Surface load effects on various geodetic variations by Green's Integral

The load Green's function is defined as the response function of the unit point mass load variation (kg/m²), and the load effect on the ground geodetic variation is equal to the convolution of the load Green's function and surface density $\sigma_w(=\rho_w h_w)$ of the ground load on the global surface. In general, similar to the Stokes' integral formula in the theory of Earth's gravity field, the load effect $F(\theta,\lambda)$ on any type of geodetic variation at the ground calculation point (θ,λ) can be expressed as the load Green's function integral under the spherical approximation:

$$F(\theta,\lambda) = R^2\rho_w \iint_\sigma h_w G(\psi) d\sigma \tag{3.1}$$

Where, σ is the unit spherical surface, R is the mean radius of the Earth, ψ is the spherical angular distance from the surface load area-element $d\sigma$ to the ground calculation point (θ,λ) and $G(\psi)$ is the load Green's function with the spherical angular distance ψ as the independent variable, and its form is related to the effect type.

The load Green's function integral $F(\theta,\lambda)$ here is divided into two parts. The first part is the direct influence of load, and the second part is the indirect influence of load.

$$F(\theta,\lambda) = F^d(\theta,\lambda) + R^2\rho_w \iint_\sigma h_w G^i(\psi) d\sigma \tag{3.2}$$

In the formula (3.2), (θ,λ) is the spherical coordinates of the ground calculation point. $F^d(\theta,\lambda)$ is the direct influence of load effect at the ground calculation point, which can be calculated by the load equivalent water height variation h_w according to the rigorous integral. $G^i(\psi)$ is called as the indirect influence of load Green's function.

8.3.1 The integral of direct influence of the load effect on ground geodetic variation

(1) The integral formula of direct influence of the load effect on geopotential

Given the surface load equivalent water height (EWH) variation h_w, whose direct effect V_w on the geopotential near Earth space directly given by the universal gravitation formula

$$V^d(r,\theta,\lambda) = G\rho_w \iint_S \frac{h_w}{L} dS, \quad L = \sqrt{r^2 + r'^2 - 2rr'cos\psi} \tag{3.3}$$

Where, L is the spatial distance between the calculation point (r,θ,λ) near Earth space and center (r',θ',λ') of integral area-element dS on the surface S, r,θ,λ are the spherical geocentric coordinates of the calculation point, namely distance from geocenter, co-latitude and longitude, respectively. G is Newton's gravitational constant, $\rho_w = 1000kg/m^3$ is the water density. ψ is the spherical angle between the calculation point (r,θ,λ) and center (r',θ',λ') of the area-element.

$$cos\psi = cos\theta cos\theta' + sin\theta sin\theta' cos(\lambda' - \lambda),$$

$$sin\psi = sin\theta cos\theta' + cos\theta sin\theta' cos(\lambda' - \lambda) \tag{3.4}$$

$$sin\psi cos\alpha = sin\theta cos\theta' - cos\theta sin\theta' cos(\lambda' - \lambda),$$

$$sin\psi sin\alpha = sin\theta' sin(\lambda' - \lambda) \quad (3.5)$$

$$\frac{\partial\psi}{\partial\theta} = -\frac{\partial\psi}{\partial\varphi} = cos\alpha, \ \frac{\partial\psi}{\partial\lambda} = -sin\alpha sin\theta \quad (3.6)$$

Here, α is the geodetic azimuth of ψ.

Considering $d\sigma = \psi d\psi d\alpha$, when the calculation point is located on the surface and overlaps with the center of integral area-element, we have

$$L = r\psi, \ r - r'cos\psi = r\psi^2/2 \quad (3.7)$$

$$A = dS = r^2 \int_{\alpha=0}^{2\pi} \int_0^{\psi_0} \psi d\psi d\alpha = \pi r^2 \psi_0^2 \rightarrow \psi_0 = \frac{1}{r}\sqrt{\frac{A}{\pi}} \quad (3.8)$$

Where, $A = dS$ is the area of integral area-element. In this case, the formula (3.3) on the calculation point is an integral singularity. From the formulas (3.7) and (3.8), the singular value of the integral can be obtained:

$$V_d^0(r,\theta,\lambda) = G\rho_w r^2 \int_{\alpha=0}^{2\pi} \int_0^{\psi_0} \frac{h_w}{r\psi} \psi d\psi d\alpha = 2\pi G\rho_w h_w r\psi_0 \quad (3.9)$$

(2) The integral formula of direct influence of the load effect on gravity disturbance

According to the definition of gravity disturbance, from Formula (3.3), the direct influence of the load effect on gravity disturbance at the calculation point (r,θ,λ) is:

$$\delta g^d(r,\theta,\lambda) = -\frac{\partial V^d(r,\theta,\lambda)}{\partial r} = -G\rho_w \iint_S h_w \frac{\partial}{\partial r}\left(\frac{1}{L}\right) dS = G\rho_w \iint_S h_w \frac{r - r'cos\psi}{L^3} dS \quad (3.10)$$

When the calculation point is located on the surface and overlaps with the center of integral area-element, the formula (3.10) on the calculation point is an integral singularity, and the singular value of the integral is

$$\delta g_0^d(r,\theta,\lambda) = 2\pi G\rho_w h_w \int_0^{\psi_0} \frac{\psi^2/2}{\psi^3} \psi d\psi = \pi G\rho_w h_w \psi_0 \quad (3.11)$$

(3) The integral formula of direct influence of the load effect on vertical deflection

According to the definition of vertical deflection, from Formula (3.3), the direct influence of the load effect on vertical deflection at the calculation point (r,θ,λ) is:

$$\Theta^d(r,\theta,\lambda) = \frac{1}{\gamma r}\frac{\partial V^d(r,\theta,\lambda)}{\partial\psi} = \frac{G\rho_w}{\gamma r} \iint_S h_w \frac{\partial}{\partial\psi}\left(\frac{1}{L}\right) dS = -\frac{G\rho_w}{\gamma} \iint_S h_w r' \frac{sin\psi}{L^3} dS \quad (3.12)$$

$$\xi^d(r,\theta,\lambda) = \Theta^d(r,\theta,\lambda)\frac{\partial\psi}{\partial\theta} = -\frac{G\rho_w}{\gamma} \iint_S h_w r' \frac{sin\psi}{L^3} cos\alpha dS,$$

$$\eta^d(r,\theta,\lambda) = -\Theta^d(r,\theta,\lambda)\frac{\partial\psi}{\partial\lambda} = -\frac{G\rho_w}{\gamma} sin\theta \iint_S h_w r' \frac{sin\psi}{L^3} sin\alpha dS \quad (3.13)$$

Where, γ is the normal gravity on the calculation point, and $\Theta^d(r,\theta,\lambda)$ is the direct influence of the load effect on total vertical deviation at the calculation point.

(4) The integral formula of direct influence of the load effect on radial gravity gradient

According to the definition of radial gravity gradient, we have

$$T_{rr} = \frac{\partial^2 V_w}{\partial r^2} = G\rho_w \int_S h_w \frac{\partial}{\partial r}\left(\frac{r - r'cos\psi}{L^3}\right) dS = G\rho_w \int_S h_w \left[\frac{1}{L^3} - \frac{3(r - r'cos\psi)^2}{L^5}\right] dS \quad (3.14)$$

$$\frac{\partial}{\partial r}\left(\frac{r - r'cos\psi}{L^3}\right) = \frac{1}{L^3} - \frac{3(r - r'cos\psi)}{L^4}\frac{\partial}{\partial r}L = \frac{1}{L^3} - \frac{3(r - r'cos\psi)^2}{L^5}, \quad \frac{\partial}{\partial r}L = \frac{r - r'cos\psi}{L} \quad (3.15)$$

When the calculation point is located on the surface and overlaps with the center of integral area-element, the formula (3.14) on the calculation point is an integral singularity, and the singular value of the integral is

$$T_{rr}^0 = -2\pi G\rho_w h_w r^2 \int_0^{\psi_0}\left(\frac{1}{r^3\psi^3} - \frac{3\psi^4}{4r^3\psi^5}\right)\psi d\psi$$

$$= -\frac{2\pi G\rho_w h_w}{r}\int_0^{\psi_0}\left(\frac{1}{\psi^2} - \frac{3}{4}\right) d\psi \approx \frac{\pi G\rho_w h_w}{r\psi_0^2} \quad (3.16)$$

(5) The integral formula of direct influence of the load effect on horizontal gravity gradient

$$\Gamma\Gamma = \frac{\partial^2 V_w}{r^2\partial\psi^2} = -\frac{G\rho_w}{r}\int_S h_w r' \frac{\partial}{\partial\psi}\left(\frac{sin\psi}{L^3}\right) dS = -\frac{G\rho_w}{r}\int_S h_w r'\left(\frac{cos\psi}{L^3} - \frac{3rr'sin^2\psi}{L^5}\right) dS \quad (3.17)$$

$$T_{NN} = \Gamma\frac{\partial^2\psi}{\partial\theta^2} = \frac{G\rho_w}{r}\int_S h_w r'\left(\frac{cos\psi}{L^3} - \frac{3rr'sin^2\psi}{L^5}\right) ctg\psi(1 - cos\alpha) dS \quad (3.18)$$

$$T_{WW} = -\Gamma\frac{\partial^2\psi}{\partial\lambda^2} = \frac{G\rho_w}{r}\int_S h_w r'\left(\frac{cos\psi}{L^3} - \frac{3rr'sin^2\psi}{L^5}\right)$$

$$\left[ctg\psi - ctg\psi(sin\theta sin\alpha)^2 - \frac{cos\theta cos\theta'}{sin\psi}\right] dS \quad (3.19)$$

$$\frac{\partial^2\psi}{\partial\theta^2} = \frac{\partial}{\partial\theta}cos\alpha = \frac{\partial}{\partial\theta}\frac{sin\theta cos\theta' - cos\theta sin\theta' cos(\lambda' - \lambda)}{sin\psi} = ctg\psi(1 - cos^2\alpha) \quad (3.20)$$

$$\frac{\partial^2\psi}{\partial\lambda^2} = -sin\theta\frac{\partial}{\partial\lambda}sin\alpha$$

$$= -sin\theta sin\theta'\frac{\partial}{\partial\lambda}\frac{sin(\lambda' - \lambda)}{sin\psi} = sin\theta sin\theta'\left[\frac{cos(\lambda' - \lambda)}{sin\psi} - \frac{sin(\lambda' - \lambda)cos\psi}{sin^2\psi}sin\alpha sin\theta\right]$$

$$= \frac{cos\psi - cos\theta cos\theta'}{sin\psi} - \frac{cos\psi}{sin\psi}(sin\theta sin\alpha)^2 = (1 - sin^2\theta sin^2\alpha)ctg\psi - \frac{cos\theta cos\theta'}{sin\psi} \quad (3.21)$$

8.3.2 Green's function integral of the indirect influence of the load effect

Substituting the load spherical harmonic coefficient $\{\Delta\bar{C}_{nm}^w, \Delta\bar{S}_{nm}^w\}$ into Formula (2.7), the indirect influence $\Delta V^i(r,\theta,\lambda)$ of load effect on geopotential can be obtained:

$$\Delta V^i = \frac{GM}{r}\frac{3\rho_w}{\rho_e}\sum_{n=0}^{\infty}\frac{k_n'}{2n+1}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}(\Delta\bar{C}_{nm}^w cosm\lambda + \Delta\bar{S}_{nm}^w sinm\lambda)\bar{P}_{nm}(cos\theta) \quad (3.22)$$

Let $\boldsymbol{e} = (\theta,\lambda)$ be the spherical coordinate of point on the unit spherical surface, then the formula (3.22) can be expressed as a linear combination of normalized spherical basis functions $\{\bar{Y}_{nm}(\boldsymbol{e}) = \bar{Y}_{nm}(\theta,\lambda)\}$ as follows:

$$\Delta V^i(r,\theta,\lambda) = \frac{GM}{r}\frac{3\rho_w}{\rho_e}\sum_{n=0}^{\infty}\frac{k'_n}{2n+1}\left(\frac{a}{r}\right)^n\sum_{m=-n}^{n}\bar{F}^w_{nm}\bar{Y}_{nm}(\boldsymbol{e}) \tag{3.23}$$

Where, $\bar{F}^w_{nm} = \Delta\bar{C}^w_{nm}, m \geq 0$ and $\bar{F}^w_{nm} = \Delta\bar{S}^w_{n|m|}, m < 0$. Let

$$Y^w_n(\boldsymbol{e}) = \sum_{m=-n}^{n}\bar{F}^w_{nm}\bar{Y}_{nm}(\boldsymbol{e}) \tag{3.24}$$

then Formula (3.23) can be expressed as:

$$\Delta V^i(r,\theta,\lambda) = \frac{GM}{r}\frac{3\rho_w}{\rho_e}\sum_{n=0}^{\infty}\frac{k'_n}{2n+1}\left(\frac{a}{r}\right)^n Y^w_n(\boldsymbol{e}) \tag{3.25}$$

The load equivalent water height spherical harmonic expansion (2.3) can be also expressed by the linear combination of the normalized spherical basis functions $\{\bar{Y}_{nm}(\boldsymbol{e}) = \bar{Y}_{nm}(\theta,\lambda)\}$ as:

$$h_w(r \approx R,\theta,\lambda) = h_w(\boldsymbol{e}) = R\sum_{n=1}^{\infty}\left(\frac{a}{R}\right)^n\sum_{m=-n}^{n}\bar{F}^w_{nm}\bar{Y}_{nm}(\boldsymbol{e})$$

$$= R\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\bar{F}^w_{nm}\bar{Y}_{nm}(\boldsymbol{e}) = a\sum_{n=1}^{\infty}Y^w_n(\boldsymbol{e}) \tag{3.26}$$

According to the theory of spherical function expansion, from the formula (3.25):

$$Y^w_n(\boldsymbol{e}) = \frac{2n+1}{4\pi a}\iint_{\sigma} h_w(\boldsymbol{e}')P_n(\psi)d\sigma \tag{3.27}$$

Here, ψ is the spherical angular distance from the integral area-element $\boldsymbol{e}'$ on the sphere to the calculation point $\boldsymbol{e}$.

Considering $dS = R^2 d\sigma$, the formula (3.27) is substituted into the formula (3.26), and the summation and integral are exchanged to obtain:

$$\Delta V^i(r,\theta,\lambda) = \frac{1}{R^2}\iint_S \rho_w h_w(\boldsymbol{e}')\frac{GM}{4\pi ra}\frac{3}{\rho_e}\sum_{n=0}^{\infty}\left(\frac{a}{r}\right)^n k'_n P_n(\psi)\,dS$$

$$= \rho_w\iint_S h_w(\boldsymbol{e}')G^i_V(\psi)dS \tag{3.28}$$

Where,

$$G^i_V(\psi) = \frac{GM}{4\pi R^2 ra}\frac{3}{\rho_e}\sum_{n=0}^{\infty}\left(\frac{a}{r}\right)^n k'_n P_n(\psi) \tag{3.29}$$

is the general form of the load Green's function of indirect influence to geopotential, which represents the indirect influence of unit mass load (kg/m²) to geopotential.

When the calculation point is also located on the ground, that is, $r \approx a \approx R$ (R is the mean radius of the Earth), considering the total mass $M = 4\pi R^3\rho_e/3$ of the Earth, the formula (3.29) can be simplified as follows:

$$G^i_V(\psi) = \frac{GM}{4\pi R^4}\frac{3}{\rho_e}\sum_{n=0}^{\infty}k'_n P_n(\psi) = \frac{G}{R}\sum_{n=1}^{\infty}k'_n P_n(\psi) \tag{3.30}$$

Similarly, the load Green's function of indirect influence to height anomaly can be obtained as follows:

$$G_{\zeta}^{i}(\psi) = \frac{R}{M}\sum_{n=0}^{\infty} k'_n P_n(\psi) \tag{3.31}$$

The load Green's function of indirect influence to ground gravity is◉:

$$G_{g}^{i}(\psi) = \frac{g_0}{M}\sum_{n=0}^{\infty}(n+1)\left(\frac{2}{n}h'_n - \frac{n+1}{n}k'_n\right)P_n(\psi) \tag{3.32}$$

Where, $g_0 = GM/R^2$.

The load Green's function of indirect influence to gravity (disturbance) is:

$$G_{\delta g}^{i}(\psi) = -\frac{g_0}{M}\sum_{n=0}^{\infty}(n+1)k'_n P_n(\psi) \tag{3.33}$$

The load Green's function of indirect influence to ground tilt is◉:

$$G_{t}^{i}(\psi) = \frac{1}{M}\sum_{n=0}^{\infty}(k'_n - h'_n)\frac{\partial P_n(\psi)}{\partial \psi} \tag{3.34}$$

The load Green's function of indirect influence to vertical deflection is:

$$G_{\Theta}^{i}(\psi) = -\frac{1}{M}\sum_{n=0}^{\infty} k'_n \frac{\partial P_n(\psi)}{\partial \psi} \tag{3.35}$$

The load Green's function of indirect influence to ground horizontal displacement is◉:

$$G_{l}^{i}(\psi) = \frac{R}{M}\sum_{n=0}^{\infty} l'_n \frac{\partial P_n(\psi)}{\partial \psi} \tag{3.36}$$

The load Green's function of indirect influence to ground radial displacement is◉:

$$G_{r}^{i}(\psi) = \frac{R}{M}\sum_{n=0}^{\infty} h'_n \frac{\partial P_n(\psi)}{\partial \psi} \tag{3.37}$$

The load Green's function of indirect influence to horizontal gravity gradient is:

$$G_{V_{rr}}^{i} = \frac{g_0}{RM}\sum_{n=0}^{\infty}(n+1)(n+2)k'_n P_n(\psi) \tag{3.38}$$

The load Green's function of indirect influence to radial gravity gradient is:

$$G_{V_{ss}}^{i}(\psi) = \frac{g_0}{RM}\sum_{n=0}^{\infty} k'_n \frac{\partial^2 P_n(\psi)}{\partial \psi^2} \tag{3.39}$$

Guo (2001) furtherly derived the asymptotic approach formula of the ground load Green's functions to suppress the high-degree oscillation of load Green's functions. Where, The load Green's functions of indirect influence are taken as follows:

$$G_{\zeta}^{i}(\psi) = \frac{R}{M}\frac{k'_\infty}{2sin\frac{\psi}{2}} + \frac{R}{M}\sum_{n=0}^{\infty}(k'_n - k'_\infty)P_n(\psi) \tag{3.40}$$

$$G_{g}^{i}(\psi) = -\frac{g_0}{M}\frac{k'_\infty - 2h'_\infty}{2sin\frac{\psi}{2}} - \frac{g_0}{M}\sum_{n=0}^{\infty}[(n+1)k'_n - k'_\infty - 2(h'_n - h'_\infty)]P_n(\psi) \tag{3.41}$$

$$G_{\delta g}^{i}(\psi) = -\frac{g_0}{M}\frac{k'_\infty}{2sin\frac{\psi}{2}} - \frac{g_0}{M}\sum_{n=0}^{\infty}[(n+1)k'_n - k'_\infty]P_n(\psi) \tag{3.42}$$

$$G_{t}^{i}(\psi) = -\frac{1}{M}\frac{h'_\infty cos\frac{\psi}{2}}{4sin^2\frac{\psi}{2}} + \frac{1}{M}\frac{k'_\infty cos\frac{\psi}{2}\left(1+2sin\frac{\psi}{2}\right)}{2sin\frac{\psi}{2}\left(1+sin\frac{\psi}{2}\right)} - \frac{1}{M}\sum_{n=1}^{\infty}\left(k'_n - \frac{k'_\infty}{n} - h'_n + h'_\infty\right)\frac{\partial P_n(\psi)}{\partial \psi} \tag{3.43}$$

$$G_{\Theta}^i(\psi) = \frac{1}{M}\frac{k'_\infty \cos\frac{\psi}{2}\left(1+2\sin\frac{\psi}{2}\right)}{2\sin\frac{\psi}{2}\left(1+\sin\frac{\psi}{2}\right)} - \frac{1}{M}\sum_{n=1}^{\infty}\left(k'_n - \frac{k'_\infty}{n}\right)\frac{\partial P_n(\psi)}{\partial\psi} \quad (3.44)$$

$$G_l(\psi) = -\frac{R}{M}\frac{l'_\infty \cos\frac{\psi}{2}\left(1+2\sin\frac{\psi}{2}\right)}{2\sin\frac{\psi}{2}\left(1+\sin\frac{\psi}{2}\right)} + \frac{R}{M}\sum_{n=1}^{\infty}\left(l'_n - \frac{l'_\infty}{n}\right)\frac{\partial P_n(\psi)}{\partial\psi} \quad (3.45)$$

$$G_r(\psi) = \frac{R}{M}\frac{h'_\infty}{2\sin\frac{\psi}{2}} + \frac{a}{M}\sum_{n=0}^{\infty}(h'_n - h'_\infty)P_n(\psi) \quad (3.46)$$

Let $\mathcal{G}^i(l) = 2R\sin\frac{\psi}{2}G^i(\psi) = lG^i(\psi)$, the load Love number is substituted into the formulas (3.31) ~ (3.39) to obtain the load Green's function of indirect influence with the integral distance under the action of unit point mass load (1 kg/m²), as shown in Tab 3.1.

Tab 3.1 Load Green's function values of the indirect influence of unit point mass

l(km)	$\mathcal{G}_\zeta^i$×10-13	$\mathcal{G}_g^i$×10-17	$\mathcal{G}_{\delta g}^i$×10-18	$\mathcal{G}_t^i$×10-14	$\mathcal{G}_\Theta^i$×10-19	$\mathcal{G}_l$×10-12	$\mathcal{G}_r$×10-11	$\mathcal{G}_{rr}^i$×10-15	$\mathcal{G}_{ss}^i$×10-15
0.1	-0.0249	-11.3315	15.8795	42.2955	-2.1192	-0.8369	-42.1264	-40.7525	20.0337
0.2	-0.0439	-9.8972	29.6981	21.1510	-8.0632	-3.1842	-41.9553	-73.6102	34.1831
0.3	-0.0625	-8.8334	39.7946	14.1058	-16.6878	-6.5901	-41.7788	-92.3770	37.9744
0.4	-0.0804	-8.2348	45.2182	10.5853	-26.3601	-10.4097	-41.5956	-93.8712	29.4189
0.5	-0.0975	-8.1095	45.8894	8.4739	-35.3064	-13.9425	-41.4057	-78.5612	9.4993
0.6	-0.1139	-8.3807	42.5773	7.0657	-41.9834	-16.5790	-41.2101	-50.3867	-18.0490
0.7	-0.1294	-8.9073	36.7009	6.0583	-45.3905	-17.9241	-41.0109	-15.8142	-47.6055
0.8	-0.1444	-9.5157	30.0034	5.3006	-45.2558	-17.8704	-40.8109	17.6468	-72.9744
1.0	-0.1727	-10.3454	20.4992	4.2343	-36.8762	-14.5596	-40.4173	55.8494	-91.9157
1.2	-0.1998	-10.1321	21.4749	3.5210	-26.2416	-10.3574	-40.0402	39.6641	-61.0517
1.4	-0.2261	-9.1669	30.0077	3.0153	-22.8895	-9.0304	-39.6752	-8.4433	-7.5471
1.6	-0.2518	-8.3519	37.0350	2.6419	-28.6871	-11.3158	-39.3091	-42.4515	24.9158
2.0	-0.3003	-8.9633	28.5858	2.1198	-40.5309	-15.9830	-38.5476	4.3817	-24.2022
2.5	-0.3570	-9.1242	24.1119	1.6843	-25.9871	-10.2232	-37.6133	17.0612	-27.2278
3.0	-0.4112	-7.9718	32.8632	1.4080	-35.2424	-13.8576	-36.7093	-28.7167	17.2271
3.5	-0.4621	-8.9437	20.3140	1.2022	-32.5321	-12.7629	-35.7866	31.1746	-40.2655
4.0	-0.5112	-7.7218	29.8481	1.0465	-28.2814	-11.0562	-34.9109	-22.8507	15.9355
5.0	-0.6036	-7.8959	22.7679	0.8291	-26.3578	-10.2305	-33.1702	5.9459	-11.1019
6.0	-0.6903	-7.8527	18.1028	0.6858	-29.9324	-11.5649	-31.5082	23.6048	-28.4842
7.0	-0.7725	-7.2943	18.8748	0.5827	-33.7803	-12.9988	-29.9389	13.5281	-18.2480
8.0	-0.8510	-6.5206	22.0921	0.5013	-33.1161	-12.6452	-28.4652	-9.3638	5.3150
10.0	-0.9991	-6.0125	18.9937	0.3784	-24.7530	-9.1540	-25.7982	-5.3162	2.8950

12.0	-1.1387	-5.9045	13.1167	0.2999	-27.9718	-10.2454	-23.5296	16.1892	-18.4692
14.0	-1.2726	-4.9048	17.3988	0.2398	-26.5722	-9.5373	-21.6664	-13.0654	11.2087
16.0	-1.4019	-4.8896	12.8941	0.1911	-21.0009	-7.2164	-20.1480	4.3047	-5.5888
20.0	-1.6520	-4.0437	14.8205	0.1306	-20.9145	-7.0582	-18.0179	-12.2601	11.2369
25.0	-1.9534	-3.6904	13.7959	0.0872	-19.8016	-6.6584	-16.5317	-10.0949	9.3198
30.0	-2.2455	-3.5544	12.9067	0.0638	-18.9897	-6.5141	-15.7982	-5.5325	4.9129
35.0	-2.5296	-3.5250	12.0811	0.0505	-18.1729	-6.4230	-15.4331	-0.0753	-0.4331
40.0	-2.8059	-3.5272	11.4345	0.0423	-17.1945	-6.2698	-15.2297	4.7358	-5.1568
50.0	-3.3365	-3.4643	11.2395	0.0322	-14.9772	-5.7725	-14.9607	8.1685	-8.4622
60.0	-3.8395	-3.2518	12.5464	0.0262	-13.6029	-5.4612	-14.6941	2.7549	-2.9775
70.0	-4.3177	-3.0073	14.0654	0.0229	-13.9783	-5.7205	-14.3923	-4.6469	4.4506
80.0	-4.7741	-2.8804	14.3310	0.0210	-15.3999	-6.3101	-14.0649	-6.2127	6.0235
100.0	-5.6311	-2.9117	11.9306	0.0171	-15.7804	-6.3810	-13.3843	4.6763	-4.8316
120.0	-6.4270	-2.6545	12.4755	0.0129	-14.0249	-5.5346	-12.7235	-0.1761	0.0607
140.0	-7.1738	-2.4359	12.7461	0.0120	-15.5946	-5.9880	-12.0989	-3.7448	3.6348
160.0	-7.8804	-2.4586	10.7233	0.0100	-14.9953	-5.5941	-11.5133	-4.4893	-4.5820
200.0	-9.1986	-2.0952	11.1758	0.0080	-15.1075	-5.3733	-10.4758	-1.7439	1.6689
250.0	-10.7136	-1.8097	10.7082	0.0058	-14.0435	-4.7072	-9.3924	-3.2869	3.2307
300.0	-12.1238	-1.5962	10.1419	0.0042	-12.9077	-4.0819	-8.5118	-3.2916	3.2481
400.0	-14.7375	-1.3210	8.9521	0.0023	-11.1503	-3.1625	-7.2265	-0.4258	0.3969
500.0	-17.1749	-1.1331	8.3207	0.0016	-10.3019	-2.7029	-6.4078	2.1612	-2.1831
600.0	-19.4980	-0.9603	8.5053	0.0014	-9.8691	-2.4641	-5.9044	2.3040	-2.3219
800.0	-23.8986	-0.6720	9.9646	0.0010	-9.0007	-2.0628	-5.4405	-0.1041	0.0908

8.3.3 Legendre function and its first and second derivatives to ψ

When calculating the load Green's function of the indirect influence to various geodetic variation, it is necessary to calculate the Legendre function $P_n(cos\psi)$ and its first and second derivatives to ψ. Here, let $t = cos\theta, u = sin\theta$, and give the fast recursive algorithm directly.

$$P_n(t) = \frac{2n-1}{n} tP_{n-1}(t) - \frac{n-1}{n} P_{n-2}(t) \tag{3.47}$$

$$P_1 = t, \quad P_2 = \frac{1}{2}(3t^2 - 1) \tag{3.48}$$

$$\frac{\partial}{\partial\psi}P_n(t)=\frac{2n-1}{n}t\frac{\partial}{\partial\psi}P_{n-1}(t)-\frac{2n-1}{n}uP_{n-1}(t)-\frac{n-1}{n}\frac{\partial}{\partial\psi}P_{n-2}(t) \tag{3.49}$$

$$\frac{\partial}{\partial\psi}P_1(t)=-u,\quad \frac{\partial}{\partial\psi}P_2(t)=-3ut \tag{3.50}$$

$$\frac{\partial^2}{\partial\psi^2}P_n(t)=\frac{2n-1}{n}\left(t\frac{\partial^2}{\partial\psi^2}P_{n-1}-2u\frac{\partial}{\partial\psi}P_{n-1}-tP_{n-1}\right)-\frac{n-1}{n}\frac{\partial^2}{\partial\psi^2}P_{n-2} \tag{3.51}$$

$$\frac{\partial^2}{\partial\psi^2}P_1(t)=-t,\quad \frac{\partial^2}{\partial\psi^2}P_2(t)=3(1-2t^2) \tag{3.52}$$

8.3.4 Calculation of load deformation field from the river-lake water variations

The changes of inland water bodies such as rivers, lakes, reservoirs, glaciers and snow mountains are represented by load equivalent water height variation grid. According to the load Green's function integral algorithm (the sum of load direct influence and indirect influence integral), the load effects on various geodetic variations at any point on the ground or in near-surface space can be calculated. The equivalent water height variation grids of multiple water bodies at the same sampling epoch can be merged into a grid directly, and then the load Green's function integral calculation is carried out.

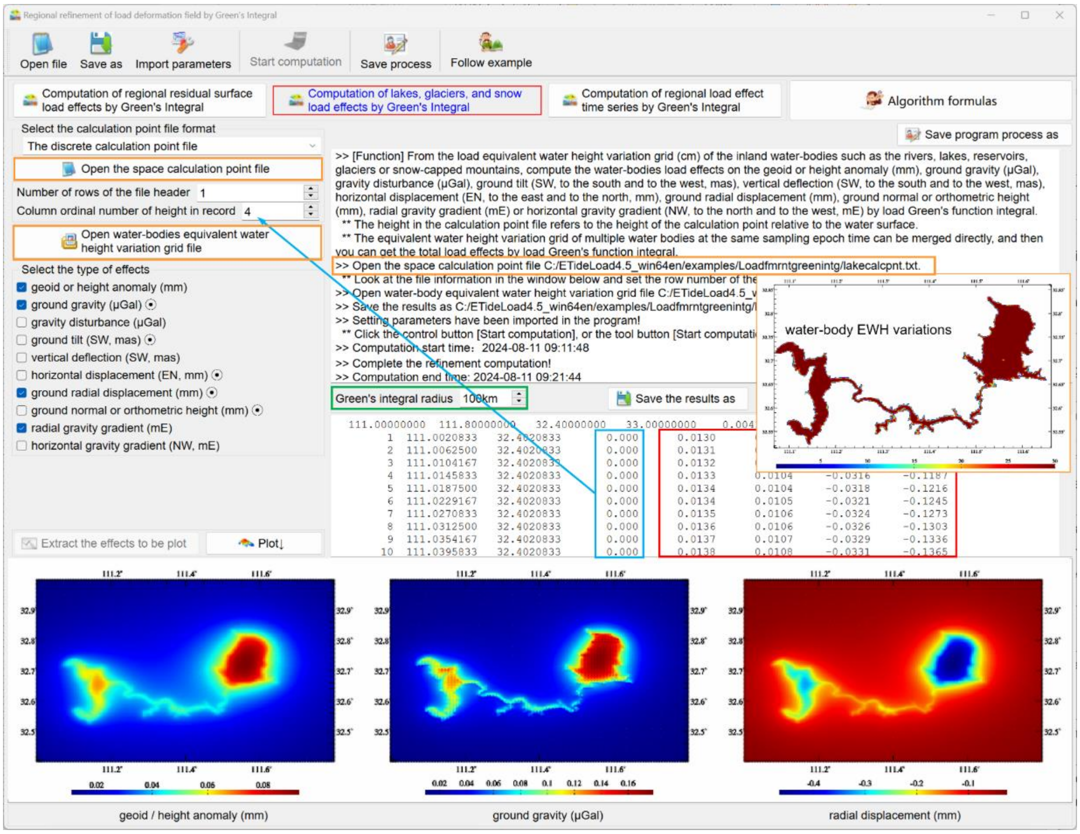

Fig 3.1 Calculation of load deformation field of water variations in rivers and lakes

Here, the river (lake) bottom topography is combined with the water-level observations on river (lake) surface, and the equivalent water height variation grid time series is constructed from the river (lake) water-level monitoring data. Then, according to the load Green's function integral algorithm, the load effect grid time series on all-element geodetic variations are calculated. Fig 3.1 is the calculation process of surface load deformation field at one sampling epoch time.

8.3.5 Regional approach of load deformation field using remove-restore scheme

When the load effects are calculated by the load Green's function integral from the load equivalent water height variation grid, the domain of the independent variable ψ of the integral kernel function should be $[0,2\pi)$ or $[-\pi,\pi)$, which is global. Direct integration need globally continuously distributed surface equivalent water height variation data, even if calculating the load effect at some a point. This is very inconvenient for the calculation and application of load deformation effect in a local area such as a country or region, and it is also inconvenient to use the advantages of surface load observations in local areas to improve the regional load deformation field.

Similar to the local gravity field approach scheme in physical geodesy, we can let the global load spherical harmonic coefficient model as the reference field, calculate and remove the reference model value from the surface load variations in local areas to obtain the residual load variations, then employ the load Green's integral to the residual load variations to refine the load deformation field in this area. This scheme can be also called as the remove-restore scheme for regional approach of load deformation field.

The process of the scheme at one epoch time is as follows: (a) The reference model value of the equivalent water height (EWH) variations in local areas are calculated from the load spherical harmonic coefficient model at the epoch. (b) From the regional high-resolution EWH variation grid, remove the reference model value to obtain the regional EWH residual value grid. This step is called as 'Remove'. (c) Using a smaller integral radius, the residual value of high-resolution load deformation field grid is calculated by the load Green's function integral. (d) The high-resolution model value grid of regional load deformation field are calculated from the load spherical harmonic coefficient model. (e) The refined value of regional high-resolution load deformation field at the epoch time are obtained by adding the high-resolution reference model value grid to residual value grid. This step is called as 'Restore'. The whole process can be called as 'Remove - load Green's function integral - Restore' scheme.

In the following, taking the refinement of the surface atmosphere load deformation field in a certain area of southern China as an example, using the 'Remove-load Green's function integral-Restore' scheme, the surface atmosphere load deformation field variation grid weekly time series are refined from the 3.75′×3.75′ surface atmospheric pressure variation (hPa) grid weekly time series whose time span is January 2018 to December 2020 with a total of 157 sampling epochs. Of which 12 epochs of surface atmospheric pressure variation grid are shown in Fig 3.2, and the upper left corner of the graph is the sampling epoch date, such as 20180214, which means February 14, 2018.

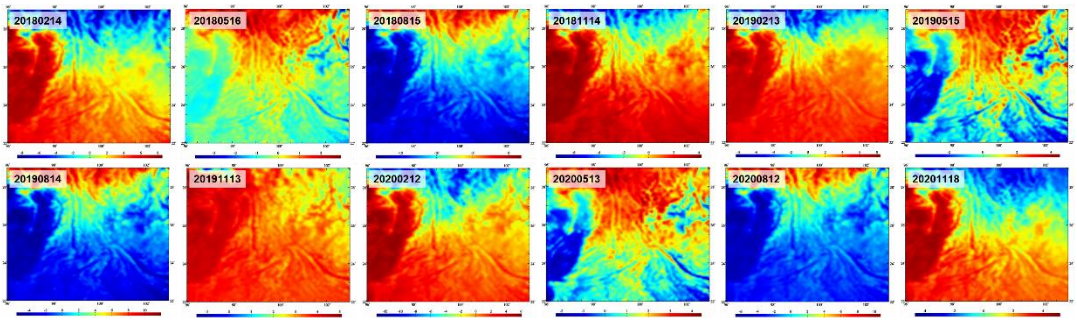

Fig 3.2 Regoinal 3.75′×3.75′ surface atmospheric pressure variation (hPa) grid weekly time series

Taking 157 180-degree global surface atmosphere load spherical harmonic coefficient model weekly time series calculated in section 8.2.5 from January 2018 to December 2020 as the load deformation field reference model time series, and similar to the regional geoid refinement scheme, the load equivalent water height grid data area (the calculation area) should be generally bigger than refine result area of the load deformation field to suppress the integral edge effect. In this case, the data area is 96°E ~ 103°E, 22°N ~ 29°N, and the result area is 98°E ~ 101°E, 24°N ~ 27°N.

Step 1: Input the 3.75′×3.75′ zero-value grid of the calculation area (zero-value means that the calculation point height relative to the ground is equal to zero), select the maximum calculation degree 180, and calculate the reference model value grid time series of the surface atmosphere variations in the calculation area from the global atmosphere load spherical harmonic coefficient model time series.

Step 2: The 3.75′×3.75′ surface atmosphere variation grid weekly time series, minus the corresponding reference model value grid weekly time series to generate the 3.75′×3.75′ surface atmosphere residual value grid weekly time series. Where, 12 epochs of residual value grid is shown in Fig 3.3.

Step 3: Input the 3.75'×3.75' zero-value grid in the result area, select the integral radius of 200 km, and calculate the 3.75'×3.75' load deformation field residual value grid weekly time series from the 3.75'×3.75' surface atmosphere residual variation grid weekly time series using the load Green's function integral.

Step 4: Input the 3.75′×3.75′ zero-value grid in the result area, select the maximum calculation degree 180, and calculate the 3.75′×3.75′ reference model value grid weekly time series of the surface atmosphere load deformation field in the result area from the surface atmosphere load spherical harmonic coefficient model weekly time series.

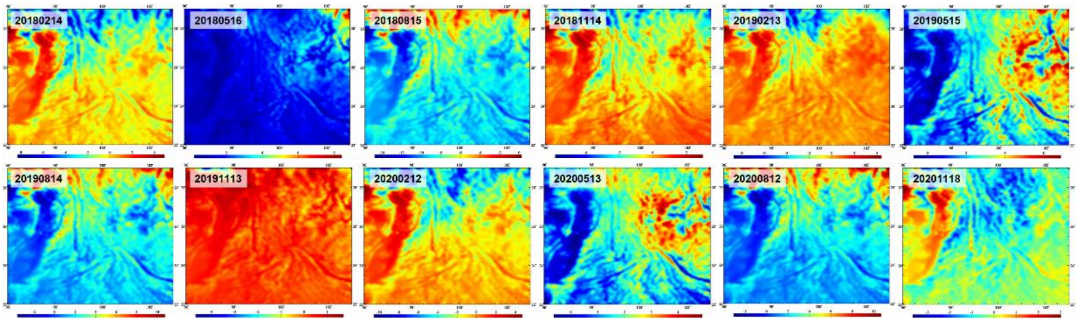

Fig 3.3 Regoinal 3.75′×3.75′ surface atmosphere residual variation (hPa) grid weekly time series

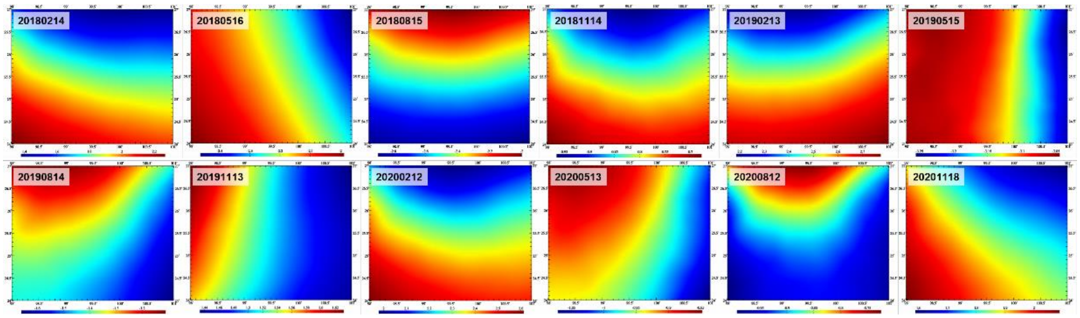

Fig 3.4 Regoinal 3.75′×3.75′ surface atmosphere load effect grid weekly time series on geoid (mm)

Step 5: The 3.75′×3.75′ residual value grid weekly time series of load deformation field in the result area is added to the 3.75′×3.75′ residual value grid weekly time series, and the 3.75′×3.75′ grid time series of surface atmosphere load deformation field in the result area are obtained. Where, 12 epochs of regional atmosphere load deformation field, including the load effects on geoid, ground gravity, ground tilt, ellipsoidal height and radial gravity gradient, are shown in Fig 3.4-Fig 3.8 respectively.

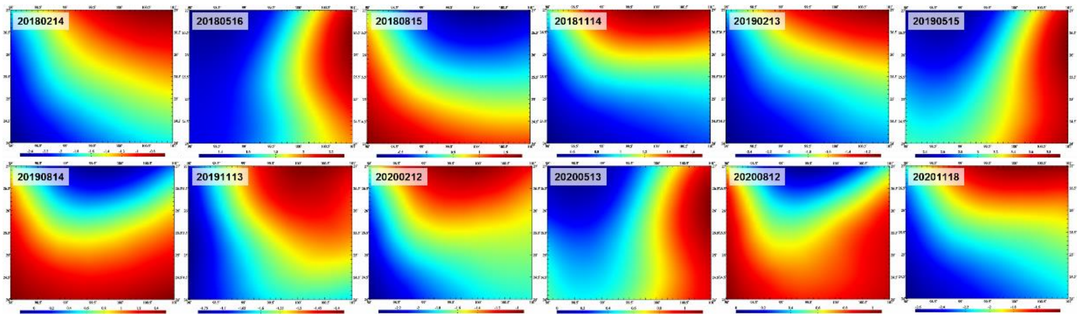

Fig 3.5 Regoinal 3.75′×3.75′ surface atmosphere load effect grid weekly time series on ground gravity (mGal)

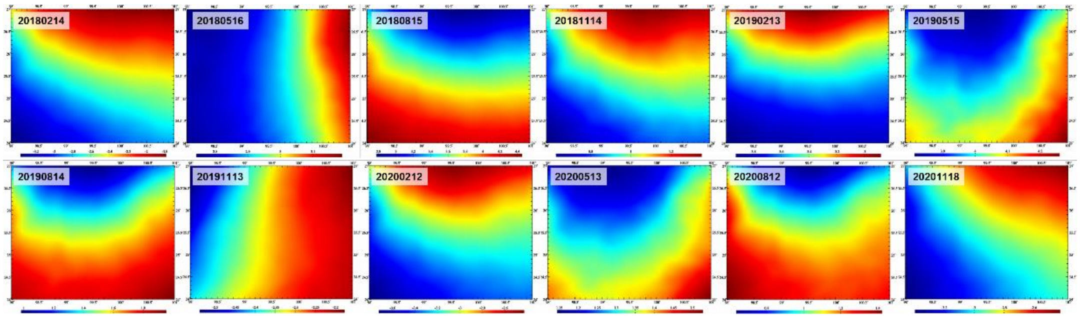

Fig 3.6 Regoinal 3.75′×3.75′ surface atmosphere load effect grid weekly time series on ellipsoidal height (mm)

In order to visually display the time-varying characteristics of surface atmosphere load effects on various geodetic variations and quantitative relationship between the surface atmosphere load effects on different types of geodetic variations in the region, the time series of surface atmosphere load effects on the geoid, ground gravity, ellipsoidal height and radial gravity gradient from January 2018 to December 2020 at the central ground point of the region are calculated using the 'Remove - load Green's function integral - Restore' scheme, as shown in Fig 3.9.

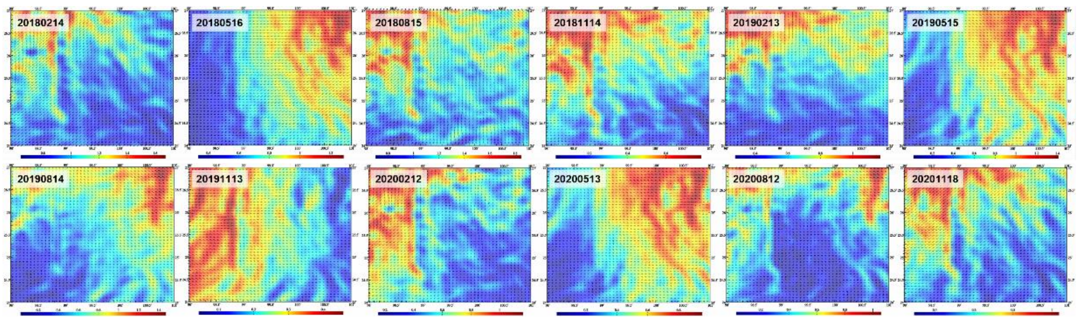

Fig 3.7 Regoinal 3.75′×3.75′ surface atmosphere load effect grid weekly time series on ground tilt (mas)

In the following, the 0.5°×0.5° global surface atmospheric pressure diurnal variations in the global reanalysis data ERA-40/ERA-Interim from the European Centre for Medium-Range Weather Forecasts (ECMWF) are employed to construct the 0.5°×0.5 surface atmosphere variation (hPa) grid weekly time series (157 sampling epochs) from January 2018 to December 2020 in chinese mainland and adjacent areas.

Taking 157 180-degree global surface atmosphere load spherical harmonic coefficient model weekly time series calculated in section 8.2.5 from January 2018 to December 2020 as the load deformation field reference model time series. Firstly, using the 'Remove-load Green's function integral - Restore' scheme and the integral radius 200 km, the surface atmosphere load effect weekly time series on the geoid, ground gravity, ellipsoidal height and radial gravity gradient are calculated at 6 CORS stations in mainland China. Then, the load Green's function integration method with the integral radius 800km is directly employed to directly calculate the surface atmosphere load effect weekly time series on the geoid, ground gravity, ellipsoidal height and radial gravity gradient are calculated at 6 CORS stations in mainland China. Finally, the two calculation results are compared.

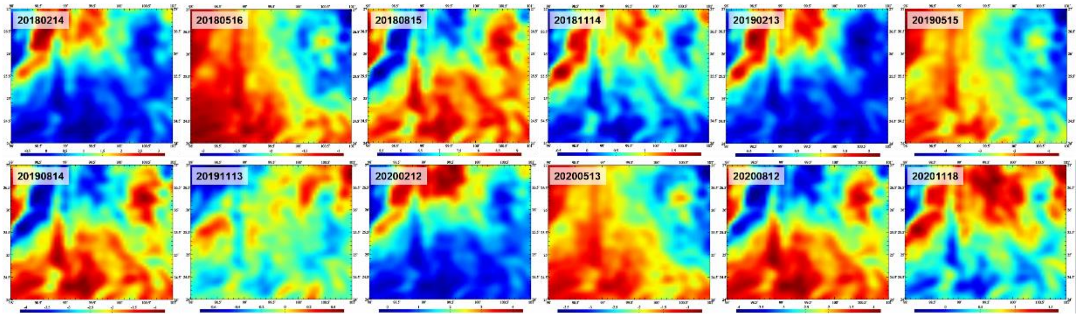

Fig 3.8 Regoinal 3.75′×3.75′ surface atmosphere load effect grid weekly time series on radial gravity gradient (mE)

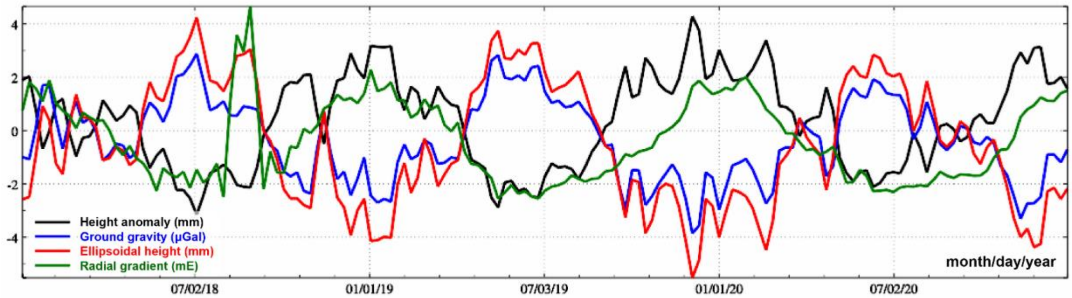

Fig 3.9 surface atmosphere load effect weekly time series on geodetic variations at the central ground point of the region

The load effect weekly time series curves at 6 CORS stations in mainland China calculated by the two methods are shown in Fig 3.10 ~ Fig 3.13. The upper figure of each figure is the calculation result using the remove -restore method, and the lower figure is the calculation result uising the load Green's function integral (direct integral) method.

Tab 3.2 gives the differences statistics between the atmosphere load effects time series calculated by the two schemes at 6 CORS stations in mainland China. In Tab 3.2, ksi, gra, hgt and grr represent the atmosphere load effect time series on the geoid, ground gravity, ellipsoidal height and radial gravity gradient, respectively.

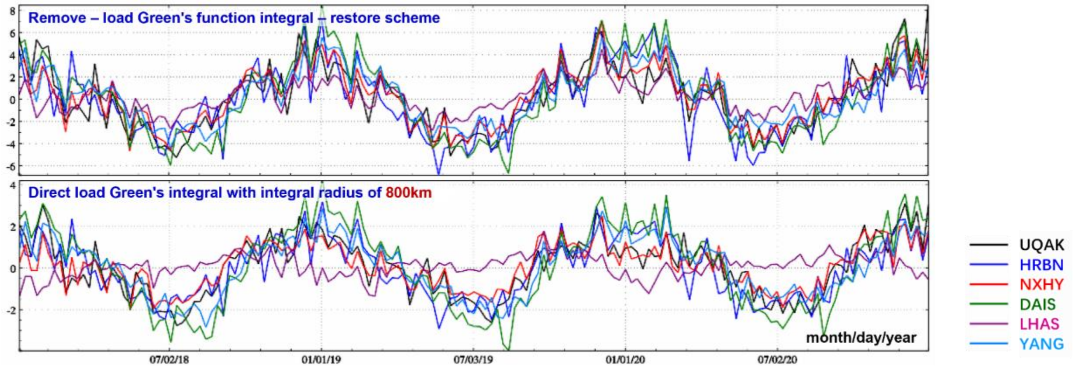

Fig 3.10 Surface atmosphere load effect time series on geoid (mm) at 6 CORS stations in mainland China using two scheme

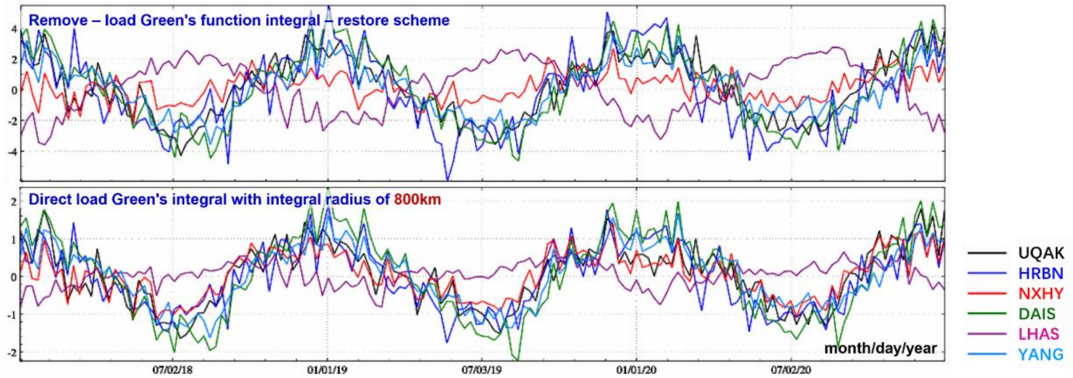

Fig 3.11 Surface atmosphere load effect time series on ground gravity (mGal) at 6 CORS stations in mainland China using two scheme

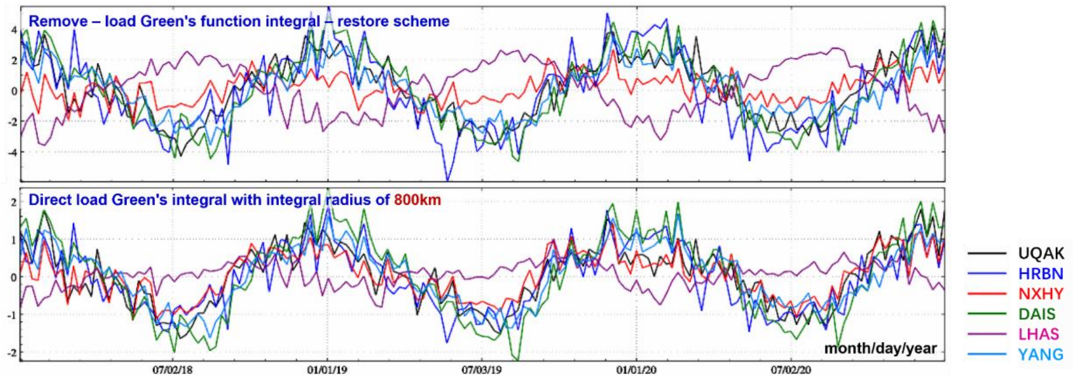

Fig 3.12 Surface atmosphere load effect time series on ellipsoidal height (mm) at 6 CORS stations in mainland China using two scheme

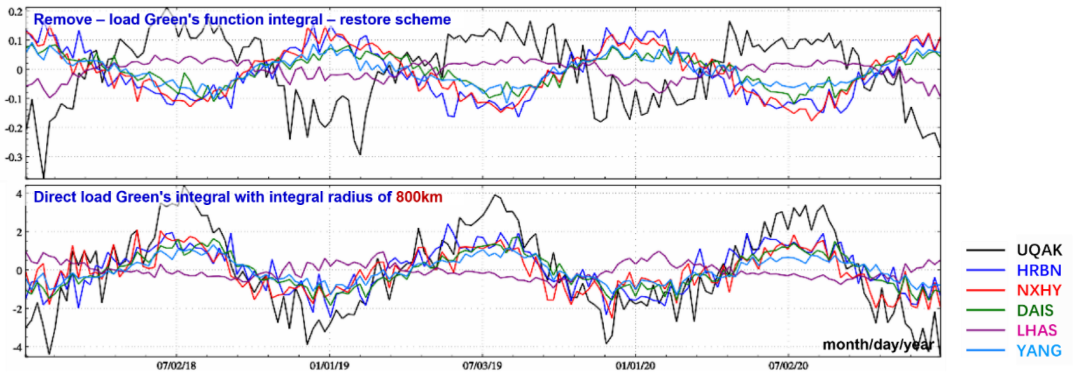

Fig 3.13 Surface atmosphere load effect time series on radial gravity gradient (mE) at 6 CORS stations in mainland China using two scheme

Tab 3.2 The differences statistics between the atmosphere load effects time series calculated by the two schemes at 6 CORS stations in mainland China

CORS station	Geodetic variation	unit	Remove-restore		Direct integral		Difference	
			mean	RSM	mean	RSM	mean	RSM
UQAK E87.97° N47.10°	ksi	mm	0.1618	2.9040	0.0669	1.3829	0.0949	1.6514
	gra	mGal	0.1363	2.0060	0.0382	0.8115	0.0982	1.2499
	hgt	mm	-0.2201	4.0837	-0.1053	2.2224	-0.1148	2.0922
	grr	mE	-0.0029	0.1224	-0.0582	2.1550	0.0553	2.0549
HRBN E126.62° N45.70°	ksi	mm	0.0335	3.1523	-0.0208	1.4114	0.0542	1.8525
	gra	mGal	-0.0628	2.5363	-0.0148	0.8494	-0.0479	1.7076
	hgt	mm	0.0053	4.3568	0.0411	2.3354	-0.0357	2.2104
	grr	mE	-0.0111	0.0883	0.0308	1.1439	-0.0419	1.2281
NXHY E105.63° N36.55°	ksi	mm	0.3069	2.6001	0.0994	1.0561	0.2075	1.6745
	gra	mGal	0.1592	0.9325	0.0576	0.6012	0.1016	0.4970
	hgt	mm	-0.3920	3.4507	-0.1589	1.6570	-0.2331	2.0348
	grr	mE	-0.0073	0.0818	-0.1068	1.0503	0.0995	1.1172
DAIS E122.20° N30.23°	ksi	mm	0.2630	3.5502	0.0907	1.9842	0.1722	1.6910
	gra	mGal	0.1284	2.4349	0.0522	1.1244	0.0763	1.3303
	hgt	mm	0.3564	4.7804	-0.1429	3.0833	-0.2135	1.9432
	grr	mE	-0.0026	0.0517	-0.0441	0.8954	0.0415	0.9444
LHAS E91.10°	ksi	mm	0.3231	1.3847	0.0826	0.5451	0.2405	1.2880
	gra	mGal	0.1167	1.6451	0.0464	0.3234	0.0703	1.4307

N29.65°	hgt	mm	-0.4104	1.8119	-0.1262	0.8929	-0.2842	1.6980
	grr	mE	-0.0043	0.0325	-0.0425	0.3946	0.0383	0.4219
YANG E109.22° N19.77°	ksi	mm	0.3055	2.4528	0.0904	1.3640	0.2151	1.2191
	gra	mGal	0.0748	1.6494	0.0499	0.7787	0.0249	0.8961
	hgt	mm	-0.4103	3.4732	-0.1365	2.1386	-0.2738	1.5685
	grr	mE	-0.0041	0.0461	-0.0344	0.6216	0.0303	0.6665

Fig 3.10 ~ Fig 3.13 show that the geometric shape of the time series curve of the load effects calculated by the two schemes is basically the same, but the numerical value is obviously different. Tab 3.2 shows that even if the integral radius reaches 800 km, the error of the direct integral of the load Green's function will exceed the magnitude of the calculated signal itself. This is because as long as the integral radius is less than $\sqrt{2}R$ (R is the mean radius of the Earth), the direct integral of the load Green's function fails to achieve global surface integral, and the calculated load effect signal is not sufficient. It can be seen that in most cases, the direct integral method of load Green's function is difficult to meet the high-precision geodesy. It is suggested to adopt the more rigorous the 'remove -load Green's function integral -Restore' scheme in theory.