Theory and algorithm of gravity field approach using spherical radial basis functions

The disturbing potential $T(x)$ at the point x outside the Earth can be expressed as a linear combination of normalized surface harmonic basis functions:

$$T(x)=\frac{GM}{r}\sum_{n=2}^{N}\left(\frac{a}{r}\right)^{n}\sum_{m=-n}^{n}\bar{F}_{nm}\bar{Y}_{nm}(e) \qquad (10.1)$$

where $x=r\cdot e=r(sin\theta cos\lambda, sin\theta sin\lambda, cos\theta)$, r,λ,θ are the geocentric distance, longitude and colatitude of the point x outside the Earth, respectively, $\bar{F}_{nm}$ are the fully normalized Stokes coefficients, also known as the geopotential coefficients, GM, a are the geocentric gravitational constant and equatorial radius of the Earth, respectively, called as the scale parameters, and $\bar{Y}_{nm}$ is the normalized surface harmonic function:

$$\bar{Y}_{nm}(e)=\bar{P}_{nm}(cos\theta)cosm\lambda,\quad \bar{F}_{nm}=\delta\bar{C}_{nm},\quad m\geq 0$$
$$\bar{Y}_{nm}(e)=\bar{P}_{n|m|}(cos\theta)sin|m|\lambda,\quad \bar{F}_{nm}=\bar{S}_{n|m|},\quad m<0 \qquad (10.2)$$

where $\bar{P}_{nm}(cos\theta)$ is the fully normalized associative Legendre function, n is called as the degree of the geopotential coefficient, and m is called as order of geopotential coefficients.

The equatorial radius a of the Earth in formula (10.1) represents the geopotential coefficients defined on the sphere whose radius is equal to the equatorial radius a of the Earth. If it is replaced by the radius $\mathcal{R}$ of the Bjerhammar sphere, the geopotential coefficients will be defined on the Bjerhammar spherical surface. In this case, the geopotential coefficient $\bar{F}_{nm}$ is also converted into $\bar{E}_{nm}$ due to the change of the scale parameter, and $a^{n}\bar{F}_{nm}=\mathcal{R}^{n}\bar{E}_{nm}$, the formula (10.1) becomes:

$$T(x)=\frac{GM}{r}\sum_{n=2}^{N}\left(\frac{\mathcal{R}}{r}\right)^{n}\sum_{m=-n}^{n}\bar{E}_{nm}\bar{Y}_{nm}(e) \qquad (10.3)$$

7.10.1 Spherical radial basis function representation of external disturbing potential

The disturbing potential $T(x)$ at any point x outside the Earth can also be expressed as a linear combination of K spherical radial basis functions (SRBFs):

$$T(x)=\frac{GM}{r}\sum_{k=1}^{K}d_{k}\Phi_{k}(x,x_{k}) \qquad (10.4)$$

where $x_{k}=\mathcal{R}\cdot e_{k}$ is the SRBF node defined on the Bjerhammar sphere, also known as the SRBF center or SRBF pole, d_{k} is the SRBF coefficient, K is the number of the

SRBF nodes, equal to the number of SRBF coefficients, $\Phi_k(x,x_k)$ is the spherical radial basis function of the disturbing potential can be abbreviated as $\Phi_k(x)=\Phi_k(x,x_k)$.

The spherical radial basis function $\Phi_k(x,x_k)$ can be furtherly expanded into the Legendre series:

$$\Phi_k(x,x_k)=\sum_{n=2}^{N}\phi_n P_n(\psi_k)=\sum_{n=2}^{N}\frac{2n+1}{4\pi}B_n\left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \qquad (10.5)$$

where ϕ_n is the degree n Legendre coefficient of SRBF, which characterizes the shape of SRBF and basically determines the spatial and spectral figures of SRBF, also known as the SRBF shape factor. When the spectral domain degree n need be not emphasized, B_n is also called as the Legendre coefficient of SRBF. $\mu=\mathcal{R}/r$ is also called as the bandwidth parameter because it is related to the spectral domain bandwidth of the radial basis function $\Phi_k(x)$.

Substitute the formula (10.5) into (10.4) to get:

$$T(x)=\frac{GM}{4\pi r}\sum_{n=2}^{N}(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n\sum_{k=1}^{K}d_k P_n(\psi_k)$$

$$=\frac{GM}{4\pi r}\sum_{k=1}^{K}d_k\sum_{n=2}^{N}(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \qquad (10.6)$$

Considering the addition theorem of spherical harmonics:

$$P_n(\psi_k)=P_n(e,e_k)=\frac{4\pi}{2n+1}\sum_{m=-n}^{n}\bar{Y}_{nm}(e)\bar{Y}_{nm}(e_k), \qquad (10.7)$$

then the formula (10.5) can be written as

$$T(x)=\frac{GM}{r}\sum_{n=2}^{N}B_n\left(\frac{\mathcal{R}}{r}\right)^n\sum_{m=-n}^{n}\sum_{k=1}^{K}d_k\bar{Y}_{nm}(e)\bar{Y}_{nm}(e_k) \qquad (10.8)$$

Comparing formulas (10.1), (10.3) and (10.8), we have:

$$\bar{F}_{nm}=\left(\frac{\mathcal{R}}{a}\right)^n\bar{E}_{nm}=B_n\left(\frac{\mathcal{R}}{a}\right)^n\sum_{k=1}^{K}d_k\bar{Y}_{nm}(e_k) \qquad (10.9)$$

Using formula (10.9), the geopotential coefficient $\bar{F}_{nm}$ can be calculated from the spherical radial basis function coefficient d_k. After that, the geopotential coefficient can be employed to calculate various anomalous gravity field elements outside the Earth.

The position, distribution and amount of the SRBF nodes (centers) $\{x_k\}$ on the Bjerhammar sphere are the key indicators for gravity field approach using spherical radial basis function, which determine the spatial degrees of freedom (spatial resolution) and spatial feature of regional gravity field, like the degree of the global geopotential coefficient model.

7.10.2 Spherical radial basis functions suitable for gravity field approach

The radial basis function employed for the gravity field approach must satisfy the Laplace equation. Some spherical radial basis kernel function such as the point mass kernel function, Poisson kernel function, radial multipole kernel function and Poisson wavelet kernel function are all harmonic.

Let x be the calculation point outside the Earth and x_k be the SRBF center on the Bjerhammar sphere $\Omega_{\mathcal{R}}$.

(1) The point mass kernel function

The point mass kernel function is an inverse multiquadric function (IMQ) proposed by Hardy (1971), which is the kernel function of the gravitational potential integral formula $V = G\iiint \frac{dm}{L}$, and its analytical expression is:

$$\Phi_{IMQ}(x, x_k) = \frac{1}{L} = \frac{1}{|x - x_k|} \quad (10.10)$$

where L is the space distance between x and x_k. Since $\Delta(1/L) = 0$, the point mass kernel function $\Phi_{IMQ}(x, x_k)$ satisfies the Laplace equation.

(2) The Poisson kernel function

The Poisson kernel function is derived from the Poisson integral formula of the anomalous gravity field element, and its analytical expression is:

$$\Phi_P(x, x_k) = -2r\frac{\partial}{\partial r}\left(\frac{1}{L}\right) - \frac{1}{L} = \frac{r^2 - r_k^2}{L^3} \quad (10.11)$$

(3) The radial multipole kernel function

The analytical expression of the radial multipole kernel function is:

$$\Phi_{RM}^m(x, x_k) = \frac{1}{m!}\left(\frac{\partial}{\partial r_k}\right)^m \frac{1}{L} \quad (10.12)$$

where m can be called as the order of the radial multipole kernel function, and the zero-order radial multipole kernel function is the point mass kernel function $\Phi_{IMQ}(x, x_k) = \Phi_{RM}^0(x, x_k)$.

(4) The Poisson wavelet kernel function

The analytical expression of the Poisson wavelet kernel function is:

$$\Phi_{PW}^m(x, x_k) = 2(\chi_{m+1} - \chi_m), \quad \chi_m = \left(r_k\frac{\partial}{\partial r_k}\right)^m \frac{1}{L} \quad (10.13)$$

The zero-order Poisson wavelet kernel function is the Poisson kernel function

$\Phi_P(x, x_k) = \Phi_{PW}^0(x, x_k)$.

(5) Calculation of spherical radial basis functions

The spherical radial basis function analytical expressions (10.10) ~ (10.13) are usually expressed in the Legendre series (10.5), and then calculated according to the Legendre expansion to highlight the spectral domain figures of the anomalous gravity field element.

PAGravf4.5 normalizes the Legendre expansion of the spherical radial basis function $\Phi_k(x, x_k)$, and then calculates the spherical radial basis function (SRBF) using the normalized Legendre expansion. When dealing with different types of observed field elements, the SRBF of various field elements are uniformly divided by the normalization coefficient of disturbing potential SRBF to maintain the analytical relationship between different types of field elements.

Let the spherical angular distance $\psi_k = 0$ from x_k to x, then $cos\psi_k = 1$, $P_n(cos\psi_k) = P_n(1) = 1$, substitute it into formula (10.5), we have the general expression of the normalization coefficient of disturbing potential SRBF:

$$\Phi^0 = \sum_{n=2}^{N} \frac{2n+1}{4\pi} B_n \mu^n \quad (10.14)$$

The Legendre expansion of the normalized spherical radial basis function of disturbing potential (height anomaly) is:

$$\Phi_k(x, x_k) = \frac{1}{\Phi^0} \sum_{n=2}^{N} \phi_n P_n(\psi_k) = \frac{1}{\Phi^0} \sum_{n=2}^{N} \frac{2n+1}{4\pi} B_n \mu^n P_n(\psi_k) \quad (10.15)$$

The above four forms of disturbing potential SRBF and their corresponding Legendre coefficients are shown in Table 2, where the constant factor 4π in the Legendre coefficient B_n has been removed in advance.

SRBF	$\Phi_k(x, x_k)$	ϕ_n	B_n
Point mass kernel	$\frac{1}{L} = \frac{1}{\lvert x - x_k \rvert}$	μ^n	$\frac{1}{2n+1}$
Poisson kernel function	$\frac{r^2 - r_k^2}{L^3}$	$(2n+1)\mu^n$	1
radial multipole kernel	$\frac{1}{m!}\left(\frac{\partial}{\partial r_k}\right)^m \frac{1}{L}$	$C_n^m \mu^{n-m} \quad (n \geq m)$	$\frac{C_n^m}{2n+1} \mu^{-m}$
Poisson wavelet kernel	$2(\chi_{m+1} - \chi_m)$	$(-nln\mu)^m (2n+1)\mu^n$	$(-nln\mu)^m$

	$\chi_m = \left(r_k \frac{\partial}{\partial r_k}\right)^m \frac{1}{L}$		

7.10.3 Spherical radial basis function representation for various gravity field elements

According to the definition of the anomalous gravity field element, the spherical radial basis function parameterized form for various anomalous gravity field elements can be derived from the disturbing potential SRBF series (the rightmost expression) of (10.6).

$$\zeta(x) = \frac{T}{\gamma} = \frac{GM}{4\pi r\gamma}\sum_{k=1}^{K} d_k \sum_{n}(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n P_n(\psi_k) \quad (10.16)$$

$$\delta g(x) = -\frac{\partial T}{\partial r} = \frac{GM}{4\pi r^2}\sum_{k=1}^{K} d_k \sum_{n}(2n+1)(n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^{n-1} P_n(\psi_k) \quad (10.17)$$

$$\Delta g(x) = -\frac{\partial T}{\partial r} - \frac{2T}{r} = \frac{GM}{4\pi r^2}\sum_{k=1}^{K} d_k \sum_{n}(2n+1)(n-1)B_n\left(\frac{\mathcal{R}}{r}\right)^{n-1} P_n(\psi_k) \quad (10.18)$$

$$\xi(x) = \frac{GM}{4\pi r^2\gamma}\sum_{k=1}^{K} d_k\, cos\alpha_k \sum_{n}(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial P_n(\psi_k)}{\partial \psi_k} \quad (10.19)$$

$$\eta(x) = \frac{GM}{4\pi r^2\gamma}\sum_{k=1}^{K} d_k\, sin\alpha_k \sum_{n}(2n+1)B_n\left(\frac{\mathcal{R}}{r}\right)^n \frac{\partial P_n(\psi_k)}{\partial \psi_k} \quad (10.20)$$

$$T_{rr}(x) = \frac{GM}{4\pi r^3}\sum_{k=1}^{K} d_k \sum_{n}(2n+1)(n+1)(n+2)B_n\left(\frac{\mathcal{R}}{r}\right)^{n-1} P_n(\psi_k) \quad (10.21)$$

where α_k is the geodetic azimuth of ψ_k.

For the regional gravity field approach, the reference geopotential model (such as the EGM2008 model) is usually employed to remove the reference model value from the observed anomalous field element, and the residual gravity field is refined by the observed residual field element.

In this case, the minimum and maximum degree range in formulas (10.16) ~ (10.21) (the spectral bandwidth of the gravity field) is closely related to the selected reference geopotential model and the feature of regional gravity field in the target area, which can only be determined after verifying and analysis from actual observations.

7.10.4 Spherical Reuter grid construction and SRBF nodes design

PAGravf4.5 adopts the global and regional consistent spherical Reuter grid, constructs the spherical radial basis function SRBF centers according to the given Reuter grid level, and then using the adaptive algorithm, make the spatial distribution of

SRBF nodes be consistent with the spatial distribution of observations everywhere.

(1) Unit spherical Reuter grid construction algorithm

Given the Reuter grid level Q (even number), the geocentric latitude interval $d\varphi$ of the unit spherical Reuter grid in the spherical coordinate system and the geocentric latitude φ_i of the center of the cell-grid i are respectively

$$d\varphi = \frac{\pi}{Q}, \quad \varphi_i = -\frac{\pi}{2} + \left(i - \frac{1}{2}\right) d\varphi, \quad 1 \le i < Q \tag{10.22}$$

The cell-grid number J_i in the prime vertical circle direction at latitude φ_i, the longitude interval $d\lambda_i$ and the side length dl_i are respectively

$$J_i = \left[\frac{2\pi cos\varphi_i}{d\varphi}\right], \quad d\lambda_i = \frac{2\pi}{J_i}, \quad dl_i = d\lambda_i cos\varphi_i \tag{10.23}$$

It is not difficult to find that $dl_i \approx d\varphi$. Let

$$\varepsilon_i = \frac{ds_i - ds}{ds} = \frac{dl_i - d\varphi}{d\varphi} = \frac{d\lambda_i}{d\varphi} cos\varphi_i - 1 \tag{10.24}$$

where ds is the cell-grid area near the equator, ds_i is the cell-grid area at the prime vertical circle φ_i, and ε_i is the relative deviation of the parallel circle cell-grid area relative to the cell-grid area near the equator.

ε_i is generally small, about a few ten-thousandth, and the value is related to the Reuter grid level Q. Near the equator, we have $ds = d\varphi \cdot d\varphi$, $\varepsilon_{Q/2} = 0$.

Given the latitude and longitude range of the local area, you can directly determine the minimum and maximum value of i according to the formula (10.22), and then calculate the maximum J_i at each prime vertical circle according to the formula (10.23), to determine the regional Reuter grid whose level is Q without calculating the global grid.

(2) Regional SRBF nodes design with adaptive observation distribution

PAGravf4.5 presents a simple Reuter grid fitting algorithm to design the SRBF centers that adapts to the spatial distribution of observations. Firstly, construct a regional Reuter grid from the given level Q, and then count the number J of effective observations in each cell Reuter grid. When J is less than a given number (as the input parameter), eliminate the SRBF center. After traversing all cell Reuter grids, generate the SRBF center set that adapts to the spatial distribution of observations.

Obviously, when the observations data are a regular grid, the SRBF centers are also regularly distributed, and when the observations are irregularly distributed, the

SRBF centers are also irregularly distributed. The denser the distribution of observations, the denser the distribution of SRBF centers. That is, the spatial distribution of SRBF centers is consistent with the spatial distribution of observations everywhere.

7.10.5 SRBF coefficients estimation and gravity field approach

After the constant $GM/(4\pi)$ removed, it does not change the analytical relationship between the anomalous gravity field elements. Therefore, formulas (10.16) ~ (10.21) are rewritten as:

$$\zeta(x) = \frac{1}{r\gamma}\sum_{k=1}^{K} d_k \sum_{n} (2n+1)B_n\mu^n P_n(\psi_k) \tag{10.25}$$

$$\delta g(x) = \frac{1}{r^2}\sum_{k=1}^{K} d_k \sum_{n} (2n+1)(n+1)B_n\mu^{n-1} P_n(\psi_k) \tag{10.26}$$

$$\Delta g(x) = \frac{1}{r^2}\sum_{k=1}^{K} d_k \sum_{n} (2n+1)(n-1)B_n\mu^{n-1} P_n(\psi_k) \tag{10.27}$$

$$\xi(x) = \frac{1}{r^2\gamma}\sum_{k=1}^{K} d_k\, cos\alpha_k \sum_{n} (2n+1)B_n\mu^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{10.28}$$

$$\eta(x) = \frac{1}{r^2\gamma}\sum_{k=1}^{K} d_k\, sin\alpha_k \sum_{n} (2n+1)B_n\mu^n \frac{\partial P_n(\psi_k)}{\partial\psi_k} \tag{10.29}$$

$$T_{rr}(x) = \frac{1}{r^3}\sum_{k=1}^{K} d_k \sum_{n} (2n+1)(n+1)(n+2)B_n\mu^{n-1} P_n(\psi_k) \tag{10.30}$$

Substituting the Legendre coefficient B_n in Table 2 into the above equations, we can obtain the basic observation equations for regional gravity field approach with the (residual) anomalous gravity field element $F(x_i)$ as the observations and the SRBF coefficients $\{d_k\}$ as the unknowns.

$$L = \{F(x_i)\}^T = A\{d_k\}^T + e \quad (i = 1,\cdots,M, k = 1,\cdots,K) \tag{10.31}$$

where A is the $M \times K$ design matrix, e is the $M \times 1$ observation error vector, M is the number of observations, K is the number of RBF centers, that is, the number of unknowns $\{d_k\}$, and x_i is the position of the observations.

PAGravf4.5 proposes an algorithm that can improve the performance of parameter estimation by suppressing edge effect. When the RBF center v is located at the margin of the calculation area, its corresponding SRBF coefficient is set to zero, that is, $d_v = 0$ as the observation equation to suppress the edge effect. The normal equation with the additional suppression of edge effect constructed by PAGravf4.5 is:

$$[A^TPA + \epsilon\Xi]\{d_k\}^T = A^TPL \tag{10.32}$$

where Ξ is a diagonal matrix, whose element is equal to 1 only when the SRBF center corresponding to its subscript is in the margin of the area, and the others are zero. ϵ is equal to the diagonal standard deviation of the matrix $A^T PA$.

In PAGravf4.5, The action distance dr of all SRBF centers is required to be equal to maintain the spatial consistency of the approach performance of gravity field. Where dr corresponds to the domain of the SRBF argument, so any observation is a linear combination of the spherical radial basis functions of the SRBF centers only within the radius dr.

PAGravf4.5 improves the ill-conditioned or singularity of $A^T PA$ by adding some observation equations that can suppresses edge effect to improve the stability and reliability of parameter estimation, to instead of the regularization of the normal equation without geophysical meaning.

You can choose the LU triangular decomposition (square root method), Cholesky decomposition or unknowns smallest norm method to solve the normal equation (10.32).

7.10.6 Regional gravity field modelling from various heterogeneous observations by SRBF

It has always been a hot and difficult issue in physical geodesy to combine various types of observations to model the regional gravity field. Like the surface harmonic coefficient expansion of anomalous gravity field elements, various types of observations can be represented by spherical radial basis function expansion, such as equations (10.25) ~ (10.30). Estimating the spherical radial basis function coefficients with equations (10.25) ~ (10.30) as observation equations, we can model gravity field from various types of observations.

(1) The crucial issues of gravity field modelling using spherical radial basis function from various types of observations

The regional gravity field modelling from various heterogeneous observations by SRBF need deal with three crucial issues, namely ① The SRBF representations from various types of observations should strictly keep the analytical relationship between different types of observations. ② How to determine the contribution of different types of observations to the SRBF coefficients $\{d_k\}$. ③ Investigate the spectral center & bandwidth of target field element, observations and SRBF, and then study the relationship between the three.

For the first issue, only the SRBF Legendre expansion of height anomaly is normalized, and the SRBF Legendre expansion of other types of observations are divided all by the SRBF normalization coefficient of height anomaly. In this way, the analytical relationships can be strictly maintained between different types of observations.

The way to deal with the second issue is to construct observation equations and normal equations from different types of observations firstly, introduce some parameters related to the error or spatial distribution of observations, and then combine these normal equations to form a new normal equation.

The third issue is related to the observation situations, the nature of gravity field and the modelling algorithm. The spectral center and bandwidth of the observations, target field elements and SRBFs need be comprehensively analyzed in different parameter combinations case, and then according to the principle of fully resolving the spectrum of the target field elements, optimize the relevant scheme and parameters.

(2) Observation contribution adjustment, edge effect suppression and Parameter estimation

PAGravf4.5 recommends a universal multi-source heterogeneous observation deep fusion method by the normalization of the normal equations and adjust the contribution of the given type of observations at the same time. The normal equation is:

$$\sum_i^{i\neq j}\left(\frac{1}{\varepsilon_i}A_i^TP_iA_i\right)+\frac{\kappa^2}{\varepsilon_j}A_j^TP_jA_j+\epsilon\Xi\{d_k\}^T=\sum_i^{i\neq j}\left(\frac{1}{\varepsilon_i}A_i^TP_iL_i\right)+\frac{\kappa^2}{\varepsilon_j}A_j^TP_jL_j \qquad (10.33)$$

where ε_j is the combination parameter for the given type of adjustable observations, ε_i is the combination parameter of the i $(i\neq j)$ observations, and κ is the contribution rate of the adjustable observations j.

PAGravf4.5 multiplies the normal equation coefficient matrix $A_j^TP_jA_j$ and constant matrix $A_j^TP_jL_j$ of the adjustable observation j by κ^2 respectively, to increase $(\kappa>1)$ or decrease $(\kappa<1)$ the contribution of the adjustable observation.

For example, the GNSS-levelling residual height anomaly in the observations can be set as the adjustable observations with the contribution rate $\kappa>1$ to improve the analytical fusion of GNSS-levelling and other observations. For another example, let the nearshore altimetric elements adjustable with the contribution rate $\kappa < 1$, we can suppress the influence of the shallow water altimetric errors and improve the separation of sea surface topography.

The above normalization method of the normal equations can effectively control the deep fusion of different types of observations using covariance structure to approach gravity field from heterogeneous observation field elements. This method completely separates the contribution of the observation system model (covariance structure) and influences of observation quality (errors or gross errors), so that the fusions are away from the observation errors (gross error), observation types and spatial distribution differences of measurement points. Which is conducive to the fusion of multiple types of observation field elements with extreme differences in spatial distribution (such as very few astronomical vertical deflection or GNSS levelling data included), and is conducive to the exact detection of observation gross errors.

In this case, the normal equation does not also need to be iteratively calculated, which conducive to improve the analytical nature of the SRBF approach algorithm.

(3) The cumulative SRBF approach method to achieve the best approach of the gravity field

The target field elements are equal to the convolution of the observations and the filter SRBFs. When the target field elements and the observations are of different types, it is difficult for one SRBF to match the spectral center and bandwidth of the observations and target field elements at the same time, which would make the spectral leakage of the target field elements. In addition, the SRBF type, the minimum and maximum degree of Legendre expansion and the SRBF center distribution also all affect the approach performance of gravity field. Therefore, only the optimal estimation of the SRBF coefficient with the burial depth as the parameter is not enough to ensure the best approach of gravity field.

PAGravf4.5 proposes a cumulative SRBF approach scheme according to the linear additivity of gravity field to solve the key troubles mentioned. Using the multiple cumulative SRBF approach, it is not necessary to determine the optimal burial depth.

When each SRBF approach of gravity field employs a SRBF with different spectral figure, the cumulative SRBF approach can fully resolve the spectral domain signal of the target field elements by combining multiple SRBF spectral centers and bandwidths, and then optimally restore the target field elements in space domain.

The character of cumulative SRBF approach scheme of gravity field: the essence of each SRBF approach is to employ the previous approach results as the reference field, and then refine the residual target field elements by remove -restore scheme.

The validity principle of once SRBF approach: (1) The residual target field element grid is continuous and differentiable (view the drawing), and whose standard deviation is as small as possible. (2) The statistical mean of residuals tends to zero with the increase of cumulative approach times, and there is no obvious reverse sign.

🌏 **The typical technical features of SRBF approach program in PAGrav4.5**

① The analytical function relationships between gravity field elements are strict, and the SRBF approach performance has nothing to do with the observation errors.

② Various heterogeneous observations in the different altitudes, cross-distribution, and land-sea coexisting cases can be directly employed to model the all-element gravity field models on or outisde the geoid without reduction, continuation and griding.

③ Can integrate very few astronomical vertical deflection or GNSS-levelling data, and effectively absorb the edge effect.

④ Has the strong capacity in the detection of observation gross errors, measurement of external accuracy indexes and control of computational performance.