the L_α are the corresponding resonance coefficients. All the parameters are complex. The σ_α and σ are expressed in cycles per sidereal day (cpsd), with the convention that positive (negative) frequencies represent retrograde (prograde) waves. (This sign convention, followed in tidal theory, is the opposite of that employed in analytical theories of nutation.) In particular, with the tidal frequency ω given in degrees per hour (°/hr), we have

$$\omega = 15\kappa\sigma, \quad \kappa = 1.002737909 \tag{1.21}$$

Here, the factor $\kappa = 1.002737909$ being the number of sidereal days per solar day. The values employed herein for the σ_α are from Mathews et al. (2002), adapted to the sign convention employed here:

$$\begin{aligned} \sigma_1 &= -0.0026010 - 0.0001361i \\ \sigma_2 &= \ \ 1.0023181 + 0.000025\ i \\ \sigma_3 &= \ \ 0.9999026 + 0.000780\ i \end{aligned} \tag{1.22}$$

Tab 1.4 shows the resonance parameters for the diurnal body tidal Love numbers $\left(k_{21}^{(0)}, k_{21}^{(+)}\right)$ calculated by the resonance formula (1.20).

Tab 1.4 Resonance parameters for diurnal Love numbers $\left(\boldsymbol{k_{21}^{(0)}}, \boldsymbol{k_{21}^{(+)}}\right)$

α	$k_{21}^{(0)}$		$k_{21}^{(+)}$	
	Re(L_α)	Im(L_α)	Re(L_α)	Im(L_α)
0	0.29954	-0.1412×10^{-2}	-0.804×10^{-3}	0.237×10^{-5}
1	-0.77896×10^{-3}	-0.3711×10^{-4}	0.209×10^{-5}	0.103×10^{-6}
2	0.90963×10^{-4}	-0.2963×10^{-5}	-0.182×10^{-6}	0.650×10^{-8}
3	-0.11416×10^{-5}	0.5325×10^{-7}	-0.713×10^{-9}	-0.330×10^{-9}

Tab 1.5 shows the resonance parameters for the diurnal load Love numbers

$(k'_{21}, h'_{21}, l'_{21})$ calculated by the resonance formula (1.20). These resonance parameters are related to the geopotential variations and Earth deformation induced by ocean tidal loads. Small imaginary parts are ignored in the Tab 1.5.

Tab 1.5 Resonance parameters for diurnal load Love numbers $(k'_{21}, h'_{21}, l'_{21})$

α	k'_{21}	h'_{21}	l'_{21}
0	-0.30808	-0.99500	0.02315
1	8.1874×10^{-4}	1.6583×10^{-3}	2.3232×10^{-4}
2	1.4116×10^{-4}	2.8018×10^{-4}	-8.4659×10^{-6}
3	3.4618×10^{-7}	5.5852×10^{-7}	1.0724×10^{-8}

The resonance effect of displacement Love numbers need to take into account the latitude dependence of the Love numbers and the resonance effect excited by the ocean tidal loads. Combining the formula (1.19) with (1.20), the resonance parameters of the displacement Love numbers and latitude dependent coefficients can be calculated, as shown in Tab 1.6.

Tab 1.6 Resonance parameters of the displacement Love numbers and latitude-dependent coefficients

α	$h_{2\mathrm{m}}$		h^{φ}	
	Re(L_α)	Im(L_α)	Re(L_α)	Im(L_α)
0	0.60671	-0.2420×10^{-2}	-0.615×10^{-3}	-0.122×10^{-4}
1	-0.15777×10^{-2}	-0.7630×10^{-4}	0.160×10^{-5}	0.116×10^{-6}
2	0.18053×10^{-3}	-0.6292×10^{-5}	0.201×10^{-6}	0.279×10^{-8}
3	-0.18616×10^{-5}	0.1379×10^{-6}	-0.329×10^{-7}	-0.217×10^{-8}
α	$l_{2\mathrm{m}}$		l^{φ}	
	Re(L_α)	Im(L_α)	Re(L_α)	Im(L_α)
0	0.84963×10^{-1}	-0.7395×10^{-3}	0.19334×10^{-3}	-0.3819×10^{-5}
1	-0.22107×10^{-3}	-0.9646×10^{-5}	-0.50331×10^{-6}	-0.1639×10^{-7}
2	-0.54710×10^{-5}	-0.2990×10^{-6}	-0.66460×10^{-8}	0.5076×10^{-9}
3	-0.29904×10^{-7}	-0.7717×10^{-8}	0.10372×10^{-7}	0.7511×10^{-9}

(2) The contributions to diurnal body tidal Love numbers of ocean tidal loads and their frequency dependent corrections

The diurnal resonance leads to the frequency dependence of load Love numbers, and causes the Earth's moment of inertia coupling change through the Earth's deformation (polar tide effect) caused by the centrifugal force of the tidal loads. The mantle anelasticity, the core-mantle coupling and the tidal friction dissipation mechanism lead to the frequency dependence of body tidal Love number for the diurnal tidal waves (mn = 21), which make the real and imaginary parts of body tidal Love numbers change slightly.

After considering the diurnal resonance effect, the diurnal load Love numbers need to be changed from (k_2', h_2', l_2') to $(k_{21}', h_{21}', l_{21}')$. The main contribution of the diurnal tide wave with a frequency of σ to the diurnal body tidal Love number of the same frequency is (Wahr and Sasao, 1981):

$$\delta\mathscr{k}_{21}^{ol}(\sigma) = [\mathscr{k}_{21}'(\sigma) - \mathscr{k}_2'] \frac{4\pi G\rho_w}{5g_0} RA_{21}(\sigma) \tag{1.23}$$

Where $\mathscr{k} = k,\ h$ or l. g_0 is the ground mean gravity, $\mathscr{k}_{21}'(\sigma)$ is the load Love number considering the diurnal resonance effect, which is a function of the tidal frequency σ, and is calculated according to Formula (1.20) using Tab 1.6. $\mathscr{k}_2' = k_2',\ h_2'$ or l_2' is degree-2 load Love number (real) of the spherically symmetric non-rotating elastic Earth. $A_{21}(\sigma)$ is the proportional factor (admittance) of the body or load Love number correction for the tide σ. Which can be calculated by the following formula after the harmonic analysis of the global ocean tide harmonic constant model:

$$A_{21}(\sigma) = H_{21}^{otide}(\sigma) / H_{21}^{TGP}(\sigma) \tag{1.24}$$

In the formula (1.24), $H_{21}^{TGP}(\sigma)$ is the global maximum amplitude (in unit of m) of the equilibrium tidal height of the celestial diurnal tide with the frequency of σ. $H_{21}^{otide}(\sigma)$ is the normalized harmonic amplitude (in unit of m) of the diurnal ocean tidal height with the same frequency of σ after normalized harmonic analysis.

The correction formula of frequency dependence for the diurnal body tidal Love numbers is as follows:

$$\delta\mathscr{k}_{21}^{\square}(\sigma) = \mathscr{k}_{21}^{(0)}(\sigma) + \delta\mathscr{k}_{21}^{ol}(\sigma) - \mathscr{k}_{21}^{\square} \tag{1.25}$$

Where $\mathscr{k} = k,\ h$ or l. $\mathscr{k}_{21}^{(0)}(\sigma)$ is the body tidal Love numbers (complex) considering the anelasticity of mantle and near-diurnal resonance, which is a function of the tidal frequency σ. Which can be calculated according to Formula (1.20) using the resonance parameters in Tab 1.5 or Tab 1.6. $\mathscr{k}_{21}^{\square}$ is the nominal diurnal Love number from Tab 1.3 or Tab 1.2.

The following is a brief summary of the three-step calculation scheme of the frequency dependent correction for diurnal tidal waves. Let degree-2 nominal load Love number $k_2' = -0.3075,\ h_2' = -1.001,\ l_2' = 0.0295$, degree-2 nominal diurnal potential Love number $k_{21}^{\square} = 0.29830 - 0.00144i$ from Tab 1.3 and degree-2 nominal diurnal displacement Love number $h_{21}^{\square} = 0.6078,\ l_{21}^{\square} = 0.0847$ from Tab 1.2.

In the first step, from Formula (1.22), Tab 1.4, Tab 1.5 and Tab 1.6, the resonance body tidal and load Love numbers $\mathscr{k}_{21}^{0}(\sigma),\ \mathscr{k}_{21}'(\sigma)$ considering the diurnal resonance effect are calculated according to Formula (1.20). The second step is to substitute the diurnal resonant load Love numbers $\mathscr{k}_{21}'(\sigma)$ into the formula (1.23) to calculate the ocean tidal load contributions $\delta\mathscr{k}_{21}^{ol}(\sigma)$ to the diurnal body tidal Love numbers. The third step is to substitute the diurnal resonance body tidal Love numbers $\mathscr{k}_{21}^{0}(\sigma)$ and the ocean tidal load contributions $\delta\mathscr{k}_{21}^{ol}(\sigma)$ into Formula (1.25) to calculate the frequency dependent corrections $\delta k_{21}(\sigma)$ for the diurnal potential Love numbers as Tab 1.7 and that $\delta h_{21}(\sigma), \delta l_{21}(\sigma)$ for the diurnal displacement Love numbers as Tab 1.8 and Tab 1.9.

Tab 1.7 The corrections for frequency dependence of $\boldsymbol{k_{2m}^{(0)}}$

Name	Doodson	ω°/hr	δk_{21}^{R}×10⁻⁵	δk_{21}^{I}×10⁻⁵	H_{21}^{TGP}×10⁻⁵m
$2Q_1$	125,755	12.85429	-29	3	-664
σ_1	127,555	12.92714	-30	3	-802
	135,645	13.39645	-45	5	-947
Q_1	135,655	13.39866	-46	5	-5020
ρ_1	137,455	13.47151	-49	5	-954
	145,545	13.94083	-82	7	-4946
O_1	145,555	13.94303	-83	7	-26221
τ_1	147,555	14.02517	-91	9	343
$N\tau_1$	153,655	14.41456	-168	14	194
	155,445	14.48520	-193	16	137
Lk_1	155,455	14.48741	-194	16	741
No_1	155,655	14.49669	-197	16	2062
	155,665	14.49890	-198	16	414
χ_1	157,455	14.56955	-231	18	394
	157,465	14.57176	-233	18	87
π_1	162,556	14.91787	-834	58	-714
	163,545	14.95673	-1117	76	137
P_1	163,555	14.95893	-1138	77	-12203
	164,554	15.00000	-1764	104	103
S_1	164,556	15.00000	-1764	104	289
	165,345	15.02958	-3048	92	7
	165,535	15.03665	-3630	195	4
	165,545	15.03886	-3845	229	-730
K_1	165,555	15.04107	-4084	262	36878
	165,565	15.04328	-4355	297	5001
	165,575	15.04548	-4665	334	-108
	166,455	15.07749	85693	21013	-0.6

	166,544	15.07993	35203	2084	1.1
ψ_1	166,554	15.08214	22794	358	293
	166,556	15.08214	22780	358	-4.5
	166,564	15.08434	16842	-85	5
	167,355	15.11392	3755	-189	18
	167,365	15.11613	3552	-182	5
ϕ_1	167,555	15.12321	3025	-160	525
	167,565	15.12542	2892	-154	-20
	168,554	15.16427	1638	-93	31
θ_1	173,655	15.51259	370	-20	395
	173,665	15.51480	369	-20	78
	175,445	15.58323	325	-17	-61
J_1	175,455	15.58545	324	-17	2062
	175,465	15.58765	323	-16	409
So_1	183,555	16.05697	194	-8	342
	185,355	16.12989	185	-7	169
Oo_1	185,555	16.13911	184	-7	1129
	185,565	16.14131	184	-7	723
	185,575	16.14352	184	-7	151
υ_1	195,455	16.68348	141	-4	216
	195,465	16.68569	141	-4	138
			$\delta k_{22}^R \times 10^{-5}$	$\delta k_{22}^I \times 10^{-5}$	$H_{22}^{TGP} \times 10^{-5}$m
N_2	245,655	28.43973	2	0	12099
M_2	255,555	28.98410	2	0	63192

Tab 1.8 The corrections for the frequency and latitude dependenc of $h_{2m}^{(0)}$

Name	Doodson	ω°/hr	$\delta h_{21}^R \times 10^{-4}$	$\delta h_{21}^I \times 10^{-4}$	$H_{21}^{TGP} \times 10^{-5}$m
$2Q_1$	125,755	12.85429	-39	-27	-664
σ_1	127,555	12.92714	-39	-26	-802
	135,645	13.39648	-42	-26	-947
Q_1	135,655	13.39866	-42	-26	-5020
ρ_1	137,455	13.47151	-43	-26	-954

	145,545	13.94082	-50	-25	-4946
O_1	145,555	13.94303	-50	-25	-26221
τ_1	147,555	14.02517	-52	-25	343
$N\tau_1$	153,655	14.41456	-67	-24	194
No_1	155,655	14.49669	-73	-23	2062
χ_1	157,455	14.56955	-80	-23	394
π_1	162,556	14.91787	-200	-15	-714
P_1	163,555	14.95893	-261	-11	-12203
S_1	164,556	15.00000	-386	-4	289
	165,545	15.03881	-795	23	-730
K_1	165,555	15.04107	-842	30	36878
	165,565	15.04333	-896	36	5001
	165,575	15.04543	-958	43	-108
ψ_1	166,554	15.08214	4491	36	293
	166,564	15.08439	3309	-50	5
ϕ_1	167,555	15.12321	567	-59	525
θ_1	173,655	15.51259	39	-30	395
J_1	175,455	15.58545	30	-30	2062
Oo_1	185,555	16.13911	2	-28	1129
			δh_{22}^{R}×10⁻⁴	δh_{22}^{I}×10⁻⁴	H_{22}^{TGP}×10⁻⁵m
M_2	255,555	28.98410	0	-22	12099

Tab 1.9 The corrections for the frequency and latitude dependence of $l_{2m}^{(0)}$

Name	Doodson	ω°/hr	δl_{21}^{R}×10⁻⁴	δl_{21}^{I}×10⁻⁴	H_{21}^{TGP}×10⁻⁵m
Q_1	135,655	13.39866	-1	-6	-5020
	145,545	13.94082	-1	-6	-4946
O_1	145,555	13.94303	-1	-6	-26221
No_1	155,655	14.49669	0	-6	2062
P_1	163,555	14.95893	6	-6	-12203
	165,545	15.03886	22	-6	-730
K_1	165,555	15.04107	23	-6	36878
	165,565	15.04328	25	-6	5001

ψ_1	166,554	15.08214	-137	-20	293
ϕ_1	167,555	15.12321	-19	-7	525
J_1	175,455	15.58545	-2	-6	395
Oo_1	185,555	16.13911	-1	-6	1129
			$\delta l_{22}^{R}\times10^{-4}$	$\delta l\times10^{-4}$	$H_{22}^{TGP}\times10^{-5}$m
M_2	255,555	28.98410	0	-7	12099

(3) Long-period body tidal Love number frequency dependent corrections of anelastic Earth

The anelasticity of the mantle further enhances the frequency dependence of body tidal Love numbers for the long-period (zonal) tidal waves $(nm = 20)$. The frequency of the long-period tidal constituent is assumed to be σ, and the long-period Love number considering the frequency dependence can be expressed as:

$$k_{20}^{\square}(\sigma) = 0.29525 - 5.796\times10^{4}\left\{ctg\frac{\epsilon\pi}{2}\left[1-\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right]+i\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right\} \tag{1.26}$$

$$h_{20}^{\square}(\sigma) = 0.5998 - 9.96\times10^{4}\left\{ctg\frac{\epsilon\pi}{2}\left[1-\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right]+i\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right\} \tag{1.27}$$

$$l_{20}^{\square}(\sigma) = 0.0831 - 3.01\times10^{4}\left\{ctg\frac{\epsilon\pi}{2}\left[1-\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right]+i\left(\frac{\sigma_m}{\sigma}\right)^{\epsilon}\right\} \tag{1.28}$$

Here, σ_m is the reference frequency with period of 200s, and $\epsilon = 0.15$.

Let $k_{20} = 0.30190, h_{20} = 0.6078, l_{20} = 0.0847$, the frequency dependent correction formula of the zonal body tidal Love numbers for the anelastic Earth are:

$$\delta\mathcal{k}_{20}^{\square}(\sigma) = \mathcal{k}_{20}^{\square}(\sigma) - \mathcal{k}_{20}^{\square},\quad (\mathcal{k} = k, h, l) \tag{1.29}$$

The formulas (1.26) ~ (1.28) is substituted into the formula (1.29) respectively, and the frequency dependent correction value of zonal body tidal Love number for the anelastic Earth are calculated as Tab 1.10 ~ Tab 1.12.

Tab 1.10 The corrections for frequency dependence of $\boldsymbol{k_{20}^{(0)}}$

Name	Doodson	ω°/hr	$\delta k_{20}^{R}\times10^{-5}$	$\delta k_{20}^{I}\times10^{-5}$	$H_{20}^{TGP}\times10^{-5}$m
Ω_1	55,565	0.00221	1347	-541	2793
Ω_2	55,575	0.00441	1124	-488	-27
S_a	56,554	0.04107	547	-349	-492
S_{sa}	57,555	0.08214	403	-315	-3100
	57,565	0.08434	398	-313	77
S_{ta}	58,554	0.12320	326	-296	-181
M_{sm}	63,655	0.47152	101	-242	-673

	65,445	0.54217	80	-237	231
M_m	65,455	0.54438	80	-237	-3518
	65,465	0.54658	79	-237	229
	65,655	0.55366	77	-236	188
M_{sf}	73,555	1.01590	-9	-216	-583
	75,355	1.08875	-18	-213	-288
M_f	75,555	1.09804	-19	-213	-6663
	75,565	1.10024	-19	-213	-2762
	75,575	1.10245	-19	-213	-258
M_{stm}	83,655	1.56956	-65	-202	-242
M_{tm}	85,455	1.64241	-71	-201	-1276
	85,465	1.64462	-71	-201	-529
M_{sqm}	93,555	2.11394	-102	-193	-204
M_{qm}	95,355	2.18679	-106	-192	-169

Tab 1.11 The corrections for the frequency and latitude dependence of $\boldsymbol{h_{20}^{(0)}}$

Name	Doodson	ω°/hr	$\delta h_{20}^{R}\times10^{-4}$	$\delta h_{20}^{I}\times10^{-4}$	$H_{20}^{TGP}\times10^{-5}$m
Ω_1	55,565	0.00221	266	-93	2793
S_{sa}	57,555	0.08214	104	-54	-3100
M_m	65,455	0.54438	48	-41	-3518
M_f	75,555	1.09804	31	-37	-6663
	75,565	1.10024	31	-37	-2762

Tab 1.12 The corrections for the frequency and latitude dependence of $\boldsymbol{l_{20}^{(0)}}$

Name	Doodson	ω°/hr	$\delta l_{20}^{R}\times10^{-4}$	$\delta l_{20}^{I}\times10^{-4}$	$H_{20}^{TGP}\times10^{-5}$m
Ω_1	55,565	0.00221	89	-28	2793
S_{sa}	57,555	0.08214	39	-16	-3100
M_m	65,455	0.54438	23	-12	-3518
M_f	75,555	1.09804	17	-11	-6663
	75,565	1.10024	17	-11	-2762

(4) The geopotential coefficient adjustments from frequency dependent corrections of potential Love number

The contribution of different frequency tidal constituent to potential Love number is different, and thus the frequency dependent correction $\delta k_{2\mathrm{m}}$ for each tidal constituent needs to be calculated one by one. The Love number correction of frequency dependence for some a constituent σ is $\delta k_{2\mathrm{m}}(\sigma)$, and the direct influence of the constituent σ on degree-2 order-m geopotential coefficient is $\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}$, then the product $k_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}\right)$ is degree-2 order-m geopotential coefficient variation caused by the Love number frequency dependent correction for the corresponding constituent σ. The geopotential coefficient adjustments $\Delta\bar{C}_{2m}^{\delta} - i\Delta\bar{S}_{2m}^{\delta}$ can be obtained by summing the contribution of frequency dependent corrections for all tidal constituents.

Let degree-2 order-m potential Love number frequency dependent correction be $\delta k_{2m} = \delta k_{2m}^{R} + i\delta k_{2m}^{I}$, then degree-2 tesseral and sector harmonic geopoential coefficient adjustments $(m = 1,2)$ caused by the diurnal and semi-diurnal Love number frequency dependent corrections are:

$$\Delta\bar{C}_{2m}^{\delta} - i\Delta\bar{S}_{2m}^{\delta} = \sum_{\sigma}\delta k_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}\right) = \eta_m\left(\sum_{\tau=1}^{\tau_{20}} A_m\delta k_{2m}^{\tau}H_{2m}^{\tau}\, e^{i\phi^{\tau}}\right) = \eta_m A_m \sum_{\tau=1}^{\tau_{2m}} H_{2m}^{\tau}[(\delta k_{2m}^{\tau R}cos\phi^{\tau} - \delta k_{2m}^{\tau I}sin\phi^{\tau}) + i(\delta k_{2m}^{\tau R}sin\phi^{\tau} + \delta k_{2m}^{\tau I}cos\phi^{\tau})] \quad (1.30)$$

$$\Delta\bar{C}_{21}^{\delta} - i\Delta\bar{S}_{21}^{\delta} = A_1\sum_{\tau=1}^{\tau_{20}} H_{21}^{\tau}[(\delta k_{21}^{\tau R}sin\phi^{\tau} + \delta k_{22}^{\tau I}cos\phi^{\tau}) - i(\delta k_{21}^{\tau R}cos\phi^{\tau} - \delta k_{21}^{\tau I}sin\phi^{\tau})] \quad (1.31)$$

$$\Delta\bar{C}_{22}^{\delta} - i\Delta\bar{S}_{22}^{\delta} = A_2\sum_{\tau=1}^{\tau_{20}} H_{22}^{\tau}[(\delta k_{22}^{\tau R}cos\phi^{\tau} - \delta k_{22}^{\tau I}sin\phi^{\tau}) + i(\delta k_{22}^{\tau R}sin\phi^{\tau} + \delta k_{22}^{\tau I}cos\phi^{\tau})] \quad (1.32)$$

Similarly, degree-2 zonal harmonic geopoential coefficient adjustments caused by the long-period Love number frequency dependent corrections are:

$$\Delta\bar{C}_{20}^{\delta} = Re\left[\sum_{\sigma}\delta k_{20}(\sigma)\Delta\bar{C}_{20}^{\sigma}\right] = Re\left(\sum_{\tau=1}^{\tau_{20}}\delta k_{20}^{\tau}A_0H_{20}^{\tau}\, e^{i\phi^{\tau}}\right) = A_0\sum_{\tau=1}^{\tau_{20}} H_{20}^{\tau}(\delta k_{20}^{\tau R}cos\phi^{\tau} - \delta k_{20}^{\tau I}sin\phi^{\tau}) \quad (1.33)$$

In the formulas (1.30) ~ (1.33), $\eta_1 = -i$, $\eta_2 = 1$,

$$A_0 = \frac{1}{R\sqrt{4\pi}} = 4.4228\times 10^{-8} \text{ (in unit of /m)}$$

$$A_m = \frac{(-1)^m}{R\sqrt{8\pi}} = (-1)^m(3.1274\times 10^{-8}) \text{ (in unit of /m)},\ m = 1,2$$

Where, $\tau_{2m}(m = 0,1,2)$ is the number of effective tidal constituents of degree-2 order-m tidal waves and in Tab 5.6, $\tau_{20} = 21$, $\tau_{21} = 48$, $\tau_{22} = 2$. H_{2m}^{τ} is the global maximum equilibrium tidal height amplitude (in unit of m) for the corresponding tidal constituent, and that is the last column in Tab 1.6. ϕ^{τ} is the astronomical argument (in unit of radian) of the tidal constituent τ, which can be calculated by six Doodson astronomical arguments or five basic Delaunay variables.

(5) Equivalent treatment of displacement Love number frequency dependent corrections

For the ground sites fixed with the solid Earth, when the solid tidal effects on geodetic variations contain the tidal deformation contribution characterized by the displacement Love number, the frequency dependent correction of the displacement Love number needs to be considered. The displacement Love number represents the displacement of the ground site excited by the tidal generating potential, and its effect on geodetic variations always appears in the form of proportional factor. From Section 8.1.1, it is not difficult to see that in the expression of solid Earth's tidal effect on radial and horizontal displacement, ground gravity and ground tilt at ground site, the displacement Love number and geopotential coefficient for the same degree-n order-m tidal waves always appears in the form of product.

According to the theory of solid Earth deformation mechanics, the solid tidal effect on ground site displacement is harmonic and can be expressed in the form of spherical harmonic series. The product of the frequency dependent correction of displacement Love number for the degree-n order-m tidal waves and the direct influence of the same degree-n order-m geopotential coefficient is the contribution of the frequency dependent correction of the displacement Love number to the degree-n order-m spherical harmonic coefficient. Thus, degree-2 spherical harmonic coefficient adjustments caused by the frequency dependent corrections of degree-2 dispalcement Love number are:

$$\Delta\hat{C}_{2m}^{\delta} - i\Delta\hat{S}_{2m}^{\delta} = \sum_{\sigma}\delta h_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}\right) \tag{1.34}$$

$$\Delta\tilde{C}_{2m}^{\delta} - i\Delta\tilde{S}_{2m}^{\delta} = \sum_{\sigma}\delta l_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}\right) \tag{1.35}$$

Comparing the algorithm formulas of geopotential coefficient adjustment from the potential Love number frequency dependent correction, degree-2 tesseral and sector spherical harmonic coefficient adjustments $(m=1,2)$ caused by the diurnal and semi-diurnal radial Love number frequency dependent corrections are:

$$\Delta\hat{C}_{2m}^{\delta} - i\Delta\hat{S}_{2m}^{\delta} = \sum_{\sigma}\delta h_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}\right) = \eta_m\left(\sum_{\tau=1}^{\tau_{20}} A_m\delta h_{2m}^{\tau}H_{2m}^{\tau}\, e^{i\phi^{\tau}}\right) = \eta_m A_m$$
$$\sum_{\tau=1}^{\tau_{20}} H_{2m}^{\tau}[(\delta h_{2m}^{\tau R}cos\phi^{\tau} - \delta h_{2m}^{\tau I}sin\phi^{\tau}) + i(\delta h_{2m}^{\tau R}sin\phi^{\tau} + \delta h_{2m}^{\tau I}cos\phi^{\tau})] \tag{1.36}$$

Degree-2 zonal spherical harmonic coefficient adjustments $(m=0)$ caused by the long-period radial Love number frequency dependent corrections are:

$$\Delta\hat{C}_{20}^{\delta} = Re\left[\sum_{\sigma}\delta h_{20}(\sigma)\Delta\bar{C}_{20}^{(\sigma)}\right] = Re\left(\sum_{\tau=1}^{\tau_{20}}\delta h_{20}^{\tau}A_0H_{20}^{\tau}\, e^{i\phi^{\tau}}\right)$$
$$= A_0\sum_{\tau=1}^{\tau_{20}} H_{20}^{\tau}(\delta h_{20}^{\tau R}cos\phi^{\tau} - \delta h_{20}^{\tau I}sin\phi^{\tau}) \tag{1.37}$$

Similarly, degree-2 tesseral and sector spherical harmonic coefficient adjustments $(m=1,2)$ caused by the diurnal and semi-diurnal horizontal Love number frequency dependent corrections are:

$$\Delta\tilde{C}_{2m}^{\delta} - i\Delta\tilde{S}_{2m}^{\delta} = \sum_{\sigma}\delta l_{2m}(\sigma)\left(\Delta\bar{C}_{2m}^{(\sigma)} - i\Delta\bar{S}_{2m}^{(\sigma)}\right) = \eta_m\left(\sum_{\tau=1}^{\tau_{20}} A_m\delta l_{2m}^{\tau}H_{2m}^{\tau}\, e^{i\phi^{\tau}}\right) = \eta_m A_m$$
$$\sum_{\tau=1}^{\tau_{20}} H_{2m}^{\tau}[(\delta l_{2m}^{\tau R}cos\phi^{\tau} - \delta l_{2m}^{\tau I}sin\phi^{\tau}) + i(\delta l_{2m}^{\tau R}sin\phi^{\tau} + \delta l_{2m}^{\tau I}cos\phi^{\tau})] \tag{1.38}$$

Degree-2 zonal spherical harmonic coefficient adjustments $(m=0)$ caused by the long-period horizontal Love number frequency dependent corrections are:

$$\Delta\tilde{C}_{20}^{\delta} = Re\left[\sum_{\sigma}\delta k_{20}(\sigma)\Delta\bar{C}_{20}^{(\sigma)}\right] = Re\left(\sum_{\tau=1}^{\tau_{20}}\delta k_{20}^{\tau}A_0H_{20}^{\tau}\, e^{i\phi^{\tau}}\right)$$
$$= A_0\sum_{\tau=1}^{\tau_{20}} H_{20}^{\tau}(\delta k_{20}^{\tau R}cos\phi^{\tau} - \delta k_{20}^{\tau I}sin\phi^{\tau}) \tag{1.39}$$

Here, the frequency dependent correction formulas of the displacement Love number is expressed as the adjustments of displacement spherical harmonic coefficients, which is the same as the frequency dependent correction formulas of potential Love number for the adjustments of geopotential coefficients. The purpose of this treatment is to standardize the computation process of the solid tidal effect on various geodetic variations, and to realize the algorithm compatibility and unified calculation of the solid tidal effects on various geometric and physical geodetic variations.

8.1.4 Unified computation scheme of the body tidal effects on all-element geodetic variations in whole Earth space

Using the analytically compatible geodetic and geodynamic algorithms with the numerical standards unified and geophysical models collaborated to calculate uniformly the solid Earth tidal effects on various geodetic variations on the ground and outside the solid Earth is an important basis and necessary condition for the collaborative monitoring of multi-geodetic technologies and deep fusion of heterogeneous Earth monitoring data.

(1) Love number frequency dependent correction algorithm for solid tidal effect

Firstly, the appropriate nominal body tidal Love numbers are selected, so that the frequency dependent corrections of Love numbers include the contribution of all imaginary parts of Love numbers, and then let the nominal Love number be real to simplify the calculation scheme of solid tidal effect.

For the reason mentioned, here let the real part of degree-2 potential Love number for the anelastic Earth in Tab 1.3 as the nominal potential Love number and the imaginary part of degree-2 potential Love number will be considered in the frequency dependent correction algorithm of the potential Love number. That is, the imaginary part of degree-2 diurnal potential Love number in Tab 1.6 is uniformly added to − 0.00144, which becomes $k_{21}^{I}(\sigma) = \delta k_{21}^{I}(\sigma) - 0.00144$, and the imaginary part of degree-2 semi-diurnal potential Love number is uniformly added to − 0.00130, which becomes $\delta k_{22}^{I}(\sigma) = \delta k_{22}^{I}(\sigma) - 0.00130$. In this way, the nominal tidal Love number value is as shown in Tab 1.13.

Tab 1.13 The values for nominal body tidal Love numbers

n	m	periods of tidal constituents	k_{nm}	h_{nm}	l_{nm}
2	0	long period	0.30190	0.6078	0.0847
2	1	diurnal	0.29830	0.6078	0.0847
2	2	semi-diurnal	0.30102	0.6078	0.0847
3	0	long period	0.093	0.2920	0.0150
3	1	diurnal	0.093	0.2920	0.0150
3	2	semi-diurnal	0.093	0.2920	0.0150
3	3	1/3-diurnal	0.094	0.2920	0.0150

The Love number frequency dependent correction algorithms for the adjustments of degree-2 geopotential coefficients have been given by Formulas (1.30) to (1.33). Substituting degree-2 geopoential coefficient adjustments $\Delta\bar{C}_{2m}^{\delta}-i\Delta\bar{S}_{2m}^{\delta}$ into the solid Earth tidal effect expression (1.4) on height anomaly, degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on height anomaly on the ground or outside solid Earth can be obtained:

$$\delta\zeta(r,\theta,\lambda)=\frac{GM}{\gamma r}\left(\frac{a}{r}\right)^2\sum_{m=0}^{2}\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda+\Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\bar{P}_{2m}(cos\theta) \quad (1.40)$$

Here, $\Delta\bar{S}_{20}^{\delta}=0$.

Similarly, degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on gravity can be obtained:

$$\delta g^{\delta}=3\frac{GM}{r^2}\left(\frac{a}{r}\right)^2\sum_{m=0}^{2}\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda+\Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\bar{P}_{2m}(cos\theta) \quad (1.41)$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on vertical deflection can be obtained:

South: $$\delta\xi=\frac{GMsin\theta}{\gamma r^2}\left(\frac{a}{r}\right)^2\sum_{m=0}^{2}\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda+\Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\frac{\partial}{\partial\theta}\bar{P}_{2m}(cos\theta) \quad (1.42)$$

West: $$\delta\eta=\frac{GM}{\gamma r^2\sin\theta}\left(\frac{a}{r}\right)^2\sum_{m=1}^{2}m\left(\Delta\bar{C}_{2m}^{\delta}sinm\lambda-\Delta\bar{S}_{2m}^{\delta}cosm\lambda\right)\bar{P}_{2m}(cos\theta) \quad (1.43)$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on gravity gradient (radial) can be obtained:

$$\delta T_{rr}=12\frac{GM}{r^3}\left(\frac{a}{r}\right)^2\sum_{m=0}^{2}\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda+\Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\bar{P}_{2m}(cos\theta) \quad (1.44)$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on horizontal gravity gradient can be obtained:

North: $$\delta T_{NN}=-\frac{GM}{r^3}\left(\frac{a}{r}\right)^2\sum_{m=0}^{2}\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda+\Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\frac{\partial^2}{\partial\theta^2}\bar{P}_{2m}(cos\theta) \quad (1.45)$$

West: $$\delta T_{WW}=\frac{GM}{r^3sin^2\theta}\left(\frac{a}{r}\right)^2\sum_{m=1}^{2}m^2\left(\Delta\bar{C}_{2m}^{\delta}cosm\lambda+\Delta\bar{S}_{2m}^{\delta}sinm\lambda\right)\bar{P}_{2m}(cos\theta) \quad (1.46)$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on site displacement can be obtained ◉ (◉ marked indicates its site fixed with the solid Earth):

East: $$\delta e=-\frac{GM}{\gamma r\sin\theta}\left(\frac{a}{r}\right)^2\sum_{m=1}^{2}m\left(\Delta\tilde{C}_{2m}^{\delta}sinm\lambda-\Delta\tilde{S}_{2m}^{\delta}cosm\lambda\right)\bar{P}_{2m}(cos\theta) \quad (1.47)$$

North: $$\delta n=-\frac{GM\sin\theta}{\gamma r}\left(\frac{a}{r}\right)^2\sum_{m=0}^{2}\left(\Delta\tilde{C}_{2m}^{\delta}cosm\lambda+\Delta\tilde{S}_{2m}^{\delta}sinm\lambda\right)\frac{\partial}{\partial\theta}\bar{P}_{2m}(cos\theta) \quad (1.48)$$

Radial: $$\delta r=\frac{GM}{\gamma r}\left(\frac{a}{r}\right)^2\sum_{m=0}^{2}\left(\Delta\hat{C}_{2m}^{\delta}cosm\lambda+\Delta\hat{S}_{2m}^{\delta}sinm\lambda\right)\bar{P}_{2m}(cos\theta) \quad (1.49)$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on ground gravity can be obtained ◉:

$$\delta g^{s}(r,\varphi,\lambda)=\frac{3GM}{r^2}\left(\frac{a}{r}\right)^2$$

$$\sum_{m=0}^{2}\left[\left(\Delta\hat{C}_{2m}^{\delta}-\frac{3}{2}\Delta\bar{C}_{2m}^{\delta}\right)cosm\lambda+\left(\Delta\hat{S}_{2m}^{\delta}-\frac{3}{2}\Delta\bar{S}_{2m}^{\delta}\right)sinm\lambda\right]\bar{P}_{2m}(cos\theta) \tag{1.50}$$

Degree-2 Love number frequency dependent correction formula of the solid Earth tidal effect on ground tilt can be obtained ◉:

South: $\delta\xi^{s}=\frac{GM\sin\theta}{\gamma r^2}\left(\frac{a}{r}\right)^2$

$$\sum_{m=0}^{2}\left[\left(\Delta\bar{C}_{2m}^{\delta}-\Delta\hat{C}_{2m}^{\delta}\right)cosm\lambda+\left(\Delta\bar{S}_{2m}^{\delta}-\Delta\hat{S}_{2m}^{\delta}\right)sinm\lambda\right]\frac{\partial}{\partial\theta}\bar{P}_{2m}(cos\theta) \tag{1.51}$$

West: $\delta\eta^{s}=\frac{GM}{\gamma r^2\sin\theta}\left(\frac{a}{r}\right)^2$

$$\sum_{m=1}^{2}m\left[\left(\Delta\bar{C}_{2m}^{\delta}-\Delta\hat{C}_{2m}^{\delta}\right)sinm\lambda-\left(\Delta\bar{S}_{2m}^{\delta}-\Delta\hat{S}_{2m}^{\delta}\right)cosm\lambda\right]\bar{P}_{2m}(cos\theta) \tag{1.52}$$

(2) The solid Earth tidal effects on degree-4 geopotential coefficients

The solid Earth tidal effects on degree-4 geopotential coefficients are calculated by the direct influence $(\Delta\bar{C}_{2m}-i\Delta\bar{S}_{2m})$ of degree-2 geopotential coefficients using the frequency dependent Love numbers $k_{2m}^{+}, (m=0,1,2)$.

$$\Delta\bar{C}_{4m}-i\Delta\bar{S}_{4m}=k_{2m}^{+}(\Delta\bar{C}_{2m}-i\Delta\bar{S}_{2m}),\quad m=0,1,2 \tag{1.53}$$

Although the solid Earth tidal effects on degree-4 geopotential coefficients are calculated by the direct influence of degree-2 geopotential coefficients according to Formula (1.53), their contributions to geodetic variations should be calculated according to the variations of degree-4 geopotential coefficients. Substituting the formula (1.53) into the formulas (1.4) ~ (1.16), the contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on the height anomaly can be obtained as follows:

$$\varepsilon\zeta(r,\varphi,\lambda)=\frac{GM}{\gamma r}\left(\frac{a}{r}\right)^4\sum_{m=0}^{2}k_{2m}^{+}(\Delta\bar{C}_{2m}cosm\lambda+\Delta\bar{S}_{2m}sinm\lambda)\bar{P}_{4m}(cos\theta) \tag{1.54}$$

The contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on gravity can be obtained as follows:

$$\varepsilon g^{\delta}=\frac{5GM}{r^2}\left(\frac{a}{r}\right)^4\sum_{m=0}^{2}k_{2m}^{+}(\Delta\bar{C}_{2m}cosm\lambda+\Delta\bar{S}_{2m}sinm\lambda)\bar{P}_{4m}(cos\theta) \tag{1.55}$$

The contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on ground tilt or vertical deflection can be obtained as follows:

South: $\varepsilon\xi=\frac{GM\sin\theta}{\gamma r^2}\left(\frac{a}{r}\right)^4\sum_{m=0}^{2}k_{2m}^{+}(\Delta\bar{C}_{2m}cosm\lambda+\Delta\bar{S}_{2m}sinm\lambda)\frac{\partial}{\partial\theta}\bar{P}_{4m}(cos\theta)$ (1.56)

West: $\varepsilon\eta=\frac{GM}{\gamma r^2\sin\theta}\left(\frac{a}{r}\right)^4\sum_{m=1}^{2}k_{2m}^{+}m(\Delta\bar{C}_{2m}sinm\lambda-\Delta\bar{S}_{2m}cosm\lambda)\bar{P}_{4m}(cos\theta)$ (1.57)

The contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on gravity gradient (radial) can be obtained as follows:

$$\varepsilon T_{rr}=\frac{30GM}{r^3}\left(\frac{a}{r}\right)^4\sum_{m=0}^{2}k_{2m}^{+}(\Delta\bar{C}_{2m}cosm\lambda+\Delta\bar{S}_{2m}sinm\lambda)\bar{P}_{4m}(cos\theta) \tag{1.58}$$

The contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on horizontal gravity gradient can be obtained as follows:

North: $$\varepsilon T_{NN}=-\frac{GM}{r^3}\left(\frac{a}{r}\right)^4\sum_{m=0}^{2}k_{2m}^{(+)}(\Delta\bar{C}_{2m}cosm\lambda+\Delta\bar{S}_{2m}sinm\lambda)\frac{\partial^2}{\partial\theta^2}\bar{P}_{4m} \tag{1.59}$$

West: $$\varepsilon T_{WW}=\frac{GM}{r^3 sin^2\theta}\left(\frac{a}{r}\right)^4\sum_{m=1}^{2}m^2\,k_{2m}^{+}(\Delta\bar{C}_{2m}sinm\lambda+\Delta\bar{S}_{2m}cosm\lambda)\bar{P}_{4m} \tag{1.60}$$

The contribution of the solid tidal effect on degree-4 geopotential coefficients to the solid tidal effect on ground displacement is always zero.

(3) The unified computation scheme of the solid Earth tidal effects on all-element geodetic variations

(a) The geopotential coefficient variations $\Delta\bar{C}_{nm}-i\Delta\bar{S}_{nm},(n=2,3,4,5,6)$ from the Earth's tidal generating potential (TGP) of celestial bodies outside the Earth are calculated directly from Formula (1.1).

(b) The nominal Love numbers from Tab 1.13 are used, and the nominal displacement Love number and the calculation point geocentric latitude φ are employed to calculate the latitude dependent displacement Love number according to formula (1.19). Then, according to formulas (1.3) to (1.16), the nominal value as x^0 of the solid tidal effects on various geodetic variations are calculated from the geopotential coefficient variations $\Delta\bar{C}_{nm}-i\Delta\bar{S}_{nm}$.

(c) The frequency dependent Love numbers $k_{2m}^{+},(m=0,1,2)$ are employed to calculate the contribution as εx of degree-4 geopotential coefficient to the solid tidal effects on geodetic variations according to formulas (1.54) to (1.60) from degree-2 geopotential coefficient variations $\Delta\bar{C}_{2m}-i\Delta\bar{S}_{2m}$.

(d) According to the formulas (1.30) and (1.31), (1.36) and (1.37), the frequency dependent corrections $\left(\Delta\bar{C}_{2m}^{\delta}-\Delta\hat{C}_{2m}^{\sigma}\right)$, $\left(\Delta\hat{C}_{2m}^{\sigma}-i\Delta\hat{S}_{2m}^{\sigma}\right)$ and $\left(\Delta\tilde{C}_{2m}^{\sigma}-i\Delta\tilde{S}_{2m}^{\sigma}\right)$ of the geopotential coefficient variations and displacement spherical harmonic coefficient variations are calculated by the frequency dependent corrections of the degree-2 Love numbers and ocean tide harmonic amplitudes. Then, according to the formula (1.40) to (1.52), degree-2 Love number frequency dependent corrections as δx of the solid tidal effects on various geodetic variations are calculated.

(e) The frequency dependent corrections of the degree-2 potential Love number and ocean tide harmonic amplitudes are shown in Tab 1.6 to Tab 1.11. The frequency dependent corrections of the potential Love numbers in these Tables are uniformly added using the imaginary parts of degree-2 Love number of the anelastic Earth in Tab 1.3, that is, $k_{21}^{I}(\sigma)=\delta k_{21}^{I}(\sigma)-0.00144$, $\delta k_{22}^{I}(\sigma)=\delta k_{22}^{I}(\sigma)-0.00130$.

(f) The sum of the nominal value x^0 of solid tidal effects, degree-4 geopotential coefficient contributions εx and degree-2 Love number frequency dependent corrections δx is the high-precision calculation result of the solid tidal effects on various geodetic variations at the calculation point on ground or outside the solid Earth.

8.1.5 Characteristics and analysis of solid Earth tidal effects in geodesy

The Earth's tidal potential and tidal force from celestial body are related to the position of the calculation point in the Earth-fixed coordinate system. In the following, the ground point P (105°N, 20°E, H100m) selected, the time-varying properties of the solid Earth tidal effects on various geodetic variations are investigated by calculating the time series of solid Earth tidal effects and the contribution time series of each part at the point P.

(1) Solid Earth tidal effects on all-element geodetic variations

Firstly, The various contributions considered in the solid Earth tidal effects, the solid Earth tidal effect time series on all-element geodetic variations at the ground point P (105°N, 20°E, H100m) are calculated, as shown in Fig 1.8. The time span is from 0 : 00 on June 1, 2020 to 24 : 00 on June 7, 2022 (7 days), with a time interval of 10 minutes.

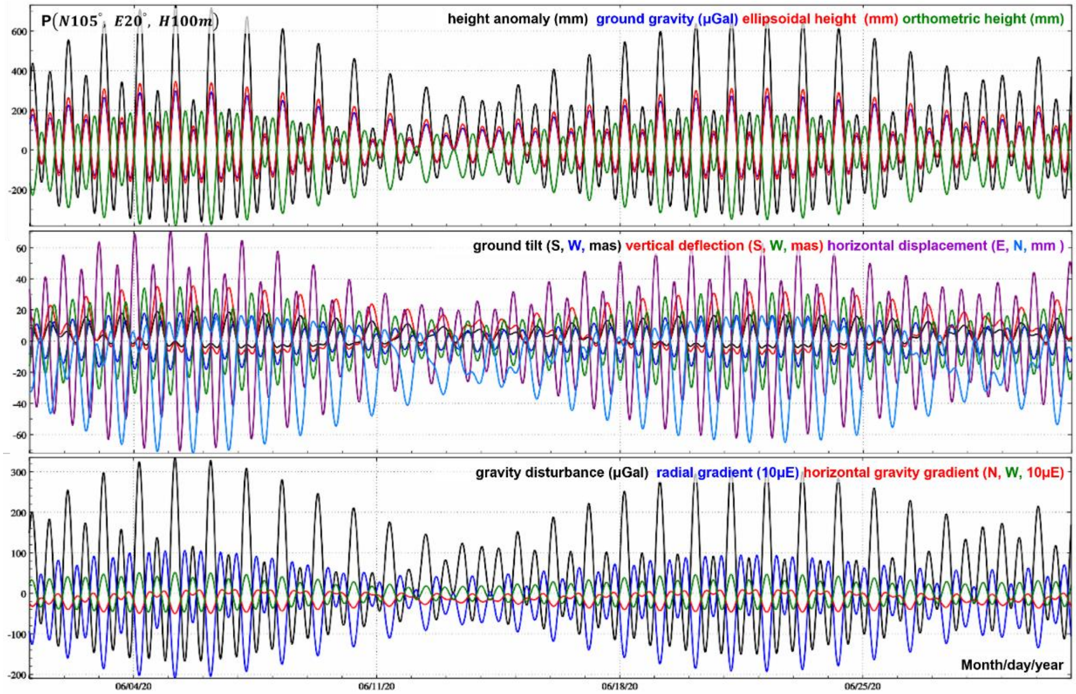

Fig 1.8 Solid Earth tidal effect time series on all-element geodetic variations

When analyzing the time-varying properties of the tidal effect time series curve, it is generally possible to focus on the amplitude variation with time, and to investigate the phase relationship between the tidal effects on different types of geodetic variations. Because the difference between the maximum and minimum values of the time series is independent of the reference time epoch, the difference between the maximum and minimum values of tidal effect time series can effectively reflect the relationship between the tidal effects on different types of geodetic variations.

Fig 1.8 shows that the difference between the maximum and minimum values of the solid tidal effects on geoid can reach 0.99m, that on ground ellipsoidal height can reach 0.51m, that on normal height can reach 0.58m, that on the ground gravity can reach 447.5μGal, that on the ground tilt can reach 45mas, that on the horizontal displacement can reach 0.16m, that on the radial gravity gradient can reach 3.20mE, and the difference between the maximum and minimum values of the solid tidal effects on the horizontal gravity gradient can reach 1.15mE.

The solid tidal effects on the ground ellipsoidal height are different from that on the normal height (the symbol is opposite at the same time epoch), the solid tidal effects on ground tilt in the south and west directions are different, and the solid tidal effects on the horizontal gravity gradient in the north and west directions are different. The solid tidal effect on the horizontal displacement, ground tilt (vertical deviation) and horizontal gravity gradient vector are generally larger in the east-west direction than that in the north-south direction.

(2) Solid tidal effects of the planets outside Earth in the solar system

Here takes all the planets outside Earth in solar system as the tidal celestial bodies, and calculates the solid tidal effect time series on all-element geodetic variations at the ground point P(105°N, 20°E, H100m), as shown in Fig 1.9. The time span is from January 1, 2020 to December 31, 2022 (12 years), with a time interval of 2 hours. The solid tidal effects from planets do not contain the contributions of the Love number frequency dependence.

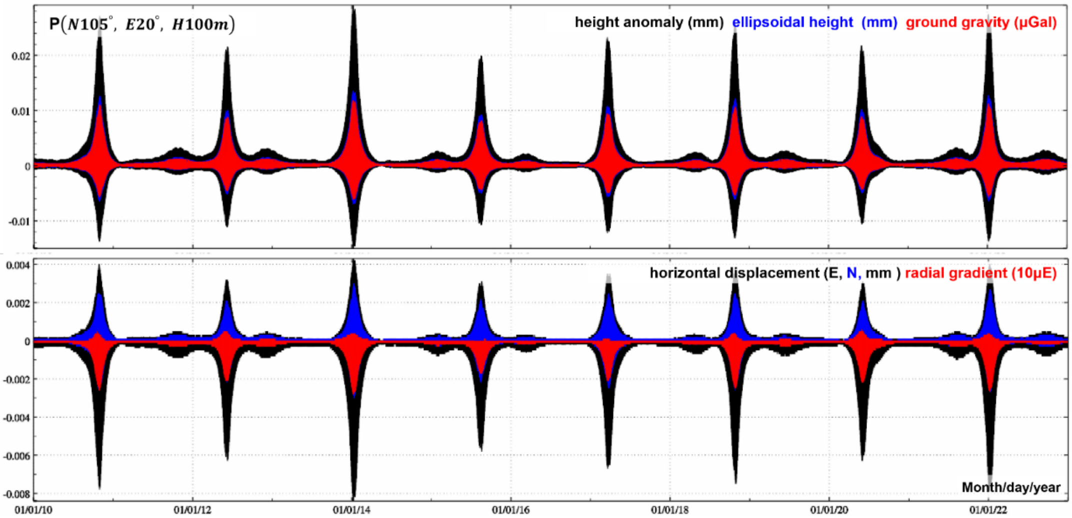

Fig 1.9 Solid tidal effect time series from the planets outside Earth

(3) The indirect influences of tidal potential to geodetic variations

The indirect influence is the contribution of tidal potential to the geodetic variations through the action of body tidal Love numbers, which is also called the solid tidal effect by some literatures. Fig 1.10 is the indirect influence time series curve of tidal potential to all-element geodetic variations at the ground point P(105°N, 20°E, H100m). The time span is from 0 : 00 on June 1, 2020 to 24 : 00 on June 7, 2022 (7 days), with a time interval of 10 minutes.

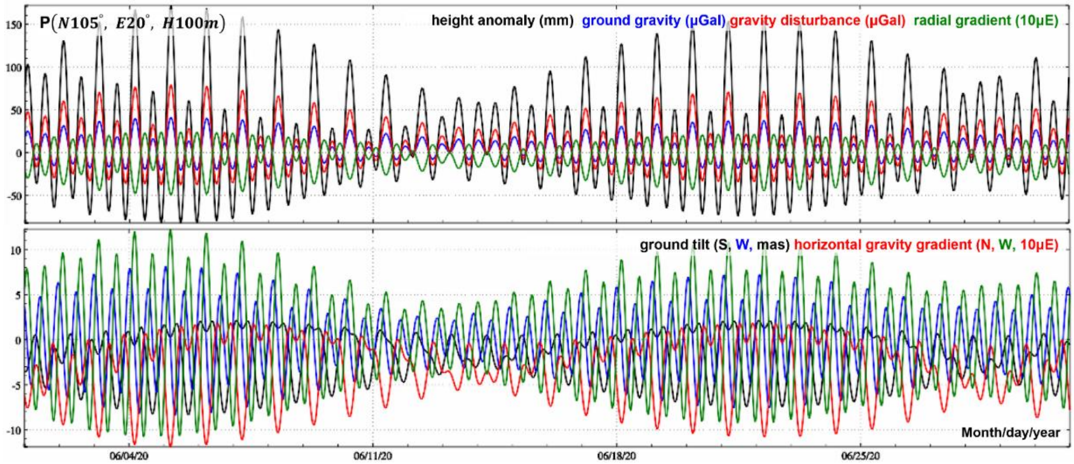

Fig 1.10 The indirect influence time series of tidal potential to geodetic variations

Figure 1.10 shows that the difference between the maximum and minimum indirect influences of tidal potential to geoid can reach 0.24 m, that to ground gravity can reach 40μGal, and the difference between the maximum and minimum indirect influences of tidal potential to radial gravity gradient can reach 0.7mE. Compared with the solid tidal effects (the sum of the direct and indirect influences of tidal potential), the phase relationship between the indirect influences to different types of geodetic variations are not completely consistent.

(4) Total contributions of Love number frequency dependent corrections to solid tidal effects

In the following, the contribution time series of potential Love number frequency dependent correction to the solid tidal effect on all-element geodetic variations at ground point P(105°N, 20°E, H100m) are calculated as Fig 1.11. The time span is from January 1, 2018 to January 31, 2018 (1 month), with a time interval of 30 minutes.

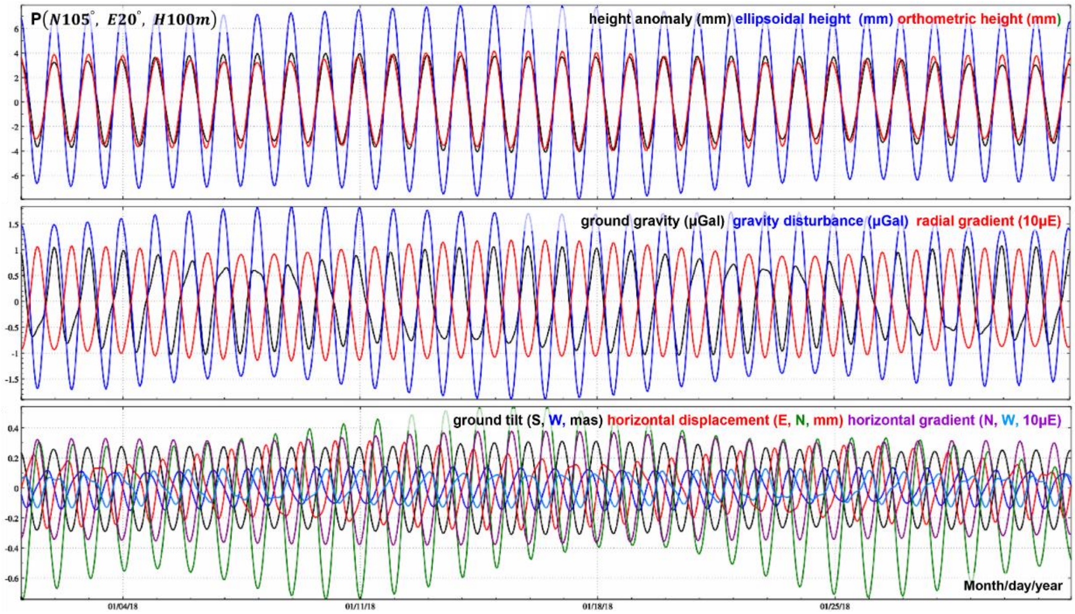

Fig 1.11 Contributions of potential Love number frequency dependent corrections