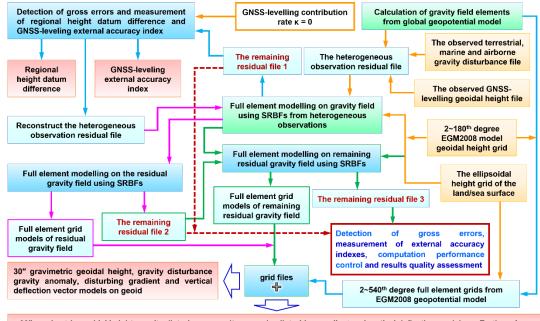
Simple process demo of full element modelling on gravity field using SRBFs in orthometric height system

Exercise purpose: From the observed terrestrial, marine and airborne gravity disturbances and GNSS-leveling geoidal heights in orthometric height system, make the full element models on gravity field using spherical radial basis functions (SRBFs) in six steps, in which all the terrain effects are not processed, to quickly master the essentials in observation analysis, computation quality control and full element modeling on regional gravity field.

After the terrain effect processing omitted, SRBF approach process of gravity field is very simple because there is no need for additional continuation reduction, gridding and GNSS leveling fusion process.



30" gravimetric geoidal height, gravity disturbance gravity anomaly, disturbing gradient and vertical deflection models on Earth surface

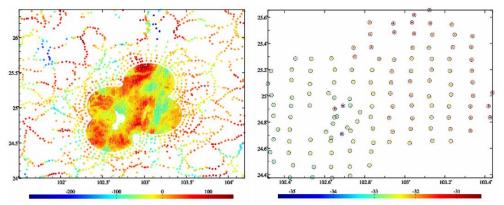
Simple process demo of full element modelling on gravity filed using SRBFs in orthometric height system

• The observed gravity disturbance and GNSS-levelling data

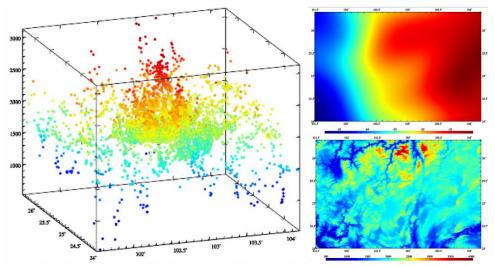
The observed terrestrial, marine and airborne gravity disturbance file obsdistgrav.txt. The file record format: ID, longitude (degree decimal), latitude, ellipsoidal height (m), observed gravity disturbance (mGal), ...

The observed GNSS-leveling geoidal height file obsGNSSIgeoid.txt in orthometric height system. The file record format: ID, longitude (degree decimal), latitude, observed geoidal height (m), ...

In the example, the observed gravity disturbance and GNSS-leveling geoidal heights are simulated from the EGM2008 model (the 2~1800th degree) in advance.



The observed gravity disturbances (mGal) and observed GNSS-levelling geoidal heights (m)



The distribution of gravity points, 2~180th degree model geoidal height and ellipsoidal height of the terrain surface

It should be noted that since the observed geoidal height by GNSS-leveling is essentially the height anomaly on the geoid in orthometric height system, the geoidal height at GNSS-leveling sites must be the geoidal height, which can be employed by the observed GNSS-leveling geoidal height or the model geoidal height from the EGM2008 model (the 2~180th degree).

• The ellipsoidal height grid of calculation surface:

The model geoidal height grid file mdlgeoidh30s.dat calculated from the 2~180th degree geopotential model, which is employed for modeling on gravity field on geoid.

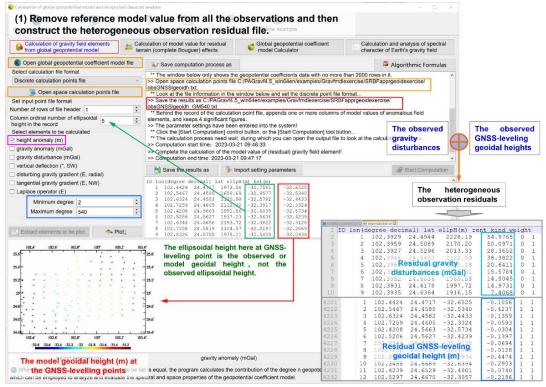
The ellipsoidal height grid file surfhgt30s.dat of the land/sea surface equal to the sum of the digital elevation model grid DEM30s.dat and the model geoidal height grid mdlgeoidh30s.dat, which is employed for modeling on ground gravity field.

Here, it is required that the grid range of the calculation surface is larger than the range of the target area to absorb edge effects.

(1) Remove reference model value from all the observations and then construct the heterogeneous observation residual file.

Call the function [Calculation of gravity field elements from global geopotential

model], let the minimum degree 2 and maximum degree 540, and input the file EGM2008.gfc, observed gravity disturbance file obsdistgrav.txt and observed GNSS-levelling geoidal height file obsGNSSIgeoid.txt, calculate and remove the 2~540th degree model value of these observations to generate the heterogeneous observation file obsresiduals0.txt according to the agreed format.



The agreed format of the heterogeneous observation file record: ID (point no / name), longitude (degree decimal), latitude, ellipsoidal height (m), observation, ..., observation type (0 ~ 5), weight, ... The order of the first five attributes is fixed by convention.

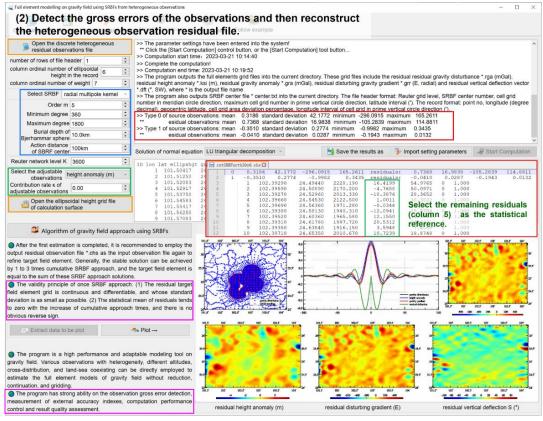
The observation types and units: 0 - residual gravity disturbance (mGal), 1 - residual geoidal height (m).

It should be noted that the ellipsoidal height of GNSS-leveling must be the geoidal height and not the ellipsoidal height at GNSS-levelling site.

(2) Detect the gross errors of the observations and then reconstruct the heterogeneous observation residual file.

Call the program [Full element modelling on gravity field using SRBFs from heterogeneous observations], select the height anomaly as the adjustable observation, let the contribution rate $\kappa = 0$, and input the heterogeneous residual file obsresiduals0.txt and model geoidal height grid file mdlgeoidh30s.dat to estimate the residual gravity field grid rntSRBFgeoidh30s0.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s0.chs.

Where, xx=ksi stands for residual geoidal height (m), xxx=rga stands for residual gravity disturbance (mGal), xxx=gra stands for residual gravity anomaly (mGal), xxx=grr stands for residual disturbing gravity gradient (radial, E) and xx=dft stands for residual vertical deflection (SW, ").



Separate the remaining residual records of the observed GNSS-leveling and observed gravity disturbance from the remaining residual file rntSRBFgeoidh30s0.chs, detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points, and then reconstruct the new heterogeneous observation residual file obsresiduals01.txt.

(3) Measure the regional height datum difference and GNSS-leveling external accuracy index.

Replace the input file obsresiduals0.txt with the new heterogeneous observation residual file obsresiduals01.txt and repeat the step (2) to re-estimate the residual gravity field grid rntSRBFdatum30s.xxx on geoid and get the new remaining residual file rntSRBFdatum30s.chs.

Since the contribution rate of GNSS-levelling $\kappa = 0$ is set in advance, it is essentially here directly to measure the external accuracy index of the observed GNSS levelling only using the observed gravity disturbances. Before and after gross error removed, the statistical results on the observation residuals are as follows.

external accura	- Juna of All								
Open the discrete heterog residual observations file	geneous >>	The parameter settir	ngs have been entered in	to the system!	2768m 👉 The	external accuracy	(index (SD)		
umber of rows of file header 1	• >>	Click the [Start Computation] control button, or the [Start Computation] control button of the 2~540 th degree model geoid							
olumn ordinal number of ellipsoid	lal e • >>	>> Computation end time: 2023-03-21 10:32:36 0 0243 m fr The external accuracy index							
height in the record blumn ordinal number of weight	ra >>>	📃 >> The program outputs the full elements grid files into the current directory. These grid files include the residual residual gravity disturbance *.rga (mGal),							
Select SRBF radial multip	*.0	*.dft (*, SW), where * is the output file name							
Order m 3		>> The program also outputs SRBF center file * center.txt into the current directory. The file header format: Reuter grid level, SRBF center number, cell grid							
Minimum degree 240	de	decimal), geocentric latitude, cell grid area deviation percentage, longitude interval of cell grid in prime vertical circle direction (').							
Maximum degree 1800		>> Type 0 of source observations: mean 0.2695 standard deviation 42.0737 minimum -296.0915 maximum 165.2611 esidual observations: mean <u>-0.5621</u> standard deviation <u>13.7996</u> minimum -80.4161 maximum 64.8276							
Burial depth of 10.0km	÷ *	> Type 1 of source observations: mean -0.3482 standard deviation 0.2768 minimum -0.9982 maximum 0.3435							
Action distance									
of SBRF center 100km	50	Solution of normal equation $~$ LU triangular decomposition $~\sim~$			Bave the results as	Import setting paramete	rs 🚽 Start Computati		
euter network level K 3600	ID ID) lon lat ellipshg 1 101.50417	gravity disturbance (mGal) height anoma		aly(m) gravity gradient(E) vertical deflection(S,W)				
elect the adjustable observations height anor	maly (m) 🗸	2 101.51250	24.00417 -35.528 24.00417 -35.519	-36.6655 -0.469	2 -36.5212 -31.91	.93 10.0940 3.9482			
ontribution rate κ of djustable observations		3 101.52083 4 101.52917	24.00417 -35.510 24.00417 -35.501		-52 4086 -60 25	14 11 5110 3 8425			
Open the ellipsoidal heigh	at arid file	5 101.53750 6 101.54583	24.00417 -35.491 24.00417 -35.481		-62 1010 -75 16	-0.0070) = -0.34			
of calculation surface	it grid nic	7 101.55417	24.00417 -35.471	-67.1305 -0.682	s 📑 👘 the meas	sured height datur	n difference		
Only using the obs	served grav	ity disturba		-79.1701 -0.762		10 13.7563 2.1200			
Algorithm of gravity			Spatial distribution of	of observations sp	oherical radial basis functio	on spatial curve residual	gravity disturbance (mGa		
d element grid is continuous a viation is as small as possible. (2 zero with the increase of cumu vious reverse sign.	?) The statistical mea	an of residuals tends	315	21.5 40 ORTinety		inghi standar inghi standar mening distantar mening dista	NC 112.5 113° 103.5 10		
Extract data to be plot The program is a high perform wity field. Various observations uss-distribution, and land-sea or imate the full element model timutation, and gridding.	mance and adaptab with heterogeneity oexisting can be d ils of gravity field	y, different altitudes, directly employed to without reduction,							
The program is a high perform wity field. Various observations se-distribution, and land-sea or imate the full element model	mance and adaptab with heterogeneity oexisting can be d is of gravity field on the observation g cy indexes, compu-	ble modeling tool on y, different altitudes, directly employed to without reduction, gross error detection,	residual height a	incomply (m)	residual disturbing grad	int (E) residual	vertical deflection S (*)		
The program is a high perform wity field. Various observations iss-distribution, and land-sea co- imate the full element model ntinuation, and gridding. The program has strong ability o asurement of external accurate	mance and adaptab with heterogeneity oexisting can be d is of gravity field on the observation g cy indexes, compu-	ble modeling tool on y, different altitudes, directly employed to without reduction, gross error detection,	number of points	nomaly (m)	residual disturbing grad standard deviation	lent (E) residua			
The program is a high perform with field. Various observations ess-distribution, and land-sea cu- imate the full element model thinuation, and gridding. The program has strong ability of assurement of external accura- trol and result quality assessme	nance and adaptab with heterogeneity existing can be d is of gravity field on the observation g cy indexes, compu- nt.	ble modeling tool on y, different altitudes, directly employed to without reduction, gross error detection,	number		standard		maximun		
The program is a high perform with field. Various observations ess-distribution, and land-sea co- imate the full element model trinuation, and gridding. The program has strong ability of assurement of external accura- tricel and result quality assessme Gravity disturbance	nance and adaptab with heterogeneity existing can be dis of gravity field on the observation g cy indexes, compu- nt. Orig resid Resid	ole modeling tool on y, different altitudes interchy employeds to without reduction, gross error detection, utation performance	number of points	mean	standard deviation	minimum	maximun 165.2611		
The program is a high perform with field. Various observations ess-distribution, and land-sea co- imate the full element model finutuation, and gridding. The program has strong ability of assurement of external accura throl and result quality assessme Gravity	nance and adaptable with heterogeneity existing can be d is of gravity field on the observation g cy indexes, compu- nt. Orrig ression Ressi- withou Rema	ole modeling tool on y, different altitudes irrectly employeds to without reduction, pross error detection, utation performance ginal duals duals	number of points 4219	mean 0.3186	standard deviation 42.1772	minimum -296.0915	vertical deflection S (1) maximum 165.2611 165.2611 64.8276		
The program is a high perform with field. Various observations ess-distribution, and land-sea co- imate the full element model trinuation, and gridding. The program has strong ability of assurement of external accura- tricel and result quality assessme Gravity disturbance	nance and adaptable with heterogeneity existing can be d is of gravity field on the observation g cy indexes, compu- nt Orrig ressic Ressi- withou Rema ressic Orrig	ble modeling tool on v, different altitudes, i without reduction, i without reduction, ginal duals duals ut error aining	number of points 4219 4215	mean 0.3186 0.2695	standard deviation 42.1772 42.0737	minimum -296.0915 -296.0915	maximun 165.2611 165.2611		
The program is a high perform with field. Various observations essidistribution, and land-see a scilatification, and land-see a trimutation, and gridding. The program has strong ability to assurement of external accura- nt and result quality assessme disturbance (mGal)	nance and adaptable with heterogeneity cexisting can be d is of gravity field on the observation g cy indexes, compu- nt. Orrig ressic Ressi Withou Rema ressic Orrig ressic Ressi	de modeling tool on , utterent attrudes, i without reduction, i without reduction, pross error detection, tutation performance ginal duals ut error aining duals ginal	number of points 4219 4215 4215	mean 0.3186 0.2695 -0.4584	standard deviation 42.1772 42.0737 13.6071	minimum -296.0915 -296.0915 -61.1040	maximun 165.2611 165.2611 64.8276		

The statistical mean (1) minus (2) of the GNSS-levelling remaining residuals in the table, that is, $-0.3482^{(1)} - (-0.0070^{(2)}) = -0.3412$ m, is the difference between the regional height datum and global height datum (gravimetric geoid). Here provides the SRBF measurement method for regional height datum difference.

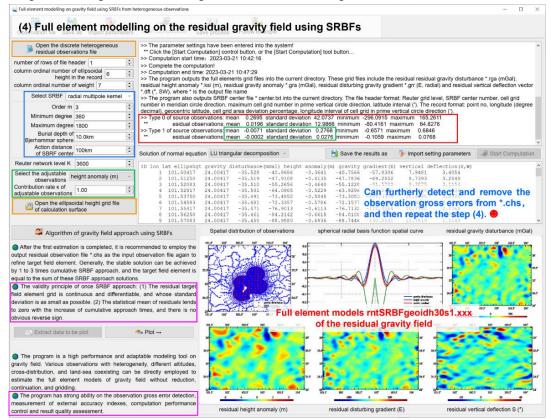
In the table, 0.0243³m is the external accuracy index of the observed GNSSlevelling expressed as standard deviation, that is, 2.43 cm. Here provides the SRBF measurement method for the external accuracy index of GNSS- leveling. The result indicates that the external accuracy of the observed GNSS-leveling is not bad than 2.43 cm (standard deviation).

In general, it is necessary to make 1 to 2 cumulative SRBF approach with *.chs as the input file to obtain the minimum of the standard deviation of the GNSS-levelling remaining residuals as the external accuracy index, and this process is omitted in this example.

After removing the regional height datum difference of -0.3411m from the GNSSlevelling residuals, the new heterogeneous observation residual file obsresiduals1.txt is reconstructed again.

(4) Full element modelling on the residual gravity field using SRBFs

Call the program [Full element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the heterogeneous residual file obsresiduals1.txt and the model geoidal height grid file mdlgeoidh30s.dat to estimate the 30" residual gravity field grid rntSRBFgeoidh30s1.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s1.chs.

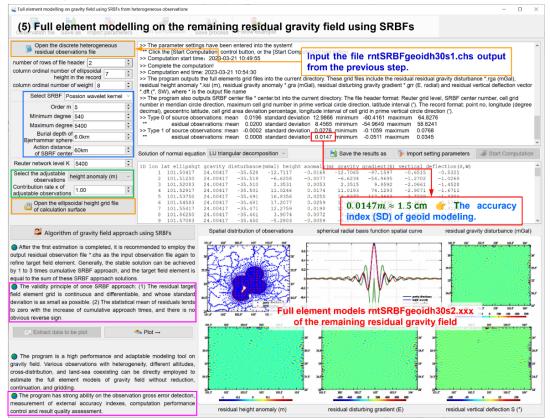


[The quality control scheme] You can furtherly detect and remove the observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file rntSRBFgeoidh30s1.chs, and

then repeat the step (4). This process is omitted in this example.

(5) Full element modelling on the remaining residual gravity field using SRBFs

Call the program [Full element modelling on gravity field using SRBFs from heterogeneous observations], let the contribution rate $\kappa = 1$, and input the remaining file rntSRBFgeoidh30s1.chs and model qeoidal height residual grid file mdlgeoidh30s.dat to estimate the 30" remaining residual gravity field grid rntSRBFgeoidh30s1.xxx on geoid, and get the remaining residual file rntSRBFgeoidh30s1.chs.



		mean	standard deviation	minimum	maximum
Residual gravity disturbance (mGal)	Residuals	0.3523	42.1561	-296.0915	165.2611
	First SRBF	0.0196	12.9866	-80.4161	64.8276
	Second SRBF	0.0200	8.4565	-54.9649	58.6241
Residual GNSS- levelling geoidal height (m)	Residuals	-0.0071	0.2768	-0.6571	0.6846
	First SRBF	-0.0002	0.0276	-0.1059	0.0768
	Second SRBF	0.0008	0.0147④	-0.0511	0.0345

In the table, $0.0147^{\textcircled{}}m = 1.5$ cm can be considered as the accuracy index of geoid modeling.

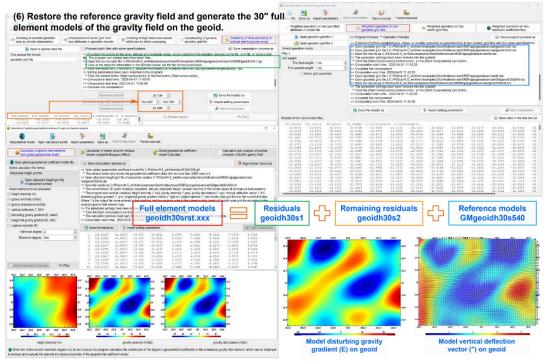
[The quality control scheme] You can furtherly detect and remove again the

observation gross error points beyond 3 times standard deviation range of the remaining residuals for the GNSS-levelling sites and beyond 5 times standard deviation range for the disturbance gravity points from the remaining residual file SRBFgeoidheight30s2.chs, and then repeat from step (4). This process is omitted in this example.

You can also do further cumulative SRBF approach to improve the results. This example omits this process.

(6) Restore the reference gravity field and generate the 30" full element models of the gravity field on the geoid.

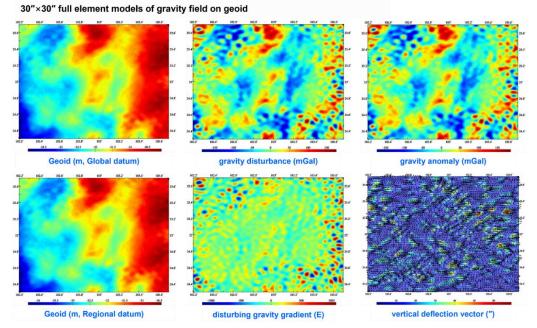
Call the function [Calculation of gravity field elements from global geopotential model], let the minimum degree 2 and maximum degree 540, input the file EGM2008.gfc, and the model geoidal height grid file mdlgeoidh30srst.dat (from mdlgeoidh30s.dat with grid edge removed), to calculate the full element grid GMgeoidh30s540.xxx of the reference gravity field on geoid.



Add the residual gravity field grid geoidh30s1.xxx (from SRBFgeoidheight30s1.xxx with grid edge removed) and remaining residual gravity field grid geoidh30s2.xxx (from SRBFgeoidheight30s2.xxx with grid edge removed) to the reference gravity field grid GMgeoidh30s540.xxx, the 30" full element gravity field models geoidh30srst.xxx on the geoid are obtained, which include the 30" gravimetric geoidal height grid (geoidh30srst.ksi, m), gravity disturbance grid (geoidh30srst.gra, mGal), gravity anomaly grid (geoidh30srst.gra, mGal), disturbing gravity gradient grid (geoidh30srst.grr, radial, E) and vertical deflection vector grid (geoidh30srst.dft, SW, ").

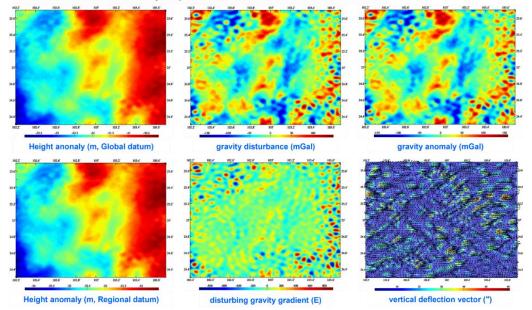
Add the regional height datum difference -0.3411m to the 30" gravimetric geoidal

height grid geoidh30srst.ksi in global height datum, the 30" gravimetric geoidal height grid geoidh30srgn.ksi in regional height datum can be obtained.



So far, the full element modeling on gravity field on the geoid have been completed. • Let the terrain surface as the calculation surface, and directly generate the

30" full element models of the gravity field on the terrain surface.



30"×30" full element models of gravity field on terrain surface

In step (3) to step (6) above, the input data file and all the parameter settings are kept the same, and only the calculation surface is changed to the terrain surface surfhgt30s.dat. Using the same computation process, you can synchronously obtain the

30" full element models surfhgt30srst.xxx of the gravity field on the terrain surface, which include the 30" gravimetric ground height anomaly grid (surfhgt30srst.ksi, m, in global height datum), ground gravity disturbance grid (surfhgt30srst.rga, mGal), ground gravity anomaly grid (surfhgt30srst.gra, mGal), ground disturbing gravity gradient grid (surfhgt30srst.grr, radial, E), ground vertical deflection vector grid (surfhgt30srst.dft, SW, ") and ground height anomaly grid (surfhgt30srgn.ksi, m, in regional height datum).