Earth's rotation polar shift effects on geodetic variations and tidal effects on EPR

The instantaneous Earth's rotation axis is inconsistent with the mean Earth's figure axis, which leads to the change of the centrifugal force potential in Earth space with time. The variation of the centrifugal force potential excites solid Earth deformation, causing the redistribution of the mass inside the Earth and generating the associated geopotential.

8.6.1 Earth's rotation polar shift effects on geodetic variations

The variation of centrifugal force potential in the Earth space caused by the Earth's rotation polar shift is the direct influence of the Earth's rotation motion to geopotential variation, which can be expressed by the direct influence of the Earth's rotation polar shift on the degree-2 tesseral harmonic geopotential coefficient variation $\Delta\bar{C}_{21}^{dr} + i\Delta\bar{S}_{21}^{dr}$. Considering the relationship between the long-peroid Love number k_0 and the degree-2 zonal geopotential coefficient $\bar{C}_{20}$, we have:

$$\Delta\bar{C}_{21}^{dr} + i\Delta\bar{S}_{21}^{dr} = \frac{\sqrt{3}}{k_0}\bar{C}_{20}(m_1 + im_2) = -\frac{1}{\sqrt{15}}\frac{\omega^2 a^3}{GM}m \tag{6.1}$$

Where, $m = m_1 + im_2$ is the complex form of Earth's rotation polar shift (in unit of radian), and ω is the angular rate of Earth's rotation.

The variation of centrifugal force potential caused by the Earth's rotation polar shift furtherly excites the deformation of the solid Earth and produces the associated geopotential, which is usually characterized by the degree-2 diurnal body tidal Love number k_{21}. The indirect influence of the centrifugal force potential to the degree-2 tesseral harmonic geopotential coefficient is as follows:

$$\Delta\bar{C}_{21}^{in} + i\bar{S}_{21}^{in} = \frac{\sqrt{3}k_{21}}{k_0}\bar{C}_{20}m = -\frac{1}{\sqrt{15}}\frac{\omega^2 a^3}{GM}k_{21}m \tag{6.2}$$

The Earth's rotation polar shift effects on the degree-2 tesseral harmonic geopotential coefficient are equal to the sum of the direct effects (non-conservative) and indirect effects (conservative) of the centrifugal force potential, that is:

$$\Delta\bar{C}_{21} + i\Delta\bar{S}_{21} = \left(\Delta\bar{C}_{21}^{dr} + \Delta\bar{C}_{21}^{in}\right) + i\left(\bar{S}_{21}^{dr} + \bar{S}_{21}^{in}\right) = -\frac{1}{\sqrt{15}}\frac{\omega^2 a^3}{GM}(1 + k_{21})m \tag{6.3}$$

Similar to the solid Earth tidal effect algorithm formulas, The Earth's rotation polar shift effect algorithm formulas on various geodetic elements can be obtained. Considering $\bar{P}_{nm}(cos\theta) = \sqrt{15}sin\theta cos\theta$, the algorithm formula of the Earth's rotation polar shift effect on height anomaly at (r,θ,λ) is:

$$\Delta\zeta(r,\theta,\lambda) = \frac{GM}{\gamma r}\left(\frac{a}{r}\right)^2(1 + k_{21})(\Delta\bar{C}_{21}^{dr}cos\lambda + \Delta\bar{S}_{21}^{dr}sin\lambda)\bar{P}_{21}(cos\theta)$$

$$= -\frac{\omega^2 a^5}{2\gamma r^3}(1 + k_{21})m^* e^{i\lambda}sin2\theta \tag{6.4}$$

Where, $e^{i\lambda}=cos\lambda+isin\lambda$ and $m^*=m_1-im_2$ is the complex conjugate of Earth's rotation polar shift m.

The algorithm formula of the Earth's rotation polar shift effect on ground gravity is◉:

$$\Delta g^s=-\frac{3}{2}\frac{\omega^2a^5}{r^4}\left(1-\frac{3}{2}k_{21}+h_{21}\right)m^*e^{i\lambda}sin2\theta \tag{6.5}$$

The algorithm formula of the Earth's rotation polar shift effect on gravity (disturbance) outside the solid Earth is:

$$\Delta g^\delta=-\frac{3}{2}\frac{\omega^2a^5}{r^4}(1+k_{21})m^*e^{i\lambda}sin2\theta \tag{6.6}$$

The algorithm formula of the Earth's rotation polar shift effect on ground tilt is◉:

$$\text{South: }\ \delta\xi^s=-\frac{\omega^2a^5}{\gamma r^4}(1+k_{21}-h_{21})m^*e^{i\lambda}sin\theta cos2\theta \tag{6.7}$$

$$\text{West: }\ \delta\eta^s=-\frac{\omega^2a^5}{\gamma r^4}(1+k_{21}-h_{21})m^*e^{i(\lambda-\pi/2)}cos\theta \tag{6.8}$$

The algorithm formula of the Earth's rotation polar shift effect on vertical deflection outside the solid Earth is:

$$\text{South: }\ \Delta\xi=-\frac{\omega^2a^5}{\gamma r^4}(1+k_{21})m^*e^{i\lambda}sin\theta cos2\theta \tag{6.9}$$

$$\text{West: }\ \Delta\eta=-\frac{\omega^2a^5}{\gamma r^4}(1+k_{21})m^*e^{i(\lambda-\pi/2)}cos\theta \tag{6.10}$$

The algorithm formula of the Earth's rotation polar shift effect on ground site displacement is◉:

$$\text{East: }\ \Delta e=\frac{\omega^2a^5}{\gamma r^3}l_{21}m^*e^{i(\lambda-\pi/2)}cos\theta \tag{6.11}$$

$$\text{North: }\ \Delta n=\frac{\omega^2a^5}{\gamma r^3}l_{21}m^*e^{i\lambda}sin\theta cos2\theta \tag{6.12}$$

$$\text{Radial: }\ \Delta r=-\frac{\omega^2a^5}{2\gamma r^3}h_{21}m^*e^{i\lambda}sin2\theta \tag{6.13}$$

The algorithm formula of the Earth's rotation polar shift effect on gravity gradient outside the solid Earth is:

$$\text{Radial: }\ \Delta T_{rr}=-\frac{6\omega^2a^5}{r^5}(1+k_{21})m^*e^{i\lambda}sin2\theta \tag{6.14}$$

$$\text{North: }\ \Delta T_{NN}=\frac{2\omega^2a^5}{r^5}(1+k_{21})m^*e^{i\lambda}sin2\theta \tag{6.15}$$

$$\text{West: }\ \Delta T_{WW}=-\frac{\omega^2a^5}{r^5}(1+k_{21})m^*e^{i\lambda}ctg\theta \tag{6.16}$$

In the above expressions, the Earth's rotation polar shift effects on the geodetic variations marked ◉ are valid only when their sites are fixed with the solid Earth, and that on the remaining geodetic variations are suitable on the ground or outside the solid Earth.

Taking the degree-2 tesseral body-tidal Love number $k_{21}=0.3077+0.0036i$, $h_{21}=0.6207$ and $l_{21}=0.0836$, the time series of Earth's rotation polar shift effects on various geodetic elements at the ground point P (105.0°E, 32.0°N, H720m) are calculated according to formulas (6.4) ~ (6.16) from the IERS Earth orientation parameters (EOP) time series, as shown in Fig 6.1. The time span of the time series is from January 2018 to December 2022 (5 years). In Fig 6.1, the 5-year mean value of the time series of Earth's rotation polar shift is removed, and the Earth's rotation polar shift (m_1, m_2) have been converted into the Earth's rotation polar coordinate variations $\left(\Delta x_p=m_1 b, \Delta y_p=-m_2 b\right)$ (in unit of m) in the ITRS (x and y axes in the Earth-fixed rectangular coordinate system).

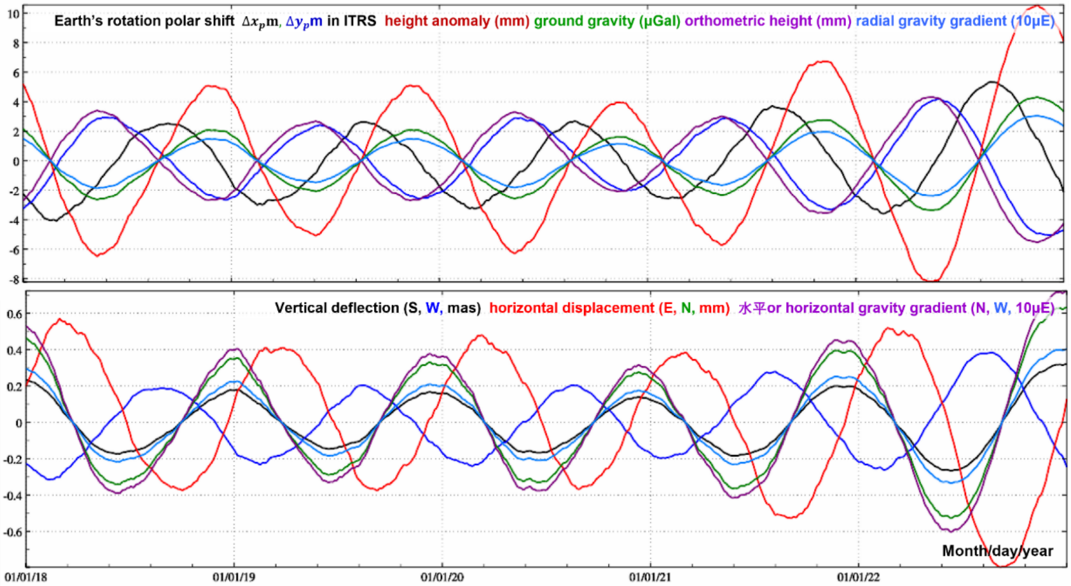

Fig 6.1 The time series of Earth's roration polar shift effects on various geodetic elements at the point P

Fig 6.1 shows that although the Earth's rotation polar shift itself can reach the meter level, the resulting effect on geoid or ground normal height is only in mm level, that on ground gravity is μGal level, that on radial gravity gradient is 10μE level, that on horizontal geodetic elements are small and can be generally ignored.

8.6.2 Self-consistent equilibrium ocean polar tide effects

It is generally believed that the ocean polar tide is the manifestation of the centrifugal force of rotation polar shift on the ocean, and its main periodic constituents are about 433 days of Chandler wobble and annual variation. In these long periods, the ocean polar tide load is expected to have an equilibrium response, that is, the displacement of the ocean surface is expected to be balanced with the equipotential surface acted by the centrifugal force.

(1) Radial displacement, sea surface height and the rotation polar shift effect on geopotential

Assuming that the centrifugal force of rotation polar shift is $\Delta\Psi_c$, the ground radial displacement is generated under the action of the radial Love number h_2, and then the Earth's rotation polar shift effect on the radial displacement can be expressed as:

$$r_p(\theta,\lambda,t)=\frac{h_2}{g_0}\Delta\Psi_c=H_pRe\big(m^*(t)h_2\bar{P}_{21}(cos\theta)e^{i\lambda}\big) \tag{6.17}$$

$$H_p=\sqrt{\frac{A}{\rho_e R}\frac{\omega^2R^2}{GM}}=\frac{\sqrt{8\pi}}{\sqrt{15}}\frac{\omega^2R^4}{GM}=\frac{\sqrt{8\pi}}{\sqrt{15}}\frac{\omega^2R^2}{g_0},\frac{8\pi}{5}R^4=\frac{3}{\rho_e R}A \tag{6.18}$$

Where, H_p is the scale factor of the Earth's rotation polar shift effect on radial displacement, $g_0=GM/R^2$ is the mean ground gravity, and $H_p=0.1385$m when the rotation polar shift parameter $m(t)$ is in unit of angular seconds (as or ″).

Similar to the expression of ocean tidal height, the Earth's rotation polar shift effect $\hbar_o(\theta,\lambda,t)$ on sea surface height can be expressed using the ocean spatial admittance function $Z(\theta,\lambda)$ as follows:

$$\hbar_o(\theta,\lambda,t)=H_pRe[m^*(t)Z(\theta,\lambda)] \tag{6.19}$$

After introducing the scale factor H_p, the ocean admittance function $Z(\theta,\lambda)$ becomes a normalized (dimensionless) spatial harmonic function, which can be decomposed into the spherical harmonic series as follows:

$$Z(\theta,\lambda)=\sum_{n=0}^{\infty}Z_n(\theta,\lambda) \tag{6.20}$$

The Earth's polar shift effect $\hbar_o(\theta,\lambda,t)$ on sea surface height also associates the re-adjustment of ocean mass and geopotential variations on sea surface. This is the direct influence of sea surface height variation induced by rotation polar shift to the geopotential, which can be expressed as:

$$U(\theta,\lambda,t)=\sum_{n=0}^{\infty}U_n(\theta,\lambda,t)=H_pg_0Re[m^*(t)\sum_{n=0}^{\infty}\alpha_nZ_n(\theta,\lambda)] \tag{6.21}$$

Here, $\alpha_n=\frac{3}{2n+1}\frac{\rho_w}{\rho_e}$.

The direct influence U_n of geopotential produces the associated potential under the action of the load potential Love number k_n', so we have:

$$U_o^a(\theta,\lambda,t)=\sum_{n=0}^{\infty}k_n'U_n(\theta,\lambda,t)=H_pg_0Re[m^*(t)\sum_{n=0}^{\infty}k_n'\alpha_nZ_n(\theta,\lambda)] \tag{6.22}$$

The ocean polar tide effect on the geopotential is equal to the sum of the direct influence of sea surface height variation induced by rotation polar shift to geopotential and the associated potential, namely

$$\begin{aligned}U_o(\theta,\lambda,t)&=\sum_{n=0}^{\infty}(1+k_n')U_n(\theta,\lambda,t)\\&=H_pg_0Re[m^*(t)\sum_{n=0}^{\infty}(1+k_n')\alpha_nZ_n(\theta,\lambda)]\end{aligned} \tag{6.23}$$

(2) Self-consistent equilibrium ocean polar tide effects on geopotential coefficients

On the two maximum long-period constituents of the solid Earth tide, the ocean is likely to have a long-wave response corresponding to the equilibrium response. As the period increases, the deviation of this response from the equilibrium state is smaller. The equilibrium ocean polar tide effect assumes that the instantaneous ocean surface is a gravity equipotential surface, that is, the instantaneous ocean surface and equipotential surface are in an equilibrium state, and then the equilibrium displacement of the ocean surface relative to the seabed is determined by subtracting the polar tide effect from the sea equipotential surface.

The equilibrium ocean polar tide admittance function $\bar{Z}^c$ is proportional to the ground tilt tidal factor (namely sea surface height tidal factor) $\gamma_2 = 1 + k_2 - h_2$, which can be expressed as the product of the normalized equilibrium admittance function $\bar{E}^c$ and ground tilt tidal factor γ_2.

$$\bar{Z}^c(\theta,\lambda) = \gamma_2 \bar{E}^c(\theta,\lambda) \tag{6.24}$$

$$\bar{E}^c(\theta,\lambda) = \sum_{n=0}^{\infty} \bar{E}_n^c(\theta,\lambda) = \mathcal{O}(\theta,\lambda)\left[\bar{P}_{21}(cos\theta)e^{i\lambda} + K^c\right] \tag{6.25}$$

Where, $\mathcal{O}(\theta,\lambda)$ is an ocean function, $\mathcal{O}(\theta,\lambda) = 1$ when (θ,λ) is located in the ocean area, and $\mathcal{O}(\theta,\lambda) = 0$ when (θ,λ) is located on land.

The complex constant K^c is employed in the equation (6.25) to maintain the mass conservation of the classical equilibrium ocean polar tide. Assuming that the ocean has a constant density, the zero-degree spherical harmonic component of the ocean polar tide should be equal to zero, namely $\bar{Z}_0^c = \bar{E}_0^c = 0$.

The self-consistent equilibrium ocean polar tide response function $\bar{Z}^s(\theta,\lambda)$ after considering the rotation polar shift centrifugal potential and its associated potential is also proportional to the ground tilt tidal factor $\gamma_2 = 1 + k_2 - h_2$, which can be expressed by the normalized self-consistent equilibrium admittance function $\bar{E}^s$ as:

$$\bar{Z}^s(\theta,\lambda) = \gamma_2 \bar{E}^s(\theta,\lambda) \tag{6.26}$$

$$\bar{E}^s(\theta,\lambda) = \sum_{n=0}^{\infty} \bar{E}_n^s = \mathcal{O}(\theta,\lambda)\left[\bar{P}_{21}(cos\theta)e^{i\lambda} + \sum_{n=0}^{\infty} \gamma_n' \alpha_n \bar{E}_n^s + K^s\right] \tag{6.27}$$

Where, K^s is a complex constant, which is employed to maintain the self-consistent balance of ocean polar tide mass conservation. $\gamma_n' = 1 + k_n' - h_n'$ is the degree-n load tidal factor of ground tilt.

The spherical harmonic components of the normalized admittance functions $\bar{E}_n^c$ and $\bar{E}_n^s$ are defined from the coefficients $(\bar{A}_{nm} + i\bar{B}_{nm})$ as the following spherical harmonic series:

$$\bar{E}(\theta,\lambda) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \bar{P}_{|n|m}(cos\theta)(\bar{A}_{nm} + i\bar{B}_{nm})e^{i\lambda} \tag{6.28}$$

The first and second terms of Equation (6.27) can be considered as the first and second terms of the self-consistent equilibrium pole tide, so the normalized admittance can be calculated by using the iterative scheme of $\bar{E}_n^s = \bar{E}_n^c$ in the first iteration.

Let $\bar{A}_{nm} = \bar{A}_{nm}^R + i\bar{A}_{nm}^I, \bar{B}_{nm} = \bar{B}_{nm}^R + i\bar{B}_{nm}^I$ be the degree-n order-m ocean polar tide load coefficients in self-consistent equilibrium state, then the direct influence of ocean polar tide loads to normalized geopotential coefficients can be expressed by the Earth's rotation polar shift parameters (m_1, m_2) (Desai, 2002) as follows:

$$\begin{bmatrix}\Delta\bar{C}_{nm}\\ \Delta\bar{S}_{nm}\end{bmatrix} = R_n\left\{\begin{bmatrix}\bar{A}^R_{nm}\\ \bar{B}^R_{nm}\end{bmatrix}(m_1\gamma_2^R + m_2\gamma_2^I) + \begin{bmatrix}\bar{A}^I_{nm}\\ \bar{B}^I_{nm}\end{bmatrix}(m_2\gamma_2^R - m_1\gamma_2^I)\right\} \tag{6.29}$$

$$R_n = \frac{\omega^2 R^4}{GM}\frac{4\pi G\rho_w}{g_0(2n+1)} = \frac{\omega^2 R^2}{g_0^2}\frac{4\pi G\rho_w}{2n+1},\ \ \gamma_2 = \gamma_2^R + i\gamma_2^I \tag{6.30}$$

Here, from the IERS Earth orientation parameters (EOP) time series and the 360-degree self-consistent equilibrium ocean polar tide load coefficient model (Desai, 2002) in IERS convertions (2010), the time series of ocean polar tide effects on various geodetic elements are calculated at the point P (121.3°E, 28.8°N, H11m) in the coastal zone as shown in Fig 6.2. The time span of the time series is from January 1,2018 to December 31,2022 (4 years), with a time interval of 1 day. Where, the calculation subroutine for formula (6.29) can be obtained from the IERS website.

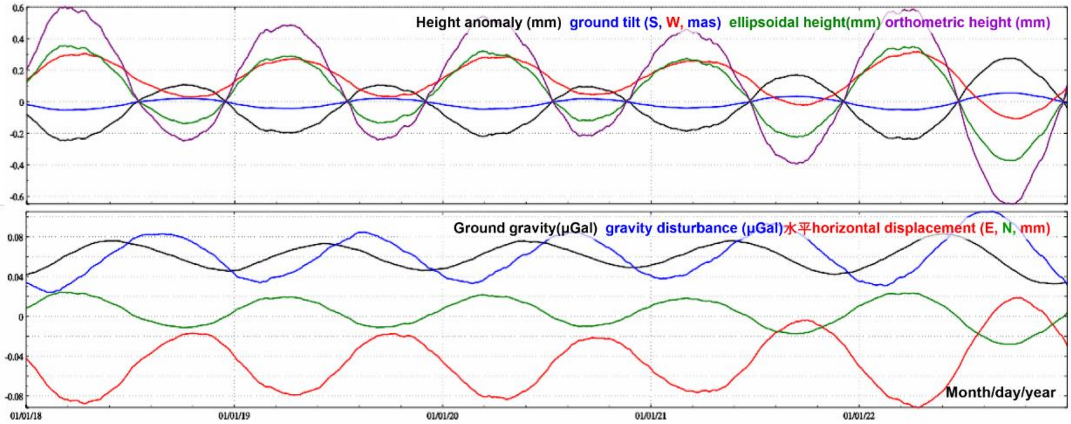

Fig 6.2 Ocean polar tide effect time series on geodetic variations at the point P in the coastal zone area

The ocean polar tide effects on geodetic variations are small, which can be ignored for general geodetic cases.

8.6.3 Various tidal effects on the Earth rotation parameters

(1) Zonal harmonic tidal effects on length of day and Earth's rotation rate

The response of the solid Earth to the zonal harmonic tidal potential causes the periodic variation of the principal moment of inertia, and then the amplitude of the Earth's rotation motion is amplified and the scale factor of the rotation rate is changed according to the conservation principle of angular momentum.

The Earth's tidal generating potential V_G from the celestial body at the ground point P (θ, λ) can be expanded into a spherical harmonic function series as:

$$V_G(P) = GM\sum_{n=2}^{\infty}\frac{a^n}{r^{n+1}}P_n(cos\psi) \tag{6.31}$$

Where, ψ is the geocentric angle distance between the ground point P (θ,λ) and celestial body (r,Θ,Λ), and (r,Θ,Λ) is the spherical coordinates of the celestial body in the Earth-fixed coordinate system, they all change with time. The degree-2 tidal potential can be decomposed into three groups of spherical harmonic functions as follows:

$$V_{G,20}(P) = GM\frac{a^2}{r^3}P_{20}(cos\theta)P_{20}(cos\Theta) \tag{6.32}$$

$$V_{G,21}(P) = \frac{1}{3}GM\frac{a^2}{r^3}P_{21}(cos\theta)P_{21}(cos\Theta)cos(\Lambda-\lambda) \tag{6.33}$$

$$V_{G,22}(P) = \frac{1}{12}GM\frac{a^2}{r^3}P_{22}(cos\theta)P_{22}(cos\Theta)cos2(\Lambda-\lambda) \tag{6.34}$$

The formulas (6.33) and (6.34) contain the tesseral and sector harmonic functions, respectively, to describe the semidiurnal and diurnal variations of short-period tides, while the formula (6.32) contains the zonal harmonic function, which only depends on the geocentric colatitude Θ of the celestial body and changes slowly, so it is employed to describe the medium and long-period tidal waves.

The main periods of the zonal harmonic tides on the Moon are 14 days M_f and 28 days M_m, and the main periods of the zonal harmonic tides from the Sun are semi-annual S_{sa} and annual S_a. These zonal harmonic tides are the largest terms that cause changes in the length of day (LOD).

Considering the frequency dependent corrections of long-period Love number $k_{20}(\sigma)$ from the mantle anelasticity, taking the scale factor $k/c_m = 0.94$ ($c_m = 0.293$ is the polar moment of inertia coefficient of the mantle), the Earth rotation long-period tidal change correction algorithm formulas (IERS convertions, 2010) with the periods of 5 days to 18.6 years are:

$$\Delta UT1 = m_3\Lambda_0 = -\sum_{i=1}^{62}(A_i sin\phi_i - B_i cos\phi_i) \tag{6.35}$$

$$\Delta LOD = \sum_{i=1}^{62}(A_i' cos\phi_i - B_i' sin\phi_i) \tag{6.36}$$

$$\Delta\omega = \sum_{i=1}^{62}(A_i'' cos\phi_i - B_i'' sin\phi_i) \tag{6.37}$$

Where, A_i, B_i, A_i', B_i', A_i'', B_i'' are the in-phase amplitude and out-of-phase amplitude of the long-period tidal constituent with frequency σ_i, respectively, as shown in column 7-12 of tab 6.1 (omit the tidal waves with all 6 coefficients less than 1.0), and ϕ_i is the astronomical argument of the long-period constituent σ_i, which is calculated by the basic Delaunay variables (columns 1~5 in the tab 6.1) or the Doodson number.

Tab 6.1 Zonal harmonic tidal effect corrections on length of day and rotation rate

Delaunay variables					Period (day)	$\Delta UT1$		ΔLOD		$\Delta\omega$	
l	l'	F	D	Ω		A_i	B_i	A_i'	B_i'	A_i''	B_i''

0	0	2	2	2	7.10	-0.1231	0.0000	1.0904	0.0000	-0.9203	0.0000
1	0	2	0	1	9.12	-0.4108	0.0000	2.8298	0.0000	-2.3884	0.0000
1	0	2	0	2	9.13	-0.9926	0.0000	6.8291	0.0000	-5.7637	0.0000
-1	0	2	2	2	9.56	-0.1974	0.0000	1.2978	0.0000	-1.0953	0.0000
0	0	2	0	0	13.61	-0.2989	0.0000	1.3804	0.0000	-1.1650	0.0000
0	0	2	0	1	13.63	-3.1873	0.2010	14.6890	0.9266	-12.3974	-0.7820
0	0	2	0	2	13.66	-7.8468	0.5320	36.0910	2.4469	-30.4606	-2.0652
2	0	0	0	0	13.78	-0.3384	0.0000	1.5433	0.0000	-1.3025	0.0000
0	0	0	2	0	14.77	-0.7341	0.0000	3.1240	0.0000	-2.6367	0.0000
-1	0	2	0	2	27.09	0.4352	0.0000	-1.0093	0.0000	0.8519	0.0000
1	0	0	0	-1	27.44	0.5339	0.0000	-1.2224	0.0000	1.0317	0.0000
1	0	0	0	0	27.56	-8.4046	0.2500	19.1647	0.5701	-16.1749	-0.4811
1	0	0	0	1	27.67	0.5443	0.0000	-1.2360	0.0000	1.0432	0.0000
-1	0	0	2	0	31.81	-1.8236	0.0000	3.6018	0.0000	-3.0399	0.0000
0	1	2	-2	2	121.75	-1.8847	0.0000	0.9726	0.0000	-0.8209	0.0000
0	0	2	-2	1	177.84	1.1703	0.0000	-0.4135	0.0000	0.3490	0.0000
0	0	2	-2	2	182.62	-49.7174	0.4330	17.1056	0.1490	-14.4370	-0.1257
0	1	0	0	0	365.26	-15.8887	0.1530	2.7332	0.0263	-2.3068	-0.0222
0	0	0	0	2	-3399.19	7.8998	0.0000	0.1460	0.0000	-0.1232	0.0000
0	0	0	0	1	-6798.38	-1617.2681	0.0000	-14.9471	0.0000	12.6153	0.0000

(2) Long-period ocean tidal correction for the Earth's rotation polar shift and effective excitation

The long-period term of the rotation polar shift mainly includes the half-Chandler, semi-annual, seasonal, month and fortnight period, etc., as well as the quasi-two-year and 300-day period. The motion equations of the unforced rotation expressed by the effective angular momentum function are:

$$\chi(t) = m^*(t) + \frac{i}{\sigma_c}\dot{m}^*(t), \quad \psi_3(t) = -m_3(t) = \frac{\Delta LOD(t)}{\Lambda_0} \tag{6.38}$$

$$\chi(t) = \chi_1(t) + i\chi_2(t), \; m(t) = m_1(t) + im_2(t) \tag{6.39}$$

$$\begin{cases} \chi_1(t) = \frac{1.608}{(C-A)\omega}[h_1(t) + (1+k_2')\omega I_{13}(t)] \\ \chi_2(t) = \frac{1.608}{(C-A)\omega}[h_2(t) + (1+k_2')\omega I_{23}(t)] \\ \chi_3(t) = \frac{0.997}{C\omega}[h_3(t) + 0.750\omega I_{33}(t)] \end{cases} \tag{6.40}$$

Here, $m^*(t)$ is the complex conjugate of $m(t)$, σ_c is the complex frequency of Chandler's wobble, $\Lambda_0 = 86400s$ is the mean day of length, $\chi(t)$ is the effective angular momentum (EAM) function of rotation polar shift, $\boldsymbol{h}(t) = [h_1(t), h_2(t), h_3(t)]$ is the relative angular momentum of matter motion in Earth's interior, C and A are the main moments of inertia of the Earth, and ω is the mean angular rate of rotation.

The effective angular momentum function $\boldsymbol{\chi}(t) = [\chi_1(t), \chi_2(t), \chi_3(t)]$ in Formula (6.40) mainly includes two parts: the change $\Delta\boldsymbol{I}$ of inertia tensor caused by the mass redistribution, and the change $\Delta\boldsymbol{h}$ of relative angular momentum caused by the velocity of matter movement in Earth's interior. Four coefficients are introduced in the formula (6.40), 1.608 is the amplitude amplification factor considering the mantle anelasticity and liquid core effect, 0.750 is the scale factor of the rotation rate change considering the ocean motion friction and drag effect of the viscosity of the mantle, 0.997 indicates that the rotation centrifugal force reduces the rotation rate by 0.3%.

Tidal correction algorithm formulas for Earth's rotation polar shift and effective angular momentum with the periods of 9 days to 18.6 years are as follows:

$$m^*(t) = m_1(t) - im_2(t) = A_p e^{i[\phi(t)+\varphi_p]} + A_r e^{i[-\phi(t)+\varphi_r]} \tag{6.41}$$

$$\chi(t) = \chi_1(t) + i\chi_2(t) = A_p e^{i[\phi(t)+\varphi_p]} + A_r e^{i[-\phi(t)+\varphi_r]} \tag{6.42}$$

Here, $\phi(t)$ is the astronomical argument, A_p, φ_p are the prograde harmonic amplitude and phase of the long-period ocean tidal effect excited by the rotation polar motion or effective angular momentum, respectively, and A_r, φ_r are the retrograde harmonic amplitude and phase of that, respectively.

Tab 6.2 Long-period ocean tidal correction for the Earth's rotation polar shift and effective excitation

	Delaunay variables					Period (day)	Correction for polar shift m				Correction for EAM χ			
	l	l'	F	D	Ω		A_pμas	φ_p°	A_rμas	φ_r°	A_pμas	φ_p°	A_rμas	φ_r°
m_{tm}	1	0	2	0	1	9.12	4.43	-112.62	5.57	21.33	205.83	67.21	269.95	21.17
M_{tm}	1	0	2	0	2	9.13	10.72	-112.56	13.48	21.3	497.59	67.27	652.59	21.14
m_f	0	0	2	0	1	13.63	27.35	-91.42	30.59	13.31	841.32	88.42	1002.12	13.15
M_f	0	0	2	0	2	13.66	66.09	-91.31	73.86	13.27	2028.73	88.53	2414.94	13.11
M_{sf}	0	0	0	2	0	14.77	5.94	-87.13	6.42	11.75	168.13	92.7	194.74	11.6
M_m	1	0	0	0	0	27.56	43.74	-56.7	31.12	-0.91	643.61	123.13	520.16	-1.06
M_{sm}	-1	0	0	2	0	31.81	8.85	-51.11	5.42	-4.21	111.62	128.72	79.23	-4.36
S_{sa}	0	0	2	-2	2	182.62	86.48	-20.3	99.77	175.57	118.56	159.42	336.32	175.46
S_a	0	1	0	0	0	365.26	17.96	-17.38	152.15	170.6	3.33	161.6	332.53	170.51
M_n	0	0	0	0	1	-6798.38	208.17	166.89	186.98	166.67	221.43	166.88	175.07	166.68

Fig 6.3 long-period tidal effect time series for the Earth's rotation motion from January 1, 2026 to December 31, 2028 (3years) are predicted according to the formulas (6.35) ~ (6.37), (6.41) and (6.42), and the time series sampling interval is 4 hours.

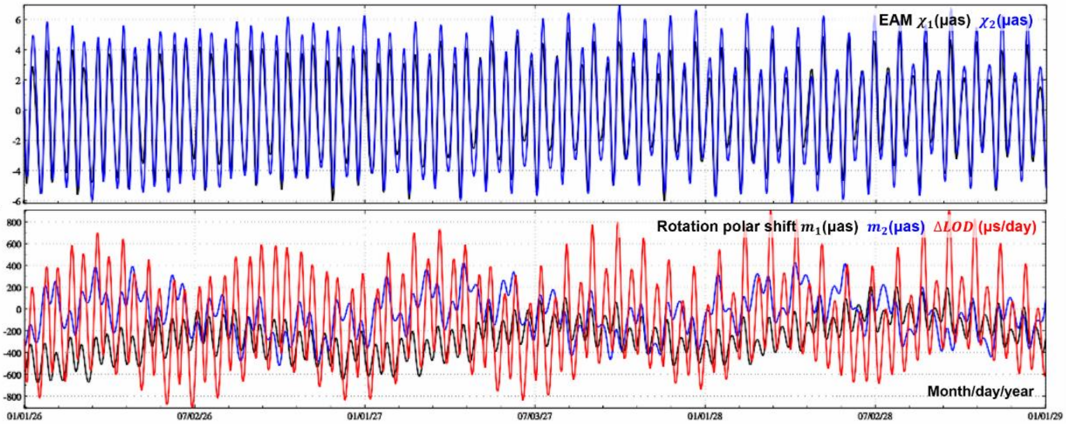

Fig 6.3 Long-period tidal effect time series for the Earth's rotation motion

(3) Diurnal and semidiurnal ocean tidal effects on the Earth's rotation parameters

The current view is that the diurnal and semi-diurnal variations of the Earth's rotation are mainly the response of the solid Earth to the effects of ocean tides and ocean currents. The centrifugal potential variation excites the deformation of solid Earth, resulting in the change of Earth's inertia tensor. The Diurnal and semidiurnal moments are mainly from the principal inertia difference $B-A$ of the three-axis Earth (the principal inertial axis coordinate system), where B is the polar moment of inertia and A is the equatorial moment of inertia, then the degree-2 sector harmonic non-normalized geopotential coefficient C_{22} is:

$$C_{22}=\frac{1}{4}MR^2(B-A) \tag{6.43}$$

In general, the principal axis of the Earth-fixed coordinate system does not coincide with the principal axis of inertia of the Earth. At this case, the difference $B-A$ in the equatorial plane becomes:

$$B-A=4MR^2\sqrt{C_{22}^2+S_{22}^2} \tag{6.44}$$

and then leads to the rotation polar shift and UT1 variation as:

$$m(t)=-\frac{0.36GM}{\omega^2R^3}\frac{B-A}{A}sin2\Theta e^{-i(\Lambda-2\lambda)} \tag{6.45}$$

$$UT1(t)=-\frac{0.3GM}{8\omega^2R^3}\frac{B-A}{C_m}sin^2\Theta sin2(\Lambda-2\lambda) \tag{6.46}$$

Where, Θ,Λ are the colatitude and longitude of the tidal celestial body in the Earth-fixed coordinate system, respectively.

It can be seen from the formulas (6.44) ~ (6.46) that C_{22} excites the semidiurnal variation of Earth's rotation. The theoretical calculation (Chao et al., 1996) shows that its magnitude is about 0.06 mas (1mas of geocentric angle distance is about 3cm away from the corresponding ground).

Similar to the excitation of the ocean tide to the Earth's rotation polar motion and rotation rate variation in Formula (6.40), the diurnal and semi-diurnal variations of the Earth's rotation excited by the ocean tide can be expressed by the harmonic function series as:

$$m_1 = \sum_{i=1}^{n}(-A_i^c cos\phi_i + A_i^s sin\phi_i) \tag{6.47}$$

$$m_2 = \sum_{i=1}^{n}(A_i^c sin\phi_i + A_i^s cos\phi_i) \tag{6.48}$$

$$\Delta UT1 = m_3\Lambda_0 = \sum_{i=1}^{n}(B_i^c cos\phi_i + B_i^s sin\phi_i) \tag{6.49}$$

At present, the Eanes2000 model and interp.f fortran code in the IERS convertions (2010) are widely employed, which can be obtained from the IERS website.

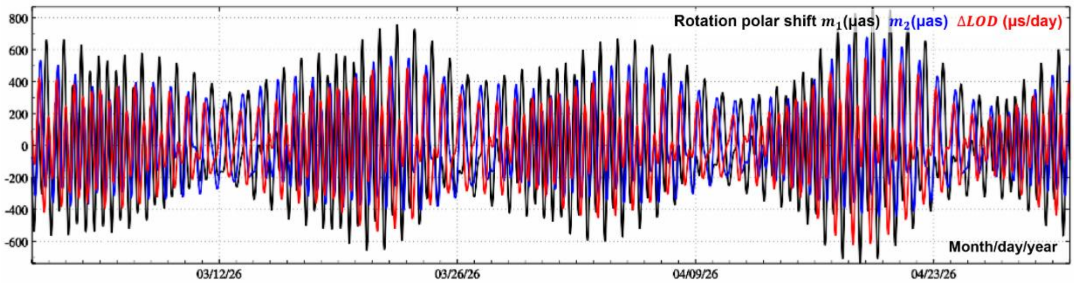

Fig 6.4 The time series of diurnal and semi-diurnal tidal effects on ERP

Fig 6.4 the time series of diurnal and semi-diurnal tidal effects on Earth's rotation parameters from March 1, 2026 to April 30, 2026 (2 months) are predicted according to the formulas (6.47) ~ (6.49), and the time series sampling interval is 15 minutes.

8.6.4 Calculation of CIP instantaneous polar coordinates in ITRS

According to the IERS convertions (2010), the instantaneous coordinates of the celestial intermediate polar (CIP) in the ITRS are expressed in the polar coordinate system, whose y-axis direction is opposite to the y-axis direction of the ITRS, denoted as (p_1, p_2), and its unit and direction are the same as the rotation polar shift (m_1, m_2). Since the forced nutation of external celestial bodies in GCRS with a period of less than 2 days is not included in the IAU2000/IAU2006 nutation model, it is necessary to consider the corresponding motion model of the Earth's rotation pole in ITRS.

(p_1, p_2) is composed of $(m_1, m_2)_{IERS}$ provided by IERS Bulletin A and B, plus ocean tides and forced nutation correction terms of external celestial bodies with periods less than 2 days in GCRS, namely

$$(p_1, p_2) = (m_1, m_2)_{IERS} + (m_1, m_2)_{OT} + (m_1, m_2)_{LIB} \tag{6.50}$$

Where, $(m_1, m_2)_{OT}$ are the diurnal and semi-diurnal variations in the rotation polar coordinates caused by ocean tides, and $(m_1, m_2)_{LIB}$ are the variations in the rotation polar coordinates corresponding to motions with periods less than two days in space that are not part of the IAU 2000 nutation model.

The high-frequency polar shift term $(m_1, m_2)_{OT}$ mainly includes the diurnal and semi-diurnal variations caused by the ocean tide, which can be calculated according to the formulas (6.47) ~ (6.49). The non-zonal harmonic oscillation terms $(m_1, m_2)_{LIB}$, including the forced diurnal and semi-diurnal rotation polar shift terms, was previously regarded as nutation and is now classified as the rotation polar shift. The non-harmonic oscillation term is due to the diurnal and semi-diurnal terms of the tidal celestial bodies, resulting in the change of the Earth's inertial tensor $\Delta\boldsymbol{I}$ with time, which in turn produces the rotation polar shift according to formula (6.1).

The long-period terms and the long-term changes caused by the torque from tidal celestial bodies are generally considered to be included in the observed rotation polar shift and do not need to be added to the $(m_1, m_2)_{IERS}$.