## Algorithms for global tidal load spherical harmonic coefficient model

The ocean tides redistribute the mass of seawater and cause the geopotential variations, which excite the deformation of the solid Earth and generate the associted geopotential through the action of load Love numbers at the same time. Similarly, atmospheric tides redistribute the density of the atmosphere, cause the geopotential variations and then generate the associted geopotential.

### 8.4.1 Construction of tidal load spherical harmonic coefficient model

Both ocean tide and solid Earth tide are generated by the tidal force from the Moon and Sun, and have the same periodic variation characteristics, such as diurnal and semi-diurnal periodic variations. Therefore, it is generally difficult to find mathematical methods to completely separate the ocean tidal load effect on geodetic variations from their solid tidal effects. The method of load Green's function and sea surface tidal height convolution, or the method of spherical harmonic analysis of ocean tide and spherical harmonic synthesis of ocean tidal load effect, is usually employed to calculate the ocean tidal load effects on geodetic variations. The ocean tidal loads are located on the surface, and the load Green's functions need to be calculated by the high-degree load Love numbers. The load spherical harmonic coefficient model also need be expressed as a high-degree or ultrahigh-degree spherical harmonic function series.

#### (1) Spherical harmonic analysis method of global ocean tide

General construction procedure of the global ocean tidal load normalized spherical harmonic coefficient model (in FES2004 format) from global ocean tidal height harmonic constant grids is generally composed of the following three steps.

**Step 1:** From the global ocean tidal harmonic constant grid model of each tidal constituent, generate the normalized spherical harmonic coefficient model of each tidal constituent by spherical harmonic analysis method.

**Step 2:** According to the astronomical tide height algorithm, convert the normalized spherical harmonic coefficients based on the harmonic constant of the tidal constituent into the normalized spherical harmonic coefficient based on the tidal load of the tidal constituent.

**Step 3:** Merging the global normalized tidal load spherical harmonic coefficients of all tidal constituents, generate the global ocean tidal load normalized spherical harmonic coefficient model in FES2004 format.

Astronomical tidal height $T(t)$ of sea surface at the epoch time $t$, expressed as the height of the astronomical tidal level relative to the local mean sea surface, is equal to the sum of $M$ tidal constituent heights

$$T(\theta,\lambda,t)=\sum_{i=1}^{M}T_i(\theta,\lambda,t)=\sum_{i=1}^{M}H_i(\theta,\lambda)cos[\phi_i(t)-g_i(\theta,\lambda)] \qquad (4.1)$$

Where, $M$ is the number of the ocean tidal constituents, $\theta_i, H_i, g_i$ are the astronomical argument, amplitude and phase at Greenwich of the tidal constituent $\sigma_i$, respectively. $T_i$ is The astronomical tide height of the tidal constituent $\sigma_i$, which can be expanded as

$$T_i(\theta,\lambda,t) = H_i(\theta,\lambda)cosg_i(\theta,\lambda)cos\phi_i(t) + H_i(\theta,\lambda)sing_i(\theta,\lambda)sin\phi_i(t)$$
$$= H_i^+(\theta,\lambda)cos\phi_i(t) + H_i^-(\theta,\lambda)sin\phi_i(t) = H_i^+cos\phi_i + H_i^-sin\phi_i \tag{4.2}$$

On the other hand, through the spherical harmonic analysis, the tidal height $T_i$ of the tidal constituent $\sigma_i$ can be also expressed as the normalized spherical harmonic series

$$T_i(\theta,\lambda,t) = \sum_{n=1}^{N}\sum_{m=0}^{n}\left[T_{i,nm}^+(\lambda,t) + T_{i,nm}^-(\lambda,t)\right]\bar{P}_{nm}(cos\theta) \tag{4.3}$$

Where,
$$T_{i,nm}^+(\lambda,t) = \bar{C}_{i,nm}^+cos(\phi_i + m\lambda) + \bar{S}_{i,nm}^+sin(\phi_i + m\lambda) \tag{4.4}$$
$$T_{i,nm}^-(\lambda,t) = \bar{C}_{i,nm}^-cos(\phi_i - m\lambda) + \bar{S}_{i,nm}^-sin(\phi_i - m\lambda) \tag{4.5}$$

In (4.4) and (4.5), the superscript + is the normalized spherical harmonic coefficient of the in-phase amplitude $(H_icosg_i)$ of the tidal constituent $\sigma_i$, and the superscript − is the normalized spherical harmonic coefficient of the out-of-phase amplitude $(H_ising_i)$ of the tidal constituent $\sigma_i$.

Expand the trigonometric functions in the formulas (4.4) and (4.5), we have

$$T_{i,nm}^+(\lambda,t) = \bar{C}^+[cos\phi_icosm\lambda - sin\phi_isinm\lambda] + \bar{S}^+[sin\phi_icosm\lambda + cos\phi_isinm\lambda]$$
$$= [\bar{C}^+cosm\lambda + \bar{S}^+sinm\lambda]cos\phi_i + [-\bar{C}^+sinm\lambda + \bar{S}^+cosm\lambda]sin\phi_i \tag{4.6}$$
$$T_{i,nm}^-(\lambda,t) = \bar{C}^-[cos\phi_icosm\lambda + sin\phi_isinm\lambda] + \bar{S}^-[sin\phi_icosm\lambda - cos\phi_isinm\lambda]$$
$$= [\bar{C}^-cosm\lambda - \bar{S}^-sinm\lambda]cos\phi_i + [\bar{C}^-sinm\lambda + \bar{S}^-cosm\lambda]sin\phi_i \tag{4.7}$$

Comparing the formula (4.2) and formula (4.3), for the tidal constituent $\sigma_i$, (the tidal constituent number $i$ is omitted below), we have

$$H^+ = \sum_{n=1}^{N}\sum_{m=0}^{n}\bar{P}_{nm}\left[(\bar{C}^+ + \bar{C}^-)cosm\lambda + (\bar{S}^+ - \bar{S}^-)sinm\lambda\right] \tag{4.8}$$
$$H^- = \sum_{n=1}^{N}\sum_{m=0}^{n}\bar{P}_{nm}\left[(\bar{S}^+ + \bar{S}^-)cosm\lambda + (-\bar{C}^+ + \bar{C}^-)sinm\lambda\right] \tag{4.9}$$
$$\bar{C}^+ = \hat{C}^+sin\varepsilon^+,\quad \bar{C}^- = \hat{C}^-sin\varepsilon^-,\quad \bar{S}^+ = \hat{C}^+cos\varepsilon^+, \bar{S}^- = \hat{C}^-cos\varepsilon^- \tag{4.10}$$

Where, $\varepsilon_i$ is the phase bias of constituent $\sigma_i$, which is defined by the sign of the maximum amplitude $H_i$ of the equilibrium tidal height of the constituent $\sigma_i$ (Cartwright & Eden, 1973), as shown in Tab 4.1. The value of $H_i$ is shown in the last column of Tab 1.7 ~ Tab 1.12.

**Table 4.1 Values of the phase bias $\varepsilon_i$ according to the sign of $H_i$**

| | | $H_i > 0$ | $H_i < 0$ |
|---|---|---|---|
| $m = 0$ | long period | $\pi$ | 0 |
| $m = 1$ | diurnal | $\pi/2$ | $-\pi/2$ |
| $m = 2$ | semi-diurnal | 0 | $\pi$ |

**(2) The direct influences of ocean tidal loads to the geopotential coefficients**

According to the universal gravitation theorem, the gopotential $V^{ot}$ directly generated by ocean tidal load can be expressed as

$$V^{ot}(\theta,\lambda,t) = G\rho_w \iint_S \frac{H(\theta',\lambda',t)}{L} dS \tag{4.11}$$

where $H$ is the instantaneous ocean tidal height, $S$ represents the whole sea surface and $L$ is the spatial distance between the calculation point $\boldsymbol{e} = (\theta,\lambda)$ and the sea surface moving area-element $\boldsymbol{e}' = (\theta',\lambda')$. $L$ can be expressed by Legendre function series as

$$\frac{1}{L} = \frac{1}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1} P_n(cos\psi) \tag{4.12}$$

From the spherical harmonic function addition theorem, we have:

$$P_n(\psi_k) = P_n(\boldsymbol{e},\boldsymbol{e}_k) = \frac{4\pi}{2n+1}\sum_{m=-n}^{n}\bar{Y}_{nm}(\boldsymbol{e})\bar{Y}_{nm}(\boldsymbol{e}_k) \tag{4.13}$$

Substituting the formula (4.13) into (4.12), and then substituting the formula (4.12) into (4.11), the following integral relationship between the global instantaneous tidal height $H$ and their direct influence to geopotential coefficient variations $(\Delta\bar{C}_{nm},\Delta\bar{S}_{nm})$ can be obtained:

$$\begin{bmatrix}\Delta\bar{C}_{nm}\\ \Delta\bar{S}_{nm}\end{bmatrix} = \frac{G\rho_w}{g_0(2n+1)}\int_0^{2\pi}\int_0^{\pi} H\bar{P}_{nm}(cos\theta)\begin{bmatrix}cosm\lambda\\ sinm\lambda\end{bmatrix} sin\theta d\theta d\lambda \tag{4.14}$$

Here, $g_0 \approx GM/R^2$ is the mean gravity on the global sea surface.

The harmonic constants (amplitude $H_i$ and phase $g_i$ at Greenwich) of the tidal constituent $\sigma_i$ are replaced by the spherical harmonic functions of the in-phase amplitude $H_i cosg_i$ and out-of-phase amplitude $H_i sing_i$, and substituted into formula (4.1), so that the instantaneous tidal height $H(\theta,\lambda,t)$ is expanded as follows:

$$H(\theta,\lambda,t) = \sum_{\sigma_i}\sum_{n=1}^{N}\sum_{m=0}^{n}\bar{P}_{nm}(sin\theta)\sum_{+}^{-} H_{i,nm}^{\pm}(\lambda,t) \tag{4.15}$$

$$H_{i,nm}^{\pm}(\lambda,t) = \bar{C}_{i,nm}^{\pm}cos(g_i+\varepsilon_i \pm m\lambda) + \bar{S}_{i,nm}^{\pm}sin(g_i+\varepsilon_i \pm m\lambda) \tag{4.16}$$

Where, $\left(\bar{C}_{i,nm}^{\pm},\bar{S}_{i,nm}^{\pm}\right)$ are called as the prograde and retrograde normalized spherical harmonic coefficients of the tidal constituent $\sigma_i$ with degree-n and order-m, which can be expressed in terms of the harmonic amplitude $\hat{C}_{i,nm}^{\pm}$ and phase bais $\varepsilon_{i,nm}^{\pm}$ as:

$$\bar{C}_{i,nm}^{\pm} = \hat{C}_{i,nm}^{\pm}sin\varepsilon_{i,nm}^{\pm},\quad \bar{S}_{i,nm}^{\pm} = \hat{C}_{i,nm}^{\pm}cos\varepsilon_{i,nm}^{\pm} \tag{4.17}$$

Substituting the formula (4.15) into (4.14), considering the formulas (4.18) and (4.19), the geopotential coefficient variations can be expressed as:

$$\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm} = \sum_{\sigma_i}\left(C_{i,nm}^{\pm}, \mp iS_{i,nm}^{\pm}\right)e^{\pm i\phi_i} \tag{4.18}$$

Comparing the formulas (4.28) and (4.16), we have:

$$C_{i,nm}^{\pm} = \frac{4\pi G\rho_w}{g_0(2n+1)}\hat{C}_{i,nm}^{\pm}sin\left(\varepsilon_{i,nm}^{\pm}+\varepsilon_i\right) \tag{4.19}$$

$$S_{i,nm}^{\pm} = \frac{4\pi G\rho_w}{g_0(2n+1)}\hat{C}_{i,nm}^{\pm}cos\left(\varepsilon_{i,nm}^{\pm}+\varepsilon_i\right) \tag{4.20}$$

Using the formulas (4.19) and (4.20), the harmonic constant grid model represented by the tidal amplitude and phase at Greenwich of the tidal constituent can be transformed into the normalized spherical harmonic coefficients of the constituent, and the direct influence of ocean tidal load to geopotential coefficients $(\Delta\bar{C}_{nm}, \Delta\bar{S}_{nm})$ can be calculated according to the formula (4.18). Furthermore, the formulas (2.6) ~ (2.20) in which the surface load spherical harmonic coefficient variations $\{\Delta\bar{C}_{nm}^{w}, \Delta\bar{S}_{nm}^{w}\}$ replaced by the direct influence of ocean tidal load to the geopotential coefficients $(\Delta\bar{C}_{nm}, \Delta\bar{S}_{nm})$ are the algorithm formulas of the ocean tidal load effects on all-element geodetic variations.

### 8.4.2 Calculation and analysis of load effects of ocean and atmosphere tide

**(1) Construction of ocean and surface atmosphere tidal load spherical harmonic coefficient model**

Taking the global ocean tide model FES2014b-extrapolated (the harmonic constant models with 34 ocean tidal constituents) from Centre national d 'études spatiales (CNES) of France as an example, here introduces the harmonic analysis process of global ocean tidal harmonic constant models and construction of ocean tidal load spherical harmonic coefficient model.

FES2014 is a global ocean tide assimilation model based on fluid dynamics launched by CNES in 2016. FES2014 assimilates a variety of satellite altimetry and global gauge measured data since 1990. Among them, FES2014b-extrapolated optimizes the coverage of sea surface altimetric data in near-shore shallow waters through satellite altimetry waveform resampling. FES2014b consist of 34 tidal constituent harmonic constant models ($2N_2$, $Eps_2$, $J_1$, $K_1$, $K_2$, $L_2$, $La_2$, $M_2$, $M_3$, $M_4$, $M_6$, $M_8$, $M_f$, $MKS_2$, $M_m$, $MN_4$, $MS_4$, $MSf$, $MSqm$, $Mtm$, $Mu_2$, $N_2$, $N_4$, $Nu_2$, $O_1$, $P_1$, $Q_1$, $R_2$, $T_2$, $S_1$, $S_2$, $S_4$, $Sa$ and $Ssa$) and the spatial resolution is 3.75′×3.75′.

**Step 1:** Gridate the 34 tidal harmonic constants of the FES2014b tidal harmonic constant models to generate 34 tidal constituents of 1°×1°, 30′×30′, 15′×15′ and 10′×10′ harmonic constant spherical coordinate grid models, respectively. Four spatial resolutions will be employed to analyze the maximum appropriate degree of the ocean tidal load spherical harmonic coefficient model.

**Step 2:** For any tidal harmonic constant spherical coordinate grid model, using the formulas (4.4) ~ (4.6), make spherical harmonic analysis on the global grid of in-phase amplitude and out-of-phase amplitude according to the FFT algorithm, and generate the 34 normalized spherical harmonic coefficient model of in-phase amplitude and out-of-phase amplitude of tidal constituent $\sigma_i$. Similar to the spherical harmonic analysis of surface load in Section 3.2, the cumulative iterative spherical harmonic analysis method can effectively improve the approach level.

**Step 3:** Using the formulas (4.20) ~ (4.22), 34 normalized spherical harmonic coefficient models $(\bar{C}_i^+, \bar{S}_i^+, \bar{C}_i^-, \bar{S}_i^-)$ of in-phase amplitude and out-of-phase amplitude of tidal constituent $\sigma_i$ are combined to obtain a normalized spherical harmonic coefficient model of global ocean tide load according to the conventional format. The ocean tidal load spherical harmonic coefficient model FES2004 format in IERS convertions (2010) is adopted here.

**Step 4:** On the basis of the above 34 tidal load spherical harmonic coefficient models, the equilibrium tide $\Omega_1$ and $\Omega_2$ load spherical harmonic coefficients are selected from the FES2004S1.dat in IERS convertions (2010), and the FES2014b ocean tidal load spherical harmonic coefficient model composed of 36 tidal constituents is constructed.

The maximum degree of the load spherical harmonic model of some a tidal constituent is equal to the number of cell-grids in the latitude direction in the harmonic constant grid of the tidal constituent. The grid resolution of different constituents or the maximum degree of their spherical harmonic coefficient models need not be consistent. The maximum degree of the ocean tidal load spherical harmonic coefficient model is the maximum degree of all the tidal constituents. The unit for ocean tidal load spherical harmonic coefficient is consistent with that for ocean tidal height. The unit in this example is cm.

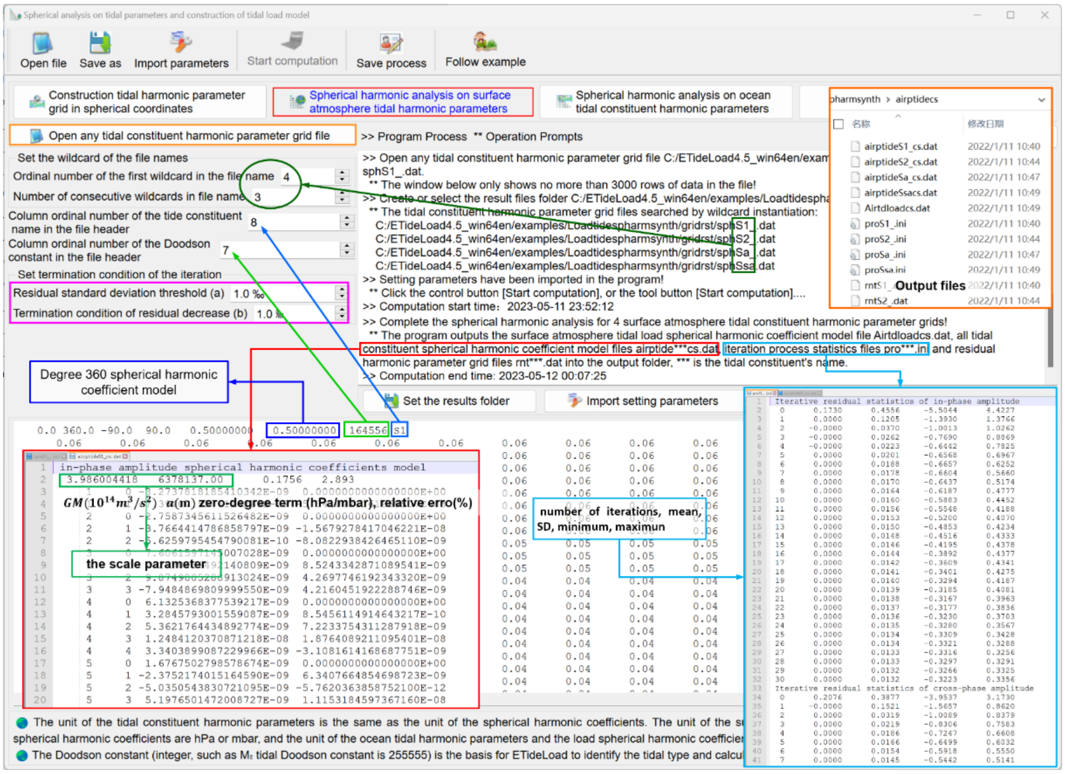
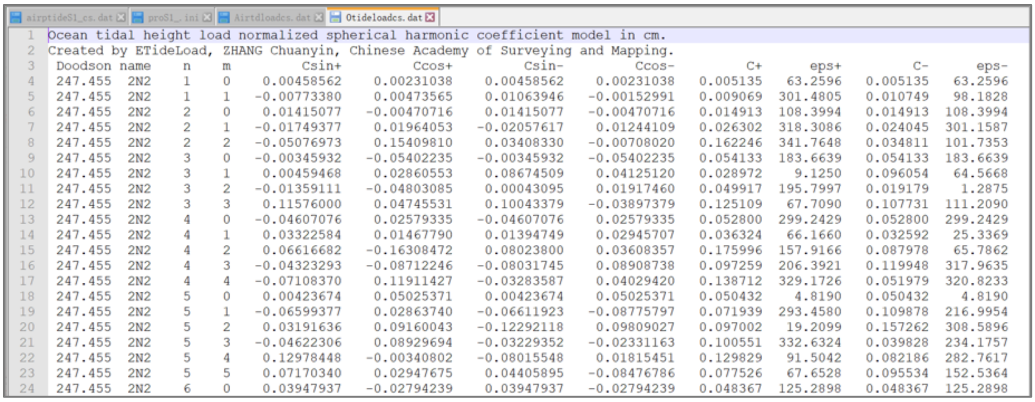



**Fig 4.1 ETideLoad4.5 program for Spherical harmonic analysis on ocean tidal constituent harmonic constants**

Fig.4.1 is the ETideLoad4.5 program for spherical harmonic analysis on ocean tidal constituent harmonic constant grid. The program reads 34 tidal 30'×30' harmonic constant spherical coordinate grids, and employs the cumulative approach method to perform spherical harmonic analysis on each tidal harmonic constant. Where, the $M_2$ tidal spherical harmonic coefficient model is shown in the lower left figure, and the iterative residual vaiations are shown in the lower right figure. The 360-degree FES2014b360cs global ocean tidal load spherical harmonic coefficient model is constructed, as shown in Fig 4.2.

The load spherical harmonic coefficient model for each tidal constituent is composed of the same in-phase amplitude spherical harmonic coefficient model and the out-of-phase amplitude spherical harmonic coefficient model. The file header are the geocentric gravitational constant GM ($\times 10^{14} m^3/s^2$), equatorial radius a (m) of the Earth, zero-degree term $a\Delta C_{00}$ (cm), relative error $\Theta$ (%).



```
airptideS1_cs.dat   proS1_.ini   Airtdloadcs.dat   Otideloadcs.dat
Ocean tidal height load normalized spherical harmonic coefficient model in cm.
Created by ETideLoad, ZHANG Chuanyin, Chinese Academy of Surveying and Mapping.
 Doodson name    n    m        Csin+         Ccos+         Csin-         Ccos-        C+      eps+        C-      eps-
 247.455  2N2    1    0   0.00458562    0.00231038    0.00458562    0.00231038  0.005135   63.2596  0.005135   63.2596
 247.455  2N2    1    1  -0.00773380    0.00473565    0.01063946   -0.00152991  0.009069  301.4805  0.010749   98.1828
 247.455  2N2    2    0   0.01415077   -0.00470716    0.01415077   -0.00470716  0.014913  108.3994  0.014913  108.3994
 247.455  2N2    2    1  -0.01749377    0.01964053   -0.02057617    0.01244109  0.026302  318.3086  0.024045  301.1587
 247.455  2N2    2    2  -0.05076973    0.15409810    0.03408330   -0.00708020  0.162246  341.7648  0.034811  101.7353
 247.455  2N2    3    0  -0.00345932   -0.05402235   -0.00345932   -0.05402235  0.054133  183.6639  0.054133  183.6639
 247.455  2N2    3    1   0.00459468    0.02860553    0.08674509    0.04125120  0.028972    9.1250  0.096054   64.5668
 247.455  2N2    3    2  -0.01359111   -0.04803085    0.00043095    0.01917460  0.049917  195.7997  0.019179    1.2875
 247.455  2N2    3    3   0.11576000    0.04745531    0.10043379   -0.03897379  0.125109   67.7090  0.107731  111.2090
 247.455  2N2    4    0  -0.04607076    0.02579335   -0.04607076    0.02579335  0.052800  299.2429  0.052800  299.2429
 247.455  2N2    4    1   0.03322584    0.01467790    0.01394749    0.02945707  0.036324   66.1660  0.032592   25.3369
 247.455  2N2    4    2   0.06616682   -0.16308472    0.08023800    0.03608357  0.175996  157.9166  0.087978   65.7862
 247.455  2N2    4    3  -0.04323293   -0.08712246   -0.08031745    0.08908738  0.097259  206.3921  0.119948  317.9635
 247.455  2N2    4    4  -0.07108370    0.11911427   -0.03283587    0.04029420  0.138712  329.1726  0.051979  320.8233
 247.455  2N2    5    0   0.00423674    0.05025371    0.00423674    0.05025371  0.050432    4.8190  0.050432    4.8190
 247.455  2N2    5    1  -0.06599377    0.02863740   -0.06611923   -0.08775797  0.071939  293.4580  0.109878  216.9954
 247.455  2N2    5    2   0.03191636    0.09160043   -0.12292118    0.09809027  0.097002   19.2099  0.157262  308.5896
 247.455  2N2    5    3  -0.04622306    0.08929694   -0.03229352   -0.02331163  0.100551  332.6324  0.039828  234.1757
 247.455  2N2    5    4   0.12978448   -0.00340802   -0.08015548    0.01815451  0.129829   91.5042  0.082186  282.7617
 247.455  2N2    5    5   0.07170340    0.02947675    0.04405895   -0.08476786  0.077526   67.6528  0.095534  152.5364
 247.455  2N2    6    0   0.03947937   -0.02794239    0.03947937   -0.02794239  0.048367  125.2898  0.048367  125.2898
```

**Fig 4.2 Ocean tidal load spherical harmonic coefficient model FES2014b360cs.dat**

For high-precision geodesy, the short-wave component of the ocean tidal load effects cannot be ignored, and a high-degree spherical harmonic coefficient model is usually required. Tab 4.2 show the residual change of ocean tidal load spherical harmonic analysis with the resolution of tidal harmonic constant grid or the maximum degree of load spherical harmonic coefficient model.

**Tab 4.2 Residual change of ocean tidal load spherical harmonic analysis with the resolution of tidal harmonic constant grid**

| Input grid resolution | Maximum degree | Name | Tidal constituent | First-degree term×10⁻⁸ | | | Relative error (%) |
|---|---|---|---|---|---|---|---|
| | | | | $\Delta\bar{C}_{10}$ | $\Delta\bar{C}_{11}$ | $\Delta\bar{S}_{11}$ | |
| 1°×1° | 180 | $K_1$ | in-phase | 6.5903 | 15.2405 | 5.7951 | 15.109 |
| | | | out-of-phase | -23.6187 | 5.4510 | 9.1115 | 13.080 |
| | | $M_2$ | in-phase | 6.4087 | 8.2092 | -3.9331 | 16.593 |
| | | | out-of-phase | 3.3741 | 0.7698 | 7.4235 | 14.206 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 30′×30′ | 360 | $K_1$ | in-phase | 6.7466 | 14.4650 | 5.6522 | 10.522 |
| | | | out-of-phase | -23.9366 | 5.5500 | 9.2329 | 9.785 |
| | | $M_2$ | in-phase | 6.3545 | 7.5901 | -4.2676 | 11.266 |
| | | | out-of-phase | 4.3474 | -0.2498 | 5.9033 | 10.673 |
| 15′×15′ | 720 | $K_1$ | in-phase | 6.7290 | 14.1161 | 5.5337 | 7.549 |
| | | | out-of-phase | -23.9978 | 5.5530 | 9.3081 | 7.069 |
| | | $M_2$ | in-phase | 6.3464 | 7.5080 | -4.5272 | 7.980 |
| | | | out-of-phase | 4.7902 | -0.6035 | 5.1936 | 7.687 |
| 10′×10′ | 1080 | $K_1$ | in-phase | 6.6860 | 14.0149 | 5.4796 | 6.161 |
| | | | out-of-phase | -23.9629 | 5.5763 | 9.3395 | 5.922 |
| | | $M_2$ | in-phase | 6.2795 | 7.5429 | -4.6921 | 6.867 |
| | | | out-of-phase | 4.9361 | -0.7832 | 4.9103 | 6.435 |

Tab 4.2 shows that the short and medium wave components of ocean tides are obvious. Considering both accuracy and computational efficiency, the appropriate maximum degree of the ocean tidal load spherical harmonic coefficient model can be selected as 720.

Similarly, from the global surface atmosphere tidal harmonic constant grid models, can construct the surface atmosphere tidal load spherical harmonic coefficient model by the spherical harmonic analysis. The 360-degree surface atmosphere tidal load spherical harmonic coefficient model ECMWF2006cs360.dat in ETideLoad4.5 were constructed according to the process above from the 0.5˚× 0.5˚ global harmonic constant grids of the four atmospheric pressure tidal constituents ($S_1, S_2, Ss_a, Sa$). The unit for surface atmosphere tidal load spherical harmonic coefficient is consistent with that surface atmospherical pressure tide. The unit in this example is hPa.

Tab 4.3 show the residual change of surface atmosphere tidal load spherical harmonic analysis with the resolution of surface atmosphere tidal harmonic constant grid or the maximum degree of load spherical harmonic coefficient model.

Tab 4.3 shows that the medium and long waves of the global surface atmospheric pressure tide are dominant. Considering both accuracy and computational efficiency, the appropriate maximum degree of the surface atmospherr tidal load spherical harmonic coefficient model can be selected from 180 to 360.

**Tab 4.3 Residual change of surface atmosphere tidal load spherical harmonic analysis with the resolution of atmosphere tidal harmonic constant grid**

| Input grid resolution | Maximum degree | Name | Tidal constituent | First-degree term×$10^{-8}$ | | | Relative error (%) |
|---|---|---|---|---|---|---|---|
| | | | | $\Delta\bar{C}_{10}$ | $\Delta\bar{C}_{11}$ | $\Delta\bar{S}_{11}$ | |
| 1°×1° | 180 | $S_1$ | in-phase | -0.3276 | -0.7396 | -5.3411 | 4.378 |
| | | | out-of-phase | 0.1765 | -4.3745 | -0.1072 | 4.335 |
| | | $S_2$ | in-phase | -0.0630 | 0.0080 | 0.3390 | 1.238 |
| | | | out-of-phase | 0.1374 | 0.5236 | -0.1086 | 1.365 |
| | | $S_{Sa}$ | in-phase | 0.6526 | -3.5846 | 1.2772 | 3.841 |
| | | | out-of-phase | 6.4837 | -2.5040 | 2.4911 | 1.158 |
| | | $S_a$ | in-phase | 8.2106 | -3.5243 | 3.5038 | 1.488 |
| | | | out-of-phase | -16.1599 | -0.8292 | -12.1651 | 2.554 |
| 30′×30′ | 360 | $S_1$ | in-phase | -0.3274 | -0.7396 | -5.3408 | 2.927 |
| | | | out-of-phase | 0.1765 | -4.3747 | -0.1074 | 2.617 |
| | | $S_2$ | in-phase | -0.0630 | 0.0077 | 0.3391 | 0.848 |
| | | | out-of-phase | 0.1374 | 0.5237 | -0.1087 | 0.903 |
| | | $S_{Sa}$ | in-phase | 0.6528 | -3.5850 | 1.2760 | 1.871 |
| | | | out-of-phase | 6.4837 | -2.5041 | 2.4916 | 0.646 |
| | | $S_a$ | in-phase | 8.2104 | -3.5242 | 3.5038 | 0.839 |
| | | | out-of-phase | -16.1596 | -0.8291 | -12.1654 | 1.276 |

**(2) Spherical harmonic synthesis and analysis of ocean tidal load effects**

The ocean tidal loads are located on the sea surface, and the height of the calculation point relative to the sea surface is orthometric (or normal) height $h$.

In the following, three calculation points are selected: $P_1$(105°E, 32°N, h720m) in the inland area above 400km from the coastline, $P_2$(121.3°E, 28.8°N, h11m) in the coastal zone and $P_3$(123.47°E, 25.75°N, h3m) on the sea island 200km away from the coastline. The 720-degree global ocean tidal load spherical harmonic coefficient model FES2014b720cs.dat is employed to calculate the ocean tidal load effect time series on all-element variations at these three ground points. The time span of the time series is from January 1, 2020 to January 31, 2020, with a time interval of 30 minutes. Comparing the similarities and differences of the ocean tidal load effect time series at three points in different regions, the spatial and time-varying characteristics of the ocean tidal load effects are analyzed.

Fig 4.3 is the ocean tidal load effect time series on geodetic variations at $P_1$ point in the inland area. It is shown in the figure that even in the inland areas more than 400km away from the coastline, the difference between the maximum and minimum values of the ocean tidal load effects on geoid can reach 8mm, that on ellipsoidal height can reach 15mm, that on normal height can reach 22mm, that on horizontal displacement can reach 8mm, that on ground tilt can reach 4.5mas, that on radial gravity gradient can reach 2.6mE and that on horizontal gravity gradient can reach 1.9mE. It can be seen that even in inland areas, centimeter-level precision geodesy should also take into account the ocean tidal load effects.

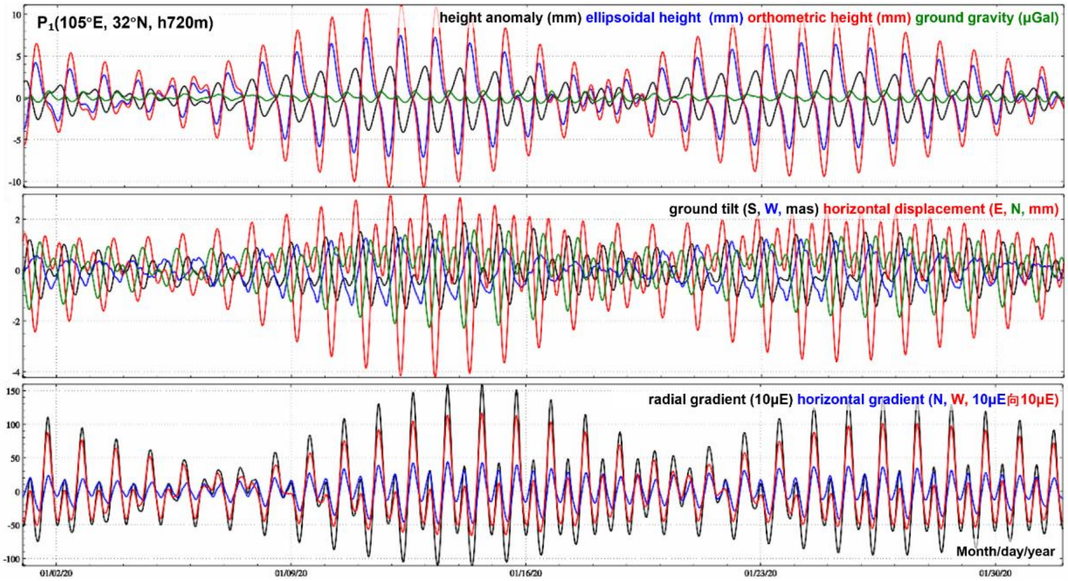



**Fig 4.3 The ocean tidal load effect time series on geodetic variations at $P_1$ point in the inland area**

Fig 4.4 is the ocean tidal load effect time series on geodetic variations at $P_2$ point on the coast. It is shown in the figure that the ocean tidal load effect in the coastal zone is generally 10 times that of the inland $P_1$ point. At $P_2$ point, the difference between the maximum and minimum values of the ocean tidal load effects on geoid can reach 6.8cm, that on normal height can reach 20cm, that on ground gravity can reach 250μGal, that on ground tilt can reach 110mas, that on horizontal displacement can reach 3.1cm, that on horizontal gravity gradient can reach 42mE and that on radial gravity gradient can reach 30.5mE.

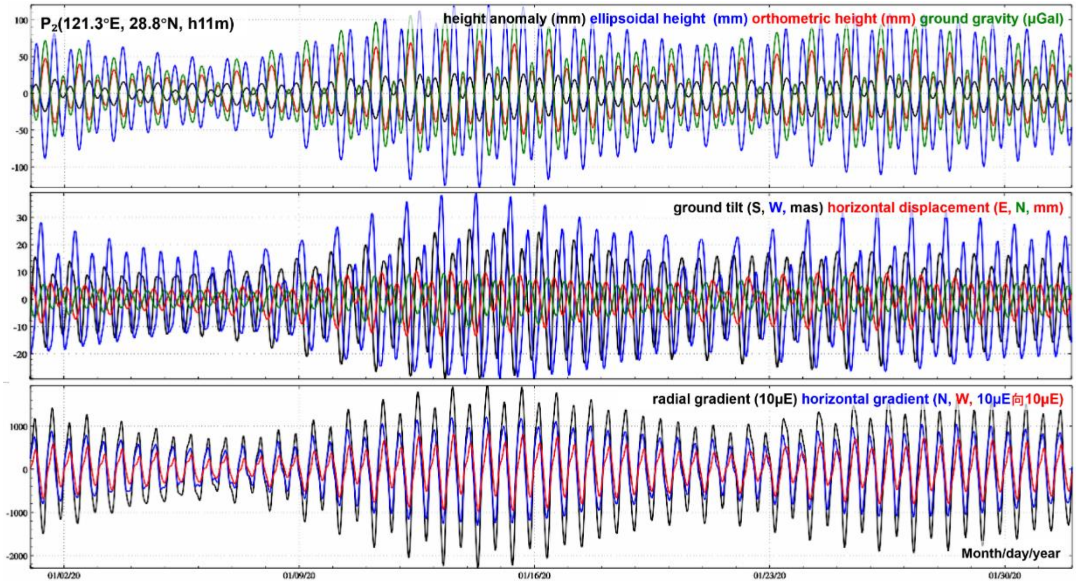



**Fig 4.4 The ocean tidal load effect time series on geodetic variations at $P_2$ point on the coastal zone**

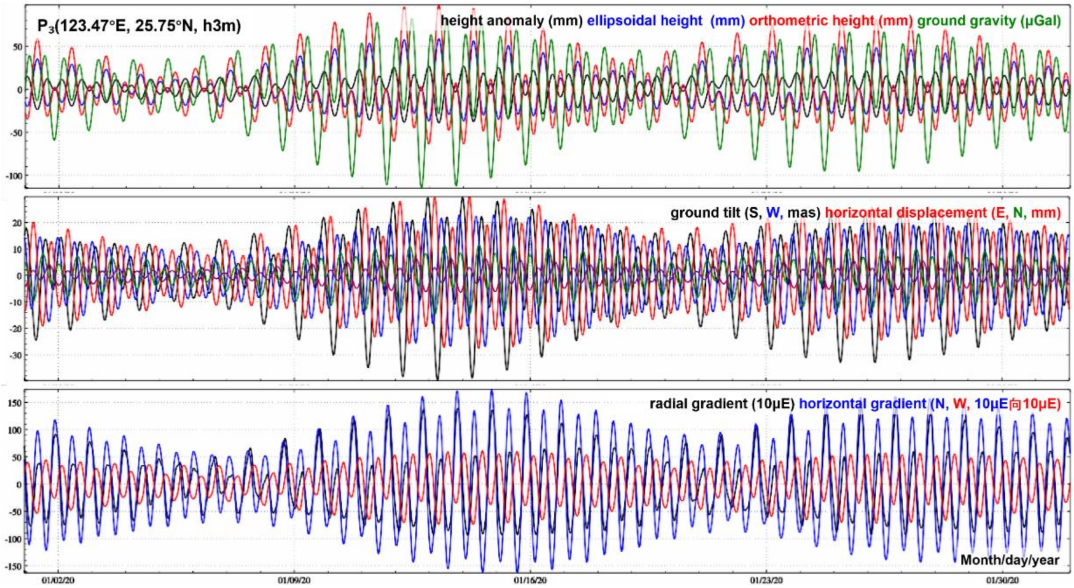



**Fig 4.5 The ocean tidal load effect time series on geodetic variations at $P_3$ point on offshore island**

Fig 4.5 is the ocean tidal load effect time series on geodetic variations at $P_3$ point on 200 km offshore island. It is shown in the figure that after a certain distance from the shore, the amplitude of the ocean tide becomes lower, the tidal wave structure is simpler than that of the near shore, and the medium and short wave parts of the ocean tidal load effect are weakened. At $P_3$ point, the difference between the maximum and minimum values of the ocean tidal load effects on geoid can reach 6.6cm, that on normal height can reach 16cm, that on ground gravity can reach 70μGal, that on ground tilt can reach 76mas, that on horizontal displacement can reach 2.8cm, that on horizontal gravity gradient can reach 2.3mE and that on radial gravity gradient can reach 3.5mE.

Different from the solid Earth tidal effects, the ocean tidal load effect on ground normal height is in phase with that on ground ellipsoidal height (the two sign are consistent). In most areas, the amplitude of ocean tidal load effect on ground normal height is about 1.5 times that on ground ellipsoidal height. In coastal waters, the ocean tidal load effects on gravity gradient and ground tilt are generally much greater than the solid Earth tidal effects on that.

**(2) Spherical harmonic synthesis and analysis of surface atmosphere tidal load effects**

In the following, using the same calculation scheme as the ocean tidal load effects, the 360-degree global surface atmosphere tidal load spherical harmonic coefficient model ECMWF2006n360cs.dat is employed to calculate the atmosphere tidal load effect time series on all-element variations at the ground point P(105°N, 20°E). The time span of the time series is from January 1,2018 to December 31,2020 (3 years), with a time interval of 30 minutes, as shown in Fig 4.6.

It is slightly different from the spherical harmonic synthesis calculation of ocean tidal load effect. When calculating the indirect influence of atmosphere loads, it is assumed that the atmosphere loads are concentrated on the ground, and the calculation point height $h$ is the height of the point relative to the ground. When calculating the direct influence of atmosphere loads to gravity and radial gravity gradient, it is assumed that there is the proportional relationship $(1-h/44330)^{5225}$ between atmospheric pressure $P_h$ at ground height h and ground atmospheric pressure $P_0$.

Fig 4.6 shows that the difference between the maximum and minimum of the atmosphere tidal load effect on ground normal height can reach 2cm. For every 1hPa increases in the ground atmospheric pressure, the ground normal height is reduced by about 1mm, that is, the atmosphere tidal load effect admittance of the ground normal height is close to −1.0mm/hPa. The annual periodic amplitude of surface atmospheric pressure tide is 3 ~ 5 times of the diurnal amplitude. In the inland area, the surace atmospheric pressure is high in winter and low in summer, which leads to the decrease of the ground in winter and the uplift in summer, resulting in the ground vertical deformation of the annual and semi-annual period, which should be taken into account in the centimeter-level geodesy.

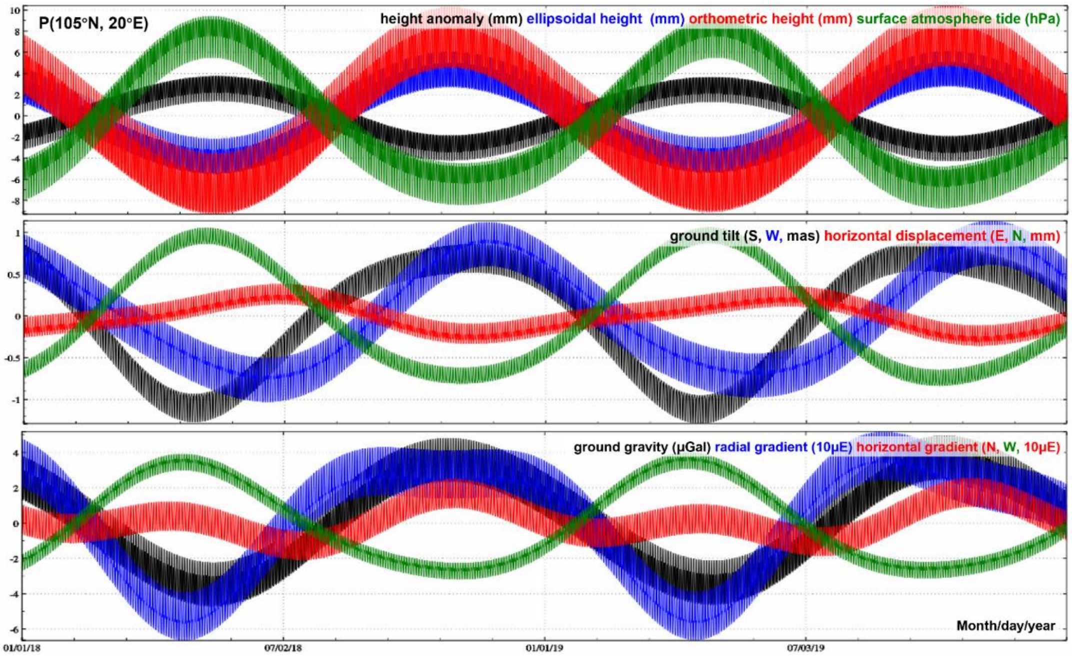



**Fig 4.6 The surface atmosphere tidal load effect time series on geodetic variations**

### 8.4.3 Regional refinement of Green's integral method for ocean tidal load effects

Taking the global ocean tidal load spherical harmonic coefficient model as the reference field and using the remove-restore scheme, the accuracy of the ocean tidal load effect can be further improved by the regional high-precision and high-resolution harmonic constant grid models. The scheme can be generally composed of the following five steps, which can be called as the ‘Remove - load Green’s function integral - Restore’ scheme.

(a) The global ocean tidal load spherical harmonic coefficient model employed as the reference field and the regional high-resolution ocean tide harmonic constant grid models are selected, and the regional ocean tide harmonic constant reference value grids are calculated from the ocean tidal load spherical harmonic coefficient model.

(b) From the regional high-resolution ocean tide harmonic constant grid model, the reference model value are removed to obtain the regional ocean tide harmonic constant residual grids. This step is called 'Remove'.

(c) Using a smaller integral radius, the residual value of the ocean tidal load effect at the target point is calculated using the load Green's function integral.

(d) The reference model value of ocean tidal load effects at the target point is calculated from the global ocean tidal load spherical harmonic coefficient model.

(e) The refine value of the ocean tidal load effects at the target point is obtained by adding the residual value of the ocean tide load effects to the reference model value. This step is called 'Restore'.

In the following, we still select two calculation points of $P_2$ (121.3°E, 28.8°N, h11m) located in the coastal zone and $P_3$ (123.47°E, 25.75°N, h3m) on the sea island 200 km away from the coastline, and the 720-degree global ocean tide load spherical harmonic coefficient model FES2014b720cs.dat is employed as the ocean tidal load reference field. From the 1.2′×1.2′ high-precision ocean tide model TMchinaR1 (J. Xu, J. Y. Bao, 2008) composed of 10 tidal harmonic constant models, the residual value and refine value time series of the ocean tidal load effect on all-element geodetic variations at these two points are calculated according to the ‘Remove - load Green's function integral – Restore’ scheme. The time span of the time series is from 0 : 00 on January 1, 2020 to 24 : 00 on January 31, 2020, with a time interval of 30 minutes.

Firstly, the 1.2′×1.2′ ocean tide harmonic constant model value grids in China offshore are calculated from the 720-degree ocean tidal load spherical harmonic coefficient model FES2014b720cs.dat. The number of tidal constituents and the grid specification are the same as those of the high-precision ocean tide model TMchinaR1 in China offshore. Then, the 10 tidal harmonic constant grids of TMchinaR1 in China offshore are removed from the corresponding tidal harmonic constant model value grids to generate the harmonic constant residual value grids of 10 tidal constituents in China offshore.

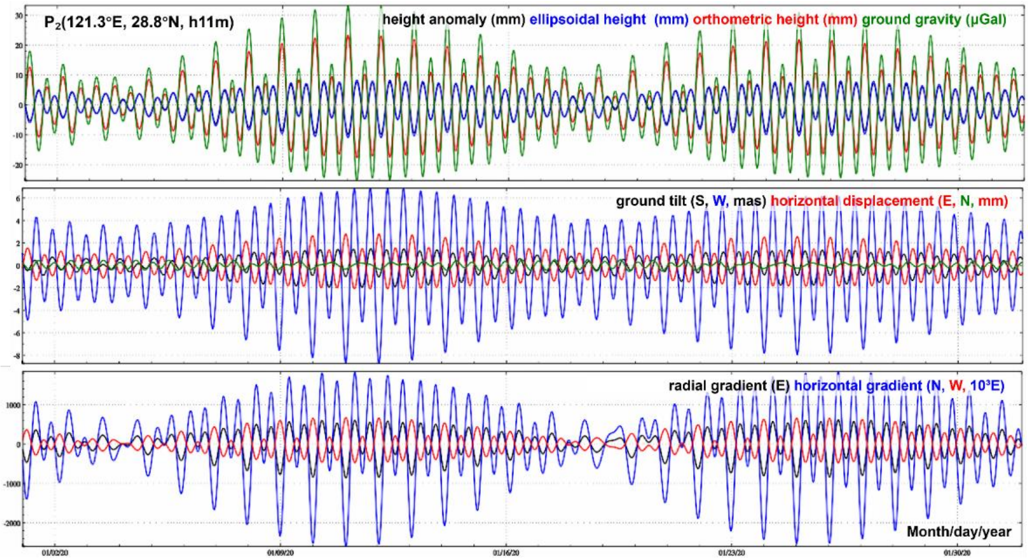



**Figure 4.7 The residual value time series of ocean tidal load effects on geodetic variations at the $P_2$ in the coastal**

Secondly, using the load Green’s function integral with the smaller integral radius (300km integral radius in this examples), the residual value time series of ocean tidal load effect on geodetic variations at $P_2$ and $P_3$ points are calculated from the 1.2′×1.2′ harmonic constant residual value grids of the 10 constituents in the coastal waters of China, as shown in Fig 4.7 and Fig 4.8.

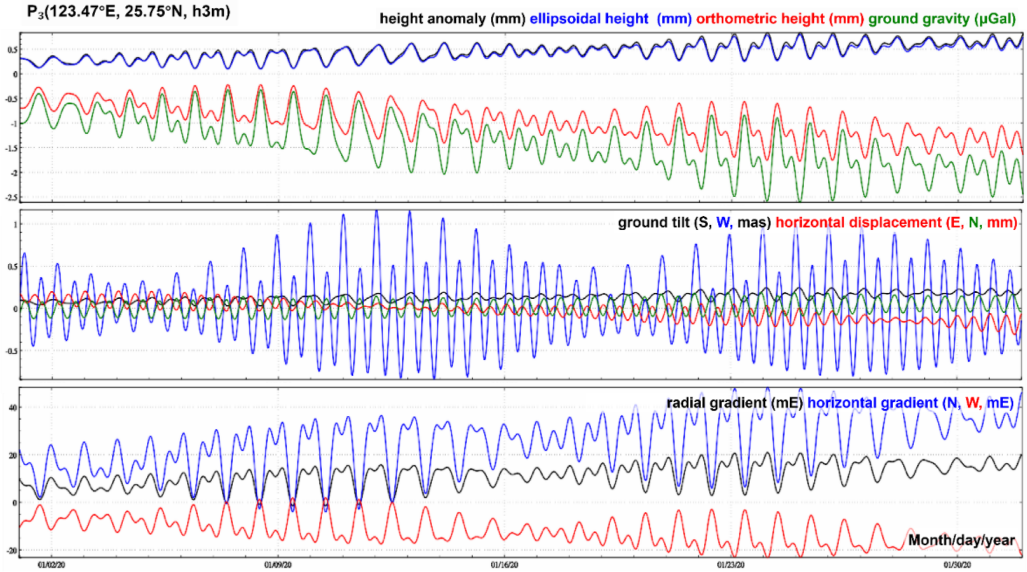



**Figure 4.8 The residual value time series of ocean tidal load effects on geodetic variations at the $P_3$ on the sea island**

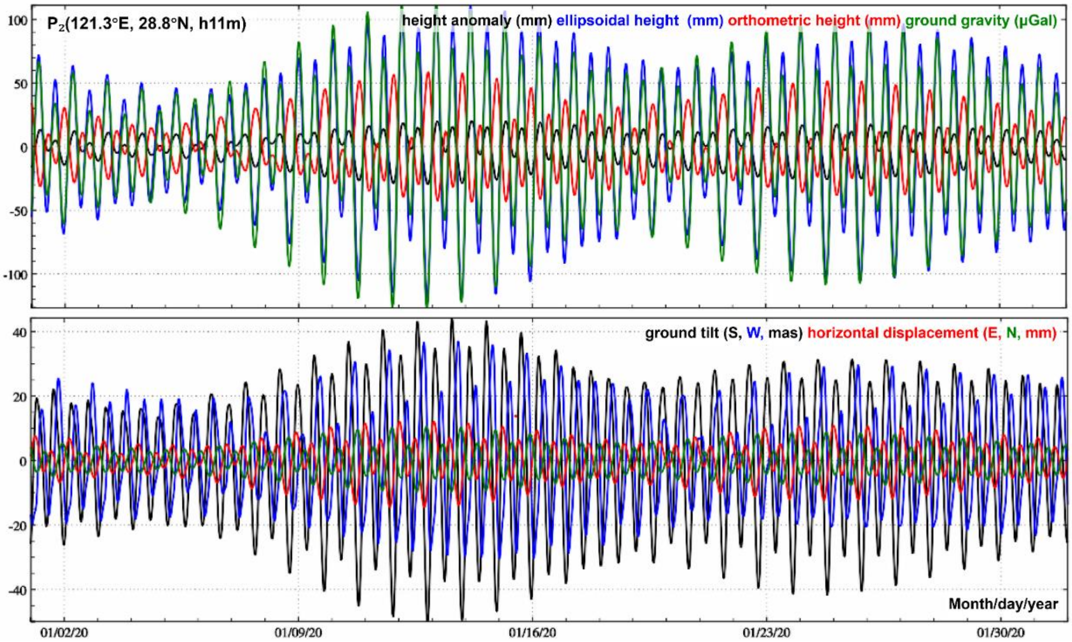



**Figure 4.9 The refine value time series of ocean tidal load effects on geodetic variations at the $P_2$ in the coastal zone**

Finally, the ocean tidal load effect model value time series at $P_2$ and $P_3$ are calculated from the global ocean tidal load spherical harmonic coefficient model, and added with the ocean tide load effect residual value time series calculated above to obtain the refine value time series of ocean tidal load effect on all-element geodetic variations at $P_2$ and $P_3$, respectively. Figure 4.9 is the refine value time series of ocean tidal load effects on geodetic variations at the $P_2$ on the coast.

It is not difficult to understand that the residual value time series of ocean tidal load effects on geodetic varitions also represents the error influence of the 720-degree FES2014b ocean tidal load spherical harmonic coefficient model to geodetic variations. Fig 4.7 and Fig 4.8 show that even for the high-degree ocean tidal load spherical harmonic coefficient model with good quality of 720-degree FES2014, the model error influence to the normal height in coastal areas can be as high as 5.9cm, and that to the geoid and ellipsoidal height can reach 1.7cm and 4.1cm, respectively, and that to the ground tilt and horizontal displacement can reach 23.6mas and 1.6cm, respectively.

In coastal areas, the error of ocean tidal spherical harmonic coefficient model has a great influence to gravity gradient, which is far beyond the magnitude of the ocean tidal load effect itself. The ocean tidal load effects on gravity gradient are dominant in the ultrashort wave parts, and the high-degree ocean tidal load spherical harmonic coefficient model FES2014b720cs cannot contain these ultrashort wave signals in coastal areas. The calculation results of the residual load effects on gravity gradient are divergent and not available using load Green's function integral.